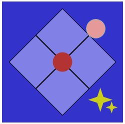
Celestial Holography in Asymptotically Flat Backgrounds

Tristan McLoughlin



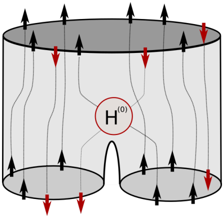
June 23, 2022

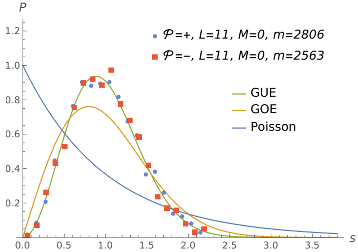
SAGEX
Scattering Amplitudes: from Geometry to Experiment

Science Foundation Ireland sfi For what's next

Based on work to appear with Riccardo Gonzo and Andrea Puhm.

SAGEX Projects: Deformed integrable models



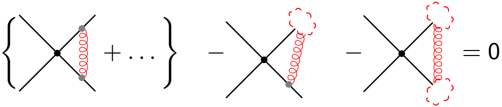


With Anne Spiering (and Raul Pereira)

- Non-planar Spectrum of $\mathcal{N} = 4$ SYM: analytical formula for one-loop anomalous dimensions via perturbed integrable model and spin-chain scalar products. [2005.14254]
- Random Matrix Theory description of statistical properties of finite-N spectrum: signature of quantum chaos. [2011.04633]
- Marginally deformed theories: found holographic, weak coupling analogue of chaotic strings in deformed AdS geometries. [2202.12075]

SAGEX Projects: Asymptotic Symmetries and Celestial Holography

With Riccardo Gonzo and Anne Spiering (and Diego Medrano)



- Faddeev-Kulish approach to QCD (á la Catani and Ciafaloni): in principle defines dressed IR finite amplitudes including collinear divergences.
- Showed conservation of asymptotic charges - defined via soft-evolution operators - corresponding to large gauge transformations and the resultant Ward identity to one-loop and leading order in IR divergences. [1906.11763]
- The study of asymptotic symmetries in gauge and gravity theories has led to recent reformulations of scattering amplitudes in alternative variables.

Celestial Holography

Reformulation of 4D Minkowskian scattering amplitudes (in scalar theory, gauge theory, gravity, ...) in the language of conformal field theory.

Reviews (and references):

- Strominger, Lectures on the Infrared Structure of Gravity and Gauge Theory
- S. Pasterski, M. Pate, and A.-M. Raclariu, "Celestial Holography," in 2022 Snowmass Summer Study
- SAGEX Review Chaper 11 with Puhm and Raclariu

Celestial Holography

Reformulation of 4D Minkowskian scattering amplitudes (in scalar theory, gauge theory, gravity, ...) in the language of conformal field theory.

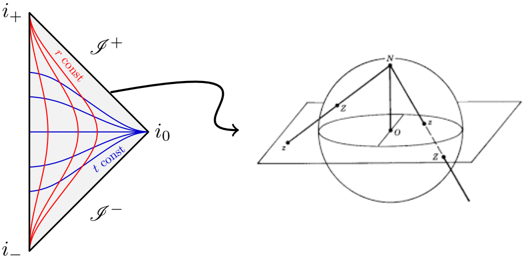
- Motivated by the group identification

$$SO^+(3,1) \simeq PSL(2,\mathbb{C}) \simeq Aut(\hat{\mathbb{C}})$$

- Such a reformulation is interesting as it can reveal new properties, connections and hidden structures (e.g. sub-leading soft theorems, memory effects, ...)
- Points to a holographic description of quantum gravity asymptotically flat space-times. Good reasons for such a description exists e.g. BH entropy formula $S_{\mathrm{BH}} = \dfrac{A}{4L_p^2}$, L_p: Planck length

