

# SUSY Overview

Why still love SUSY?

SUSY after  $g_\mu - 2$  and Run 2  
pMSSM vs GUT-inspired models

Flipped SU(5)

Prospects for discovery in Runs  $\geq 3$

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College  
LONDON

What lies beyond the Standard Model?

# Supersymmetry

New motivations  
from LHC

- **Stabilize electroweak vacuum**
- **Successful prediction for Higgs mass**
  - Should be  $< 130$  GeV in simple models
- **Successful predictions for couplings**
  - Should be within few % of SM values
- Naturalness, GUTs, string, inflation, **dark matter**, ..

# Everything about Higgs is Puzzling

$$\mathcal{L} = yH\psi\bar{\psi} + \mu^2|H|^2 - \lambda|H|^4 - V_0$$

- Pattern of Yukawa couplings  $y$ :
  - **Flavour problem**
- Magnitude of mass term  $\mu$ :
  - **Naturalness/hierarchy problem**
- Magnitude of quartic coupling  $\lambda$ :
  - **Stability of electroweak vacuum**
- Cosmological constant term  $V_0$ :
  - **Dark energy**

SUSY

SUSY

SUSY

# Supersymmetry &

$$g_\mu - 2$$

go back a long way

## SPIN-ZERO LEPTONS AND THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

John ELLIS, John HAGELIN and D.V. NANOPOULOS  
*CERN, Geneva, Switzerland*

Received 14 June 1982

The anomalous magnetic moment of the muon  $(g - 2)_\mu$  imposes constraints on the masses and mixing of spin-zero leptons (sleptons). We develop the predictions of models of spontaneous supersymmetry breaking for the slepton mass matrix, and show that they are comfortably consistent with the  $(g - 2)_\mu$  constraints.

During the present resurgence of interest in supersymmetry broken at low energies [1] new significance is attached to the classical phenomenological playgrounds of gauge theories such as the anomalous magnetic moments of the electron and muon [2], flavour-changing neutral interactions [3,5] parity [6] and CP violation [7,8] in the strong interactions. The three latter phenomena make life rather difficult [3,7] for the most general form of soft supersymmetry breaking, whereas simple models [9-11] of spontaneously broken supersymmetry naturally [3,4 7] respect the  $\Delta F \neq 0, P$  and CP violation constraints. As for the anomalous magnetic moments of the leptons, it has long been known that they vanish in an exactly supersymmetric theory [12], and Fayet [2] showed that in his model of supersymmetry breaking  $(g - 2)_\mu$  would be compatible with experiment if the spin-zero muon (smuon) masses were heavier than 15 GeV. Direct experimental searches [13] now exclude the existence of lighter smuons. Fayet's analysis [2] was in the context of a model with a very light photino  $\tilde{\gamma}$  (see fig. 1a), and Grifols and Méndez [14] have recently made the interesting observation that his analysis is significantly altered for massive gauginos (see figs. 1b, 1c). They show that there are potentially nontrivial constraints on the smuon masses in models of broken supersymmetry.

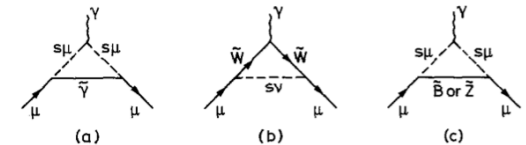


Fig. 1. One-loop diagrams contributing to  $(g - 2)_\mu$ : (a) essentially massless photino ( $\tilde{\gamma}$ ) exchange, (b)  $\tilde{W}$  and sneutrino ( $s\nu$ ) exchange, and (c)  $\tilde{B}$  or  $\tilde{Z}$  exchange.

right transition operator there is a GIM [15]-like cancellation between the smuon mass eigenstates in fig. 1c which provides a potential suppression mechanism. We analyze recent models [10,11] of spontaneous supersymmetry breaking originating in the  $D$  and  $F$  sectors, respectively. We show that in the former case  $(g - 2)_\mu$  is suppressed by near degeneracy between the smuon mass eigenstates, while in the latter case  $(g - 2)_\mu$  is suppressed by small mixing angles between the left- and right-handed smuons. We close with some remarks about  $(g - 2)_e$  and about parity violation in the strong interactions.

When they examined figs. 1a, 1b and 1c, Grifols and Méndez [14] realized that there was a fundamental difference between the (almost ?) massless  $\tilde{\gamma}$  diagram of fig. 1a and the  $\tilde{W}$  diagram of fig. 1b as compared to the massive  $\tilde{B}$  or  $\tilde{Z}$  diagram of fig. 1c. The

- One-loop contribution from smuon/neutralino loop

$$\Delta(g - 2)_\mu = -ab(\cos \alpha \sin \alpha / 4\pi^2)(m_\mu / m_{\tilde{G}})$$

$$\times \{1/(1 - \eta_1) + 2\eta_1/(1 - \eta_1)^2$$

$$+ [2\eta_1/(1 - \eta_1)^3] \log \eta_1 - (\eta_1 \leftrightarrow \eta_2)\},$$

- where  $\eta_i \equiv (m_{s\mu_i}^2 / m_{\tilde{G}}^2)$

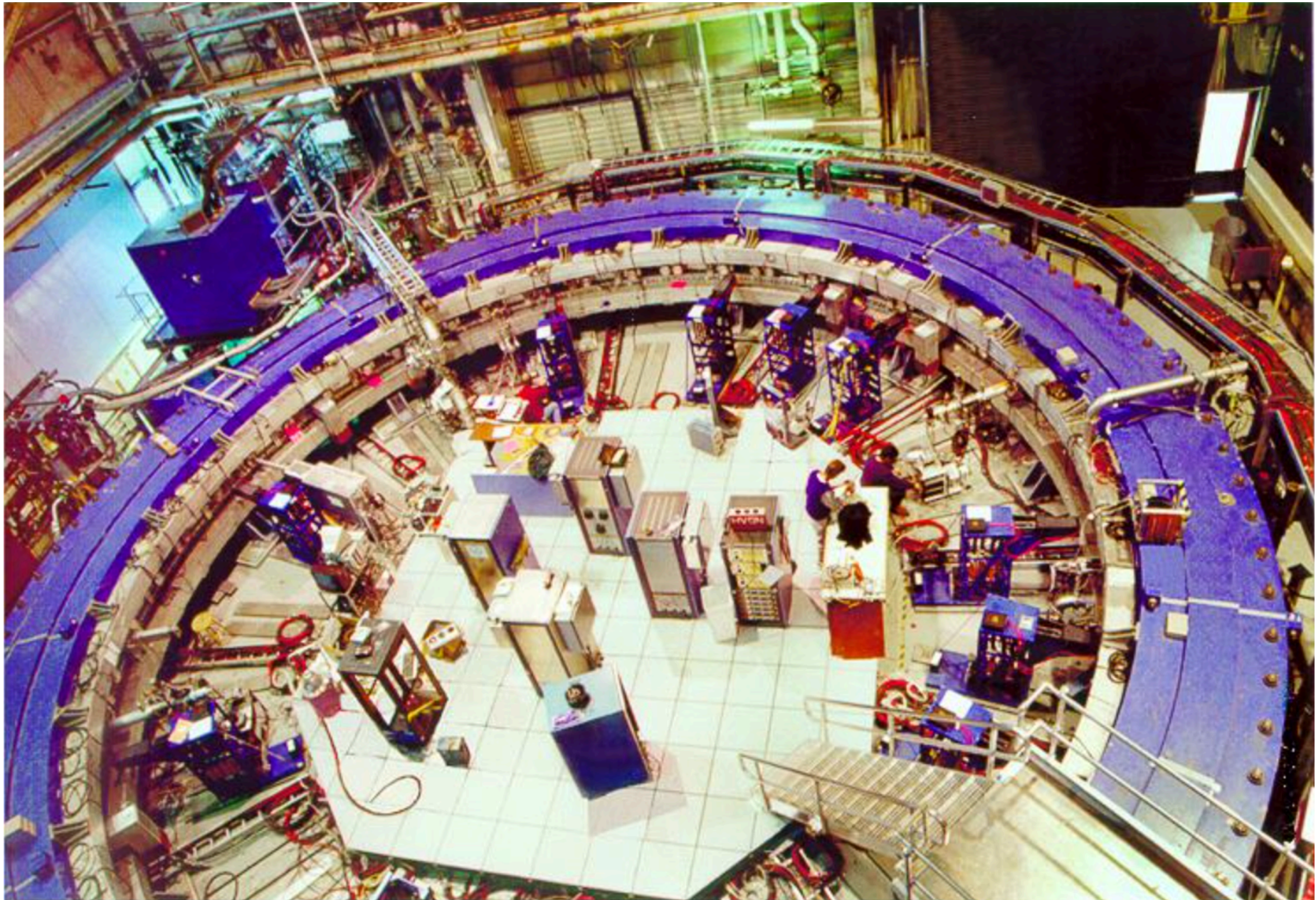
- and  $\mathcal{L} = a\sqrt{2} s_\mu \bar{\mu}_L \tilde{G} + b\sqrt{2} t_\mu \bar{\mu}_R \tilde{G}$

Before idea of supersymmetric dark matter

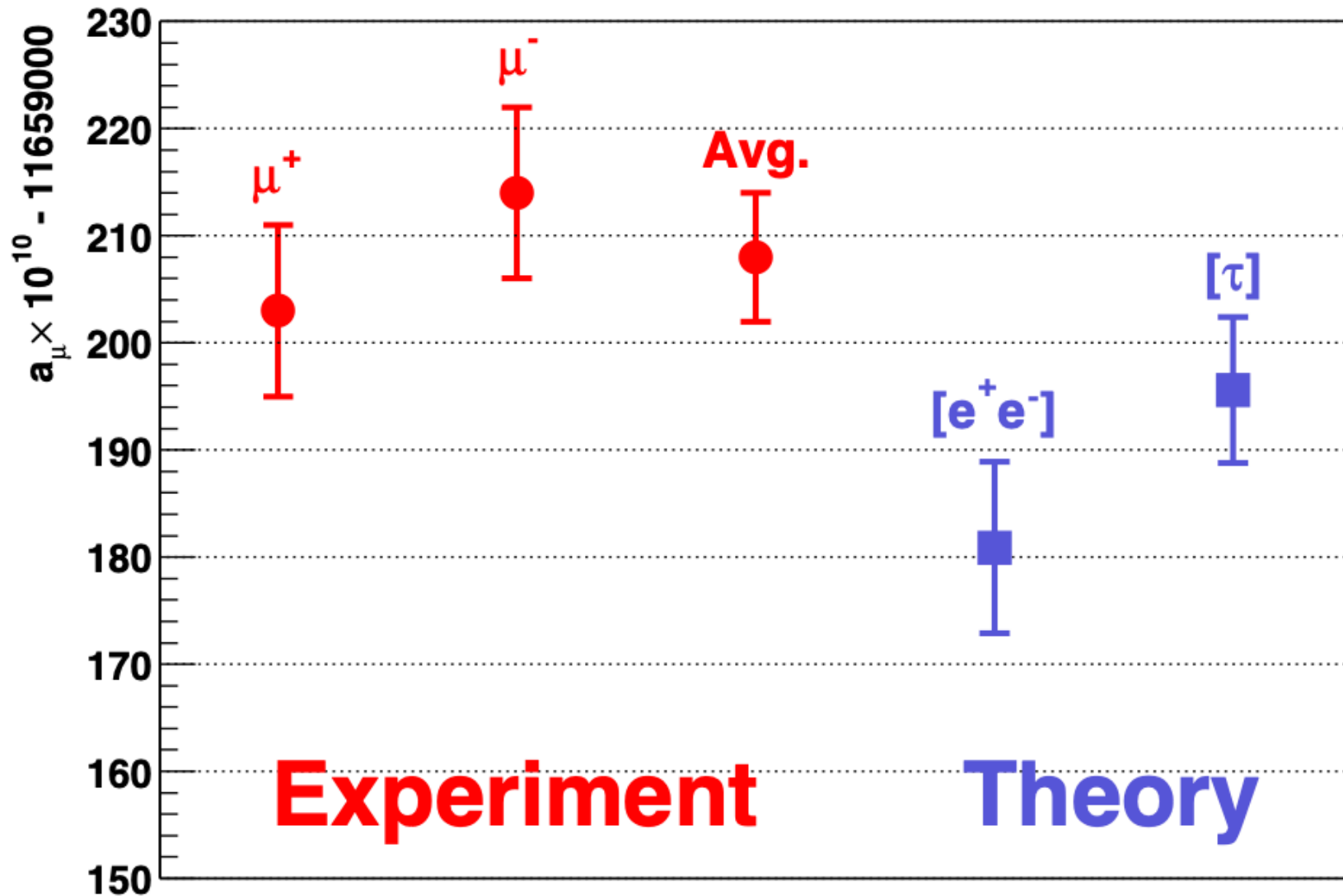


# Even before BNL Experiment

(1984 - 2003)



# Did BNL Discover Supersymmetry?



$$\delta a_\mu = \pm 0.47 \text{ ppm}$$



# $g_\mu - 2$ & DM in Supersymmetry v2: the CMSSM

## Combining the muon anomalous magnetic moment with other constraints on the CMSSM

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<sup>b</sup> Department of Physics, Texas A&M University, College Station, TX 77843, USA

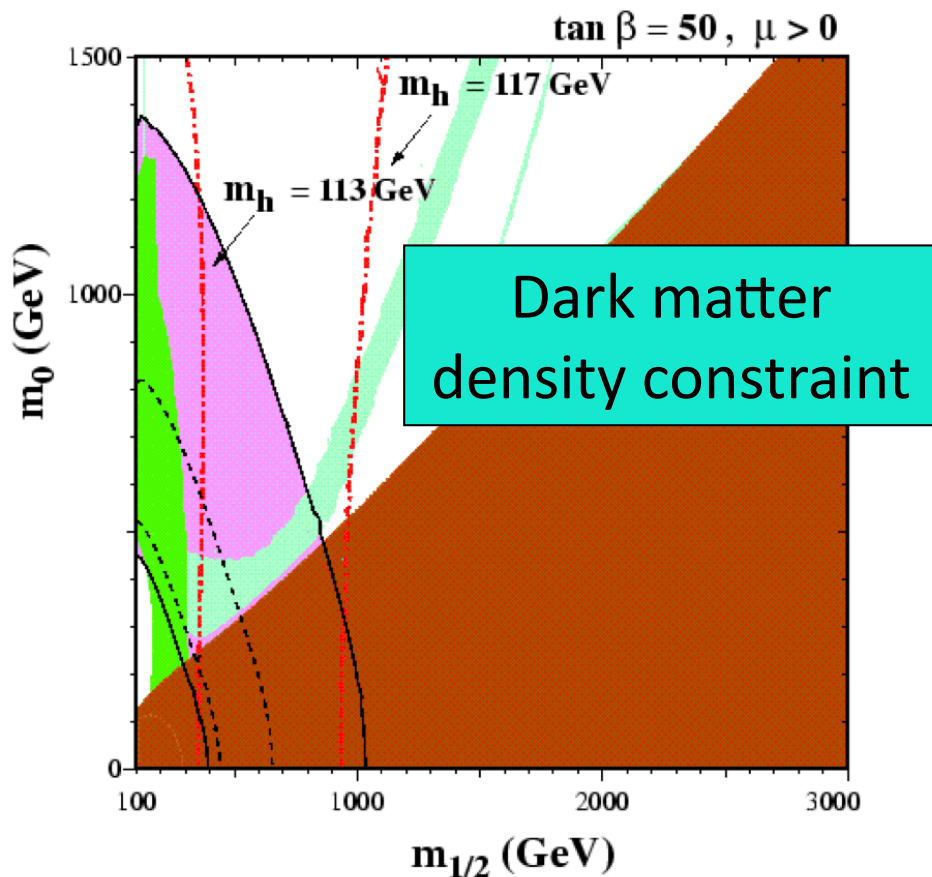
<sup>c</sup> Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA

<sup>d</sup> Chair of Theoretical Physics, Academy of Athens, Division of Natural Sciences, 28 Panepistimiou Avenue, Athens 10679, Greece

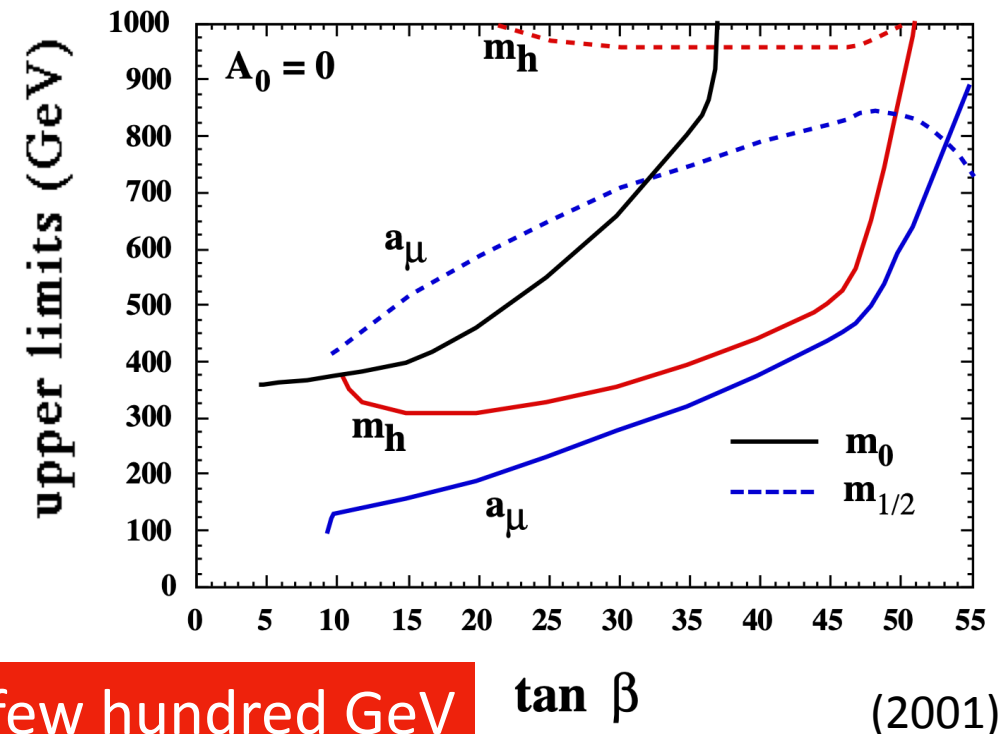
<sup>e</sup> Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA

Received 16 March 2001; accepted 10 April 2001

Editor: R. Gatto



Sparticle masses a few hundred GeV



(2001)

