Modelling the high-energy curvature radiation for the Vela pulsar using a general particle dynamics approach.

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Introduction

Standard Pulsar Model

- The large rotating magnetic field induces an electric field
- The induced electric field rips particles from the star's surface
- The last closed magnetic field lines touch the light cylinder at $R_{\rm LC} = \frac{cP}{2\pi}$
- In the braking model the change in rotational energy is equated to the luminosity radiated by a magnetic dipole in a vacuum
- This may be written as the equation $\dot{\Omega} = -k\Omega^n$

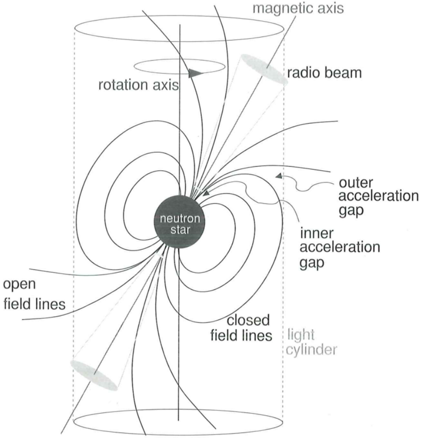
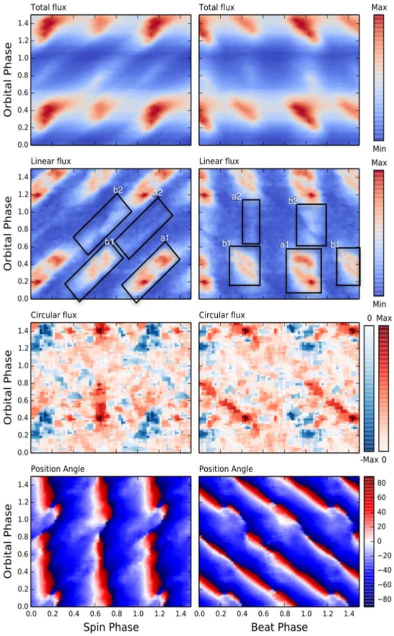


Figure: Lorimer and Kramer (2005)

Aims

Develop a new general emission model to work for a WD binary scenario:

- Solve particle dynamics generally without assuming super-relativistic particles and small pitch angles.
- Calculate the broadband light curves and spectra at different orbital phases.
- Calculate Stokes parameters, PPA, and degree of polarisation at different orbital phases.
- Calibrate our code with the millisecond pulsar emission code of Harding and collaborators.

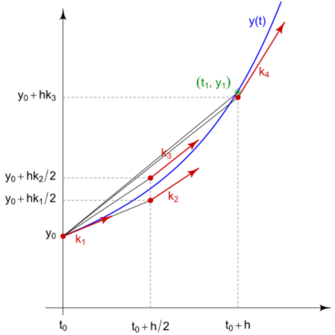


Adaptive ODE Solver

- Solve Lorentz equation:

$$\frac{d\mathbf{p}}{dt} = q\left(\mathbf{E} + \frac{c\mathbf{p}\times\mathbf{B}}{\sqrt{m^2c^4+\mathbf{p}^2c^2}}\right). \quad (1)$$

- Do n-stage evaluations to solve the ODE depending on method accuracy.
- One can calculate the next value by weighing stages, $y_{n+1} = y_n + h\sum_{i=1}^{s} b_i k_i$.
- One can use a method with embedded lower order to get a truncation error, $\tau_{n+1} = y_{n+1} - y^*_{n+1} = \sum_{i=1}^{s}(b_i - b_i^*)k_i$.
- Calculate the adaptive next step size using τ_{n+1} and a given accuracy threshold.



