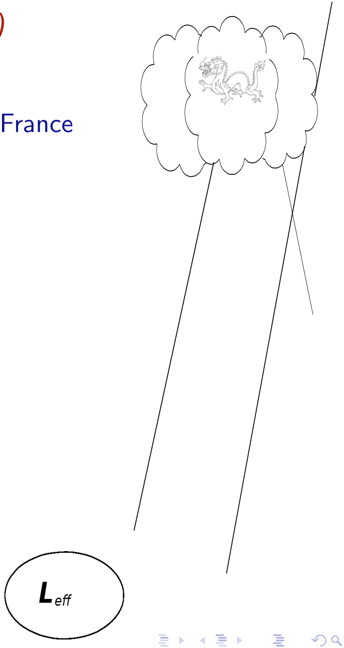
Lepton Flavour Violation in EFT
(and Leptogenesis)

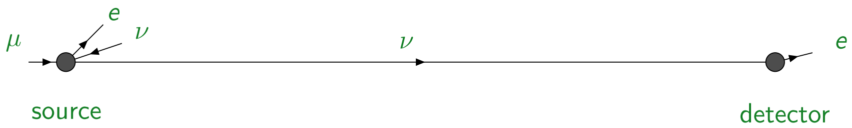
Sacha Davidson
IN2P3/CNRS,Montpellier, France



1. intro :
   - what is LFV ?
   - Why interesting ?
   - What do we know
   - (what is $\mu A \to eA$?)
2. how to calculate ?
   - worth including loops ?
   - an example LQ caln
   - models vs EFT
3. what can we learn ?

Whats is LFV ? Contact Interaction where charged lepton changes flavour

- three lepton flavours in the Standard Model : e, μ, τ (flavour $\equiv$ mass eigenstate)
  (no "flavour" for ν mass eigenstates—only detect ν when turn into charged leptons)
- Leptons change flavour : we see this in ν oscillations



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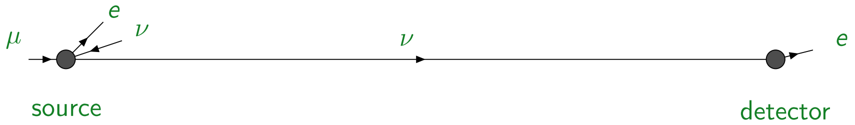
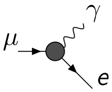


- **(C)LFV $\equiv$ charged lepton changes flavour at a point**

$$\text{eg :} \quad \frac{C}{v^2} y_\mu (\overline{\ell}_e H \sigma^{\alpha\beta} \mu) F_{\alpha\beta} \to \frac{C m_\mu}{v^2} (\overline{e} \sigma \cdot F \mu)$$



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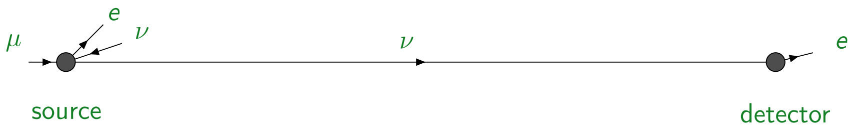

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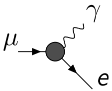

- there is also : Non Standard ν Interactions (NSI), or Generalised Neutrino Interactions (GSI) = contact interactions where a ν changes flavour $\left(f \in \{e, u, d\}\right)$

$$\text{eg} : \quad \frac{1}{\Lambda^4} (\overline{\ell}_e \tilde{H} \gamma^\alpha \tilde{H}^\dagger \ell_\mu)(\overline{f} \gamma_\alpha f) \to \frac{v^2}{\Lambda^4} (\overline{\nu}_e \gamma^\alpha \nu_\mu)(\overline{f} \gamma_\alpha f) \quad , \quad \frac{v^2}{\Lambda^4} \equiv \frac{\varepsilon^{e\mu}}{v^2}$$

Interest = BSM that νosc expts can search for if $\varepsilon \gtrsim 10^{-3}$ (also search for BSM CC operators).
Challenge = evade LFV bds ; tune dim 8 op.s with $\Lambda_{NP} \lesssim$LHC (D+GorbahnLeak)
or light Z' coupled to $\nu_{sterile}$s Farzan .... not talk about NSI

Why interesting ?

⋆ its NP that exists (if SM defined without ν_R)

- ν oscillations ⇒ charged leptons change flavour ⇒ LFV must occur !
  not observed (yet ; $[m_\nu]_{\alpha\beta}$ not predict LFV rates)
  But if m_ν via $Y_\nu \bar{\ell}\tilde{H}\nu_R \Rightarrow BR_{LFV} \lesssim 10^{-50}$ so lets assume heavy LFV NP
- there are other lepton anomalies : $(g\text{-}2)_\mu$, LFUV...
  ...LFUV not imply LFV, but often together

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$\star$ exptal sensitivity to $\mu \to e$ flavour change planned to improve :

$$BR(\mu \to e) < 10^{-12} \to BR(\mu \to e) < 10^{-16} \to 10^{-18} \to \ldots$$

BelleII (+LHC) will improve sensitivity to a multitude of decays

$$\frac{\Gamma(\tau \to l \ldots)}{\Gamma(\tau \to e\nu\bar{\nu})} \lesssim 10^{-7} \to \begin{cases} \sim 10^{-8} \\ \text{few} \times 10^{-9} \end{cases}$$

What we know now /what exptalists can learn

some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	10^{-16} (202x, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUMII)	$10^{-(16\to ?)}$ (Mu2e,COMET)
		$10^{-(18\to ?)}$ (PRISM/PRIME/E
$K^+ \to \pi^+\bar{\mu}e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)
...		
$B^+ \to \bar{\mu}\nu$	$< 1.0 \times 10^{-6}$ (Belle)	$\sim 10^{-7}$ (BelleII)
$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$(Belle-II, LHCb ?)
$\tau \to \ell\{\pi, \rho, \phi, K, ...\}$	$\lesssim$ few $\times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to$...		
$h \to \tau^\pm\ell^\mp$	$< 1.5, 2.2 \times 10^{-3}$(ATLAS/CMS)	
$h \to \mu^\pm e^\mp$	$< 6.1 \times 10^{-5}$(ATLAS/CMS)	
$Z \to e^\pm\mu^\mp$	$< 7.5 \times 10^{-7}$(ATLAS)	

