Top-Pair Events with B-hadrons at the LHC

Michał Czakon, **Terry Generet**, Alexander Mitov, René Poncelet
arXiv:2102.08267

LHC TOP WG Open Meeting

December 1, 2021
RWTH Aachen University

Table of Contents

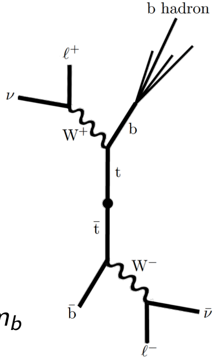
Top-pairs with B-Hadrons

- Process considered:
  $p\,p \to t(\to B\ W^+ + X)\ \bar{t}(\to \bar{b}\ W^-)$
  $\quad\quad\quad\quad\quad\quad\quad\quad \hookrightarrow \ell^+\nu_\ell \quad\quad\quad\quad\quad \hookrightarrow \ell^-\bar{\nu}_\ell$

- Measurements of B-hadrons very precise
  $\Rightarrow$ high-precision top-mass determination

- High top mass $\Rightarrow$ small power corrections in m_b

- Production of hadrons is a non-perturbative effect

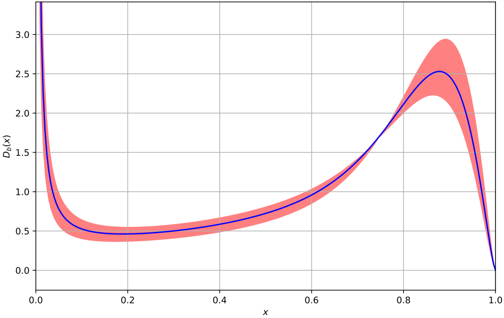


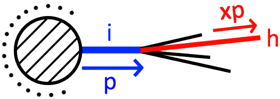
Introduction to Fragmentation

- Idea: describe production of hadrons using two steps
  1. The production of partons (gluon and quarks) using perturbation theory
  2. The (non-perturbative) fragmentation of these partons into the observed hadrons
- Transition parton $\longrightarrow$ hadron in the final state
- Hadron's momentum is measurable (parton's is not)
- Mathematically similar to transition hadron $\longrightarrow$ parton in the initial state

Fragmentation Functions

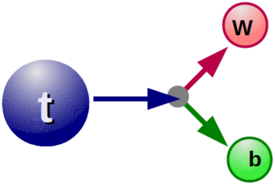
- "Probability distribution" to find a hadron h with a fraction x of the parton i's momentum: $D_{i\to h}(x)$
- Only considers longitudinal kinematics; i, h massless
- Non-perturbative: fitted to data
- Scale dependent
- Analogous to PDFs
- No parton showers used

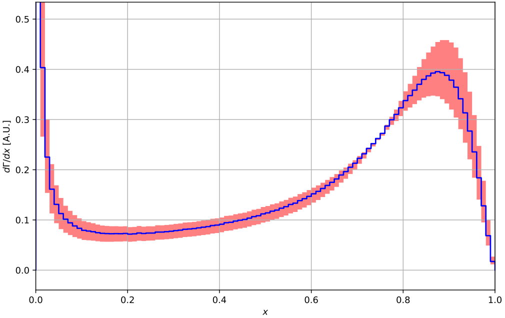




Example: Top-decay to a B-hadron at LO

- $t \longrightarrow B\ W^{+} + X$
- Partonically at LO:
  $t \longrightarrow b\ W^{+}$





Perturbative Fragmentation Functions: Introduction

- Need to fit many parameters (one function per parton)
- Reduction possible for heavy flavours using perturbative fragmentation functions (PFFs) *Mele and Nason (1991)*
- Heavy-flavoured hadrons contain heavy quarks
- The heavy-quark mass satisfies $m_Q \gg \Lambda_{\mathrm{QCD}}$
- $\Rightarrow$ Production of heavy quarks can be described perturbatively
- $\Rightarrow$ Split fragmentation into production of heavy quark and fragmentation of heavy quark into hadron