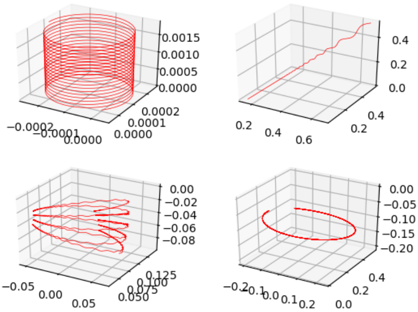
$$\begin{aligned}
k_1 &= f(t_n, y_n),\\
k_2 &= f(t_n + c_2h, y_n + h(a_{21}k_1)),\\
k_3 &= f(t_n + c_3h, y_n + h(a_{31}k_1 + a_{32}k_2)),\\
&\vdots\\
k_s &= f(t_n + c_sh, y_n + h(a_{s1}k_1 + a_{s2}k_2 + \cdots + a_{s,s-1}k_{s-1})).
\end{aligned}$$

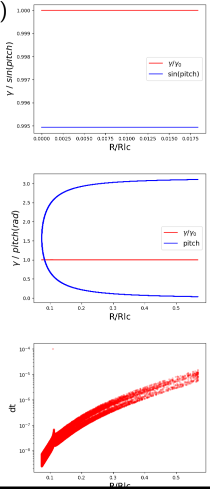
$$\begin{array}{c|cccc}
c_1 & a_{11} & a_{12} & \dots & a_{1s}\\
c_2 & a_{21} & a_{22} & \dots & a_{2s}\\
\vdots & \vdots & \vdots & \ddots & \vdots\\
c_s & a_{s1} & a_{s2} & \dots & a_{ss}\\
\hline
 & b_1 & b_2 & \dots & b_s\\
 & b_1^* & b_2^* & \dots & b_s^*
\end{array}
= \begin{array}{c|c}
\mathbf{c} & A\\
\hline
 & \mathbf{b^T}
\end{array}$$

Calibration of ODE Solver

- Set $E = 0$.
- Conserved energy and constant pitch angle for:
  - Constant B-field.
  - Changing B-field strength.
- Conserved energy for static magnetic dipole and see if the obtained magnetic mirror effect as well as drift are correct.

$$\mathbf{v}_{\nabla B} = \frac{m v_\perp}{2eB} \frac{\mathbf{B} \times \nabla \mathbf{B}}{B^2}. \quad (2)$$





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Higher-Order ODE Schemes

- The ODE schemes implemented in the code:
  - Runge-Kutta Fehlberg 4(5): 5 stage.
  - DVERK 6(5): 8 stage.
  - Prince-Dormand 8(7): 12 stage.
  - Adaptive Curtis 10(8): 18 stage.
  - Adaptive Hiroshi 12(9): 29 stage.
- Benchmark of schemes:

Tol $1e^{-12}$	RKF	DV	PD	CR	HR
Time vs RKF	1	0.82	0.36	0.32	0.65
$\delta\gamma$	0.4	0.25	0.001	0.014	0

Tol $1e^{-14}$	RKF	DV	PD	CR	HR
Time vs RKF	1	0.82	0.29	0.23	0.48
$\delta\gamma$	0.01	0.008	0	0	0

Radiation-Reaction Force

- Use equation from Landau and Lifshitz for general radiation-reaction force:

$$\begin{aligned}\mathbf{f} =& \frac{2e^3\gamma}{3mc^3}\left\{\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\mathbf{E}+\frac{1}{c}\mathbf{v}\times\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\mathbf{H}\right\} \\ &+\frac{2e^4}{3m^2c^4}\left\{\mathbf{E}\times\mathbf{H}+\frac{1}{c}\mathbf{H}\times(\mathbf{H}\times\mathbf{v})+\frac{1}{c}\mathbf{E}\left(\mathbf{v}\cdot\mathbf{E}\right)\right\} \\ &-\frac{2e^4\gamma^2}{3m^2c^5}\mathbf{v}\left\{\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{H}\right)^2-\frac{1}{c^2}\left(\mathbf{E}\cdot\mathbf{v}\right)^2\right\}.\end{aligned} \tag{3}$$

- The first term of Equation 3 requires 9, 18 or 36 evaluations of the B-field per stage to find the derivatives.
- This first term is $\sim 10^8 - 10^{10}$ times smaller than the largest component.
- The super-relativistic form of Equation 3 is given by:

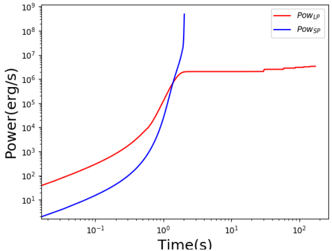
$$f_x = -\frac{2e^4\gamma^2}{3m^2c^4}\left\{(E_y - H_z)^2 + \left(E_z + H_y^2\right)\right\} \tag{4}$$

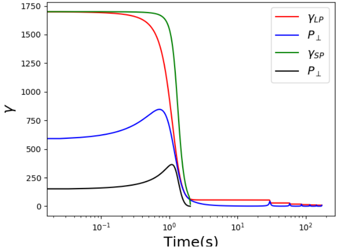
- Equation 3 and 4 converge at a Lorentz factor around $10^4 - 10^5$.

