Subleading high-energy logarithms and NLO accuracy for W+jets

Emmet Byrne
emmet.byrne@ed.ac.uk

23rd MCnet Meeting at the University of Manchester
6th December 2021

Outline: Subleading high-energy logarithms and NLO accuracy for W+jets

Introduction
High-energy logarithms
Factorisation of QCD at high energy
The High Energy Jets (HEJ) framework

W+jets [1]
Obtaining new factorised components
NLO matching
Comparison to experimental data

[1] arXiv:2012.10310 Jeppe Andersen, James Black, Helen Brooks, EB, Andreas Maier and Jennifer Smillie

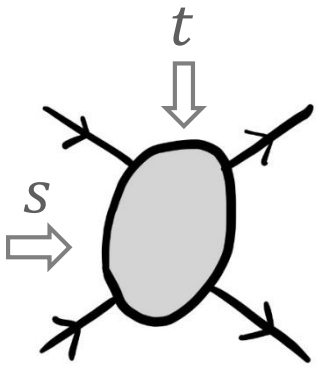
Tuesday at 11.50: HEJ+PYTHIA, Hitham Hassan

Wednesday at 9.50: HEJ Status Report, Jeppe Andersen

High-energy logarithms

At each order in perturbative QCD, large logarithms arise when the centre of mass energy is much greater than the transverse momenta of the produced partons.

For $2 \to 2$ scattering we can write the cross section as



$$L \equiv \log\left(\frac{s}{-t}\right) \gg 1$$

$$\alpha_s \ll 1$$

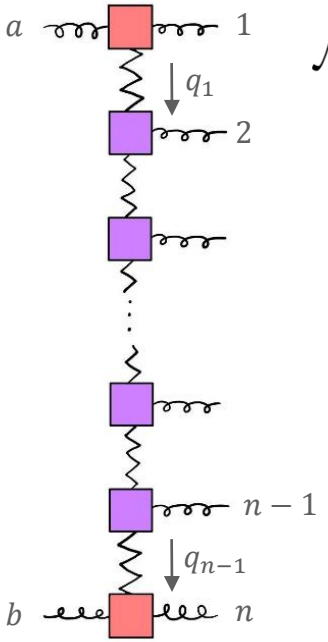
$$\alpha_s L \sim 1$$

	LL		NLL		NNLL	
$\sigma^{(0)}/\sigma^{(0)} =$	1					
$\sigma^{(1)}/\sigma^{(0)} =$	$\alpha_s L c_0^{(1)}$	$+$	$\alpha_s c_1^{(1)}$			
$\sigma^{(2)}/\sigma^{(0)} =$	$\alpha_s^2 L^2 c_0^{(2)}$	$+$	$\alpha_s^2 c_1^{(2)} L$	$+$	$\alpha_s^2 c_2^{(2)}$	
$\sigma^{(3)}/\sigma^{(0)} =$	$\alpha_s^3 L^3 c_0^{(3)}$	$+$	$\alpha_s^3 L^2 c_1^{(3)}$	$+$	$\alpha_s^3 L c_2^{(3)}$	$+ \ldots$
$\vdots$	$\vdots$		$\vdots$		$\vdots$	

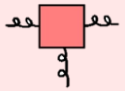
Let's begin with a description of QCD amplitudes which include the leading logarithmic terms at all orders in α_s.

QCD amplitudes at LL accuracy

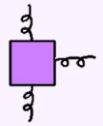
At LL accuracy, $2 \to n$ amplitudes in QCD take a simple factorised form:[2]



$$\mathcal{M}^{\mathrm{LL}}_{gg\to(n-2)g} = 2s_{12}\left[ig_s f^{a_1c_1a_a} C^{g(0)}(p_a^{\nu_a}, p_1^{\nu_a})\right]$$
$$\times \prod_{i=2}^{n-1}\left[g_s f^{c_{i-1}c_ia_1} V^{g(0)}(q_{i-1}, p_i^{\nu_i}, q_i)\right]$$
$$\times \prod_{i=1}^{n-1} \frac{-1}{|q_{i\perp}|^2} e^{\alpha^{(1)}(q_{i\perp})\log\left(\frac{s_{i,i+1}}{\tau}\right)}$$
$$\times \left[ig_s f^{a_bc_{n-1}a_n} C^{g(0)}(p_b^{\nu_b}, p_n^{\nu_n})\right]$$



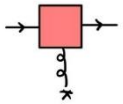
$$C^{g(0)}(p_a^{\nu_a}, p_1^{\nu_1}) = \delta^{\nu_a\nu_1}$$



$$V^{g(0)}(q_1, p^{\oplus}, q_2) = \frac{q_{1\perp}^* q_{2\perp}}{p_\perp}$$

$$\alpha^{(1)}(q_\perp) = \frac{\alpha_s}{4\pi}\kappa_\Gamma C_A \frac{2}{\epsilon}\left(\frac{\mu^2}{-|q_\perp|^2}\right)^\epsilon$$

The scattering of quarks is described by the replacement of $f^{a_ica_j} \to T^c_{a_ia_j}$

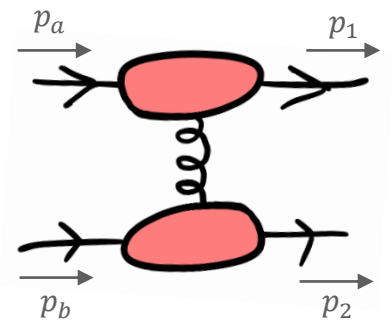


This approximation becomes exact in the Multi-Regge Kinematic (MRK) limit $y_1 \gg y_2 \gg \cdots \gg y_n$

[2] Sov. Phys. JETP, Vol.44, No. 3 (1976) Kuraev, Lipatov, Fadin

Return to LO $qQ \to qQ$

At LO the process is *already factorised:*

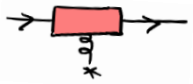


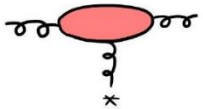
$$\mathcal{M}^{(0)}_{qQ\to qQ} = [ig_s T^c \bar{u}^{\nu_a}(p_a)\gamma^\mu u^{\nu_1}(p_1)] \left(\frac{-i}{t}\right) [ig_s T^c \bar{u}^{\nu_b}(p_b)\gamma^\mu u^{\nu_2}(p_2)]$$

This motivates us to take the quark current itself as our impact factor [3]:

$$j^\alpha_{q\to q}(p_a, p_1) = \bar{u}^{\nu_a}(p_a)\gamma^\alpha u^{\nu_1}(p_1)$$

rather than making the approximation $j^\alpha_{q\to q}(p_a, p_1) \xrightarrow[\text{MRK}]{} 2p^\alpha \delta^{\nu_a \nu_1}$ that we saw earlier



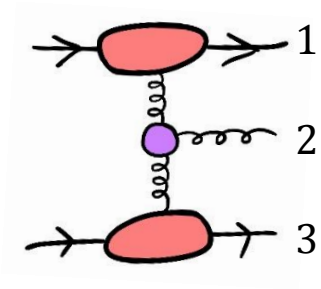


The dominant helicity configurations of $qg \to qg$ can also be described exactly by spinor currents, leading to an improved gluon impact factor [4].

[3] arXiv:0908.2786 Andersen, Smillie; [4] arXiv:0910.5113 Andersen, Smillie

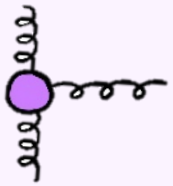
Real emissions

The process $qQ \to qgQ$ factorises in the MRK limit. However, we make minimal approximations to the LO amplitude in order to retain as much of the LO process as possible.



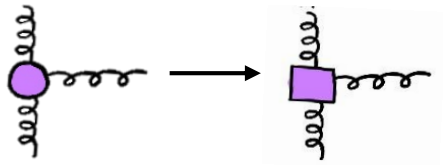
$$\mathcal{M}^{(0)}_{qQ\to qgQ} = \left[ig_s T^{c_1} j^{\mu}_{q\to q}\right]\left(\frac{-i}{t_1}\right)\left[f^{c_1 c_2 a_2} V^{\gamma}_{g} \epsilon^{*}_{\gamma}\right]\left(\frac{-i}{t_2}\right)\left[ig_s T^{c_2} j_{Q\to Q\ \mu}\right]$$

This lets us define the HEJ Lipatov vertex [3]:



$$V^{\rho}_{g}(q_i, q_{i+1}) = -(q_i + q_{i+1})^{\rho} + \frac{p^{\rho}_a}{2}\left(\frac{q_i^2}{s_{i+1,a}} + \frac{s_{i+1,b}}{s_{a,b}} + \frac{s_{i+1,n}}{s_{a,n}}\right) + p_a \leftrightarrow p_1$$
$$- \frac{p^{\rho}_b}{2}\left(\frac{q_{i+1}^2}{s_{i+1,b}} + \frac{s_{i+1,a}}{s_{ba}} + \frac{s_{i+1,1}}{s_{b1}}\right) - p_b \leftrightarrow p_n.$$

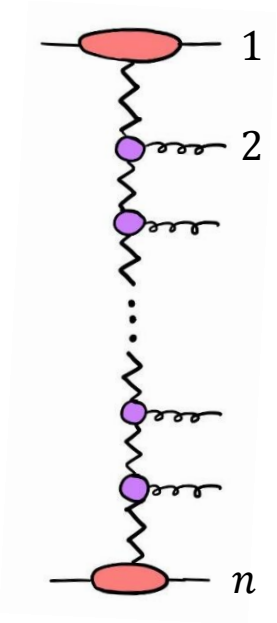
which agrees with the FKL form upon further approximation



[3] arXiv:0908.2786 Andersen, Smillie

LL HEJ amplitude for QCD

With these building blocks we can write the LL HEJ QCD amplitudes:



$$\left|\mathcal{M}^{\text{LL HEJ}}_{f_a f_b \to f_a \cdot (n-2) g \cdot f_b}\right|^2 = \frac{1}{4(N_C^2-1)} \left|j_{f_a\to f_a} \cdot j_{f_b \to f_b}\right|^2 \left(g_s^2 K_{f_a}(p_a,p_1)\frac{1}{t_1}\right)$$
$$\times \left(\prod_{i=1}^{n-1} e^{\alpha(t_i)(y_{j+1}-y_j)}\right)$$
$$\times \left(\prod_{i=1}^{n-2} \frac{-g_s^2 C_A}{t_i t_{i+1}} \left|V_g(q_i,q_{i+1})\right|^2\right)$$
$$\times \left(g_s^2 K_{f_b}(p_b,p_n)\frac{1}{t_{n-1}}\right),$$

