Avoiding the Use of Complex Numbers: Didactic Problems Regarding the Uncertainty Principle?

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Abstract. The uncertainty principle can be interpreted via different approaches to quantum mechanics. In this presentation some consequences for the general uncertainty principle are shown in an approach avoiding the use of complex numbers. These are strange to students who are familiar with the position-momentum uncertainty relation. If we deal with real vector spaces, like most of the two-state approaches do in secondary schools, there is no sense to talk about any uncertainty inequality, even if the two quantities do not have common eigenstates. This presentation provides a form of the uncertainty principle valid in two-state problems, too, and also presents misconceptions and didactic proposals.

Introduction

In QM, statistical description is used because of the probabilistic nature of phenomena. A physical quantity is uncertain if the variance assigned to the quantity is not zero because the state of system is in a superposition state regarding the measured quantity, so there are different outcomes of the measurements with given probabilities. The general uncertainty relation gives a lower limit to the product of variances of any two physical quantities (A and B represented by operators $\widehat{A}$ and $\widehat{B}$) in a quantum state $|\psi\rangle$ [1-2]:

$$(\Delta A)^2(\Delta B)^2 \geq \frac{1}{4}\left|\langle\psi|\widehat{A}\widehat{B} - \widehat{B}\widehat{A}|\psi\rangle\right|^2. \tag{1}$$

The position-momentum uncertainty relation $\Delta x \Delta p \geq \hbar/2$ can be deduced from (1) and the relation is usually taught in secondary school in several countries [3] based on the wave approach of QM.

However, two-state approaches have been becoming popular [4] which use only real numbers in secondary schools [5-10] because complex numbers are not part of the curricula. I present that, the unavoidable restriction to real vector spaces and the consideration of the simultaneous variances of two quantities leads to the conclusion that it is meaningless to talk about any uncertainty inequality. Namely, the general uncertainty relation (1) always yields the lower limit 0 for the product of the variance of any two *real* operators even if these quantities are *incompatible*. Relation (1) is then a triviality since the variances are positive anyhow. We can only talk about uncertainty principle in such cases [10] (not relation); and should avoid writing any inequality.

Mixing the uncertainty principle and the uncertainty relation might cause didactical confusion, however, if a teacher would like to teach e.g., a two-state approach to QM, students will be confronted with two interpretations described with the same (or nearly the same) term but with different meanings in countries where position-momentum uncertainty relation is taught. [10-14]

Methods and results

The presentation is based on interviews and tests regarding uncertainty of quantities and uncertainty principle. They were done by university students (prospective physics teachers), in-service physics teachers and secondary school students. The test indicated that in order to explore the uncertainty principle, students must understand that the uncertainty of physical quantities can be quantified. We have to make clear that every physical quantity can only be measured without uncertainty if the state of system is an eigenstate. However, many students and teachers argued that a quantity can never be certain in QM, that is the uncertainty relation forbids the existence zero variances. As interviews show, this misunderstanding can come from the position and momentum relation since they find infinite uncertainty unrealistic. In addition, students found the essence of uncertainty relation in the reciprocal dependence of the variances of the measured quantities. It seems that, the mere knowledge of the position-momentum uncertainty relation can lead to wrong interpretations about the general nature of quantum uncertainty. The presentation also offers didactic proposals:

- Use different expressions for different interpretations. The terms “uncertainty relation”, “uncertainty principle”, “general uncertainty relation” are some examples.
- The uncertainty principle can, should be taught on qualitative level, as it only states that there are quantity pairs that can never be measured without uncertainty simultaneously.
- Emphasize that the position-momentum uncertainty relation is a special case of the general uncertainty principle as it also implies the reciprocal dependence. So, one can distinguish the complementary (quantities with reciprocal dependence) and incompatible properties.

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