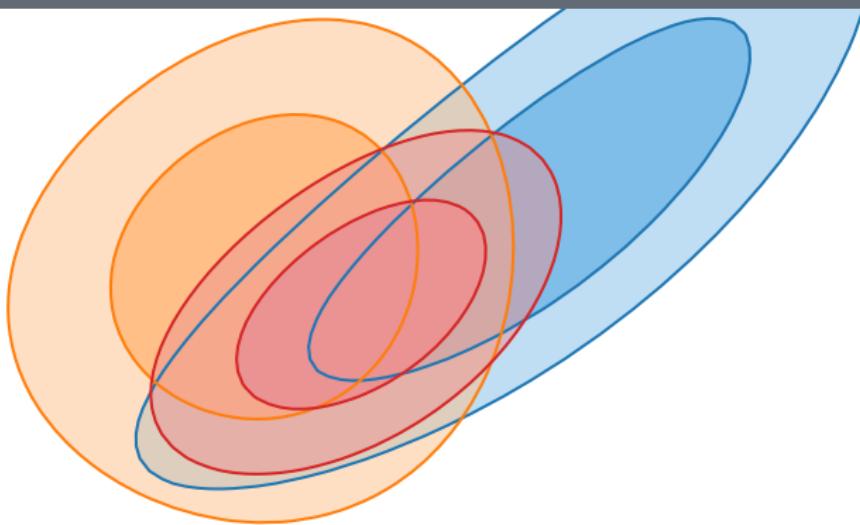


# From the global $b \rightarrow s\ell\ell$ fit to models of new physics

Peter Stangl AEC & ITP University of Bern



# The $b \rightarrow s\ell\ell$ anomalies

# $b \rightarrow s \mu^+ \mu^-$ anomaly

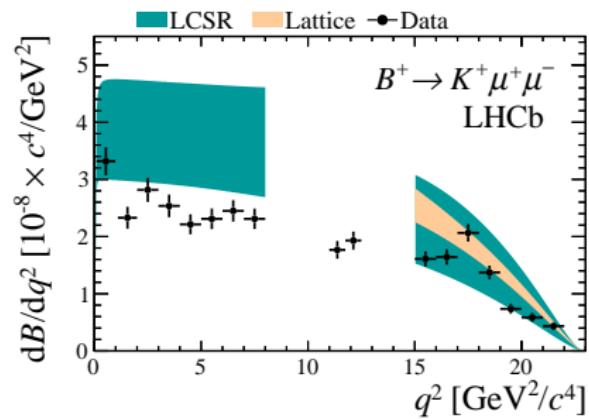
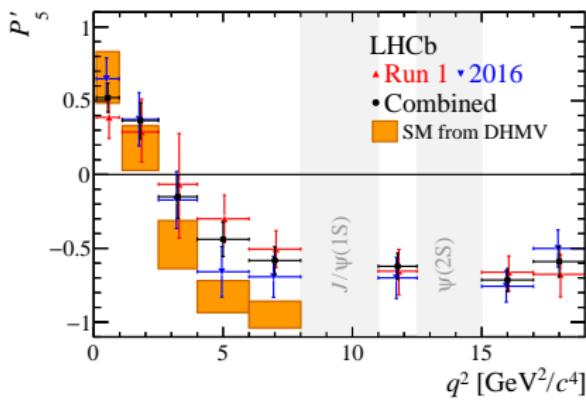
Several LHCb measurements deviate from Standard model (SM) predictions\* by  $2\text{-}3\sigma$ :

- ▶ Angular observables in  $B \rightarrow K^* \mu^+ \mu^-$ .

LHCb, arXiv:2003.04831, arXiv:2012.13241

- ▶ Branching ratios of  $B \rightarrow K \mu^+ \mu^-$ ,  $B \rightarrow K^* \mu^+ \mu^-$ , and  $B_s \rightarrow \phi \mu^+ \mu^-$ .

LHCb, arXiv:1403.8044, arXiv:1506.08777, arXiv:1606.04731, arXiv:2105.14007



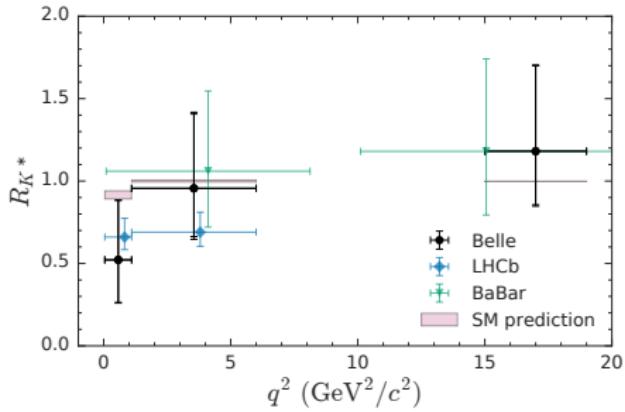
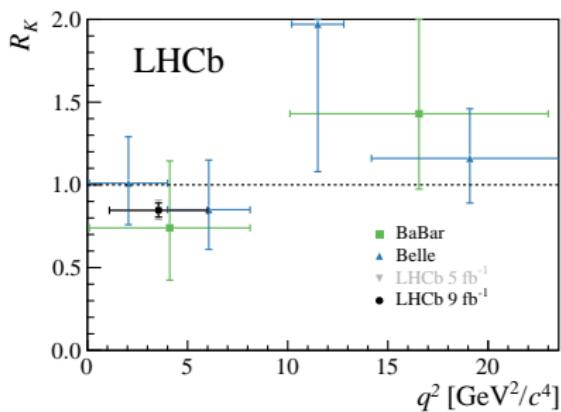
\*: based on hadronic assumptions on which there is no theory consensus yet

# Hints for LFU violation in $b \rightarrow s \ell^+ \ell^-$ decays

Measurements of lepton flavor universality (LFU) ratios  $R_{K^*}^{[0.045,1.1]}, R_{K^*}^{[1.1,6]}, R_K^{[1,6]}$  show deviations from SM by 2.3, 2.5, and  $3.1\sigma$ .

LHCb, arXiv:1705.05802, arXiv:2103.11769  
Belle, arXiv:1904.02440, arXiv:1908.01848

$$R_{K^{(*)}} = \frac{BR(B \rightarrow K^{(*)}\mu^+\mu^-)}{BR(B \rightarrow K^{(*)}e^+e^-)}$$



# Combination of $B_{s,d} \rightarrow \mu^+ \mu^-$ measurements

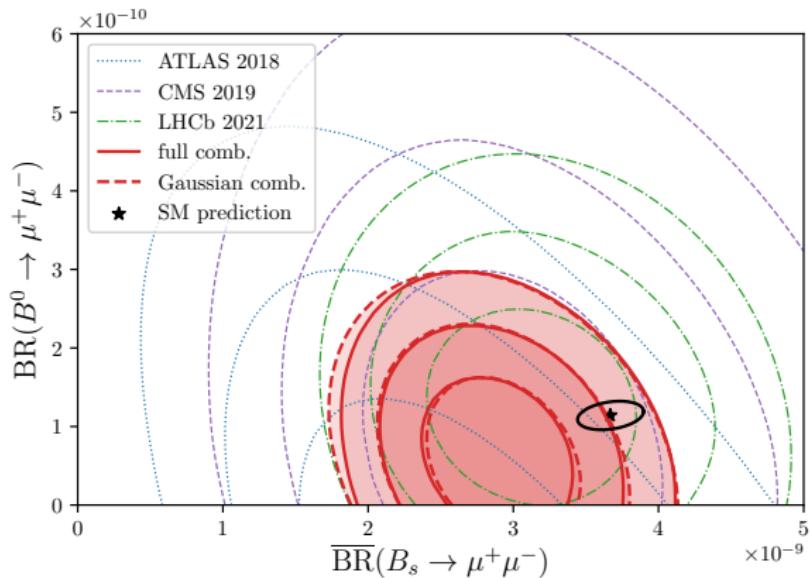
Measurements of  $\text{BR}(B_{s,d} \rightarrow \mu^+ \mu^-)$  by LHCb, CMS, and ATLAS show combined deviation from SM predictions\* by about  $2\sigma$ .

ATLAS, arXiv:1812.03017

CMS, arXiv:1910.12127

LHCb, arXiv:2108.09283, arXiv:2108.09284

Altmannshofer, PS, arXiv:2103.13370



\*: depends on parameters like  $V_{cb}$  but tension persists in  $V_{cb}$ -free ratio  $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)/\Delta M_s$

Bobeth, Buras, arXiv:2104.09521

# The $b \rightarrow c \ell \nu$ anomalies

# Hints for LFU violation in $b \rightarrow c \ell \nu$ decays

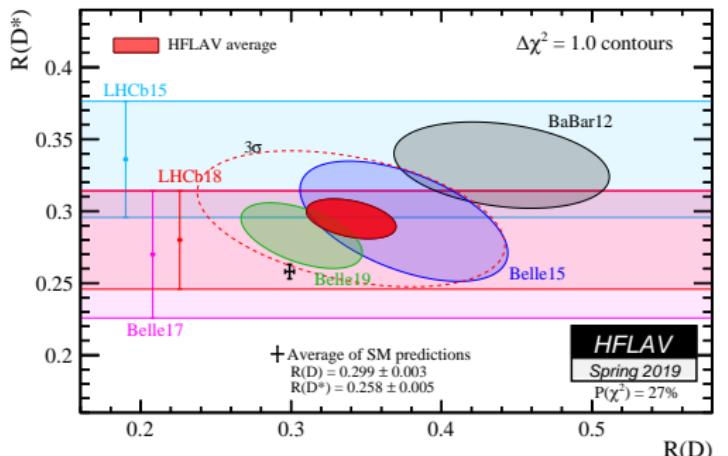
Measurements of LFU ratios  $R_D$  and  $R_{D^*}$  by BaBar, Belle, and LHCb show combined deviation from SM by about  $3\text{-}4\sigma$ .