Reduction of Non-Perturbative Parameters

- Split fragmentation function into a non-perturbative FF (NPFF) and PFFs:

$$D_{i\to h} = D_{i\to Q} \otimes D_{Q\to h}$$

- $D_{i\to Q}$ calculable $\Rightarrow$ only need to fit $D_{Q\to h}$ (single function)
- Without PFFs: gluon FF poorly constrained by e^+e^--colliders
- $\Rightarrow$ Large uncertainties at the LHC

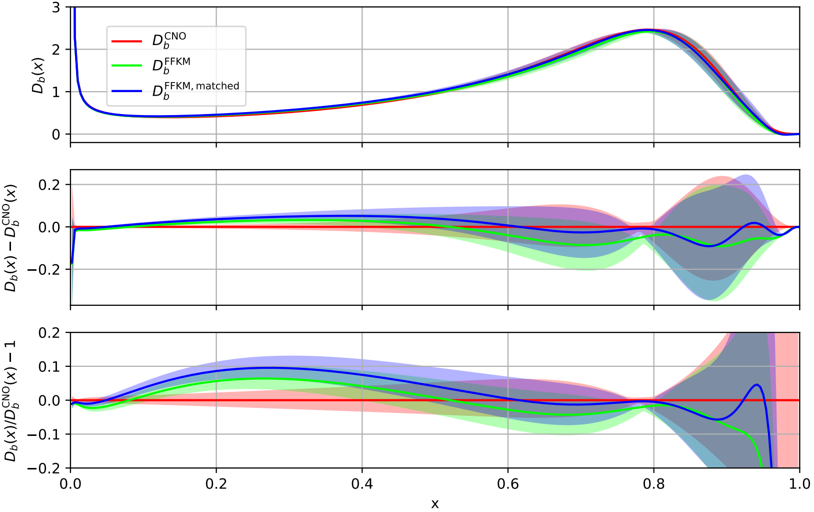
Fragmentation Functions Used in the Paper

- At present: no fits based on PFF approach available at NNLO
- Three different FF sets based on three different compromises
- Two based on NNLO calculation within SCET/HQET

  M. Fickinger, S. Fleming, C. Kim and E. Mereghetti (2016)
- One based on NLO calculation within PFF approach

  M. Cacciari, P. Nason and C. Oleari (2006)
- $\ln \frac{\mu_{Fr}^2}{m_Q^2}$ is resummed at NNLL, $\ln(1-x)$ at (NN)NLL.

Fragmentation Functions Used in the Paper



The Software

- Calculations were performed using C++ library STRIPPER
- Many NNLO firsts over the years. Recently:
  - Three-jet production at the LHC *Czakon, Mitov, Poncelet (2021)*
  - Diphoton + jet at the LHC *Chawdhry, Czakon, Mitov, Poncelet (2021)*
  - Exact top-mass effects in Higgs production at the LHC *Czakon, Harlander, Klappert, Niggetiedt (2021)*
  - Top-pairs with B-hadrons at the LHC *Czakon, T.G., Mitov, Poncelet (2021)*
  - W + c-jet at the LHC *Czakon, Mitov, Pellen, Poncelet (2020)*
  - ...
- This work: first implementation of fragmentation in a general code for NNLO cross sections
- Fully general implementation; not limited to cases presented in this talk

Isolated Top Decay: Setup

- Previously considered through NLO

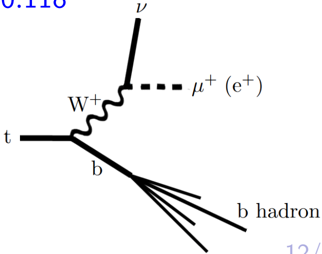
  S. Biswas, K. Melnikov and M. Schulze (2010)

- On-shell W^+ (narrow width approximation)

