1 Adventure 1: Interpreting the low-energy data

1.1 A simple fit to B-physics data

In this project we are going to reproduce part of the results presented in Ref [1] (with updated experimental inputs). By the time of this reference, it was clear that there was an anomaly in B-physics data, so an EFT and simplified NP interpretation was needed. Our goal is to rediscover the results of this reference.

Consider the following EFT Lagrangian,

$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm SM} - \frac{1}{v^2}\,\lambda^q_{ij}\lambda^\ell_{\alpha\beta}\left[C_T\,(\bar q^i_L\gamma_\mu\tau^I q^j_L)(\bar\ell^\alpha_L\gamma^\mu\tau^I\ell^\beta_L) + C_S\,(\bar q^i_L\gamma_\mu q^j_L)(\bar\ell^\alpha_L\gamma^\mu\ell^\beta_L)\right]\,, \tag{1.1}$$

where τ^I are the Pauli matrices, $i,j\,(\alpha,\beta)$ are quark (lepton) flavor indices and

$$q^i_L = \begin{pmatrix} V^*_{ji}\,u^j_L \\ d^i_L \end{pmatrix}\,, \qquad \ell^\alpha_L = \begin{pmatrix} \nu^\alpha_L \\ e^\alpha_L \end{pmatrix}\,, \tag{1.2}$$

with V_{ij} being the CKM matrix elements. We are going to do a χ^2 fit to the following data

Observable	Experiment/constraint	SM prediction
$\{R^{\tau\ell}_D,\, R^{\tau\ell}_{D^*}\}$	$\{0.337\pm0.030, 0.298\pm0.014\}$, $\rho=-0.42$ [2]	$\{0.299\pm0.003, 0.258\pm0.005\}$ [3]
$\{C^\mu_9 = -C^\mu_{10},\, C^U_9\}$	$\{-0.30\pm0.12, -0.74\pm0.22\}$, $\rho=-0.44$ [4]	0
$R^{\mu e}_D$	0.978 ± 0.035 [5,6]	1
R^ν_K	2.4 ± 0.9 [7]	1
$R^\nu_{K^*}$	0.0 ± 1.6 [8]	1
$(g_\tau/g_{e,\mu})_{\ell,\pi,K}$	1.0012 ± 0.0012 [9]	1
$\delta(\Delta m_{B_s})$	0.0 ± 0.1	0

Table 1.1: Low-energy data for the fit.

1. Assuming negligible NP contributions to electrons and setting $\lambda^q_{bb} = \lambda^\ell_{\tau\tau} = 1$ for simplicity, find that the tree-level linearized EFT predictions for the low-energy observables in the table are:

$$\begin{aligned} R_{D^{(*)}} &= R_{D^{(*)}}\big|_{\rm SM}\left[1 + 2C_T(1-\lambda^q_{sb}V^*_{tb}/V^*_{ts})(1-\lambda^\ell_{\mu\mu}/2)\right]\,,\\ C^\mu_9 = -C^\mu_{10} &= -\frac{\pi}{\alpha_{\rm em}V_{tb}V^*_{ts}}\lambda^\ell_{\mu\mu}\lambda^q_{sb}(C_T+C_S)\,, \qquad C^U_9 = 0\,,\\ R^{\mu e}_D &= 1 + 2C_T(1-\lambda^q_{sb}V^*_{tb}/V^*_{ts})\lambda^\ell_{\mu\mu}\,,\\ R^\nu_{K^{(*)}} &= 1 + \frac{2}{3}\frac{\pi}{\alpha_{\rm em}V_{tb}V^*_{ts}C^{\rm SM}_\nu}(C_T - C_S)\,\lambda^q_{sb}(1+\lambda^\ell_{\mu\mu})\,, \end{aligned} \tag{1.3}$$

with $C^{\rm SM}_\nu = -6.4$.

2. Do a χ^2 fit to the data in the table using the expressions in (1.3), and imposing the following constraints on the fit parameters

$$-0.1 < C_S < 0.1\,, \quad -0.1 < C_T < 0.1\,, \quad -0.5 < \lambda^q_{sb} < 0.5\,, \quad -5\times10^{-2} < \lambda^\ell_{\mu\mu} < 0\,. \tag{1.4}$$

Once the fit is working produce analogous plots to those in Ref [1].