$\mu A \to eA \equiv \mu$ in $1s$ state of nucleus A converts to e

highlights of that table(for this talk)

1. restrictive bounds on three $\mu \to e$ processes, with

$$\frac{\Gamma(\mu \to 3e)}{\Gamma(\mu \to e\bar{\nu}\nu)} \lesssim 10^{-12} \Rightarrow \Lambda_{NP} \gtrsim 10^3 \langle v \rangle \sim 100 \text{ TeV}$$

tree level
$\mathcal{O}(1)$ cplings

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2. bounds on a multitude of $\tau \to \{e, \mu\}$ processes

$$\frac{\Gamma(\tau \to 3e)}{\Gamma(\tau \to e\bar{\nu}\nu)} \times .2 \lesssim \text{few} \times 10^{-8} \Rightarrow \Lambda_{NP} \gtrsim 55 \langle v \rangle \gtrsim 10 \text{ TeV}$$

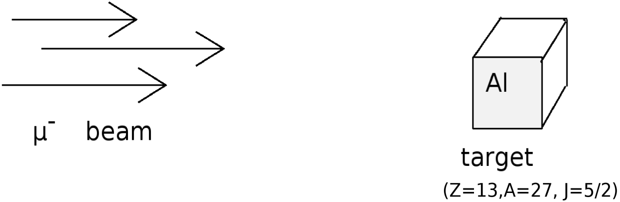
BelleII will improve sensitivities to $\sim 10^{-9}$.
promising for identification of LFV NP : complementary observables constrain most/all SMEFT LFV/QFC coefficients. (Provided NP not to heavy.)

3. (bds on lepton and quark FC interactions : independent info)
4. (heavy particle LFV decays : independent info)

(What is $\mu A \rightarrow eA$?

(a minor Marzia-monster ?)

What is $\mu \to e$ conversion ?



- μ^- captured by Al nucleus, tumbles down to 1s. ($r \sim Z\alpha/m_\mu \gtrsim r_{Al}$)
- in SM : muon capture $\mu + p \to \nu + n$

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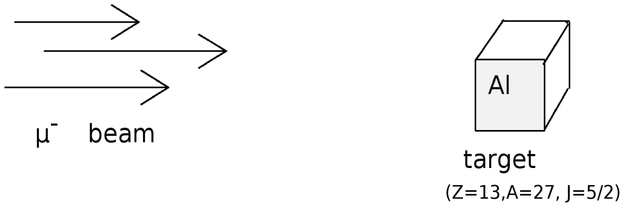

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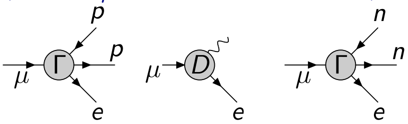

$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V, A, T\}$$

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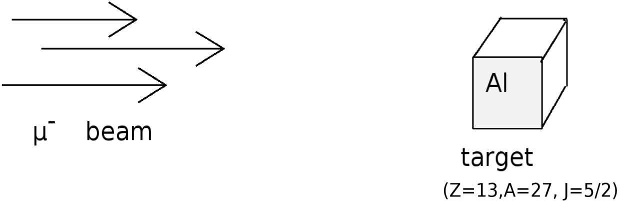

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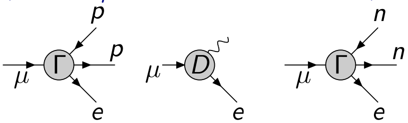

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$\approx$ WIMP scattering on nuclei

1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)

2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (Σ nucleons $\propto$ spin of only unpaired nucleon)

Spin Indep $\mu \to e$ conversion

$$\mathrm{BR}_{SI}(A\mu \to Ae) \propto \left| \widetilde{C}^{pp}_{V,R} V^{(p)}_A + \widetilde{C}'^{pp}_{S,L} S^{(p)}_A + \widetilde{C}^{nn}_{V,R} V^{(n)}_A + \widetilde{C}'^{nn}_{S,L} S^{(n)}_A + C_{D,L} D \right|^2 + \{L \leftrightarrow R\}$$

$$\mathrm{BR}_{SD}(A\mu \to Ae) \sim \left| \widetilde{C}^{NN}_{A,L} + 2\widetilde{C}^{NN}_{T,R} \right|^2 + \left| \widetilde{C}^{NN}_{A,R} + 2\widetilde{C}^{NN}_{T,L} \right|^2 \quad (N \text{ odd})$$

Can distinguish SD vs SI, L vs R. But if observe SI conversion, how to know if is due to scalar/vector operator on n or p?

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Can distinguish SD vs SI, L vs R. But if observe SI conversion, how to know if is due to scalar/vector operator on n or p ?Change nucleus ? KitanoKoikeOkada

$$S^{(p)}_A, V^{(p)}_A \sim \int d^3x \rho_p(x) \overline{\psi_e} \{1, \gamma_0\} \psi^{1s}_\mu$$

so Au poor target with pulsed beam...

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construct $\vec{v}_A \equiv \left(V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)}, D_A \right)$ for different nuclear targets.

$\{\vec{v}_A\}$ "live" in coefficient space, like $\vec{C} = (\widetilde{C}^{pp}_V, \widetilde{C}^{pp}_S, \widetilde{C}^{nn}_V, \widetilde{C}^{nn}_S, (D))$

1.1st exptal search (eg Gold) probes $\vec{C} \parallel \vec{v}_{Au}$

2. chose next target with suff large component $\perp$ Gold (eg Ti, or Al)

$\Rightarrow$ three (suitable) nuclear targets (+improve theory caln) could probe 3 combinations of $\{\widetilde{C}^{pp}_V, \widetilde{C}^{pp}_S, \widetilde{C}^{nn}_V, \widetilde{C}^{nn}_S\}$

DKunoSaporta
DKunoYamanaka

SD more difficult to calculate CiriglianoDKuno, HoferichterEtal

Why is $\mu \to e$ conversion interesting ?