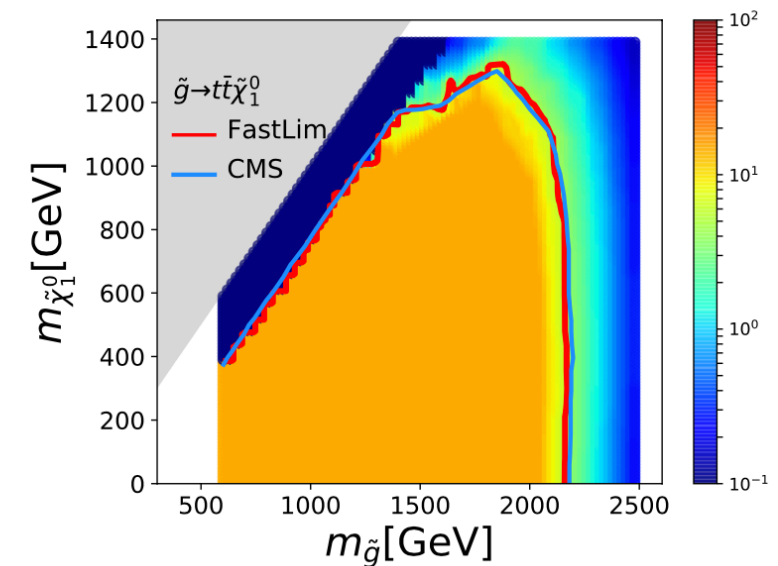
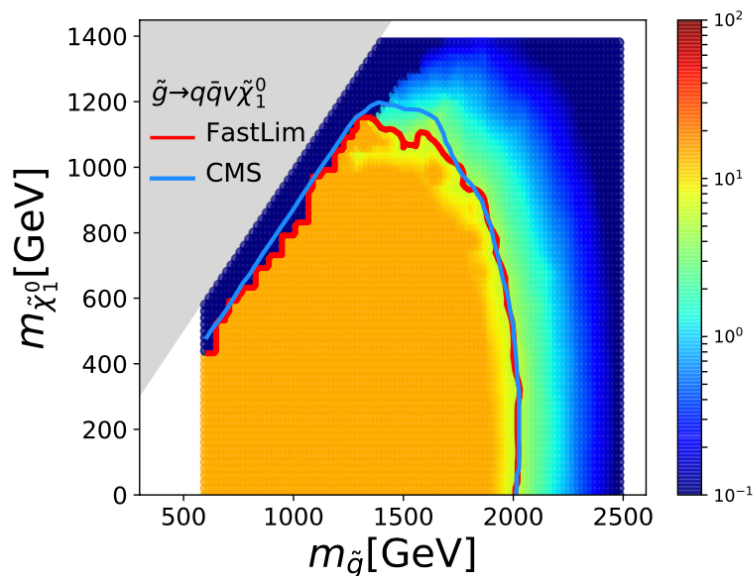
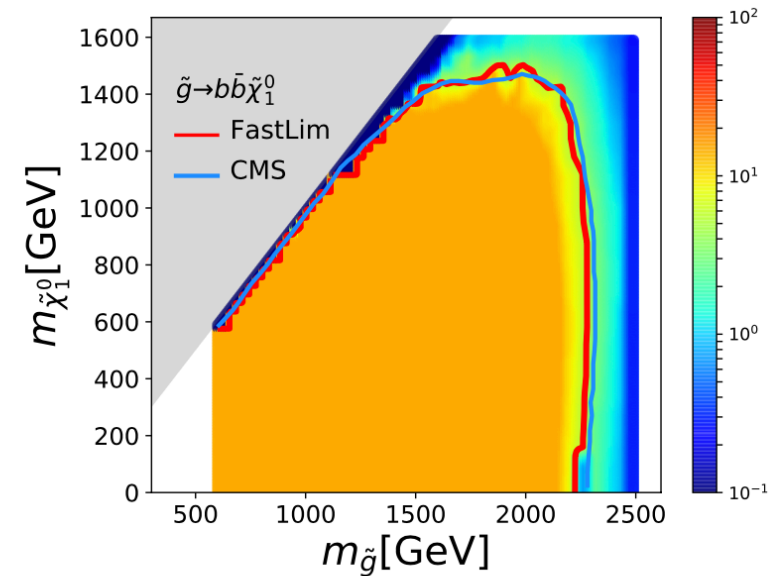
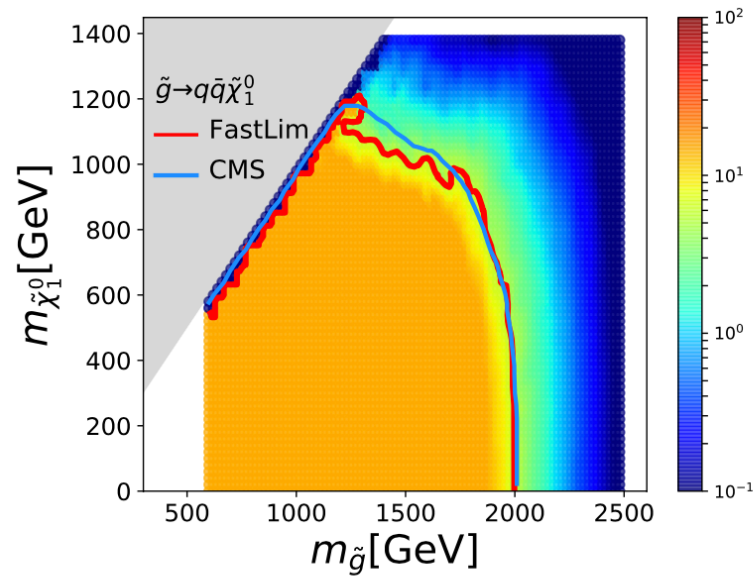
### Abstract

We combine the constraint suggested by the recent BNL E821 measurement of the anomalous magnetic moment of the muon on the parameter space of the constrained MSSM (CMSSM) with those provided previously by LEP, the measured rate of  $b \rightarrow s\gamma$  decay and the cosmological relic density  $\Omega_\chi h^2$ . Our treatment of  $\Omega_\chi h^2$  includes carefully the direct-channel Higgs poles in annihilation of pairs of neutralinos  $\chi$  and a complete analysis of  $\chi - \tilde{\ell}$  coannihilation. We find excellent consistency between all the constraints for  $\tan\beta \gtrsim 10$  and  $\mu > 0$ , for restricted ranges of the CMSSM parameters  $m_0$  and  $m_{1/2}$ . All the preferred CMSSM parameter space is within reach of the LHC, but may not be accessible to the Tevatron collider, or to a first-generation  $e^+e^-$  linear collider with centre-of-mass energy below 1.2 TeV. © 2001 Published by Elsevier Science B.V.

# Modelling LHC Constraints

## Glino decays:

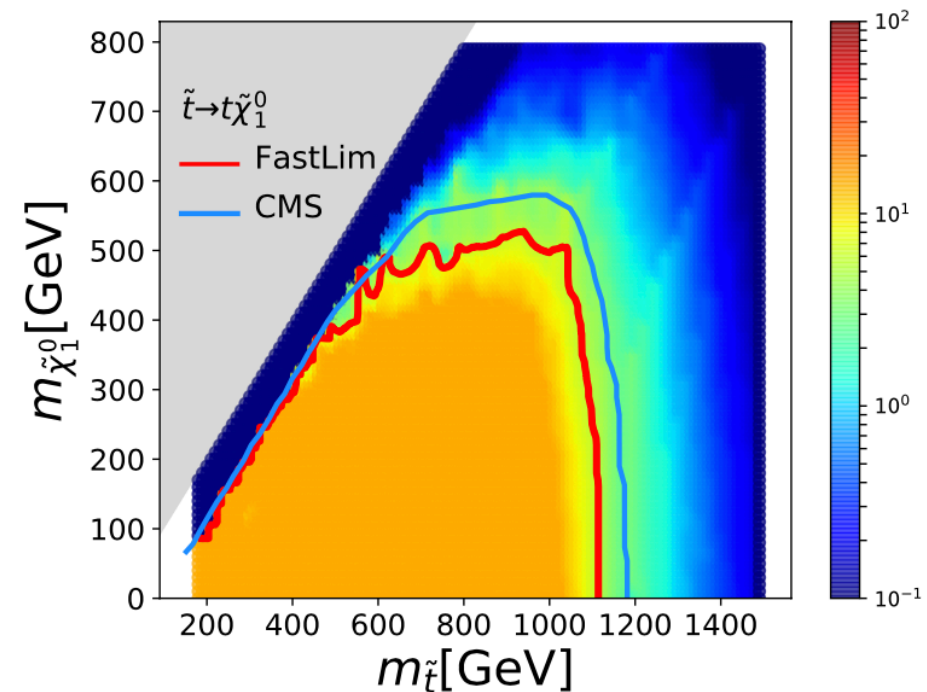
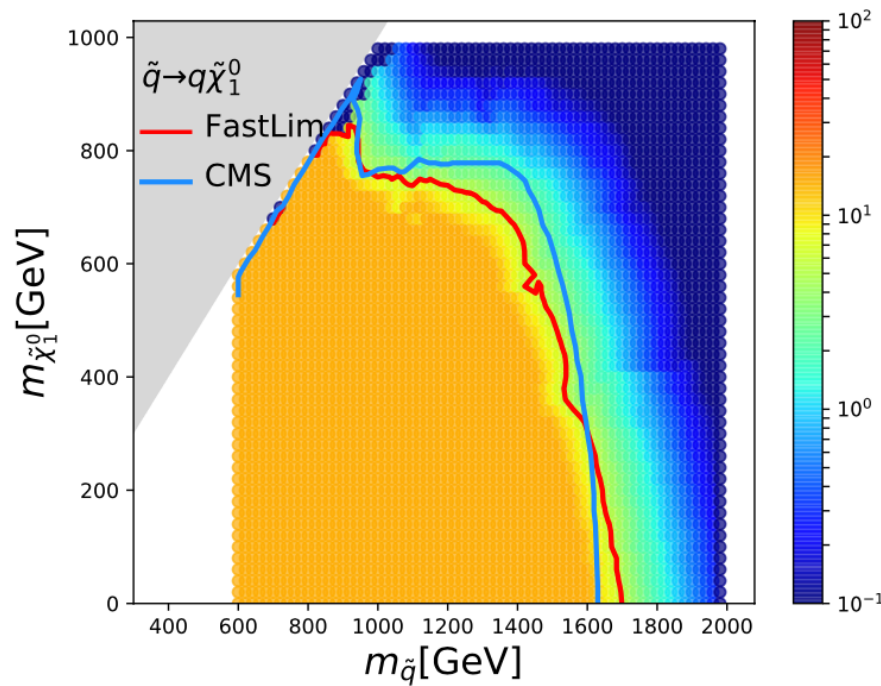
Reproduce LHC constraints using FastLim (does what it says on the tin)



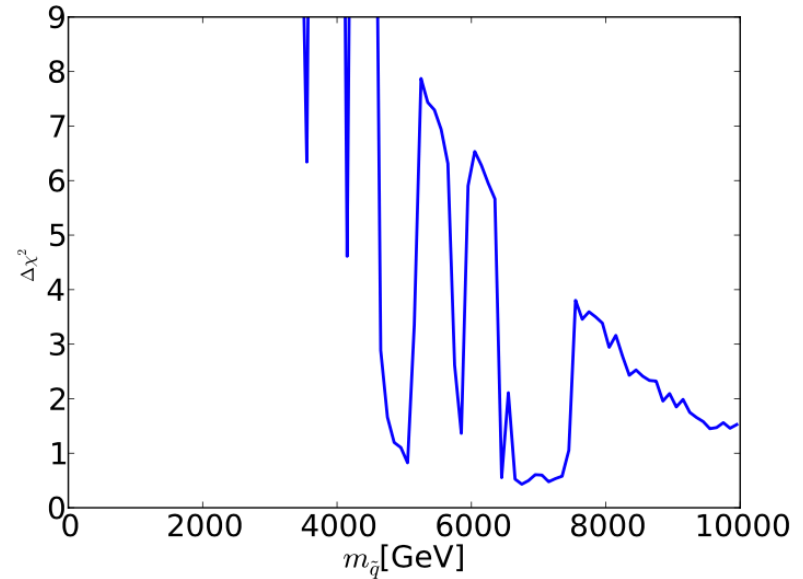
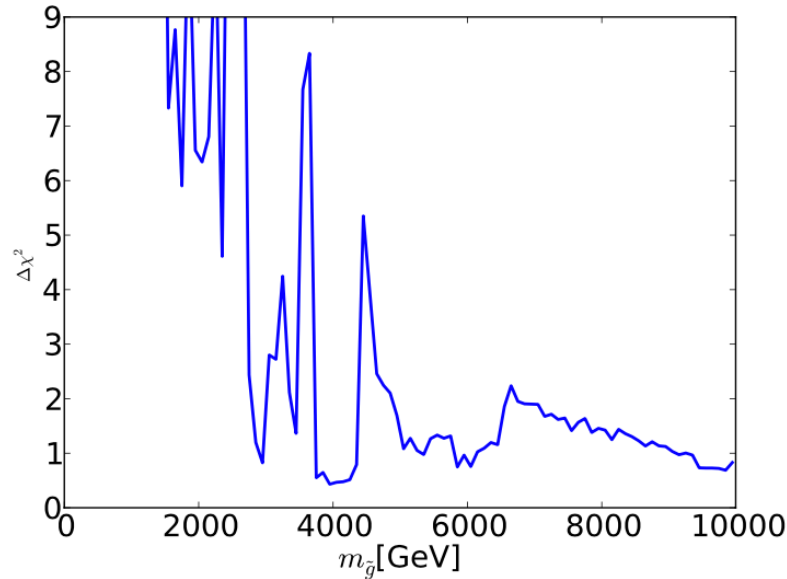
# Modelling LHC Constraints

## Squark and stop decays:

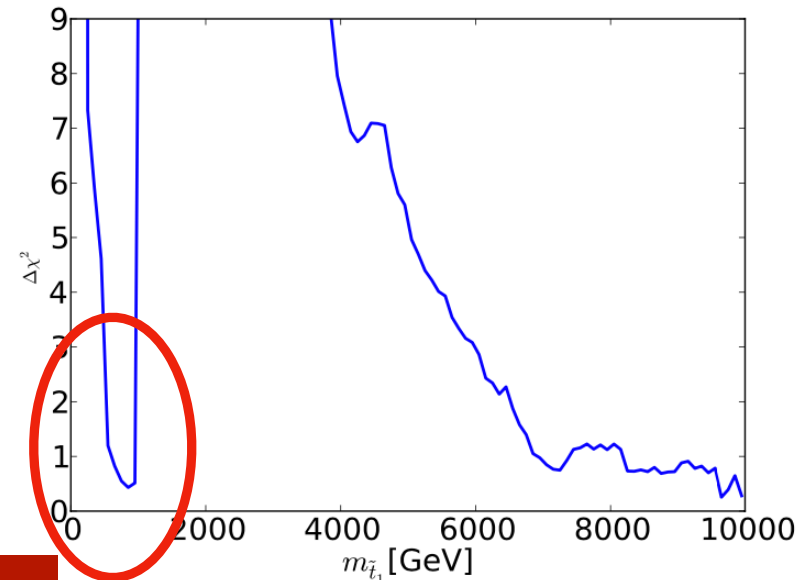
Reproduce LHC constraints using FastLim (does what it says on the tin)



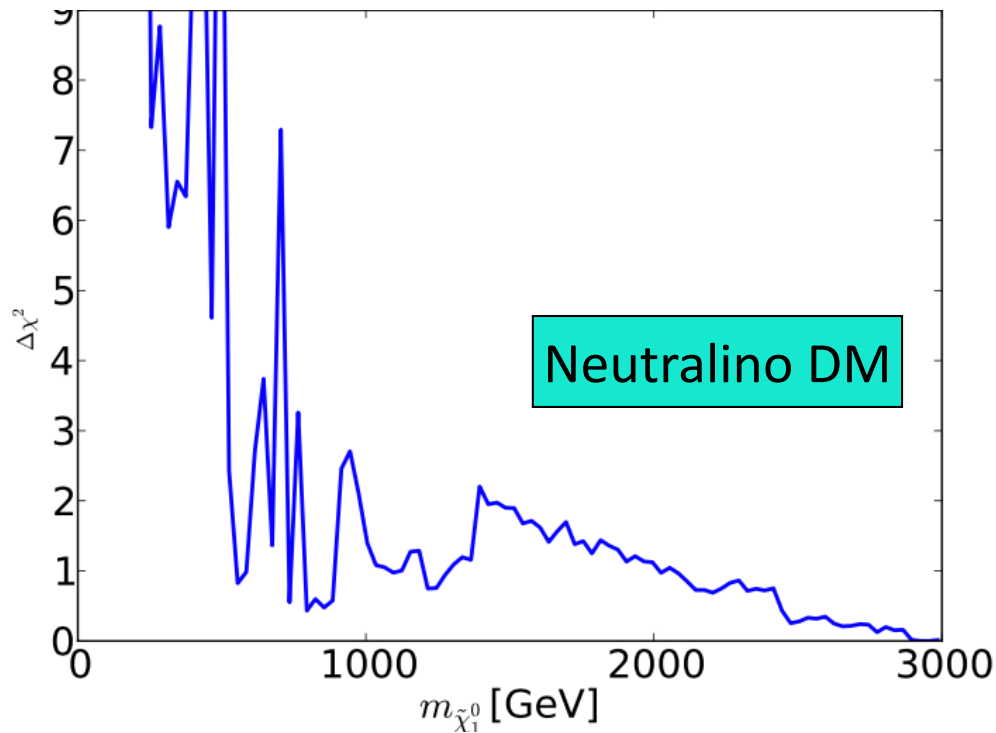
# 1D Profiled Likelihoods in CMSSM



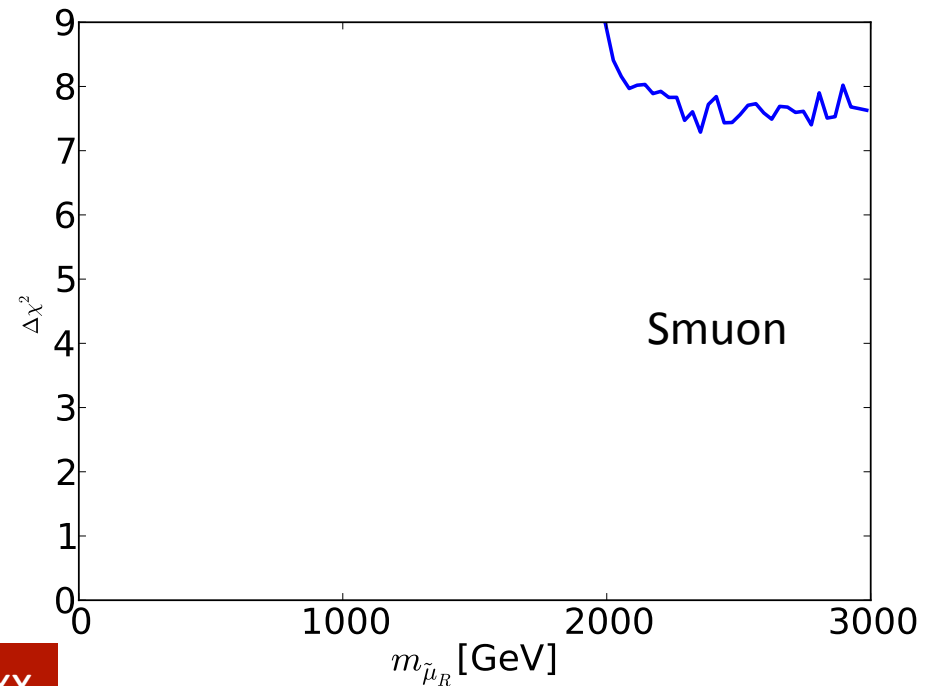
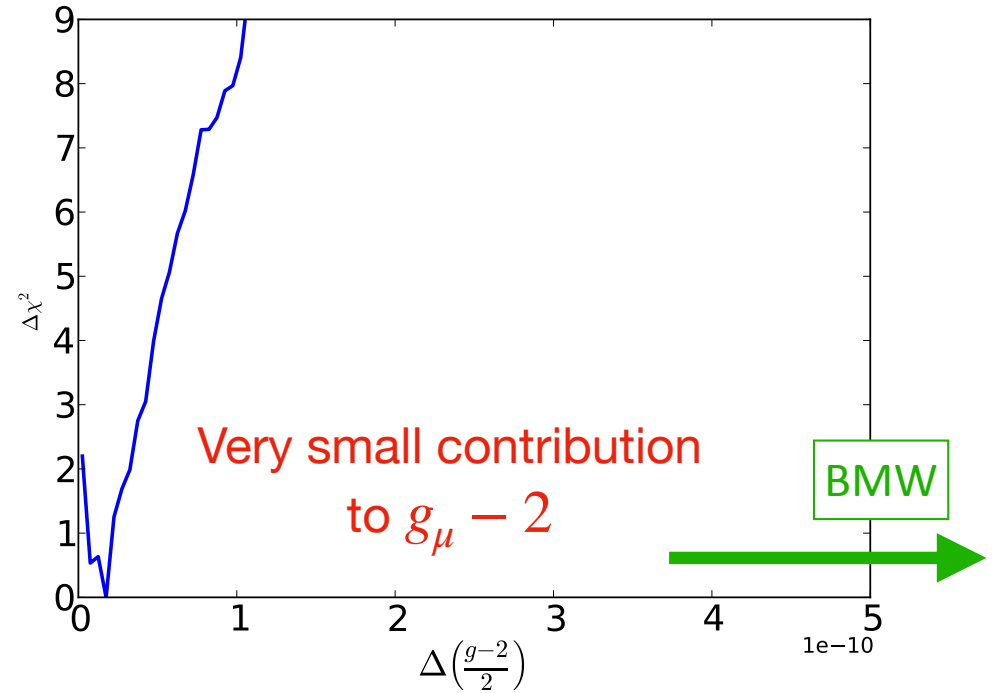
- Heavy squarks
- Very heavy gluon
- **Possibility of stop < 1 TeV**