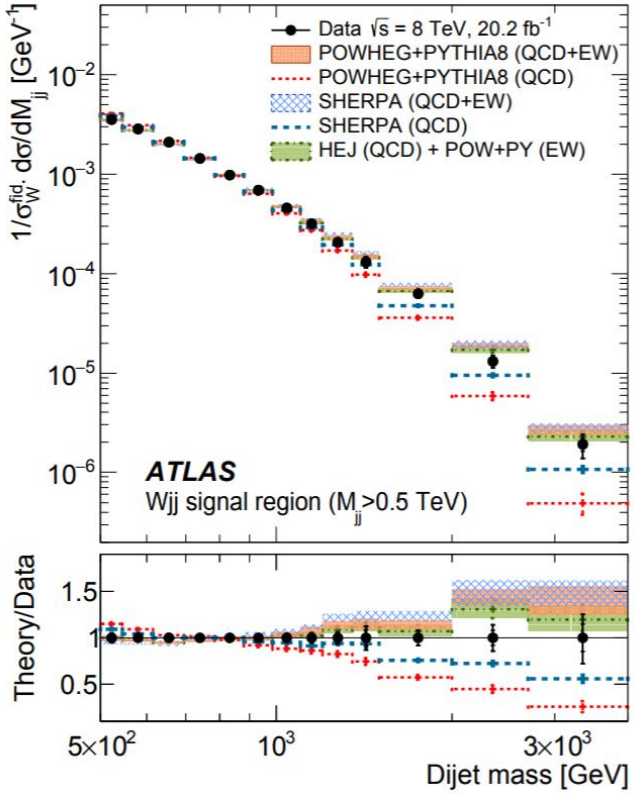
Approximations are only performed at amplitude level.
Phase space integration is performed via Monte Carlo sampling.

As of HEJ 2.0, LO matched by default: LO events can be used as input where the are available.
HEJ Fixed Order Generator can be used where these are unavailable or computationally expensive.

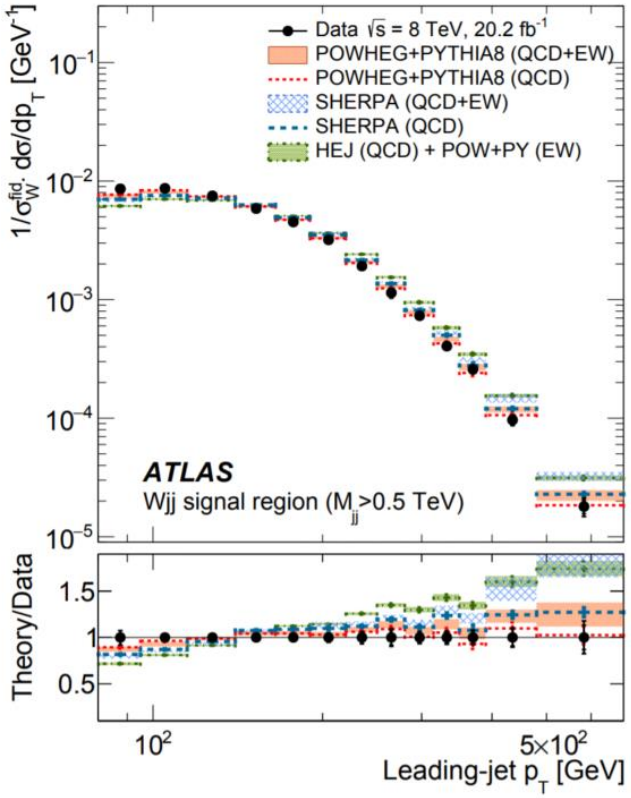
See hej.hepforge.org for more details.

Systematic improvements to the HEJ framework

In regions correlated with the MRK, LL+LO HEJ is seen to give a good description of data [6] :



Observables such as transverse momentum distributions are typically not well described as this violates the MRK conditions:



The HEJ framework can by systematically improved in two directions:

1) Improved logarithmic accuracy
2) Higher fixed-order matching

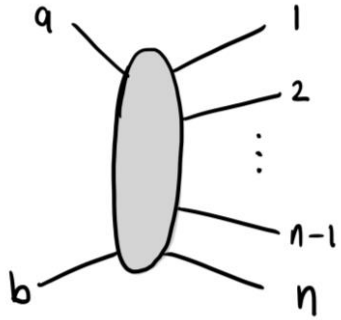
We have made progress in both of these directions in a recent study of W+jets [1], leading to a significant improvement in the description of transverse observables.

[1] arXiv:2012.10310 Jeppe Andersen, James Black, Helen Brooks, EB, Andreas Maier and Jennifer Smillie

Regge Scaling

We can use ideas from Regge theory to anticipate the importance of a given rapidity ordering of an event.

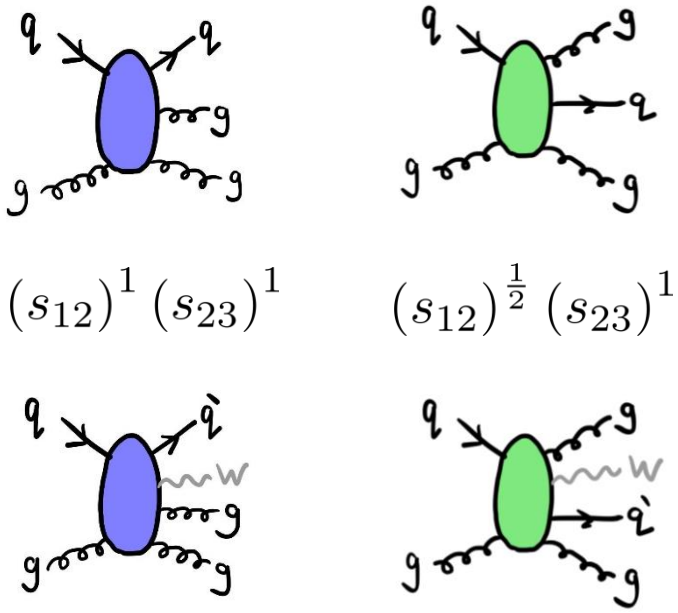
This gives us a useful classification of events. This classification is not altered by production of colour singlets.



Regge scaling in MRK[7]:

$$\mathcal{M}^{\mathrm{MRK}} \propto s_{12}^{J_1(t_1)} s_{23}^{J_2(t_2)} \ldots s_{n-1,n}^{J_{n-1}(t_{n-1})} \gamma$$

$\mathcal{M}^{(0)} \propto$

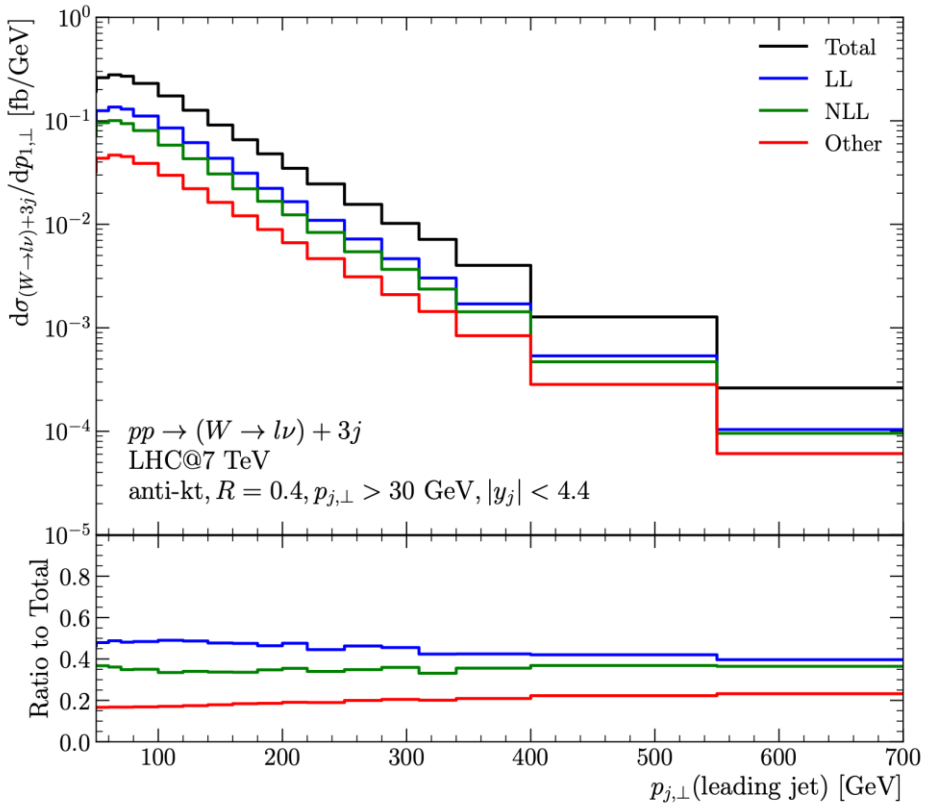


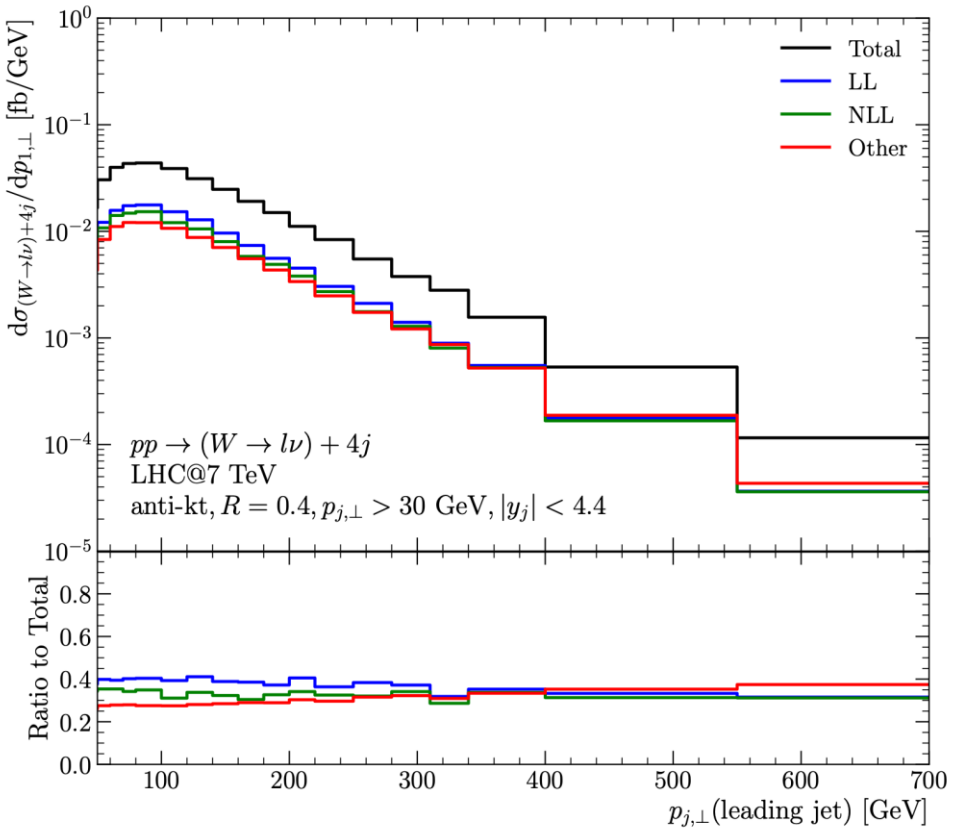
$(s_{12})^1 (s_{23})^1$	$(s_{12})^{\frac{1}{2}} (s_{23})^1$	$(s_{12})^{\frac{1}{2}} (s_{23})^{\frac{1}{2}}$
LL config.	NLL config.	NNLL config.