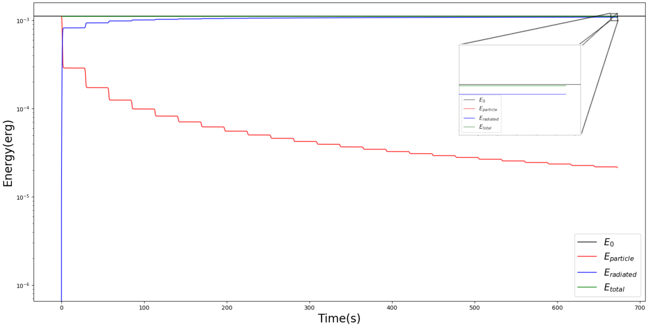
$$\begin{aligned}P_{rad} =& \mathbf{F}_{rad}\cdot\mathbf{v}, \\ E_{rad} =& \int \mathbf{F}_{rad}\cdot\mathbf{v}.dt\end{aligned} \tag{5}$$

Radiation-Reaction Force

Results







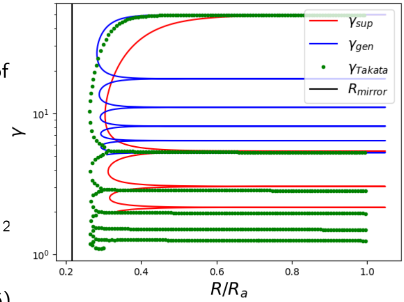
Comparison with Takata et al. (2019)

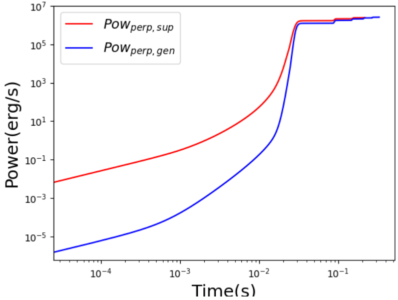
- Takata uses rewritten forms of equations from Harding et al. (2005).

$$\frac{d\gamma}{dt} = -\frac{P_\perp^2}{t_s}$$

$$\frac{d}{dt}\left(\frac{P_\perp^2}{B}\right) = -2\frac{B}{t_s\gamma}\left(\frac{P_\perp^2}{B}\right)^2 \quad (6)$$

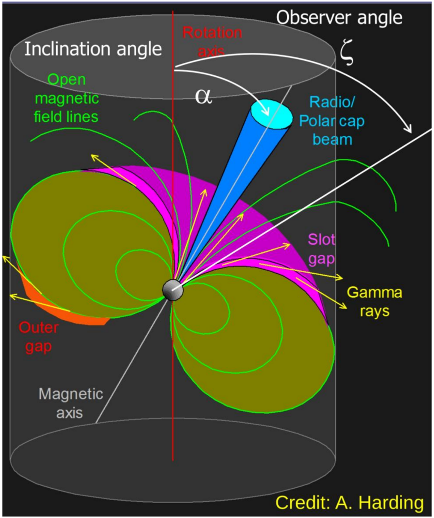
- Where $t_s = 3m_e^3c^5/2e^4B^2$.
- They predict the mirror at $r_m \sim a\sin^{2/3}\theta_p$
- The super-relativistic case agrees with Takata's γ_{loss} but not the mirror point.
- The general case disagrees largely with Takata's results.





