Introduction to Accelerator Physics

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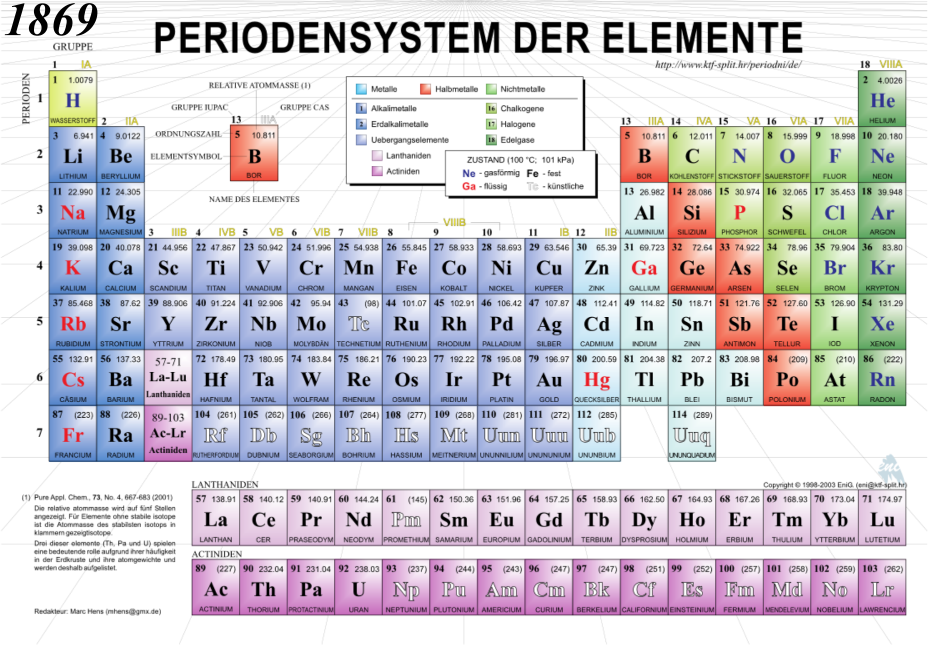
A Short Introduction ...

In the end and after all ... : We try to exp[lain the structure of "hadronic matter" in the universe.

In short words: "What is going on, up there ???"

$E = mc^2, \lambda = h / p$

1869



PERIODENSYSTEM DER ELEMENTE

http://www.ktf-split.hr/periodni/de/

GRUPPE

PERIODEN

RELATIVE ATOMMASSE (1)
GRUPPE IUPAC
GRUPPE CAS
ORDNUNGSZAHL
ELEMENTSYMBOL
NAME DES ELEMENTES

13 IIIA
5 10.811
B
BOR

🟦 Metalle
🟥 Halbmetalle
🟩 Nichtmetalle

1 Alkalimetalle
2 Erdalkalimetalle
Uebergangselemente
Lanthaniden
Actiniden
16 Chalkogene
17 Halogene
18 Edelgase

ZUSTAND (100 °C; 101 kPa)
Ne - gasförmig
Fe - fest
Ga - flüssig
Tc - künstliche

	1 IA	2 IIA	3 IIIB	4 IVB	5 VB	6 VIB	7 VIIB	8 VIIIB	9	10	11 IB	12 IIB	13 IIIA	14 IVA	15 VA	16 VIA	17 VIIA	18 VIIIA
1	1 1.0079 H WASSERSTOFF																	2 4.0026 He HELIUM
2	3 6.941 Li LITHIUM	4 9.0122 Be BERYLLIUM											5 10.811 B BOR	6 12.011 C KOHLENSTOFF	7 14.007 N STICKSTOFF	8 15.999 O SAUERSTOFF	9 18.998 F FLUOR	10 20.180 Ne NEON
3	11 22.990 Na NATRIUM	12 24.305 Mg MAGNESIUM											13 26.982 Al ALUMINIUM	14 28.086 Si SILIZIUM	15 30.974 P PHOSPHOR	16 32.065 S SCHWEFEL	17 35.453 Cl CHLOR	18 39.948 Ar ARGON
4	19 39.098 K KALIUM	20 40.078 Ca CALCIUM	21 44.956 Sc SCANDIUM	22 47.867 Ti TITAN	23 50.942 V VANADIUM	24 51.996 Cr CHROM	25 54.938 Mn MANGAN	26 55.845 Fe EISEN	27 58.933 Co KOBALT	28 58.693 Ni NICKEL	29 63.546 Cu KUPFER	30 65.39 Zn ZINK	31 69.723 Ga GALLIUM	32 72.64 Ge GERMANIUM	33 74.922 As ARSEN	34 78.96 Se SELEN	35 79.904 Br BROM	36 83.80 Kr KRYPTON
5	37 85.468 Rb RUBIDIUM	38 87.62 Sr STRONTIUM	39 88.906 Y YTTRIUM	40 91.224 Zr ZIRKONIUM	41 92.906 Nb NIOB	42 95.94 Mo MOLYBDÄN	43 (98) Tc TECHNETIUM	44 101.07 Ru RUTHENIUM	45 102.91 Rh RHODIUM	46 106.42 Pd PALLADIUM	47 107.87 Ag SILBER	48 112.41 Cd CADMIUM	49 114.82 In INDIUM	50 118.71 Sn ZINN	51 121.76 Sb ANTIMON	52 127.60 Te TELLUR	53 126.90 I IOD	54 131.29 Xe XENON
6	55 132.91 Cs CÄSIUM	56 137.33 Ba BARIUM	57-71 La-Lu Lanthaniden	72 178.49 Hf HAFNIUM	73 180.95 Ta TANTAL	74 183.84 W WOLFRAM	75 186.21 Re RHENIUM	76 190.23 Os OSMIUM	77 192.22 Ir IRIDIUM	78 195.08 Pt PLATIN	79 196.97 Au GOLD	80 200.59 Hg QUECKSILBER	81 204.38 Tl THALLIUM	82 207.2 Pb BLEI	83 208.98 Bi BISMUT	84 (209) Po POLONIUM	85 (210) At ASTAT	86 (222) Rn RADON
7	87 (223) Fr FRANCIUM	88 (226) Ra RADIUM	89-103 Ac-Lr Actiniden	104 (261) Rf RUTHERFORDIUM	105 (262) Db DUBNIUM	106 (266) Sg SEABORGIUM	107 (264) Bh BOHRIUM	108 (277) Hs HASSIUM	109 (268) Mt MEITNERIUM	110 (281) Uun UNUNNILIUM	111 (272) Uuu UNUNUNIUM	112 (285) Uub UNUNBIUM		114 (289) Uuq UNUNQUADIUM				

LANTHANIDEN

57 138.91 La LANTHAN	58 140.12 Ce CER	59 140.91 Pr PRASEODYM	60 144.24 Nd NEODYM	61 (145) Pm PROMETHIUM	62 150.36 Sm SAMARIUM	63 151.96 Eu EUROPIUM	64 157.25 Gd GADOLINIUM	65 158.93 Tb TERBIUM	66 162.50 Dy DYSPROSIUM	67 164.93 Ho HOLMIUM	68 167.26 Er ERBIUM	69 168.93 Tm THULIUM	70 173.04 Yb YTTERBIUM	71 174.97 Lu LUTETIUM

ACTINIDEN

89 (227) Ac ACTINIUM	90 232.04 Th THORIUM	91 231.04 Pa PROTACTINIUM	92 238.03 U URAN	93 (237) Np NEPTUNIUM	94 (244) Pu PLUTONIUM	95 (243) Am AMERICIUM	96 (247) Cm CURIUM	97 (247) Bk BERKELIUM	98 (251) Cf CALIFORNIUM	99 (252) Es EINSTEINIUM	100 (257) Fm FERMIUM	101 (258) Md MENDELEVIUM	102 (259) No NOBELIUM	103 (262) Lr LAWRENCIUM

(1) Pure Appl. Chem., **73**, No. 4, 667-683 (2001)

Die relative atommasse wird auf fünf Stellen angezeigt. Für Elemente ohne stabile isotope ist die Atommasse des stabilsten isotops in klammern gezeigtisotope.

Drei dieser elemente (Th, Pa und U) spielen eine bedeutende rolle aufgrund ihrer häufigkeit in der Erdkruste und ihre atomgewichte und werden deshalb aufgelistet.

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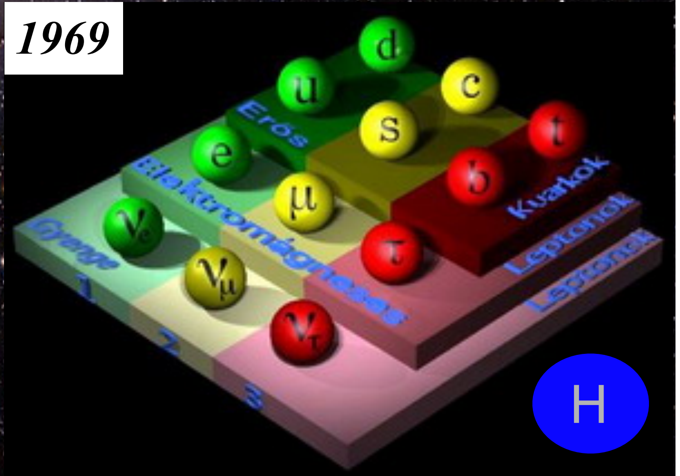
1969



the beta function is usually obtained via the matrix element „m12", which is in Twiss form for the undistorted case

$$m_{12} = \beta_0 \sin 2\pi Q$$

$$(1) \quad m_{12}^* = \beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta k ds$$

and including the error:

$$\underbrace{m_{12}^*}_{} = b_{11} a_{12} + b_{12} a_{22} - \cancel{b_{12} a_{12} \Delta k ds}$$

$$m_{12} = \beta_0 \sin 2\pi Q$$

As M* is still a matrix for one complete turn we still can express the element m_{12} in twiss form:

$$(2) \quad m_{12}^* = (\beta_0 + d\beta) * \sin 2\pi (Q + dQ)$$

$$- a_{12} b_{12} \Delta k ds = \beta_0 2\pi dQ \cos 2\pi Q + d\beta_0 \sin 2\pi Q$$

Equalising (1) and (2) and assuming a small error

$$dQ = \frac{\Delta k \beta_1 ds}{4\pi}$$

$$\beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta k ds = (\beta_0 + d\beta) * \sin 2\pi (Q + dQ)$$

What we will NOT do .....

$$\beta_0 \sin 2\pi Q - a_{12} b_{12} \Delta k ds = (\beta_0 + d\beta) * \sin 2\pi Q \underbrace{\cos 2\pi dQ}_{\approx 1} + \cos 2\pi Q \underbrace{\sin 2\pi dQ}_{\approx 2\pi dQ}$$

$$- a_{12} b_{12} \Delta k ds = \frac{\beta_0 \Delta k \beta_1 ds}{2} \cos 2\pi Q + d\beta_0 \sin 2\pi Q$$

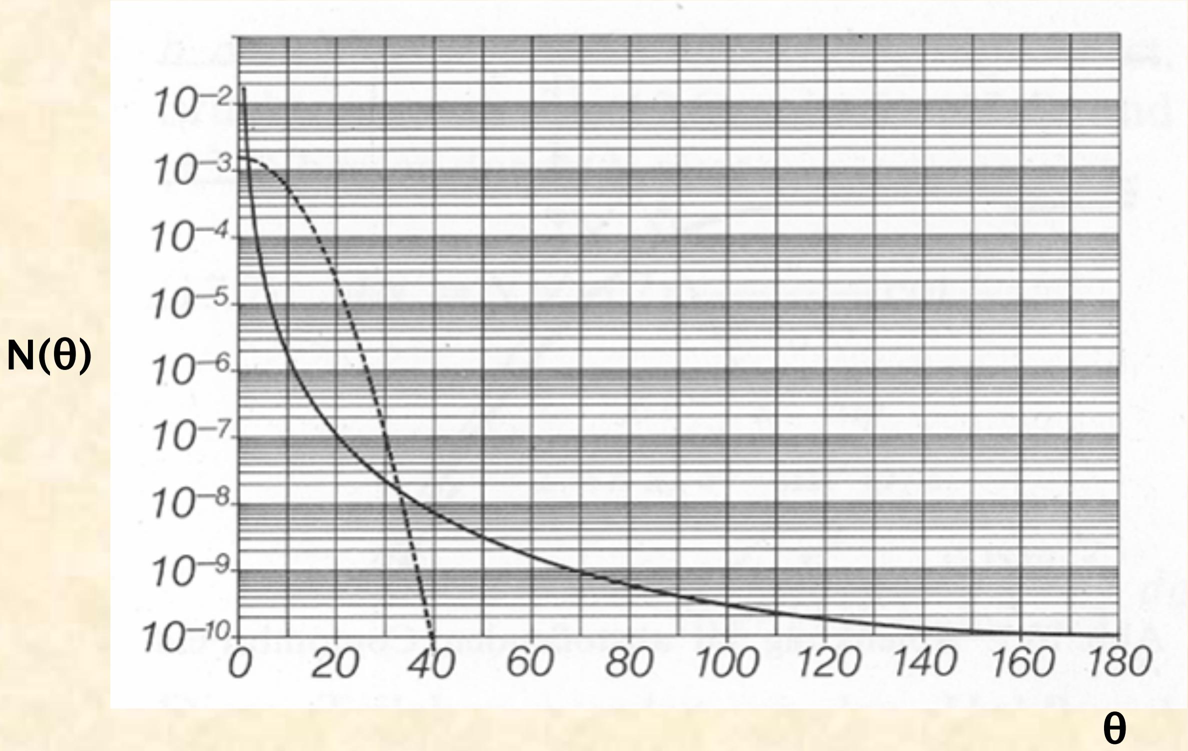
$$\beta_0 \sin 2\pi Q - \cancel{a_{12} b_{12} \Delta k ds} = \beta_0 \sin 2\pi Q + \cancel{\beta_0 2\pi dQ \cos 2\pi Q} + d\beta_0 \sin 2\pi Q + d\beta_0 2\pi dQ \cos 2\pi Q$$

$$d\beta_0 = \frac{-1}{2 \sin 2\pi Q} \{2 a_{12} b_{12} + \beta_0 \beta_1 \cos 2\pi Q\} \Delta k ds$$

$$M = \begin{pmatrix} \sqrt{\frac{\beta_s}{\beta_0}} (\cos\psi_s + \alpha_0 \sin\psi_s) & \sqrt{\beta_s \beta_0} \sin\psi_s \\ \frac{(\alpha_0 - \alpha_s)\cos\psi_s - (1 + \alpha_0 \alpha_s)\sin\psi_s}{\sqrt{\beta_s \beta_0}} & \sqrt{\frac{\beta_0}{\beta s}} (\cos\psi_s - \alpha_s \sin\psi_s) \end{pmatrix}$$

I.) A Bit of History

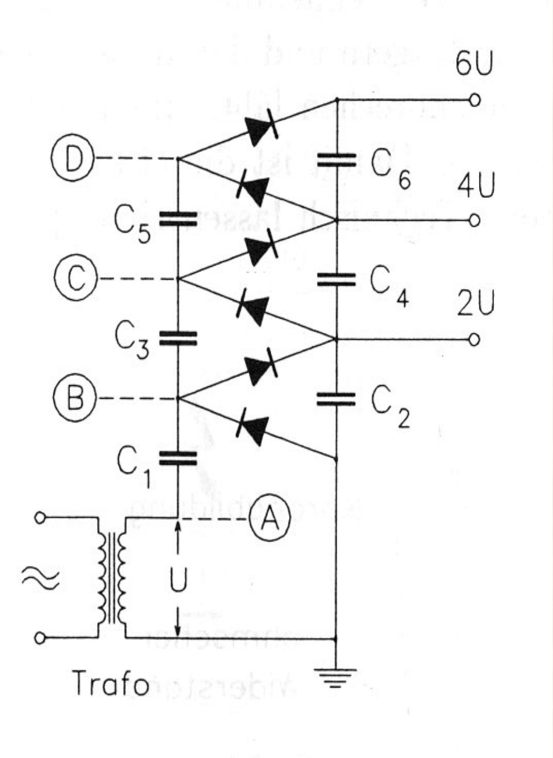
$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta / 2)}$$



Rutherford Scattering, 1911

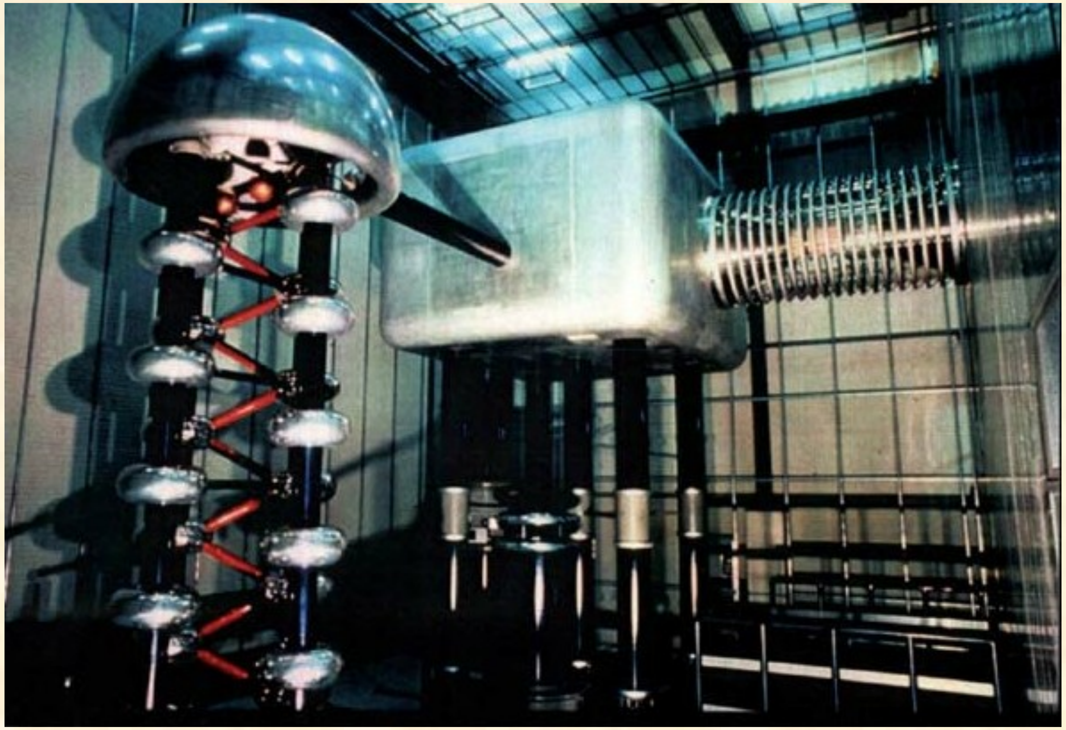
Using radioactive particle sources:
α-particles of some MeV energy

1.) Electrostatic Machines:
The Cockcroft-Walton Generator



1928: Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam

1932: First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV



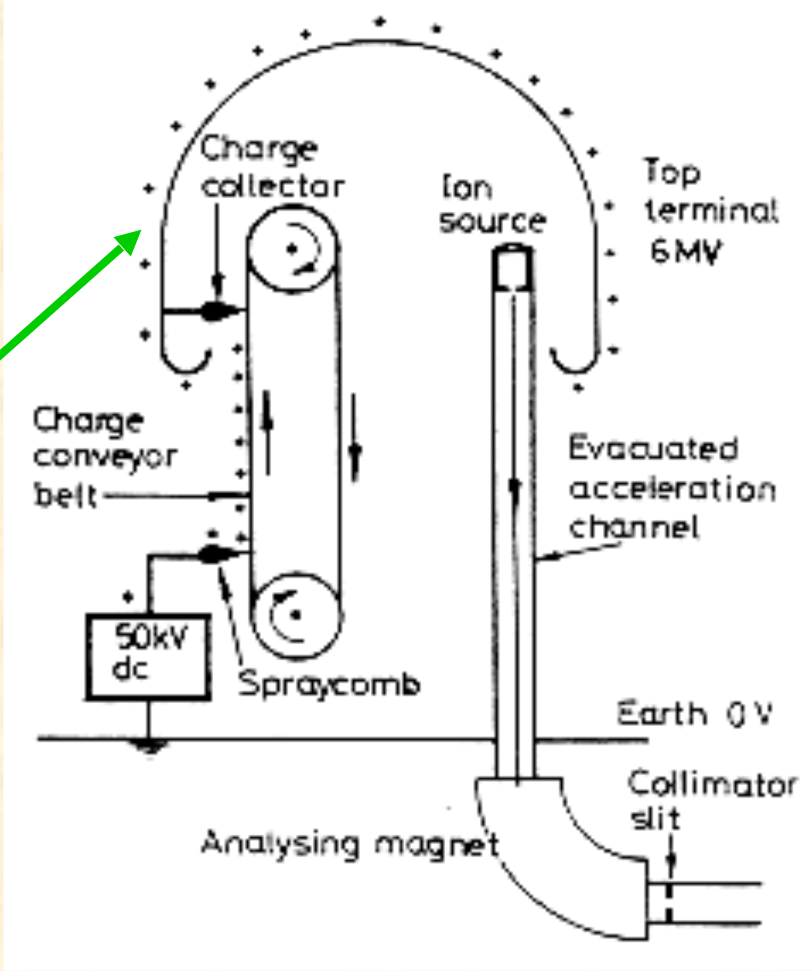
Particle source: Hydrogen discharge tube on 400 kV level
Accelerator: evacuated glas tube
Target: Li-Foil on earth potential

Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

Problem:
DC Voltage can only be used once

2.) Electrostatic Machines:
(Tandem –) van de Graaff Accelerator (1930 ...)

creating high voltages by mechanical transport of charges



** Terminal Potential: U ≈ 12 ...28 MV using high pressure gas to suppress discharge (SF_6)*

*Problems: * Particle energy limited by high voltage discharges*
** high voltage can only be applied once per particle ...*
... or twice ?