BaBar, arXiv:1205.5442, arXiv:1303.0571  
LHCb, arXiv:1506.08614, arXiv:1708.08856

Belle, arXiv:1507.03233, arXiv:1607.07923, arXiv:1612.00529, arXiv:1904.08794

$$R_{D^{(*)}} = \frac{BR(B \rightarrow D^{(*)}\tau\nu)}{BR(B \rightarrow D^{(*)}\ell\nu)}$$

$$\ell \in \{e, \mu\}$$



HFLAV, hflav.web.cern.ch

# Theoretical Framework

# $b \rightarrow s\ell\ell$ in the weak effective theory

- Effective Hamiltonian at scale  $m_b$ :  $\mathcal{H}_{\text{eff}}^{\text{bs}\ell\ell} = \mathcal{H}_{\text{eff, sl}}^{\text{bs}\ell\ell} + \mathcal{H}_{\text{eff, had}}^{\text{bs}\ell\ell}$
- **Semileptonic operators:** ( $\mathcal{N} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{16\pi^2} \approx (34 \text{ TeV})^{-2}$ )

$$\mathcal{H}_{\text{eff, sl}}^{\text{bs}\ell\ell} = -\mathcal{N} \left( C_7^{\text{bs}} O_7^{\text{bs}} + C_7'^{\text{bs}} O_7'^{\text{bs}} + \sum_{\ell} \sum_{i=9,10,S,P} \left( C_i^{\text{bs}\ell\ell} O_i^{\text{bs}\ell\ell} + C_i'^{\text{bs}\ell\ell} O_i'^{\text{bs}\ell\ell} \right) \right) + \text{h.c.}$$

$$O_9^{(\prime)\text{bs}\ell\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell), \quad C_9^{\text{SM}} \approx -4.1$$

$$O_{10}^{(\prime)\text{bs}\ell\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell), \quad C_{10}^{\text{SM}} \approx +4.2$$

$$O_7^{(\prime)\text{bs}} = \frac{m_b}{e} (\bar{s}\sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu}, \quad C_7^{\text{SM}} \approx -0.3$$

$$O_S^{(\prime)\text{bs}\ell\ell} = m_b (\bar{s}P_{R(L)} b)(\bar{\ell}\ell),$$

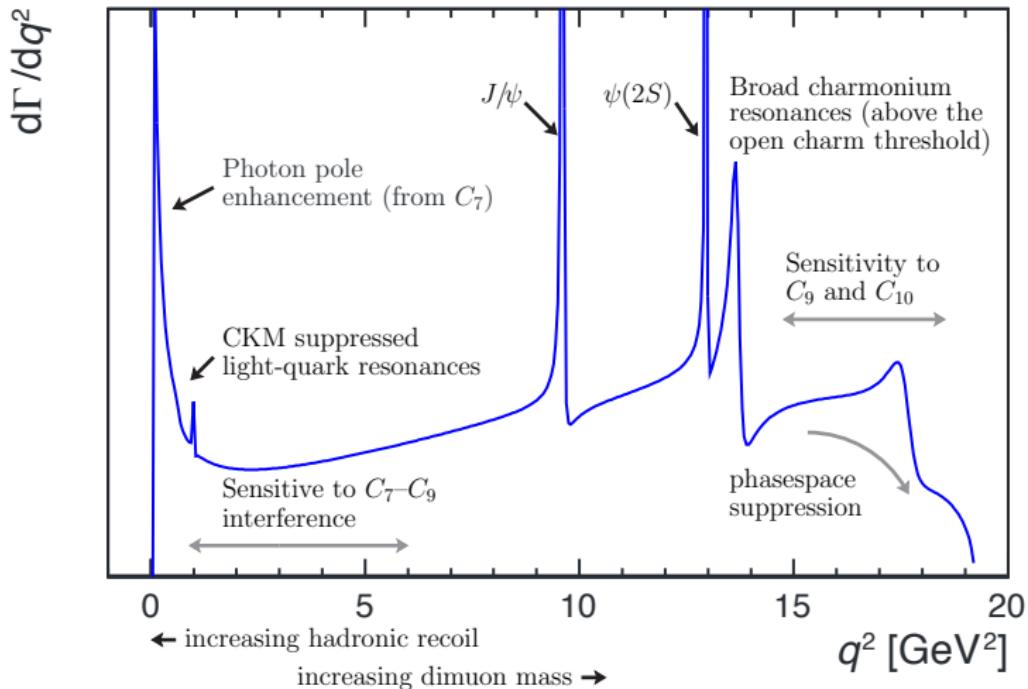
$$O_P^{(\prime)\text{bs}\ell\ell} = m_b (\bar{s}P_{R(L)} b)(\bar{\ell}\gamma_5 \ell).$$

- **Hadronic operators:**

$$\mathcal{H}_{\text{eff, had}}^{\text{bs}\ell\ell} = -\mathcal{N} \frac{16\pi^2}{e^2} \left( C_8^{\text{bs}} O_8^{\text{bs}} + C_8'^{\text{bs}} O_8'^{\text{bs}} + \sum_{i=1..6} C_i^{\text{bs}\ell\ell} O_i^{\text{bs}} \right) + \text{h.c.}$$

e.g.  $O_1^{\text{bs}} = (\bar{s}\gamma_\mu P_L T^a c)(\bar{c}\gamma^\mu P_L T^a b), \quad O_2^{\text{bs}} = (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma^\mu P_L b).$

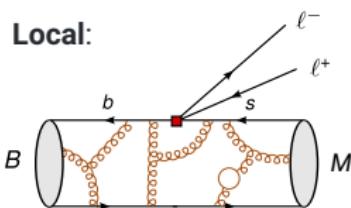
# Cartoon: $q^2$ dependence of $B \rightarrow K^* \ell^+ \ell^-$



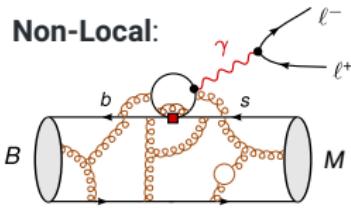
Blake, Lanfranchi, Straub, arXiv:1606.00916

# Theory of $B \rightarrow M\ell\ell$ decays ( $M = K, K^*, \phi$ )

$$\begin{aligned}\mathcal{M}(B \rightarrow M\ell\ell) &= \langle M\ell\ell | \mathcal{H}_{\text{eff}}^{bs\ell\ell} | B \rangle \\ &= \mathcal{N} \left[ (\mathcal{A}_V^\mu + \mathcal{H}^\mu) \bar{u}_\ell \gamma_\mu v_\ell + \mathcal{A}_A^\mu \bar{u}_\ell \gamma_\mu \gamma_5 v_\ell + \mathcal{A}_S \bar{u}_\ell v_\ell + \mathcal{A}_P \bar{u}_\ell \gamma_5 v_\ell \right]\end{aligned}$$



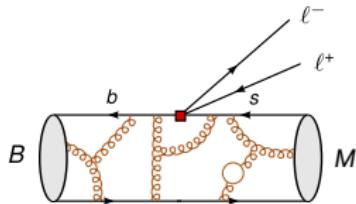
$$\begin{aligned}\mathcal{A}_V^\mu &= -\frac{2im_b}{q^2} C_7 \langle M | \bar{s} \sigma^{\mu\nu} q_\nu P_R b | B \rangle + C_9 \langle M | \bar{s} \gamma^\mu P_L b | B \rangle \\ &\quad + (P_L \leftrightarrow P_R, C_i \rightarrow C'_i) \\ \mathcal{A}_A^\mu &= C_{10} \langle M | \bar{s} \gamma^\mu P_L b | B \rangle + (P_L \leftrightarrow P_R, C_i \rightarrow C'_i) \\ \mathcal{A}_{S,P} &= C_{S,P} \langle M | \bar{s} P_R b | B \rangle + (P_L \leftrightarrow P_R, C_i \rightarrow C'_i)\end{aligned}$$



$$\begin{aligned}\mathcal{H}^\mu &= \frac{-16i\pi^2}{q^2} \sum_{i=1..6,8} C_i \int dx^4 e^{iq \cdot x} \langle M | T\{ j_{\text{em}}^\mu(x), O_i(0) \} | B \rangle \\ j_{\text{em}}^\mu &= \sum_q Q_q \bar{q} \gamma^\mu q\end{aligned}$$

- **Wilson coefficients**  $C_i = C_i^{\text{SM}} + C_i^{\text{NP}}$ :  
perturbative, short-distance ( $q^2$ -independent), parameterize heavy new physics
- **local and non-local hadronic matrix elements**:  
non-perturbative, long-distance ( $q^2$ -dependent), **main source of uncertainty** see talk by Marzia Bordone

# Local matrix elements



$$\begin{aligned}\mathcal{A}_V^\mu &= -\frac{2im_b}{q^2} \mathbf{C}_7 \langle M | \bar{s} \sigma^{\mu\nu} q_\nu P_R b | B \rangle + \mathbf{C}_9 \langle M | \bar{s} \gamma^\mu P_L b | B \rangle \\ &\quad + (P_L \leftrightarrow P_R, C_i \rightarrow C'_i) \\ \mathcal{A}_A^\mu &= \mathbf{C}_{10} \langle M | \bar{s} \gamma^\mu P_L b | B \rangle + (P_L \leftrightarrow P_R, C_i \rightarrow C'_i) \\ \mathcal{A}_{S,P} &= \mathbf{C}_{S,P} \langle M | \bar{s} P_R b | B \rangle + (P_L \leftrightarrow P_R, C_i \rightarrow C'_i)\end{aligned}$$

- Matrix elements  $\langle M | \bar{s} \Gamma_i b | B \rangle$  can be parameterized by:

- 3 form factors for each **spin zero** final state,  $M = K$
- 7 form factors for each **spin one** final state,  $M = K^*, \phi$

- Determination of form factors

- high  $q^2$ : **Lattice QCD**

HPQCD, arXiv:1306.2384  
Fermilab, MILC, arXiv:1509.06235  
Horgan, Liu, Meinel, Wingate, arXiv:1310.3722, arXiv:1501.00367

