Subleading high-energy logarithms and NLO accuracy for $W$+jets

Emmet Byrne
emmet.byrne@ed.ac.uk

23rd MCnet Meeting at the University of Manchester
6th December 2021
Outline: Subleading high-energy logarithms and NLO accuracy for W+jets

Introduction
High-energy logarithms
Factorisation of QCD at high energy
The High Energy Jets (HEJ) framework

W+jets [1]
Obtaining new factorised components
NLO matching
Comparison to experimental data


Tuesday at 11.50: HEJ+PYTHIA, Hitham Hassan

Wednesday at 9.50: HEJ Status Report, Jeppe Andersen
High-energy logarithms

At each order in perturbative QCD, large logarithms arise when the centre of mass energy is much greater than the transverse momenta of the produced partons.

For \( 2 \to 2 \) scattering we can write the cross section as

\[
\frac{\sigma^{(0)}/\sigma^{(0)}}{\sigma^{(1)}/\sigma^{(0)}} = 1 \quad \text{LL} \quad \alpha_s L c_0^{(1)} + \frac{\sigma^{(2)}/\sigma^{(0)}}{\sigma^{(3)}/\sigma^{(0)}} = \alpha_s^2 L^2 c_0^{(2)} + \frac{\sigma^{(3)}/\sigma^{(0)}}{\sigma^{(4)}/\sigma^{(0)}} = \alpha_s^3 L^3 c_0^{(3)} + \ldots \quad \text{NLL} \quad \alpha_s c_1^{(1)} + \alpha_s^2 c_1^{(2)} L + \alpha_s^3 L c_2^{(3)} \quad \text{NNLL} \quad + \ldots
\]

Let's begin with a description of QCD amplitudes which include the leading logarithmic terms at all orders in \( \alpha_s \).
**QCD amplitudes at LL accuracy**

At LL accuracy, $2 \to n$ amplitudes in QCD take a simple factorised form:[2]

\[
\mathcal{M}^{\text{LL}}_{gg \to (n-2)g} = 2s_{12} \left[ ig_s f^{a_1 c_1 a_2} C^{g(0)} (p_1^{\nu_a}, p_2^{\nu_a}) \right] \\
\times \prod_{i=2}^{n-1} \left[ g_s f^{c_{i-1} c_i a_1} V^{g(0)} (q_{i-1}, p_i^{\nu_i}, q_i) \right] \\
\times \prod_{i=1}^{n-1} \left( -\frac{1}{|q_i|} \right)^2 e^{\alpha(1)_{q_i}} \log \left( \frac{s_{i,i+1}}{\tau} \right) \\
\times \left[ ig_s f^{a b c n-1 a_n} C^{g(0)} (p_b^{\nu_b}, p_n^{\nu_n}) \right]
\]

The scattering of quarks is described by the replacement of $f^{a_i c a_j} \to T_{a_i a_j}^{\text{c}}$.

This approximation becomes exact in the Multi-Regge Kinematic (MRK) limit $y_1 \gg y_2 \gg \cdots \gg y_n$

Return to LO $qQ \rightarrow qQ$

At LO the process is already factorised:

\[ \mathcal{M}_{qQ \rightarrow qQ}^{(0)} = [ig_s T^c \bar{u}^a(p_a) \gamma^\mu u^\nu_1(p_1)] \left( \frac{-i}{t} \right) [ig_s T^c \bar{u}^b(p_b) \gamma^\mu u^\nu_2(p_2)] \]

This motivates us to take the quark current itself as our impact factor\[\text{[3]}:\]

\[ j_\alpha_{q \rightarrow q}(p_a, p_1) = \bar{u}^a(p_a) \gamma^\alpha u^\nu_1(p_1) \]

rather than making the approximation \[ j_\alpha_{q \rightarrow q}(p_a, p_1) \xrightarrow{\text{MRK}} 2p^\alpha \delta^{\nu_a \nu_1} \] that we saw earlier.