The „Tandem principle“: Apply the accelerating voltage twice ...

... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

***Example for such a „steam engine“:** 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg*

Gretchen Frage (J.W. Goethe, Faust)

Fallen die Dinger eigentlich runter ?

$l_{vdG} = 30m$

$v \approx 10\,\%\,c \approx 3 * 10^7\ m/s$

$\Delta t = 1\mu s$

Free Fall in Vacuum:

$$s = \frac{1}{2} g t^2$$

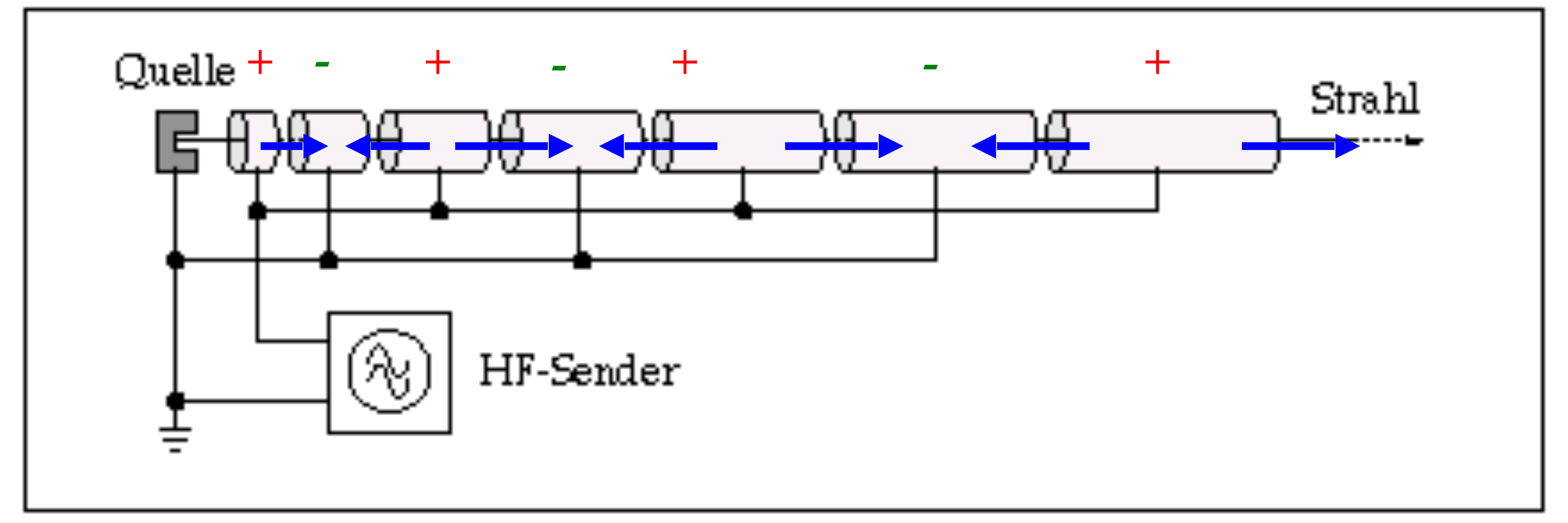
$$s = \frac{1}{2} \cdot 10 \frac{m}{s^2} \cdot (1\mu s)^2$$

$$s = 5 \cdot 10^{-12} m = 5pm$$

3.) The first RF-Accelerator: „Linac“

1928, Wideroe: ***how can the acceleration voltage be applied several times to the particle beam***

schematic Layout:



Energy gained after n acceleration gaps

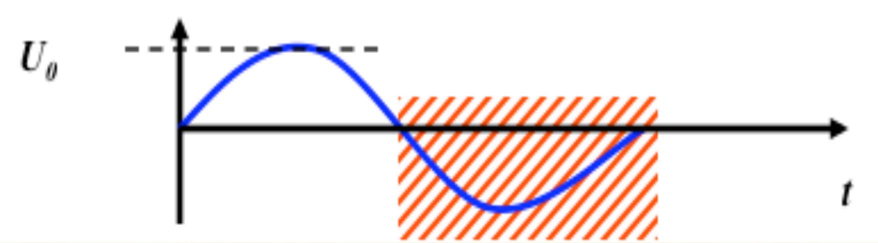
$$E_n = n \cdot q \cdot U_0 \cdot sin\ \psi_s$$

n *number of gaps between the drift tubes*
q *charge of the particle*
U_0 *Peak voltage of the RF System*
Ψ_S *synchronous phase of the particle*

**** acceleration of the proton in the first gap***
**** voltage has to be „flipped“ to get the right sign in the second gap → RF voltage***
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{rf}/2$

Length of the Drift Tube: $l_n = v_n \cdot \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_n = \frac{1}{2} m v^2$

$$v_n = \sqrt{2E_n/m}$$

$$l_n = \frac{1}{f_{rf}} \cdot \sqrt{\frac{n \cdot q \cdot U_0 \cdot sin\psi_s}{2m}}$$

valid for non relativistic particles ...

Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: ≈ 20 MeV per Nucleon β ≈ 0.04 … 0.6, Particles: Protons/Ions

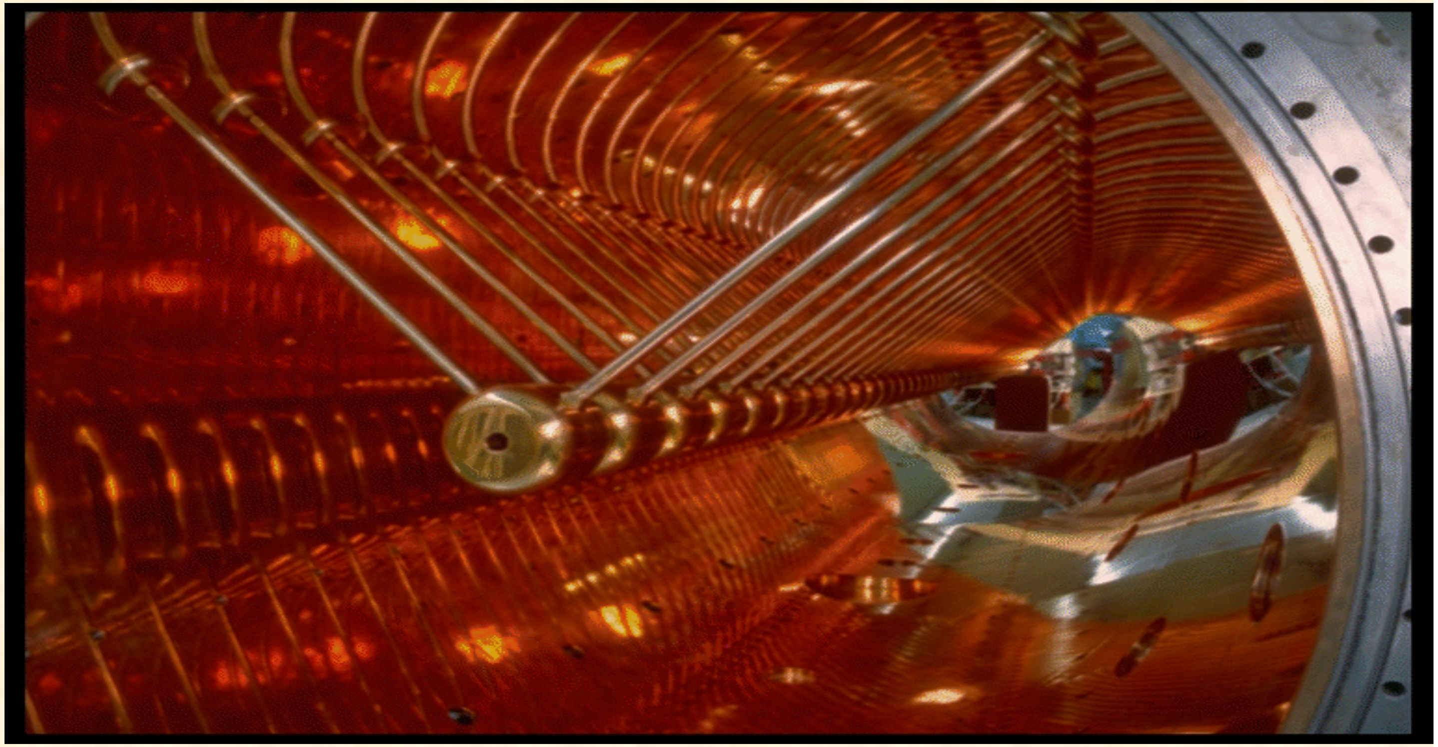
Accelerating structure of a Proton Linac (DESY Linac III)

$$E_{total} = 988\ MeV$$

$$m_0c^2 = 938\ MeV$$

$$E_{kin} = 50\ MeV$$

$$p = 310\ MeV/c$$



Beam energies

Energy Gain per „Gap“: $\boxed{W = q U_0 \sin \omega_{RF} t}$

1.) reminder of some relativistic formula

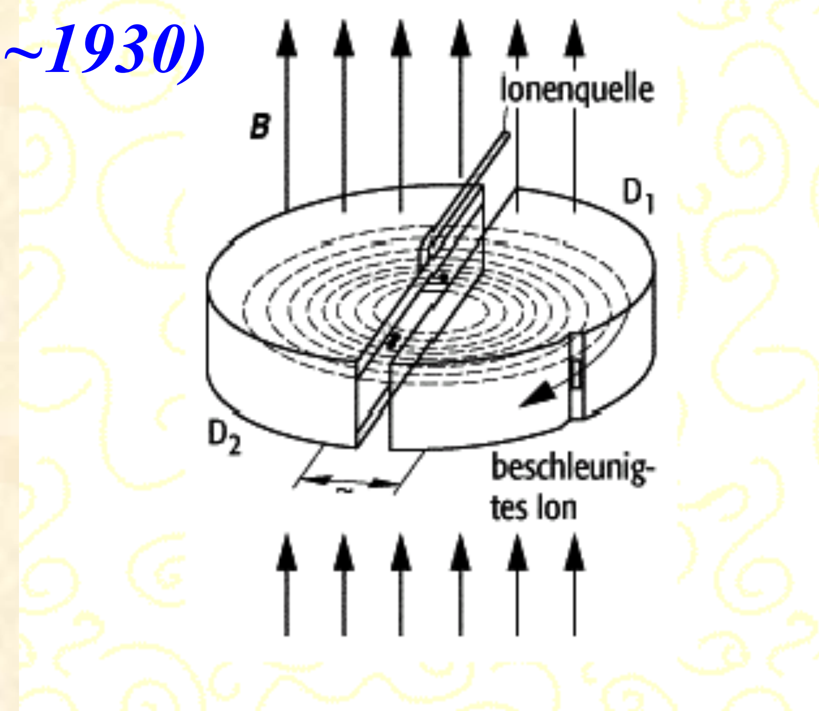
rest energy $E_0 = m_0c^2$

total energy $E = \gamma \cdot E_0 = \gamma \cdot m_0c^2$

kinetic energy $E_{kin} = E_{total} - m_0c^2$

momentum $E^2 = p^2c^2 + m_0^2c^4$

3.) The Cyclotron: (Livingston / Lawrence ~1930)



Idea: B = const, RF = const
Synchronisation particle / RF via orbit

Lorentzforce

$$\vec{F} = q * (\vec{v} \times \vec{B}) = q * v * B$$

circular orbit

$$q * v * B = \frac{m * v^2}{R} \quad \longrightarrow \quad B * R = p / q$$

increasing radius for
increasing momentum
➔ ***Spiral Trajectory***

revolution frequency

$$\omega_z = \frac{v}{R} = \frac{q}{m} * B_z$$

the cyclotron (rf-) frequency
is independent of the momentum

rf-frequency = h* revolution frequency, h = "harmonic number"

Cyclotron:

exact equation for revolution frequency:

$$\omega_z = \frac{v}{R} = \frac{q}{\gamma * m} * B_z$$

1.) *if* $v << c \Rightarrow \gamma \cong 1$

2.) γ *increases with the energy*
$\Rightarrow$ *no exact synchronism*

Cyclotron SPIRAL at GANIL

"synchronisation" with the acceleration potential is established via the spiraling orbit length

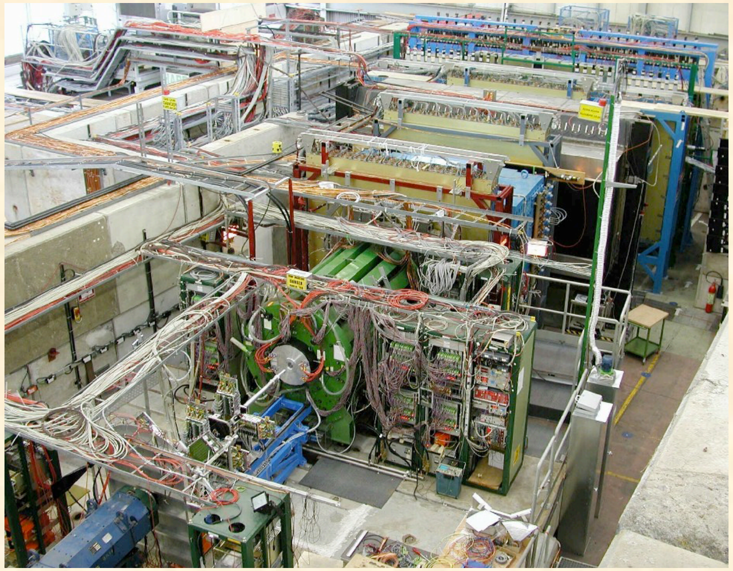
Syn

$B = constant$
$\gamma\, \omega_{RF} = constant$
ω_{RF} *decreases with time*

$$\omega_s(t) = \omega_{rf}(t) = \frac{q}{\gamma(t) \cdot m_0} B$$

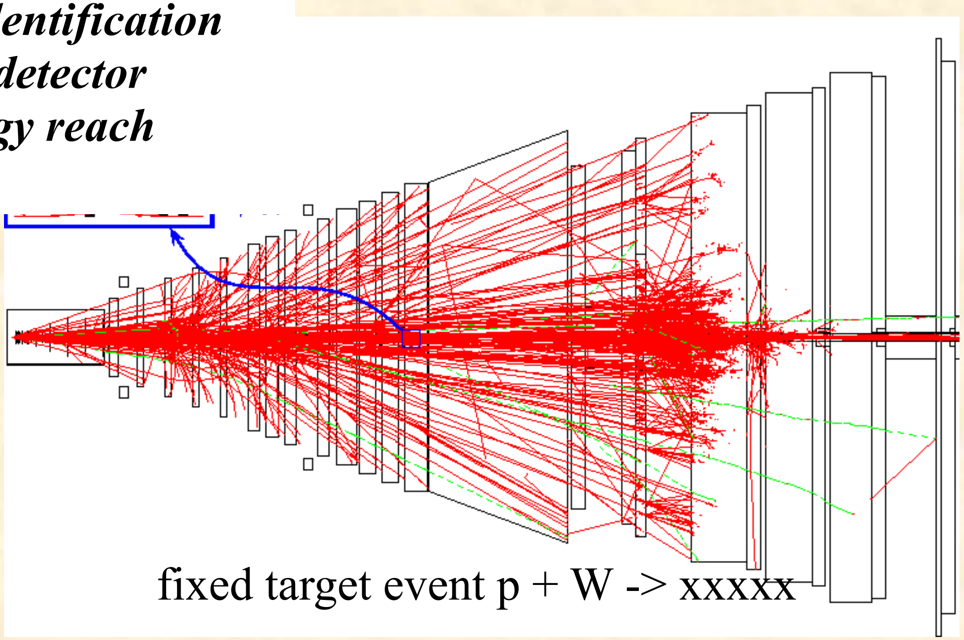
keep the synchronisation condition by varying the rf frequency

Fixed target experiments:



HARP Detector, CERN

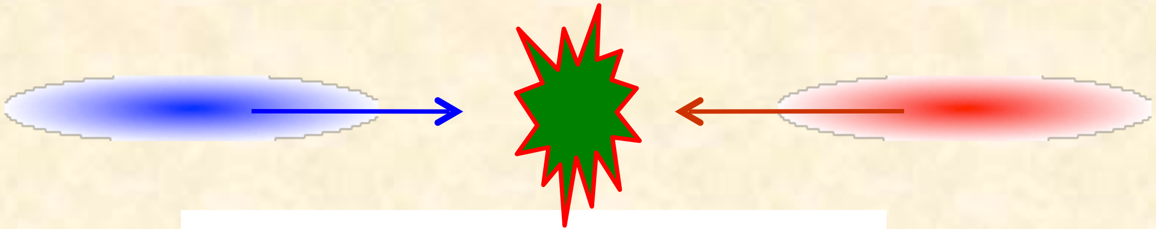
high event rate
easy track identification
asymmetric detector
limited energy reach



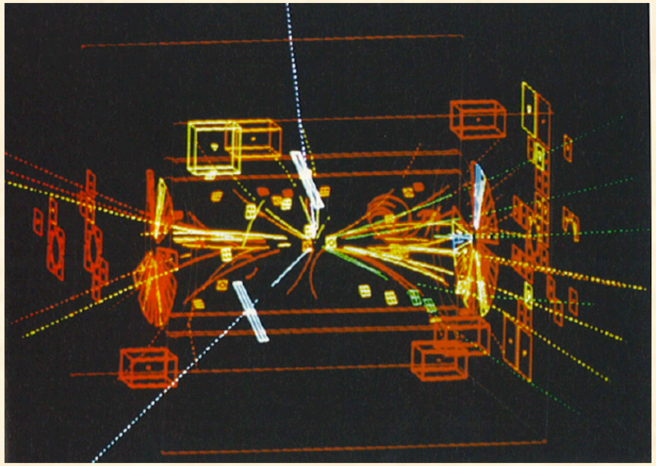
fixed target event p + W -> xxxxx

Collider experiments:

E=mc²

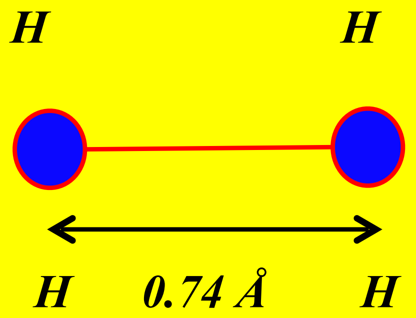


low event rate (luminosity)
challenging track identification
symmetric detector
$E_{lab} = E_{cm}$



Z_0 boson discovery at the UA2 experiment (CERN).
The Z_0 boson decays
into a e+e− pair, shown as white dashed lines.