1. maybe where $\mu \to e$ could be discovered ?
   - experimental sensitivity to BR will improve : $10^{-12} \to 10^{-16}$ (expts under construction) $\to 10^{-18}$ (planned expts)
     (future sensitivity to $BR(\mu \to e\bar{e}e) \sim 10^{-16}$)
   - $\Gamma_{SI}(\mu A \to eA) \propto A^2$, whereas $\Gamma(\mu A \to \nu A')$ is not coherently enhanced. So

$$BR_{SI}(\mu A \to eA) \equiv \frac{\Gamma_{SI}(\mu A \to eA)}{\Gamma(\mu A \to \nu A')} \propto A^2 |\sum C|^2$$

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2. To calculate SI $\mu A \to eA$ rate due to LFV operator built of $\{e, \mu, \gamma, q, g\}$:
   - match gg, $q\bar{q}$ ops to hadrons at 2 GeV (big lattice-EFT discrepancies here)
   - include QED+χPT corrections $\to m_\mu$ (QED is LO RGEs ; χPT is at LO)
   - build nucleus out of nucleons (approx. n distributions in nuclei in SI)

   Many aspects of theory calculations to improve !
   (SD needs nuclear model —differs from WIMP calns due to $Q^2 \neq 0$, lepton wavefns)

End of minor-monster $\boldsymbol{\mu}A \to \boldsymbol{e}A$) .

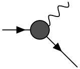
How to calculate LFV ?

We are in discovery mode for LFV $\Rightarrow$ **Question # 1** : can one just calculate observables at tree level in models ?
Its simple...and QED loops are small, right ? (QCD not mix LFV operators)

Does one need which loop calculations for $\mu \to e$?

Ardu+D

Two dipole operators contribute to $\mu \to e\gamma$:



$$\delta\mathcal{L}_{meg} = -\frac{4G_F}{\sqrt{2}} m_\mu \left(C_{D,R}\overline{\mu R}\sigma^{\alpha\beta} e_L F_{\alpha\beta} + C_{D,L}\overline{\mu L}\sigma^{\alpha\beta} e_R F_{\alpha\beta} \right)$$

$$BR(\mu \to e\gamma) = 384\pi^2(|C_{D,R}|^2 + |C_{D,L}|^2) < 4.2 \times 10^{-13}$$

$$\Rightarrow |C_X^D| \lesssim 10^{-8}$$

MEG expt, PSI

How big does one expect $C_{D,X}$ to be ?

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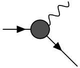

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$$\frac{m_\mu}{v^2} C_{D,X} \sim \frac{e m_\mu}{(16\pi^2)^n \Lambda^2}$$

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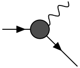

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			$n=1$	$n=2$
$\frac{m_\mu}{v^2} C_{D,X} \sim \frac{e m_\mu}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 100$ TeV	10 TeV
$\frac{m_\mu}{v^2} C_{D,X} \sim \frac{e v}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 3000$ TeV	300 TeV

$\Rightarrow \mu \to e$ expts probe multi-loop effects with $\Lambda_{NP} \gg$ reach of LHC

In particular : penguin, vector4f ops mix first to dipole at 2-loop. Included in RGEs below m_W ; some SMEFT anom dims in talk of Clara Fernandez Castaner.

QED loops are $\mathcal{O}(\alpha/4\pi)$... a negligeable correction to tree ?

Work top-down = suppose a model that gives only tensor operator at m_W :

$$2\sqrt{2}G_F\ C_T(\overline{u}\sigma u)(\overline{e}\sigma P_Y\mu)$$

1 :forget RGEs Match to nucleons $N\in\{n,p\}$: $\widetilde{C}_T^{NN} \simeq \langle N|\bar{u}\sigma u|N\rangle C_T^{uu} \lesssim \frac{3}{4}C_T^{uu}$

nucl. matrix ele. : EngelRTO KlosMGS

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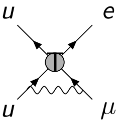
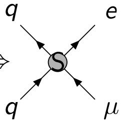
$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$ 

$+ \ldots \quad \Rightarrow$ 

$64 \frac{\alpha_e}{4\pi} \log \frac{m_W}{m_\tau} \; C_T^{uu} \; (\overline{u}u)(\overline{e}P_Y \mu)$

$\Delta C_S^{uu}(m_\tau) \sim \frac{1}{7} C_T^{uu}(m_W)$

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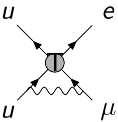
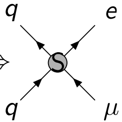
2 : include RGEs

$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$ 

$+ \ldots \quad \Rightarrow$ 

$64\frac{\alpha_e}{4\pi} \log \frac{m_W}{m_\tau} \; C_T^{uu} \; (\overline{u}u)(\overline{e}P_Y\mu)$

$\Delta C_S^{uu}(m_\tau) \sim \frac{1}{7} C_T^{uu}(m_W)$

Then match to nucleons : $\widetilde{C}_S^{NN} = \langle N|\bar{u}u|N\rangle \Delta C_S^{uu} \sim C_T^{uu}$ so $\widetilde{C}_S^{pp} \gtrsim \widetilde{C}_T^{pp}$,

$$\boldsymbol{BR(\mu A \to eA) \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 10^3 BR_{SD}}$$

loops can change Lorentz structure/external legs ⇒ different operator whose coefficient better constrained

But what about loops in τ LFV ?

In the $\tau \to \{e, \mu\}$ sector
1. comparable BRs for a multitude of processes $\Leftrightarrow$ probe most dim6 SMEFT operators involving $\{e, \mu, u, d, s\}$ at tree level.
2. current exptal reach $\sim$10 TeV ; if induced via a loop, NP could be in tension with LHC.