# $g_\mu - 2$ in CMSSM

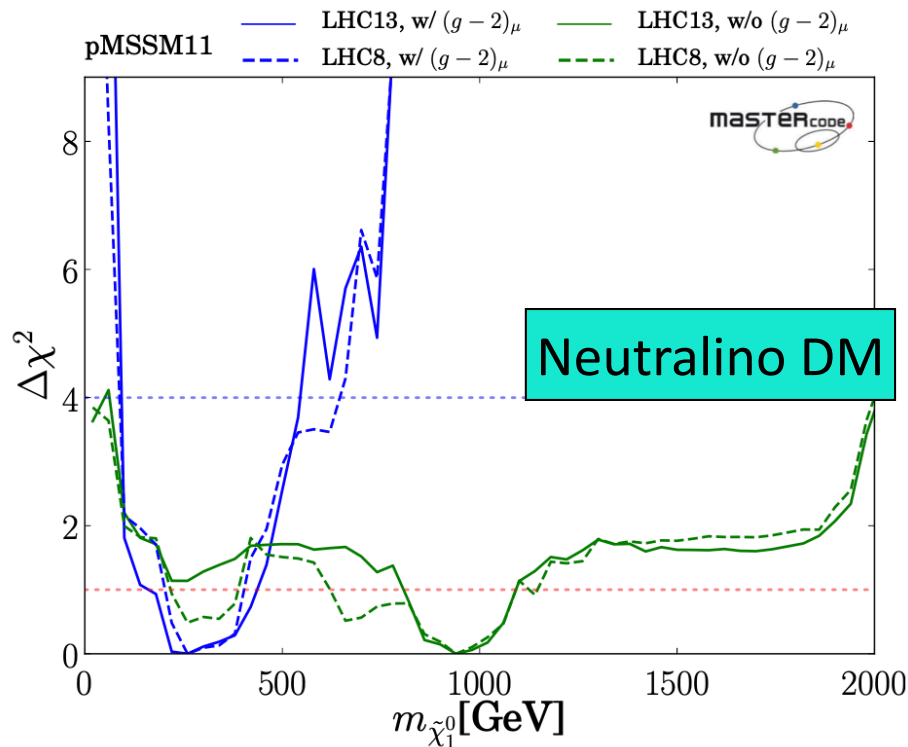


Scenario relates squark & gluino masses to sleptons and neutralino  
 Cannot accommodate BNL/FNAL result  
 Smuon masses  $\gtrsim 4$  TeV





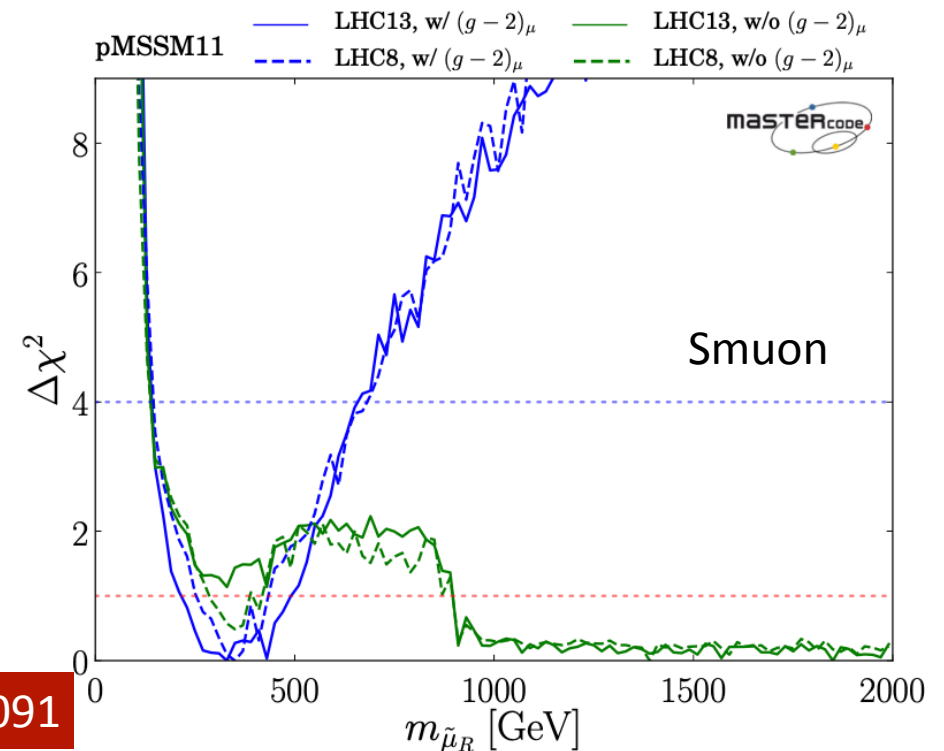
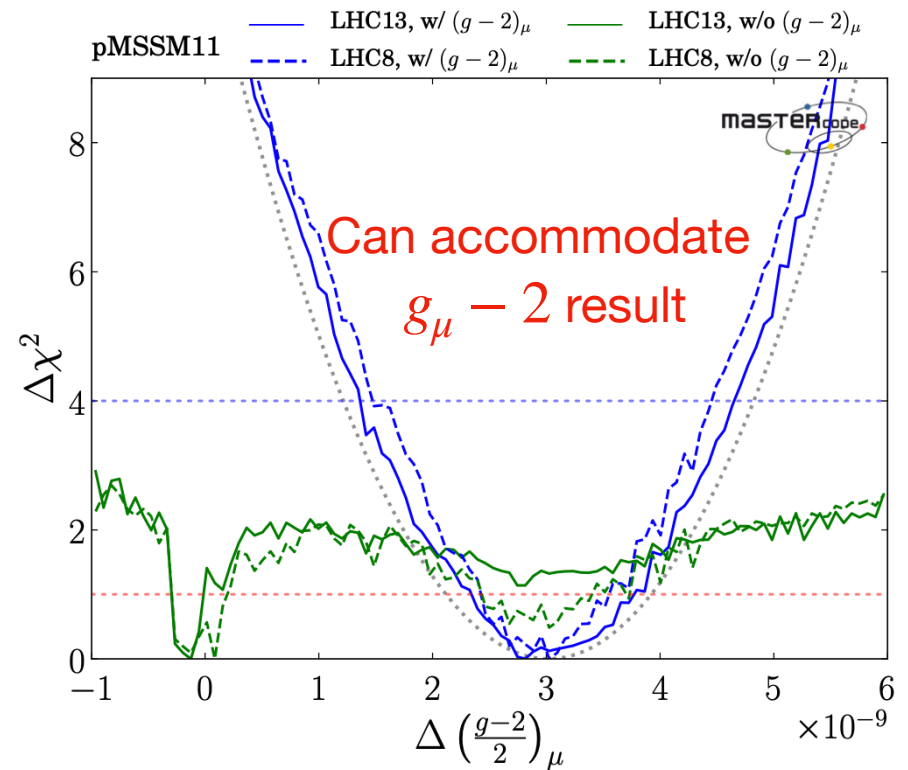
# $g_\mu - 2$ in Phenomenological Supersymmetry (pMSSM11)



No relation between squark & gluino masses and sleptons and neutralino

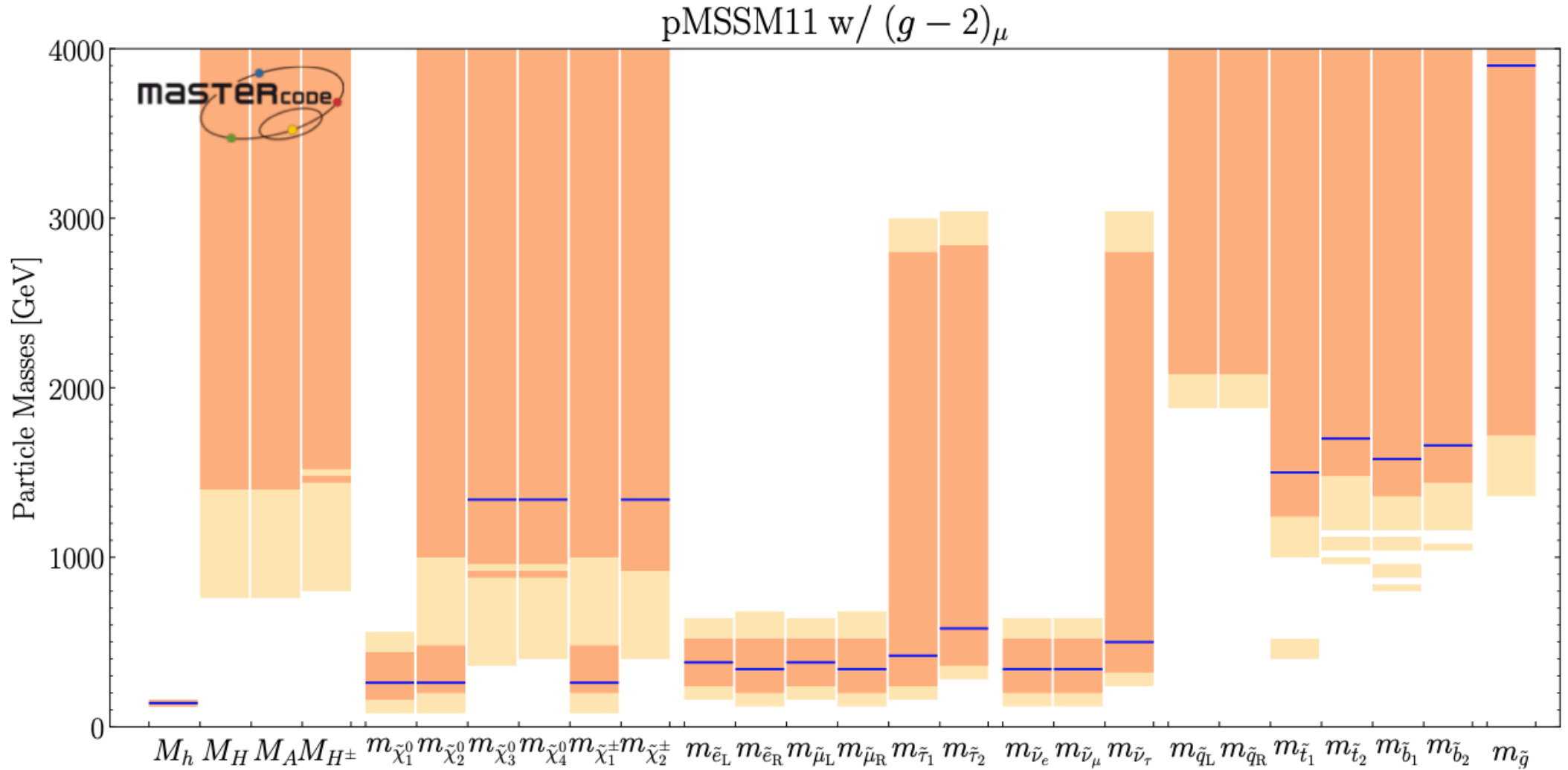
**No problem accommodating BNL/FNAL result**

Neutralino DM, smuon masses  $\sim 300/400$  GeV



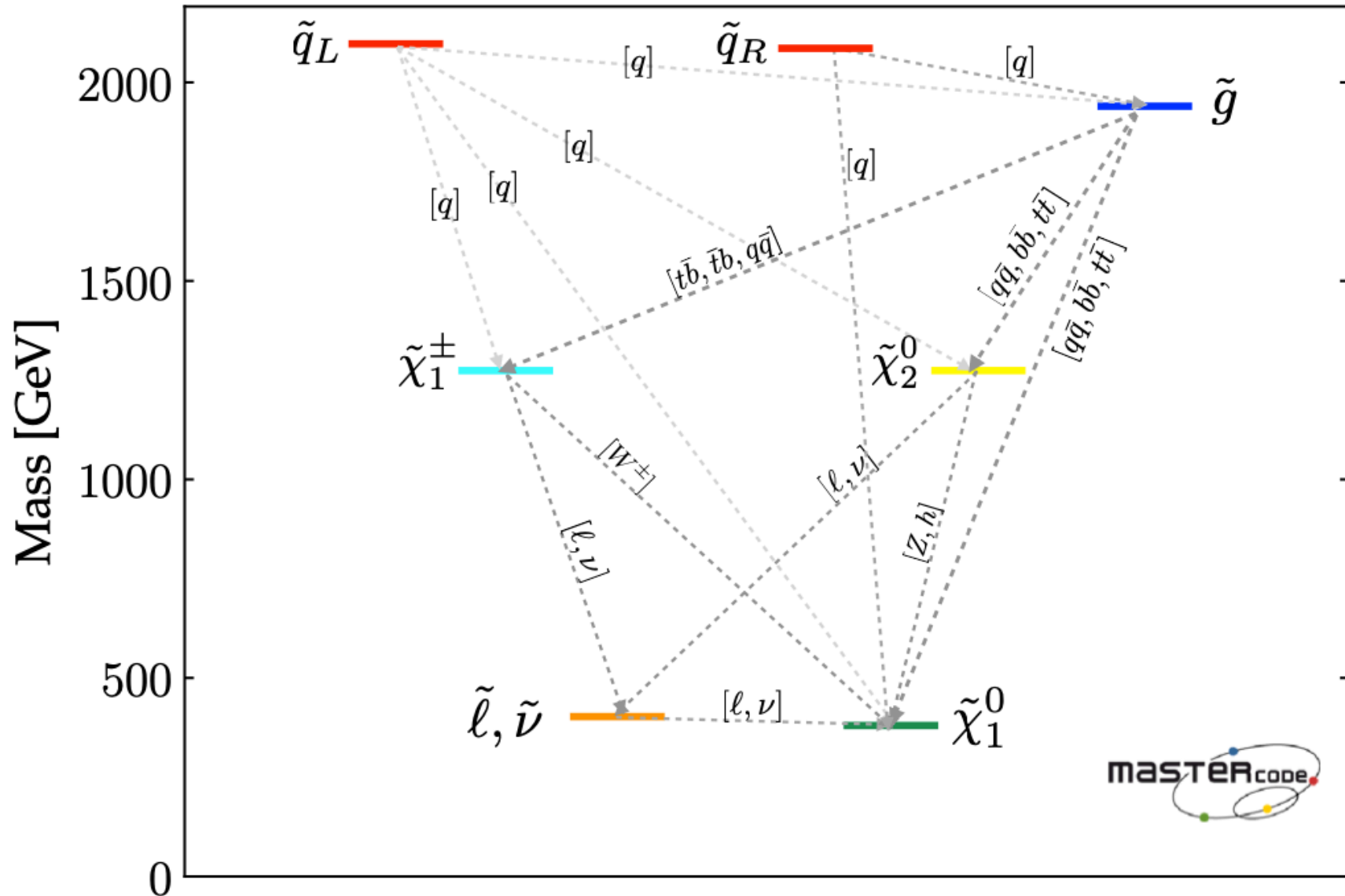


# Particle Spectrum with $g_\mu - 2$ Constraint



● Possibilities for the LHC?