Particle Density in matter

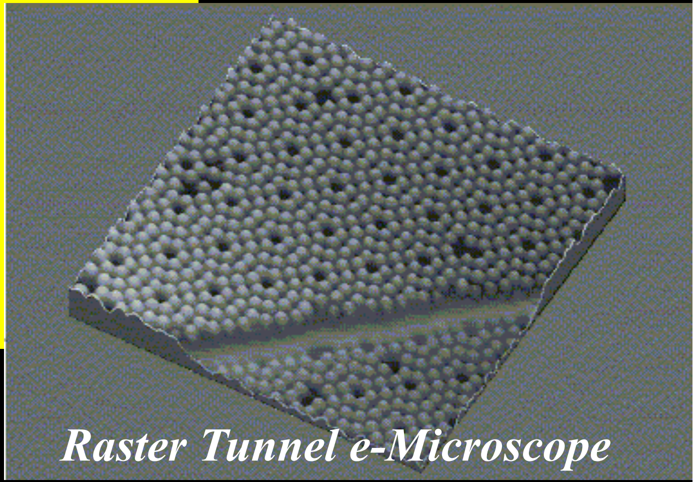


Atomic Distance in Hydrogen Molecule

$R_B \approx 0.5$ Å

in solids / fluids $\lambda \approx 1 \dots 3$ Å

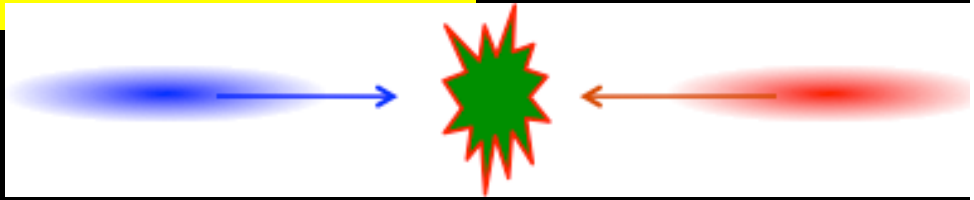
in gases $\lambda \approx 35$ Å $= 3.5$ nm



Raster Tunnel e-Microscope

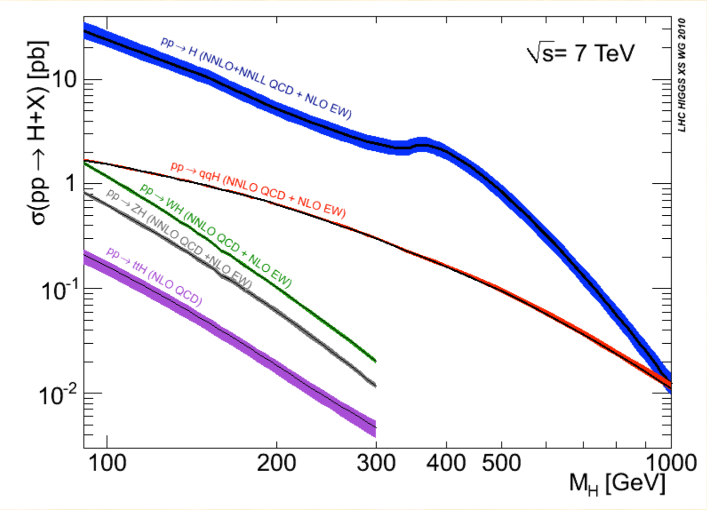
Particle Distance in Accelerators: $\lambda \approx 600$ nm (Arc) … 300nm (IP LEP)

$= 3000$ Å

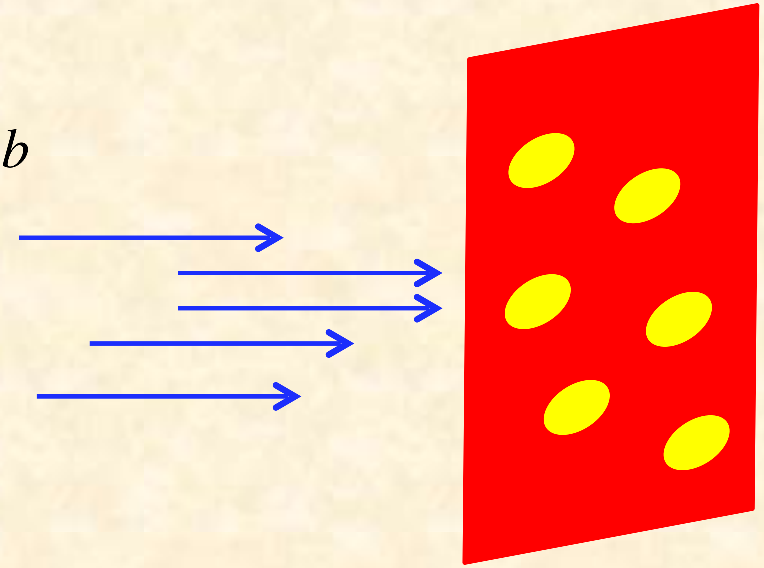


Problem: Our particles are VERY small !!

Overall cross section of the Higgs:



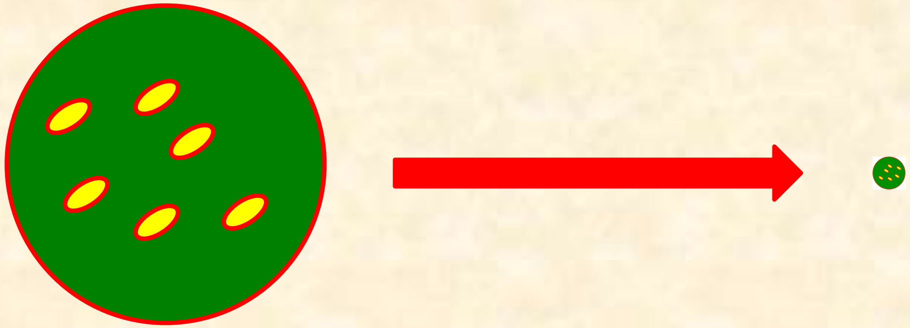
$$\Sigma_{react} \approx 1pb$$



$$1b = 10^{-24} cm^2 = \frac{1}{mio} \cdot \frac{1}{mio} \cdot \frac{1}{mio} \cdot \frac{1}{10000} \quad mm^2 \longrightarrow 1pb = 10^{-12} b \approx ZERO$$

The particles are "very small"

The only chance we have:
compress the transverse beam size … at the IP

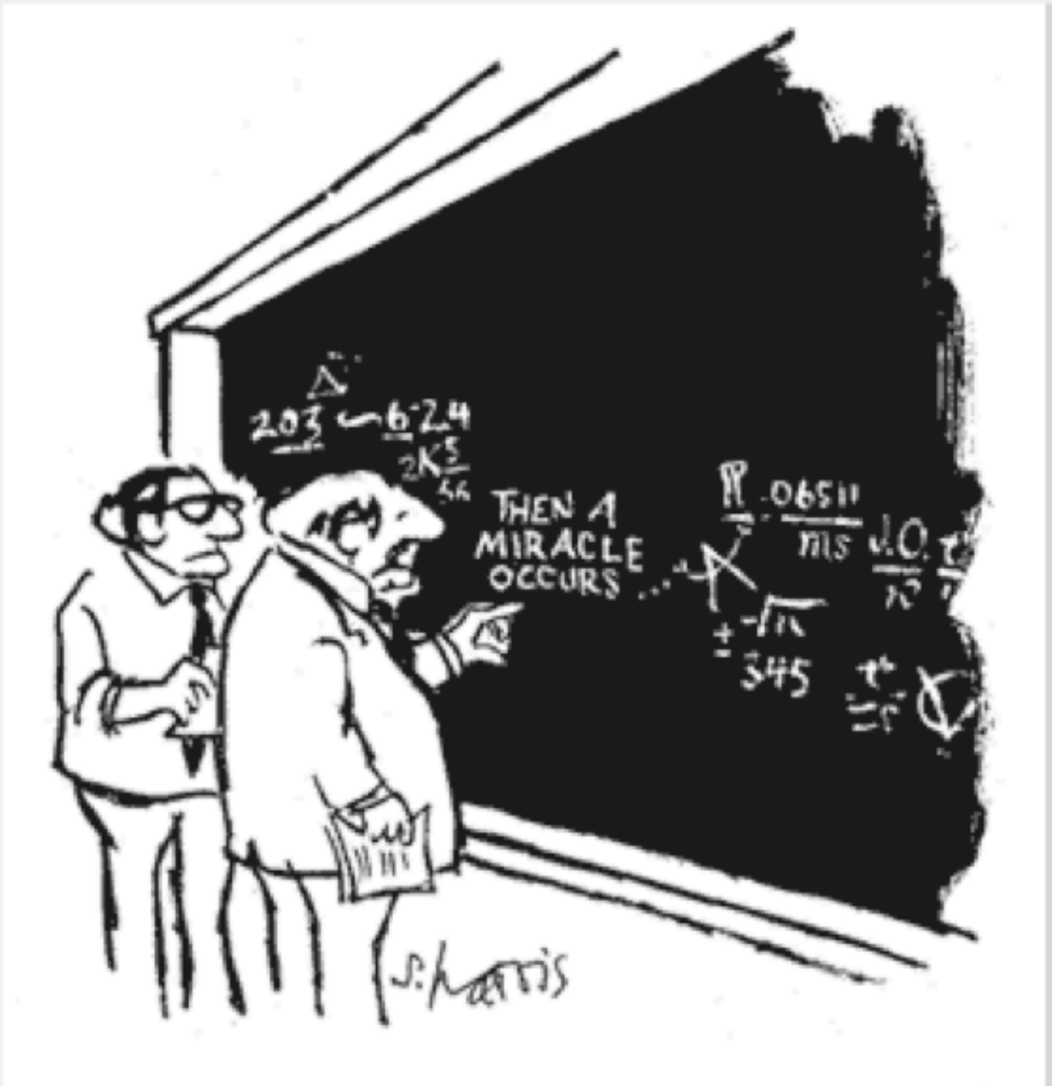


LHC typical:

σ = 0.1 mm → 16 µm

II.) A Bit of Theory

The big storage rings: „Synchrotrons“



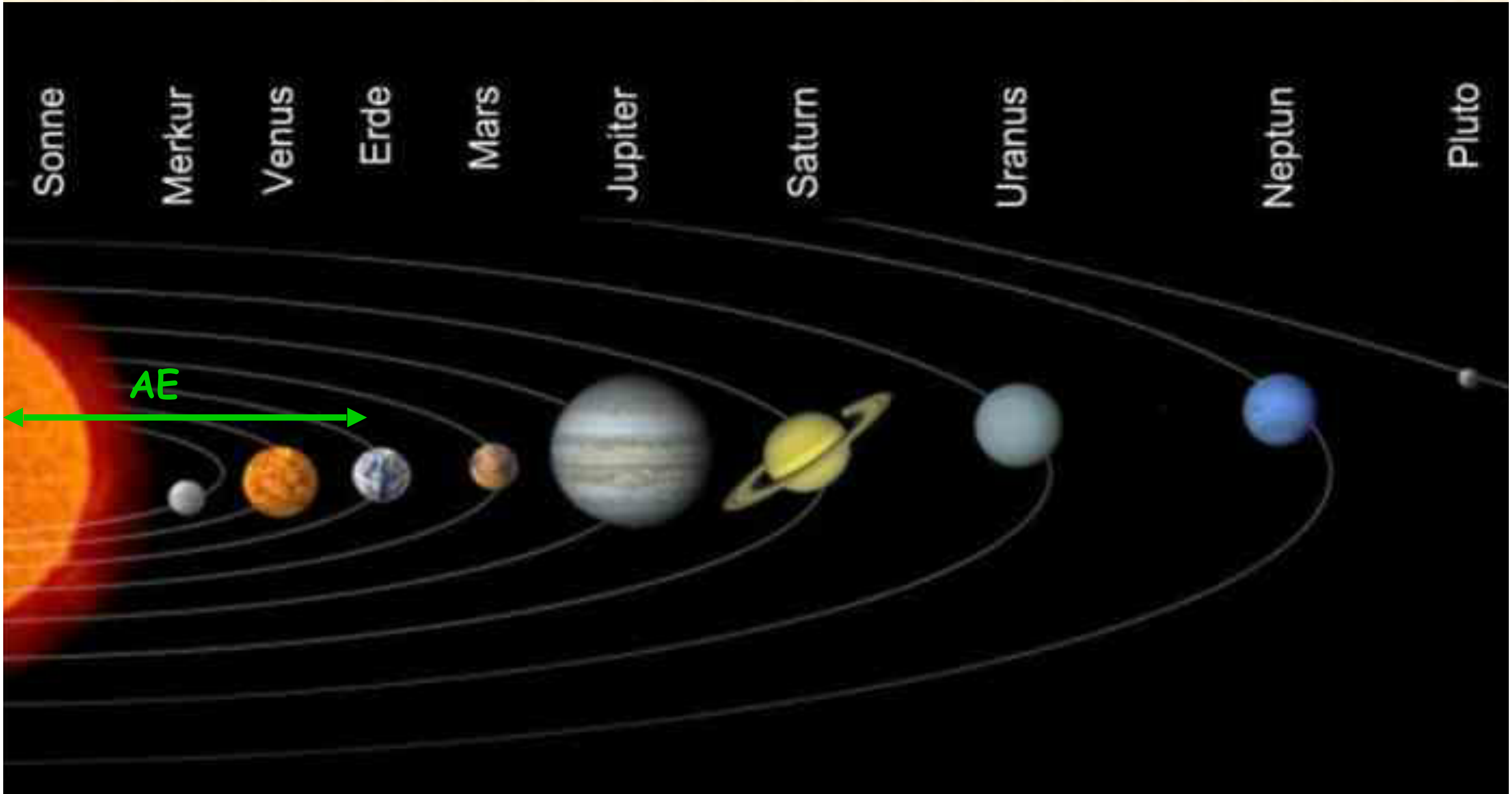
"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

Largest storage ring: The Solar System

astronomical unit: average distance earth-sun

1AE $\approx$ 150 $*10^6$ km

Distance Pluto-Sun $\approx$ 40 AE



1.) Introduction and Basic Ideas

„ … in the end and after all it should be a kind of circular machine“

→ *need transverse deflecting force*

Lorentz force

$$\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$$

typical velocity in high energy machines:

$$v \approx c \approx 3 * 10^{8}\ {}^{m}\!/_{s}$$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q * 3 * 10^{8}\,\frac{m}{s} * 1\,\frac{Vs}{m^{2}}$$

$$F = q * \underbrace{300\,\frac{MV}{m}}_{E}$$

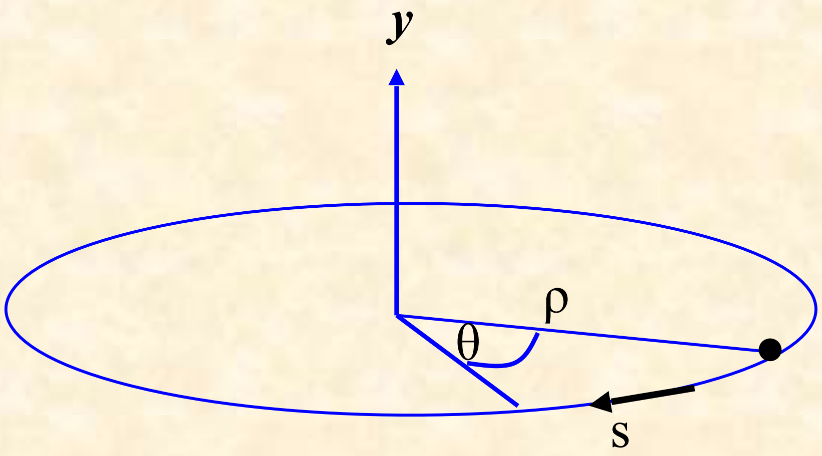
equivalent electrical field: E

Technical limit for electrical fields:

$$E \leq 1\,\frac{MV}{m}$$

Pearl of Wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force

$$F_L = e\,v\,B$$

centrifugal force

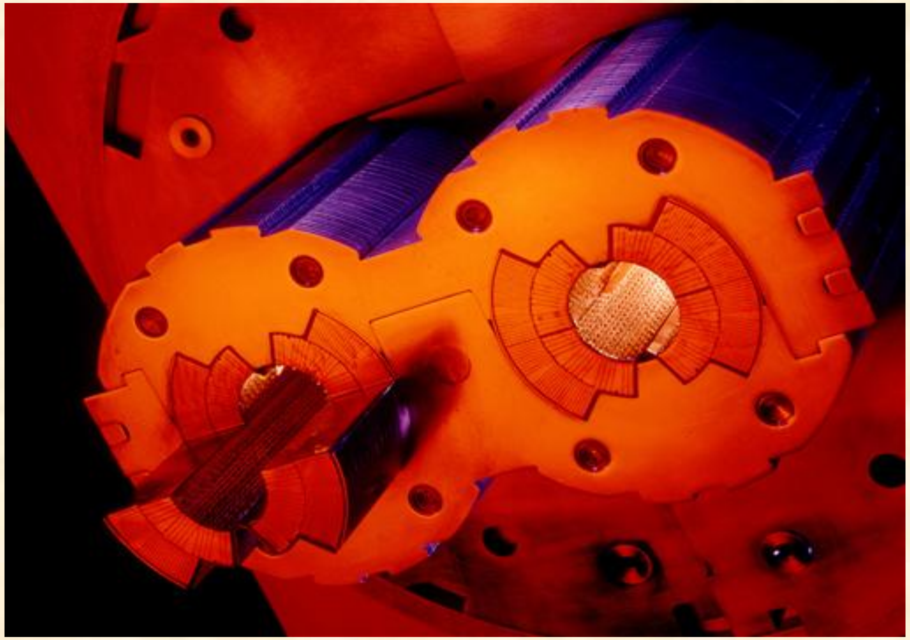
$$F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$$

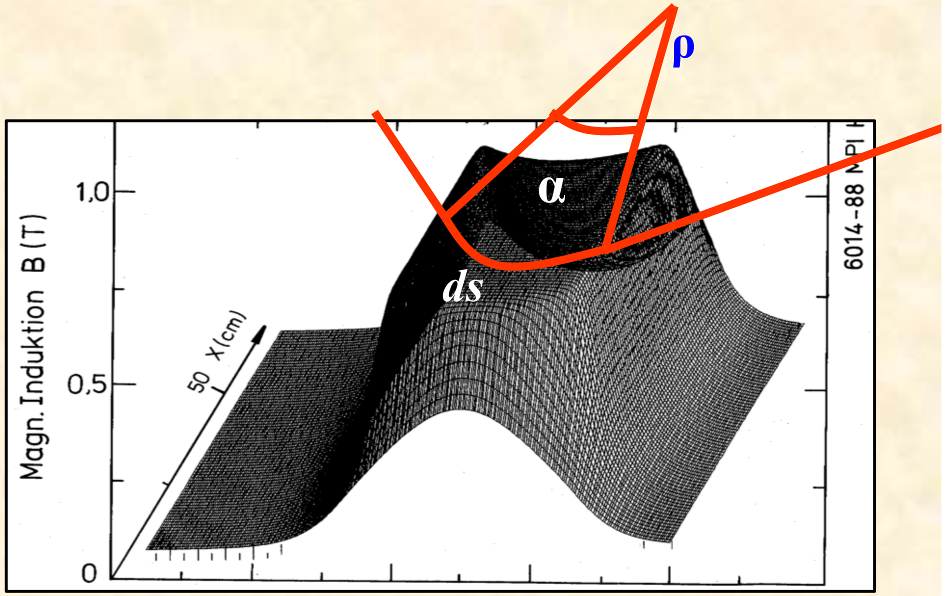
$$\frac{\gamma\, m_0\, v^{\cancel{2}}}{\rho} = e\,\cancel{v}\,B$$

$$\frac{p}{e} = B\,\rho$$

$B\,\rho$ = "beam rigidity"

The Magnetic Guide Field





field map of a storage ring dipole magnet

$\rho = 2.8\ km$ ⟶ $2\pi\rho = 17.6\ km$

$\approx 66\%$

$B \approx 1 \dots 8\ T$

nota bene: $\frac{\Delta B}{B} \approx 10^{-4}$

rule of thumb: $\boxed{\frac{1}{\rho} \approx 0.3 \frac{B[T]}{p[GeV/c]}}$

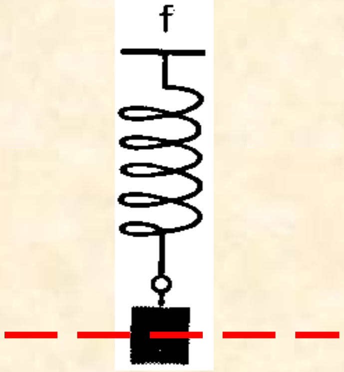
„normalised bending strength“

2.)Focusing Forces: Hook's law

... keeping the flocs together:
In addition to the pure bending of the beam
we have to keep 10^{11} particles close together

focusing force

And here we borrow the idea from classical mechanics:
The pendulum



there is a restoring force, proportional to the elongation x:

$$F = m * a = -\,const * x$$

$$F = m * \frac{d^2x}{dt^2} = -\,const * x$$

general solution:
free harmonic oscillation

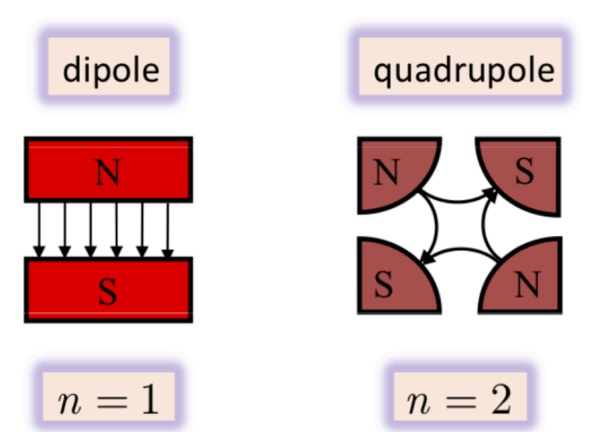
$$x(t) = A * \cos(\omega t + \varphi)$$

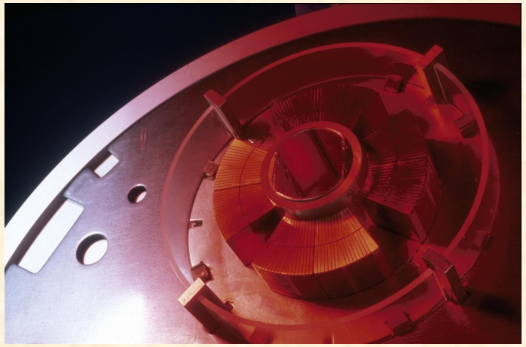
...this is how grandma's Kuckuck's clock is working!!!