Hint: Do not hesitate to ask for help with the fit if this is your first time doing them!

3. It turns out, as it was discovered at the time, that loop-induced contributions have a significant impact in the fit to the data. Show that the RGE-induced contributions to $(g_\tau/g_{e,\mu})_{\ell,\pi,K}$ [10] and C_9^U [11] are given by (set $\Lambda = 2$ TeV)

$$\begin{aligned}(g_\tau/g_{e,\mu})_{\ell,\pi,K} &\approx 1 - \frac{6y_t^2}{16\pi^2} C_T \log \frac{\Lambda}{m_t} \,, \\ C_9^U(m_b) &\approx \frac{1}{V_{ts}^* V_{tb}} \frac{1}{3} (C_T + C_S)\, \lambda_{sb}^q \ln \frac{\Lambda^2}{m_b^2} \,.\end{aligned} \tag{1.5}$$

Hint: There are dedicated tools that can help you obtaining the RG evolution of the EFT coefficients, such as `DsixTools` (Mathematica) [12] or `Wilson` (Python) [13]. Ask for help if you would like to learn more about these tools. They allow you to perform the full RGE resummation yielding more accurate results than just the leading logarithmic contribution.

4. Repeat the χ^2 now including the RGE-induced contributions and the theoretical constraint (with cutoff $\Lambda_{bs} = 1$ TeV, see *Adventure 3*)

$$\delta(\Delta m_{B_s}) = \left| \frac{\Lambda_{bs}^2}{2m_W^2\, S_0(x_t)} \left\{ \left[\frac{\lambda_{sb}^q\,(C_T + C_S)}{2V_{tb}^* V_{ts}} \right]^2 + \left[\frac{\lambda_{sb}^q\,(C_T - C_S)}{2V_{tb}^* V_{ts}} \right]^2 \right\} \right| \,, \tag{1.6}$$

with $x_t = m_t^2/m_W^2$ and $S_0(x_t) \approx 2.37$ [14].

5. A list of all possible tree-level mediators that can contribute to the EFT in (1.1), together with their matching conditions, can be found in Ref. [15]. Based on the EFT fit, use this list to determine which mediator or pair of mediators would provide a good explanation to the data.

   Hint: Note that colorless mediators would also contribute to B_s mixing (the $\delta(\Delta m_{B_s})$ constraint in the table) at tree-level with a strength proportional to λ_{sb}^q.

1.2 A fit to EW precision and Higgs data using `smelli`. The *W* mass anomaly.

In this second part, we are going to use the power of existing tools to perform a fit to EW precision and Higgs data, and study the recent W mass anomaly from a model building perspective.

1. Follow Ref. [16] (sections 3.1, 3.2.1, and 3.2.5) to produce the current constraints (excluding the recent CDF W mass measurement) on the S and T parameters using the Python package `smelli`.

   Hint: You can find the package maintainer in the audience, feel free to ask him for help if you need to!

2. In order to modify the measurements that are taken into account by `smelli`, you should install it in development mode:

   - If you have `git` installed on your system, simply clone the `smelli` GitHub repository. To this end, open a terminal, change to the folder in which you want to put the smelli repository, and enter

     ```
     git clone https://github.com/smelli/smelli.git
     ```

     Alternatively, you can also download a zip file from https://github.com/smelli/smelli/archive/refs/heads/master.zip and extract it in a folder of your choice.

   - Now change to the newly created folder and install the package using the command

     ```
     python3 -m pip install -e . --user
     ```

The `-e` switch means that the package is installed in "development mode", so you can make modifications to the downloaded code and do not have to reinstall.