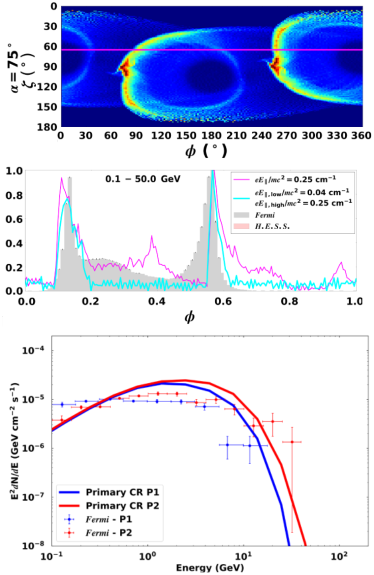
Harding and Collaborators' code

- Tracing out the particle trajectory incorporating $\mathbf{E} \times \mathbf{B}$ drift from Kalapotharakos et al. (2014).
- $\mathbf{v}/c = \mathbf{E} \times \mathbf{B}/(B^2 + E_0^2) + f\mathbf{B}/B$.
- Solving transport equations from Harding et al (2005) to calculate emission.
- $d\gamma/dt = eE_{\|}/mc - 2e^4B^2p_{\perp}^2/3m^3c^5$,
- $dp_{\perp}/dt = -3cp_{\perp}/2r - 2e^4B^2p_{\perp}^3/3m^3c^5\gamma$.
- Gyrocentre approach with average particle pitch angle.



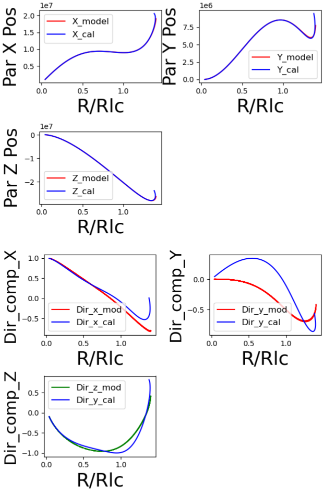
Emission Map Calculations

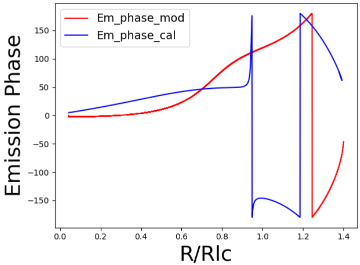
- We are only looking at curvature radiation for the calibration.
- We temporarily use the analytical formula for the static dipole curvature radius.
- The perpendicular E-field for a co-rotating plasma in the retarded-dipole scenario is given by:
- $\mathbf{E}_{\perp} = (-\mathbf{\Omega} \times \mathbf{r})/\mathbf{c} \times \mathbf{B}$.
- The phase corrections are given by:
- $\phi_{obs} = \phi_{em} - \mathbf{r_{em}} \times \eta_{\mathbf{em}}/\mathbf{R_{LC}} - \mathbf{\Delta}\phi_{\mathbf{rot}}$
- Drift velocity is calculated using:
- $\mathbf{v}/c = \mathbf{E} \times \mathbf{B}/B^2$.
- Figures from Barnard et al.(2021).