2.)Focusing Forces: Quadrupole Fields

Apply this concept to magnetic forces: ***we need a*** *Lorentz force* ***that rises as a function of the*** *distance to .... the design orbit*

$$F(x) = q * v * B(x)$$





Dipoles: Create a constant field $B_y = const$

Quadrupoles: Create a linear increasing magnetic field: $B_y = g \cdot x, \quad B_x = g \cdot y$

Focusing forces and particle trajectories:

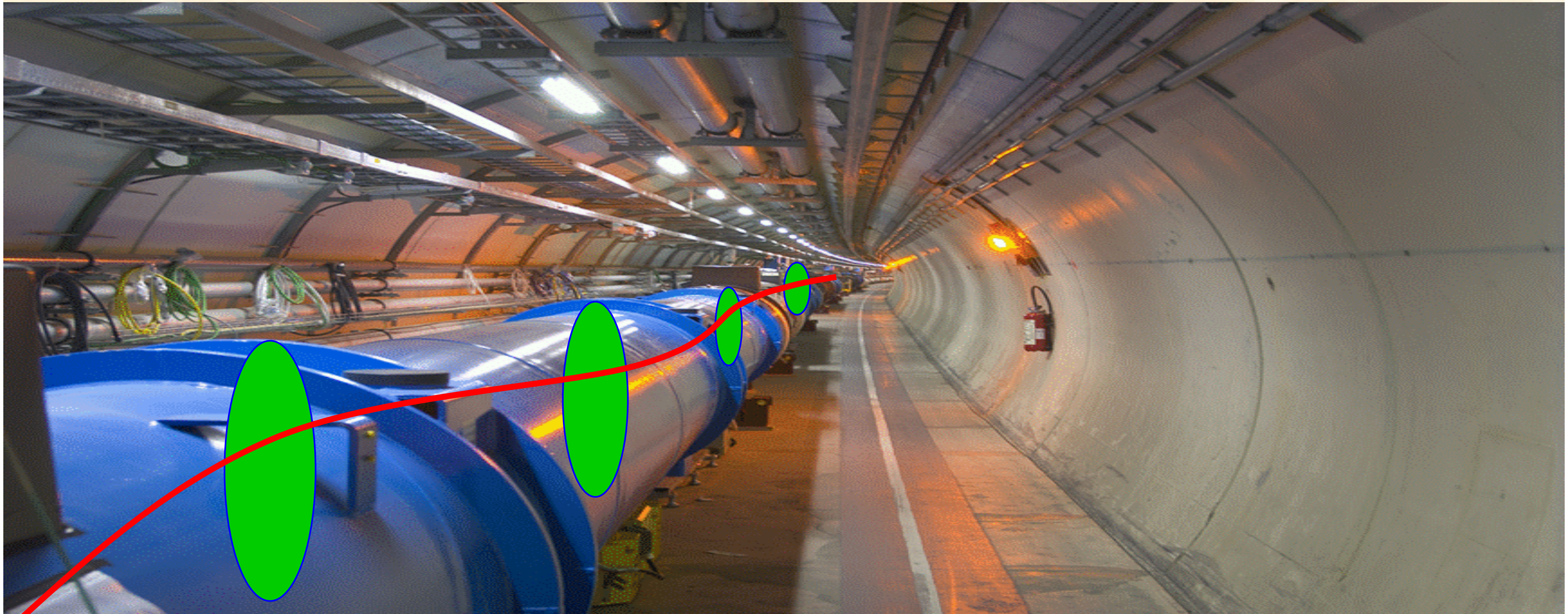
normalise magnet fields to momentum
(remember: ***B*ρ = p / q*** *)*

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet

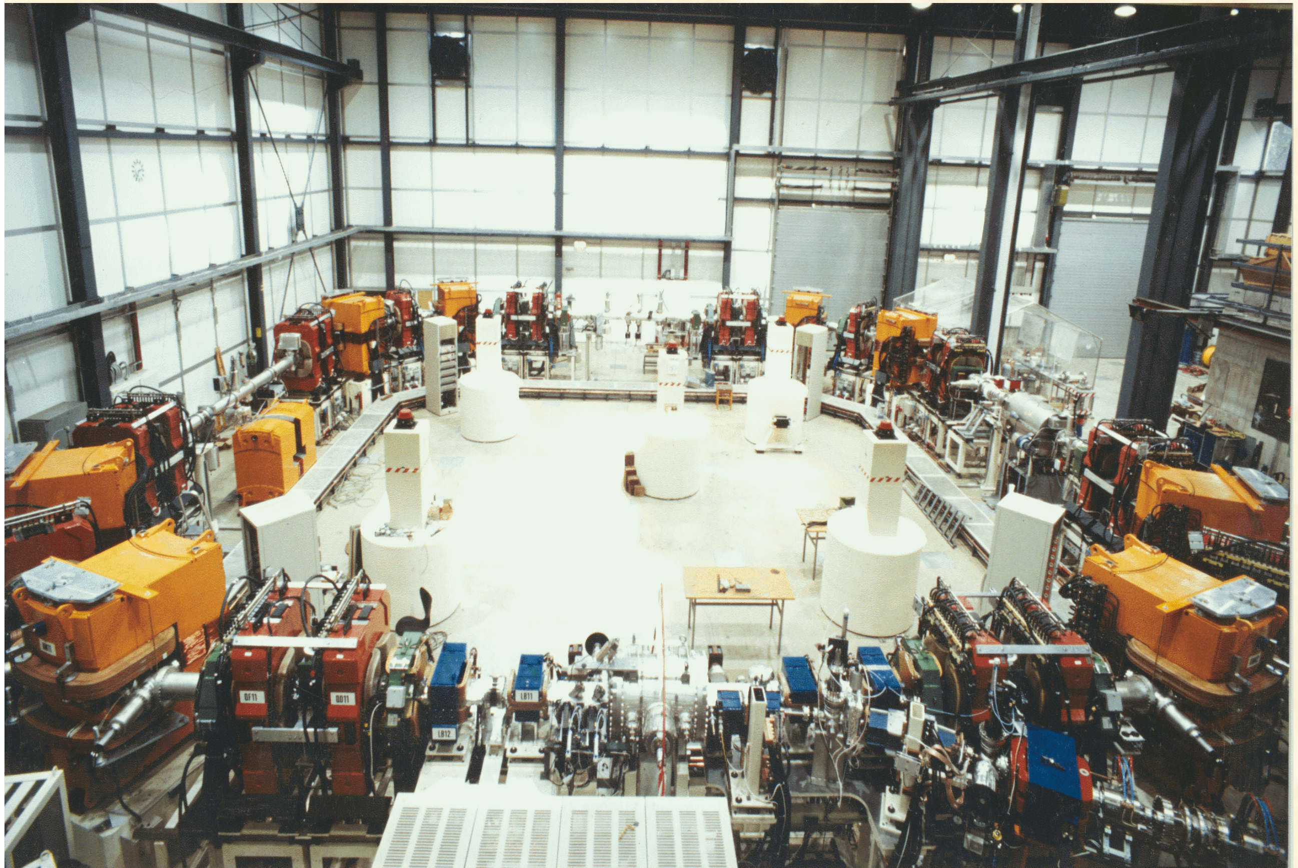
$$k := \frac{g}{p/q}$$



3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \cancel{\frac{1}{2!}\,m\,x^2} + \cancel{\frac{1}{3!}\,n\,x^3} + \dots$$

only terms linear in x, y taken into account *dipole fields*
quadrupole fields



Separate Function Machines:

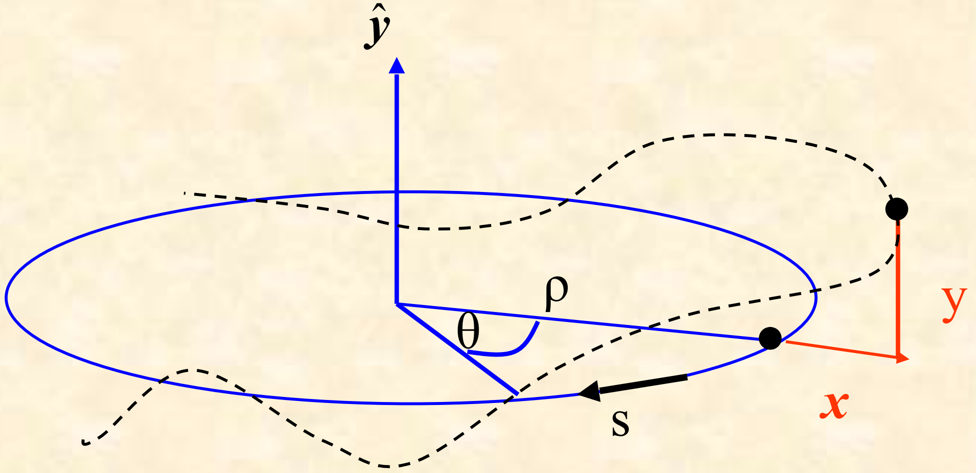
Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

*
man sieht nur
dipole und quads → linear

The Equation of Motion:



* ***Equation for the horizontal motion:***

$$x'' + x \cdot (\frac{1}{\rho^2} + k) = 0$$

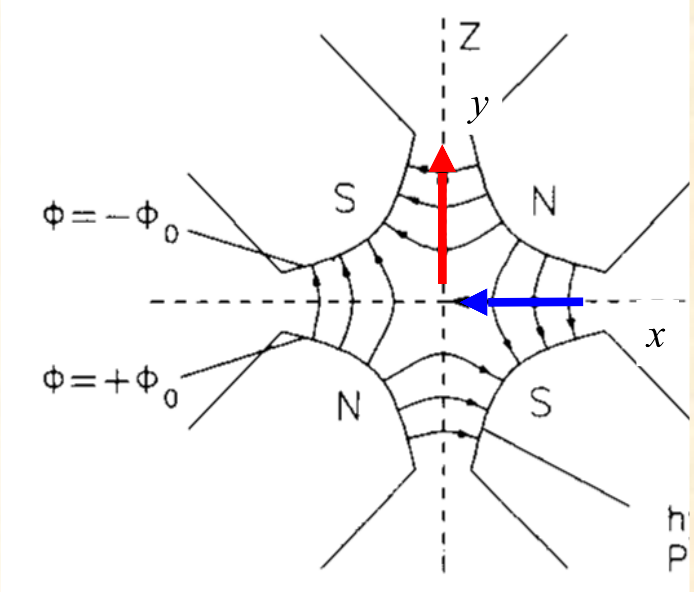
x =   particle amplitude
x' = angle of particle trajectory (wrt ideal path line)

* ***Equation for the vertical motion:***

$$\frac{1}{\rho^2} = 0$$ ***no dipoles … in general …***

$k \quad \leftrightarrow \quad -k$ ***quadrupole field changes sign***

$$y'' - k \cdot y = 0$$



4.) Solution of Trajectory Equations

Define … hor. plane: $K = 1/\rho^2 + k$

… vert. Plane: $K = -k$

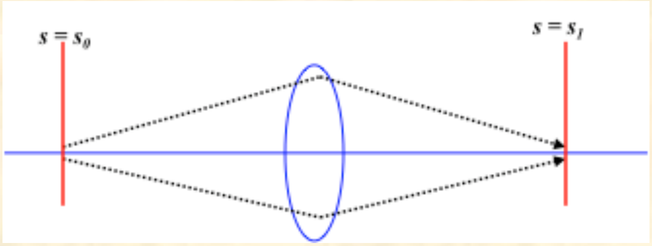
$$\left.\begin{array}{l} \text{hor. plane: } K = 1/\rho^2 + k \\ \text{vert. Plane: } K = -k \end{array}\right\} \qquad \boxed{x'' + K\, x = 0}$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: *Hor. Focusing Quadrupole K > 0:*

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x_0' \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x_0' \cdot \cos(\sqrt{|K|}s)$$

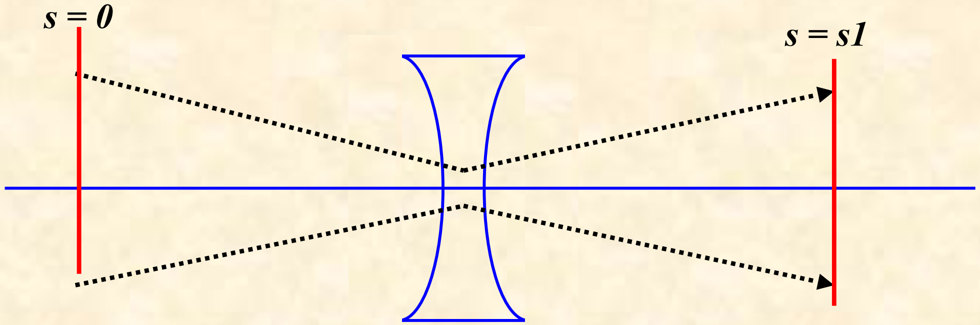


For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$M_{foc} = \begin{pmatrix} \cos\left(\sqrt{|K|}l\right) & \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}l\right) \\ -\sqrt{|K|} \sin\left(\sqrt{|K|}l\right) & \cos\left(\sqrt{|K|}l\right) \end{pmatrix}$$

hor. defocusing quadrupole:



$$x'' - K\ x = 0$$

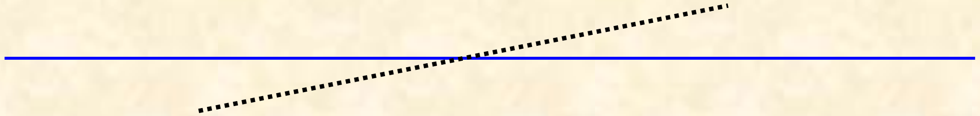
Ansatz: ***Remember from school***

$$x(s) = a_1 \cdot \cosh(\omega\, s) + a_2 \cdot \sinh(\omega\, s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$

drift space:

K = 0



$$x(s) = x_0' \cdot s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

! ***with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“***

Ok … ok ... it’s a bit complicated and cosh and sinh and all that is a pain.
BUT ... compare ...

Weak Focusing / Strong Focusing

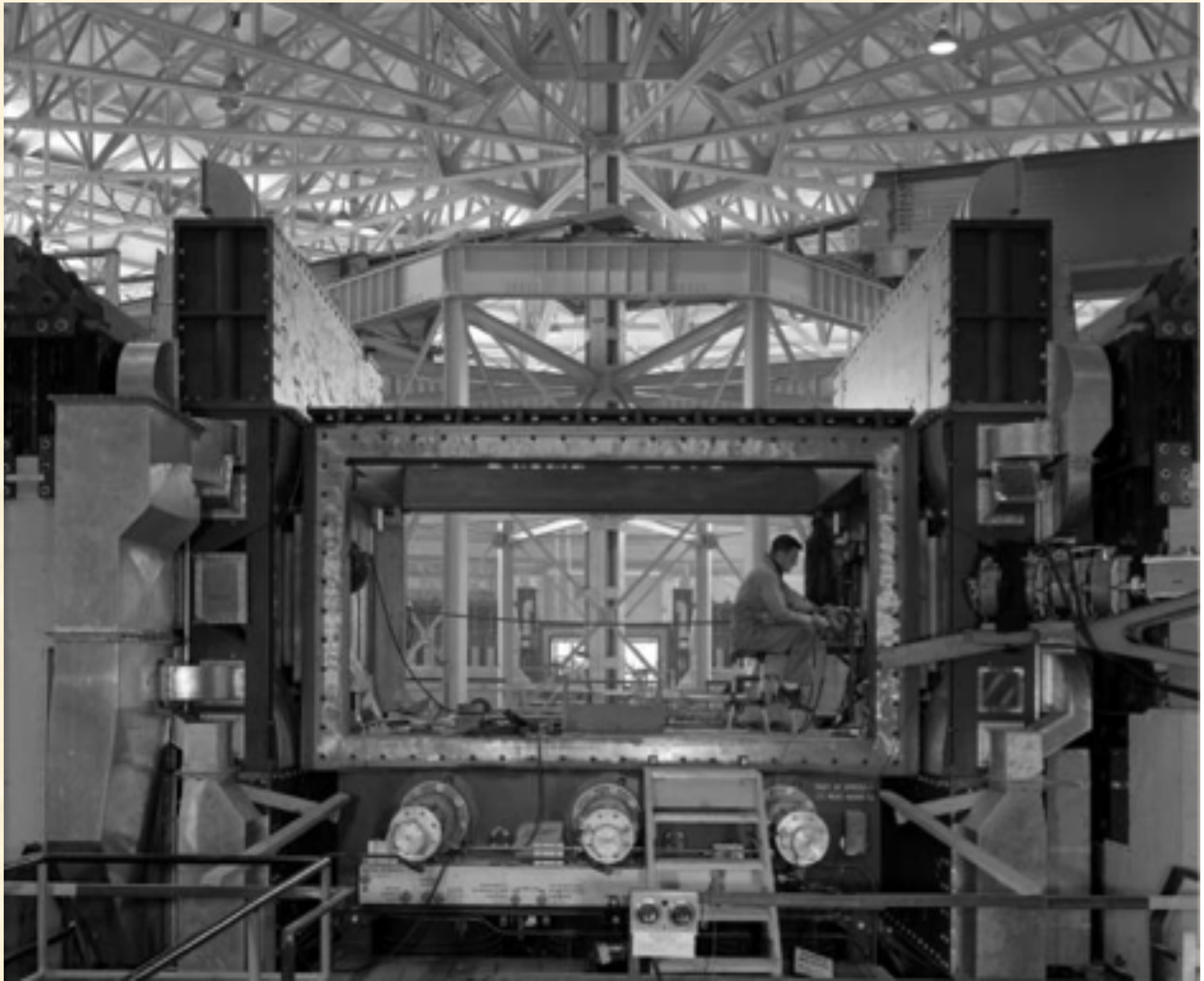
weak focusing term = 1/ρ²

$$x'' + x\left(\frac{1}{\rho^2} + \cancel{k}\right) = 0$$

Problem: the higher the energy, the larger the machine

The last weak focusing
high energy machine …
BEVATRON

→ *large apertures needed*
→ *very expensive magnets*

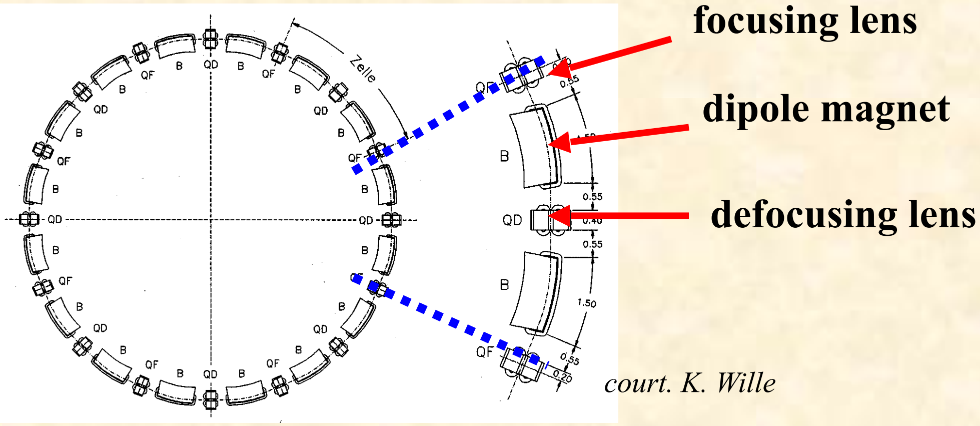


Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

$$M_{total} = M_{QF} * M_{D} * M_{QD} * M_{Bend} * M_{D*.....}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



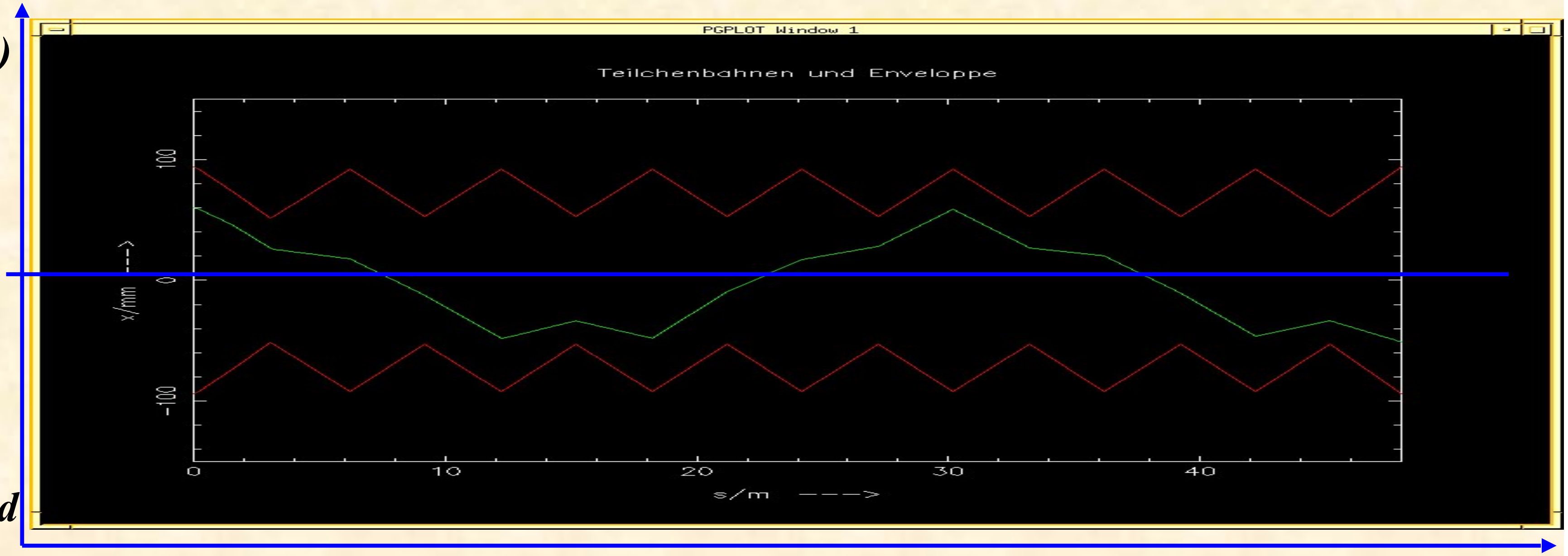
focusing lens

dipole magnet

defocusing lens

court. K. Wille

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



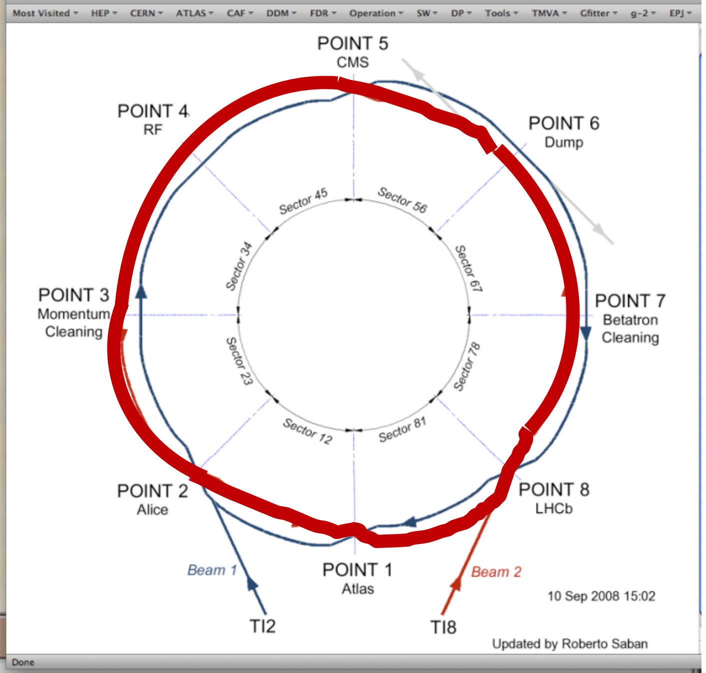
typical values in a strong foc. machine:

$x \approx mm$, $x' \leq mrad$

… just as Big Ben

… and just as any harmonic pendulum

LHC Operation: Beam Commissioning



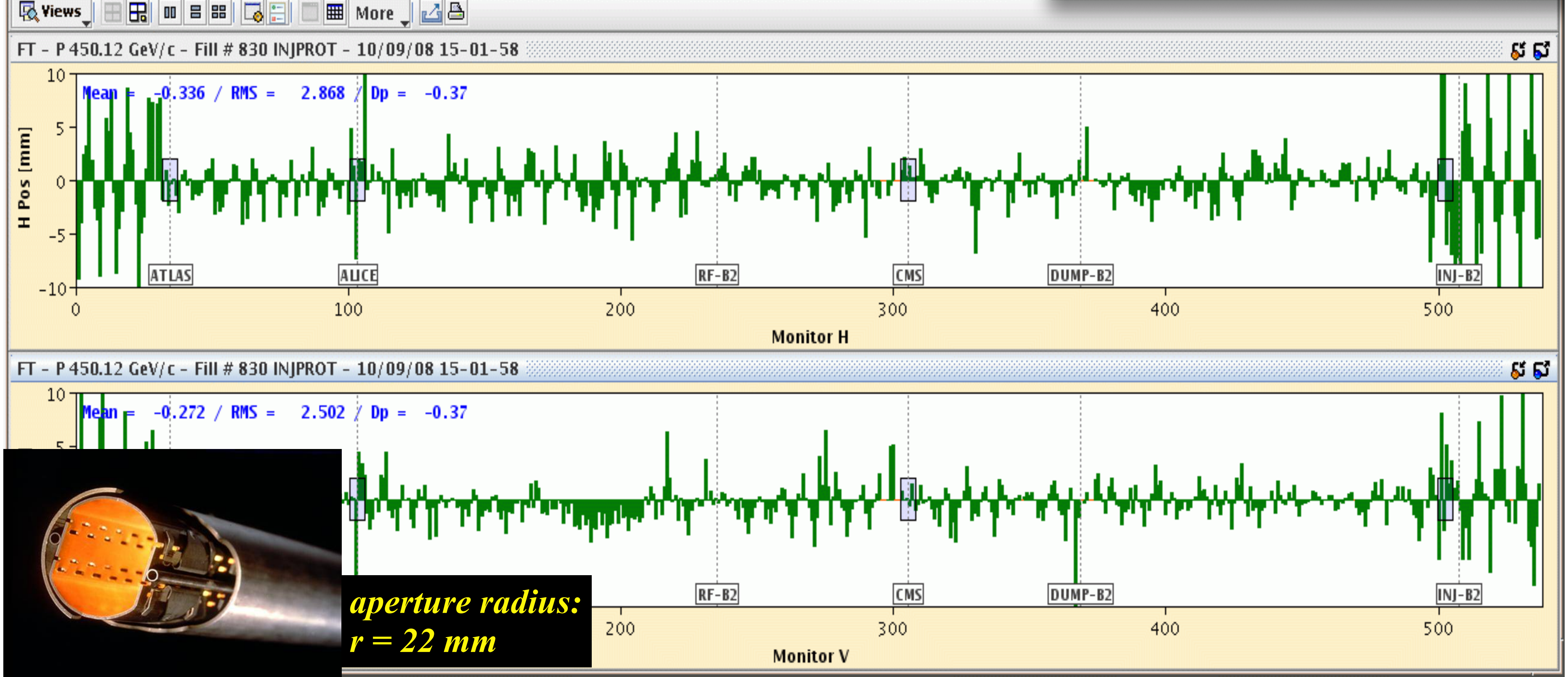
10 Sep 2008 15:02

Updated by Roberto Saban

*The **transverse focusing fields** create a **harmonic oscillation of the particles** with a well defined "Eigenfrequency" which is called **tune***

First turn steering "by sector:"

YASP DV LHCRING / INJ-TEST-NB / beam 2



FT - P 450.12 GeV/c - Fill # 830 INJPROT - 10/09/08 15-01-58

Mean = -0.336 / RMS = 2.868 / Dp = -0.37

FT - P 450.12 GeV/c - Fill # 830 INJPROT - 10/09/08 15-01-58

Mean = -0.272 / RMS = 2.502 / Dp = -0.37

aperture radius:
r = 22 mm

“Once more unto the breach, dear friends, once more”
(W. Shakespeare, Henry 5)

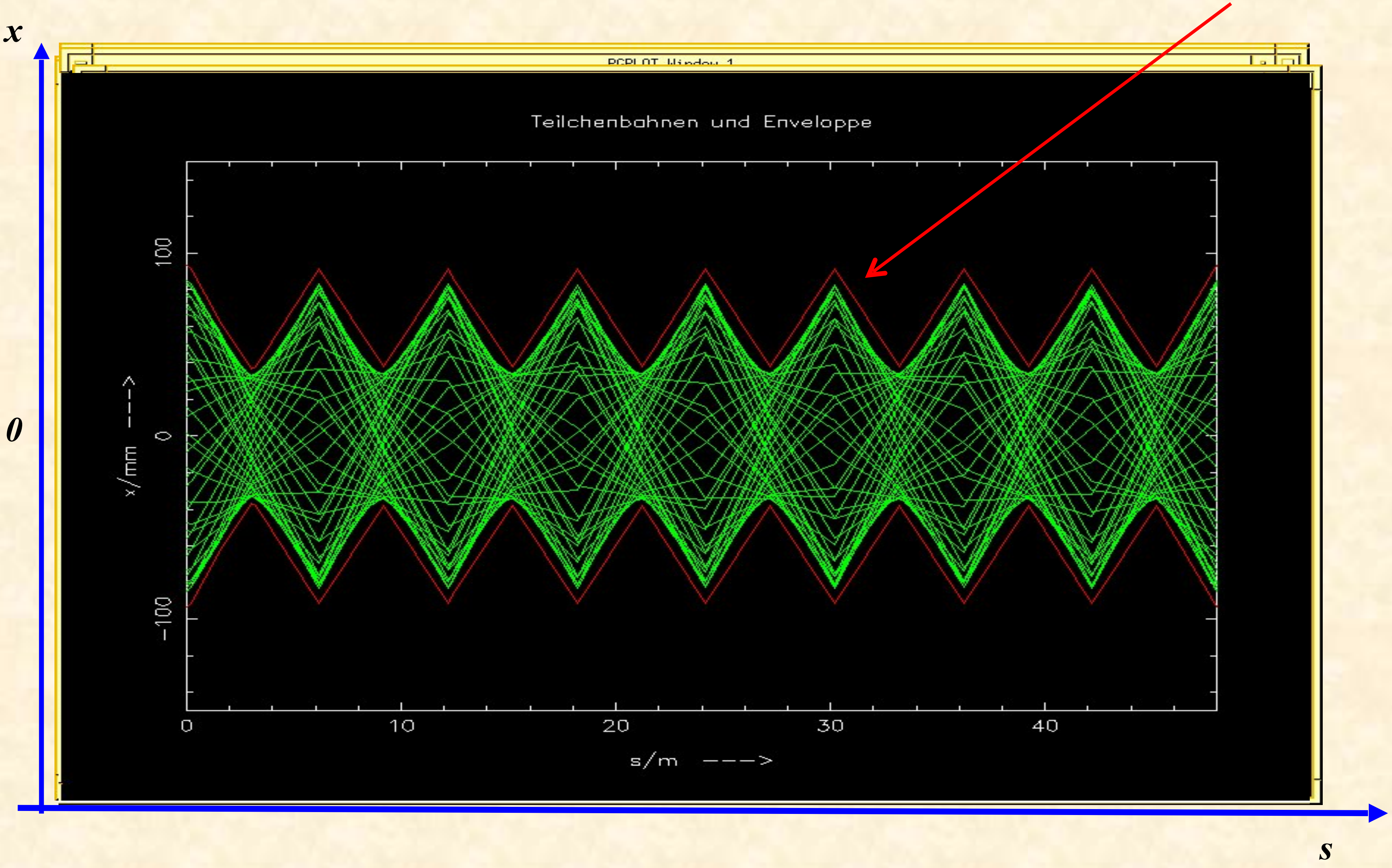
“Do they actually drop ?”

Answer: No

Question: ***what will happen, if the particle performs a second turn ?***

... or a third one or ... 10^{10} ***turns***

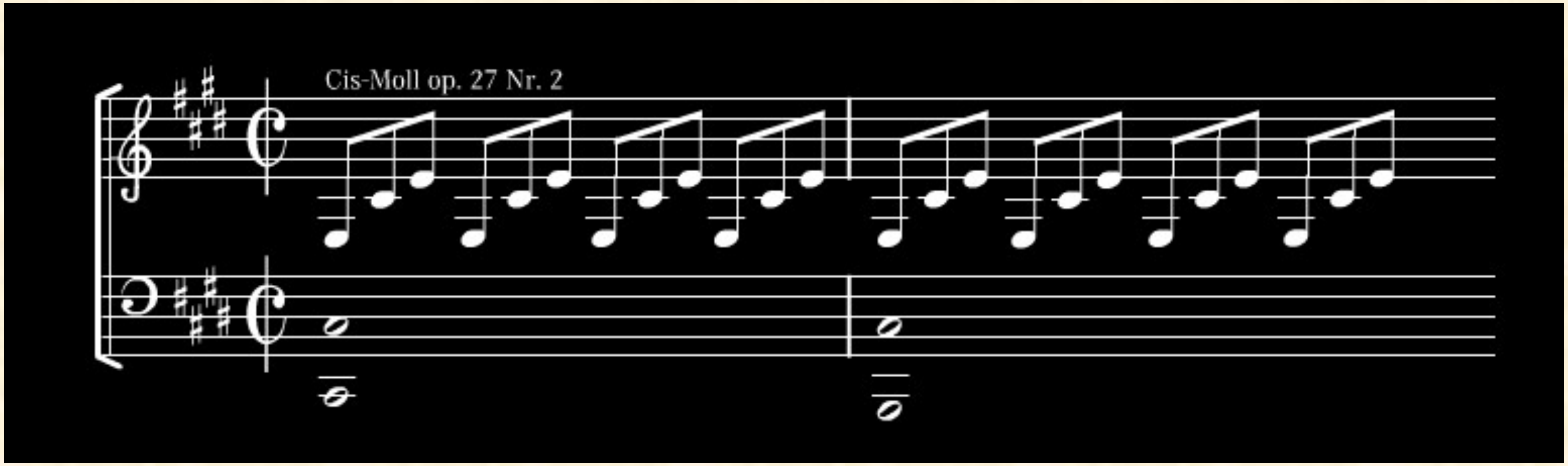
$$\sigma = \sqrt{\varepsilon\beta}$$



19th century:

Ludwig van Beethoven: „Mondschein Sonate“

Sonate Nr. 14 in cis-Moll (op. 27/II,   1801)



Astronomer Hill:

differential equation for motions with periodic focusing properties „Hill's equation“

Example: particle motion with periodic coefficient

equation of motion: $x''(s) - k(s)x(s) = 0$

$$\left.\begin{array}{l}\textit{restoring force} \neq \textit{const,} \\ \quad k(s) = \textit{depending on the position } s \\ \quad k(s+L) = k(s), \quad \textit{periodic function}\end{array}\right\}$$

we expect a kind of quasi harmonic oscillation: amplitude & phase will depend on the position s in the ring.

Amplitude of a particle trajectory:

$$x(s) = \sqrt{\varepsilon} * \sqrt{\beta(s)} * \cos(\psi(s) + \varphi)$$

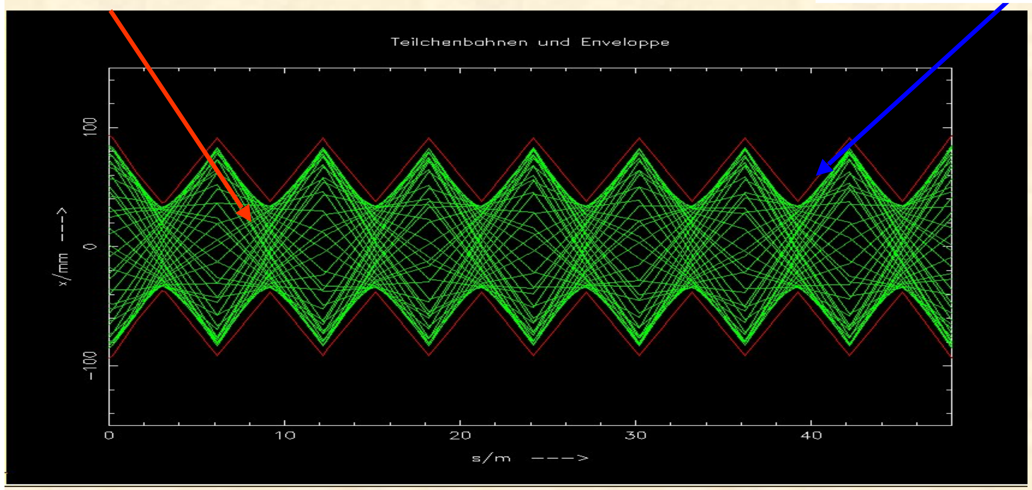
Maximum size of a particle amplitude

$$\hat{x}(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}$$

Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}\cdot\cos(\Psi(s)+\phi)$$

$$\hat{x}(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}$$



single particle trajectories, $N \approx 10^{11}$ per bunch

Gauß Particle Distribution:

$$\rho(x) = \frac{N\cdot e}{\sqrt{2\pi}\,\sigma_x}\cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$$

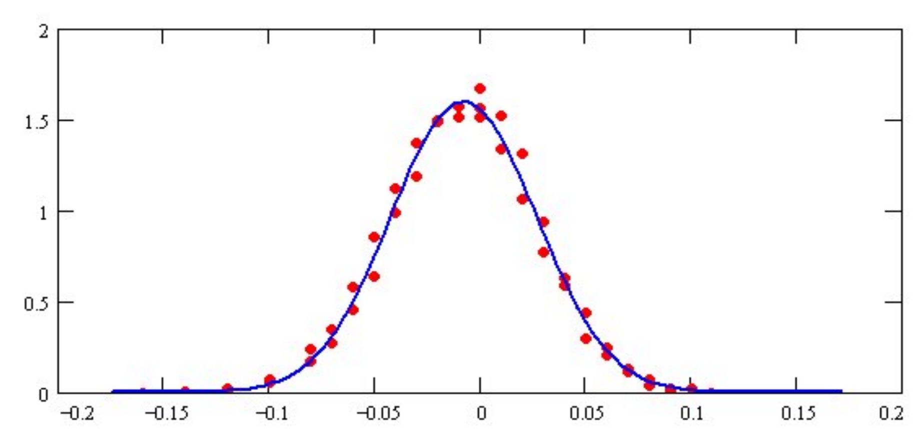
particle at distance 1 σ from centre
↔ 68.3 % of all beam particles

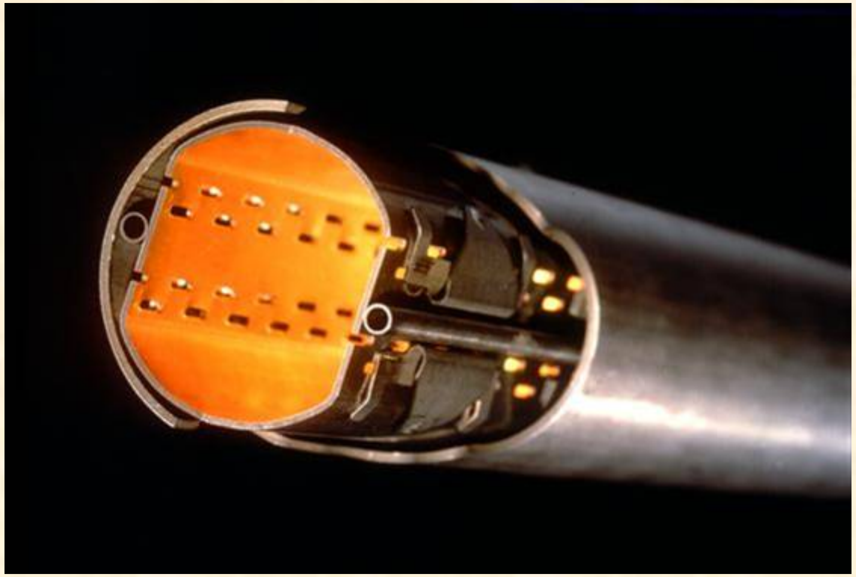
LHC:

$$\beta = 180\,m$$

$$\varepsilon = 5*10^{-10}\,m\,rad$$

$$\sigma = \sqrt{\varepsilon*\beta} = \sqrt{5*10^{-10}\,m*180\,m} = 0.3\,mm$$

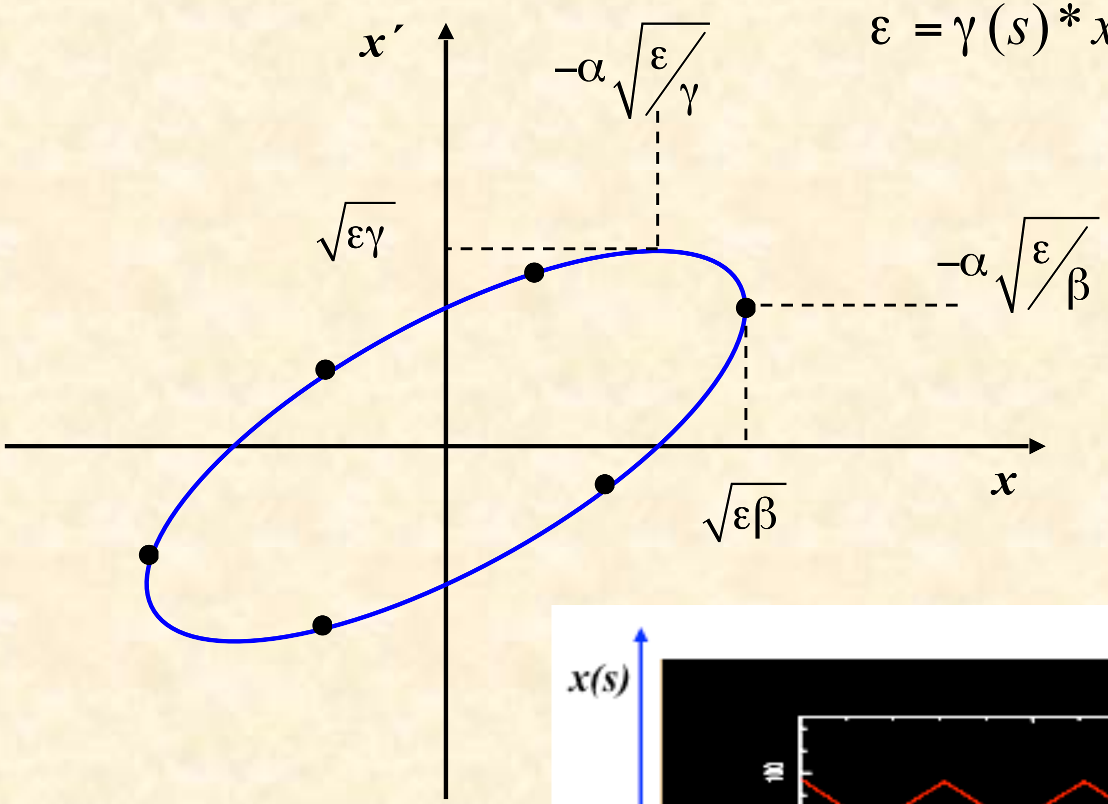




aperture requirements: $r_0 = 17 * \sigma$

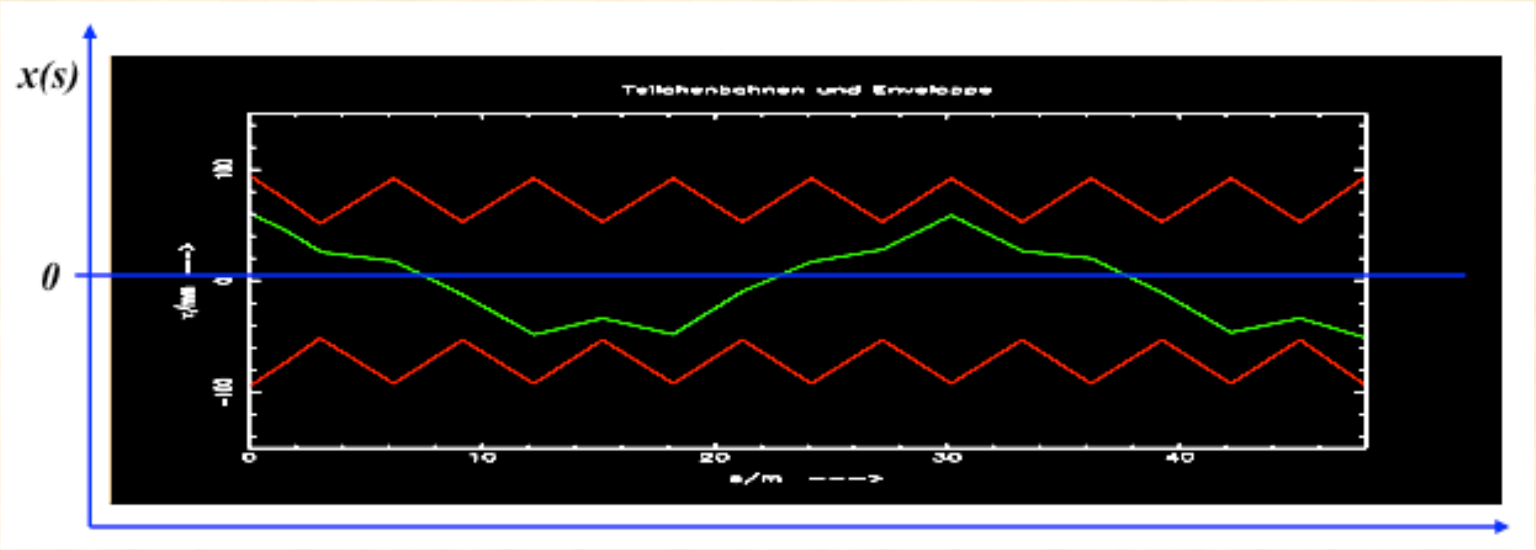
Beam Emittance and Phase Space Ellipse

$$\varepsilon = \gamma(s) * x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$



Liouville: ***in reasonable storage rings***
area in phase space is constant.