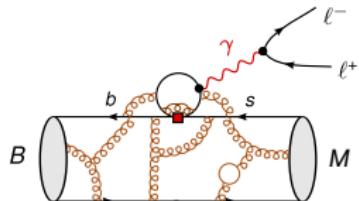
- low  $q^2$ : **Light-cone sum rules (LCSR)**

Bharucha, Straub, Zwicky, arXiv:1503.05534  
Khodjamirian, Mannel, Pivovarov, Wang, arXiv:1006.4945  
Gubernari, Kokulu, van Dyk, arXiv:1811.00983  
Ball, Zwicky, arXiv:hep-ph/0406232

- low + high  $q^2$ : Combined fit **LCSR + lattice**

Bharucha, Straub, Zwicky, arXiv:1503.05534  
Gubernari, Kokulu, van Dyk, arXiv:1811.00983  
Altmannshofer, Straub, arXiv:1411.3161

# Non-local matrix elements



$$\mathcal{H}^\mu = \frac{-16i\pi^2}{q^2} \sum_{i=1..6,8} C_i \int dx^4 e^{iq \cdot x} \langle M | T\{ j_{\text{em}}^\mu(x), O_i(0) \} | B \rangle$$

$$j_{\text{em}}^\mu = \sum_q Q_q \bar{q} \gamma^\mu q$$

- ▶ Contributions at low  $q^2$  from QCD factorization (QCDF)  
Beneke, Feldmann, Seidel, arXiv:hep-ph/0106067
- ▶ **Beyond-QCDF contributions the main source of uncertainty**
- ▶ Non-local contributions can mimic New Physics in  $C_9$
- ▶ Several approaches to estimate beyond-QCDF contributions at low  $q^2$ 
  - ▶ fit of sum of resonances to data Blake, Egede, Owen, Pomery, Petridis, arXiv:1709.03921
  - ▶ direct fit to angular data Ciuchini, Fedele, Franco, Mishima, Paul, Silvestrini, Valli, arXiv:1512.07157
  - ▶ Light-Cone Sum Rules estimates Khodjamirian, Mannel, Pivovarov, Wang, arXiv:1006.4945  
Gubernari, van Dyk, Virto, arXiv:2011.09813
  - ▶ analyticity + experimental data on  $b \rightarrow s c \bar{c}$  Bobeth, Chrzaszcz, van Dyk, Virto, arXiv:1707.07305  
Gubernari, van Dyk, Virto, arXiv:2011.09813

## "Cleanliness" of $b \rightarrow s\ell\ell$ observables in the SM

	parametric uncertainties	local hadr. matrix elements	non-local hadr. matrix elements
$\mathcal{B}(B \rightarrow M\ell\ell)$	✗	✗	✗
angular observables	✓	✗	✗
$\overline{\mathcal{B}}(B_s \rightarrow \ell\ell)$	✗	✓	✓ (N/A)
LFU observables	✓	✓	✓

# New physics interpretation of $b \rightarrow s\ell\ell$ anomalies

# New physics in $b \rightarrow s\ell\ell$ in the weak effective theory

- Effective Hamiltonian at scale  $m_b$ :  $\mathcal{H}_{\text{eff}}^{bs\ell\ell} = \mathcal{H}_{\text{eff, SM}}^{bs\ell\ell} + \mathcal{H}_{\text{eff, NP}}^{bs\ell\ell}$

$$\mathcal{H}_{\text{eff, NP}}^{bs\ell\ell} = -\mathcal{N} \sum_{\ell=e,\mu} \sum_{i=9,10,S,P} \left( C_i^{bs\ell\ell} O_i^{bs\ell\ell} + C_i'^{bs\ell\ell} O_i'^{bs\ell\ell} \right) + \text{h.c.}$$

- Operators considered here ( $\ell = e, \mu$ )

$$O_9^{(\prime)bs\ell\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell),$$

$$O_{10}^{(\prime)bs\ell\ell} = (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell).$$

- Not considered here

- Scalar operators: can only reduce tension in  $B_s \rightarrow \mu\mu$
- Dipole operators: strongly constrained by radiative decays
- Four quark operators: dominant effect from RG running above  $m_B$

e.g. Paul, Straub, arXiv:1608.02556

Jäger, Leslie, Kirk, Lenz, arXiv:1701.09183

# Setup

- ▶ Quantify agreement between theory and experiment by  $\chi^2$  function

$$\chi^2(\vec{C}) = \left( \vec{O}_{\text{exp}} - \vec{O}_{\text{th}}(\vec{C}) \right)^T \left( C_{\text{exp}} + C_{\text{th}} \right)^{-1} \left( \vec{O}_{\text{exp}} - \vec{O}_{\text{th}}(\vec{C}) \right).$$

- ▶ **theory errors** and **correlations** in covariance matrix  $C_{\text{th}}$
- ▶ **experimental errors** and available **correlations** in covariance matrix  $C_{\text{exp}}$
- ▶ Theory errors depend on new physics (NP) Wilson coefficients  $C_{\text{th}}(\vec{C})$
- ▶  $\Delta\chi^2$  and pull

$$\text{pull}_{1D} = 1\sigma \cdot \sqrt{\Delta\chi^2}, \quad \text{where } \Delta\chi^2 = \chi^2(\vec{0}) - \chi^2(\vec{C}_{\text{best fit}}).$$

$$\text{pull}_{2D} = 1\sigma, 2\sigma, 3\sigma, \dots \quad \text{for } \Delta\chi^2 \approx 2.3, 6.2, 11.8, \dots$$

- ▶ New physics scenarios in **Weak Effective Theory (WET)** at scale 4.8 GeV

# Setup

- ▶ Quantify agreement between theory and experiment by  $\chi^2$  function

$$\chi^2(\vec{C}) = \left( \vec{O}_{\text{exp}} - \vec{O}_{\text{th}}(\vec{C}) \right)^T \left( C_{\text{exp}} + C_{\text{th}}(\vec{C}) \right)^{-1} \left( \vec{O}_{\text{exp}} - \vec{O}_{\text{th}}(\vec{C}) \right).$$

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- ▶ **experimental errors** and available **correlations** in covariance matrix  $C_{\text{exp}}$
- ▶ Theory errors depend on new physics (NP) Wilson coefficients  $C_{\text{th}}(\vec{C})$  \*NEW\*
- ▶  $\Delta\chi^2$  and pull

Altmannshofer, PS, arXiv:2103.13370

$$\text{pull}_{1\text{D}} = 1\sigma \cdot \sqrt{\Delta\chi^2}, \quad \text{where } \Delta\chi^2 = \chi^2(\vec{0}) - \chi^2(\vec{C}_{\text{best fit}}).$$

$$\text{pull}_{2\text{D}} = 1\sigma, 2\sigma, 3\sigma, \dots \quad \text{for } \Delta\chi^2 \approx 2.3, 6.2, 11.8, \dots$$

- ▶ New physics scenarios in **Weak Effective Theory (WET)** at scale 4.8 GeV

# Observables in global $b \rightarrow s\ell\ell$ analysis

- ▶ Inclusive decays
  - ▶  $B \rightarrow X_s \ell^+ \ell^- (\mathcal{B})$
- ▶ Exclusive leptonic decays
  - ▶  $B_{s,d} \rightarrow \ell^+ \ell^- (\mathcal{B})$
- ▶ Exclusive semileptonic decays
  - ▶  $B^{(0,+)} \rightarrow K^{(0,+)} \ell^+ \ell^- (\mathcal{B}_\mu, R_K, \text{angular observables})$
  - ▶  $B^{(0,+)} \rightarrow K^{*(0,+)} \ell^+ \ell^- (\mathcal{B}_\mu, R_{K^{*0}}, \text{angular observables})$
  - ▶  $B_s \rightarrow \phi \mu^+ \mu^- (\mathcal{B}, \text{angular observables})$
  - ▶  $\Lambda_b \rightarrow \Lambda \mu^+ \mu^- (\mathcal{B}, \text{angular observables})$
- ▶ Fits include  $\sim 200$  observables  $\Rightarrow$  **global  $b \rightarrow s\ell\ell$  analysis**

# Results of global fit

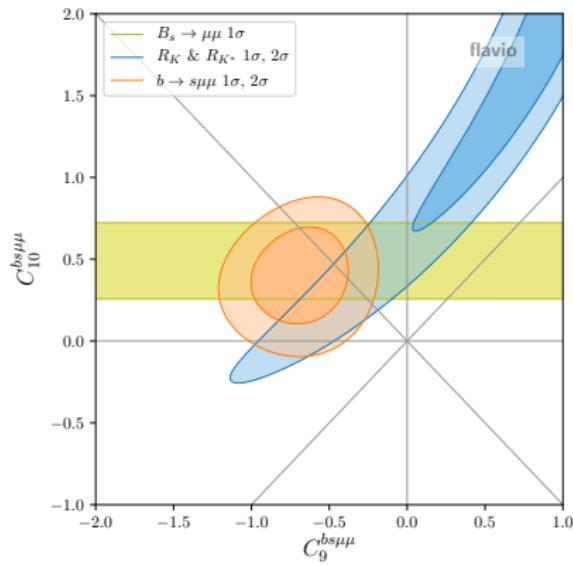
based on Altmannshofer, PS, arXiv:2103.13370 (+  $B_s \rightarrow \phi\mu^+\mu^-$  angular observables, LHCb arXiv:2107.13428)  
see also similar fits by other groups:

Geng et al., arXiv:2103.12738      Algueró et al., arXiv:2104.08921      Hurth et al., arXiv:2104.10058  
Ciuchini et al., arXiv:2110.10126      Alok et al., arXiv:1903.09617,   Datta et al., arXiv:1903.10086,  
Kowalska et al., arXiv:1903.10932,   D'Amico et al., arXiv:1704.05438,   Hiller et al., arXiv:1704.05444,   ...