2. In the `smelli` folder, modify the yaml file that specifies which measurements are used for the EW precision observables, `smelli/data/yaml/measurements_ewpt.yaml`. This file already includes two measurements of m_W, `m_W ATLAS 2017` and `m_W Tevatron`, which correspond to the 2017 ATLAS measurement [17] and the 2013 combination of CDF and D0 measurements [18]. For simplicity, we will replace these measurements by a recent combination [19] which is obtained by averaging all W mass measurements and adding a scaling factor to the error to account for the internal discrepancy. Comment out the two lines containing `m_W ATLAS 2017` and `m_W Tevatron` by prepending them with `#` and add the following line to the yaml file (starting with a dash and a space):

```
- m_W combination 2022
```

This is our name for the 2022 combination, which we will implement in the next step. After modifying the yaml file, its last three lines should look like

```
#- m_W ATLAS 2017
#- m_W Tevatron
- m_W combination 2022
```

3. In your Python session (or Jupyter notebook), implement the 2022 combination of m_W measurements by entering the following lines:

```
import flavio
m = flavio.Measurement('m_W combination 2022')
m.set_constraint('m_W', '80.4133 +- 0.015')
```

This imports the `flavio` package (which is used by `smelli` and provides the theory predictions and a database of measurements), creates a new `Measurement` instance with name `m_W combination 2022`, and sets the corresponding value for the observable `m_W` to the combination from Ref. [19].

3. Now redo the fit form step 1, which will now include the newly added measurement in any newly created `GlobalLikelihood` instance.

4. Use the following relation between the EW and SMEFT parameters

$$C_{\phi WB} = \frac{g_L\, g_Y}{16\,\pi\, v^2}\, S\,, \qquad C_{\phi D} = -\frac{g_L^2\, g_Y^2}{2\,\pi\,(g_L^2 + g_Y^2)\, v^2}\, T. \tag{1.7}$$

and the NP-SMEFT dictionary in Ref. [15] to determine which mediators could potentially account for the anomaly at tree level.

Disclaimer: Note that the W mass can receive corrections from other SMEFT parameters as well. For a more comprehensive discussion see Ref. [20].

2 Adventure 2: Matching UV models to their EFTs

In this project, we will delve a little deeper into specific UV completions. Two SM model extensions that successfully reproduce the B-physics data, discussed in *Adventure 1*, are those containing a U_1 vector leptoquark, and those with an S_1 and S_3 scalar leptoquarks. The Lagrangians of those two simplified models are

$$\begin{aligned}\mathcal{L}_{U_1} &= \mathcal{L}_{\rm SM} - \frac{1}{2}\,U_{\mu\nu}^\dagger\,U^{\mu\nu} + M_U^2\,U_\mu^\dagger\,U^\mu - ig_s\,(1-\kappa_U)\,U_\mu^\dagger\,T^a\,U_\nu\,G^{a\,\mu\nu} - ig_Y\,\frac{2}{3}\,(1-\tilde{\kappa}_U)\,U_\mu^\dagger\,U_\nu\,B^{\mu\nu} \\ &\quad + \frac{g_U}{\sqrt{2}}\,[U^\mu\,(\beta_L^{ij}\,\bar{q}_L^i\gamma_\mu\ell_L^j + \beta_R^{ij}\,\bar{d}_R^i\gamma_\mu e_R^j) + \text{h.c.}]\,, \qquad (2.1)\\ \mathcal{L}_{S_1+S_3} &= \mathcal{L}_{\rm SM} + |D_\mu S_1|^2 + |D_\mu S_3|^2 - M_1^2|S_1|^2 - M_3^2|S_3|^2 + \left[\lambda_{1L}^{i\alpha}(\bar{q}_i^c\epsilon\ell_\alpha) + \lambda_{1R}^{i\alpha}(\bar{u}_i^c e_\alpha)\right]S_1 \\ &\quad + \lambda_{3L}^{i\alpha}(\bar{q}_i^c\epsilon\sigma^I\ell_\alpha)S_3^I + \text{h.c.} - \lambda_{H1}|H|^2|S_1|^2 - \lambda_{H3}|H|^2|S_3^I|^2 - \lambda_{\epsilon H3} i\epsilon^{IJK}(H^\dagger\tau^I H)S_3^{J\dagger}S_3^K \\ &\quad - \left[\lambda_{H13}(H^\dagger\tau^I H)S_3^{I\dagger}S_1 + \text{h.c.}\right]\,, \qquad (2.2)\end{aligned}$$

with $U_{\mu\nu} = D_\mu U_\nu - D_\nu U_\mu$ and the $SU(2)_L$ doublets decomposing as in (1.2).