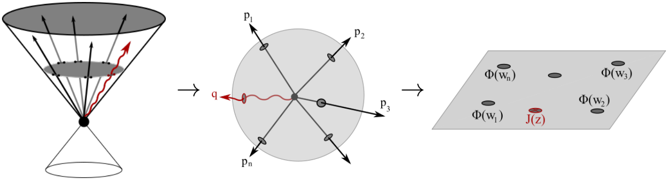
Celestial Holography

Boundary description of quantum field theory in Minkowski space-time:



Massless momentum can be parameterized:

$$p_i^\mu = \eta_i \omega_i (1 + |z_i|^2, z_i + \bar{z}_i, -i(z_i - \bar{z}_i), 1 - |z_i|^2)$$



Celestial Conformal Field Theory

Good observables in gravity (and other theories!) are S-matrix elements:

$$ {}_{\text{boost}}\langle \text{out}|S|\text{in}\rangle_{\text{boost}} = \langle \mathcal{O}^{\pm}_{\Delta}(z_1, \bar{z}_1) \ldots \mathcal{O}^{\pm}_{\Delta}(z_n, \bar{z}_n)\rangle_{CCFT} $$

- Each massless momentum labels a point at $\mathscr{I}^{\pm}$
- Transform asymptotic states from momentum states to boost eigenstates
- Operators labelled by position on 2-sphere, $SL(2, \mathbb{C})$ conformal dimensions Δ_i corresponding to boost eigenvalue and spins J_i

$$ L_0 = -\tfrac{i}{2}(J_3 + iK_3)\ , \quad \bar{L}_0 = -\tfrac{i}{2}(-J_3 + iK_3) $$

Conformal Primary Wavefunctions

Define conformal primary scalar wavefunctions by transformation properties:

$$\Phi_\Delta(\Lambda^\mu_\nu X^\nu, \tfrac{az+b}{cz+d}) = |cz+d|^{2\Delta}\Phi_\Delta(X^\mu, z)$$

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e.g. Mellin transform of plane-waves

$$\phi^\pm_\Delta(X; z) = \int_0^\infty d\omega\ \omega^{\Delta-1} e^{\pm i\omega q\cdot X} = \frac{(\mp i)^\Delta \Gamma(\Delta)}{(-q\cdot X_\pm)^\Delta}$$

- $q^\mu = (1+|z_i|^2, z_i+\bar{z}_i, -i(z_i-\bar{z}_i), 1-|z_i|^2)$

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- Solution of $\Box\phi = 0$
- Arbitrary complex Δ; when $\Delta \in 1 + i\ \mathbb{R}$ (principle continuous series of $SL(2,\mathbb{C})$) form a complete δ-function normalizable basis
- Given amplitude of massless particles construct CCFT correlator by taking Mellin transform on all external legs:

$$\tilde{\mathcal{A}}_n(\Delta_i, z_i) = \mathcal{M}[\mathcal{A}_n] = \prod_{k=1}^n \int d\omega_k \omega_k^{\Delta_k-1}\mathcal{A}_n(\omega_i, z_i)$$

Transforms with definite weights (Δ_i, J_i) under $SL(2,\mathbb{C})$.

Conformal Primary Wavefunctions

Define conformal vectors, metrics, ... etc by transformation properties:

$$\Phi^s_{\Delta,J}(\Lambda^\mu_\nu X^\nu, \tfrac{az+b}{cz+d}) = (cz+d)^{\Delta+J}(cz+d)^{*\,\Delta-J} D(\Lambda)_s \Phi^s_\Delta(X^\mu, z)$$

e.g. Shockwaves: $\phi_{\Delta=1}(X;z) = \log X^2 \delta(q\cdot X)$,

$$A^\mu_{\Delta=0,J=0}(X;z) = q^\mu \phi_{\Delta=1}(X;z),$$
$$h^{\mu\nu}_{\Delta=-1,J=0}(X;z) = q^\mu q^\nu \phi_{\Delta=1}(X;z)$$

- Solution of massless wave equation with massless point source

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- Solution of massless wave equation with massless point source
- Spin-1 version is solution of Maxwell equations and spin-2 version is Kerr-Schild form of Aichelberg-Sexl metric (exact solution of Einstein equations)