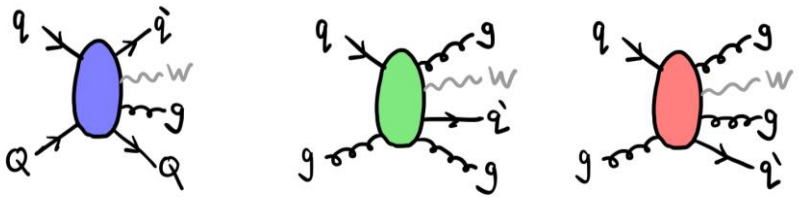
[7] Phys. Rept. 14 (1974) Brower, De Tar, Weis

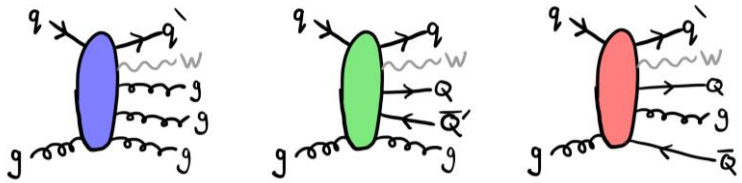
Impact of NLL configurations at LO

A LO analysis demonstrates the importance of also including LL resummation for NLL configurations:



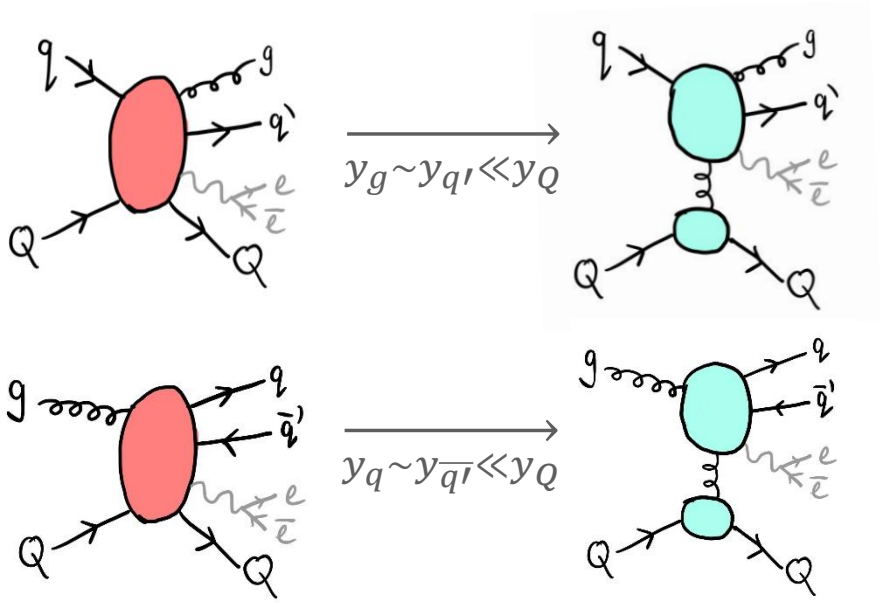


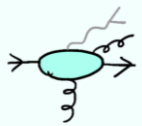




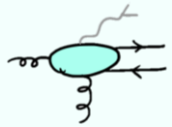
New factorised components for W+3j

In order to apply LL resummation to all NLL configurations for W+2j, we need several new factorised expressions. For 3j events we can extract the required pieces from LO amplitudes:





$$j^{\mu}_{q\to W g q'}(p_q; p_\ell, p_{\bar{\ell}}, p_g, p_{q'})$$



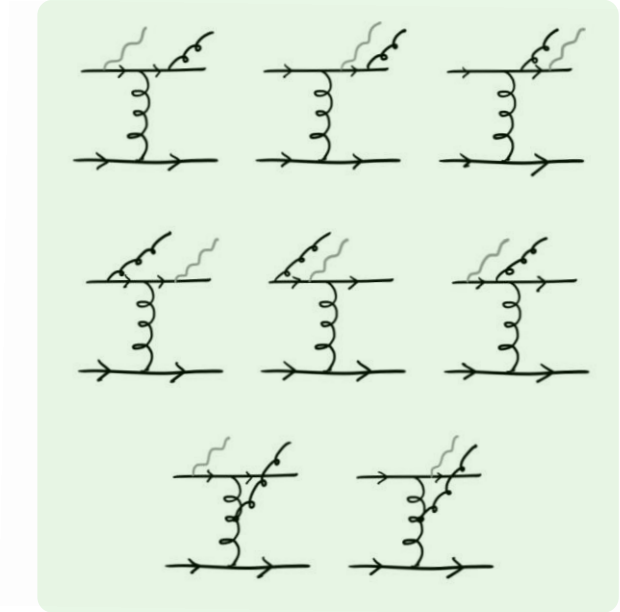
$$j^{\mu}_{g\to W q \bar{q}'}(p_g; p_\ell, p_{\bar{\ell}}, p_q, p_{\bar{q}'})$$

The approximations required to factorise a quark current from these LO amplitudes are so mild that the expressions derived still possess crossing symmetry:

$$j^{\mu}_{q\to W g q'}(p_q; p_\ell, p_{\bar{\ell}}, p_g, p_{q'}) = j^{\mu}_{g\to W q \bar{q}'}(-p_g; p_\ell, p_{\bar{\ell}}, -p_q, p_{\bar{q}'})$$

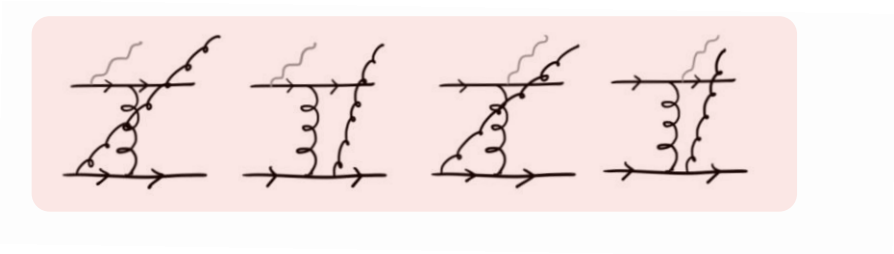
Obtaining the factorised expressions

8/12 Feynman diagrams are already factorised:



We make no kinematic approximations to these terms.

4/12 Feynman diagrams are not already factorised:

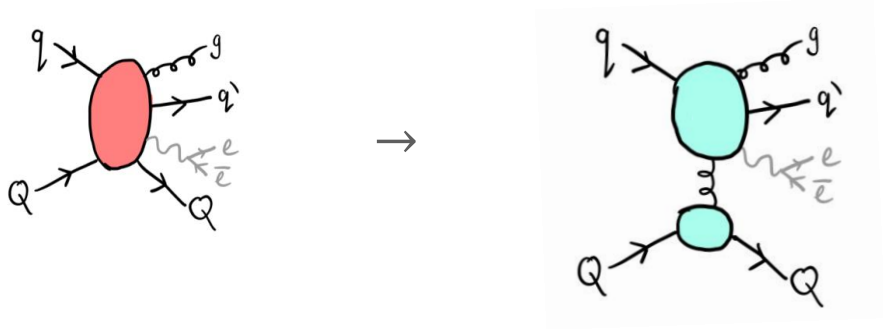


We make gauge invariant approximations to these four diagrams, valid in the QMRK

Note that the approximation only involves the opposite spinor string: this is the reason why crossing symmetry is retained.