So we need to do loop calns; how to calculate?

top down :

model-build ...if you can imagine how the world works?
Decades of literature doing "fixed order" calns (expand in α,α^2,...), with parameter scans.
Could match-to/start-with EFT at Λ_{NP}, and run down.
Rarely done... not required for strong interactions ($\leq 2q$ legs= rescale ops, not mix), 1-loop SMEFT RGEs more recent...

bottom-up = EFT :

start from data, no imagination required.
Sensible in $\mu \leftrightarrow e$ sector, where few restrictive exptal constrants that change rarely.

Lets try to do this...

To calculate 1-loop in EFT

Buras@Houches,chap5, hep-ph/9806471
Manohar@Houches, 1804.05863

regularise with dim.reg., renormalise with $\overline{MS}$, + Fierz and γ_5 in 4-d because in EFT will only look for $1/\epsilon$ poles.

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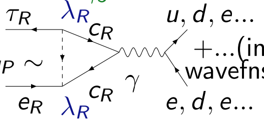
$\mathcal{M}_{NP} \sim$



$+ ...$(include wavefns)

- Allows to calculate $\mathcal{M}_{NP}(\tau \to e\rho)$ (also $\tau \to 3e$), which is finite

$$\langle e(p_e, s_e), \rho(p_\rho)|\boldsymbol{S}|\tau(\vec{P} = 0, S_\tau)\rangle = i\tilde{\delta}^4(P - p_e - p_\rho)\mathcal{M}_{NP}$$

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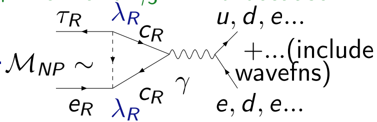

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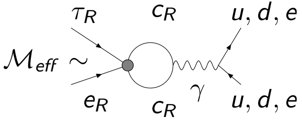
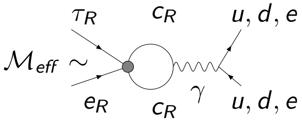
- But $\Lambda_{NP} \gg m_\tau$?
  1 : just calculate *relevant* dynamics = SM.
  2 : $\mathcal{O}(m_\tau^2/\Lambda^2)$ part of $\mathcal{M}_{NP}$ accurate enough ?

At tree level, in $\mathcal{L}$, replace LQ $\to \underbrace{\frac{\lambda_R \lambda_R}{2\Lambda_{NP}^2}}_{\text{coeff. } C_{LQ}} \underbrace{(\overline{e}\gamma P_R \tau)(\overline{c}\gamma P_R c)}_{\text{operator } O_{LQ}}$

- And calculate $\mathcal{M}_{eff}$



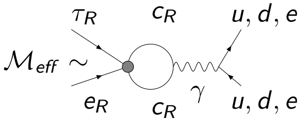
To calculate, ctd



- *Yeeks !* $\mathcal{M}_{eff}$ diverges...
  ...regularise+add counterterms (for all operators generated)
  $(\Rightarrow \delta\mathcal{L} : C_{LQ}\mathcal{O}_{LQ} \to \mu^{2\epsilon} \sum_N C_{LQ} Z_{LQ,N} Z_\psi^{n/2} \mathcal{O}_N \ )$

- With renormalised $\mathcal{L}$, obtain finite $\mathcal{M}_{eff,ren}$ ...except, still logμ in $\mathcal{M}_{eff,ren}$.

To calculate, ctd



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- With renormalised $\mathcal{L}$, obtain finite $\mathcal{M}_{eff,ren}$ ...except, still logμ in $\mathcal{M}_{eff,ren}$.

- cancel μ-dependence of time-ordered-product-of-fields in usual way :
  require coupling constants (= operator coefficients) to be μ-dependent
  $\Rightarrow$ RGEs : $\mu \frac{d}{d\mu} \vec{C} = \vec{C} \left(\mu \frac{d}{d\mu} [Z] \right) Z^{-1} \equiv \vec{C} \cdot [\Gamma]$

- *Eureeka !* $\mathcal{M}_{eff,ren}$ with running coefficients $C(\mu)$, is finite + μ-indep.
  But :had to renormalise operators—prefer result *independent* of μ and scheme (no operators in renormalisable models),*so only calculate*
  scheme-indep, μ-indep quantities in EFT
  $\Rightarrow$ *at one loop, coeff of* $(1/\epsilon + \log)$ is scheme-indep
  (but finite parts might not be ; their scheme-dep. cancels vs 2-loop anom.dim.s)
  $\Rightarrow$ allows to obtain *all* $(\frac{\log}{16\pi^2})^n$ terms !

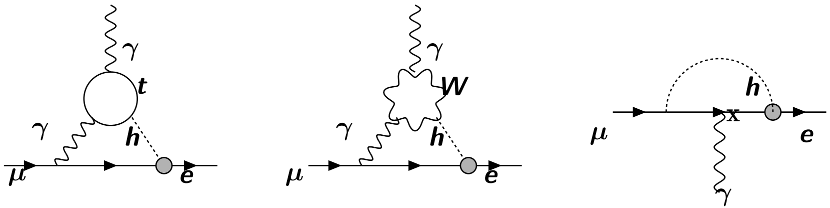
what does "matching at tree level" mean ? (depends on the author)

if you want results indep of operator renormalisation :

- run at N-loop, eg $N = 1$
- match at $N-1$-loop, eg **tree in the EFT** Buras@Houches,chap5, hep-ph/9806471

  (finite part of EFT loops might be scheme-dep, finite part of "model" loops should be fine)

So suppose I am matching SMEFT (with $\mathcal{O}_{EH} = H^\dagger H\overline{\ell}He$) onto EFT-below-$m_W$ at m_W :



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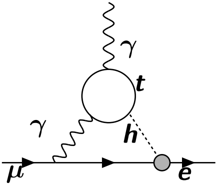

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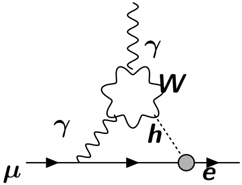

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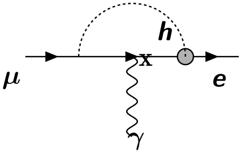
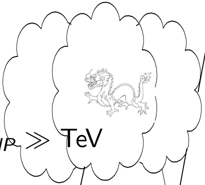


but NOT include !