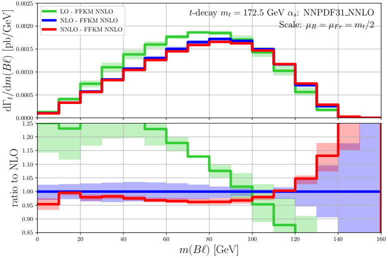
- Parameters:
  $m_t = 172.5$ GeV, $m_W = 80.385$ GeV, $\Gamma_W = 2.0928$ GeV,
  $m_b = (4.66$ GeV, 4.75 GeV$)$, $\alpha_s(M_Z) = 0.118$

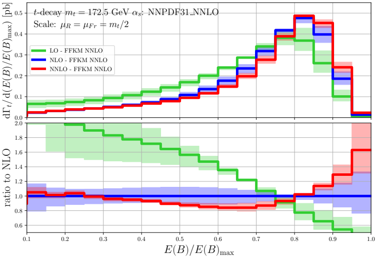
- 7-point scale variation with
  central scales $\mu_R = \mu_{Fr} = m_t/2$

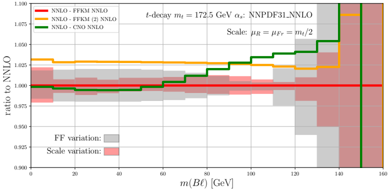
- Single cut: $E(B) > 5$ GeV

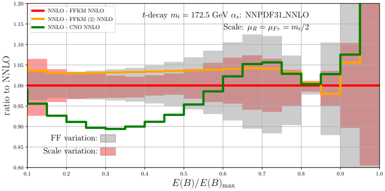


Isolated Top Decay: Plots









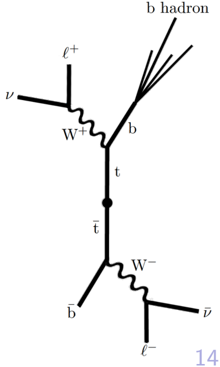
Top-Pair Events with B-hadrons at the LHC: Setup

- Previously studied at NLO

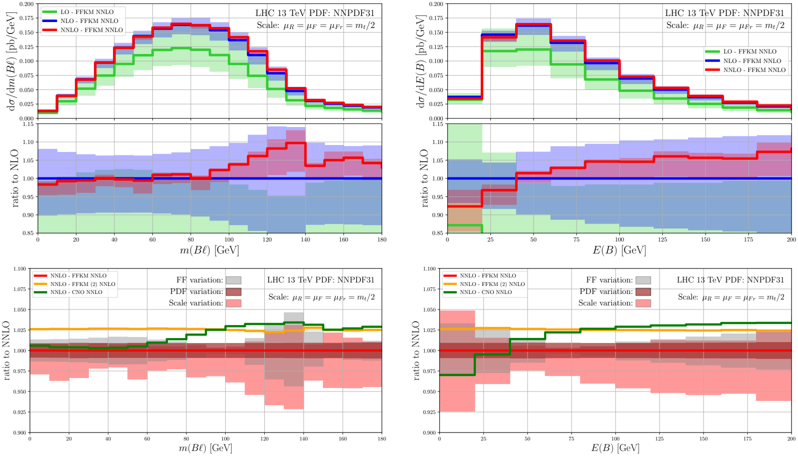
  A. Kharchilava (2000), S. Biswas, K. Melnikov and M. Schulze (2010)

  K. Agashe, R. Franceschini and D. Kim (2013), K. Agashe, R. Franceschini, D. Kim and M. Schulze (2016)

- On-shell W^+ (narrow width approximation)
- Parameters as before
- 15-point scale variation with central scales $\mu_R = \mu_F = \mu_{Fr} = m_t/2$ and $1/2 \leq \mu_i/\mu_j \leq 2$
- PDF set: NNPDF3.1
- $p_T(B) > 10$ GeV and $|\eta(B)| < 2.4$