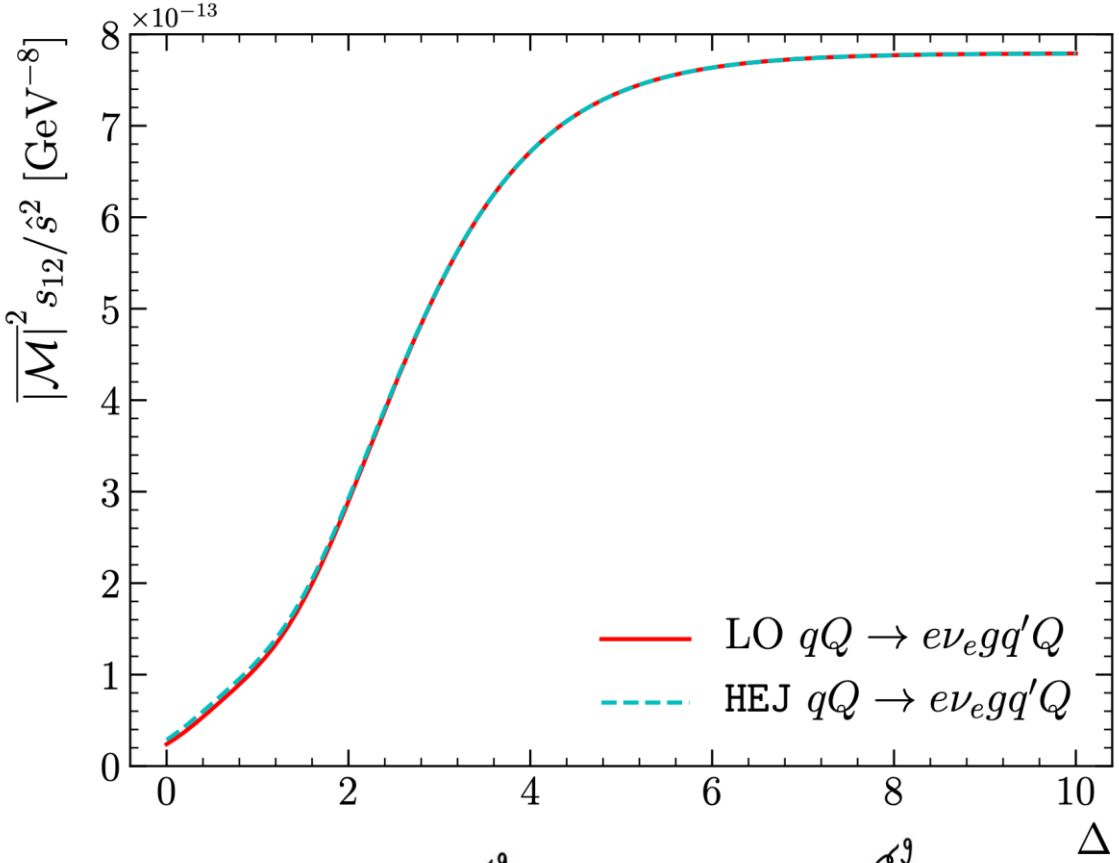
We may then write the amplitude as

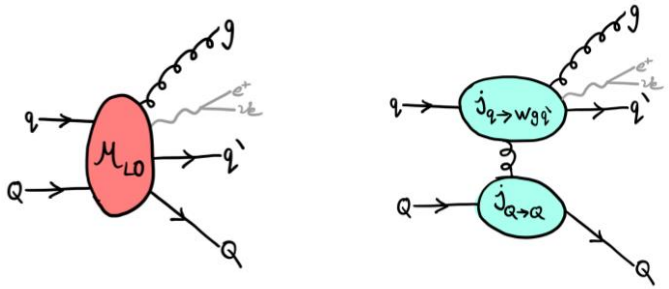
$$\mathcal{M}_{qQ\to Wgq'Q} \xrightarrow[y_g\sim y_{q'}\ll y_Q]{} j^{\mu}_{q\to Wgq'}\left(\frac{1}{t}\right) j_{Q\to Q\ \mu}$$

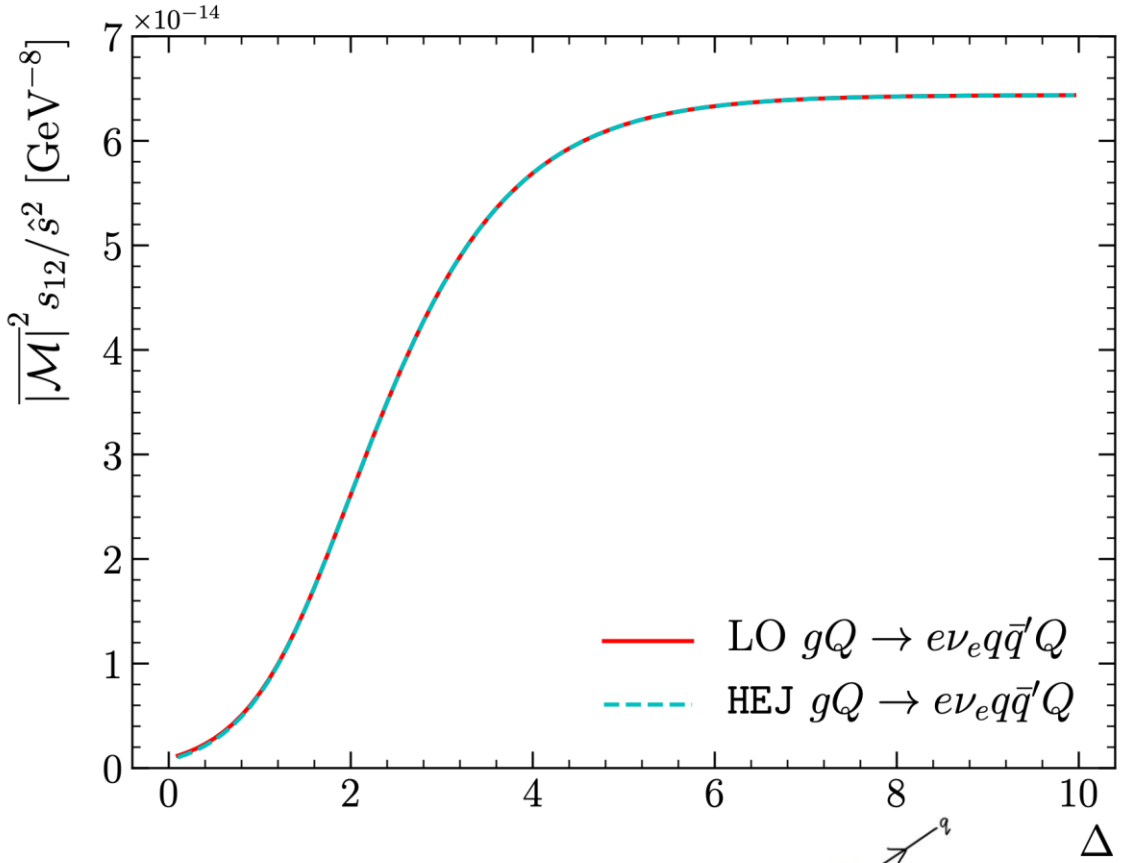


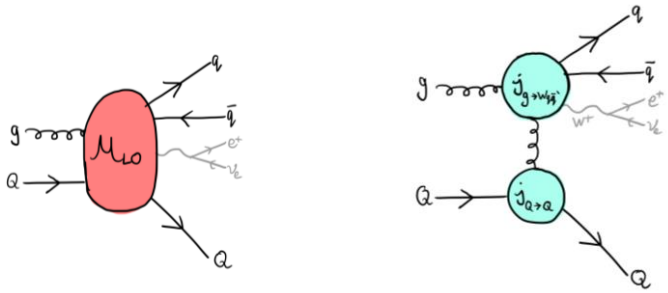
New factorised components for W+3j

These expressions obey the expected scaling, and provide a good approximation to the LO amplitude:



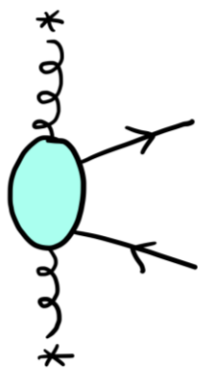




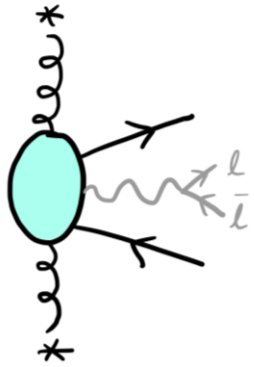


New factorised components for W+4j

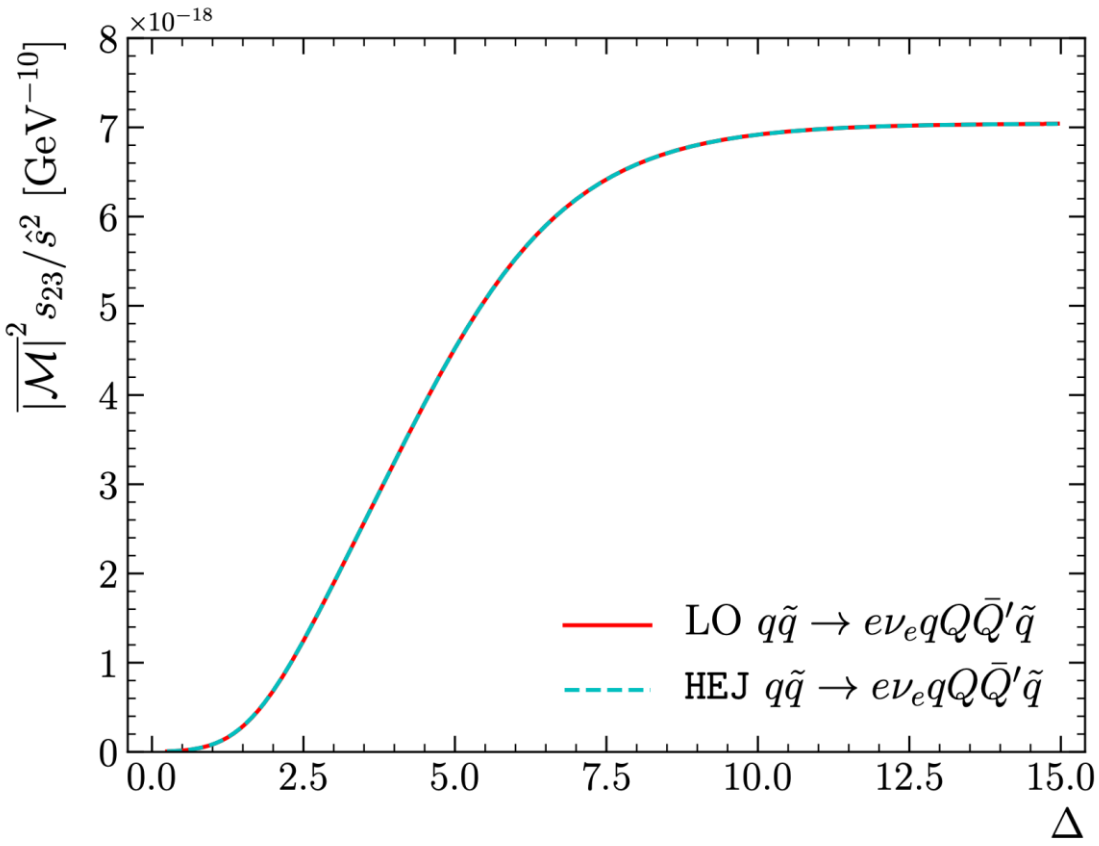
Finally, we need effective vertices for the emission of a central $Q\bar{Q}$ pair in order to describe all NLL configurations. These are required for ≥4j events.