1. Perform the tree-level matching of these mediators to the SMEFT and use Fierz relations to relate your results to the Lagrangian in (1.1). As you will see, new operators other than those in (1.1) are generated, but as soon as only left-handed currents are considered (e.g. $\beta_R = \lambda_{1R} = 0$) we recover this Lagrangian. Show that the U_1 leptoquark further predicts $C_S = C_T$ at tree-level, as required by fit to the low-energy data (see *Adventure 1*). On the contrary, a tuning of S_1 and S_3 contributions is needed to keep $C_S \approx C_T$.

2. We are going to study the potential of these two models for explaining the $(g-2)_\mu$ anomaly. To this end, we need to obtain the one-loop matching contributions to the dipole operators

$$O_{eW}^{\alpha\beta} = \frac{g_L}{16\pi^2}(\bar{\ell}_L^\alpha\sigma^{\mu\nu}T^I e_R^\beta)HW_{\mu\nu}^I\,, \qquad O_{eB}^{\alpha\beta} = \frac{g_Y}{16\pi^2}(\bar{\ell}_L^\alpha\sigma^{\mu\nu}e_R^\beta)HB_{\mu\nu}\,, \qquad (2.3)$$

   where $T^I = \tau^I/2$ are the $SU(2)_L$ generators. Compute the contribution of S_1 and U_1 to these operators in the limit in which only $y_{t,b}$ are non-vanishing, assuming that both right-handed currents (e.g. β_R and λ_{1R}) and left-handed currents (e.g. β_L and λ_{1L}) are present and sizable.

   Hint: In order to obtain a finite contribution for the U_1 loop, you need to take $\tilde{\kappa}_U = 0$. This is the value you would obtain when the U_1 arises from a gauge theory (see *Adventure 4*), and it points to the need of a UV completion to perform loop calculations with this mediator.[1]

   Disclaimer: If you do not have enough time to compute the loops, you can find the results in Refs. [21] and [22] for U_1 and S_1, respectively.

3. Show that the simultaneous presence of $\beta_L^{s\mu}$ (as required to explain the $b \to s\mu^+\mu^-$ anomalies, see *Adventure 1*) and $\beta_R^{b\mu}$ implies a chiral-enhanced contribution to $\mathcal{B}(B_s \to \mu^+\mu^-)$. This prevents the U_1 from providing a simultaneous explanation to B and $(g-2)_\mu$ anomalies. Show that this is not the case for the S_1 mediator. As discussed in Ref. [23], the $S_{1,3}$ extension is able to simultaneously accommodate B and $g-2$ anomalies (at the cost of having a NP flavor structure that significantly departs from that of the SM Yukawas).

   Hint: You can find the expression of $\mathcal{B}(B_s \to \mu^+\mu^-)$ in terms of the LEFT Wilson coefficients by looking at (D.5), (D.6) and (D.16) of Ref [24].

[1] Actually, you would also need to consider additional particles from the U_1 UV completion such as would-be Goldstone or additional gauge bosons and fermions to consistently compute loops, see Ref. [21] for more details. However, the y_b-enhanced dipole contribution is one of the few exceptions where these additional particles are not needed.

4. As you saw in point 1, the U_1 mediates a tree-level contribution to the EFT operator

$$O_{LR}^{ij\alpha\beta} = (\bar{q}_L^i \gamma_\mu \ell_L^\alpha)(\bar{e}_R^\beta \gamma^\mu d_R^j)\,, \tag{2.4}$$

which is related by Fierz identities to the operator $Q_{ledq}^{\alpha\beta ij} = (\bar{\ell}_L^\alpha \gamma_\mu e_R^\beta)(\bar{d}_R^i \gamma^\mu q_L^j)$ as follows

$$O_{LR}^{ij\alpha\beta} = -2\left[Q_{ledq}^\dagger\right]^{\beta\alpha ij}. \tag{2.5}$$

Show that $O_{LR}^{ij\alpha\beta}$ gives a contribution to the $e^\alpha \to e^\beta\gamma$ dipole amplitude while Q^{ledq} does not. This result shows that Fierz identities are not valid at one-loop order and need to be supplemented by extra (evanescent) contributions.