What do we learn about NP in $\mu \to e$ sector ?



$\Lambda_{NP} \gg$ TeV

RGEs : 1-loop SMEFT

$\{Z, W, \gamma, g, h, t, f\}$

$\vec{C}_{above} = \vec{C}_{below}\, \boldsymbol{V}$ $\quad m_W \sim m_h \sim m_t$

$\{\gamma, g, e\mu, \tau, u, d, c, s, b\}$

RGEs : $\mu \frac{\partial}{\partial \mu} \vec{C} = \vec{C}\boldsymbol{\Gamma}$ $\quad$,$\boldsymbol{\Gamma}$ @LO=1loop+V→D

$\Rightarrow \vec{C}(m_\tau) \sim \vec{C}(m_W)\, \exp\{-\boldsymbol{\Gamma} \log\}$

GeV $\sim m_c, m_b, m_\tau$

$\{\gamma, e, \mu, p, n, (\pi)\}$ $\quad$ $\boxed{\mu \to e \text{ data}}$

Lagrangian at "low E" (for precisely measured $\mu \to e_L$ observables)

Focus on operators with $\mu \to e_L$ bilinear ($\mu \to e_R$ similar), individually constrained by $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and $\mu A \to eA$

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$$\delta\mathcal{L} = 2\sqrt{2}G_F\Big[C_{DR}(m_\mu \overline{e}\sigma \cdot FP_R\mu) + C_{SRR}(\overline{e}P_R\mu)(\overline{e}P_Re) + C_{VLR}(\overline{e}\gamma^\alpha P_L\mu)(\overline{e}\gamma_\alpha P_Re)$$
$$+ C_{VLL}(\overline{e}\gamma^\alpha P_L\mu)(\overline{e}\gamma_\alpha P_Le) + C_{Al}\mathcal{O}_{Al} + C_{Au\perp}\mathcal{O}_{Au\perp}\Big]$$

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What are $\mathcal{O}_{Al}$, $\mathcal{O}_{Au\perp}$? (PSI expts used Titanium (≈Aluminium) and Gold)
Recall a target sensitive to a linear combo. of coeffs, determined by "overlap integrals",eg

$$\mathcal{O}_{Al} \approx \frac{1}{4}(\overline{e}P_R\mu\overline{p}p + \overline{e}P_R\mu\overline{n}n + \overline{e}\gamma^\alpha P_L\mu\overline{p}\gamma_\alpha p + \overline{e}\gamma^\alpha P_L\mu\overline{n}\gamma_\alpha n)$$

$\mathcal{O}_{Au\perp}$ = combo of ops probed by Au, not Al.

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$\mathcal{O}_{Au\perp}$ = combo of ops probed by Au, not Al.

NB : a zoo of "flat directions" ! 12 coefficients constrained, but ~ 90 relevant $\mu \leftrightarrow e$ operators below m_W, otherwise flavour-diag.
Should we worry ?

Take "constrainable coefficients" to the weak scale (tractable analytic expressions here)

line up all operator coefficients in row vector $\vec{C}$, satifies $\mu \frac{\partial}{\partial \mu} \vec{C} = \vec{C} \mathbf{\Gamma}$. Solution :

$$\vec{C}(m_\mu) = \vec{C}(m_W) \boldsymbol{G}$$

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$$\begin{aligned} C_{D,X}(m_\mu) = \; & C_{D,X}(m_W)\left(1 - 16\frac{\alpha_e}{4\pi}\ln\frac{m_W}{m_\mu}\right) \\ & -\frac{\alpha_e}{4\pi e}\ln\frac{m_W}{m_\mu}\left(-8\frac{m_\tau}{m_\mu}C^{\tau\tau}_{T,XX} + C^{\mu\mu}_{S,XX} + C_{2loop}\right) \\ & +16\frac{\alpha_e^2}{2e(4\pi)^2}\ln^2\frac{m_W}{m_\mu}\left(\frac{m_\tau}{m_\mu}C^{\tau\tau}_{S,XX}\right) \\ & -8\lambda^{a_T}\frac{\alpha_e}{4\pi e}\ln\frac{m_W}{2\ \mathrm{GeV}}\left(-\frac{m_s}{m_\mu}C^{ss}_{T,XX} + 2\frac{m_c}{m_\mu}C^{cc}_{T,XX} - \frac{m_b}{m_\mu}C^{bb}_{T,XX}\right) f_{TD} \\ & +16\frac{\alpha_e^2}{3e(4\pi)^2}\ln^2\frac{m_W}{2\ \mathrm{GeV}}\left(\sum_{u,c}4\frac{m_q}{m_\mu}C^{qq}_{S,XX} + \sum_{d,s,b}\frac{m_q}{m_\mu}C^{qq}_{S,XX}\right) \end{aligned}$$

$\lambda = \alpha_s(m_W)/\alpha_s(2\mathrm{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

2. Repeat for other coefficients...

Take observable-motivated subspace rest of way from m_W to Λ_{NP} ?

1. At m_W, find in LO RGEs that almost "all" Cs contribute to ≥ 1 observable with at most suppression $\sim 10^{-3}$.
Yippee—modulo cancellations, if $\mu \to e$ flavour change is there, should see it in $\mu \to e\gamma$, $\mu \to e\bar{e}e$, and/or $\mu A \to eA$.

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Wee trouble : $C^{ij}_{EH} H^\dagger H \bar{\ell}_i H e_j$ — see next slide

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4. check if your favourite model can sit anywhere inside the ellipse (consistent with any future $\mu \to e\gamma$,$\mu \to e\bar{e}e$, $\mu A \to eA$ observations), or only part of it (makes testable predictions).

This 6-d subspace should be$\approx$ finite-eigenvalue subspace of correlation matrix.

Efficient to take rare, slowly-improving data to Λ_{NP}, rather than computing observables across param space of many models ?