The dominant helicity configurations of $qq \rightarrow qq$ can also be described exactly by spinor currents, leading to an improved gluon impact factor\[\text{[4]}:\]

Real emissions

The process $qQ \rightarrow qgQ$ factorises in the MRK limit. However, we make minimal approximations to the LO amplitude in order to retain as much of the LO process as possible.

\[
M_{qQ\rightarrow qgQ}^{(0)} = \left[ ig_s T_{c1}^{c1} j_{q\rightarrow q}^{\mu} \right] \left( \frac{-i}{t_1} \right) \left[ f_{c1} c_2 a_2 V_{g}^{\gamma} c_1^* \right] \left( \frac{-i}{t_2} \right) \left[ ig_s T_{c2}^{c2} j_{Q\rightarrow Q} \mu \right]
\]

This lets us define the HEJ Lipatov vertex\[^1\]:

\[
V_g^\rho (q_i, q_{i+1}) = -(q_i + q_{i+1})^\rho + \frac{p_a^\rho}{2} \left( \frac{q_i^2}{s_{i+1,a}} + \frac{s_{i+1,b}}{s_{a,b}} + \frac{s_{i+1,n}}{s_{a,n}} \right) + p_a \leftrightarrow p_1
\]

\[
- \frac{p_b^\rho}{2} \left( \frac{q_{i+1}^2}{s_{i+1,b}} + \frac{s_{i+1,a}}{s_{b,a}} + \frac{s_{i+1,1}}{s_{b,1}} \right) - p_b \leftrightarrow p_n.
\]

which agrees with the FKL form upon further approximation

\[^1\] arXiv:0908.2786 Andersen, Smillie
LL HEJ amplitude for QCD

With these building blocks we can write the LL HEJ QCD amplitudes:

\[
\left| \mathcal{M}_{f_a f_b \rightarrow f_a \cdot (n-2) g \cdot f_b}^{\text{LL HEJ}} \right|^2 = \frac{1}{4(N_C^2 - 1)} \left| j_{f_a \rightarrow f_a} \cdot j_{f_b \rightarrow f_b} \right|^2 \left( g_s^2 K_{f_a} (p_a, p_1) \frac{1}{t_1} \right) \\
\times \left( \prod_{i=1}^{n-1} e^{\alpha(t_i)(y_{j+1} - y_j)} \right) \\
\times \left( \prod_{i=1}^{n-2} \frac{-g_s^2 C_A}{t_i t_{i+1}} \left| V_g(q_i, q_{i+1}) \right|^2 \right) \\
\times \left( g_s^2 K_{f_b} (p_b, p_n) \frac{1}{t_{n-1}} \right),
\]

Approximations are only performed at amplitude level. Phase space integration is performed via Monte Carlo sampling.

As of HEJ 2.0, LO matched by default: LO events can be used as input where they are available. HEJ Fixed Order Generator can be used where these are unavailable or computationally expensive.

See hepforge.org for more details.
Systematic improvements to the HEJ framework

In regions correlated with the MRK, LL+LO HEJ is seen to give a good description of data\footnote{[6] arXiv:1703.04362 ATLAS Collaboration}:

Observables such as transverse momentum distributions are typically not well described as this violates the MRK conditions:

The HEJ framework can by systematically improved in two directions:

1) Improved logarithmic accuracy
2) Higher fixed-order matching

We have made progress in both of these directions in a recent study of W+jets\footnote{[1] arXiv:2012.10310 Jeppe Andersen, James Black, Helen Brooks, EB, Andreas Maier and Jennifer Smillie}, leading to a significant improvement in the description of transverse observables.
Regge Scaling

We can use ideas from Regge theory to anticipate the importance of a given rapidity ordering of an event.

This gives us a useful classification of events. This classification is not altered by production of colour singlets.

Regge scaling in MRK\cite{1}:

\[ M_{\text{MRK}} \propto s_{12}^{J_1(t_1)} s_{23}^{J_2(t_2)} \ldots s_{n-1,n}^{J_{n-1}(t_{n-1})} \gamma \]

\[ M^{(0)} \propto (s_{12})^1 (s_{23})^1 \]

\[ (s_{12})^{\frac{1}{2}} (s_{23})^1 \]

\[ (s_{12})^{\frac{1}{2}} (s_{23})^{\frac{1}{2}} \]

LL config.  NLL config.  NNLL config.