Celestial Amplitudes

The four-point amplitude can be written as a universal prefactor times a function of the cross-ratios $z, \bar{z}$

$$\tilde{\mathcal{A}}_4(\Delta_i, z_i, \bar{z}_i) = \frac{\left(\frac{z_{24}}{z_{14}}\right)^{h_{12}} \left(\frac{z_{14}}{z_{13}}\right)^{h_{34}}}{z_{12}^{h_1+h_2} z_{34}^{h_3+h_4}} \frac{\left(\frac{\bar{z}_{24}}{\bar{z}_{14}}\right)^{\bar{h}_{12}} \left(\frac{\bar{z}_{14}}{\bar{z}_{13}}\right)^{\bar{h}_{34}}}{\bar{z}_{12}^{\bar{h}_1+\bar{h}_2} \bar{z}_{34}^{\bar{h}_3+\bar{h}_4}} \tilde{A}_4(z, \bar{z})$$

- Conformal weights $(h_i, \bar{h}_j) = \frac{1}{2}(\Delta_i + J_i, \Delta_i - J_i)$ with $h_{ij} = h_i - h_j$

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- Two scalars with photon exchange: $(\Delta_i = 1 + i\nu_i,\ \nu_i \in \mathbb{R},\ J_i = 0)$

$$\tilde{A}_4(z, \bar{z}) \propto e_{\phi_1} e_{\phi_2} \delta(i\bar{z} - iz)(z-1)^{1-2h_2-2h_3} z^{2h-2}(1+\bar{z}) \int d\omega \omega^{\sum \Delta_i - 5}$$

We use $\mathcal{I}(s) = \int_0^\infty d\omega \omega^{s-1} = 2\pi\delta(\mathrm{Im}(s))$ with $\mathrm{Re}(s) = 0$.

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- Two scalars with **graviton** exchange: $(\Delta_i = 1 + i\nu_i,\ \nu_i \in \mathbb{R},\ J_i = 0)$

$$\tilde{A}_4(z, \bar{z}) = \frac{i\kappa^2}{2}(-1)^{2h}\delta(i\bar{z} - iz)(z-1)^{-2h_2-2h_3} z^{2h}\mathcal{I}(\sum_i \Delta_i - 2)$$

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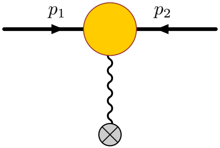
Amplitudes on Backgrounds

We want to extend the computation of celestial quantities to amplitudes on non-trivial backgrounds.



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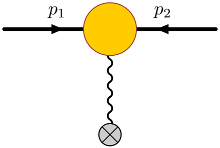

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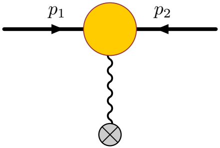

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- Use method of Boulware and Brown: classical solution gives tree-level generating functional of connected correlation functions $W[J]$ as $\Phi_{cl}[J] = \delta W/\delta J$.

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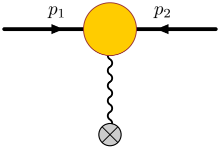

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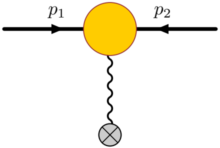
- Use method of Boulware and Brown: classical solution gives tree-level generating functional of connected correlation functions $W[J]$ as $\Phi_{cl}[J] = \delta W/\delta J$.
- Amplitudes are found using LSZ prescription:

$$\mathcal{A}(p_1, \ldots, p_n) \equiv \lim_{p_n^2 \to 0} ip_n^2 \prod_{i=1}^{n-1} \lim_{p_i^2 \to 0} ip_i^2 \; \frac{\delta \tilde{\Phi}_{cl}(-p_n)}{\delta \tilde{J}(p_i)} \bigg|_{J=0}$$