Troubles with $\mathcal{O}_{EH}^{ij} \equiv H^\dagger H \bar{\ell}_i H e_j$

$\left.\begin{array}{l}\text{Below}\\ \text{Above}\end{array}\right\}$ m_W, $\mathcal{L}_{ren}$ is flav.-diag. in $\left\{\begin{array}{c}\text{mass eigenstate}\\ \text{diagonal Yukawa}\end{array}\right\}$ basis(for charged leptons)

But with $\mathcal{O}_{EH}^{ij}$:

$$[m]_{ij} = [Y_e]^{ij} v + [C_{EH}]^{ij} \frac{v^3}{\Lambda^2}$$

so not the same. What to do ?

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bottom-up : stay in mass eigenstate basis above m_W (problem that not know C^{ij}_{EH}). OK because $h \to e_i \bar{e}_j$ bds ensure that off-diagonal elements of Y_e in SMEFT RGEs are negligeable, at accuracy required for upcoming expts. Ardu+D

top-down : ? if use diagonal Y_e basis in SMEFT, run flavour-changing and diagonal matrix elements, and diagonalise $[m_e]$ at m_W ?

Summary

Lepton Flavour Violation is BSM that exists, and experimental sensitivities set to improve by orders of magnitude ($\to BR(\mu \leftrightarrow e) \sim 10^{-16 \to -18}$, $BR(\tau \to l) \sim 10^{-9}$).

If LFV induced by heavy New Physics, can parametrise with EFT. Find

in $\boldsymbol{\mu} \leftrightarrow \boldsymbol{e}$ sector :

• almost every $\mu \to e$ interaction below m_W (otherwise flav. diag., ≤ 4 legs) contributes at $\gtrsim \mathcal{O}(10^{-3})$ to $\mu \to e\gamma$, $\mu \to e\bar{e}e$ and/or $\mu A \to eA$ (possible exceptions :$\bar{e}\mu G\widetilde{G}$, $\bar{e}\mu F\widetilde{F}$, $\bar{e}\gamma\mu F\partial F$...)

• current data constrains 12-d subspace of Wilson coeffs. (zoo of flat directions). Efficient to construct observable-motivated basis for this subspace, and project models onto allowed ellipse at Λ_{NP} ?

in $\boldsymbol{\tau} \leftrightarrow \boldsymbol{l}$ sector :

• comparable constraints on multitude of observables ; constrain all SMEFT coefficients with light quarks to $\Lambda_{NP} \gtrsim 10 - 100$ TeV

Many calculations rudimentary : *lots to do*

differs from WIMP scattering in that μ and nucleus charged

1. suppose start with $\mu \leftrightarrow e$ operators involving gluons, γ, u, d, s, c, b
2. match quark/gluon operators onto nucleon ($N \in \{n, p\}$) operators :
$$\bar{q}(x)\Gamma_O q(x) \to G_O^{N,q}\bar{N}(x)\Gamma_O N(x)$$
Gs in Appendix

eg, $\langle N|\bar{q}(x)q(x)|N\rangle = G_O^{N,q}\langle N|\bar{N}(x)N(x)|N\rangle = G_O^{N,q}\overline{u_N}(P_f)u_N(P_i)e^{-i(P_f-P_i)x}$

To calculate SI $\boldsymbol{\mu}-\boldsymbol{e}$ conversion (at LO in χPT...)

intro for QFTists
+SDep in DKunoSaporta

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3. *imagine* to build the atom as a bound state of nucleus and muon in 1s state

in P+S

$$|\mu A(\vec{P}_i = 0)\rangle = \sqrt{\frac{2(M_A + m_\mu)}{4M_A m_\mu}} \sum_s \int \frac{d^3k}{(2\pi)^3} \widetilde{\psi}_\mu(\vec{k})\, |A(-\vec{k})\rangle \otimes |\mu(\vec{k}, s)\rangle$$

then build nucleus as bd state of nucleons (app. B of 1203.3542),gives :

SD overlap int : guess from SD DM targets

$$\langle e, A|\widetilde{C}_O O|\mu A\rangle \propto \widetilde{C}_O(\bar{u}_e\Gamma_O u_\mu) \int d^3x \psi_\mu^{1s}|f_N(x)|^2\psi_e(\bar{N}\Gamma_O N)$$

where "overlap integral" over nucleus, of muon wavefn ($\widetilde{\psi}_\mu^{1s}$), nucleon density ($|f_N(x)|^2$), e wavefn ($\psi_e \sim e^{-iqx}$) and operator performed in KitanoKoikeOkada.

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Gs in Appendix

   eg, $\langle N|\bar{q}(x)q(x)|N\rangle = G_O^{N,q}\langle N|\bar{N}(x)N(x)|N\rangle = G_O^{N,q}\overline{u_N}(P_f)u_N(P_i)e^{-i(P_f - P_i)x}$

3. look up rate in KitanoKoikeOkada, PRD (2002), eqn 14, check your operator normalisation against KKO eqn 1, read numerical value of overlap integrals from table I, and divide by capture rate in table VIII of KKO.

Shortcut to calculate $\mu - e$ conversion

shortcut for current bounds (Gold and Titanium) :write

$$BR_{SI}(\mu A \to eA) = B_A \left[|\hat{v}_A \cdot \vec{C}_L|^2 + |\hat{v}_A \cdot \vec{C}_R|^2 \right]$$

where

$$\vec{C}_L = (\widetilde{C}_{D,R}, \widetilde{C}^{pp}_{S,R}, \widetilde{C}^{pp}_{V,L}, \widetilde{C}^{nn}_{S,R}, \widetilde{C}^{nn}_{V,L})$$

$$B_A \equiv \frac{32 G_F^2 m_\mu^5 |\vec{v}_A|^2}{\Gamma_{cap}(A)} = \begin{cases} 250 & Ti \\ 300 & Au \\ 142 & Al \end{cases}$$