3 Adventure 3: Interpreting high-p_T data and making predictions

Disclaimer: To do this project, you need to have `MadGraph` installed in your computer (feel free to ask for help if you do not know how to it!). A `FeynRules` installation in Mathematica is also advised, though not strictly necessary.

In this project, we will study the U_1 mediator in the context of high-p_T LHC searches. To this end, we will consider a process where this mediator is produced on-shell at LHC, U_1 pair production.[2] We will also study a novel search for vector-like leptons based on a model prediction of U_1 UV completions (see *Adventure 4*).

1. We will start by generating the Universal Feynman Output (UFO) for the U_1 model. This is the basic input for `MadGraph` simulations, and can be generated using the Mathematica package `FeynRules`. Check the ancillary files *SM.fr* and *vector_LQ.fr* and make sure you understand how they work (otherwise feel free to ask!). If you have `FeynRules` installed in you computer, use the ancillary file *model_gen.nb* to generate the UFO. Otherwise, you can use the UFO already pre-generated (the folder *vector_LQ_UFO*).[3]

2. We are going to focus on the following parameter benchmark that is well-motivated by the fit to the B-physics anomalies (see U_1 Lagrangian in (2.1))

$$\begin{aligned} &\textbf{Benchmark I:} \quad g_U = 3\,, \quad \kappa_U = 0\,, \quad \beta_L^{b\tau} = 1\,, \quad \beta_R^{b\tau} = 0\,, \\ &\textbf{Benchmark II:} \quad g_U = 3\,, \quad \kappa_U = 0\,, \quad \beta_L^{b\tau} = 1\,, \quad \beta_R^{b\tau} = -1\,, \end{aligned} \tag{3.1}$$

   and all other couplings set to zero. Use `MadGraph` to compute the partial decay widths of the U_1 for these two benchmarks.

   Hint: If you do not know how to run `MadGraph`, you can follow the instructions in the ancillary file *MG_script.dat*.

3. Since the U_1 is a colored particle, it could be directly produced in pairs at the LHC via gluon fusion, represented by Feynman diagrams such as:

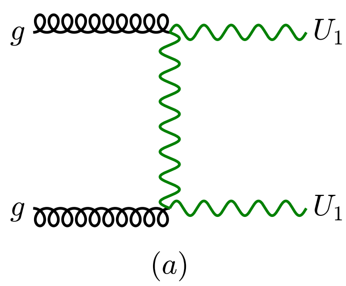

Figure 3.1: Representative diagram for U_1 pair production.

   As it happens with the diagram shown above, leptoquark pair-production cross sections at the LHC are dominated by QCD dynamics and thus are largely insensitive to the leptoquark couplings to fermions, whose main effect is on the leptoquark branching fraction to the different

[2] Similar searches to scalar leptoquarks, like the $S_{1,3}$ considered in *adventure 2*, can be done in complete analogy to the ones discussed here. The main difference is that, begin scalars, these mediators have a lower production cross section, which translates into lower mass limits from pair-production searches [25].

[3] All these files are also publicly available at the following url: `https://feynrules.irmp.ucl.ac.be/wiki/LeptoQuark`.

final states. We are going to use the UFO from the previous point to reproduce the theory predictions of the CMS dedicated search for $pp \to U_1 \bar{U}_1 \to b\tau t\nu$ [26]. Load the UFO file in `MadGraph` and compute the cross section as a function of the U_1 mass for $pp \to U_1 \bar{U}_1$ for benchmark I in (3.1). Compare the cross sections you obtain with the blue line in the first plot of Figure 5 in Ref. [26], and determine the mass limit from the crossing point between your cross-section and the solid black line.