A = π*ε=const

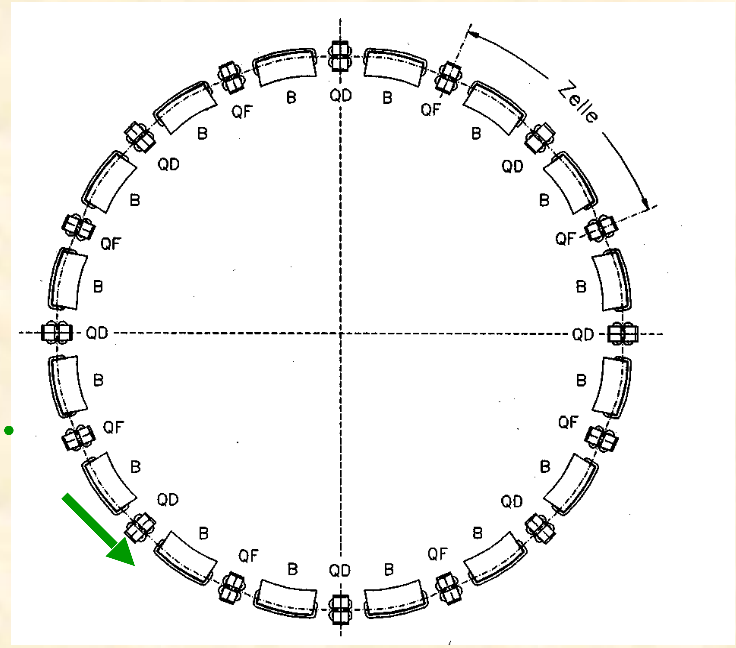


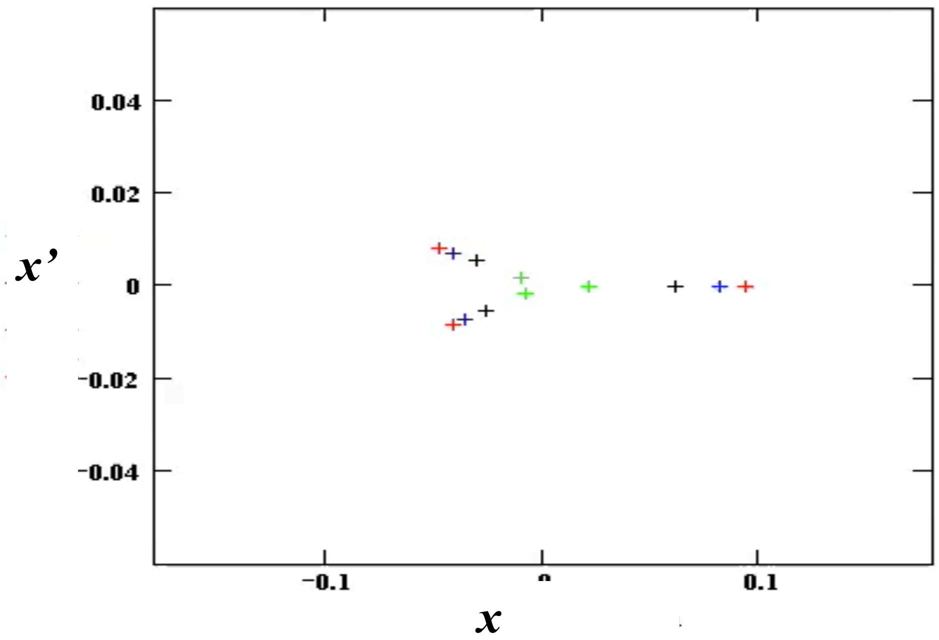
ε beam emittance = woozilycity of the particle ensemble, intrinsic beam parameter,
cannot be changed by the foc. properties.
Scientifiquely spoken: area covered in transverse x, x´ phase space … and it is constant !!!

Particle Tracking in a Storage Ring

Calculate x, x´ for each accelerator element according to matrix formalism and plot x, x´at a given position „s“ in the phase space diagram

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{turn} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$





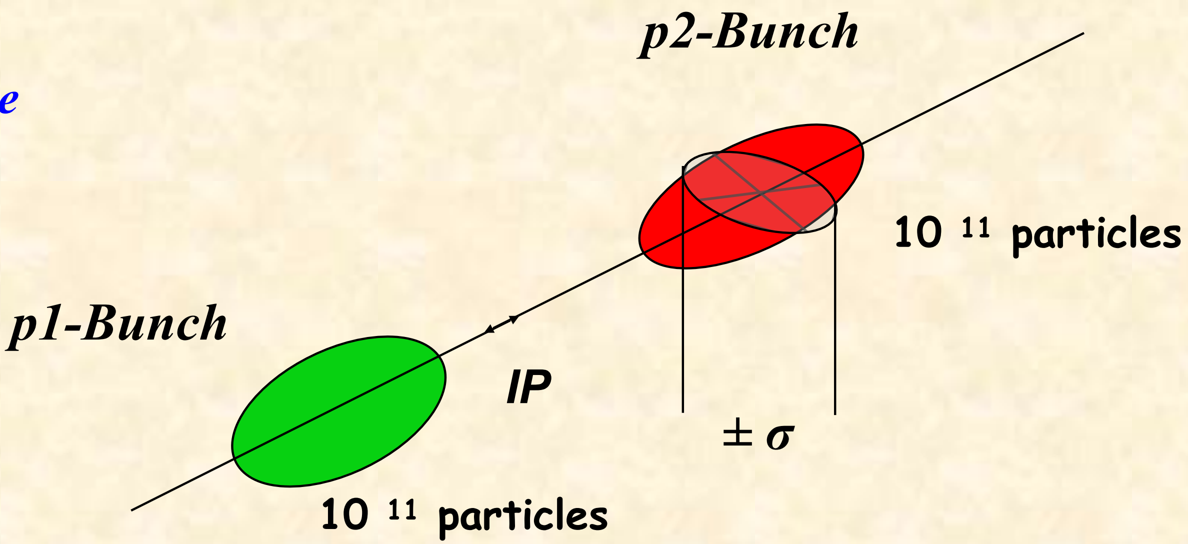
A beam of 4 particles

– each having a slightly different emittance:

5.) Luminosity

Ereignis Rate:"Physik" pro Sekunde

$$R = L \cdot \Sigma_{react}$$



Example: Luminosity run at LHC

$$\beta_{x,y} = 0.55\, m \qquad f_0 = 11.245\, kHz$$

$$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m \qquad n_b = 2808$$

$$\sigma_{x,y} = 17\, \mu m$$

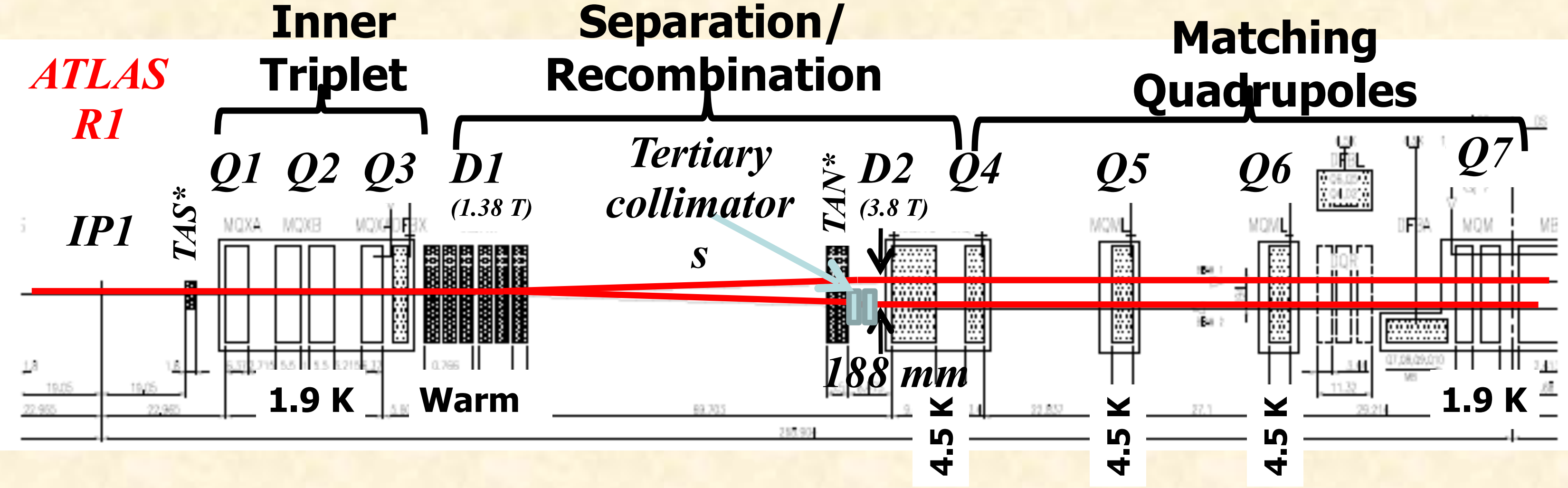
$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

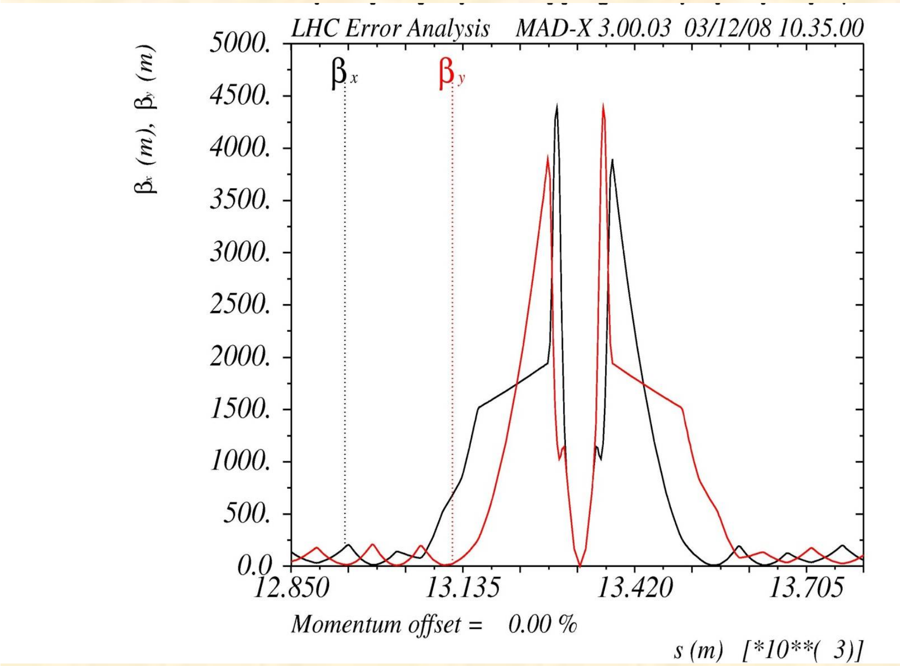
$$I_p = 584\, mA$$

$$L = 1.0 * 10^{34}\ {}^{1}\!/_{cm^2 s}$$

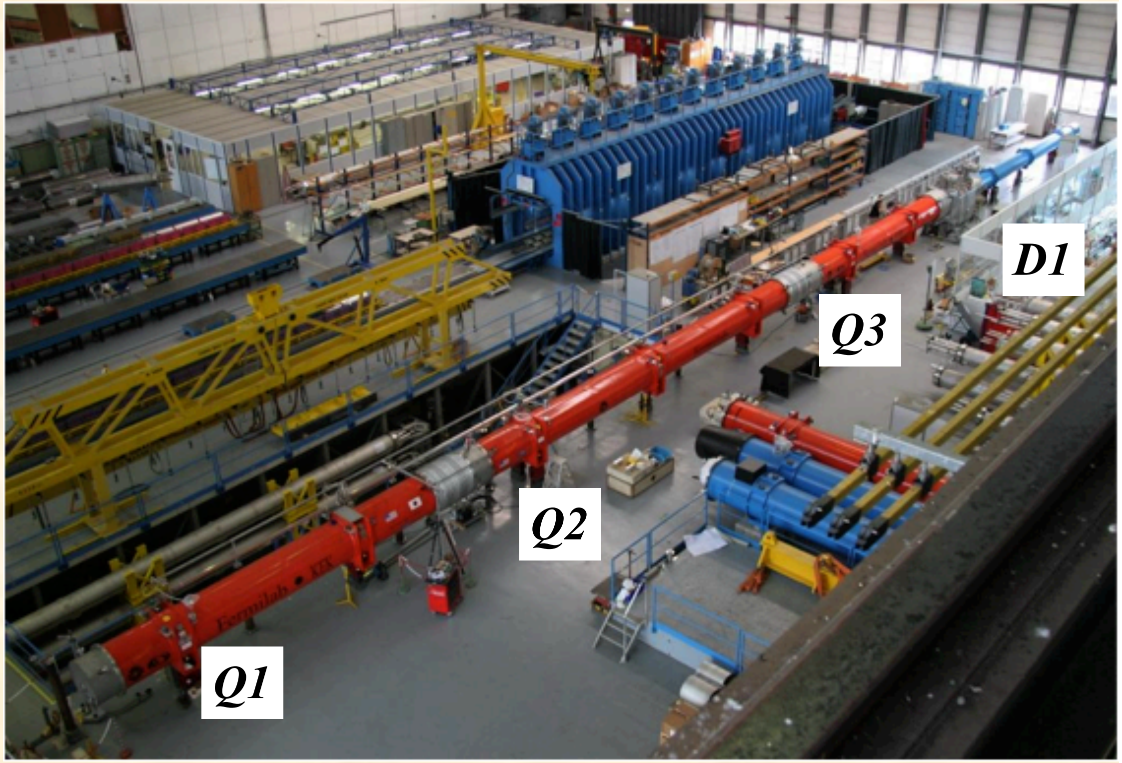
beam sizes in the order of my cat’s hair !!

The LHC Mini-Beta-Insertions



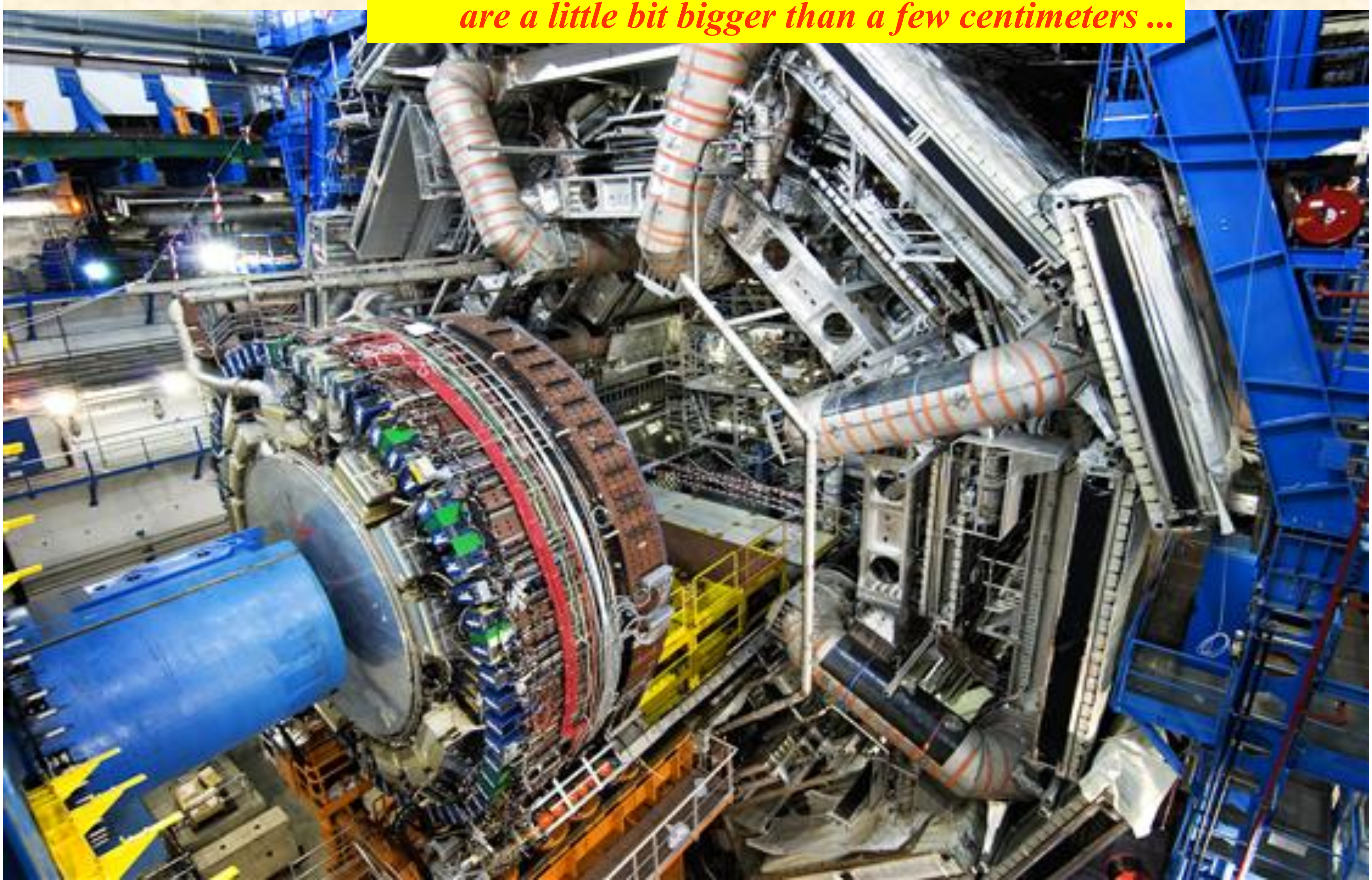


mini β optics



... clearly there is an

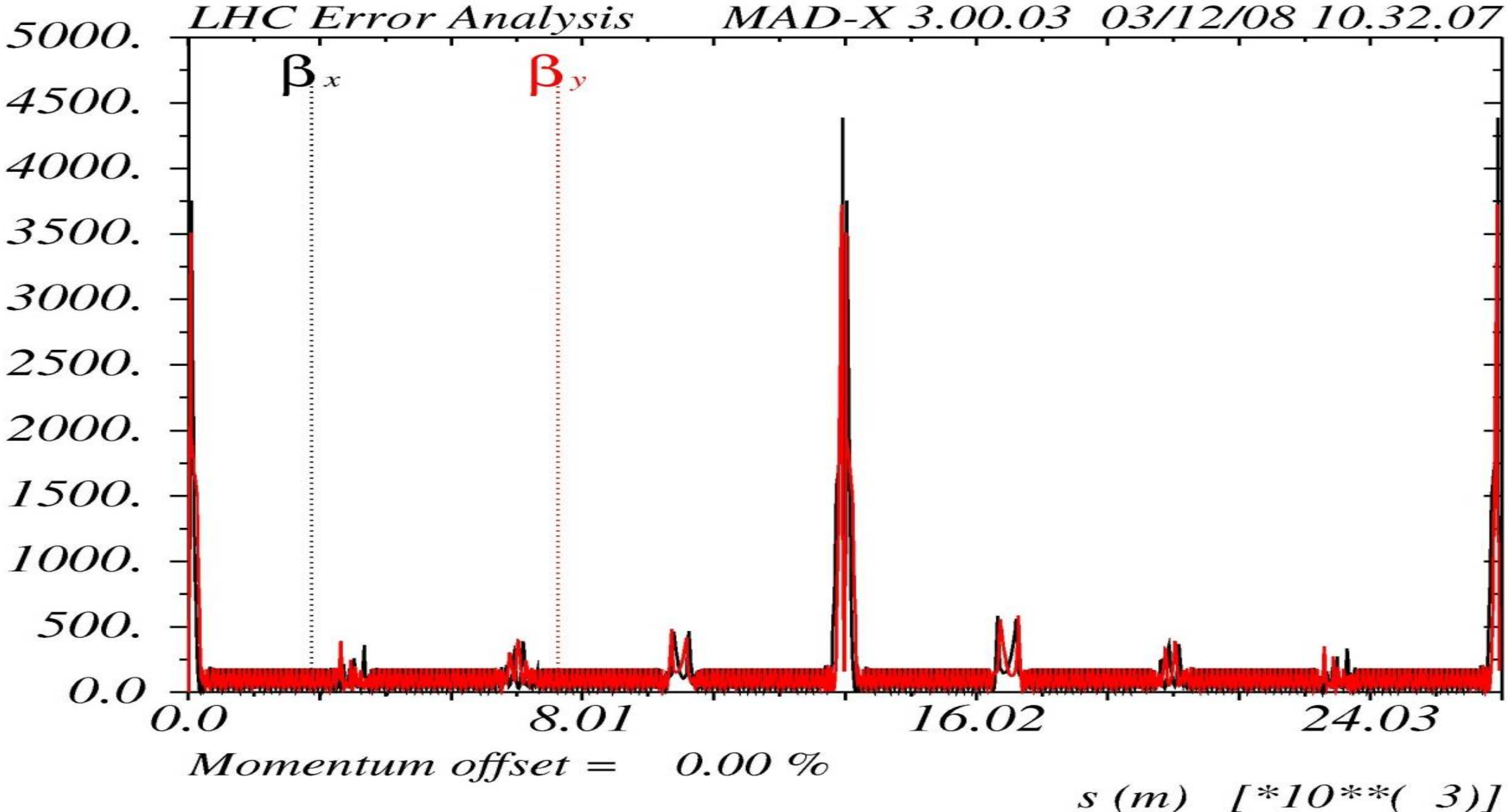
... unfortunately ... in general
high energy detectors that are
installed in that drift spaces
are a little bit bigger than a few centimeters ...