# Scenarios with a single Wilson coefficients

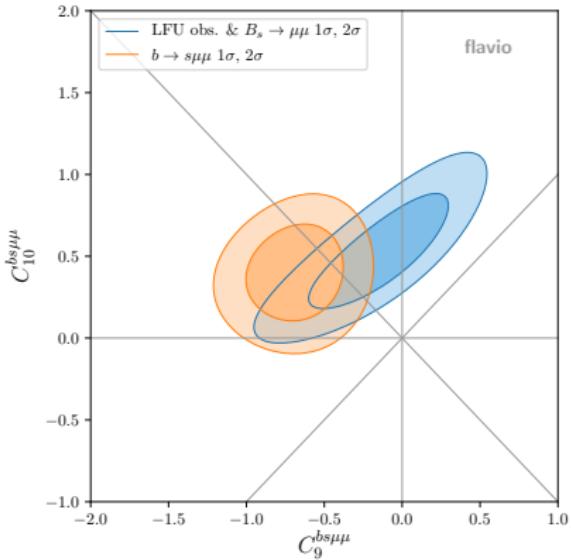
Wilson coefficient	$b \rightarrow s\mu\mu$ best fit	$b \rightarrow s\mu\mu$ pull	LFU, $B_s \rightarrow \mu\mu$ best fit	LFU, $B_s \rightarrow \mu\mu$ pull	all rare $B$ decays best fit	all rare $B$ decays pull
$C_9^{bs\mu\mu}$	$-0.70^{+0.21}_{-0.22}$	<b><math>3.3\sigma</math></b>	$-0.74^{+0.20}_{-0.21}$	$4.1\sigma$	$-0.71^{+0.15}_{-0.15}$	<b><math>5.1\sigma</math></b>
$C_{10}^{bs\mu\mu}$	$+0.45^{+0.22}_{-0.23}$	$1.9\sigma$	$+0.60^{+0.14}_{-0.14}$	<b><math>4.7\sigma</math></b>	$+0.54^{+0.12}_{-0.12}$	$4.8\sigma$
$C_9'^{bs\mu\mu}$	$+0.15^{+0.24}_{-0.24}$	$0.6\sigma$	$-0.32^{+0.16}_{-0.17}$	$2.0\sigma$	$-0.19^{+0.13}_{-0.13}$	$1.5\sigma$
$C_{10}'^{bs\mu\mu}$	$-0.09^{+0.15}_{-0.15}$	$0.6\sigma$	$+0.07^{+0.11}_{-0.13}$	$0.5\sigma$	$+0.04^{+0.10}_{-0.09}$	$0.4\sigma$
$C_9^{bs\mu\mu} = C_{10}^{bs\mu\mu}$	$-0.16^{+0.14}_{-0.14}$	$1.1\sigma$	$+0.43^{+0.18}_{-0.18}$	$2.4\sigma$	$+0.05^{+0.11}_{-0.11}$	$0.5\sigma$
$C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu}$	$-0.55^{+0.13}_{-0.13}$	<b><math>3.8\sigma</math></b>	$-0.35^{+0.08}_{-0.08}$	<b><math>4.6\sigma</math></b>	$-0.39^{+0.07}_{-0.07}$	<b><math>5.6\sigma</math></b>
$C_9^{bsee}$			$+0.74^{+0.20}_{-0.19}$	$4.1\sigma$	$+0.75^{+0.20}_{-0.19}$	$4.1\sigma$
$C_{10}^{bsee}$			$-0.67^{+0.17}_{-0.18}$	$4.2\sigma$	$-0.66^{+0.17}_{-0.18}$	$4.3\sigma$
$C_9'^{bsee}$			$+0.36^{+0.18}_{-0.17}$	$2.1\sigma$	$+0.40^{+0.19}_{-0.18}$	$2.3\sigma$
$C_{10}'^{bsee}$			$-0.32^{+0.16}_{-0.16}$	$2.1\sigma$	$-0.31^{+0.15}_{-0.16}$	$2.1\sigma$
$C_9^{bsee} = C_{10}^{bsee}$			$-1.39^{+0.26}_{-0.26}$	$4.0\sigma$	$-1.28^{+0.24}_{-0.23}$	$4.1\sigma$
$C_9^{bsee} = -C_{10}^{bsee}$			$+0.37^{+0.10}_{-0.10}$	$4.2\sigma$	$+0.37^{+0.10}_{-0.10}$	$4.3\sigma$

# Scenarios with two Wilson coefficients



WET at 4.8 GeV

# Scenarios with two Wilson coefficients



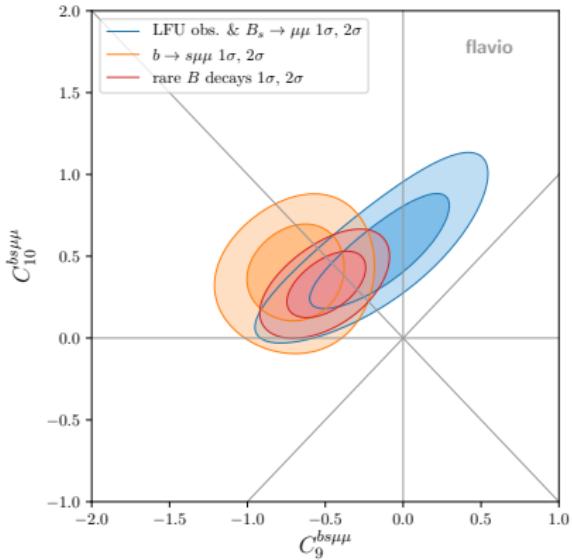
Combination of  $B_s \rightarrow \mu^+ \mu^-$  and LFU observables ( $R_K, R_{K^*}, D_{P_{4',5'}}$ )

- ▶ LFU obs. &  $B_s \rightarrow \mu\mu$ : very clean theory prediction, insensitive to universal  $C_9^{\text{univ.}}$ .
- ▶  $b \rightarrow s\mu\mu$  sensitive to univ. coeff. possibly afflicted by underestimated hadr. uncert.
- ▶ Agreement between  $b \rightarrow s\mu\mu$  observables and  $R_K$  &  $R_{K^*}$  could be further improved by **LFU** contribution to  $C_9^{\text{univ.}}$ .

possible connection to  $b \rightarrow c\ell\nu$  anomalies  
see backup slides

WET at 4.8 GeV

# Scenarios with two Wilson coefficients



WET at 4.8 GeV

Combination of  $B_s \rightarrow \mu^+ \mu^-$  and LFU observables ( $R_K, R_{K^*}, D_{P_{4',5'}}$ )

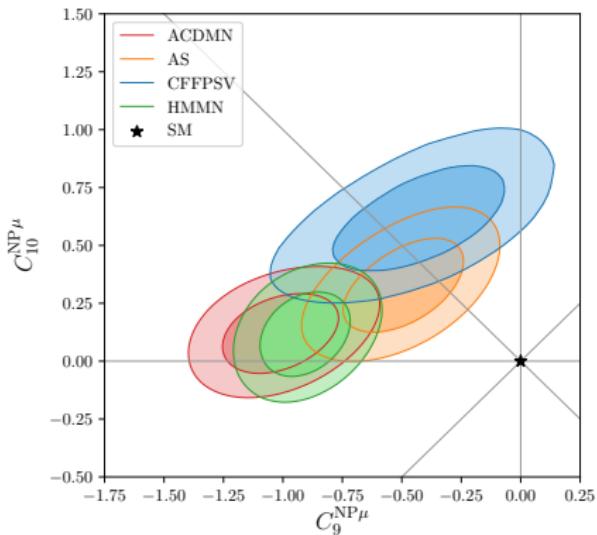
- ▶ LFU obs. &  $B_s \rightarrow \mu\mu$ : very clean theory prediction, insensitive to universal  $C_9^{\text{univ.}}$ .
- ▶  $b \rightarrow s \mu\mu$  sensitive to univ. coeff. possibly afflicted by underestimated hadr. uncert.
- ▶ Agreement between  $b \rightarrow s \mu\mu$  observables and  $R_K$  &  $R_{K^*}$  could be further improved by **LFU** contribution to  $C_9^{\text{univ.}}$

possible connection to  $b \rightarrow c \ell \nu$  anomalies  
see backup slides

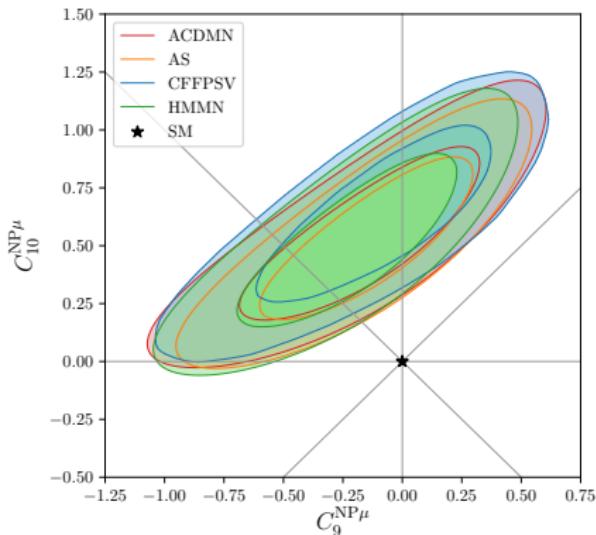
Global fit in  $C_9^{\text{bs}\mu\mu}$ - $C_{10}^{\text{bs}\mu\mu}$  plane prefers negative  $C_9^{\text{bs}\mu\mu} = -C_{10}^{\text{bs}\mu\mu}$

# Robustness of global fits

Capdevila, Fedele, Neshatpour, PS



global fit



fit to LFU observables +  $B_s \rightarrow \mu\mu$

$\Lambda\text{CDMN}$  (Algueró, Capdevila, Descotes-Genon, Matias, Novoa-Brunet), arXiv:2104.08921