Two-point Amplitude in Scalar Electrodynamics



Solve wave equation

$$\partial^2\Phi - 2ieA^\mu\partial_\mu\Phi - ie\partial_\mu A^\mu\Phi - e^2A_\mu A^\mu\Phi = J$$

perturbatively in e

$$\tilde{\Phi}_{cl}(p) = \sum_{n=0}^{\infty}\tilde{\Phi}_{cl}^{(n)}(p)$$

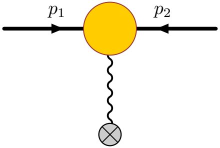
with

$$\tilde{\phi}^{(0)}(p) = -\frac{\tilde{J}(p)}{p^2}\ , \quad \tilde{\phi}^{(1)}(p) = \frac{e}{p^2}\int\frac{d^4k}{(2\pi)^4}\tilde{A}^\mu(p-k)(p_\mu+k_\mu)\tilde{\phi}^{(0)}(k)\ , \ldots$$

gives the leading amplitude

$$\mathcal{A}_2^{(1)}(p_1,p_2) = e(p_1-p_2)_\mu\tilde{A}^\mu(p_1+p_2)$$

Ex 1: Two-point Amplitude on Coulomb Background



So with Coulomb potential $A_\mu(x) = \dfrac{Q}{4\pi r} q_\mu$ with $q^\mu = (1, \dfrac{\vec{x}}{r})$ the amplitude is

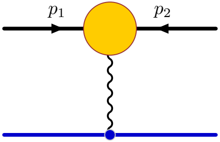
$$\mathcal{A}^{(1)}_{2,Coulomb}(p_1, p_2) = -4\pi e Q \frac{p_1^0 \delta(p_1^0 + p_2^0)}{(p_1 + p_2)^2}$$

and Mellin transforming we have

$$\widetilde{\mathcal{A}}^{(1)}_{2,Coulomb}(\Delta_1, \Delta_2) = (2\pi)^2 \frac{eQ}{4\pi} \frac{1}{|z_{12}|^2} \left(\frac{1 + |z_1|^2}{1 + |z_2|^2} \right)^{\Delta_2 - 1} \mathcal{I}(\Delta_1 + \Delta_2 - 2),$$

- No kinematic delta-function; delta-function support for dimensions on principle series
- Distinctive $|z_{12}|^{-2}$ two-point function dependence
- Similar result for Schwarzschild but integral is not convergent on principle series, $\mathcal{I}(\Delta_1 + \Delta_2 - 1)$

Ex 2: Two-point Amplitude on Point Source Background



Charged particle corresponding to a current $j^\mu(x) = -\int d\tau \; q^\mu(\tau)\delta^{(4)}(x-y)$ generates a potential with two-point amplitude:

$$\mathcal{A}_2^{(1)}(p_1, p_2) = -eQ \int d\tau \; \frac{(p_1 - p_2)\cdot q}{(p_1+p_2)^2} e^{-i(p_1+p_2)\cdot y}$$

For a massless source (shockwave background) with 4-velocity

$$q^\mu = (1 + |z_{sw}|^2, z_{sw} + \bar{z}_{sw}, i(\bar{z}_{sw} - z_{sw}), 1 - |z_{sw}|^2)$$

The Mellin transformed amplitude

$$\tilde{\mathcal{A}}_2^{(1)}(\Delta_1, \Delta_2) = \frac{eQ(2\pi)^3 \delta(i(\Delta_1 + \Delta_2 - 2))}{|z_{12}|^{\Delta_1+\Delta_2} |z_{1sw}|^{\Delta_1-\Delta_2} |z_{2sw}|^{\Delta_2-\Delta_1}}$$

Has the form of a standard CFT three point amplitude!