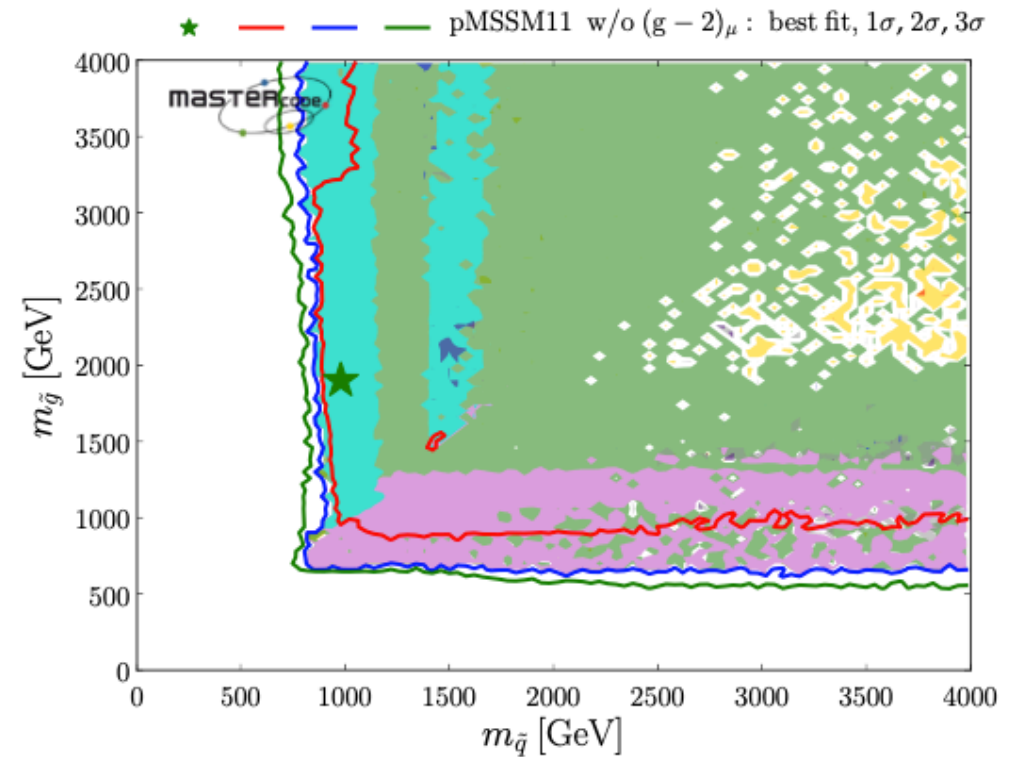
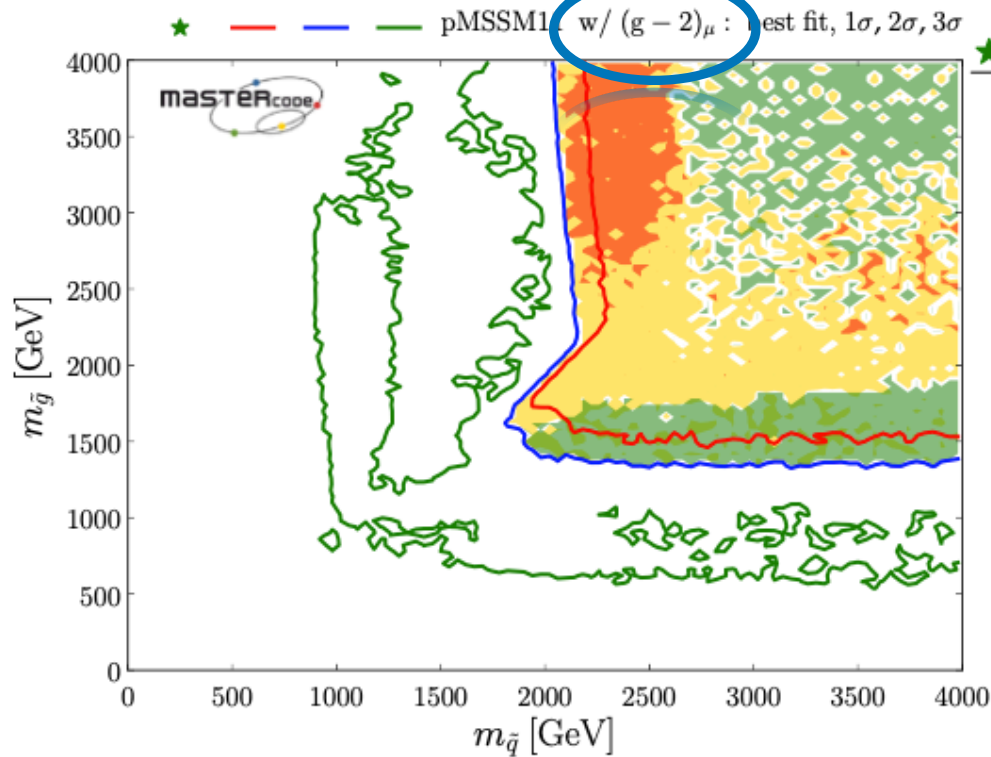
# Particle Decays with $g_\mu - 2$ Constraint



- Possibilities for the LHC?

## LHC Constraints on Squarks & Gluinos

- Squarks and gluinos **with** and without  $g_\mu - 2$  constraint



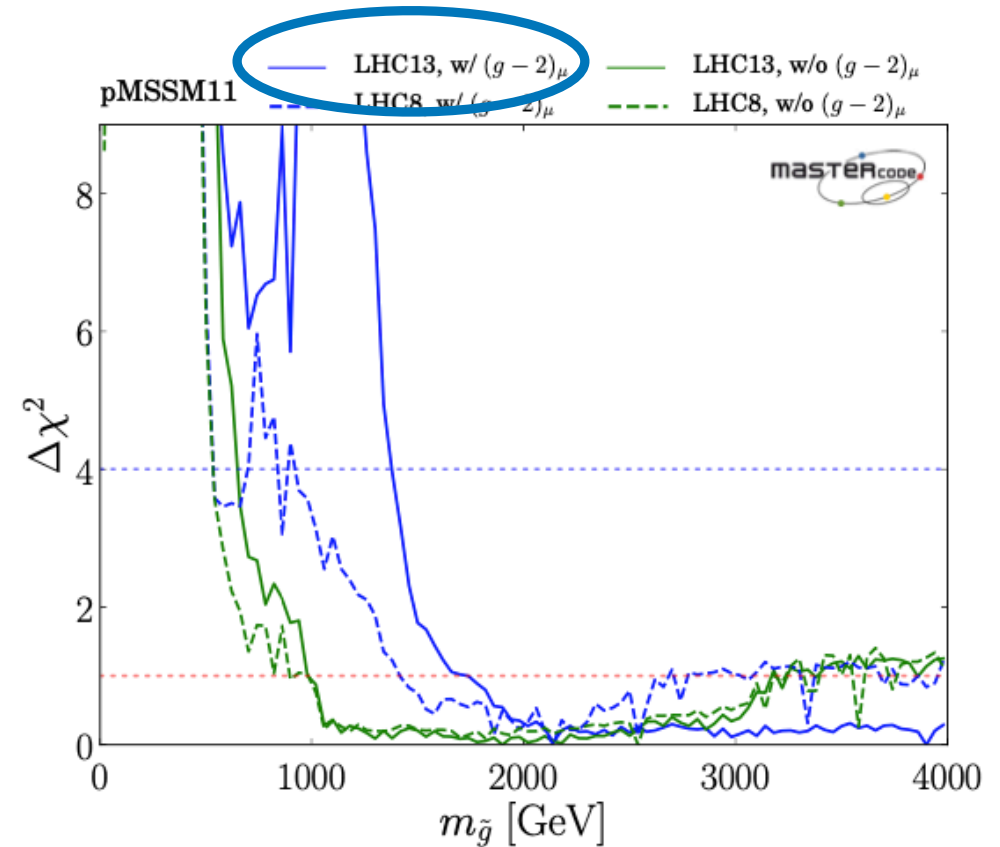
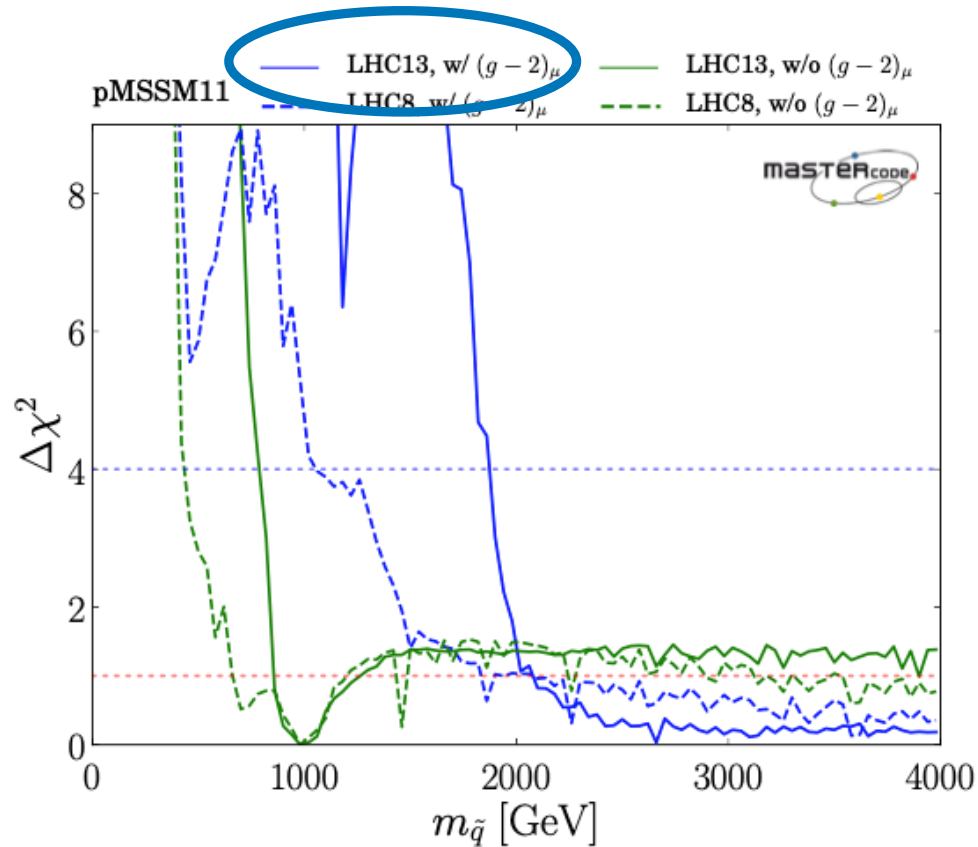
- Colour coded for dominant dark matter mechanism



- Possibilities for the LHC?**

## LHC Constraints on Squarks & Gluinos

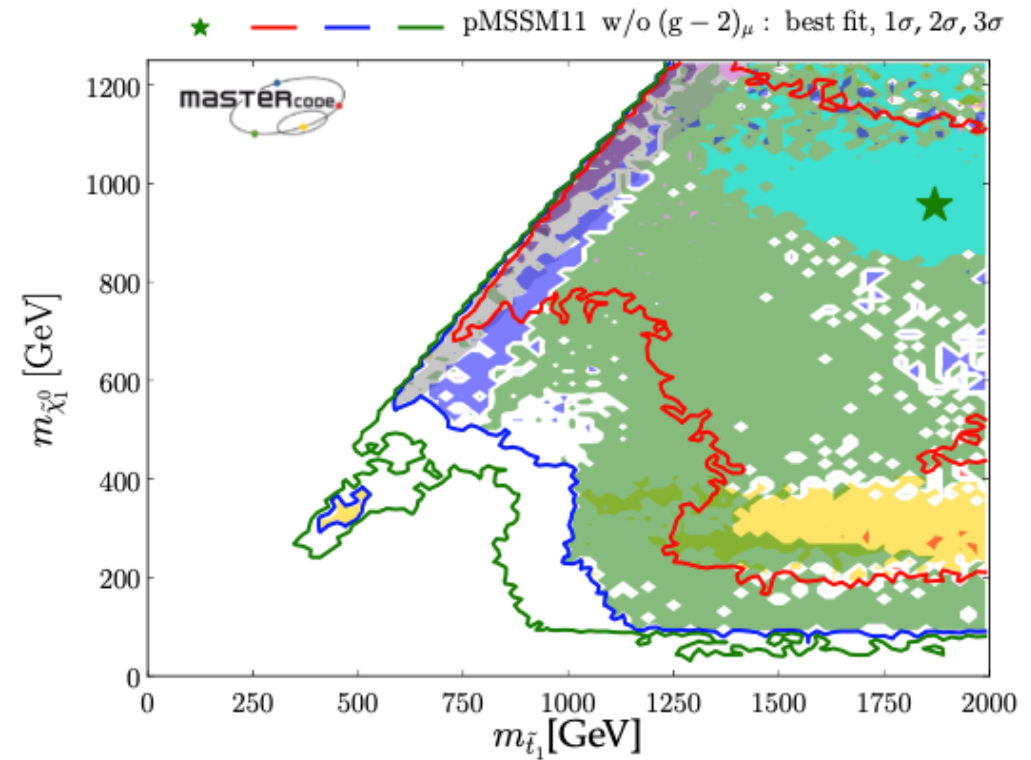
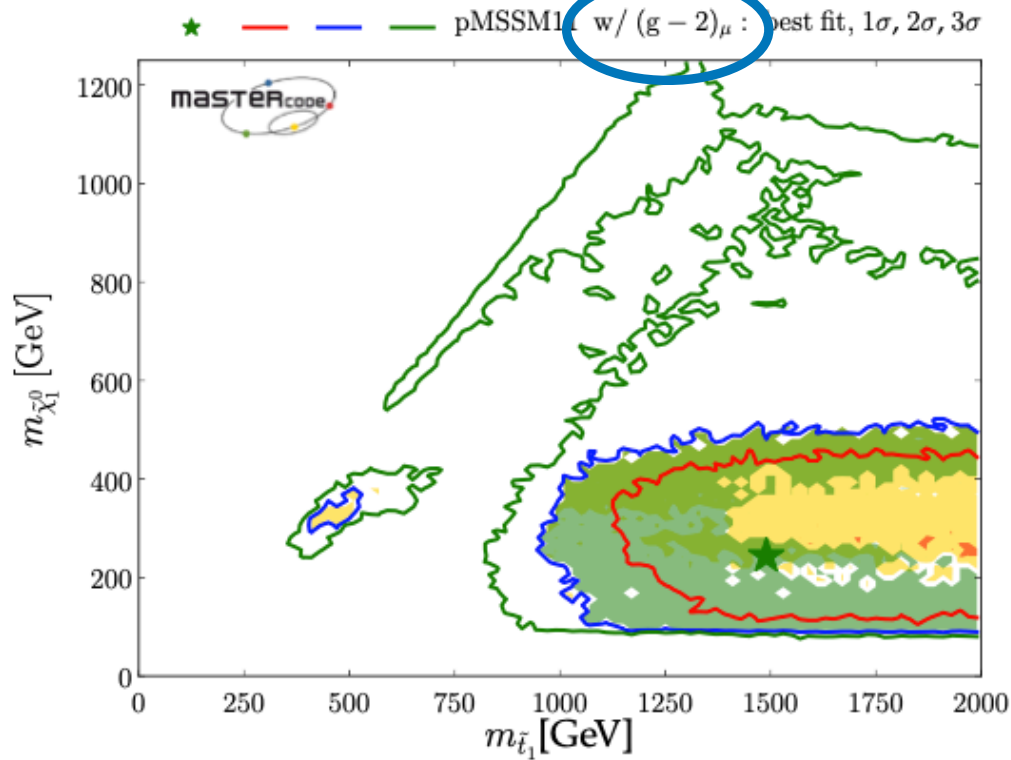
- Squarks and gluinos **with** and without  $g_\mu - 2$  constraint



- **Possibilities for the LHC?**

## LHC Constraints on Stop & Neutralino

- Stop and neutrino **with** and without  $g_\mu - 2$  constraint



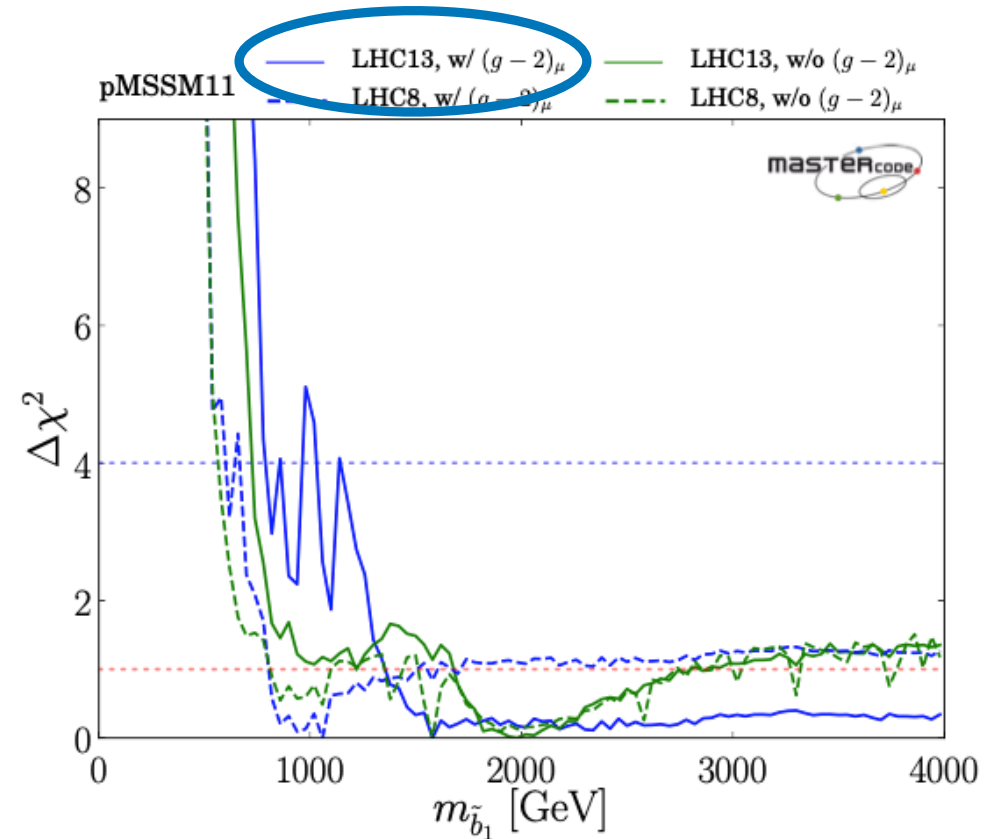
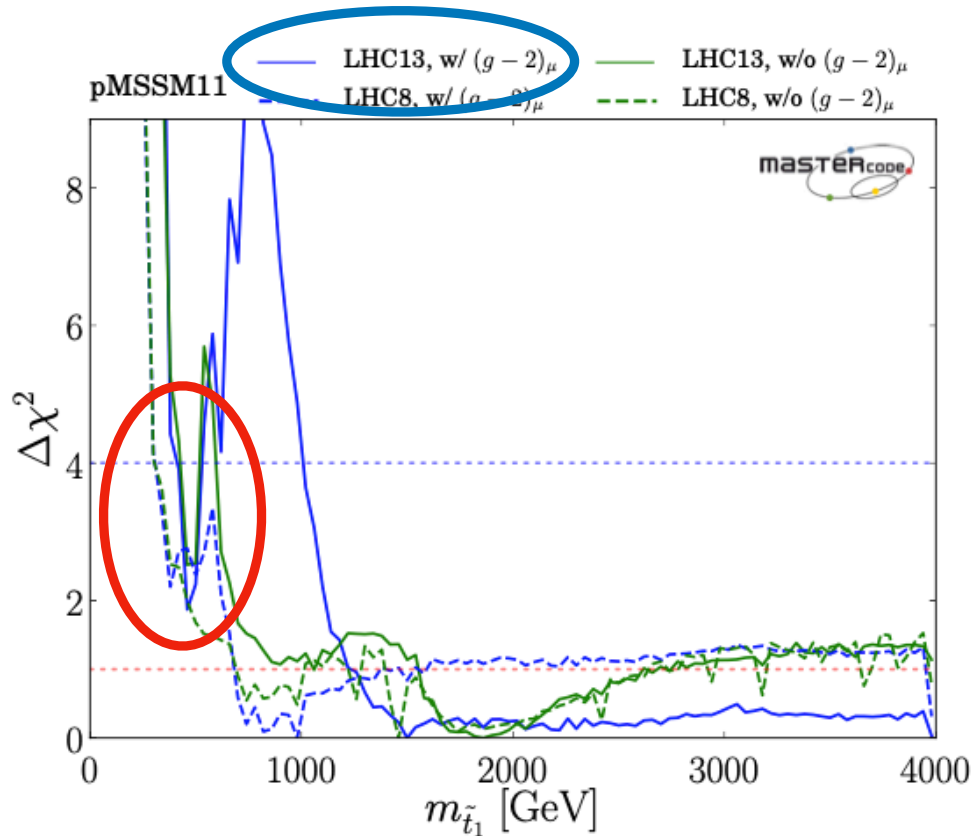
- Colour coded for dominant dark matter mechanism