AS (Altmannshofer, PS), arXiv:2103.13370

CFFPSV (Ciuchini, Fedele, Franco, Paul, Silvestrini, Valli), arXiv:2011.01212

HMMN (Hurth, Mahmoudi, Martínez-Santos, Neshatpour), arXiv:2104.10058

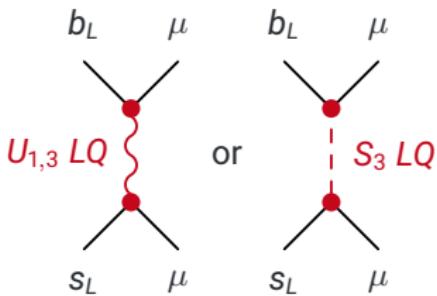
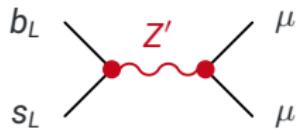
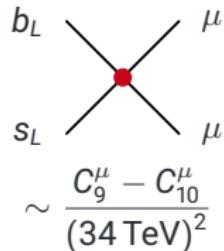
# New particles to explain $b \rightarrow s\ell\ell$ anomalies

# New particles to explain $b \rightarrow s\ell\ell$ anomalies

Global fits suggest

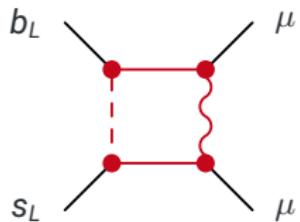
$$C_9^\mu - C_{10}^\mu \approx -0.7, \quad 0 \gtrsim \frac{C_{10}^\mu}{C_9^\mu} \gtrsim -1$$

$$O_9^\mu = (\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu \mu), \quad O_{10}^\mu = (\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu \gamma_5 \mu)$$



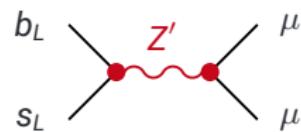
$$\sim \frac{g_{bs} g_{\mu\mu}}{m_{Z'}^2}$$

$$\sim \frac{g_{b\mu} g_{s\mu}}{m_{LQ}^2}$$

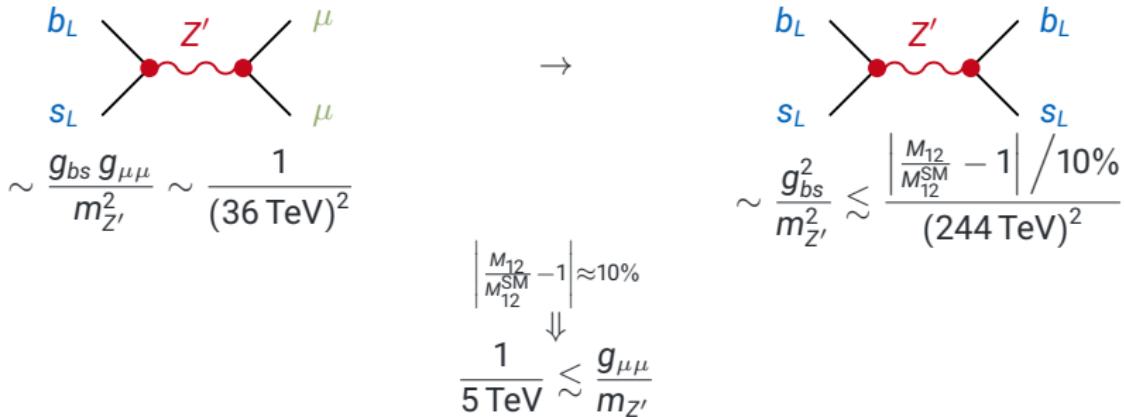


$$\sim \frac{g_b g_s g_{\mu,1} g_{\mu,2}}{16 \pi^2 m_{NP}^2}$$

$Z'$



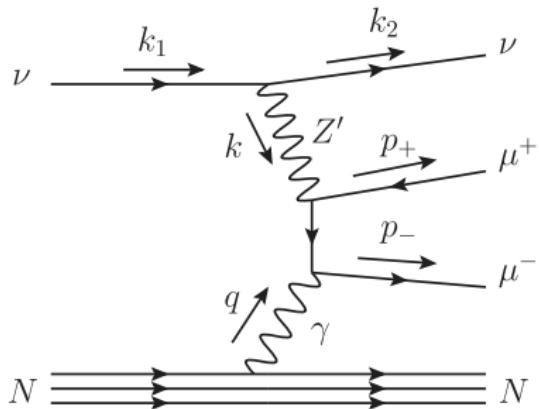
# $Z'$ : Constraints from $B_s$ - $\bar{B}_s$ mixing



Ways around:

- ▶ imaginary part of  $g_{bs} \rightarrow$  constraints from  $CP$  violating observables
- ▶  $Z'$  coupling to  $(\bar{s}\gamma_\mu P_R b) \rightarrow$  constraint from  $R_K \approx R_{K^*}$
- ▶ ...

# $Z'$ : Constraints from neutrino trident production



Altmannshofer, Gori, Pospelov, Yavin, arXiv:1406.2332

- ▶  $\mu^+\mu^-$  production induced by neutrino in Coulomb field of heavy nucleus
- ▶ Cross section with  $Z'$  contribution

$$\frac{\sigma}{\sigma_{SM}} \simeq \frac{1 + \left(1 + 4 s_W^2 + 2 v^2 \frac{g_{Z'}^2}{m_{Z'}^2}\right)^2}{1 + (1 + 4 s_W^2)^2}$$

⇓

$$\frac{g_{\mu\mu}}{m_{Z'}} \lesssim \frac{1}{0.5 \text{ TeV}}$$

# $Z'$ : Constraints from $B_s$ - $\bar{B}_s$ mixing and neutrino trident

Example: Gauged  $L_\mu - L_\tau$

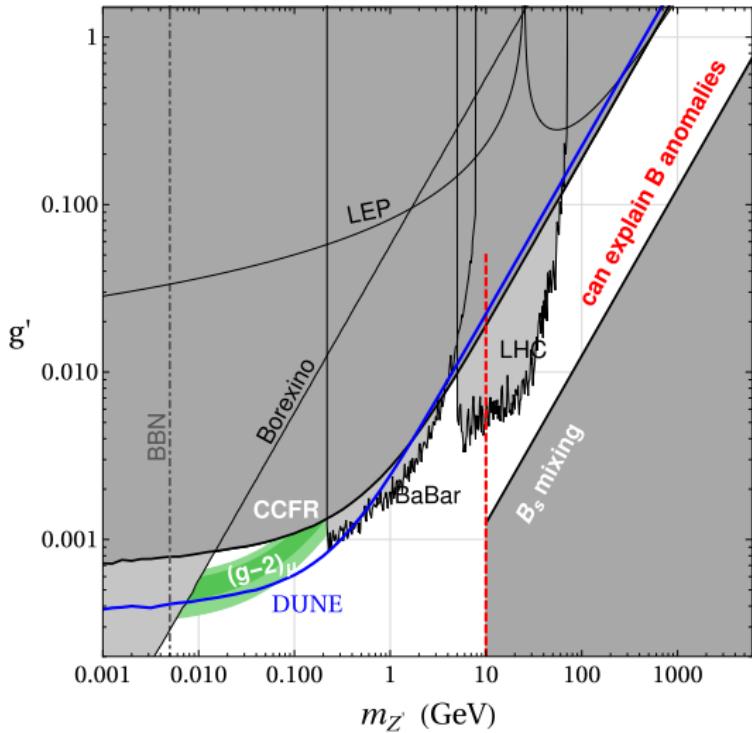
Combined constraints from

- $B_s$ - $\bar{B}_s$  mixing

$$\frac{1}{5 \text{ TeV}} \lesssim \frac{g_{\mu\mu}}{m_{Z'}}$$

- neutrino trident production

$$\frac{g_{\mu\mu}}{m_{Z'}} \lesssim \frac{1}{0.5 \text{ TeV}}$$



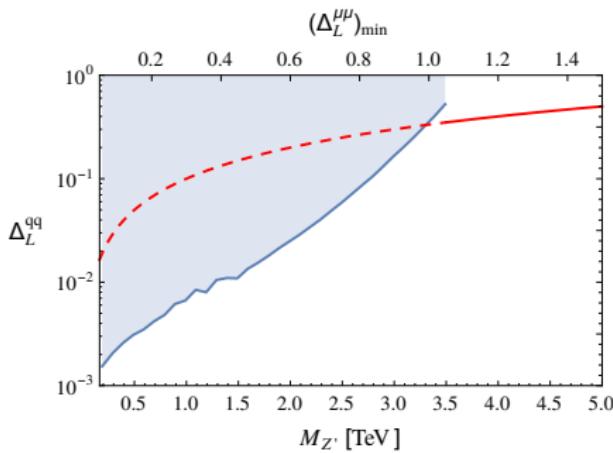
Altmannshofer, Gori, Martin-Albo, Sousa, Wallbank, arXiv:1902.06765

## $Z'$ : Constraints from $pp \rightarrow \mu\mu$

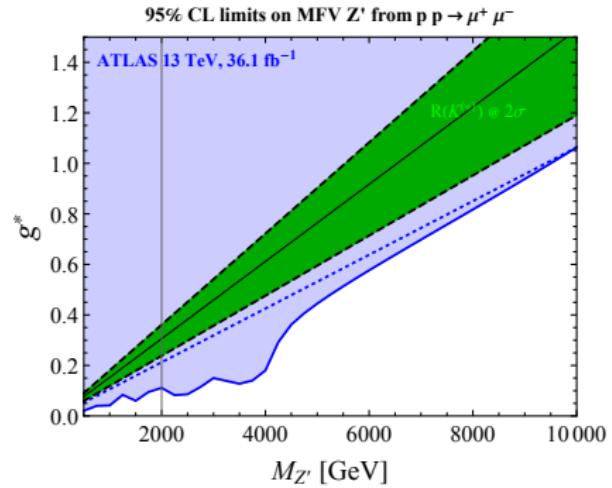


- ▶ Direct searches for a  $Z'$  resonance
- ▶ Searches for quark-lepton contact interactions