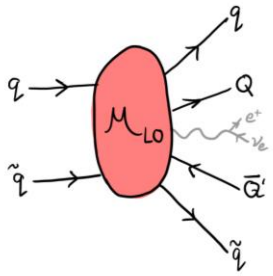
$V^{\mu\nu}_{Q\overline{Q}}$

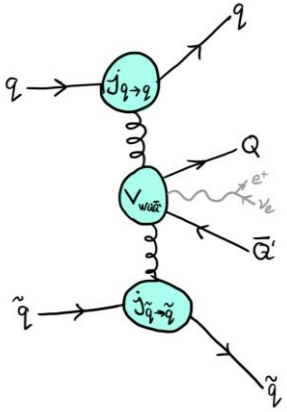


$V^{\mu\nu}_{WQ\overline{Q'}}$

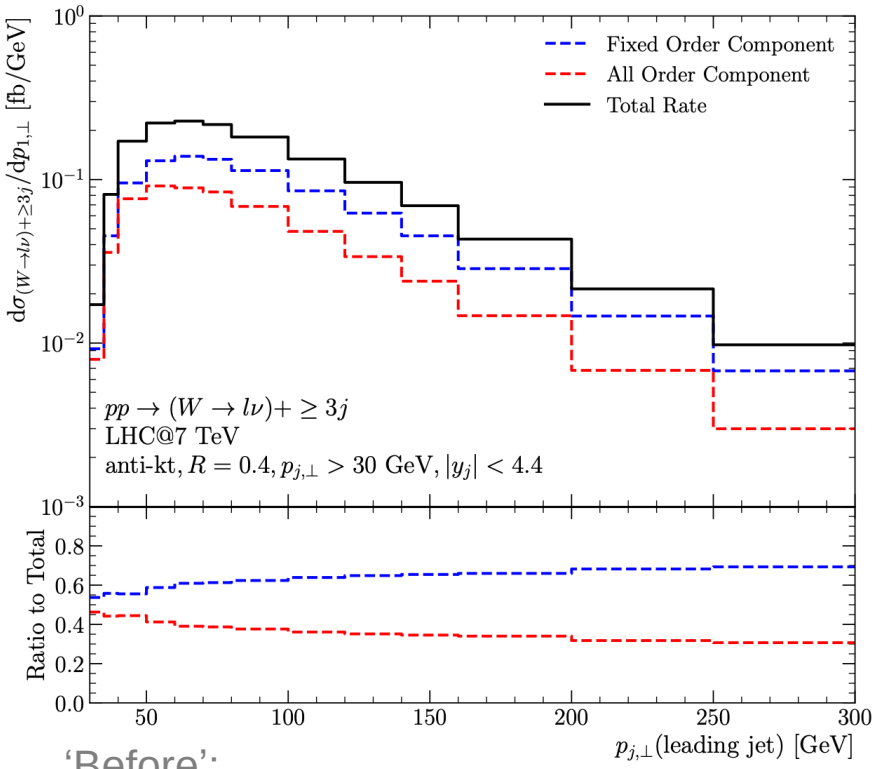


See appendix of [1] arXiv:2012.10310 for the derivation of these factorised expressions.

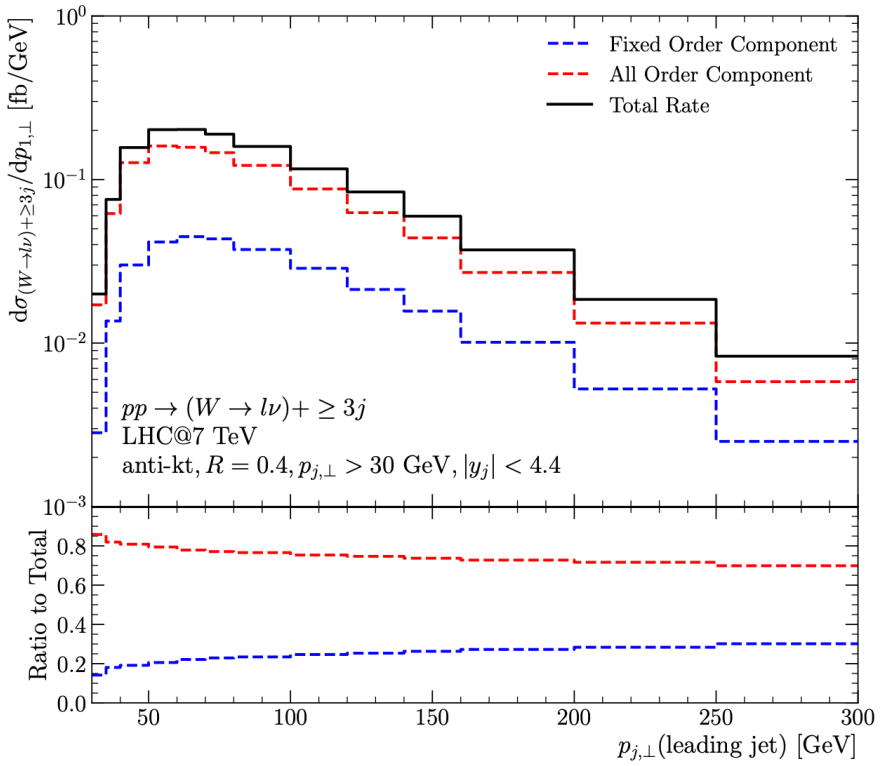




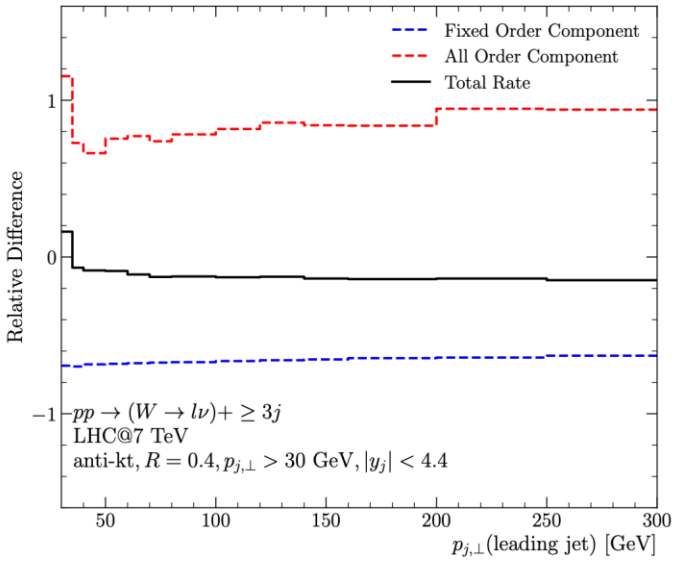
Numerical impact of LL resummation to NLL configurations



'Before':
Resummation applied to LL configs
All other configs included at LO.



'After':
Resummation applied to LL and NLL configs
All other configs included at LO.



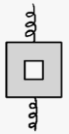
$$\frac{A-B}{B}$$

NLO matching procedure

HEJ provides an all-orders description which may be truncated to any desired order in α_s

In particular we can obtain HEJ@NLO by restricting to a single real emission, and truncating the all-orders virtual corrections:

$$\frac{1}{t}e^{\tilde{\alpha}(q_\perp)\Delta y} \to \frac{1}{t}\left(1+\tilde{\alpha}(q_\perp)\Delta y\right) \quad (1+$$



$)$

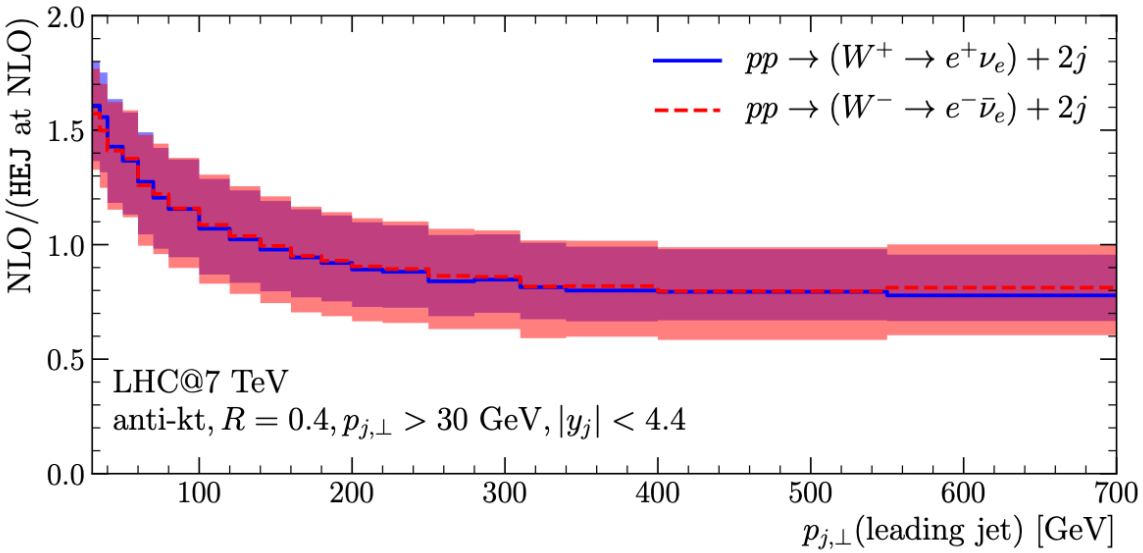
We can match HEJ bin-by-bin to W+2j NLO results to obtain NLO accuracy while maintaining the logarithmic accuracy of HEJ.

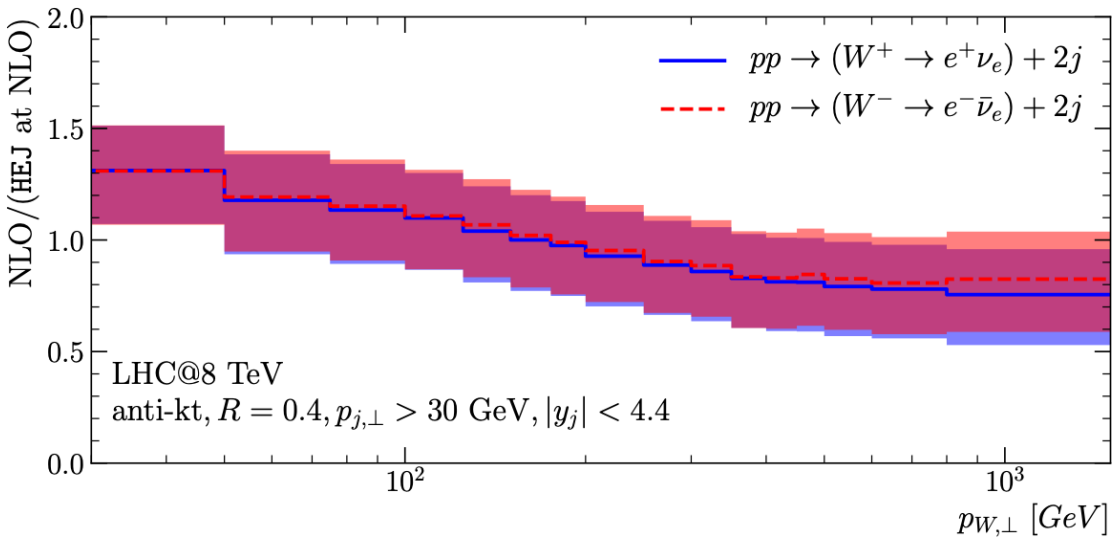
We calculate the bin-by-bin reweighting factors:

$$R = \frac{w_{\mathrm{NLO}}}{w_{\mathtt{HEJ@NLO}}}$$

The final NLO-matched bin-weight is then:

$$w_{\mathtt{HEJ2NLO}} = R(w_{\mathtt{HEJ2}}) + w_{\mathrm{FO}\ W+\geq 4j}$$

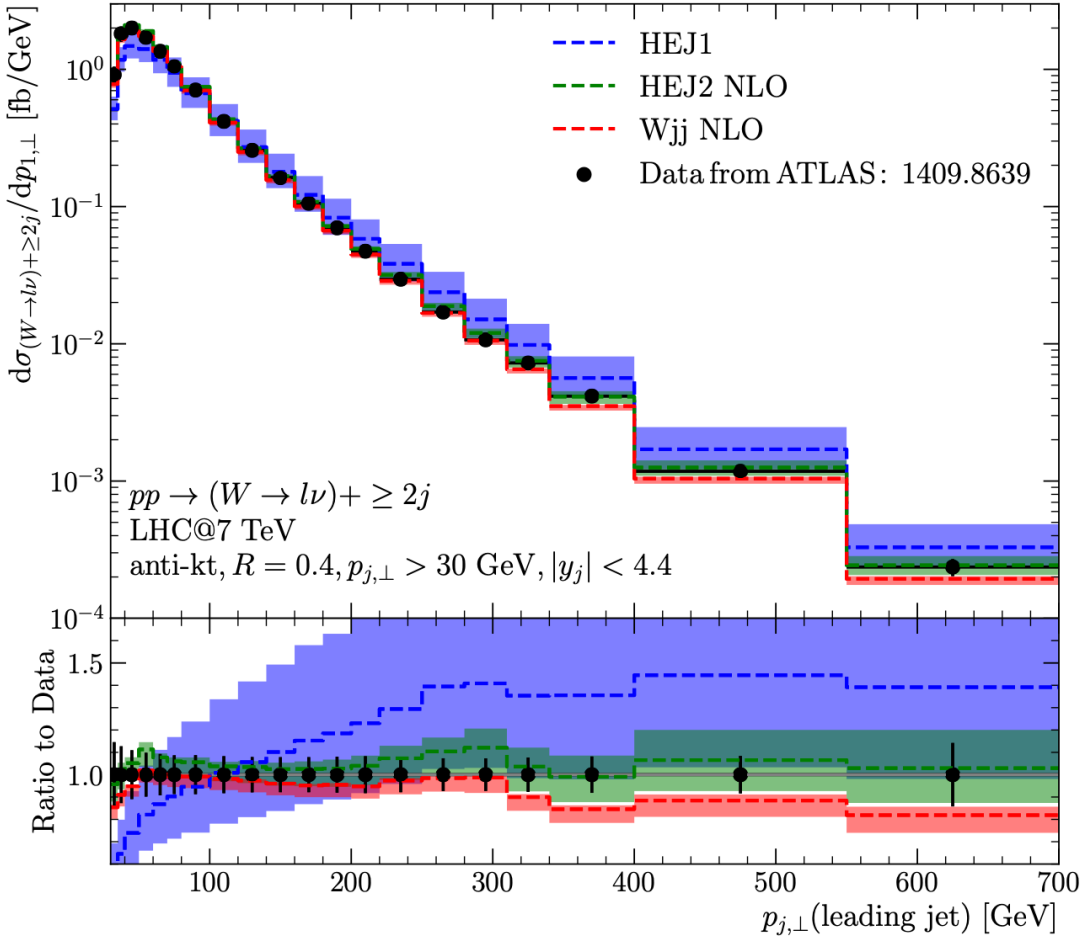


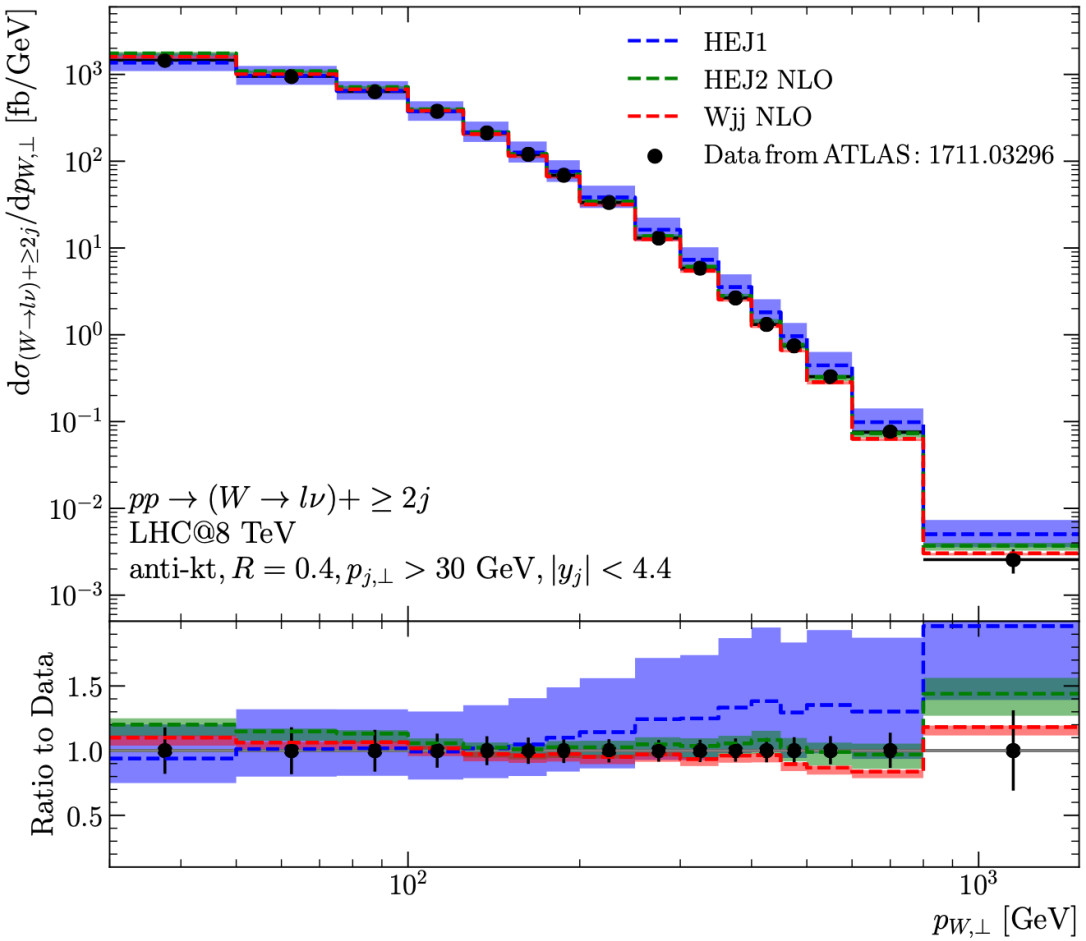


[8] arXiv:1409.8639 ATLAS W+jets study at 7TeV
[9] arXiv:1711.03296 ATLAS W+jets study at 8TeV

Comparison to W+2j data

LL HEJ (HEJ1) does not give a good description of observables uncorrelated with the high energy limit, as we expect: This breaks the MRK condition upon which they were based.





HEJ with the inclusion of the new NLL configurations, and bin-by-bin matching to NLO (HEJ2 NLO), gives a good description of data.

Wjj NLO obtained with Sherpa (arXiv:0811.4622) + OpenLoops (arXiv:1111.5206)

Summary

- For W+jets we have included a class of NLL corrections, which required new factorised expressions.
- We have also performed the first matching of HEJ to NLO.
- These improvements are seen to give a good description of LHC data, and a significant improvement to observables such as $p_\perp$ distributions.
- Work is ongoing to extend the HEJ framework to full NLL accuracy.

Thanks for your attention!

Tuesday at 11.50: HEJ+PYTHIA, Hitham Hassan

Wednesday at 9.50: HEJ Status Report, Jeppe Andersen

Backup Slides

Return to LO $qg \to qg$

For $qg \to qg$ the dominant helicity configurations are also exactly expressible as a contraction of quark currents over a t-channel pole, e.g. [4]

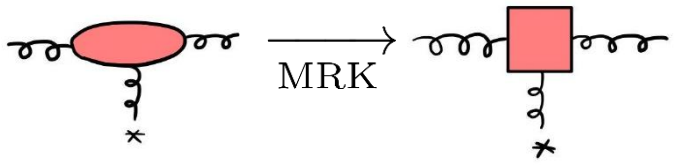
$$\left|\mathcal{M}^{(0)}(q^{\ominus}g^{\oplus} \to q^{\ominus}g^{\oplus})\right|^2 = \frac{K_g(p_2, p_b)}{C_F}\left|\mathcal{M}^{(0)}(qQ \to qQ)\right|^2$$

$$K_g(p_b, p_2) \equiv \frac{1}{2}\left(C_A - \frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-}\right) + \frac{1}{C_A}.$$

Here we see that in the MRK approximation $p_b^- \approx p_2^-$ we obtain

$$K_g(p_b^-, p_2^-) \xrightarrow[\text{MRK}]{} C_A$$

in agreement with the approximation of switching the representation of the generator in the impact factor.