Impact of NLL configurations at LO

A LO analysis demonstrates the importance of also including LL resummation for NLL configurations:
New factorised components for W+3j

In order to apply LL resummation to all NLL configurations for W+2j, we need several new factorised expressions. For 3j events we can extract the required pieces from LO amplitudes:

\[ j_{q \rightarrow W g q'}^\mu(p_q; p_\ell, p_{\bar{\ell}}, p_g, p_{q'}) \]

\[ j_{g \rightarrow W q q'}^\mu(p_g; p_\ell, p_{\bar{\ell}}, p_q, p_{q'}) \]

The approximations required to factorise a quark current from these LO amplitudes are so mild that the expressions derived still possess crossing symmetry:

\[ j_{q \rightarrow W g q'}^\mu(p_q; p_\ell, p_{\bar{\ell}}, p_g, p_{q'}) = j_{g \rightarrow W q q'}^\mu(-p_g; p_\ell, p_{\bar{\ell}}, -p_q, p_{q'}) \]
Obtaining the factorised expressions

8/12 Feynman diagrams are already factorised:

We may then write the amplitude as

\[ \mathcal{M}_{qQ \rightarrow Wgq'Q} \xrightarrow{y_g \sim y_{q'} \ll y_Q} j_q^\mu ( \frac{1}{t} ) j_{Q \rightarrow Q } \mu \]

4/12 Feynman diagrams are not already factorised:

We make gauge invariant approximations to these four diagrams, valid in the QMRK.

Note that the approximation only involves the opposite spinor string: this is the reason why crossing symmetry is retained.
New factorised components for W+3j

These expressions obey the expected scaling, and provide a good approximation to the LO amplitude:
New factorised components for W+4j

Finally, we need effective vertices for the emission of a central $Q\bar{Q}$ pair in order to describe all NLL configurations. These are required for $\geq 4j$ events.

$$V_{Q\bar{Q}}^{\mu\nu} \quad V_{WQQ'}^{\mu\nu}$$

Numerical impact of LL resummation to NLL configurations

‘Before’:
Resummation applied to LL configs
All other configs included at LO.

‘After’:
Resummation applied to LL and NLL configs
All other configs included at LO.
NLO matching procedure

HEJ provides an all-orders description which may be truncated to any desired order in $\alpha_s$

In particular we can obtain HEJ@NLO by restricting to a single real emission, and truncating the all-orders virtual corrections:

$$\frac{1}{t} e^{\tilde{\alpha}(q_\perp) \Delta y} \rightarrow \frac{1}{t} \left( 1 + \tilde{\alpha}(q_\perp) \Delta y \right) \left( 1 + \frac{1}{3} \right)$$

We can match HEJ bin-by-bin to $W + 2j$ NLO results to obtain NLO accuracy while maintaining the logarithmic accuracy of HEJ.

We calculate the bin-by-bin reweighting factors:

$$R = \frac{w_{NLO}}{w_{HEJ@NLO}}$$

The final NLO-matched bin-weight is then:

$$w_{HEJ2NLO} = R(w_{HEJ}) + w_{FO} W + \geq 4j$$
Comparison to W+2j data

LL HEJ (HEJ1) does not give a good description of observables uncorrelated with the high energy limit, as we expect: This breaks the MRK condition upon which they were based.

HEJ with the inclusion of the new NLL configurations, and bin-by-bin matching to NLO (HEJ2 NLO), gives a good description of data.

Summary

• For W+jets we have included a class of NLL corrections, which required new factorised expressions.

• We have also performed the first matching of HEJ to NLO.

• These improvements are seen to give a good description of LHC data, and a significant improvement to observables such as $p_\perp$ distributions.

• Work is ongoing to extend the HEJ framework to full NLL accuracy.

Thanks for your attention!