- Possibilities for the LHC?**

## LHC Constraints on Stop & Bottom

- Stop and bottom **with** and without  $g_\mu - 2$  constraint

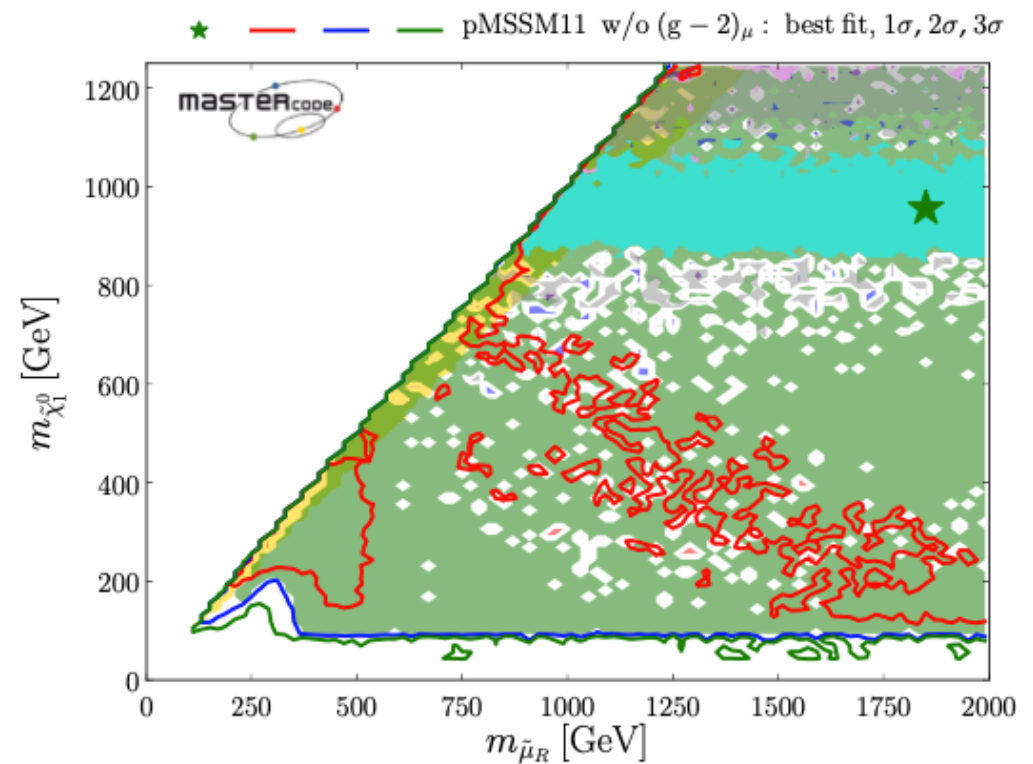
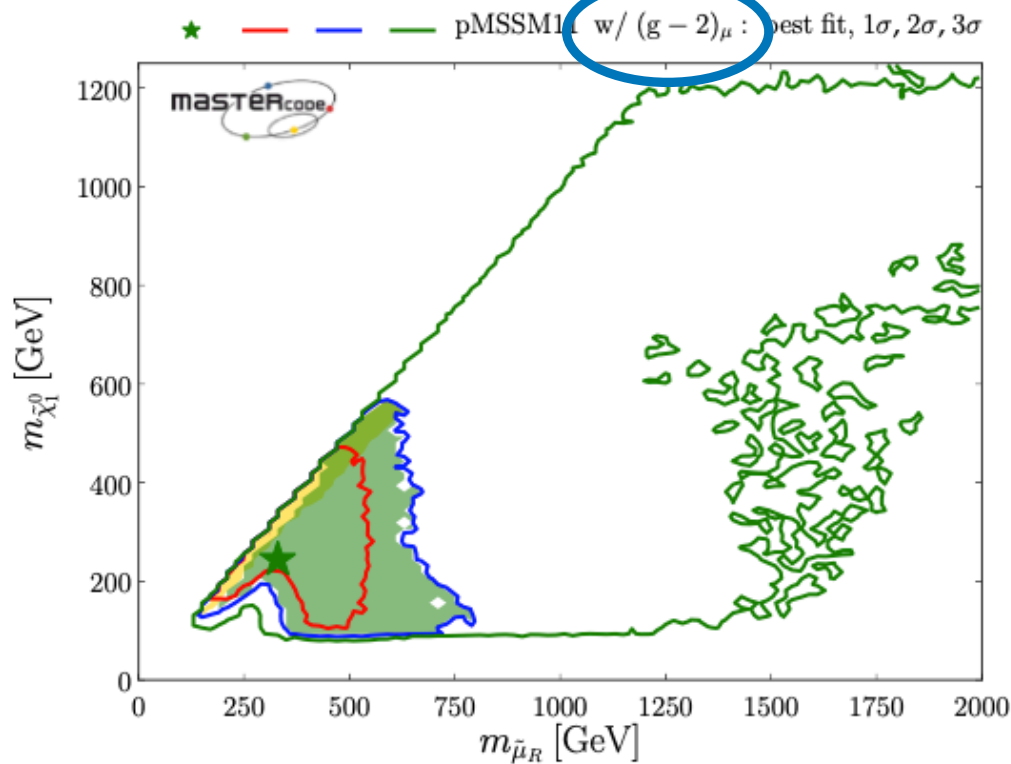


- Possibilities for the LHC?



## LHC Constraints on Smuon & Neutralino

- Smuon and neutralino **with** and without  $g_\mu - 2$  constraint

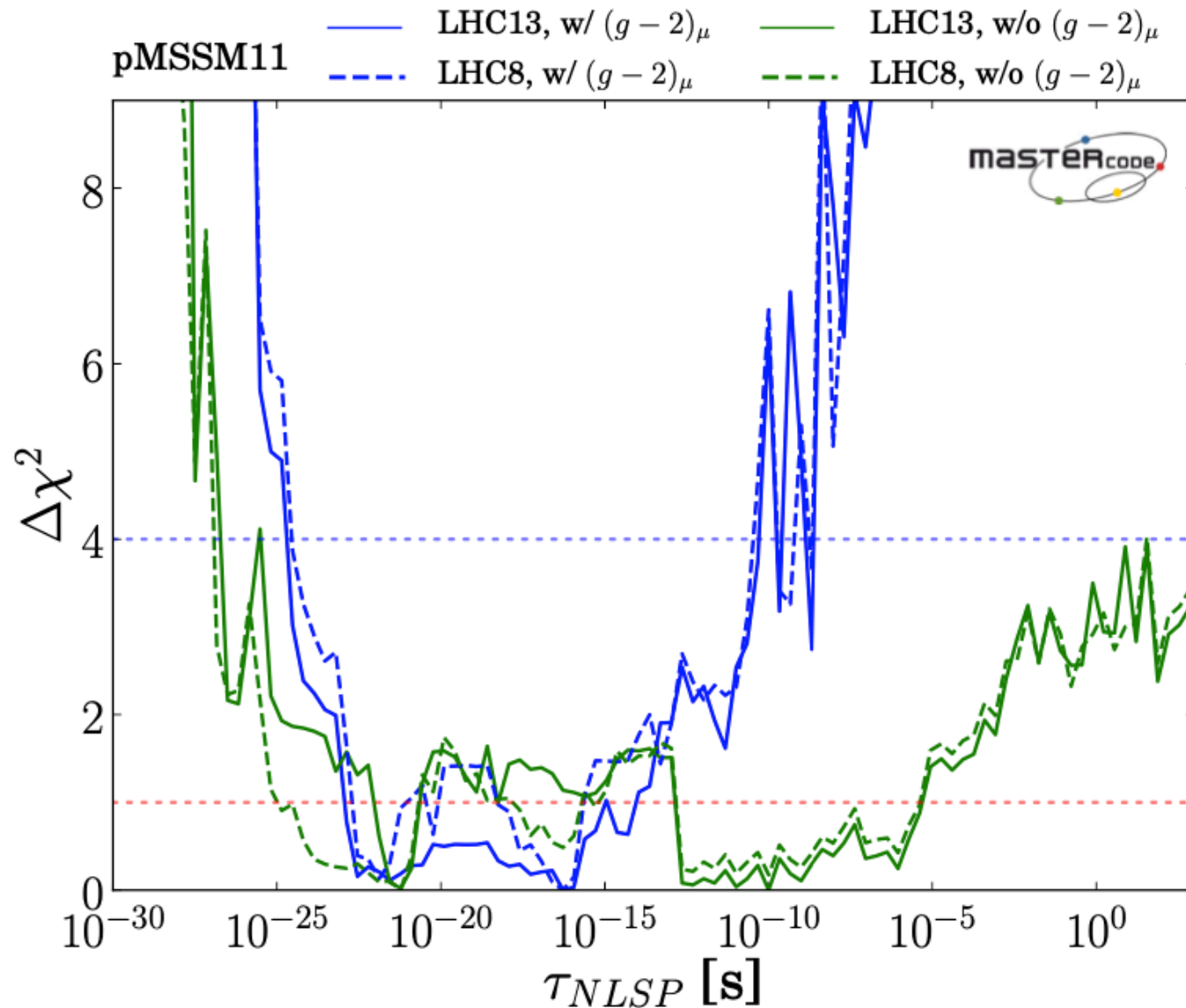


- Colour coded for dominant dark matter mechanism



- **Slepton detection at the LHC?**

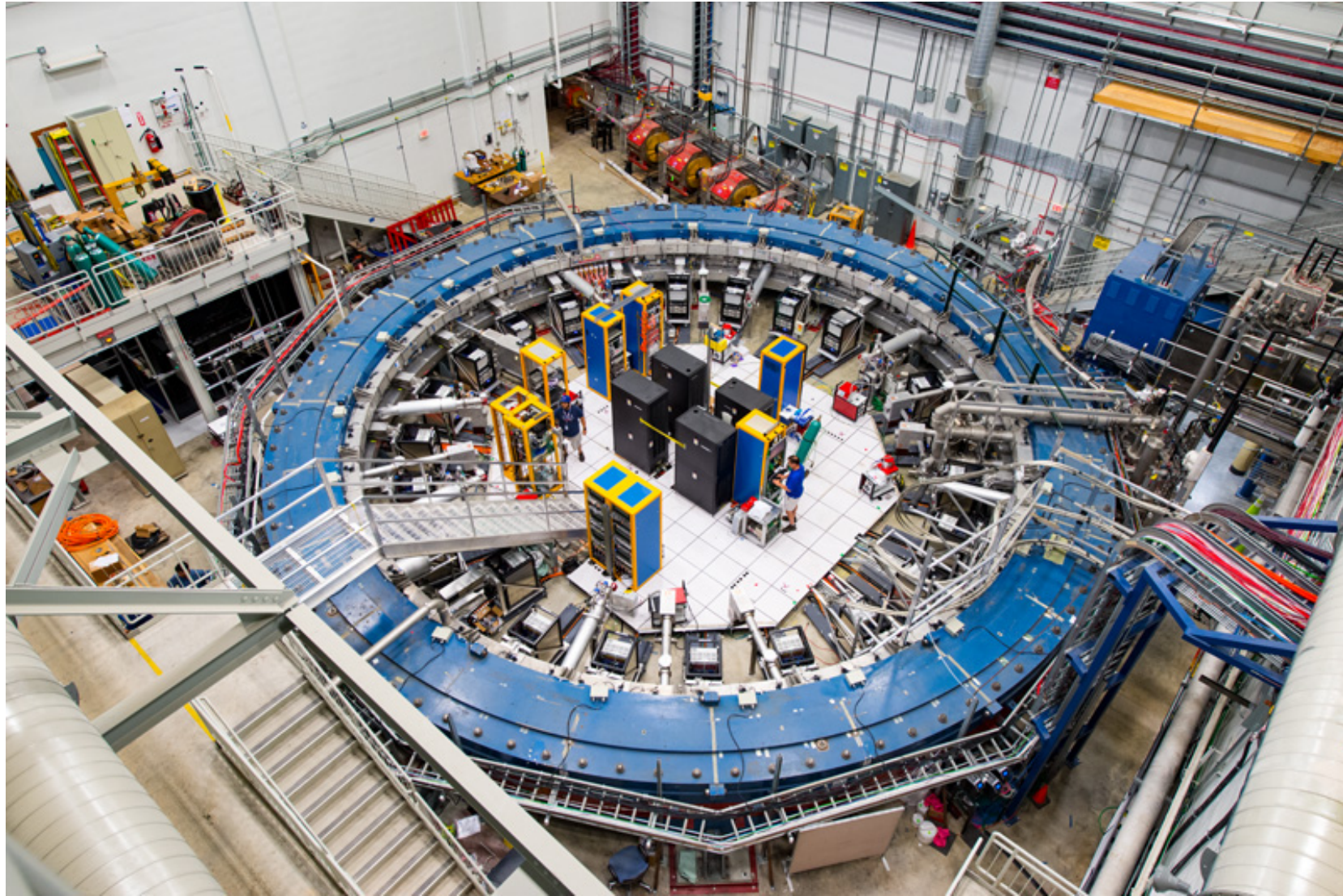
# Long-Lived NLSP?



- Possibilities for the LHC?



# Fermilab Experiment



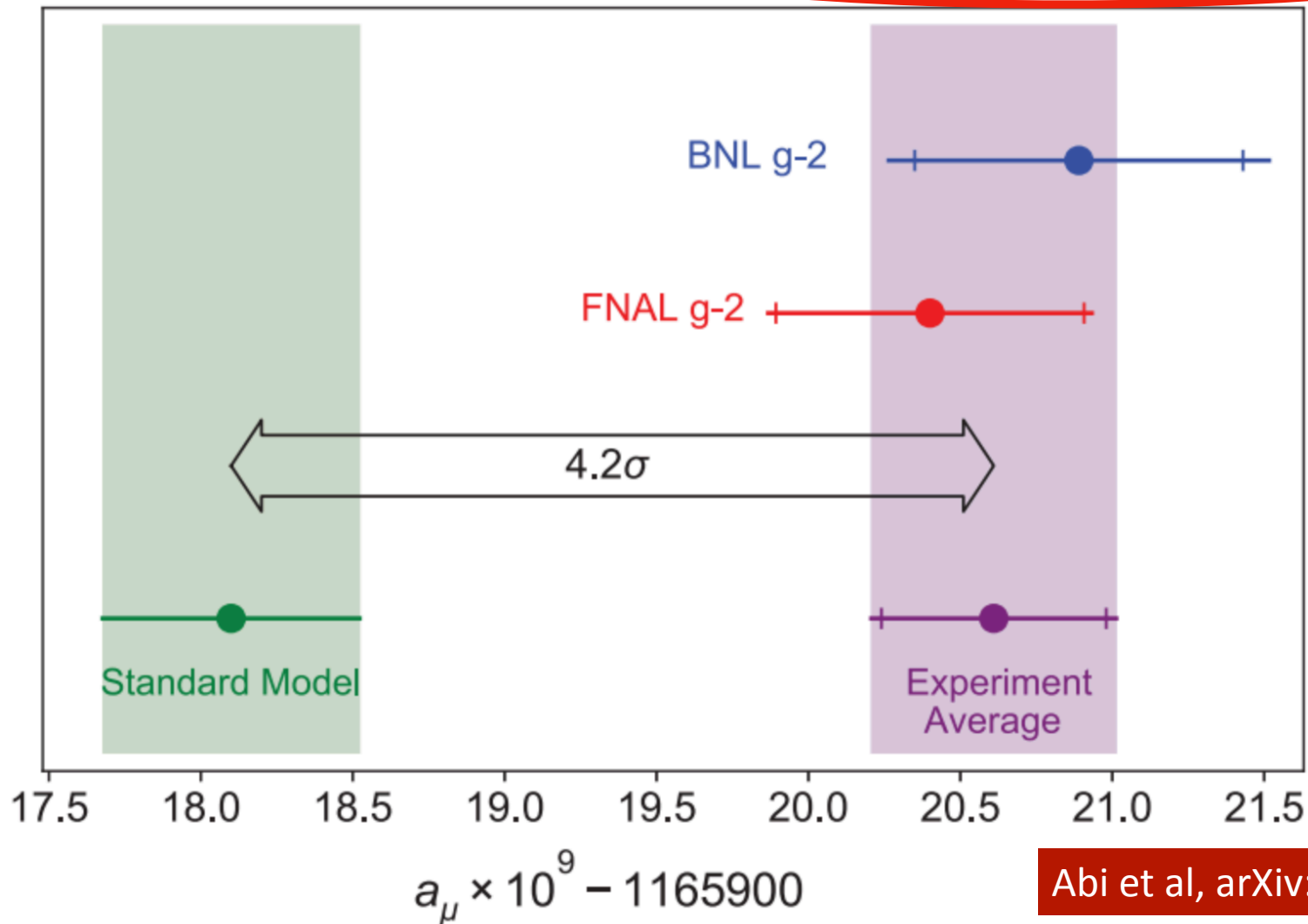
Does the magnet look familiar?