3. As it was first noticed in [27], the quadratic divergence of the U_1 box amplitude contributing to B_s mixing (Figure 3.2) is saturated by the mass of additional vector-like leptons that are required for a consistent UV completion (see *Adventure 4*).

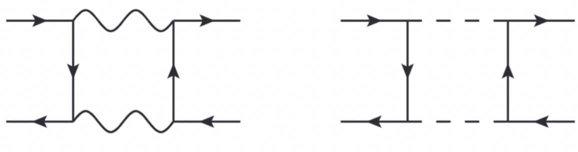

Figure 3.2: Loop diagrams contributing to B_s mixing. The wiggle line represents the U_1 leptoquark while the dashed line is the would-be Goldstone boson that is eaten by the U_1. The fermion lines in the loop include both SM and vector-like leptons.

Such unitarization of the U_1 box amplitude is analogous to the one happening in the SM W box amplitude mediating K-$\bar{K}$ mixing, usually referred to as GIM-mechanism, which gave rise to the prediction of the c quark and provided an upper limit to its mass. Much in the same way, we can use the U_1 contribution to B_s mixing to infer an upper limit on the vector-like lepton mass. Using the same conventions as in Lagrangian (2.1), and following the analysis in Ref. [21], the (finite) loop contribution to the $B_s - \bar{B}_s$ mass difference is found to be

$$\delta(\Delta m_{B_s}) \approx \left| \frac{4M_U^2}{m_W^2\, S_0(x_t)} C_U^2 \left[\frac{(\beta_L^{s\tau})^*}{V_{tb}^* V_{ts}} \right]^2 F_{\Delta F=2}(x_L) \right| . \tag{3.2}$$

where $C_U = g_U^2 v^2/(4M_U^2)$, $x_L = M_L^2/M_U^2$, with M_L being the vector-like lepton mass, and the loop function is given by

$$F_{\Delta F=2}(x) = \frac{x(x+4)(x^2-1)}{8(x-1)^3} \left[\frac{1}{2} + \frac{x \ln x}{1-x^2} \right] , \tag{3.3}$$

Using the expansion $F_{\Delta F=2}(x) = x/4 + \mathcal{O}(x^2)$, the effective cutt-off scale Λ_{bs} introduced in (1.6) can be identified with $\Lambda_{bs} = \sqrt{2}\, M_L$ in the small x_L limit (e.g. $M_L \ll M_U$). Using the parameter values from a low-energy fit not much different from that in *Adventure 1*, it is found that M_L should be at most $1-2$ TeV [28], and therefore within the reach of the LHC.

Using this information, we are going to characterize the search for these vector-like leptons. In particular, the vector-like leptons transform under the SM gauge group as $L \sim (\mathbf{1}, \mathbf{2})_{1/2}$ and the relevant Lagrangian for this search is

$$\mathcal{L}_{\rm L} = (\bar{L}\, i \not{D} L) + \left[\frac{g_U}{\sqrt{2}}\, U_\mu\, \beta_L^{qL}\, (\bar{q}_L^3 \gamma^\mu L_L) + \text{h.c.} \right] , \tag{3.4}$$

where the vector-like $SU(2)_L$ doublet L decomposes as $L = (N\ E)^\intercal$. Due to $SU(2)_L$ invariance, the fermions E and N are mass degenerate to a good approximation.

You are asked to:

3.1 Extend the U_1 `FeynRules` model file from point 1 (*vector_LQ.fr*) to include this new particle and generate the corresponding UFO.

Hint: If you get stuck or do not have `FeynRules` installed, you can use the UFO provided in the ancillary material (the folder *vector_LQ_wVLLs_UFO*).

3.2 With this UFO, use `MadGraph` to reproduce the plots in Figure 14 of Ref [27] (EW production only).

3.3 Note that, with the Lagrangian above, the vector-like leptons can only decay via an off-shell U_1: $E \to b\, U_1^*(b\tau, t\nu)$, $N \to t\, U_1^*(b\tau, t\nu)$. Use `MadGraph` to obtain the partonic events, including the decay of the vector-like leptons using benchmark I.