Ex 2: Three-point Correlator from Shockwave Background

Start from form factor of electromagnetic current

$$\mathcal{A}_{3;\mu}(p_1, p_2, p) = \langle p_1|\tilde{j}_\mu(p)|p_2\rangle = e(2\pi)^4\delta^{(4)}(p_1 + p_2 + p)(p_{1\mu} - p_{2\mu})$$

Mellin transform massless on-shell legs and use electromagnetic shockwave wavefunction

$$\begin{aligned} A^{sw}_{0,0;\mu}(x,q) &= -Qq_\mu \log(x^2)\delta(q\cdot x) \\ &= \mathcal{S}_\mu[e^{ip\cdot x}] = 8\pi^2 Qq_\mu \int \frac{d^4p}{(2\pi)^4}\frac{\delta(p\cdot q)}{p^2}e^{ip\cdot x} \end{aligned}$$

so that

$$\begin{aligned} \tilde{\mathcal{A}}_3(\Delta_1,\Delta_2,\Delta_{sw}=0) &\equiv \mathcal{M}[\mathcal{S}_\mu[\mathcal{A}_3^\mu(p_1,p_2,p)]] \\ &\propto \frac{eQ\delta(i(\Delta_1+\Delta_2-2))}{|z_{12}|^{\Delta_1+\Delta_2}|z_{1sw}|^{\Delta_1-\Delta_2}|z_{2sw}|^{\Delta_2-\Delta_1}} \end{aligned}$$

i.e. same as $\tilde{\mathcal{A}}_2^{(1)}$. The two-point function in the shockwave background is a three-point function with the background state created by a shockwave operator.

Ex 3: Two-point Amplitude on Gyraton Background

Consider scalar field gravitationally minimally coupled to metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

with $h_{\mu\nu}$ taken to be small. Equation of motion

$$\Box\phi - h^{\mu\nu}\partial_\mu\partial_\nu\phi - \partial_\mu(h^{\mu\nu} - \tfrac{1}{2}h^\lambda{}_\lambda\eta^{\mu\nu})\partial_\nu\phi = J$$
$$\Rightarrow \mathcal{A}_2^{(1)}(p_1, p_2) = -\left[(p_1)_\mu(p_2)_\nu - \tfrac{1}{2}\eta_{\mu\nu}p_1\cdot p_2\right]\tilde{h}^{\mu\nu}(p_1+p_2)$$

Spinning particle infinitely boosted **along** axis of rotation (gyraton)

$$\begin{aligned} h_{\mu\nu} &= -q_\mu q_\nu r_0\delta(q\cdot x)\log(x^2 - a^2) \\ &= 4\pi^3 q_\mu q_\nu r_0 ia\int d^4p\frac{\delta(p\cdot q)}{|p|}H_{-1}^{(2)}(a|p|)e^{ip\cdot x} \equiv \mathcal{S}^a_{\mu\nu}[e^{ip\cdot x}] \end{aligned}$$

- $h_{\mu\nu}$ is conformal primary metric of dimenson $\Delta = -1$, spin $J = 0$.
- Take $q^\mu = (1, 0, 0, 1)$ for convenience and $p_i^- = \dfrac{1}{2}(p^0 - p^3)$.

Ex 3: Two-point amplitude on Gyraton Background

Amplitude is given by Hankel function:

$$\mathcal{A}_2^{(1)} = -8\pi^3 r_0 i a H_{-1}^{(2)}(a|p_1 + p_2|) \frac{p_1^- p_2^- \delta(p_1^- + p_2^-)}{|p_1 + p_2|}$$

to leading order in G (via r_0), all orders in a. The Mellin transformed celestial two-point function is:

$$\widetilde{\mathcal{A}}_{2,gyraton}^{(1)}(\Delta_1, \Delta_2) = (2\pi)^2 r_0 \frac{a^{1-\Delta_1-\Delta_2}|z_2|^2}{|z_{12}|^{\Delta_1+\Delta_2+1}} \left(\frac{|z_1|^2}{|z_2|^2}\right)^{\frac{\Delta_2-\Delta_1+1}{2}} \times \mathcal{I}'(\Delta_1 + \Delta_2 - 1)$$

- Integral is finite and smooth for a range of dimensions:

$$\mathcal{I}'(s) = -\frac{i\pi}{2} \frac{\Gamma(1+s/2)}{\Gamma(1-s/2)} (1 + i\cot(\pi s/2)), \quad 0 < \mathrm{Re}(s) < \tfrac{1}{2}.$$

Spin "softens" high-energy behaviour, exactly same seen in electromagnetic analogue ($\mathcal{I}'(\Delta_1 + \Delta_2 - 2)$); somewhat similar effect for Kerr/spinning charged particle.