# Fermilab Measurement

FNAL result:  $a_\mu(\text{FNAL}) = 116\,592\,040(54) \times 10^{-11}$  (0.46 ppm)

Combined result:  $a_\mu(\text{Exp}) = 116\,592\,061(41) \times 10^{-11}$  (0.35 ppm)

Difference from Standard Model:  $a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}$





# Interpretation Papers

2104.05685	Vector LQ	B	Du		
5656	$L_\mu - L_\tau$	DM	Borah		
5006	$B_q - L_\mu$	B	Cen		Leptoquarks
4494	LFV	LFV	Li		
4503	Pseudoscalar	DM, H decays	Lu		Extra U(1)
4456	2HDM	DM	Arcadi		
3542	B-LSSM	H decays	Yang		Extra Higgs
3701	Leptophilic spin 0	H factory	Chun		
3839	SUSY	HL-LHC	Aboubrahim		Supersymmetry
3691	Survey	DM, LHC	Athron		
3705	Seesaw	$g_e$	Escribano		Axion
3699	Gauged 2HDM	B	Chen		
3239	SUSY	Gravitino DM	Gu		
3284	NMSSM	DM	Cao		
3262	GUT-constrained SUSY	DM, LHC	Wang		
3292	MSSM	CPV	Han		
3296	lepton mass matrix	Flavour	Calibbi		
3280	$Z_d$	Cs weak charge	Cadeddu		
3334	$E_6$ 3-3-1	H stability	Li		
3242	$\mu$ - $\tau$ -philic H	$\tau$ decays, LHC	Wang		
3259	Anomaly mediation	DM	Yin		
3245	pMSSM	DM, fine-tuning	Van Beekveld		
3274	NMSSM	DM, AMS-02 pbar	Abdughani		
3290	MSSM	DM	Cox		
3367	2HDM	V-like leptons	Ferreira		
3267	Axion	Low-scale	Buen-Abad		
3340	$L_\mu - L_\tau$	AMS-02 positrons	Zu		
3282	ALP	V-like fermions	Brdar		
3301	Lepton portal	DM	Bai		
3276	Dark axion portal	Dark photon	Ge		
3491	GmSUGRA	LHC	Ahmed		
3227	2HDM	LHC	Han		
3302	SUSY	small $\mu$	Baum		
3238	Scalar	DM, $p$ radius	Zhu		
3489	$\mu$ $\nu$ SSM	B, H decays	Zhang		
3287	pMSSM	ILC	Chakraborti		
3228	DM	B, H decays	Arcadi		

890	Radiative seesaw				Chiang
2103.13991	Scalar LQ	B, H decays			Greljo
2012.11766	DM				D'Agnolo
2012.07894	Axions				Darmé
1812.06851	Charmphilic LQ				Kowalska
2104.04458	GUT-constrained SUSY	DM			Chakraborti
5730	LQ + charged singlet	B, Cabibbo			Marzocca
6320	L-R symmetry				Boyarkin
6858	$L_\mu - L_\tau$	$\nu$ masses			Zhou
6854	D-brane	U(1), Regge			Anchordoqui
6656	vector LQ	B			Ban
7597	SUSY	LHC, landscape			Baer
7047	3HDM	Fermion masses			Carcamo
7680	Leptophilic Z'	Global analysis			Buras
8289	Custodial symmetry	Light scalar + pseudoscalar			Balkin
9205	U(1)D	Neutrino mass			Dasgupta
8819	Lepton non-universality	Naturalness			Cacciapaglia
8640	$2 \times 2 \times 1$	Higgses, heavy nus			Boyarkina
8293	Multi-TeV sleptons in FSSM	Extended H, tau decays			Altmannshofer
10114	SO(10)	Yukawa unification			Aboubrahim
7681	U(1)B-L	DUNE			Dev
10324	Gauged lepton number	Dark matter			Ma
10175	2HDM	Lighter Higgs?			Jueid
11229	LQ	Matter unification			Fileviez
15136	U(1)	HE neutrinos, H tension			Alonso
2105.00903	Anomalous 3-boson vertex	W mass			Arbuzov
7655	U(1)T3R	RK(*)			Dutta
8670	Leptoquark	$\nu$ mass, LFV			Zhang

# $g_\mu - 2$ in Supersymmetry



- Muon  $\psi_f$ , 4 neutralinos  $\psi_i$ , 2 smuons  $\phi_k$  ( $\tilde{\mu}_{L,R}$ )

$$- \mathcal{L}_{int} = \sum_{ik} \bar{\psi}_f \left( K_{ik} \frac{1 - \gamma_5}{2} + L_{ik} \frac{1 + \gamma_5}{2} \right) \psi_i \phi_k + H.c.$$

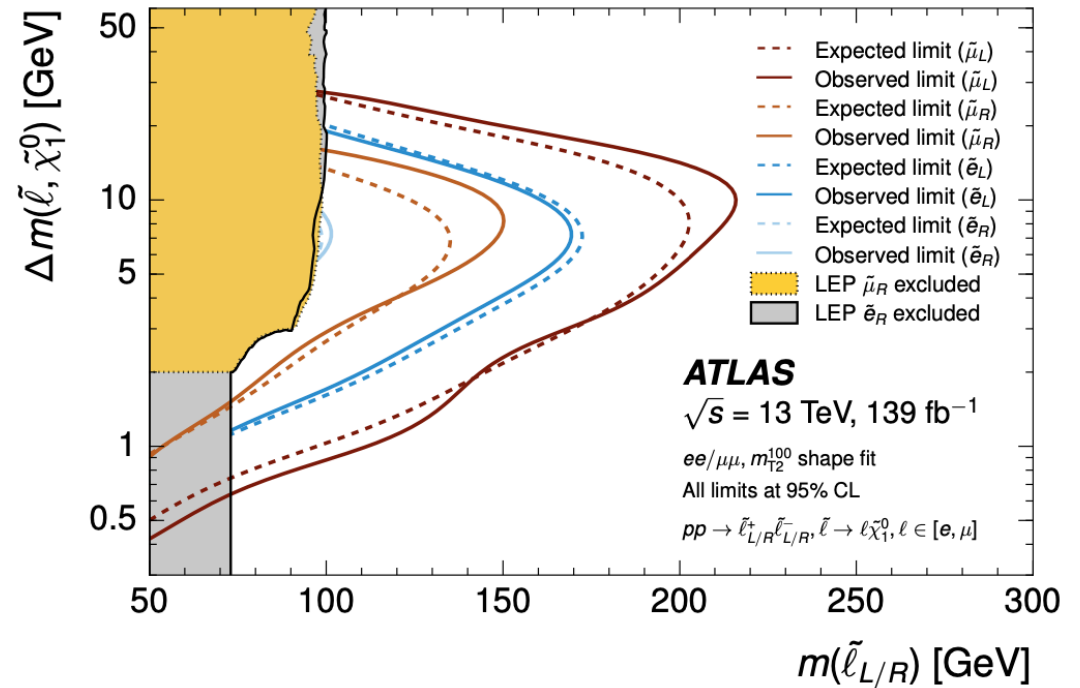
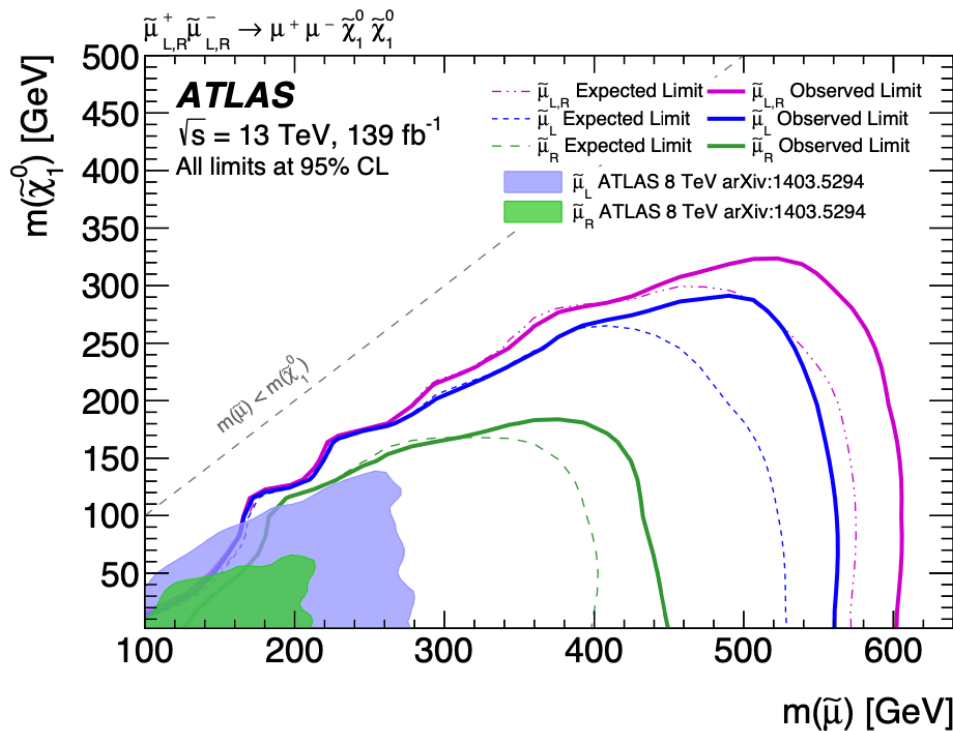
- One-loop contributions from smuon/neutralino loops:

- Left-right mixing: 
$$a_f^{11} = \sum_{ik} \frac{m_f}{8\pi^2 m_i} \text{Re}(K_{ik} L_{ik}^*) I_1\left(\frac{m_f^2}{m_i^2}, \frac{m_k^2}{m_i^2}\right)$$

- Unmixed: 
$$a_f^{12} = \sum_{ik} \frac{m_f^2}{16\pi^2 m_i^2} (|K_{ik}|^2 + |L_{ik}|^2) I_2\left(\frac{m_f^2}{m_i^2}, \frac{m_k^2}{m_i^2}\right)$$

# LHC vs Sleptons

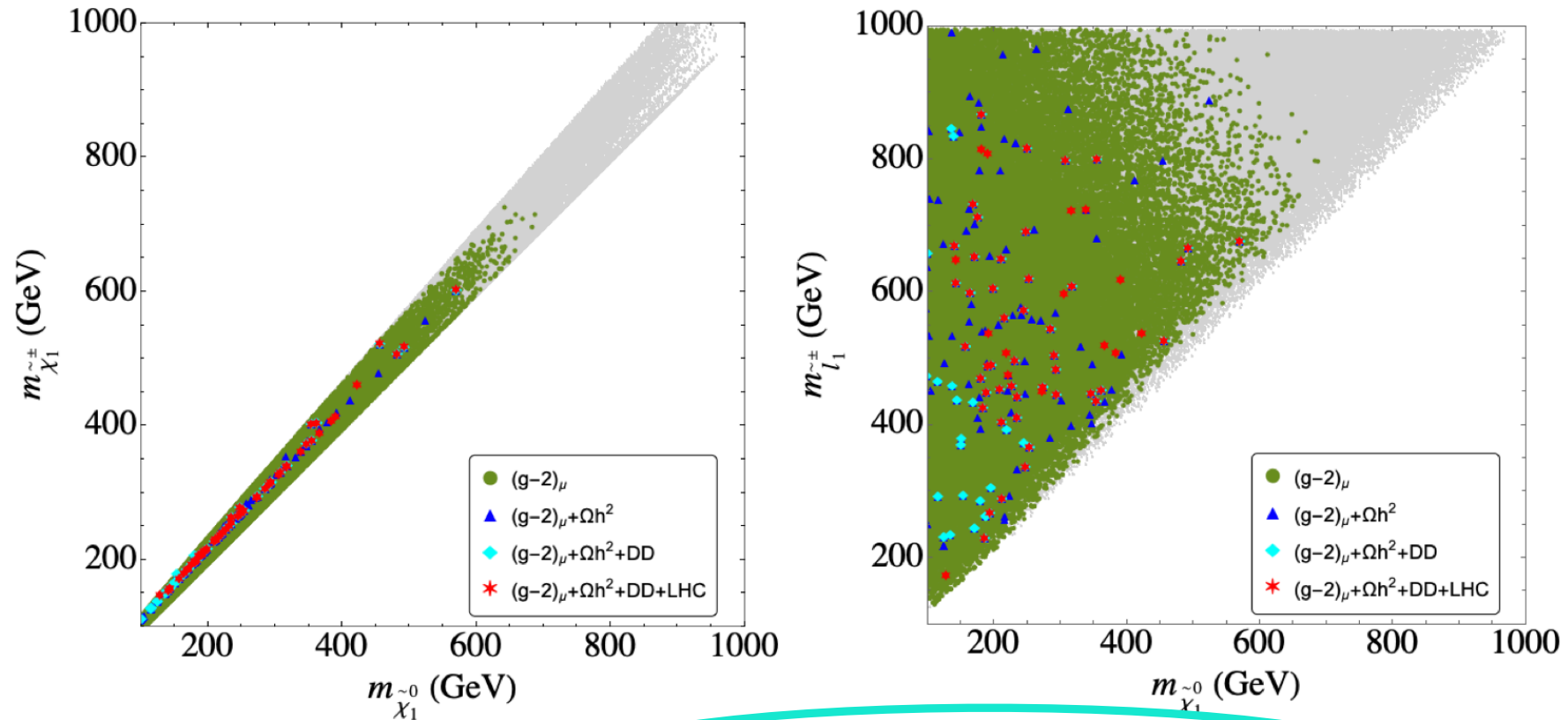
- LHC does not exclude (relatively) light electroweakly-interacting particles, e.g., sleptons



- LHC favours squarks & gluinos  $> 2 \text{ TeV}$  (but loopholes)

# Light Supersymmetry & $g_\mu - 2$

- $g_\mu - 2$ -friendly scenario with light neutralino, chargino & slepton



- Red star points include all dark matter density, direct scattering & LHC constraints

- Prospects for the LHC?

# $g_\mu = 2$ in Flipped SU(5) GUT

- Extend GUT SU(5) with additional U(1)

Antoniadis, JE, Hagelin, Nanopoulos, 1980s

- “Flipped” fermion assignments to representations:

$$\bar{f}_i(\bar{\mathbf{5}}, -3) = \{U_i^c, L_i\} \quad , \quad F_i(\mathbf{10}, 1) = \{Q_i, D_i^c, N_i^c\} \quad , \quad l_i(\mathbf{1}, 5) = E_i^c \quad , \quad i = 1, 2, 3$$

- Break GUT symmetry with 10-dimensional Higgses, electroweak symmetry with 5-dimensional Higgses:

$$H(\mathbf{10}, 1) = \{Q_H, D_H^c, N_H^c\} \quad , \quad \bar{H}(\bar{\mathbf{10}}, -1) = \{\bar{Q}_H, \bar{D}_H^c, \bar{N}_H^c\}$$

$$h(\mathbf{5}, -2) = \{T_{H_c}, H_d\} \quad , \quad \bar{h}(\bar{\mathbf{5}}, 2) = \{\bar{T}_{\bar{H}_c}, H_u\}$$

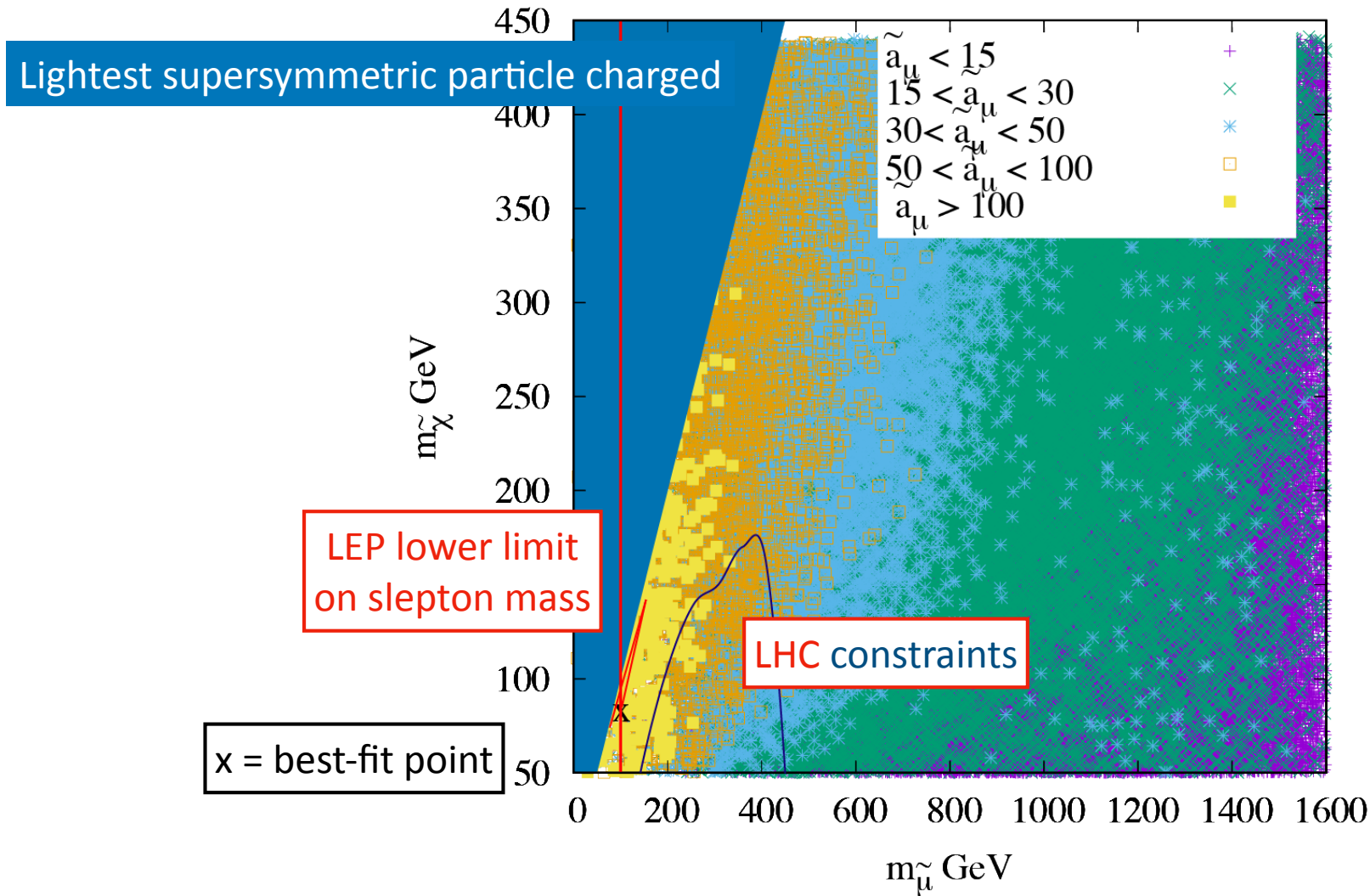
- Superpotential:

$$W = \lambda_1^{ij} F_i F_j h + \lambda_2^{ij} F_i \bar{f}_j \bar{h} + \lambda_3^{ij} \bar{f}_i \ell_j^c h + \lambda_4 H H h + \lambda_5 \bar{H} \bar{H} \bar{h} \\ + \lambda_6^{ia} F_i \bar{H} \phi_a + \lambda_7^a h \bar{h} \phi_a + \lambda_8^{abc} \phi_a \phi_b \phi_c + \mu_\phi^{ab} \phi_a \phi_b \quad ,$$

- Scan free parameters of model:

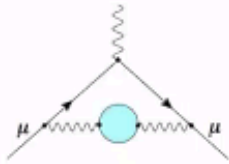
$$M_5, M_{X1}, m_{10}, m_5, m_1, \mu, M_A, A_0, \tan \beta$$