The search for this type of vector-like leptons has recently been pursued by the CMS Collaboration, you can find their results in Ref. [29].

4 Adventure 4: Going beyond simplified models

In this project, we will revisit the steps that took to the realization that the gauge model $SU(4) \times SU(3)' \times SU(2)_L \times U(1)_X$, commonly denoted as the *4321 model*, is the minimal UV-complete description of a TeV-scale U_1 leptoquark.

1. **The need for a Z'.** Independently on whether the U_1 originates from a gauge theory or strong dynamics, the fields that it couples to need be well-defined representations of a given Lie group $G_{\rm NP}$. For left-handed fermions, we denote this representation ψ_L and, without loss of generality, decompose it as

$$\psi_L = \psi_L^{\rm SM} + \psi_L^{\rm VL} , \qquad \psi_L^{\rm SM} = \begin{pmatrix} q_L^\beta \\ \ell_L \end{pmatrix} , \tag{4.1}$$

where we accounted for possible mixing with vector-like fermions (see below) by including $\psi_L^{\rm VL}$. In this notation, the left-handed current in (2.1) can be written as $(J_U^L)_\mu^\alpha = \bar\psi_L^{\rm SM}(T_+^\alpha)\gamma_\mu\psi_L^{\rm SM}$ with the following explicit expression for the action of the $G_{\rm NP}$ generators on the SM projection of ψ_L:

$$T_+^\alpha = \begin{pmatrix} 0 & \delta_{\alpha\beta} \\ 0 & 0 \end{pmatrix} . \tag{4.2}$$

Show that the closure of the algebra of the six generators $T_\pm^\alpha$, associated with the six components of U_1 and $U_1^\dagger$, implies the need of the following additional (color-neutral) generator

$$T_{B-L} = \begin{pmatrix} \frac{1}{3}\delta_{\beta\gamma} & 0 \\ 0 & -1 \end{pmatrix} . \tag{4.3}$$

Hint: T_-^α can be obtained, similarly to T_+^α, from the hermitian conjugate of J_U^L. To the closure of the algebra relation you need to prove the following identity

$$\frac{1}{3}\sum_{\alpha,\delta=1}^{3}[T_+^\alpha, T_-^\delta] = T_{B-L} . \tag{4.4}$$

The same conclusion is reached by looking at the right-handed current in (2.1). Hence we see that any consistent UV theory containing a U_1 should also contain a Z' associated to the $U(1)_{\rm B-L}$ group.

2. **The Z' problem.** One can follow similar closure of the algebra relations to deduce that the minimal Lie group containing generators associated to the SM representation $U_1 \sim (\mathbf{3}, \mathbf{1}, 2/3)$ is

$$G_{\rm NP}^{\rm min} = SU(4) \times SU(2)_L \times U(1)_{T_R^3} , \tag{4.5}$$

i.e. the subgroup of the Pati-Salam group $G_{\rm PS} = SU(4) \times SU(2)_L \times SU(2)_R$ [30] where $SU(2)_R$ is broken to $U(1)_{T_R^3}$. The coset $G_{\rm NP}^{\rm min}/G_{\rm SM}$ contains seven generators: the six $T_\pm^\alpha$ associated to the $U_1 \sim (\mathbf{3}, \mathbf{1}, 2/3)$, and T_{B-L} associated to the $Z' \sim (\mathbf{1}, \mathbf{1}, 0)$.

Note that since $SU(4)$ contains the SM gluons, the interaction strengths of both U_1 and Z' are unambiguously related to the QCD and hypercharge couplings. Find these relations and show that the couplings of the Z' to SM fermions are necessarily flavor universal even in the presence of fermion mixing.

One can further show that it is not possible to make the Z' significantly heavier than the U_1, no matter how $G_{\rm NP}^{\rm min}$ is broken to the SM (see App. A.5 in Ref. [27]). A TeV scale Z' that strongly couples to SM fermions in a flavor-universal manner would be excluded by LHC direct searches. This is the Z' problem.