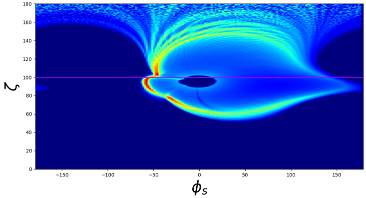


Emission Map Calibrations

Retarded dipole

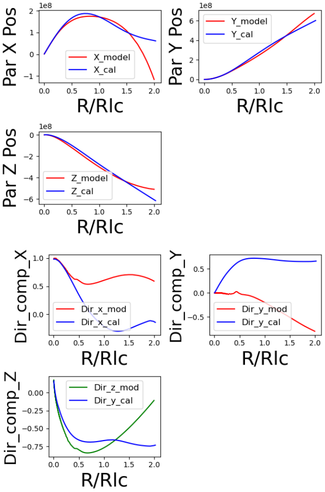


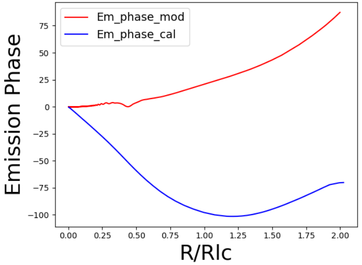


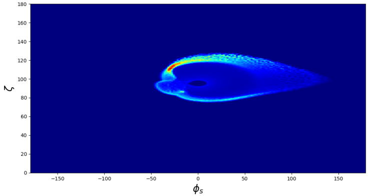


Emission Map Calibrations

Force-Free Fields



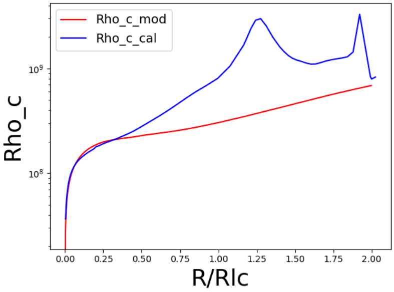


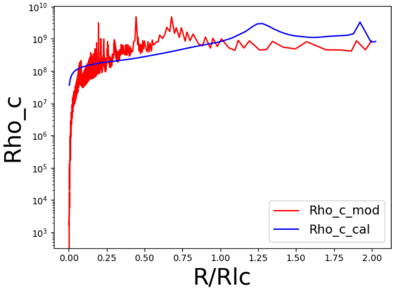


Emission Map Calibrations

Curvature Radius Calculations

- The particle curvature radius can be calculated using:
- $\rho_c = 1/\sqrt{(x'')^2 + (y'')^2 + (z'')^2}$.
- Using numerical differentiation we calculate x_i'' using:
- $x_i'' = (-3x_{i-1}' + 4x_i' - x_{i+1}')/2ds$.
- What difference does including the gyroradius have on the curvature radius?
- Use the effective curvature radius calculation employed in synchro-curvature radiation models.





Future Work

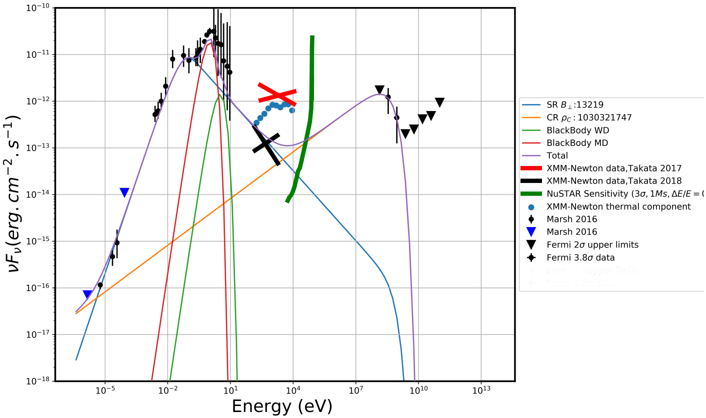
- Write up results for ApJ/MNRAS article.
- Calibrate with Harding and collaborators' emission maps and particle trajectories for pulsar scenario.
- Use appropriate E-field (force-free fields) to get $\mathbf{E} \times \mathbf{B}$ drift. Study effect of new WD scenario on model outputs.
- Implement polarisation calculations to produce phase plots.
- Determine how to scale particles' emission to have significant statistics. Invoke magic trickery to get code running in a reasonable time.
- Run code for orbital time scale, investigate different B-fields and E-fields, and investigate different particle pitch-angle distributions.

Bibliography

Geng, J.-J., Zhang, B., and Huang, Y.-F. (2016). A Model of White Dwarf Pulsar AR Scorpii. *Astrophys. J. Lett.*, 831:L10.

Extra

Plots



10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
$\nu F_\nu (erg.cm^{-2}.s^{-1})$
10^{-5}
10^{-2}
10^{1}
10^{4}
10^{7}
10^{10}
10^{13}
Energy (eV)
SR $B_\perp$:13219
CR ρ_C: 1030321747
BlackBody WD
BlackBody MD
Total
XMM-Newton data,Takata 2017
XMM-Newton data,Takata 2018
NuSTAR Sensitivity ($3\sigma, 1Ms, \Delta E/E = ($
XMM-Newton thermal component
Marsh 2016
Marsh 2016
Fermi 2σ upper limits
Fermi 3.8σ data