# $g_\mu = 2$ in Flipped SU(5)

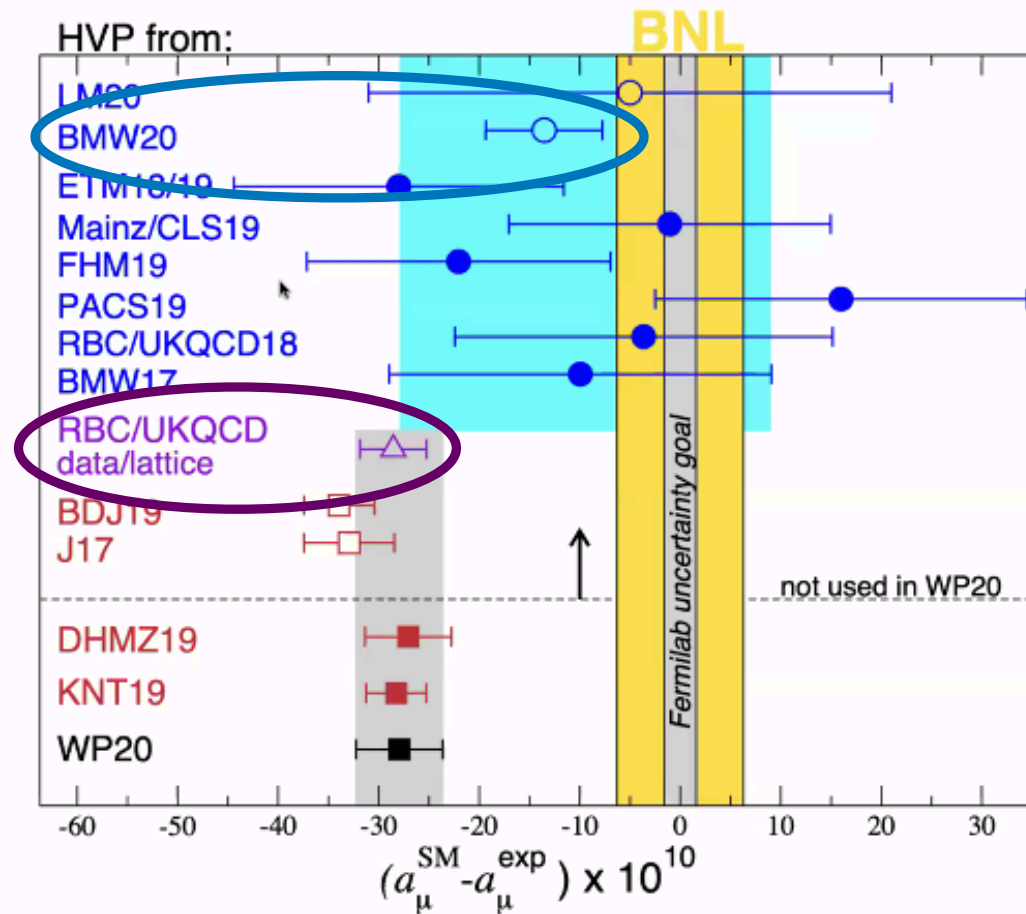




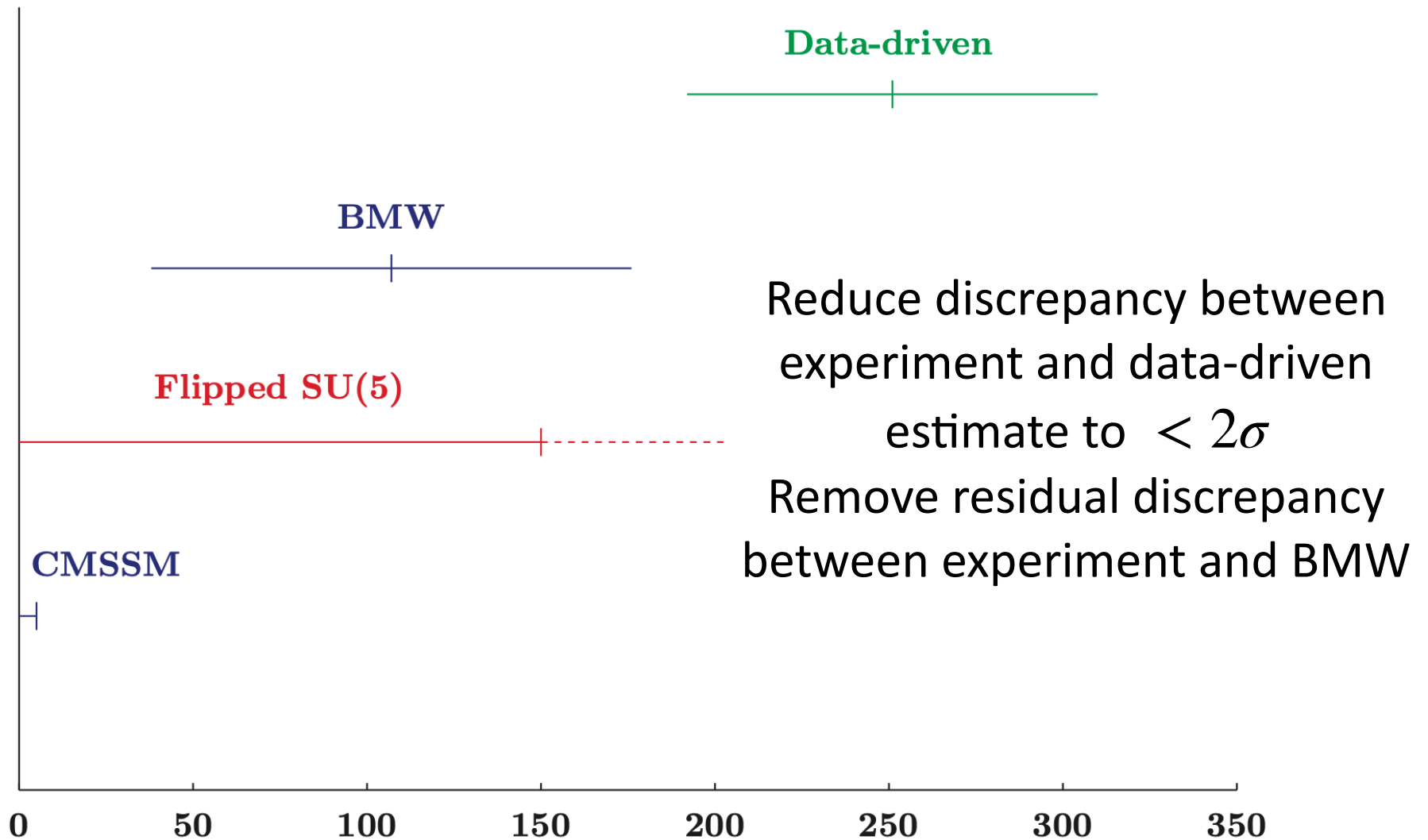
# Comparison of Calculations of Hadronic Vacuum Polarization



$$a_{\mu}^{\text{HVP}} + [a_{\mu}^{\text{QED}} + a_{\mu}^{\text{Weak}} + a_{\mu}^{\text{HLbL}}] \Rightarrow a_{\mu}^{\text{SM}}$$



# $g_\mu - 2$ in CMSSM & Flipped SU(5) vs Lattice, Data-Driven Calculation



$\Delta a_\mu$  ( $\times 10^{11}$ ): GUT models vs Standard Model calculations

# $g_\mu - 2$ in Flipped SU(5)

## Parameters & predictions at best-fit point

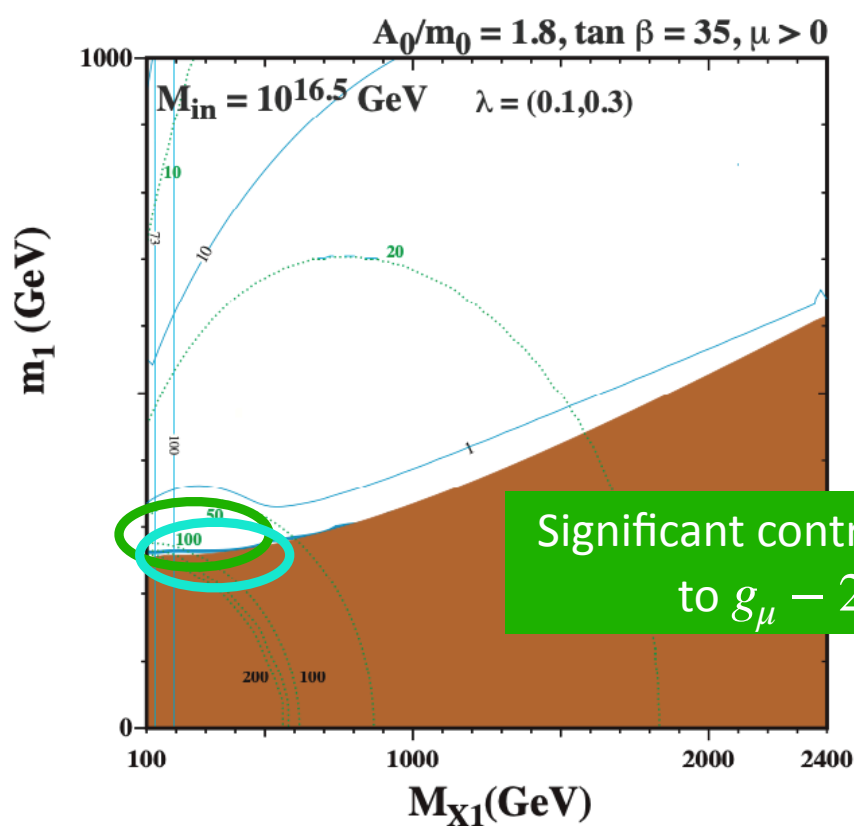
Input GUT parameters (masses in units of $10^{16}$ GeV)		
$M_{GUT} = 1.00$	$M_X = 0.79$	$V = 1.13$
$\lambda_4 = 0.1$	$\lambda_5 = 0.3$	$\lambda_6 = 0.001$
$g_5 = 0.70$	$g_X = 0.70$	$m_{\nu_3} = 0.05$ eV
Input supersymmetry parameters (masses in GeV units)		
$M_5 = 2460$	$M_1 = 240$	$\mu = 4770$
$m_{10} = 930$	$m_{\bar{5}} = 450$	$m_1 = 0$
$M_A = 2100$	$A_0/M_5 = 0.67$	$\tan \beta = 35$
MSSM particle masses (in GeV units)		
$m_\chi = 84$	$m_{\tilde{t}_1} = 4030$	$m_{\tilde{g}} = 5090$
$m_{\chi_2} = 2160$	$m_{\chi_3} = 5080$	$m_{\tilde{\chi}_1^0} = 101$
$m_{\tilde{\mu}_R} = 101$	$m_{\tilde{\mu}_L} = 1600$	$m_{\tilde{\nu}_\tau} = 101$
$m_{\tilde{q}_L} = 4470$	$m_{\tilde{d}_R} = 4250$	$m_{\tilde{u}_L} = 1600$
$m_{\tilde{t}_2} = 4410$	$m_{\tilde{b}_1} = 4170$	$m_{\tilde{d}_L} = 1600$
$m_{\chi^\pm} = 2160$	$m_{H,A} = 2100$	$m_{\tilde{b}_2} = 2100$
		$m_{H^\pm} = 2100$
Other observables		
$\Delta a_\mu = 150 \times 10^{-11}$	$\Omega_\chi h^2 = 0.13$	$m_h = 122$ GeV
Normal-ordered $\nu$ masses:	$\tau_{p \rightarrow e^+ \pi^0} _{NO} = 1.1 \times 10^{36}$ yrs	$\tau_{p \rightarrow \mu^+ \pi^0} _{NO} = 1.1 \times 10^{36}$ yrs
Inverse-ordered $\nu$ masses:	$\tau_{p \rightarrow e^+ \pi^0} _{IO} = 3.2 \times 10^{37}$ yrs	$\tau_{p \rightarrow \mu^+ \pi^0} _{IO} = 2.3 \times 10^{36}$ yrs

Opportunities to search for light smuon, neutralino at LHC  
Other sparticles too heavy?

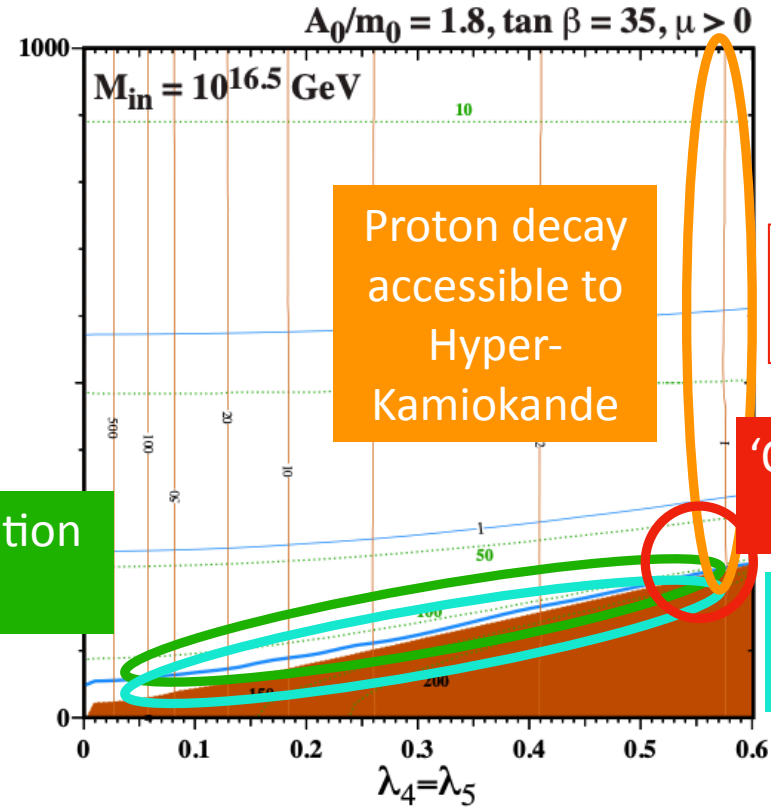
Get good CDM density without even trying!

# Flipped Supersymmetry, $g_\mu - 2$ & Dark Matter

- Exploration of  $g_\mu - 2$ , dark matter and proton decay possibilities



Soft supersymmetry-breaking parameters specific to flipped SU(5)



Sensitivity to superpotential parameters of flipped SU(5): effect on  $p \rightarrow e^+ \pi^0$  decay rate

# Supersymmetry: Nil carborundum

“Don’t let the bastards grind you down”

- Supersymmetry is (for me) still the best-motivated BSM scenario
- The BNL/FNAL measurements of  $g_\mu - 2$  may have taken us (finally) beyond the Standard Model
- Supersymmetry could resolve the tension between experiment and the data-driven SM theory estimate of  $g_\mu - 2$
- And simultaneously provide the cold dark matter
- Interesting possible LHC signatures (light sleptons, ...)





We still believe in supersymmetry

You must be joking!

# Summary

Visible matter

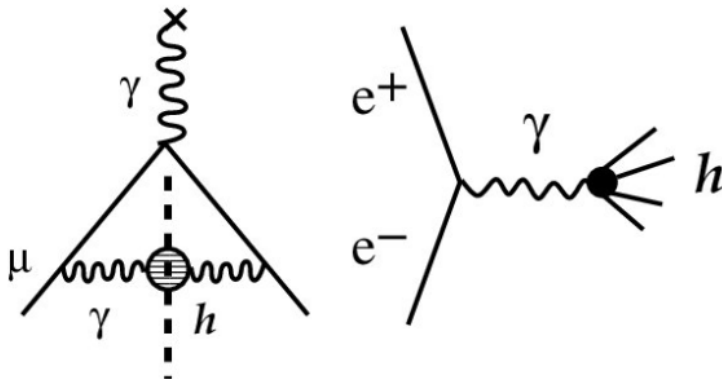
Standard Model

$g_\mu = 2$   
& DM:  
SUSY?



# Theory Initiative

- Comprehensive review of calculations of the Standard Model contributions to  $g_\mu - 2$
- Including discussion of the uncertainties
- Particularly in calculation of leading-order vacuum polarisation



Aoyama et al, arXiv:2006.04822



## The anomalous magnetic moment of the muon in the Standard Model

T. Aoyama<sup>1,2,3</sup>, N. Asmussen<sup>4</sup>, M. Benayoun<sup>5</sup>, J. Bijnens<sup>6</sup>, T. Blum<sup>7,8</sup>, M. Bruno<sup>9</sup>, I. Caprini<sup>10</sup>, C.M. Carloni Calame<sup>11</sup>, M. Cè<sup>9,12,13</sup>, G. Colangelo<sup>14,\*</sup>, F. Curciarello<sup>15,16</sup>, H. Czyż<sup>17</sup>, I. Danilkin<sup>12</sup>, M. Davier<sup>18,\*</sup>, C.T.H. Davies<sup>19</sup>, M. Della Morte<sup>20</sup>, S.I. Eidelman<sup>21,22,\*</sup>, A.X. El-Khadra<sup>23,24,\*</sup>, A. Gérardin<sup>25</sup>, D. Giusti<sup>26,27</sup>, M. Golterman<sup>28</sup>, Steven Gottlieb<sup>29</sup>, V. Gülpers<sup>30</sup>, F. Hagelstein<sup>14</sup>, M. Hayakawa<sup>31,2</sup>, G. Herdoíza<sup>32</sup>, D.W. Hertzog<sup>33</sup>, A. Hoecker<sup>34</sup>, M. Hoferichter<sup>14,35,\*</sup>, B.-L. Hoid<sup>36</sup>, R.J. Hudspith<sup>12,13</sup>, F. Ignatov<sup>21</sup>, T. Izubuchi<sup>37,8</sup>, F. Jegerlehner<sup>38</sup>, L. Jin<sup>7,8</sup>, A. Keshavarzi<sup>39</sup>, T. Kinoshita<sup>40,41</sup>, B. Kubis<sup>36</sup>, A. Kupich<sup>21</sup>, A. Kupś<sup>42,43</sup>, L. Laub<sup>14</sup>, C. Lehner<sup>26,37,\*</sup>, L. Lellouch<sup>25</sup>, I. Logashenko<sup>21</sup>, B. Malaescu<sup>5</sup>, K. Maltman<sup>44,45</sup>, M.K. Marinković<sup>46,47</sup>, P. Masjuan<sup>48,49</sup>, A.S. Meyer<sup>37</sup>, H.B. Meyer<sup>12,13</sup>, T. Mibe<sup>1,\*</sup>, K. Miura<sup>12,13,3</sup>, S.E. Müller<sup>50</sup>, M. Nio<sup>2,51</sup>, D. Nomura<sup>52,53</sup>, A. Nyffeler<sup>12,\*</sup>, V. Pascalutsa<sup>12</sup>, M. Passera<sup>54</sup>, E. Perez del Rio<sup>55</sup>, S. Peris<sup>48,49</sup>, A. Portelli<sup>30</sup>, M. Procura<sup>56</sup>, C.F. Redmer<sup>12</sup>, B.L. Roberts<sup>57,\*</sup>, P. Sánchez-Puertas<sup>49</sup>, S. Serednyakov<sup>21</sup>, B. Schwartz<sup>21</sup>, S. Simula<sup>27</sup>, D. Stöckinger<sup>58</sup>, H. Stöckinger-Kim<sup>58</sup>, P. Stoffer<sup>59</sup>, T. Teubner<sup>60,\*</sup>, R. Van de Water<sup>24</sup>, M. Vanderhaeghen<sup>12,13</sup>, G. Venanzoni<sup>61</sup>, G. von Hippel<sup>12</sup>, H. Wittig<sup>12,13</sup>, Z. Zhang<sup>18</sup>, M.N. Achasov<sup>21</sup>, A. Bashir<sup>62</sup>, N. Cardoso<sup>47</sup>, B. Chakraborty<sup>63</sup>, E.-H. Chao<sup>12</sup>, J. Charles<sup>25</sup>, A. Crivellin<sup>64,65</sup>, O. Deineka<sup>12</sup>, A. Denig<sup>12,13</sup>, C. DeTar<sup>66</sup>, C.A. Dominguez<sup>67</sup>, A.E. Dorokhov<sup>68</sup>, V.P. Druzhinin<sup>21</sup>, G. Eichmann<sup>69,47</sup>, M. Fael<sup>70</sup>, C.S. Fischer<sup>71</sup>, E. Gámiz<sup>72</sup>, Z. Gelzer<sup>23</sup>, J.R. Green<sup>9</sup>, S. Guellati-Khelifa<sup>73</sup>, D. Hatton<sup>19</sup>, N. Hermansson-Truedsson<sup>14</sup>, S. Holz<sup>36</sup>, B. Hörz<sup>74</sup>, M. Knecht<sup>25</sup>, J. Koponen<sup>1</sup>, A.S. Kronfeld<sup>24</sup>, J. Laiho<sup>75</sup>, S. Leupold<sup>42</sup>, P.B. Mackenzie<sup>24</sup>, W.J. Marciano<sup>37</sup>, C. McNeile<sup>76</sup>, D. Mohler<sup>12,13</sup>, J. Monnard<sup>14</sup>, E.T. Neil<sup>77</sup>, A.V. Nesterenko<sup>68</sup>, K. Ottnad<sup>12</sup>, V. Pauk<sup>12</sup>, A.E. Radzhabov<sup>78</sup>, E. de Rafael<sup>25</sup>, K. Raya<sup>79</sup>, A. Risch<sup>12</sup>, A. Rodríguez-Sánchez<sup>6</sup>, P. Roig<sup>80</sup>, T. San José<sup>12,13</sup>, E.P. Solodov<sup>21</sup>, R. Sugar<sup>81</sup>, K. Yu. Todyshev<sup>21</sup>, A. Vainshtein<sup>82</sup>, A. Vaquero Avilés-Casco<sup>66</sup>, E. Weil<sup>71</sup>, J. Wilhelm<sup>12</sup>, R. Williams<sup>71</sup>, A.S. Zhevlakov<sup>78</sup>

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E-mail address: [MUON-GM2-THEORY-SC@fnal.gov](mailto:MUON-GM2-THEORY-SC@fnal.gov) (G. Colangelo, M. Davier, S.I. Eidelman, A.X. El-Khadra, M. Hoferichter, C. Lehner, T. Mibe, A. Nyffeler, B.L. Roberts, T. Teubner).