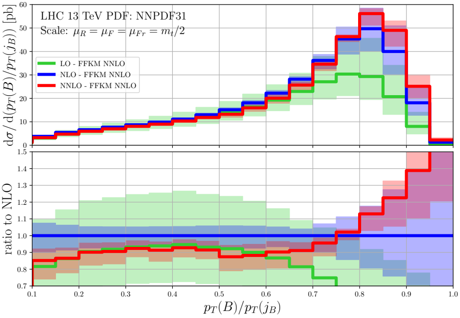


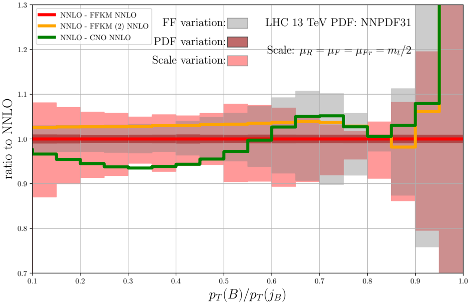
Top-Pair Events with B-hadrons at the LHC: Plots



Top-Pair Events with B-hadrons at the LHC: Jet Ratio

- Jet algorithm: anti-k_T with $R = 0.8$

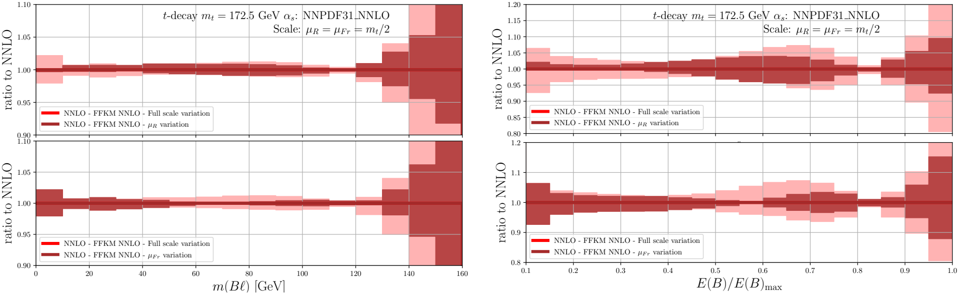




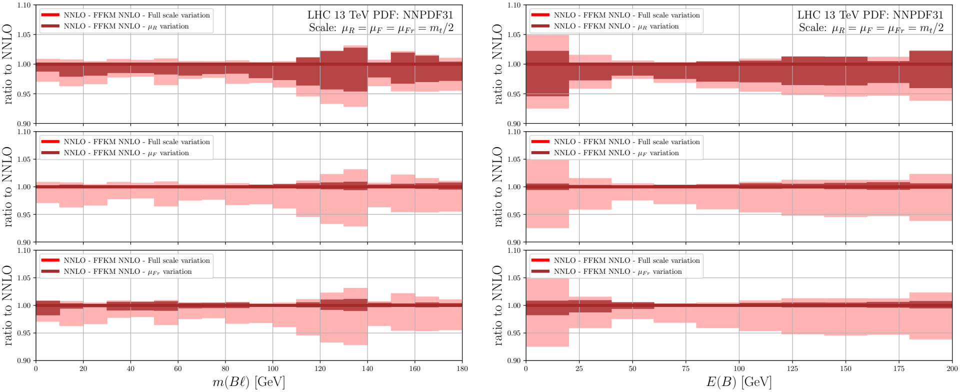
Conclusion and Outlook

- Fragmentation has been implemented in STRIPPER
- First application: top-quark pairs at the LHC
- Big reduction in scale uncertainties from NLO to NNLO
- $\Rightarrow$ potential for more accurate top-mass determination
- PDF-insensitive extraction of FFs at LHC plausible
- Framework completely general: can describe the production of any hadron in any process at NNLO

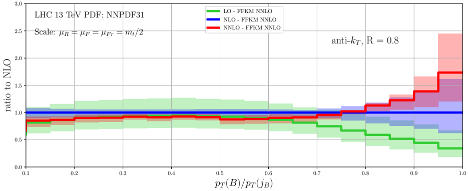
Isolated Top Decay: Separated Scale Dependence