Tuesday at 11.50: HEJ+PYTHIA, Hitham Hassan

Wednesday at 9.50: HEJ Status Report, Jeppe Andersen
Backup Slides
Return to LO $qg \rightarrow qg$

For $qg \rightarrow qg$ the dominant helicity configurations are also exactly expressible as a contraction of quark currents over a t-channel pole, e.g.\cite{4}

\[
|\mathcal{M}^{(0)}(q^\ominus g^\oplus \rightarrow q^\ominus g^\oplus)|^2 = \frac{K_g(p_2, p_b)}{C_F} \left|\mathcal{M}^{(0)}(qQ \rightarrow qQ)\right|^2
\]

\[
K_g(p_b, p_2) \equiv \frac{1}{2} \left(C_A - \frac{1}{C_A}\right) \left(\frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-}\right) + \frac{1}{C_A}.
\]

Here we see that in the MRK approximation $p_b^- \approx p_2^-$ we obtain

\[
K_g(p_b^-, p_2^-) \xrightarrow{\text{MRK}} C_A
\]

in agreement with the approximation of switching the representation of the generator in the impact factor.

\cite{4} arXiv:0910.5113 Andersen, Smillie
Comparison at LO

The HEJ amplitudes capture more of the LO physics relevant to LHC phase space than the strictly LL amplitudes:

\[ qQ \rightarrow qgQ \]

\[ M^2 / (2^8 \pi^5 s^2) \times 10^{-14} \text{[GeV]}^6 \]

\[ qQ \rightarrow qgQ \]

\[ M^2 / (2^12 \pi^8 s^2) \times 10^{-19} \text{[GeV]}^8 \]

Details of these slices through phase space are given in [3] arXiv:0908.2786 Andersen, Smillie
Impact of NLL configurations at LO

A LO analysis demonstrates the importance of also including LL resummation for NLL configurations:

\[ pp \rightarrow (W \rightarrow l\nu) + 3j \]
LHC@7 TeV
anti-kt, \( R = 0.4, p_{T,j} > 30 \text{ GeV}, |y_j| < 4.4 \]

\[ pp \rightarrow (W \rightarrow l\nu) + 4j \]
LHC@8 TeV
anti-kt, \( R = 0.4, p_{T,j} > 30 \text{ GeV}, |y_j| < 4.4 \]

Numerical impact of LL resummation to NLL configurations

‘Before’: Resummation applied to LL configs
All other configs included at LO.

‘After’: Resummation applied to LL and NLL configs
All other configs included at LO.
NLO matching procedure

We can write the $2j$ inclusive NLO cross section as:

$$\sigma_{2j}^{\text{NLO}} = f_{2j}^{(2)} \alpha_s^2 + \left(f_{2j}^{(3)} + f_{3j}^{(3)}\right) \alpha_s^3.$$

By contrast, HEJ provides an all-orders description which may be truncated to any desired order in $\alpha_s$:

$$\sigma_{2j}^{\text{HEJ}} = h_{2j}^{(2)} \alpha_s^2 + \left(h_{2j}^{(3)} + h_{3j}^{(3)}\right) \alpha_s^3 + \left(h_{2j}^{(4)} + h_{3j}^{(4)} + h_{4j}^{(4)}\right) \alpha_s^4 + O(\alpha_s^5).$$

In particular we can truncate the HEJ predictions to NLO:

$$\sigma_{2j}^{\text{HEJ}@\text{NLO}} = h_{2j}^{(2)} \alpha_s^2 + \left(h_{2j}^{(3)} + h_{3j}^{(3)}\right) \alpha_s^3.$$

We can then construct the ratio:

$$\frac{\sigma_{2j}^{\text{NLO}}}{\sigma_{2j}^{\text{HEJ}@\text{NLO}}} = 1 + \left(f_{2j}^{(3)} - h_{2j}^{(3)}\right) \prod_{n=0}^{\infty} (-1)^n \alpha_s^{(n+1)} \frac{(f_{3j}^{(3)} + h_{2j}^{(3)})^n}{(f_{2j}^{(2)}(n+1))}.$$

which ensures the inclusive HEJ prediction agrees with the fixed-order result up to $\alpha_s^3$:

$$\sigma_{2j}^{\text{HEJ}} \left(\frac{\sigma_{2j}^{\text{NLO}}}{\sigma_{2j}^{\text{HEJ}@\text{NLO}}}\right) = f_{2j}^{(2)} \alpha_s^2 + \left(f_{2j}^{(3)} + f_{3j}^{(3)}\right) \alpha_s^3 + \left(h_{2j}^{(4)} + h_{3j}^{(4)} + f_{4j}^{(4)}\right) \alpha_s^4 + O(\alpha_s^5).$$