Ex 3: Three-point Correlator from Gyraton Background

Consider form factor of stress-energy tensor

$$
\begin{aligned}
\mathcal{A}_{3;\mu\nu}(p_1,p_2,p) &= \langle p_1|\tilde{T}_{\mu\nu}(p)|p_2\rangle \\
&= -\kappa(2\pi)^4\delta^{(4)}(p_1+p_2-p)\left[p_{1\mu}p_{2\nu}-\tfrac{1}{2}\eta_{\mu\nu}p_1\cdot p_2\right]
\end{aligned}
$$

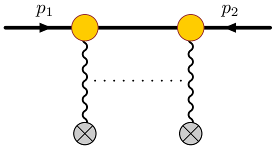
Mellin transform massless on-shell legs and use spinning shockwave wavefunction
So that again taking $q^\mu=(1+|z_{ssw}|^2, z_{ssw}+\bar{z}_{ssw}, i(\bar{z}_{ssw}-z_{ssw}), 1-|z_{ssw}|^2)$

$$
\begin{aligned}
\tilde{\mathcal{A}}_3(\Delta_1,\Delta_2,\Delta_{ssw}=-1) &\equiv \mathcal{M}[\mathcal{S}^a_{\mu\nu}[\mathcal{A}_3^{\mu\nu}(p_1,p_2,p)]] \\
&\propto \frac{a^{1-\Delta_1-\Delta_2}\mathcal{I}'(\Delta_1+\Delta_2-1)}{|z_{12}|^{\Delta_1+\Delta_2+1}|z_{1ssw}|^{\Delta_1-\Delta_2-1}|z_{2ssw}|^{\Delta_2-\Delta_1-1}}
\end{aligned}
$$

- Agrees with two-point amplitude computed in gyraton background: Shockwave backgrounds have CCFT operator interpretation. Not obvious for “massive” backgrounds e.g. Schwarzschild & Kerr.

All Order Amplitudes and IR divergences

All previous results are linear order in the background.



- We can solve the wave-equation iteratively

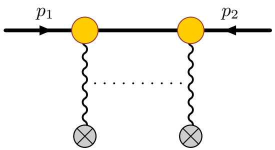
$$\frac{\delta\tilde{\Phi}(-p_2)}{\delta J(p_1)} = -\frac{1}{p_2^2}\sum_{n=1}^{\infty}\int\prod_{\ell=1}^{n-1}\frac{d^4k^{(\ell)}}{(2\pi)^4}\,\frac{\mathcal{A}_2^{(1)}(-p_2,-k^{(1)})}{k^{(1)2}}\cdots\frac{\mathcal{A}_2^{(1)}(k^{(n-1)},-p_1)}{p_1^2}$$

- Resum in the eikonal approximation: $1/(k^{(1)}+p_1)^2 \to 1/2k^{(1)}\cdot p_1$ and dropping powers of $k^{(\ell)}$ in numerators, so the amplitude becomes

$$\mathcal{A}_2^{eik}(p_1,p_2) = \boxed{\exp\left[e\int\frac{d^4k}{(2\pi)^4}\,\frac{\tilde{A}(-k)\cdot p_2}{k\cdot p_2}\right]}\,\mathcal{A}_2^{(1)}(p_1,p_2)\ .$$

Familiar Wilson line result in eikonal limit. Similar result in gravity.

All Order Amplitudes and IR divergences

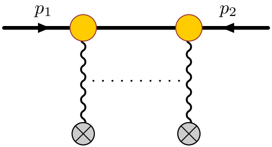


- For point particle backgrounds eikonal phase factor produces IR divergent prefactor and factorization persists for celestial amplitudes:

$$\tilde{\mathcal{A}}_2^{IR}(\Delta_1, \Delta_2) \quad = \int \prod_{i=1}^{2} d\omega_i \omega_i^{\Delta_i - 1} \mathcal{A}_2^{IR,eik}(\omega_1, \omega_2, \hat{p}_1, \hat{p}_2) = \tilde{\mathcal{A}}_2^{soft} \tilde{\mathcal{A}}_2^{(1)}$$

IR divergent prefactor is operator valued in gravity.