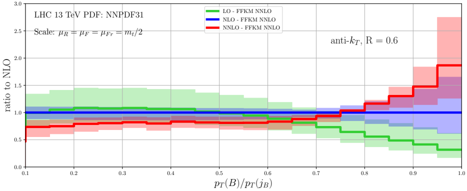


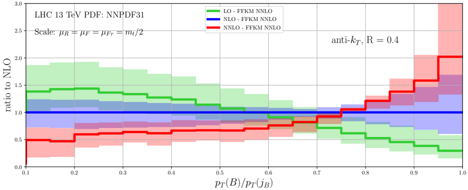
Top-Pair Events with B-hadrons at the LHC: Separated Scale Dependence

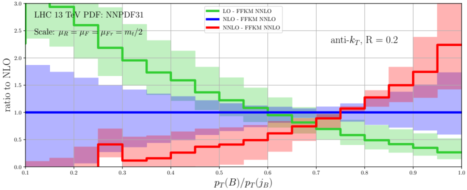


Jet Ratio: R-Dependence

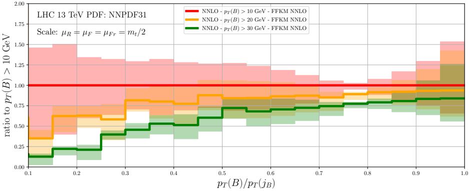


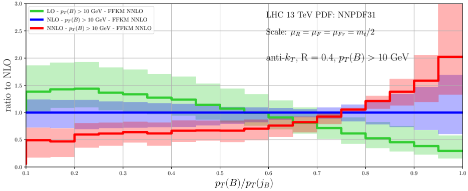


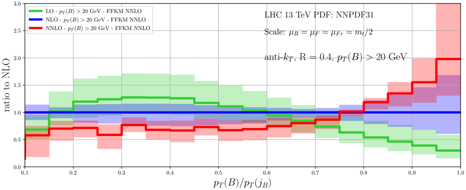


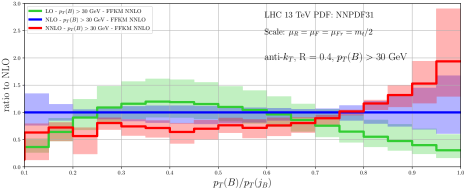


Jet Ratio: p_T-Cut-Dependence

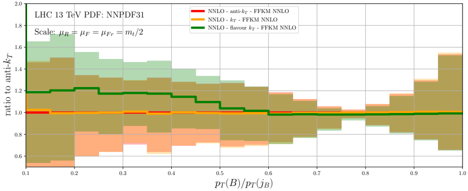


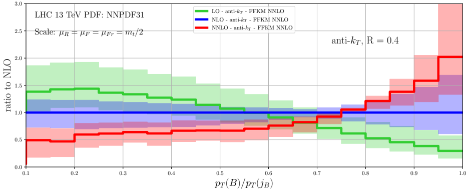


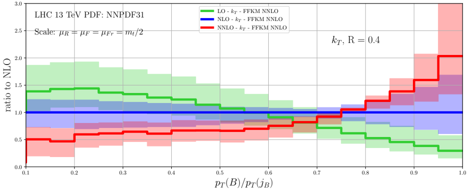


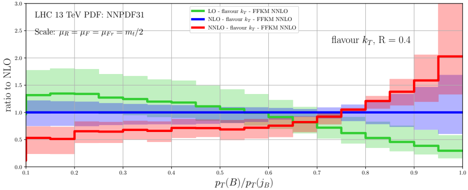


Jet Ratio: Jet-Algorithm-Dependence









Perturbative Fragmentation Function Formalism

- Factorise production of massive quarks into production of massless partons and fragmentation:

$$\frac{d\sigma_Q}{dE_Q} = \sum_i \left(\frac{d\sigma_i}{dE_i}(m_Q = 0) \otimes D_{i \to Q} \right)$$