The Beta Function: Lattice Design & Beam Optics

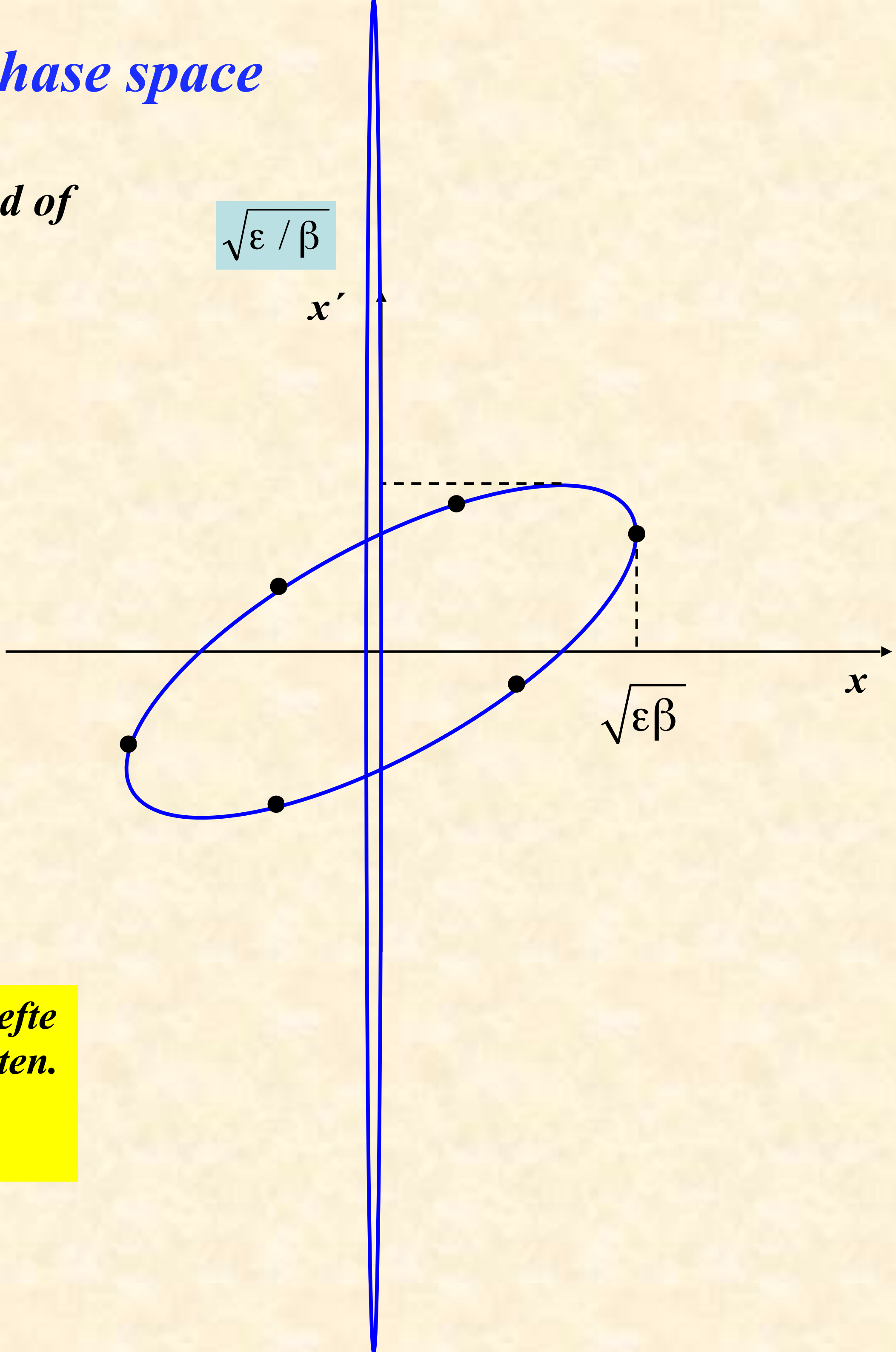
The beta function determines the maximum amplitude a single particle trajectory can reach at a given position in the ring.
It is determined by the focusing properties of the lattice and follows the periodicity of the machine.

LHC mini beta optics at 7000 GeV



Mini-Beta-Insertions in phase space

A mini-β insertion is always a kind of
special symmetric drift space.
→ greetings from Liouville



the smaller the beam size
the larger the bam divergence

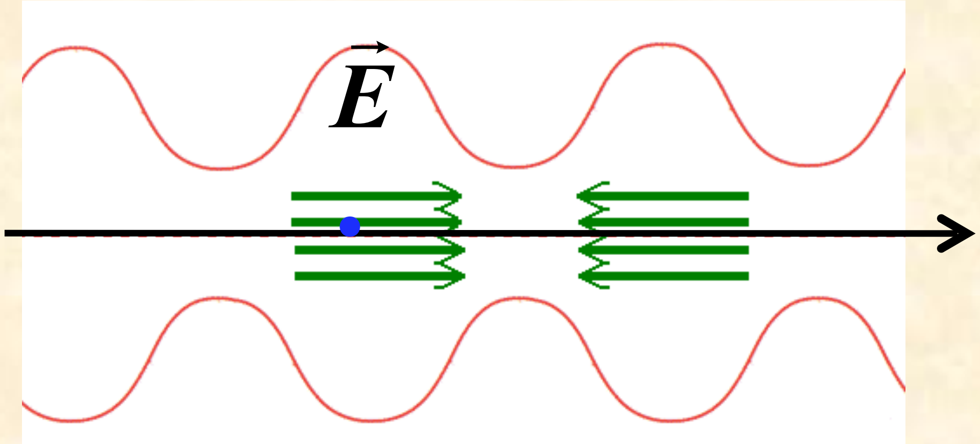
***Liouville:* im Falle konservativer Kraefte bleibt die Phasenraum-Flaeche erhalten.**

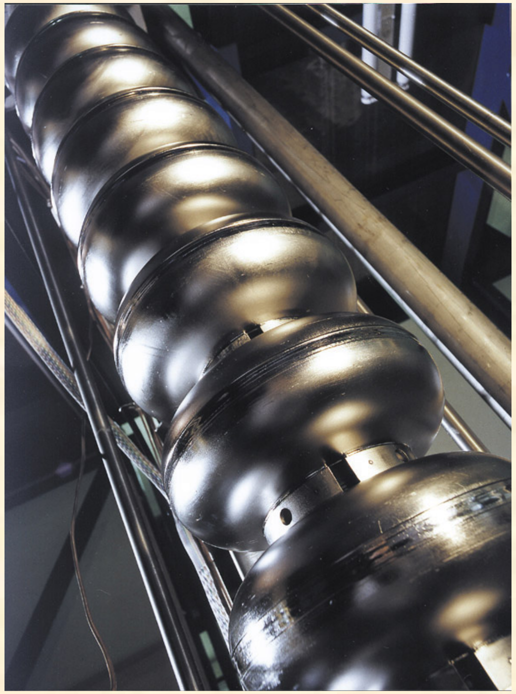
$$A = \pi * \varepsilon = const$$

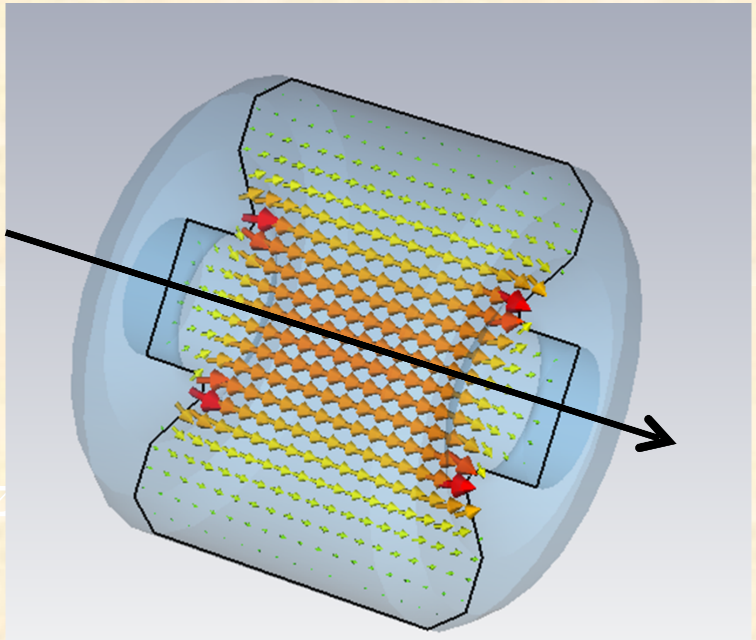
III. The Acceleration

Where is the acceleration?

Install an RF accelerating structure in the ring:





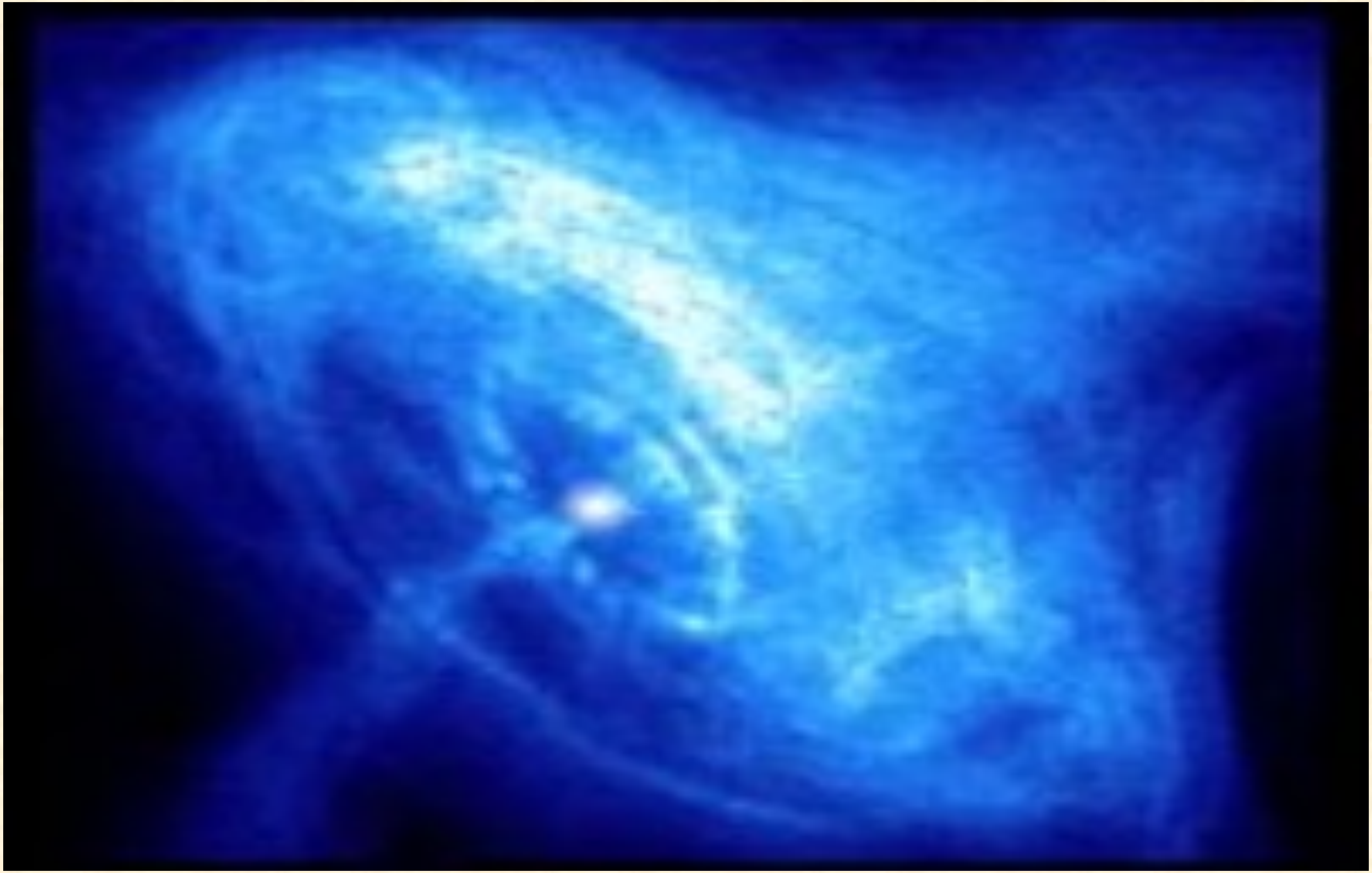


B. Salvant
N. Biancacci

A real (!) Particle Accelerator

$E=10^{20}$ eV

→ 100*1Mio*LHC



crab nebula, burst of charged particles

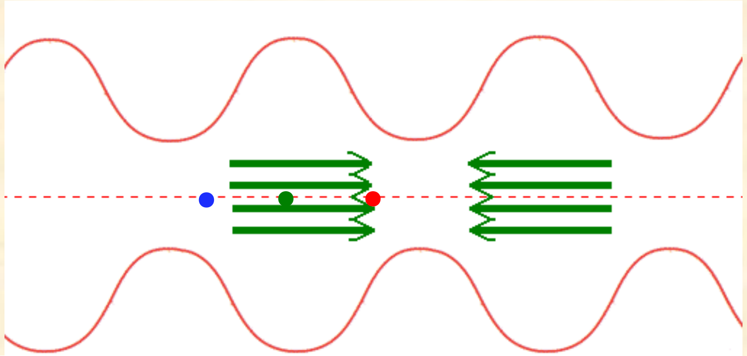
The Acceleration & “Phase Focusing”