and normalised overlap integrals of KKO are lined up in target vectors

$$\hat{v}_{Ti} = (0.250, 0.426, 0.458, 0.503, 0.541)$$

$$\hat{v}_{Au} = (0.222, 0.289, 0.458, 0.432, 0.686)$$

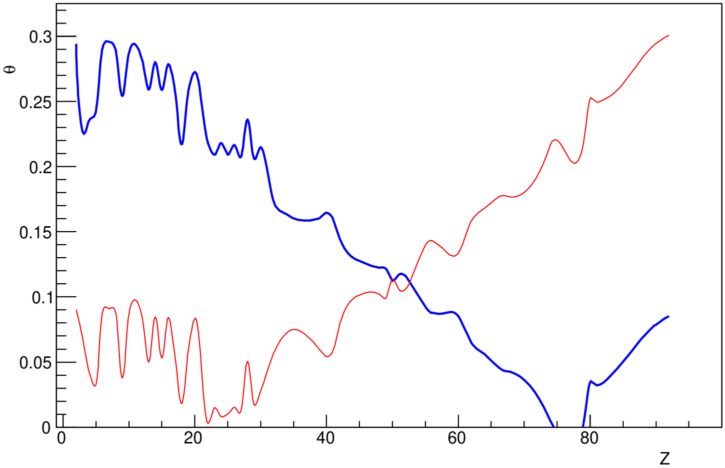
(Spin Dep : ?likely in noise of SI signal—RGEs of QED mix T,A$\to$ S, V.
To calculate, need nuclear caln, see discussion inCiriglianoDKuno, DKunoSaporta)

Gs

$G_V^{p,u} = G_V^{n,d} = 2$	$G_V^{p,d} = G_V^{n,u} = 1$	$G_V^{p,s} = G_V^{n,s} = 0$
$G_A^{p,u} = G_A^{n,d} = 0.84$ $G_A^{p,u} = G_A^{n,d} = 0.863$	$G_A^{p,d} = G_A^{n,u} = -0.43$ $G_A^{p,d} = G_A^{n,u} = -0.345$	$G_A^{p,s} = G_A^{n,s} = -.085$ $G_A^{p,s} = G_A^{n,s} = -.0240$
$G_S^{p,u} = 5.9$ $(G_S^{p,u} = 9.0)$ $G_S^{n,u} = 5.0$ $(G_S^{n,u} = 8.1)$ $G_S^{N,c} = \frac{2m_N}{17m_c}$	$G_S^{p,d} = 5.0$ $(G_S^{p,d} = 8.2)$ $G_S^{n,d} = 6.0$ $(G_S^{n,d} = 9.0)$ $(G_S^{N,b} = \frac{2m_N}{17m_b}$	$G_S^{p,s} = 0.42$ $G_S^{n,s} = 0.42$ $(G_S^{n,s} = 0.42)$
$G_P^{p,u} = 144 = G_P^{n,d}$	$G_P^{p,d} = -150 = G_P^{n,u}$	$G_P^{p,s} = -4.9 = G_P^{n,s}$
$G_T^{p,u} = G_T^{n,d} = 0.77(7)$	$G_T^{p,d} = G_T^{n,u} = -0.23(3)$	$G_T^{p,s} = G_T^{n,s} = .008(9)$
$G_S^{N,gg} = -8\pi m_N/(9\alpha_s v)$		

Table – Matching coefficients between nucleon and operators and gluon or light-quark(flavour-diagonal) operators. References in DKunoSaporta. Scalar Gs in parentheses are EFT caln, otherwise from lattice. In all cases, the $\overline{\mathrm{MS}}$ quark masses at $\mu = 2$ GeV are taken as $m_u = 2.2$ MeV, $m_d = 4.7$ MeV, and $m_s = 96$ MeV. The nucleon masses are $m_p = 938$ MeV and $m_n = 939.6$ MeV.

Current data+ theory uncertainty $\sim 10\%$: two targets give $\Delta\theta > 0.2$



$$\vec{v}_A = (V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)}), \text{ and } BR \propto |\vec{v}_A \cdot \vec{C}|^2$$
$$\vec{v}_{Au} \cdot \vec{v}_Z \equiv |\vec{v}_{Au}||\vec{v}_Z| \cos\theta \text{ ...plot } \theta \text{ on vertical axis}$$

Counting bds from $\mu \to e\bar{e}e$

$\mu \to e_L\bar{e}e$: add

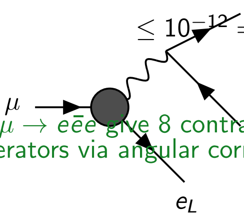
$$2\sqrt{2}G_F\Big[C_{V,LL}(\bar{e}\gamma^\alpha P_L\mu)(\bar{e}\gamma_\alpha P_L e) + C_{V,LR}(\bar{e}\gamma^\alpha P_L\mu)(\bar{e}\gamma_\alpha P_R e) + C_{S,RR}(\bar{e}P_R\mu)(\bar{e}P_R e)\Big]$$

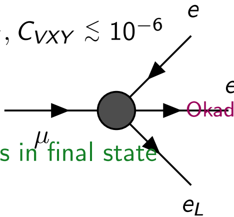
$$BR = \frac{|C_{S,RR}|^2}{8} + 2|C_{V,LL} + 4eC_{D,R}|^2 + |C_{V,LR} + 4eC_{D,R}|^2 + (64\ln\frac{m_\mu}{m_e} - 136)|eC_{D,R}|^2 + \{L \leftrightarrow R\}$$

(e relativistic ≈ chiral, neglect interference between e_L, e_R)

$$\leq 10^{-12} \Rightarrow C_{SXX}, C_{VXY} \lesssim 10^{-6}$$

SINDRUM, PSI





OkadaOkomuraShimizu

$\Rightarrow \mu \to e\gamma + \mu \to e\bar{e}e$ give 8 contraints
distinguish operators via angular correlations in final state

I use Dirac spinors, with 4 degrees of freedom(dof) labelled by $\{\pm E, \pm s\}$, in *chiral* decomposition