# $Z'$ : Constraints from $pp \rightarrow \mu\mu$



Altmannshofer, Straub, arXiv:1411.3161

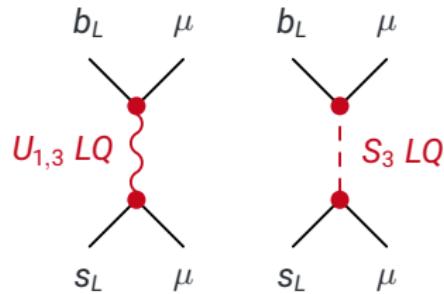


Greljo, Marzocca, arXiv:1704.09015

- Couplings to light quarks must be suppressed for  $m_{Z'} < 4.5$  TeV

- MFV-like  $Z'$ -quark couplings already excluded

# Leptoquarks



# Overview of Leptoquarks

Scenario	Spin	$G_{\text{SM}}$	$\mathcal{L}_{\text{int}}$
$S_1$	0	$(\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	$\hat{\lambda}_L (\bar{q}_L^c \cdot \epsilon \cdot l_L) \phi + \hat{\lambda}_R \bar{u}_R^c \ell_R \phi + \hat{\lambda}_{qq}^1 (\bar{q}_L \cdot \epsilon \cdot q_L^c) \phi + \hat{\lambda}_{qq}^2 \bar{d}_R u_R^c \phi$
$\tilde{S}_1$	0	$(\bar{\mathbf{3}}, \mathbf{1})_{\frac{4}{3}}$	$\hat{\lambda}_R \bar{d}_R^c \ell_R \phi + \hat{\lambda}_{qq} \bar{u}_R u_R^c \phi$
$R_2$	0	$(\mathbf{3}, \mathbf{2})_{\frac{7}{6}}$	$\hat{\lambda}_L (\bar{q}_L \cdot \phi) \ell_R + \hat{\lambda}_R \bar{u}_R (l_L \cdot \epsilon \cdot \phi)$
$\tilde{R}_2$	0	$(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	$\hat{\lambda}_R \bar{d}_R (l_L \cdot \epsilon \cdot \phi)$
$S_3$	0	$(\bar{\mathbf{3}}, \mathbf{3})_{\frac{1}{3}}$	$\hat{\lambda}_L (\bar{q}_L^c \cdot \epsilon \cdot \tau^a \cdot l_L) \phi^a + \hat{\lambda}_{qq} (\bar{q}_L \cdot \epsilon \cdot \tau^a \cdot q_L^c) \phi^a$
$U_1$	1	$(\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	$\hat{\lambda}_L (\bar{q}_L \gamma^\mu l_L) \phi_\mu + \hat{\lambda}_R \bar{d}_R \gamma^\mu \ell_R \phi_\mu$
$\tilde{U}_1$	1	$(\mathbf{3}, \mathbf{1})_{\frac{5}{3}}$	$\hat{\lambda}_R \bar{u}_R \gamma^\mu \ell_R \phi_\mu$
$V_2$	1	$(\bar{\mathbf{3}}, \mathbf{2})_{\frac{5}{6}}$	$\hat{\lambda}_L (\bar{q}_L^c \cdot \epsilon \cdot \phi_\mu) \gamma^\mu \ell_R + \hat{\lambda}_R \bar{d}_R^c \gamma^\mu (l_L \cdot \epsilon \cdot \phi_\mu) + \hat{\lambda}_{qq} \bar{u}_R \gamma^\mu (q_L^c \cdot \phi_\mu)$
$\tilde{V}_2$	1	$(\bar{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}}$	$\hat{\lambda}_R \bar{u}_R^c \gamma^\mu (l_L \cdot \epsilon \cdot \phi_\mu) + \hat{\lambda}_{qq} (\bar{q}_L \cdot \phi_\mu) \gamma^\mu d_R^c$
$U_3$	1	$(\mathbf{3}, \mathbf{3})_{\frac{2}{3}}$	$\hat{\lambda}_L (\bar{q}_L \cdot \tau^a \cdot \gamma^\mu l_L) \phi_\mu^a$

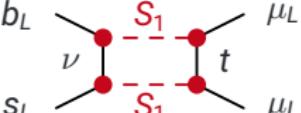
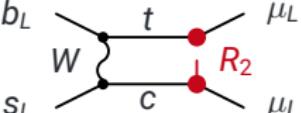
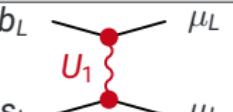
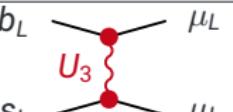
Table from Christoph Niehoff, PhD thesis

# Leptoquark contributions to WET Wilson coefficients

	$C_9^{\text{NP}}$	$C_{10}^{\text{NP}}$	$C'_9$	$C'_{10}$	$C_S$	$C_P$	$C'_S$	$C'_P$
$S_1$	—	—	—	—	—	—	—	—
$\tilde{S}_1$	—	—	$-\frac{1}{2}\lambda_R^{b\ell}\lambda_R^{s\ell*}$	$+C'_9$	—	—	—	—
$R_2$	$\frac{1}{2}\lambda_L^{s\ell}\lambda_L^{b\ell*}$	$+C_9^{\text{NP}}$	—	—	—	—	—	—
$\tilde{R}_2$	—	—	$-\frac{1}{2}\lambda_R^{s\ell}\lambda_R^{b\ell*}$	$-C'_9$	—	—	—	—
$S_3$	$\frac{3}{4}\lambda_L^{b\ell}\lambda_L^{s\ell*}$	$-C_9^{\text{NP}}$	—	—	—	—	—	—
$U_1$	$-\frac{1}{2}\lambda_L^{s\ell}\lambda_L^{b\ell*}$	$-C_9^{\text{NP}}$	$-\frac{1}{2}\lambda_R^{s\ell}\lambda_R^{b\ell*}$	$+C'_9$	$\lambda_L^{s\ell}\lambda_R^{b\ell*}m_b^{-1}$	$-C_S$	$-\lambda_R^{s\ell}\lambda_L^{b\ell*}m_b^{-1}$	$+C'_S$
$\tilde{U}_1$	—	—	—	—	—	—	—	—
$V_2$	$-\frac{1}{2}\lambda_L^{b\ell}\lambda_L^{s\ell*}$	$+C_9^{\text{NP}}$	$\frac{1}{2}\lambda_R^{b\ell}\lambda_R^{s\ell*}$	$-C'_9$	$\lambda_L^{b\ell}\lambda_R^{s\ell*}m_b^{-1}$	$-C_S$	$-\lambda_R^{b\ell}\lambda_L^{s\ell*}m_b^{-1}$	$+C'_S$
$\tilde{V}_2$	—	—	—	—	—	—	—	—
$U_3$	$-\frac{3}{2}\lambda_L^{b\ell}\lambda_L^{s\ell*}$	$-C_9^{\text{NP}}$	—	—	—	—	—	—

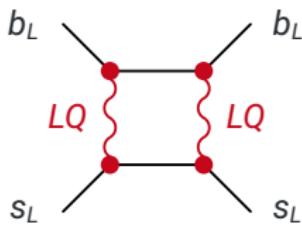
Table from Christoph Niehoff, PhD thesis

# Leptoquarks: possible solutions for $b \rightarrow s\mu\mu$

Spin	$G_{\text{SM}}$	Name	Characteristic process	First time used for $b \rightarrow s\mu\mu$
0	$(\bar{3}, 1)_{1/3}$	$S_1$		Bauer, Neubert, arXiv:1511.01900
0	$(\bar{3}, 3)_{1/3}$	$S_3$		Hiller, Schmaltz, arXiv:1408.1627
0	$(3, 2)_{7/6}$	$R_2$		Bećirević, Sumensari, arXiv:1704.05835
1	$(3, 1)_{2/3}$	$U_1$		Barbieri et al., arXiv:1512.01560
1	$(3, 3)_{2/3}$	$U_3$		Fajfer, Košnik, arXiv:1511.06024

# Leptoquarks: $B_s$ - $\bar{B}_s$ mixing loop-suppressed

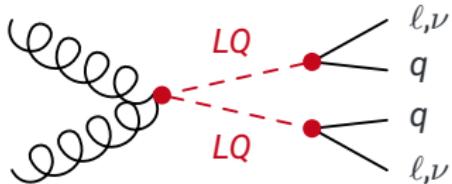
- Generic strong constraint on  $Z'$  models is loop-suppressed for leptoquark models



- Big advantage compared to  $Z'$

# Leptoquarks: direct constraints

- QCD pair production
- Direct searches with  $jj\ell\ell$  or  $jj\nu\nu$  final states



Decays	Scalar LQ limits	Vector LQ limits	$\mathcal{L}_{\text{int}} / \text{Ref.}$
$jj\tau\bar{\tau}$	—	—	—
$b\bar{b}\tau\bar{\tau}$	1.0 (0.8) TeV	1.5 (1.3) TeV	$36 \text{ fb}^{-1}$ [39]
$t\bar{t}\tau\bar{\tau}$	1.4 (1.2) TeV	2.0 (1.8) TeV	$140 \text{ fb}^{-1}$ [40]
$jj\mu\bar{\mu}$	1.7 (1.4) TeV	2.3 (2.1) TeV	$140 \text{ fb}^{-1}$ [41]
$b\bar{b}\mu\bar{\mu}$	1.7 (1.5) TeV	2.3 (2.1) TeV	$140 \text{ fb}^{-1}$ [41]
$t\bar{t}\mu\bar{\mu}$	1.5 (1.3) TeV	2.0 (1.8) TeV	$140 \text{ fb}^{-1}$ [42]
$jj\nu\bar{\nu}$	1.0 (0.6) TeV	1.8 (1.5) TeV	$36 \text{ fb}^{-1}$ [43]
$b\bar{b}\nu\bar{\nu}$	1.1 (0.8) TeV	1.8 (1.5) TeV	$36 \text{ fb}^{-1}$ [43]
$t\bar{t}\nu\bar{\nu}$	1.2 (0.9) TeV	1.8 (1.6) TeV	$140 \text{ fb}^{-1}$ [44]

Angelescu, Bećirević, Faroughy, Jaffredo, Sumensari, arXiv:2103.12504

# Leptoquarks: still viable solutions for $b \rightarrow s\mu\mu$

Spin	$G_{\text{SM}}$	Name	Characteristic process	$R_{K^{(*)}}$	
0	$(\bar{3}, 1)_{1/3}$	$S_1$		X	requires too large couplings
0	$(\bar{3}, 3)_{1/3}$	$S_3$		✓	
0	$(3, 2)_{7/6}$	$R_2$		X	tension with LHC limits
1	$(3, 1)_{2/3}$	$U_1$		✓	
1	$(3, 3)_{2/3}$	$U_3$		✓	

cf. Angelescu, Bećirević, Faroughy, Jaffredo, Sumensari, arXiv:2103.12504

# Model building challenges

# Model building challenges

Single-particle tree-level explanation possible with neutral  $Z'$  vector boson, scalar  $S_3$  or vector  $U_1$  (or  $U_3$ ) leptoquark.