- Initially used to resum mass logarithms ($\ln(p_T/m_Q)$)
- Added benefit: massive cross section from massless ones
- PFFs already known through NNLO

  NLO: Mele and Nason (1991)

  NNLO: Melnikov and Mitov (2004, 2005)

The NLO Perturbative Fragmentation Functions

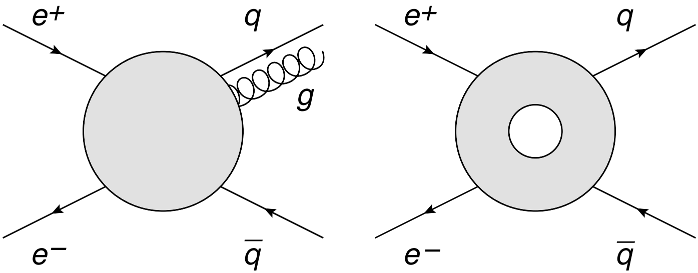
$$D_{Q\to Q}(x,\mu_{Fr},m_Q) = \delta(1-x)+\frac{\alpha_s C_F}{2\pi}\left[\frac{1+x^2}{1-x}\left(\ln\frac{\mu_{Fr}^2}{m_Q^2}-2\ln(1-x)-1\right)\right]_+$$

$$\int_0^1 f(x)g_+(x)dx = \int_0^1 (f(x)-f(1))g(x)dx$$

- New and arbitrary 'renormalisation' scale μ_{Fr}
- Two kinds of logarithm could spoil perturbative convergence
  $\Rightarrow$ Resummation

Fragmentation and Collinear Divergences

- Reminder: $\frac{d\sigma_h}{dE_h} = \sum_i \frac{d\sigma_i}{dE_i} \otimes D_{i\to h}$
- $d\sigma_i$ is infrared-unsafe
- No cancellation of divergences by KLN theorem



Collinear Renormalisation

- Solved by collinear renormalisation:

$$D_i^B(x) = \sum_j \left(Z_{ij} \otimes D_j\right)(x)$$

- Analogous to coupling renormalisation
- Yields RGEs for FFs (DGLAP equations):

$$\mu_{Fr}^2 \frac{dD_{i\to h}}{d\mu_{Fr}^2}(x, \mu_{Fr}) = \sum_j \left(P_{ij}^{\mathrm{T}} \otimes D_{j\to h}\right)(x, \mu_{Fr})$$

- $\Rightarrow$ Only need to fit NPFFs at a single scale
- μ_{Fr}-dependence known $\Rightarrow$ can resum $\ln \frac{\mu_{Fr}^2}{m_Q^2}$ in PFFs

Collinear Renormalisation

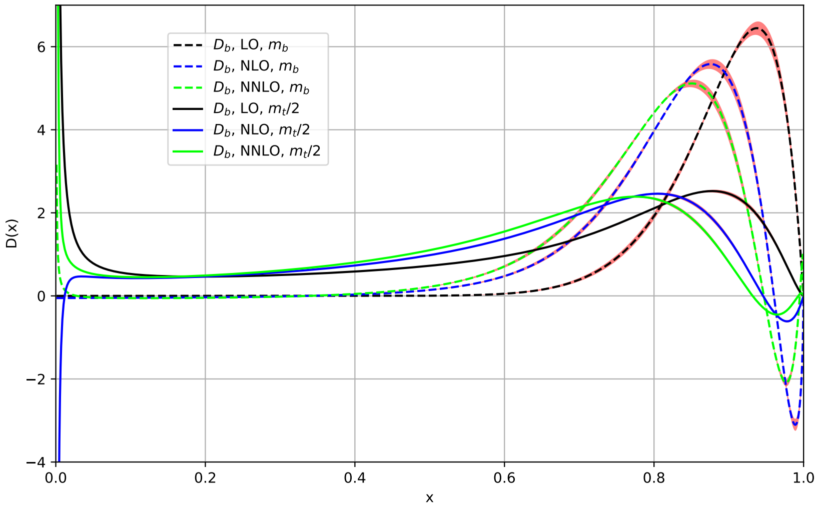
$$D_i^B(x) = \sum_j \left(Z_{ij} \otimes D_j\right)(x) \,, \quad (f \otimes g)(x) = \int_x^1 \frac{dz}{z} f\left(\frac{x}{z}\right) g(z)$$