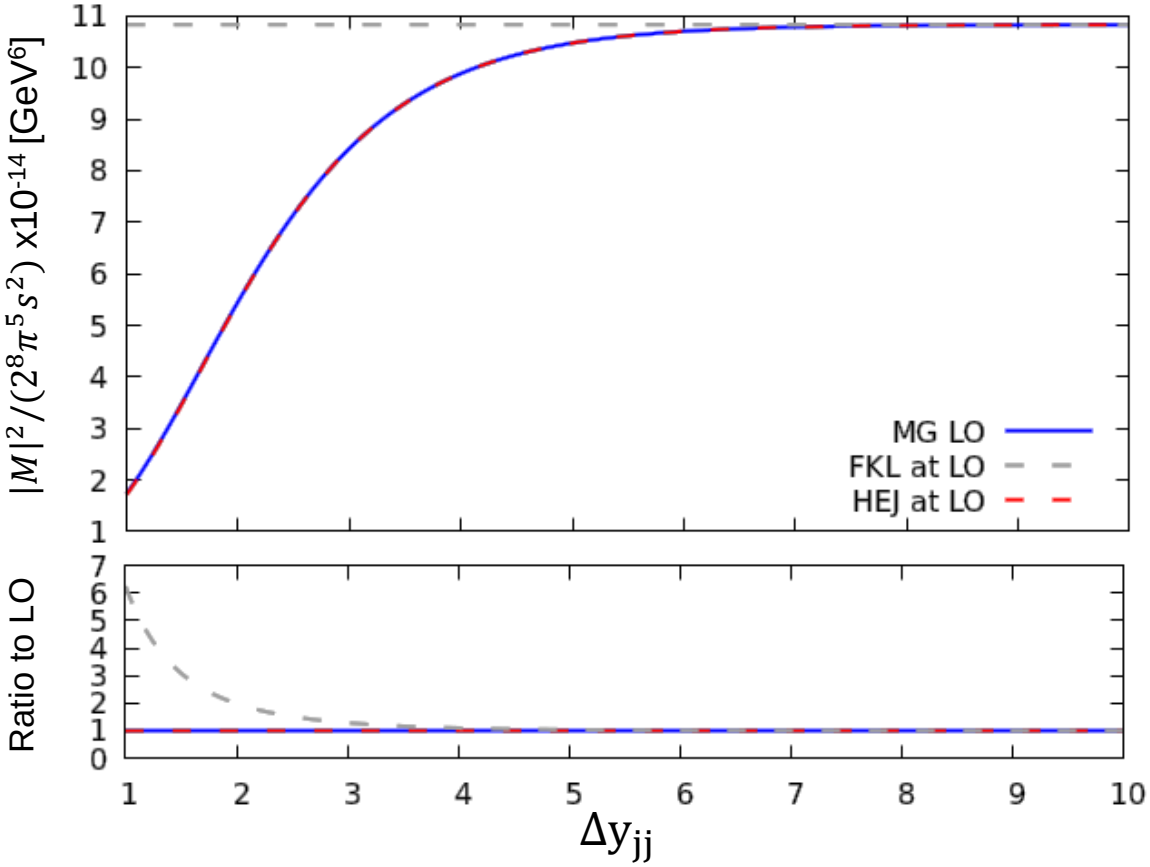


[4] arXiv:0910.5113 Andersen, Smillie

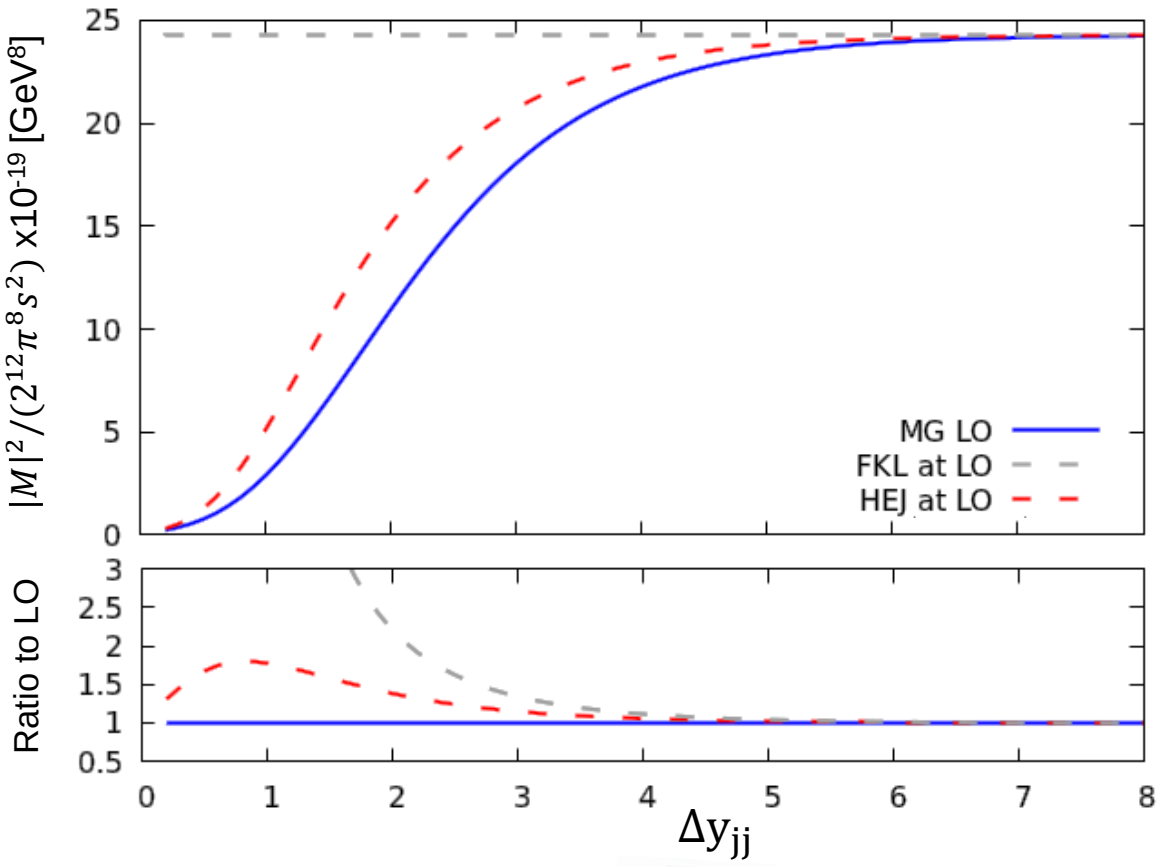
Comparison at LO

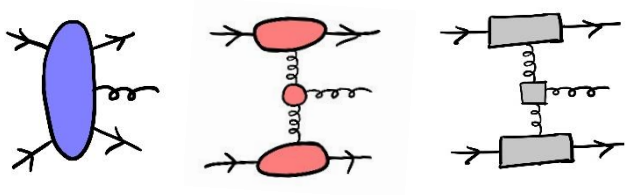
The HEJ amplitudes capture more of the LO physics relevant to LHC phase space than the strictly LL amplitudes:

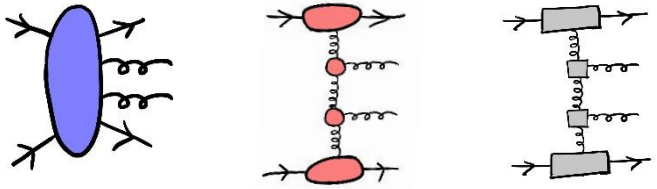
$qQ \to qgQ$



$qQ \to qggQ$



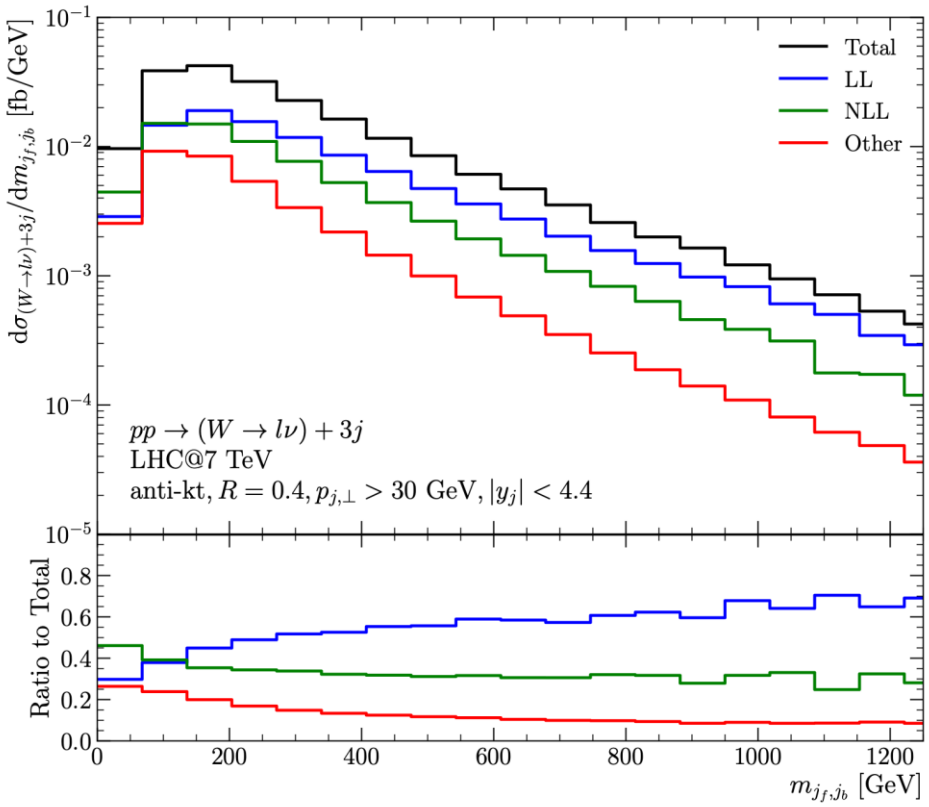


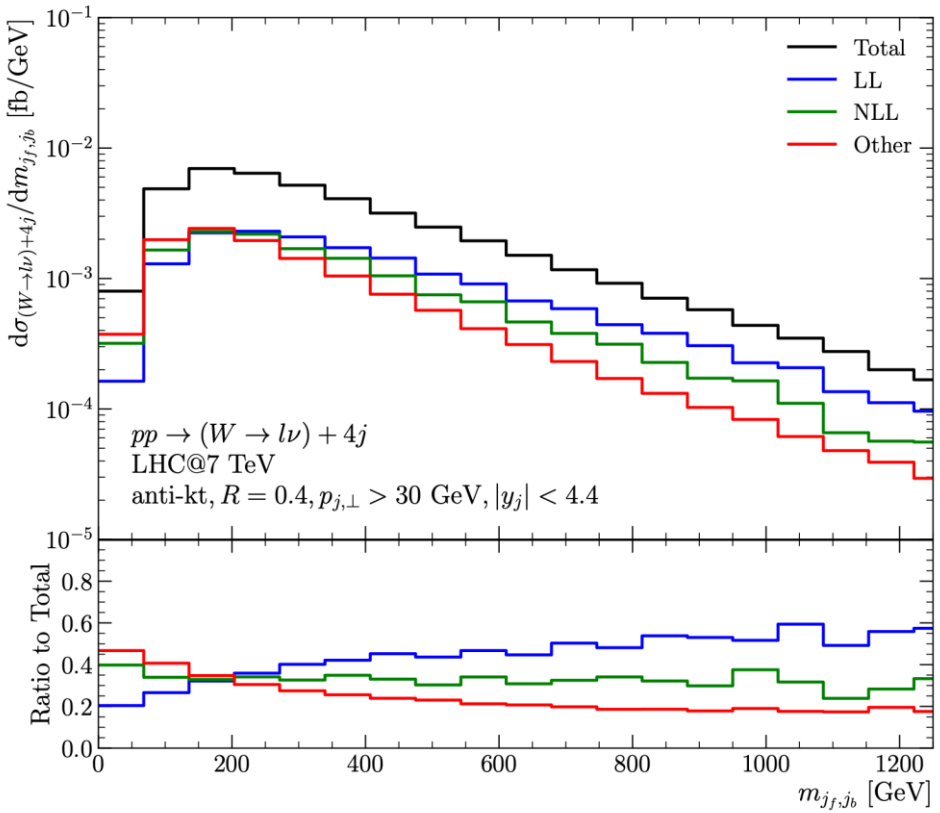


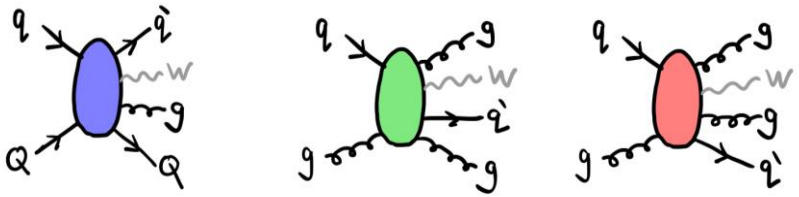
Details of these slices through phase space are given in [3] arXiv:0908.2786 Andersen, Smillie

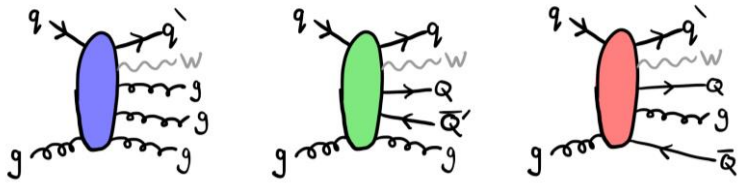
Impact of NLL configurations at LO

A LO analysis demonstrates the importance of also including LL resummation for NLL configurations:



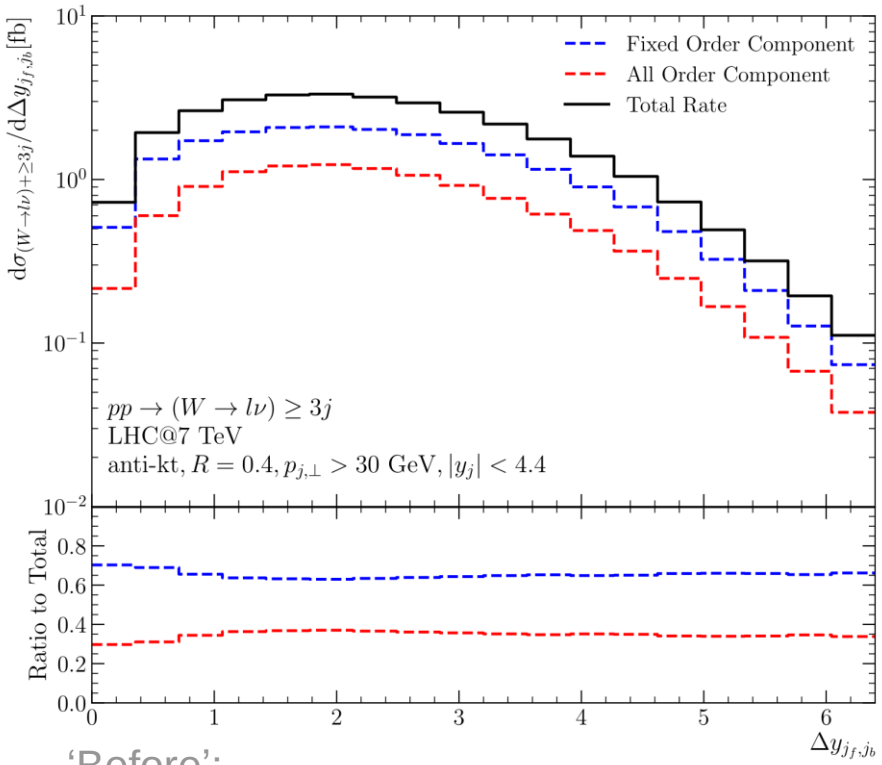




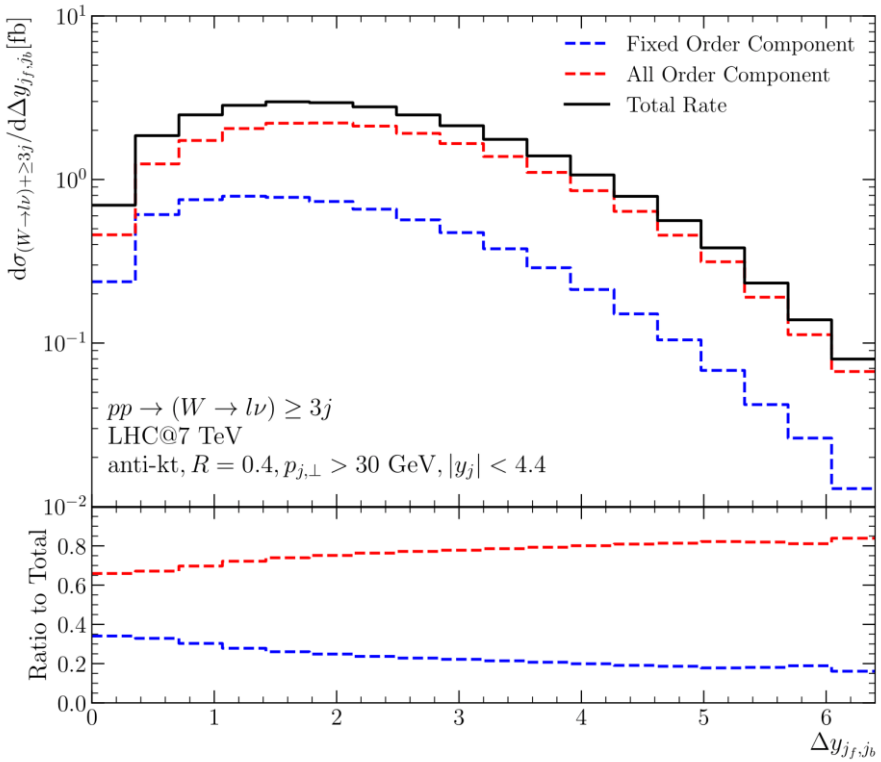


[19] arXiv:2012.10310 Jeppe Andersen, James Black, Helen Brooks, EB, Andreas Maier and Jennifer Smillie

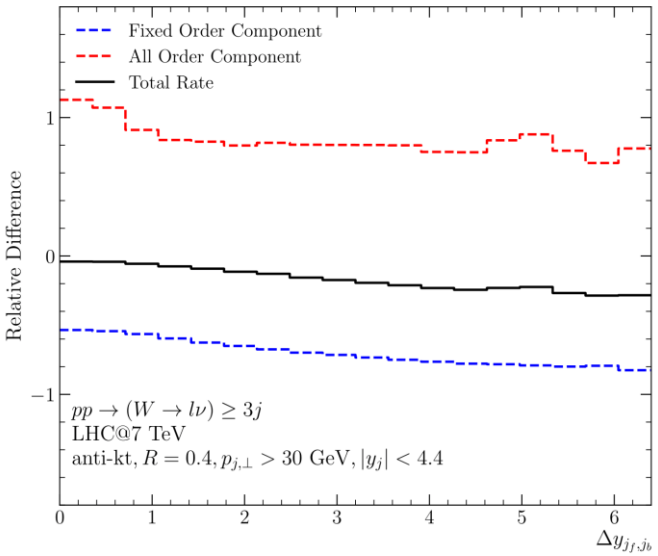
Numerical impact of LL resummation to NLL configurations



'Before':
Resummation applied to LL configs
All other configs included at LO.



'After':
Resummation applied to LL and NLL configs
All other configs included at LO.



$$\frac{A-B}{B}$$

NLO matching procedure

We can write the $2j$ inclusive NLO cross section as:

$$\sigma_{2j}^{\mathrm{NLO}} = f_{2j}^{(2)}\alpha_s^2 + \left(f_{2j}^{(3)} + f_{3j}^{(3)}\right)\alpha_s^{(3)}.$$

By contrast, HEJ provides an all-orders description which may be truncated to any desired order in α_s:

$$\sigma_{2j}^{\mathrm{HEJ}} = h_{2j}^{(2)}\alpha_s^2 + (h_{2j}^{(3)} + h_{3j}^{(3)})\alpha_s^3 + (h_{2j}^{(4)} + h_{3j}^{(4)} + h_{4j}^{(4)})\alpha_s^4 + \mathcal{O}(\alpha_s^5).$$

In particular we can truncate the HEJ predictions to NLO:

$$\sigma_{2j}^{\mathrm{HEJ@NLO}} = h_{2j}^{(2)}\alpha_s^2 + (h_{2j}^{(3)} + h_{3j}^{(3)})\alpha_s^3.$$

We can then construct the ratio:

$$\frac{\sigma_{2j}^{\mathrm{NLO}}}{\sigma_{2j}^{\mathrm{HEJ@NLO}}} = 1 + (f_{2j}^{(3)} - h_{2j}^{(3)}) \prod_{n=0}^{\infty} (-1)^n \alpha_s^{(n+1)} \frac{(f_{3j}^{(3)} + h_{2j}^{(3)})^n}{(f_{2j}^{(2)})^{(n+1)}}.$$

which ensures the inclusive HEJ prediction agrees with the fixed-order result up to α_s^3

$$\sigma_{2j}^{\mathrm{HEJ}}\left(\frac{\sigma_{2j}^{\mathrm{NLO}}}{\sigma_{2j}^{\mathrm{HEJ@NLO}}}\right) = f_{2j}^{(2)}\alpha_s^2 + \left(f_{2j}^{(3)} + f_{3j}^{(3)}\right)\alpha_s^3 + \left(h_{2j}^{(4)} + h_{3j}^{(4)} + f_{4j}^{(4)}\right)\alpha_s^4 + \mathcal{O}(\alpha_s^5),$$