But we need to **build UV-complete models!**

- ▶ Vector boson  $Z'$  or  $U_1$  LQ:
    - ▶ UV completion for massive vector boson: **gauge boson** or **composite state**
    - ▶ Quark-flavor violating coupling might require quarks mixing with **vector-like quarks**
    - ▶ **Find proper symmetry group** that **contains  $Z'$  or  $U_1$**  (gauged and gauge-anomaly free for gauge boson or global group for composite state)  
E.g.  $G \supset U(1)'$  for  $Z'$  or  $G \supset SU(4)$  for  $U_1$
  - ▶ Scalar  $S_3$  LQ:
    - ▶  $S_3$  generically **couples to all lepton flavors** and to **diquarks**
    - ▶ Without protection mechanism: **excessive contributions** to **LFV** and **proton decay!**
    - ▶ Charge  $S_3$  under **new  $U(1)$  symmetry** to forbid diquark couplings and allow only second-generation lepton couplings
- Hambye, Heeck, arXiv:1712.04871  
Davighi, Kirk, Nardecchia, arXiv:2007.15016
- ⇒ Leptoquark coupling only to muons: "**muoquark**"      Greljo, PS, Thomsen, arXiv:2103.13991

More model building challenges in interactive session by Javier Fuentes-Martin

# Conclusions

# Conclusions

- ▶ Discrepancies in numerous  $b \rightarrow s\ell\ell$  observables can be **consistently explained by NP**
- ▶ Global fits show preference for NP contributions to  $C_9^\mu$  and/or  $C_{10}^\mu$
- ▶ Main source of theory uncertainties due to **non-local hadronic contributions**
- ▶ SM predictions of **LFU observables** very well under control  
⇒ experimental observation of discrepancy in these observables would be **clear sign of NP**
- ▶ Interpretation in terms of a single **new particle** possible:  
neutral  $Z'$  vector bosons, scalar  $S_3$  or vector  $U_1$  (or  $U_3$ ) leptoquark
- ▶ **NP models** imply **effects in many other observables** in indirect ( $B_s$ - $\bar{B}_s$  mixing, LFV, etc.) and direct (e.g.  $pp \rightarrow \ell\ell, pp \rightarrow jj\ell\ell$ ) searches.

# Backup slides

# *p*-value of the SM fit

# $p$ -value of the SM fit

$p$ -value of goodness-of-fit from Wilks' theorem

$$p_{SM} = 1 - F(\chi^2_{SM}; n_{obs})$$

with  $F(\chi^2; n_{obs})$  the  $\chi^2$  CDF and  $n_{obs}$  the number of independent observables (measurements of an observable by different experiments counted separately).

- **ACDMN** (Algueró, Capdevila, Descotes-Genon, Matias, Novoa-Brunet), arXiv:2104.08921

$$\begin{aligned} \textit{Global fit} : n_{obs} &= 246 &\Rightarrow p &= 1.1\% \\ \textit{LFU fit*} : n_{obs} &= 22 &\Rightarrow p &= 1.4\% \end{aligned}$$

- **AS** (Altmannshofer, PS), arXiv:2103.13370

$$\begin{aligned} \textit{Global fit} : n_{obs} &= 191 &\Rightarrow p &= 1.2\% \\ \textit{LFU fit*} : n_{obs} &= 21 &\Rightarrow p &= 0.5\% \end{aligned}$$

- **HMMN** (Hurth, Mahmoudi, Martínez-Santos, Neshatpour), arXiv:2104.10058

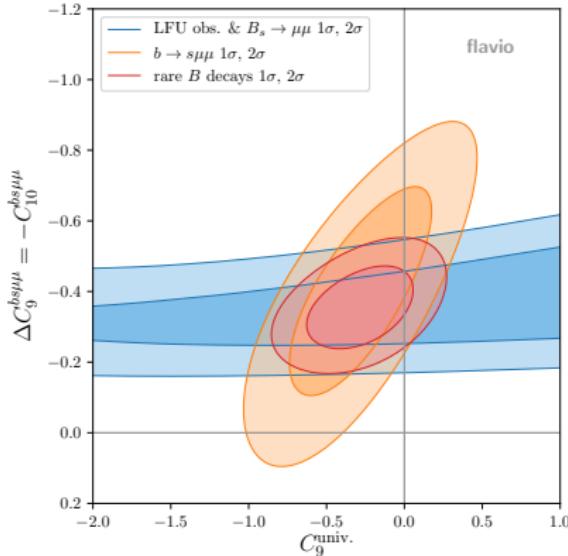
$$\begin{aligned} \textit{Global fit} : n_{obs} &= 173 &\Rightarrow p &= 0.4\% \\ \textit{LFU fit*} : n_{obs} &= 7 &\Rightarrow p &= 0.02\% \end{aligned}$$

\* LFU fit: all the measured LFU observables +  $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$  (all groups)

+ effective  $B_s \rightarrow \mu \mu$  lifetime + radiative decays +  $\mathcal{B}(B_s \rightarrow X_s \mu^+ \mu^-)$  (depending on the group)

# Scenario with universal $C_9$

# Scenarios with two Wilson coefficients



WET at 4.8 GeV

- ▶ Perform two-parameter fit in space of  $C_9^{\text{univ.}}$  and  $\Delta C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu}$ :

$$C_9^{\text{bssee}} = C_9^{bs\tau\tau} = C_9^{\text{univ.}}$$

$$C_9^{bs\mu\mu} = C_9^{\text{univ.}} + \Delta C_9^{bs\mu\mu}$$

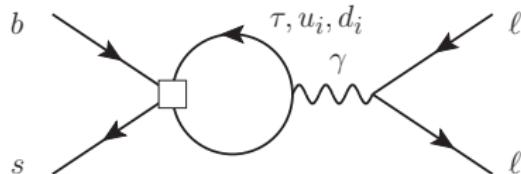
$$C_{10}^{\text{bssee}} = C_{10}^{bs\tau\tau} = 0$$

$$C_{10}^{bs\mu\mu} = -\Delta C_9^{bs\mu\mu}$$

scenario first considered in  
Algueró et al., arXiv:1809.08447

- ▶ Slight preference for **non-zero  $C_9^{\text{univ.}}$**

- ▶ could be mimicked by hadronic effects
- ▶ can arise from RG effects:

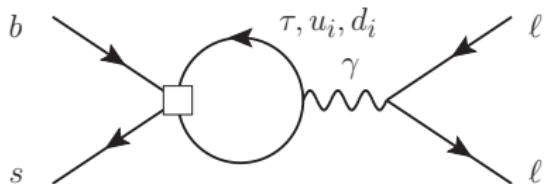


Bobeth, Haisch, arXiv:1109.1826  
Crivellin, Greub, Müller, Saturnino, arXiv:1807.02068

# RG effect in SMEFT

RG effects require scale separation

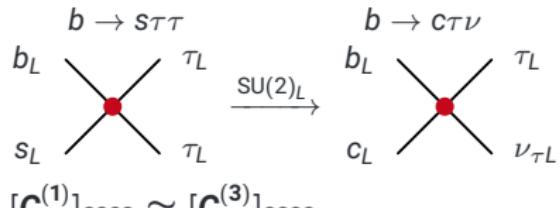
- ▶ Consider **SMEFT**



Possible operators:

- ▶  $[O_{lq}^{(3)}]_{3323} = (\bar{l}_3 \gamma_\mu \tau^a l_3)(\bar{q}_2 \gamma^\mu \tau^a q_3)$ :  
Might also **explain  $R_D^{(*)}$  anomalies!**

- ▶  $[O_{lq}^{(1)}]_{3323} = (\bar{l}_3 \gamma_\mu l_3)(\bar{q}_2 \gamma^\mu q_3)$ :  
Strong constraints from  $B \rightarrow K \nu \nu$  require  $[C_{lq}^{(1)}]_{3323} \approx [C_{lq}^{(3)}]_{3323}$



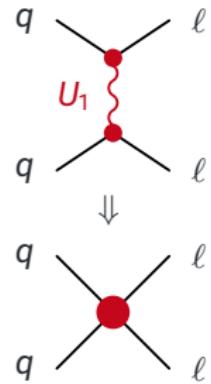
Buras et al., arXiv:1409.4557

- ▶  **$U_1$  vector leptoquark  $(3, 1)_{2/3}$**  couples LH fermions

$$\mathcal{L}_{U_1} \supset g_{lq}^{ji} \left( \bar{q}^i \gamma^\mu l^j \right) U_\mu + \text{h.c.}$$

- ▶ Generates **semi-leptonic operators at tree-level**

$$[C_{lq}^{(1)}]_{ijkl} = [C_{lq}^{(3)}]_{ijkl} = -\frac{g_{lq}^{jk} g_{lq}^{il*}}{2M_U^2}$$