All Order Amplitudes and IR divergences



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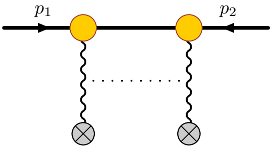
IR divergent prefactor is operator valued in gravity.

- Wave equation result can be matched to eikonal limit of 4-pt amplitude and IR divergences are related to known all-order results for amplitudes/celestial correlators.

All Order Amplitudes and IR divergences



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  IR divergent prefactor is operator valued in gravity.

- Wave equation result can be matched to eikonal limit of 4-pt amplitude and IR divergences are related to known all-order results for amplitudes/celestial correlators.
- Interpretation as vertex operator correlation functions of Goldstone bosons for large gauge symmetry/supertranslations and a composite operator for background.

IR divergences from Vertex Operators

For massless source/shockwave we introduce two bosons $\Phi^{(+)}$ and $\Phi^{(-)}$ which have the two-point functions

$$\langle \Phi^{(a)}(z_i)\Phi^{(b)}(z_j)\rangle = -\frac{1}{8\pi^2\epsilon}(\ln|z_{ij}|^2 + i\delta^{ab}\pi)$$

We define the dressing factor for in-/out-going particles as $R_k^{(\mp)} = \eta_k e_k \Phi^{(\mp)}(z_k)$ and background dressing operator the appropriately normalised normal ordered product

$$e^{-iR_{sw}} =: e^{-\frac{i}{2}R_{P_A}} e^{-\frac{i}{2}R_{P_B}} := \exp\Big[-i\tfrac{Q}{2}(\Phi^{(+)} - \Phi^{(-)})\Big]$$

Soft factor of the two-point amplitude/three-point correlator can be written as

$$\tilde{\mathcal{A}}_2^{soft} = \langle e^{iR_{sw}} e^{iR_1} e^{iR_2}\rangle$$

Contractions between R_1 and R_2 are sub-leading and neglected.

IR Finite Correlators

IR finite celestial amplitudes between massless scalars are obtained by dressing the conformal primary operator for outgoing or incoming states:

$$\hat{\mathcal{O}}^{(\pm)}_{\Delta_k+\alpha e_k^2}(z) = \lim_{z\to w} |z-w|^{-2\alpha e_k^2} :e^{-ie_k\eta_k\Phi(z)}::\mathcal{O}^{(\pm)}_{\Delta_k}(w): \ .$$

Note shifted dimension.
Similarly define a dressed shockwave operator

$$\hat{\mathcal{O}}_{sw}(z) = \lim_{z\to w} :e^{-iQ(\Phi^{(+)}(z)-\Phi^{(-)}(z)}::\mathcal{O}_{sw}(w):$$

The IR finite two-point amplitude in the shockwave background is then

$$\tilde{\mathcal{A}}_2^{\text{dressed}} = \langle \hat{\mathcal{O}}_{sw}(z_{sw})\hat{\mathcal{O}}^{(-)}_{\Delta_1}(z_1)\hat{\mathcal{O}}^{(+)}_{\Delta_2}(z_2)\rangle$$

where contractions between the Goldstone bosons cancel the IR divergent phases

Conclusions

- Computed tree-level two-point amplitudes in various Kerr-Schild backgrounds and their celestial counterparts.
- For backgrounds which are conformal primary potentials/metrics two-point amplitudes can be interpreted as three-point functions
  - Can this be extended to other backgrounds?
  - To “massive” backgrounds e.g. Schwarzschild?
- Can we include higher-order results? Included all-order eikonal phase factors for point particle backgrounds.
  - Can we incorporate next-to-eikonal? Genuine quantum corrections?
  - IR divergences for AS shockwaves have natural interpretation in CCFT, can this be extended to spin? to loop effects? to massive backgrounds?
- Can we learn anything interesting about quantum gravity?