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad , \{\gamma^\alpha\} = \left\{ \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} \right\}$$

$$\{\sigma_i\} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\psi_L = P_L \psi \quad \text{avec} \quad P_L = \frac{(1-\gamma_5)}{2} \quad , \psi_R = P_R \psi$$

chirality is *not* an observable ($\rightarrow$ helicity $= \pm \hat{s} \cdot \hat{k} = \pm 1/2$ in relativistic limit), but $P_{L,R}$ simple to calculate with :)

notation : $\overline{(\psi_R)} = (P_R \psi)^\dagger \gamma_0 = \psi^\dagger P_R \gamma_0 = \psi^\dagger \gamma_0 P_L = (\overline{\psi})_L$

$(\psi^c)_L = P_L(-i\gamma_0\gamma_2\gamma_0\psi^*) = -i\gamma_0\gamma_2\gamma_0\psi^*_R$

$$\mathcal{O}_{\ell q}^{(1)\alpha\beta nm} = \frac{1}{2}(\overline{\ell}_\alpha \gamma^\mu \ell_\beta)(\overline{q}_n \gamma_\mu q_m) \qquad \mathcal{O}_{\ell q}^{(3)\alpha\beta nm} = \frac{1}{2}(\overline{\ell}_\alpha \gamma^\mu \tau^a \ell_\beta)(\overline{q}_n \gamma_\mu \tau^a q_m)$$

$$\mathcal{O}_{eq}^{\alpha\beta nm} = \frac{1}{2}(\overline{e}_\alpha \gamma^\mu e_\beta)(\overline{q}_n \gamma_\mu q_m)$$

$$\mathcal{O}_{\ell u}^{\alpha\beta nm} = \frac{1}{2}(\overline{\ell}_\alpha \gamma^\mu \ell_\beta)(\overline{u}_n \gamma_\mu u_m) \qquad \mathcal{O}_{\ell d}^{\alpha\beta nm} = \frac{1}{2}(\overline{\ell}_\alpha \gamma^\mu \ell_\beta)(\overline{d}_n \gamma_\mu d_m)$$

$$\mathcal{O}_{eu}^{\alpha\beta nm} = \frac{1}{2}(\overline{e}_\alpha \gamma^\mu e_\beta)(\overline{u}_n \gamma_\mu u_m) \qquad \mathcal{O}_{ed}^{\alpha\beta nm} = \frac{1}{2}(\overline{e}_\alpha \gamma^\mu e_\beta)(\overline{d}_n \gamma_\mu d_m)$$

$$\mathcal{O}_{\ell equ}^{\alpha\beta nm} = (\overline{\ell}_\alpha^A e_\beta)\varepsilon_{AB}(\overline{q}_n^B u_m) \qquad \mathcal{O}_{\ell edq}^{\alpha\beta nm} = (\overline{\ell}_\alpha e_\beta)(\overline{d}_n q_m)$$

$$\mathcal{O}_{T,\ell equ}^{\alpha\beta nm} = (\overline{\ell}_\alpha^A \sigma^{\beta\nu} e_\beta)\varepsilon_{AB}(\overline{q}_n^B \sigma_{\beta\nu} u_m)$$

where ℓ, q are doublets and e, u are singlets, n, m are quark family indices, taken equal, and A, B are SU(2) indices.

SMEFT ops, ctd :4-lepton+ penguins/dipoles

$$\mathcal{O}_{\ell\ell}^{\alpha\beta\rho\sigma} = \frac{1}{2}(\overline{\ell}_\alpha \gamma^\mu \ell_\beta)(\overline{\ell}_\rho \gamma_\mu \ell_\sigma)$$

$$\mathcal{O}_{\ell e}^{\alpha\beta\rho\sigma} = \frac{1}{2}(\overline{\ell}_\alpha \gamma^\mu \ell_\beta)(\overline{e}_\rho \gamma_\mu e_\sigma) \qquad \mathcal{O}_{ee}^{\alpha\beta\rho\sigma} = \frac{1}{2}(\overline{e}_\alpha \gamma^\mu e_\beta)(\overline{e}_\rho \gamma_\mu e_\sigma)$$

$$\mathcal{O}_{eH}^{\alpha\beta} = (H^\dagger H)(\overline{\ell}_\alpha H e_\beta) \qquad \mathcal{O}_{He}^{\alpha\beta} = \frac{i}{2}(H^\dagger \overleftrightarrow{D}_\mu H)(\overline{e}_\alpha \gamma^\mu e_\beta)$$

$$\mathcal{O}_{H\ell(1)}^{\alpha\beta} = \frac{i}{2}(H^\dagger \overleftrightarrow{D}_\mu H)(\overline{\ell}_\alpha \gamma^\mu \ell_\beta) \qquad \mathcal{O}_{H\ell(3)}^{\alpha\beta} = \frac{i}{2}(H^\dagger \overleftrightarrow{D}^a_\mu H)(\overline{\ell}_\alpha \gamma^\mu \tau^a \ell_\beta)$$

$$\mathcal{O}_{eW}^{\alpha\beta} = y_\beta(\overline{\ell}_\alpha \tau^a H \sigma^{\mu\nu} e_\beta) W^a_{\mu\nu} \qquad \mathcal{O}_{eB}^{\alpha\beta} = y_\beta(\overline{\ell}_\alpha H \sigma^{\mu\nu} e_\beta) B_{\mu\nu}$$

where y_β = charged lepton Yukawa, and
$i(H^\dagger \overleftrightarrow{D}^a_\mu H) \equiv i(H^\dagger \tau^a D_\mu H) - i(D_\mu H)^\dagger \tau^a H$.

For 4-lepton ops, can have $\Delta L_i = 1, 2$. Also, Warsaw basis supposes that flavour change across the bilinears is allowed ; if impose one unit of flavour change in first bilinear, should add triplet operator $\mathcal{O}_{\ell\ell}^{\mathbf{3},e\mu ee}$. If in $\mathcal{L}$ sum all indices over all flavours, there are 2s for $\mathcal{O}_{ee}$ and $\mathcal{O}_{\ell\ell}$ ($(\overline{e}\gamma^\mu \mu)(\overline{\tau}\gamma_\mu \tau) = (\overline{\tau}\gamma^\mu \tau)(\overline{e}\gamma_\mu \mu)$) :