3. **The 4321 solution.** To avoid the Z' problem, $T^{\alpha}_{\pm}$, T_{B-L}, and the QCD generators T^A, should not be unified in a single $SU(4)$ group. Given the commutation rules between $T^{\alpha}_{\pm}$ and T^A, the next-to-minimal option is obtained with [31] (see also [32, 33])

$$(G_{\rm NP}^{\rm min})' = SU(4)\times SU(3)'\times SU(2)_L\times U(1)_X\,, \tag{4.6}$$

where $SU(3)_c$ and $U(1)_Y$ are the diagonal subgroups of $SU(4)\times SU(3)'$ and $SU(4)\times U(1)_X$, respectively. The $(G_{\rm NP}^{\rm min})'$ group is broken to the SM by the vev of the scalar fields $\Omega_1 \sim (\bar{\mathbf{4}},\mathbf{1},\mathbf{1},-1/2)$ and $\Omega_3 \sim (\bar{\mathbf{4}},\mathbf{3},\mathbf{1},1/6)$

$$\langle\Omega_1^{\intercal}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\0\\0\\\omega_1\end{pmatrix}\,,\quad \langle\Omega_3^{\intercal}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\omega_3&0&0\\0&\omega_3&0\\0&0&\omega_3\\0&0&0\end{pmatrix}\,. \tag{4.7}$$

Show that the coset $(G_{\rm NP}^{\rm min})'/G_{\rm SM}$ contains now 15 generators: the six $T^{\alpha}_{\pm}$ associated to the $U_1\sim(\mathbf{3},\mathbf{1},2/3)$, T_{B-L} associated to the $Z'\sim(\mathbf{1},\mathbf{1},0)$, and eight new generators $T^A_{G'}$ associated to a massive gluon (usually denoted as coloron) $G'\sim(\mathbf{8},\mathbf{1},0)$. Find the interaction strengths and masses of U_1, Z' and G' in terms of $SU(4)$ and SM gauge couplings and the $\Omega_{1,3}$ vevs, and obtain the values of κ_U and $\tilde{\kappa}_U$ in (2.1) for this model.

4. **Introducing flavor non-universality.** Now that we have separated the SM gluons from the new gauge bosons, we can generate flavor non-universal couplings for U_1, Z' and G' either via mixing with vector-like fermions [31], and/or with a flavor-dependent assignment of the $SU(4)\times SU(3)'$ quantum numbers [34–36]. In this project, we will follow the first path. To this end, we consider three families of SM-like fermions, transforming as in the SM under $SU(3)'\times SU(2)_L\times U(1)_X$, together with three families of vector-like fermions transforming under $(G_{\rm NP}^{\rm min})'$ as $\chi_{L,R} = (Q\ L)^{\intercal}\sim(\mathbf{4},\mathbf{1},\mathbf{2},0)$.[4] The same scalars that break $(G_{\rm NP}^{\rm min})'$ down to the SM gauge group also give rise to the following Yukawa interactions

$$\mathcal{L}_{\rm mix} = -\bar{q}_L'\,\lambda_q\,\Omega_3^T\chi_R - \bar{\ell}_L'\,\lambda_\ell\,\Omega_1^T\chi_R - \bar{\chi}_L M_\chi\chi_R + {\rm h.c.} \tag{4.8}$$

Assume that the Yukawa couplings $\lambda_{q,\ell}$ and vector-like mass M_χ are of the form

$$\lambda_{q,\ell} = \hat{\lambda}_{q,\ell}\,W_{q,\ell}\,,\qquad M_\chi = M\,\mathbb{1}\,, \tag{4.9}$$

with $\hat{\lambda}_{q,\ell}$ diagonal 3×3 matrices, $W_{q,\ell}$ unitary 3×3 matrices, and M a positive number. Study the structure of the fermion couplings to U_1, Z' and G', and show that flavor violations among SM fields appear only in the U_1 interactions and that they are proportional to $W = W_q^\dagger\,W_\ell$. Note that the W matrix is the analogue of the CKM matrix but in quark-lepton space!

[4] Note that the vector-like lepton doublet L is the one we are searching for at high-p_T in *Adventure 3*.

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