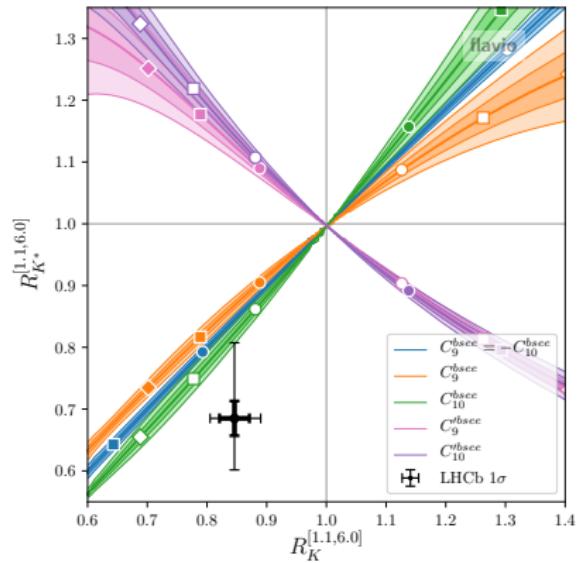
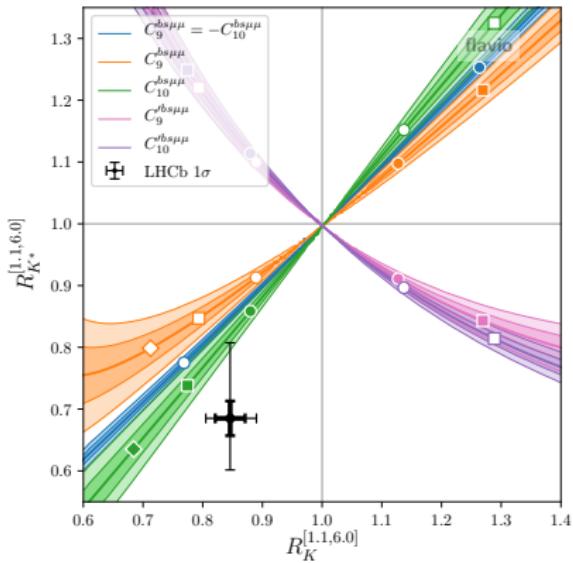
# Theory uncertainties in presence of NP

# Scenarios with a single Wilson coefficients

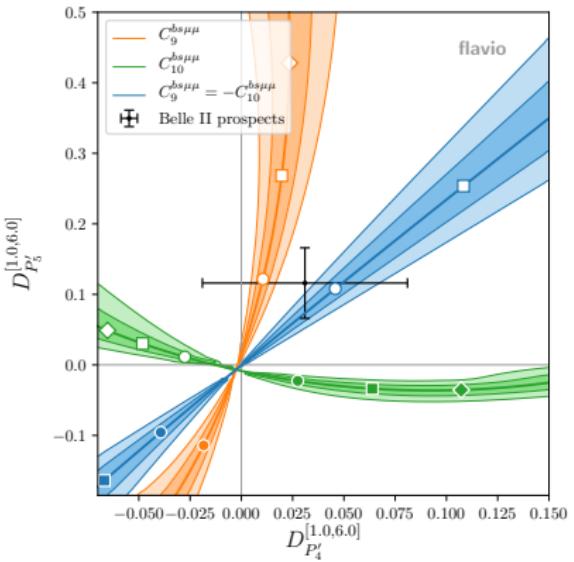
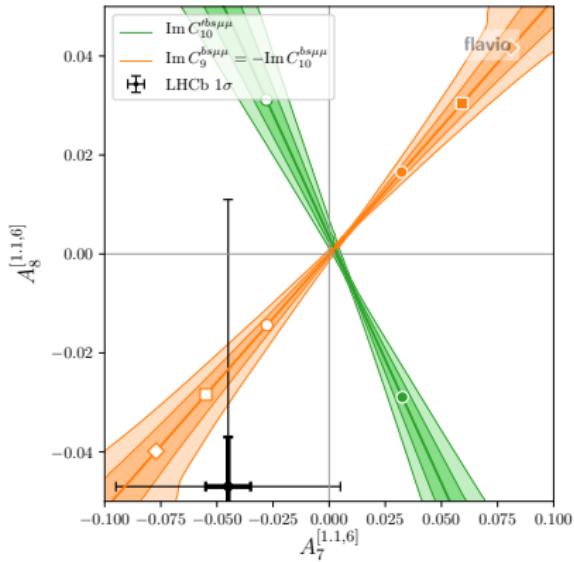
	Wilson coefficient	$b \rightarrow s\mu\mu$		LFU, $B_s \rightarrow \mu\mu$		all rare $B$ decays	
		best fit	pull	best fit	pull	best fit	pull
NP err.	$C_9^{bs\mu\mu}$	$-0.70^{+0.21}_{-0.22}$	<b>3.3<math>\sigma</math></b>	$-0.74^{+0.20}_{-0.21}$	$4.1\sigma$	$-0.71^{+0.15}_{-0.15}$	<b>5.1<math>\sigma</math></b>
	$C_{10}^{bs\mu\mu}$	$+0.45^{+0.22}_{-0.23}$	$1.9\sigma$	$+0.60^{+0.14}_{-0.14}$	<b>4.7<math>\sigma</math></b>	$+0.54^{+0.12}_{-0.12}$	$4.8\sigma$
	$C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu}$	$-0.55^{+0.13}_{-0.13}$	<b>3.8<math>\sigma</math></b>	$-0.35^{+0.08}_{-0.08}$	<b>4.6<math>\sigma</math></b>	$-0.39^{+0.07}_{-0.07}$	<b>5.6<math>\sigma</math></b>
SM err.	$C_9^{bs\mu\mu}$	$-0.83^{+0.22}_{-0.20}$	<b>3.6<math>\sigma</math></b>	$-0.74^{+0.20}_{-0.21}$	$4.1\sigma$	$-0.77^{+0.15}_{-0.15}$	<b>5.3<math>\sigma</math></b>
	$C_{10}^{bs\mu\mu}$	$+0.45^{+0.21}_{-0.20}$	$2.3\sigma$	$+0.60^{+0.14}_{-0.14}$	<b>4.7<math>\sigma</math></b>	$+0.54^{+0.12}_{-0.12}$	$4.9\sigma$
	$C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu}$	$-0.60^{+0.17}_{-0.18}$	<b>3.8<math>\sigma</math></b>	$-0.35^{+0.08}_{-0.08}$	<b>4.6<math>\sigma</math></b>	$-0.39^{+0.07}_{-0.07}$	<b>5.6<math>\sigma</math></b>

Visible effect of theory errors depending on new physics, in particular for  $C_9^{bs\mu\mu}$

# Theory uncertainties in presence of NP



# Theory uncertainties in presence of NP



# Parameterisation of beyond-QCDF contributions

# Parameterisation of beyond-QCDF contributions for $B \rightarrow K$

$$C_9^{\text{eff}}(q^2) \rightarrow C_9^{\text{eff}}(q^2) + a_K + b_K(q^2 / \text{GeV}^2) \quad \text{at low } q^2 ,$$

$$C_9^{\text{eff}}(q^2) \rightarrow C_9^{\text{eff}}(q^2) + c_K \quad \text{at high } q^2 ,$$

$$\text{Re}(a_K) = 0.0 \pm 0.08 , \quad \text{Re}(b_K) = 0.0 \pm 0.03 , \quad \text{Re}(c_K) = 0.0 \pm 0.2 ,$$

$$\text{Im}(a_K) = 0.0 \pm 0.08 , \quad \text{Im}(b_K) = 0.0 \pm 0.03 , \quad \text{Im}(c_K) = 0.0 \pm 0.2 .$$

$1\sigma$  uncertainties enclose the effects considered in [Khodjamirian et al. arXiv:1006.4945](#),  
[Beylich et al. arXiv:1101.5118](#), [Khodjamirian et al. arXiv:1211.0234](#)

# Parameterisation of beyond-QCDF contributions for $B \rightarrow K^*$ and $B_s \rightarrow \phi$

$$\begin{aligned} C_7^{\text{eff}}(q^2) &\rightarrow C_7^{\text{eff}}(q^2) + a_{0,-} + b_{0,-}(q^2/\text{GeV}^2) && \text{at low } q^2, \\ C'_7 &\rightarrow C'_7 + a_+ + b_+(q^2/\text{GeV}^2) \end{aligned}$$

$$C_9^{\text{eff}}(q^2) \rightarrow C_9^{\text{eff}}(q^2) + c_\lambda \quad \text{at high } q^2,$$

$\text{Re}(a_+) = 0.0 \pm 0.004$ ,	$\text{Re}(b_+) = 0.0 \pm 0.005$ ,	$\text{Re}(c_+) = 0.0 \pm 0.3$ ,
$\text{Im}(a_+) = 0.0 \pm 0.004$ ,	$\text{Im}(b_+) = 0.0 \pm 0.005$ ,	$\text{Im}(c_+) = 0.0 \pm 0.3$ ,
$\text{Re}(a_-) = 0.0 \pm 0.015$ ,	$\text{Re}(b_-) = 0.0 \pm 0.01$ ,	$\text{Re}(c_-) = 0.0 \pm 0.3$ ,
$\text{Im}(a_-) = 0.0 \pm 0.015$ ,	$\text{Im}(b_-) = 0.0 \pm 0.01$ ,	$\text{Im}(c_-) = 0.0 \pm 0.3$ ,
$\text{Re}(a_0) = 0.0 \pm 0.12$ ,	$\text{Re}(b_0) = 0.0 \pm 0.05$ ,	$\text{Re}(c_0) = 0.0 \pm 0.3$ ,
$\text{Im}(a_0) = 0.0 \pm 0.12$ ,	$\text{Im}(b_0) = 0.0 \pm 0.05$ ,	$\text{Im}(c_0) = 0.0 \pm 0.3$ .

$1\sigma$  uncertainties enclose the effects considered in [Khodjamirian et al. arXiv:1006.4945](#),  
[Beylich et al. arXiv:1101.5118](#)