$$\begin{aligned} Z_{ij}(x) = \delta_{ij}\delta(1-x) &+ \frac{1}{\epsilon}\left(\frac{\mu_R^2}{\mu_{Fr}^2}\right)^{\epsilon} \frac{\alpha_s}{2\pi} P_{ij}^{(0)\mathrm{T}}(x) \\ &+ \left(\frac{\alpha_s}{2\pi}\right)^2 \left[\frac{1}{2\epsilon}\left(\frac{\mu_R^2}{\mu_{Fr}^2}\right)^{2\epsilon} P_{ij}^{(1)\mathrm{T}}(x)\right. \\ &\qquad + \frac{1}{2\epsilon^2}\left(\frac{\mu_R^2}{\mu_{Fr}^2}\right)^{2\epsilon} \sum_k (P_{ik}^{(0)\mathrm{T}} \otimes P_{kj}^{(0)\mathrm{T}})(x) \\ &\qquad \left. + \frac{\beta_0}{4\epsilon^2}\left\{\left(\frac{\mu_R^2}{\mu_{Fr}^2}\right)^{2\epsilon} - 2\left(\frac{\mu_R^2}{\mu_{Fr}^2}\right)^{\epsilon}\right\} P_{ij}^{(0)\mathrm{T}}(x)\right] \end{aligned}$$

Reference Observables

- Momentum fraction of subtraction terms not fully constrained
- Must be the same distribution for full/integrated subtraction terms
- Must match fraction of real contribution in relevant singular limit
- $\Rightarrow$ Can use freedom to improve numerical convergence
- Idea: rescale fractions per event to make all terms land in the same histogram bin
- Significantly reduce poor convergence due to "missed binning"
- Process requires "reference observable"

A Fragmentation Function Through NNLO



Introduction to Subtraction Schemes

- Strategy for numerical integration of cross sections
- Cross sections contain singularities in $d = 4$ (soft, collinear)
- In $d = 4 - 2\epsilon$, cross sections behave like

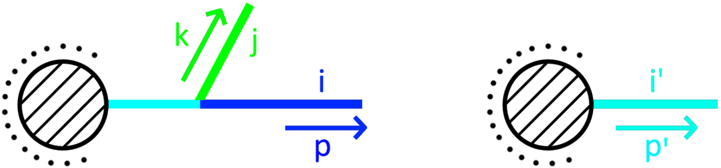
$$\sigma = \int_0^1 \frac{f_\epsilon(x)}{x^{1-a\epsilon}} dx$$

- Idea: subtract divergences differentially (subtraction terms), add them in integrated form (integrated subtraction terms):

$$\sigma = \underbrace{\int_0^1 \left(\frac{f_\epsilon(x)}{x^{1-a\epsilon}} - \frac{f_\epsilon(0)}{x^{1-a\epsilon}} \right)}_{\text{regular at } x=0} dx + f_\epsilon(0) \underbrace{\int_0^1 \frac{1}{x^{1-a\epsilon}} dx}_{1/(a\epsilon)}$$

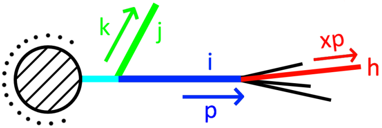
Introduction to Subtraction Schemes

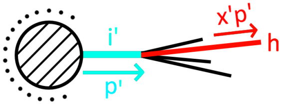
- $\sigma = \underbrace{\int_0^1 \left(\frac{f_\epsilon(x)}{x^{1-a\epsilon}} - \frac{f_\epsilon(0)}{x^{1-a\epsilon}} \right) dx}_{\text{expand in } \epsilon \text{ around } d=4} + \frac{f_\epsilon(0)}{a\epsilon}$
- Can perform numerical integration in $d = 4$
- Subtraction term can in principle be any function, but:
- Both value and kinematics of subtraction term must match cross section in singular limit