<https://doi.org/10.1016/j.physrep.2020.07.006>

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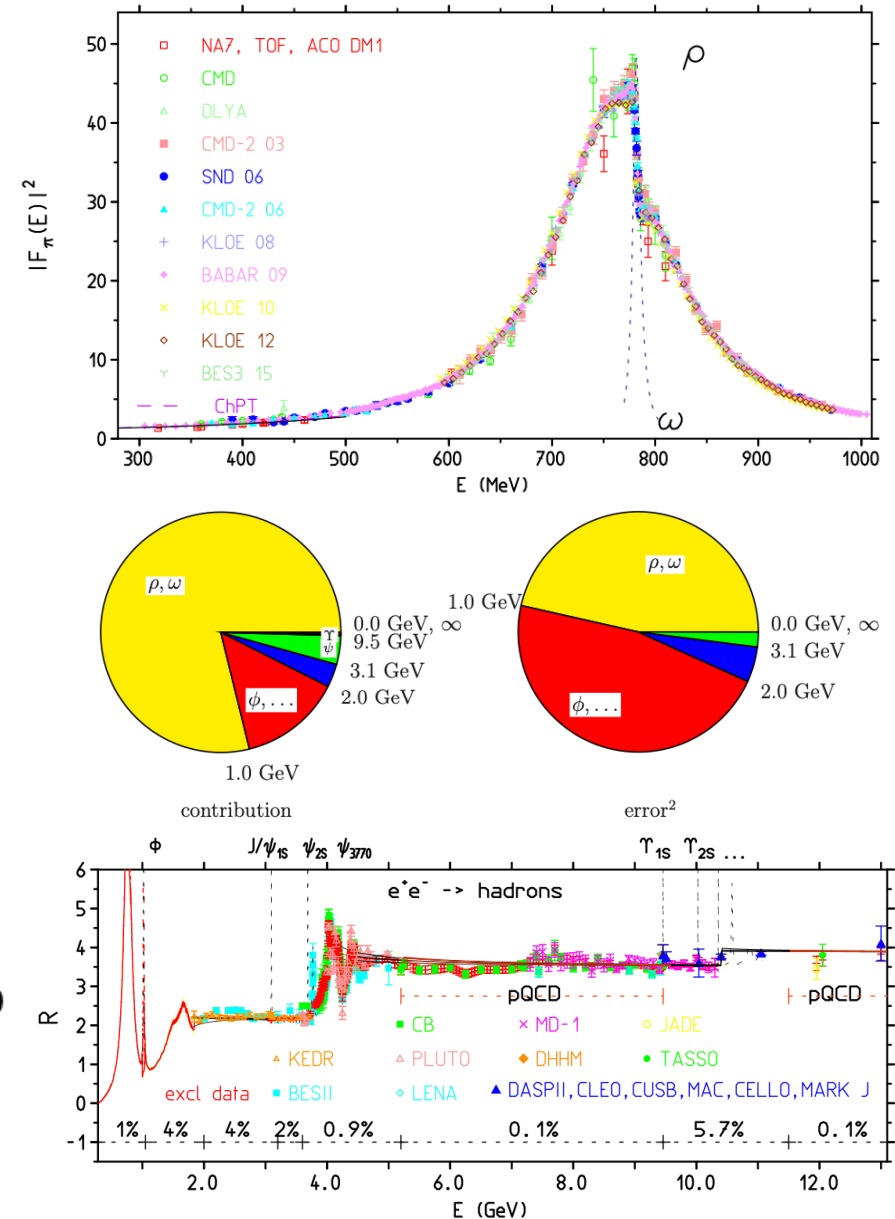


# Hadronic Vacuum Polarization

- Most important contribution is from low energies  $\lesssim 1$  GeV, dominated by  $\rho$  and  $\omega$  peaks, taking account of interference effects
- Uncertainties dominated by  $\rho$  and  $\omega$  region, and by region between 1 and 2 GeV ( $\phi$ , etc.)
- High energies under good control from perturbative QCD

$$\begin{aligned}
 a_{\mu}^{\text{HVP, LO}} &= 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10} \\
 &= 693.1(4.0) \times 10^{-10}.
 \end{aligned}$$

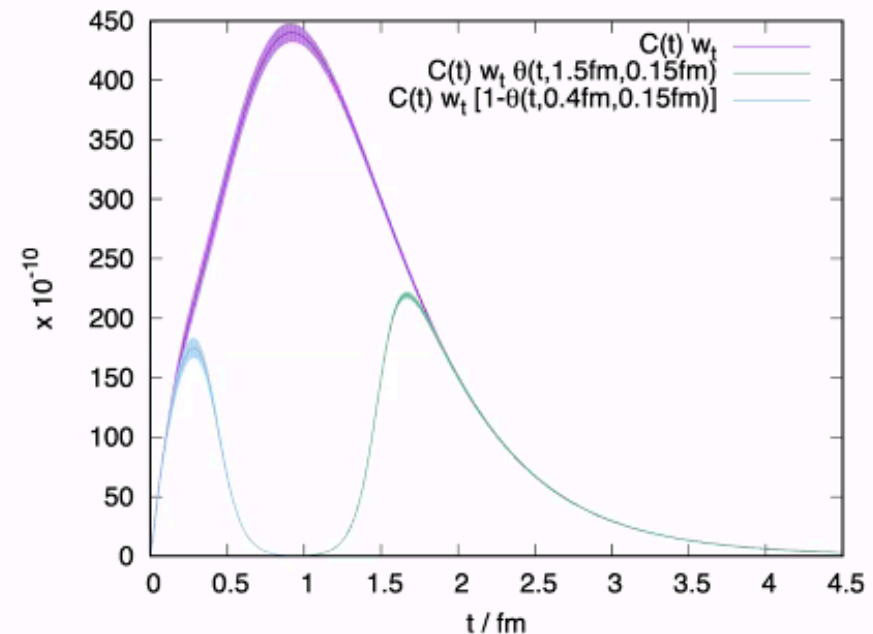
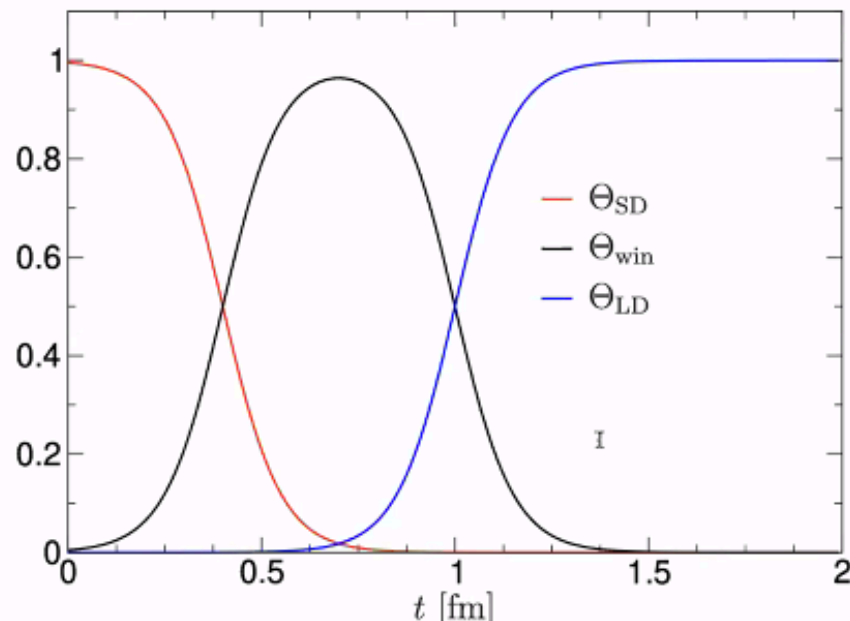
Aoyama et al, arXiv:2006.04822



# RBC/UKQCD Hybrid Method

Replace lattice data at very short and long distances  
by experimental e+e- scattering data

- Convert R-ratio data to Euclidean correlation function (via the dispersive integral) and compare with lattice results for windows in Euclidean time
- intermediate window:  
expect reduced FV effects and discretization errors



BUT

# BMW Lattice Calculation

Isospin-symmetric



Connected light

$$633.7(2.1)_{\text{stat}}(4.2)_{\text{syst}}$$



Connected strange

$$53.393(89)_{\text{stat}}(68)_{\text{syst}}$$



Connected charm

$$14.6(0)_{\text{stat}}(1)_{\text{syst}}$$



Disconnected

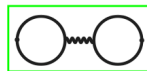
$$-13.36(1.18)_{\text{stat}}(1.36)_{\text{syst}}$$

QED isospin breaking: valence



Connected

$$-1.23(40)_{\text{stat}}(31)_{\text{syst}}$$



Disconnected

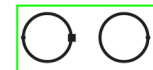
$$-0.55(15)_{\text{stat}}(10)_{\text{syst}}$$

Strong-isospin breaking



Connected

$$6.60(63)_{\text{stat}}(53)_{\text{syst}}$$



Disconnected

$$-4.67(54)_{\text{stat}}(69)_{\text{syst}}$$

QED isospin breaking: sea



Connected

$$0.37(21)_{\text{stat}}(24)_{\text{syst}}$$



Disconnected

$$-0.040(33)_{\text{stat}}(21)_{\text{syst}}$$

Other

Bottom; higher-order; perturbative

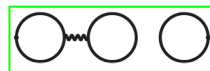
$$0.11(4)_{\text{tot}}$$

QED isospin breaking: mixed



Connected

$$-0.0093(86)_{\text{stat}}(95)_{\text{syst}}$$



Disconnected

$$0.011(24)_{\text{stat}}(14)_{\text{syst}}$$

Finite-size effects

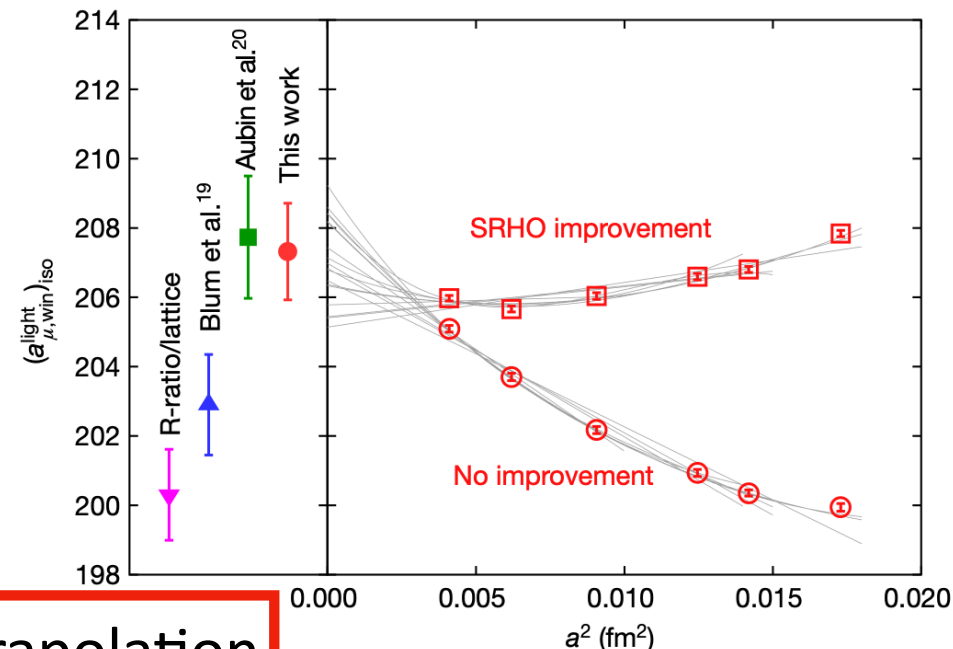
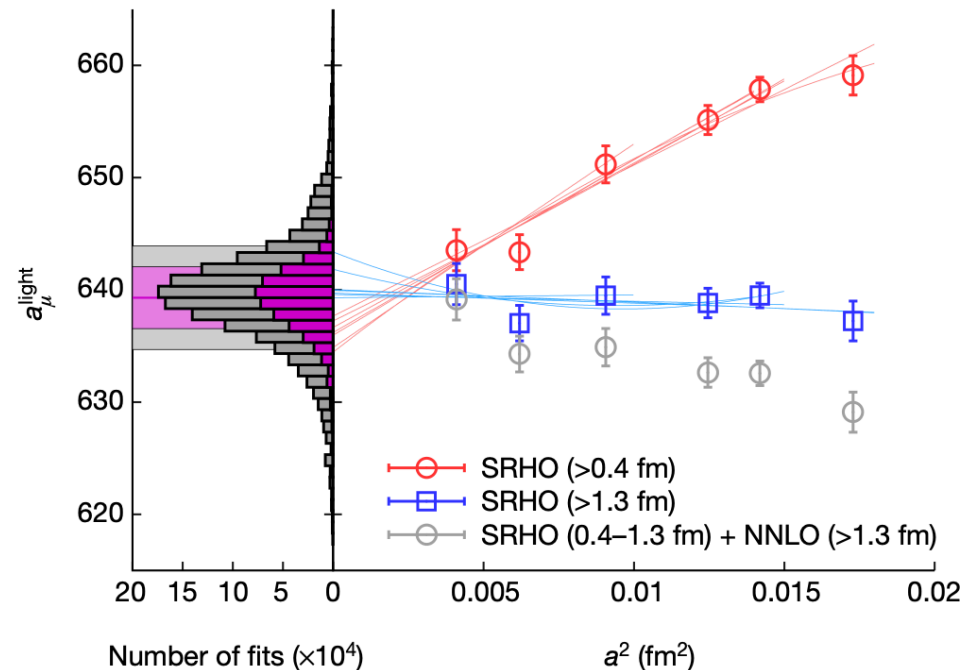
Isospin-symmetric

$$18.7(2.5)_{\text{tot}}$$

Isospin-breaking

$$0.0(0.1)_{\text{tot}}$$

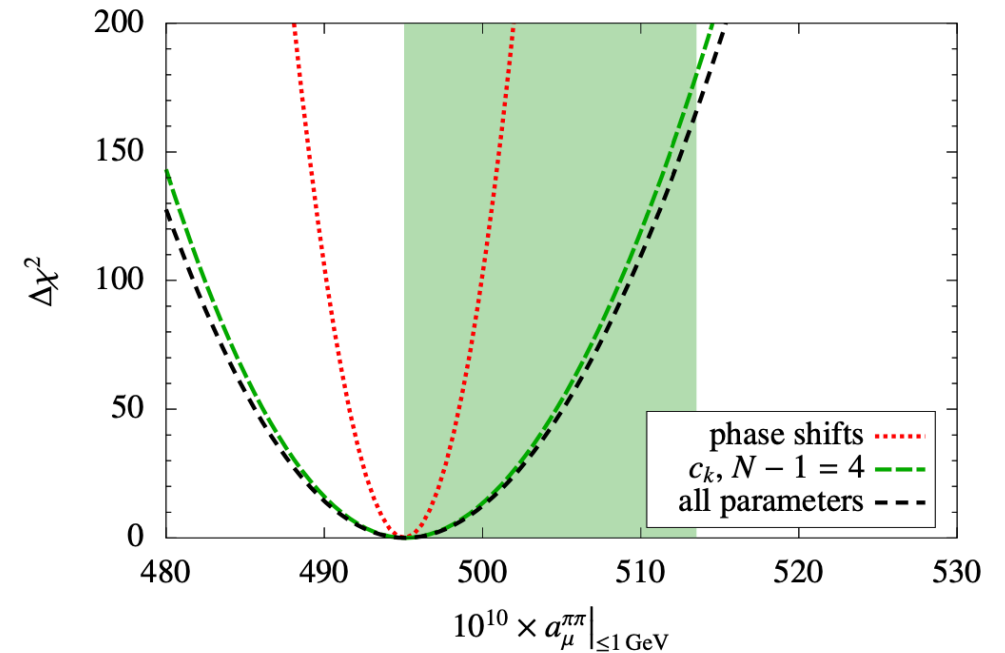
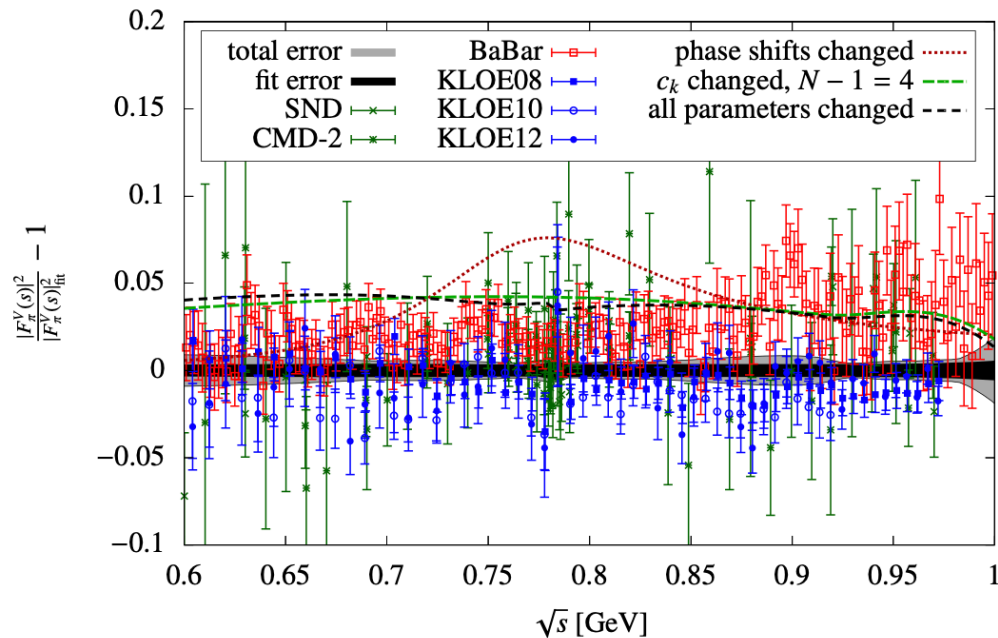
$$a_{\mu}^{\text{LO-HVP}} (\times 10^{10}) = 707.5(2.3)_{\text{stat}}(5.0)_{\text{syst}}(5.5)_{\text{tot}}$$



High statistics, accurate continuum extrapolation

$$a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{BMW}} = 107(70) \times 10^{-11}$$

# How to Accommodate BMW?



- Analyticity and unitarity constrain increase in  $\pi^+\pi^-$  cross section  $< 1$  GeV
- Maximum allowed conflicts with data, does not change greatly prediction for  $a_\mu$
- Increase in cross section at higher energies affects electroweak observables