Subtraction Schemes and Fragmentation

- Without fragmentation: cannot distinguish collinear quark-pair $q(p_1) + \overline{q}(p_2)$ from $g(p_1 + p_2)$
- With fragmentation: both momentum of fragmenting particle and flavour matter
  $\Rightarrow$ must store flavour and e.g. $p_1^0/(p_1^0 + p_2^0)$
- Introduce concept of reference observables: match reference observable for cross section and subtraction term by rescaling the momentum fraction





Subtraction Schemes and Fragmentation

- Without fragmentation: cannot distinguish $q(p) + g(0)$ from $q(p)$
- With fragmentation: cannot remove gluon if it is the fragmenting particle
- Usually: have to recalculate integrated subtraction terms

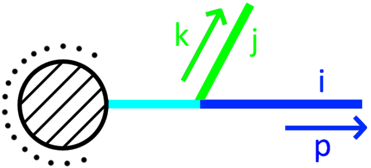
$$\mathcal{V}_{ij}(\epsilon) = \int_0^1 d\tilde{z}_i \, (\tilde{z}_i(1-\tilde{z}_i))^{-\epsilon} \int_0^1 \frac{dy}{y} \, (1-y)^{1-2\epsilon} \, y^{-\epsilon} \, \frac{< \boldsymbol{V}_{ij,k}(\tilde{z}_i; y) >}{8\pi\alpha_S \mu^{2\epsilon}}$$

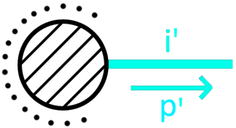
with $\downarrow$ fragmentation

$$\overline{\mathcal{V}}_{ij}(z;\epsilon) = \Theta(z)\Theta(1-z) \, \frac{z^{1-\epsilon}}{(1-z)^{1+\epsilon}} \int_0^1 d\tilde{z}_i \, (\tilde{z}_i(1-\tilde{z}_i))^{-\epsilon} \frac{< \boldsymbol{V}_{ij,a}(\tilde{z}_i; 1-z) >}{8\pi\alpha_S \mu^{2\epsilon}}$$

Subtraction Schemes and Fragmentation

- Calculation of integrated subtraction terms laborious
- Important observation: not necessary if each subtraction term cancels only one singularity
- Exceptionally the case for the **sector-improved residue subtraction scheme**
- $\Rightarrow$ Major simplification of fragmentation implementation





Sector-Improved Residue Subtraction Scheme

- Implemented in C++ library STRIPPER
- Many NNLO firsts over the years. Recently:
  - Diphoton + jet at the LHC *Chawdhry, Czakon, Mitov, Poncelet (2021)*
  - Exact top-mass effects in Higgs production at the LHC *Czakon, Harlander, Klappert, Niggetiedt (2021)*
  - Top-pairs with B-hadrons at the LHC *Czakon, T.G., Mitov, Poncelet (2021)*
  - W + c-jet at the LHC *Czakon, Mitov, Pellen, Poncelet (2020)*
  - ...
- Edge over competitors due to minimisation of analytic integration
- First implementation of fragmentation in a general NNLO subtraction scheme
- Fully general implementation; not limited to cases presented in this talk

Tests of the Implementation

- Implementation has been tested in several ways
- Cancellation of divergences
- A sum rule for top-decay:

$$\Gamma = \int_0^1 x \frac{d\Gamma}{dx} dx \qquad \text{if} \qquad \int_0^1 z\, D_{i\to h}(z) dz = 1$$

- Comparison of $e^+e^- \longrightarrow B + X$ to an analytic calculation
- All tests confirm the implementation is correct

Cross-Check: B-Hadrons at LEP