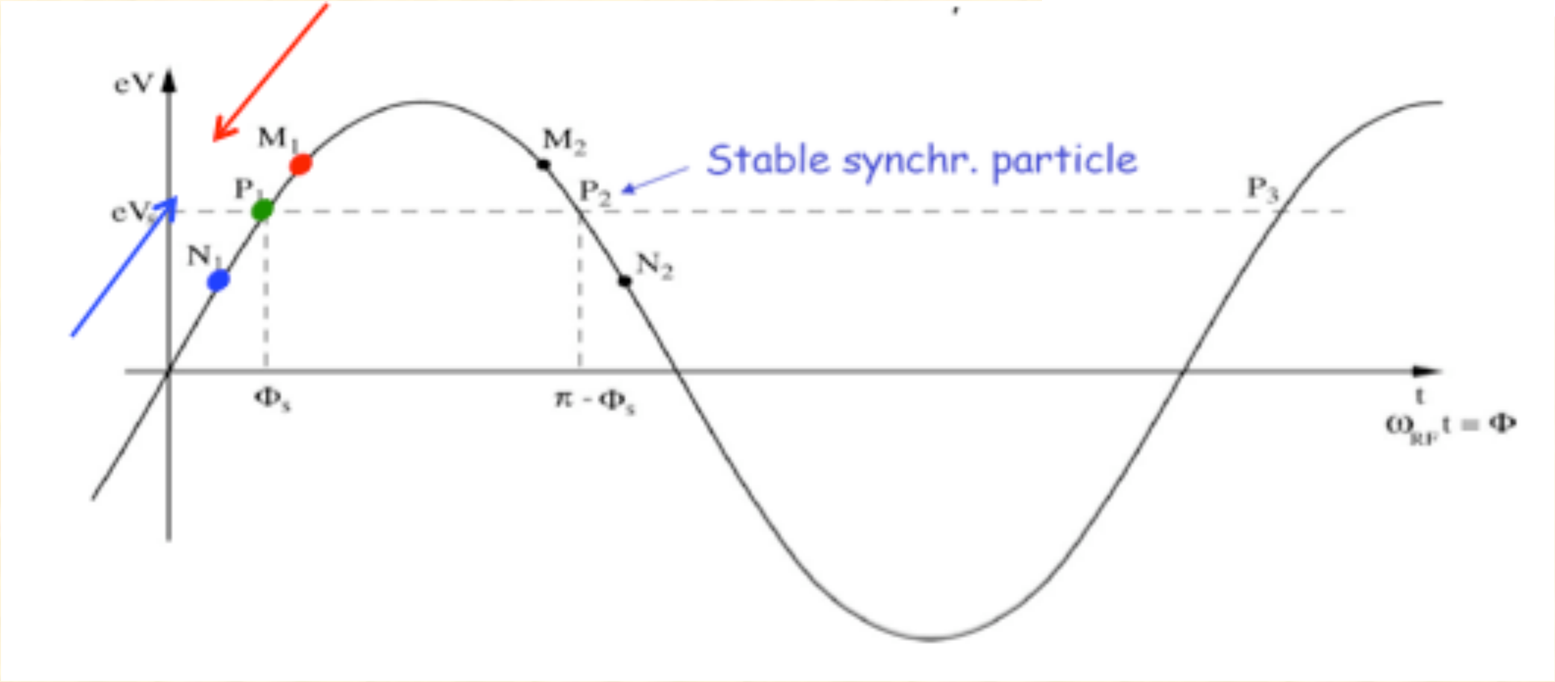
$\Delta p/p \neq 0$ below transition

ideal particle ●

particle with Δp/p > 0 ● *faster*

particle with Δp/p < 0 ● *slower*



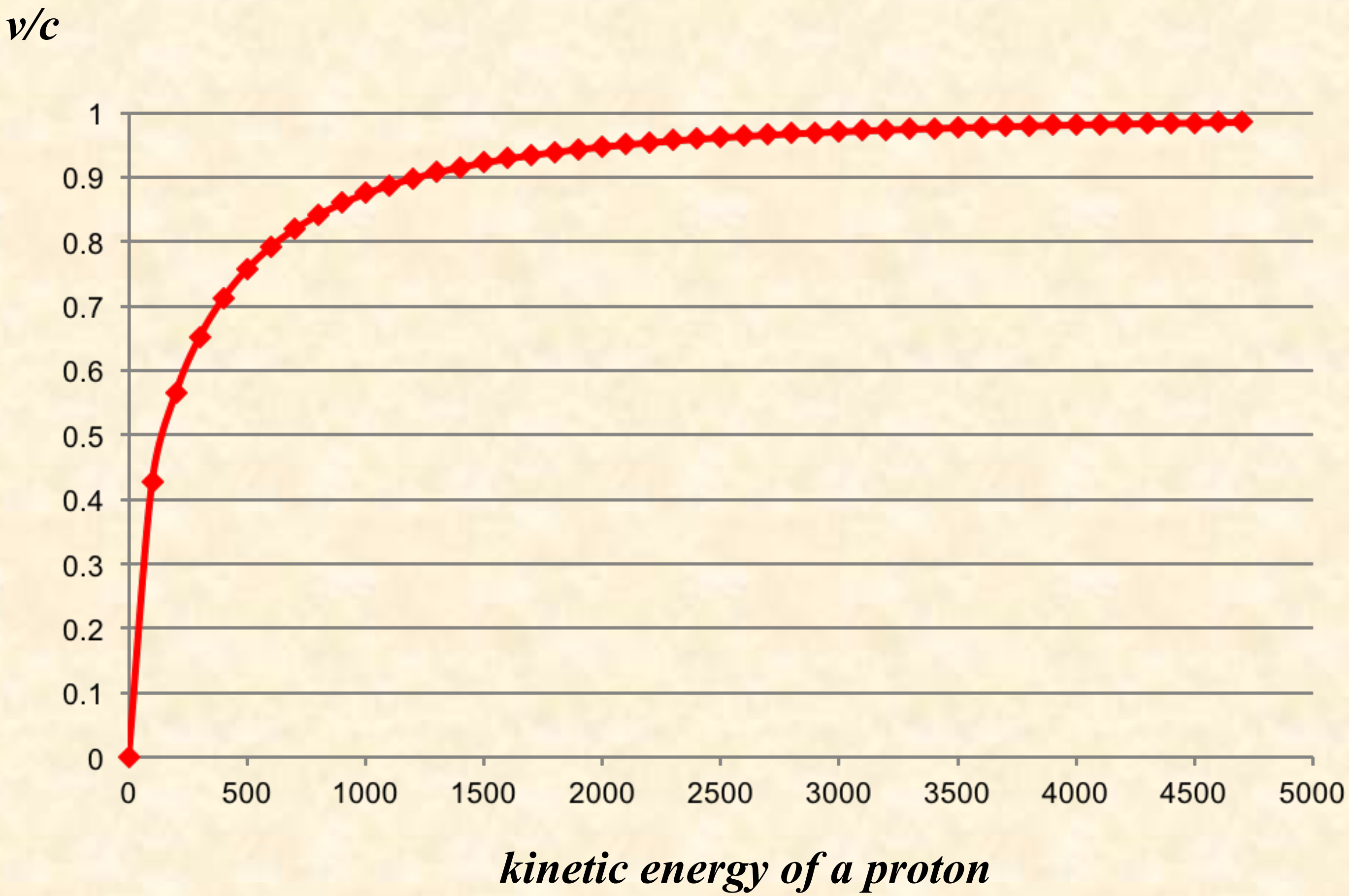


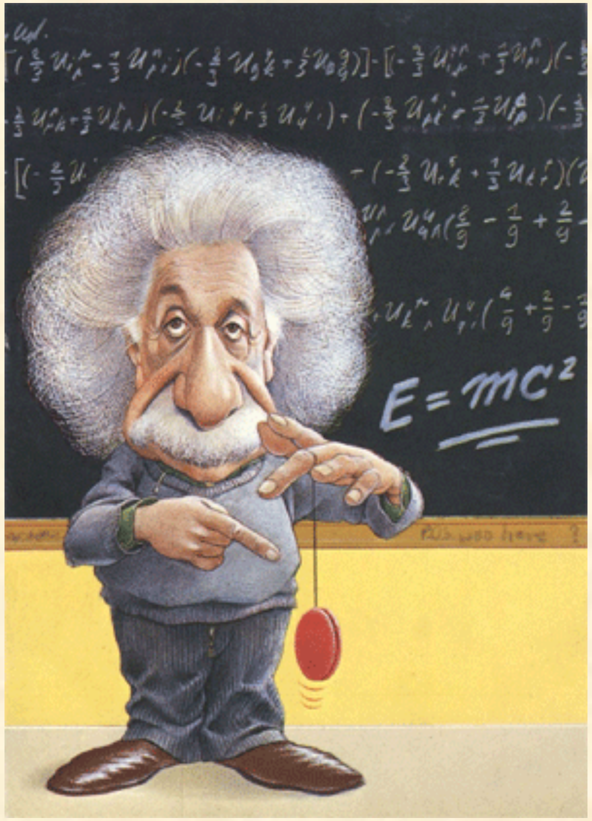
Focussing effect in the longitudinal direction keeping the particles close together ... forming a “bunch”

oscillation frequency: $f_s = f_{rev}\sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0\cos\phi_s}{E_s}}$ *≈ some Hz*

... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow \frac{v}{c} = \sqrt{1-\frac{mc^2}{E^2}}$$





... some when the particles do not get faster anymore

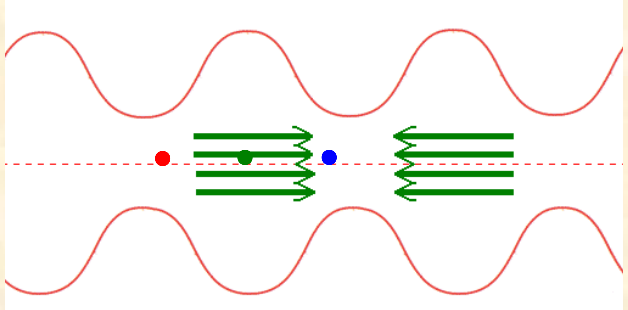
.... but heavier !

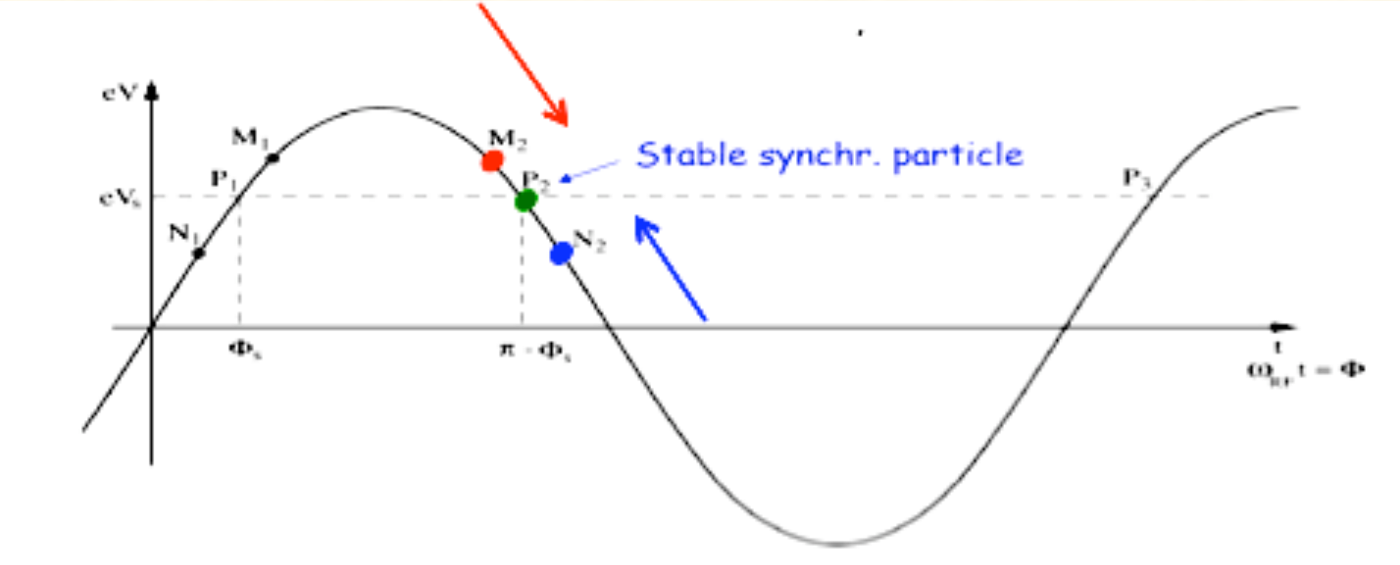
The Acceleration above transition

ideal particle ●

particle with Δp/p > 0 ● *heavier*

particle with Δp/p < 0 ● *lighter*



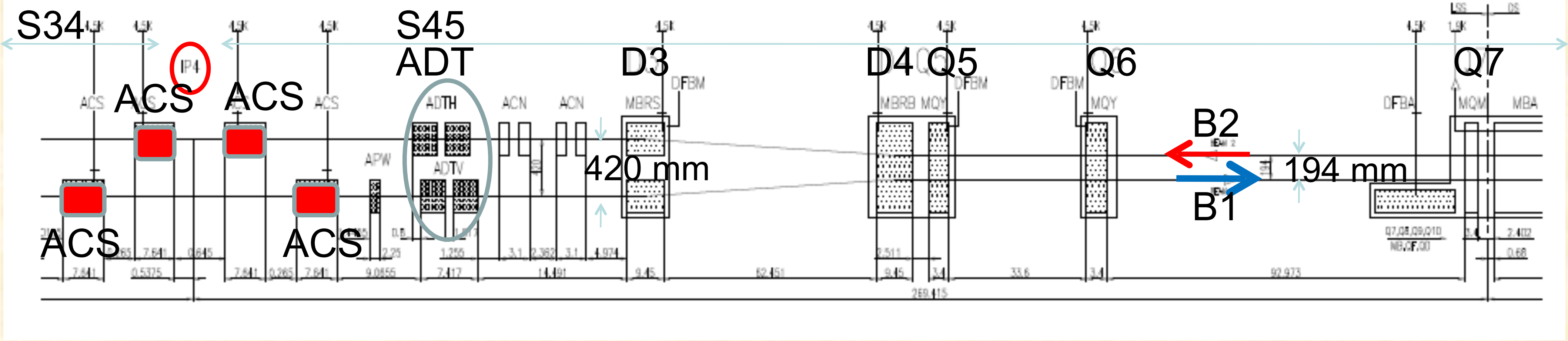


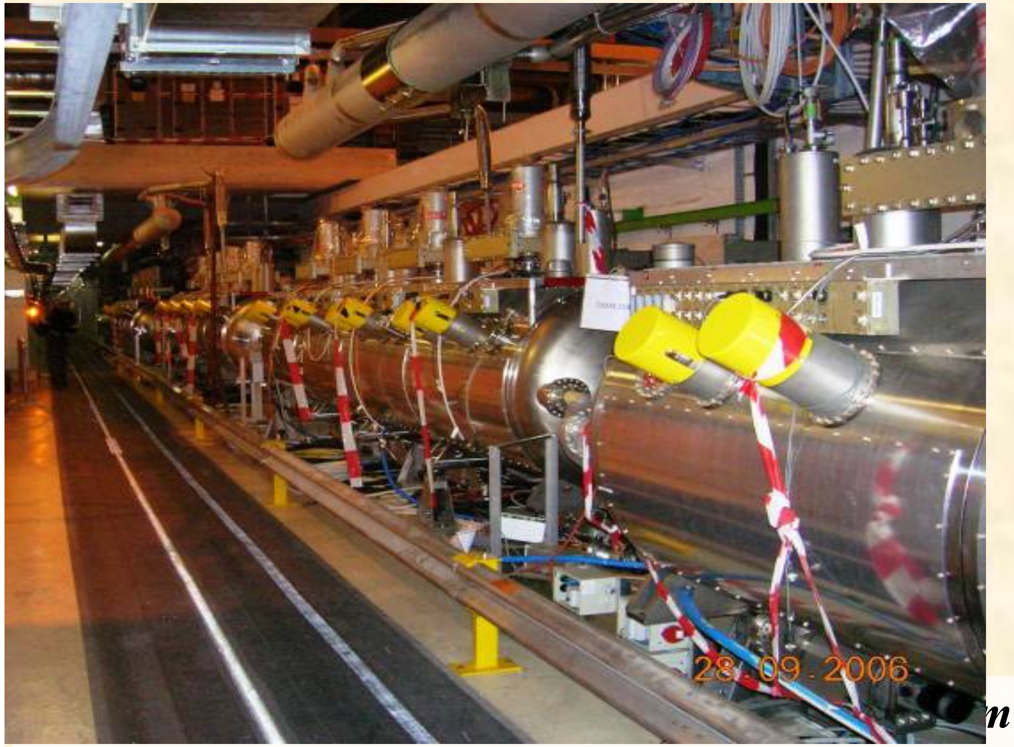
Focussing effect in the longitudinal direction
keeping the particles close together ... forming a "bunch"

... and how do we accelerate now ???

with the dipole magnets !

The RF system: IR4





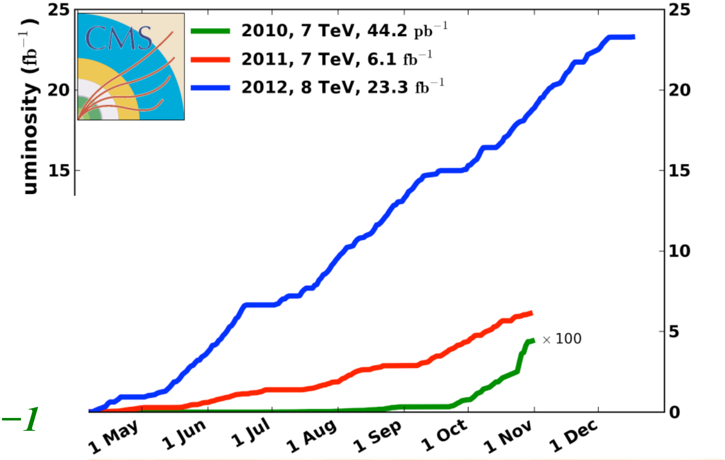
Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

Bunch length (4σ)	ns	1.06
Energy spread (2σ)	10^{-3}	0.22
Synchr. rad. loss/ turn	keV	7
Synchr. rad. power	kW	3.6
RF frequency	MHz	400
Harmonic number		35640
RF voltage/beam	MV	16
Energy gain/turn	keV	485
Synchrotron frequency	Hz	23.0

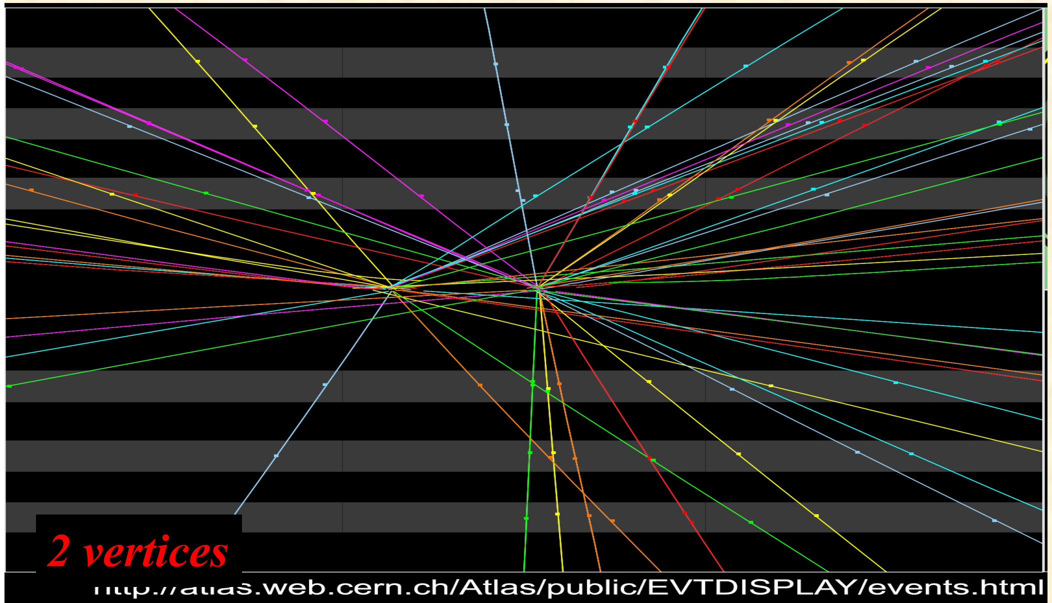
1.) Where are we ?

**** Standard Model of HEP***
**** Higgs discovery***

And still...
The LHC Performance in Run 1

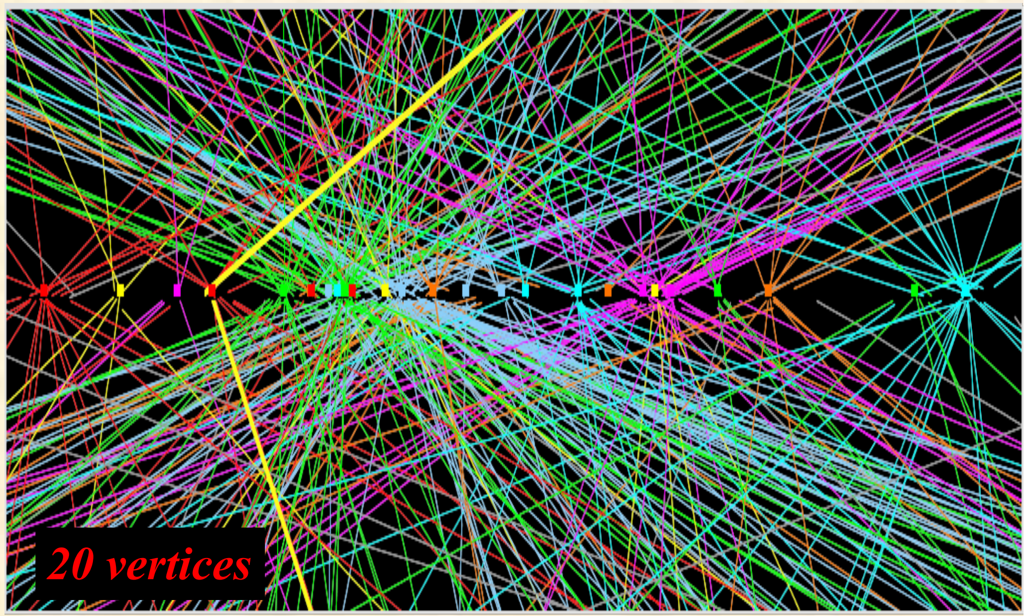


	Design	2012
Momentum at collision	7 TeV /c	4 TeV/c
Luminosity	10^{34} cm $^{-2}$ s^{-1}	7.7*10^{33} cm $^{-2}$ s^{-1}
Protons per bunch	1.15 × 10^{11}	1.50 × 10^{11}
Number of bunches/beam	2808	1380
Nominal bunch spacing	25 ns	50ns
beta *	55 cm	60 cm
rms beam size IP	17 µm	20 µm



2 vertices

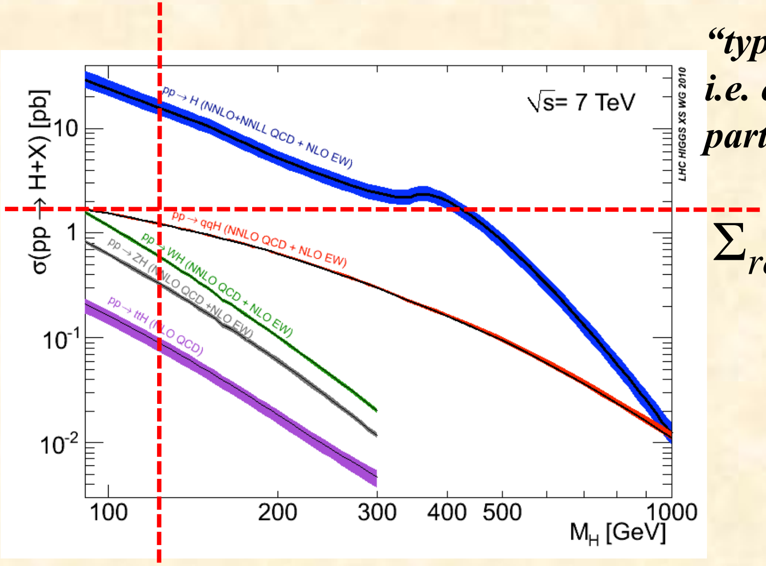
http://atlas.web.cern.ch/Atlas/public/EVTDISPLAY/events.html



20 vertices

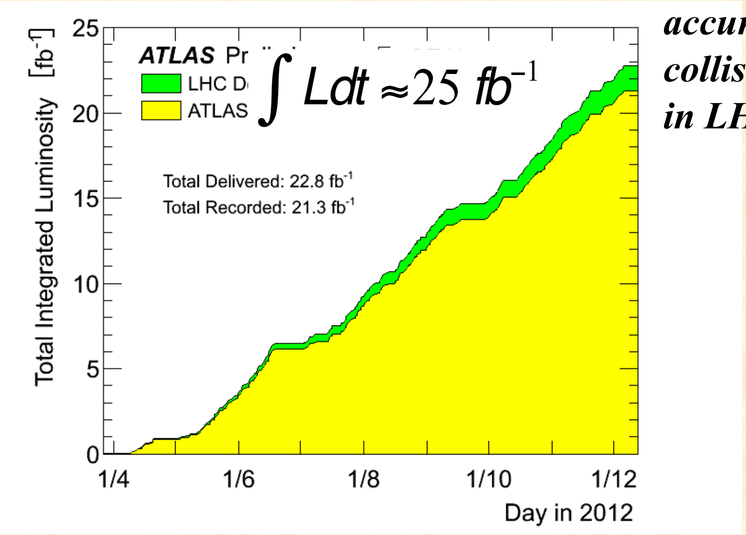
The High light of the year

production rate of events** is **determined by** the **cross section Σ_{react}
and a parameter L that is given by the design of the accelerator:
… the luminosity



"typical particle size"
i.e. cross section for
particle production

$$\Sigma_{react} \approx 1pb$$



$$\int Ldt \approx 25\ fb^{-1}$$

accumulated
collision rate
in LHC run 1

$$1b = 10^{-24}cm^2 = \frac{1}{mio}\cdot\frac{1}{mio}\cdot\frac{1}{mio}\cdot\frac{1}{10000}\quad mm^2$$

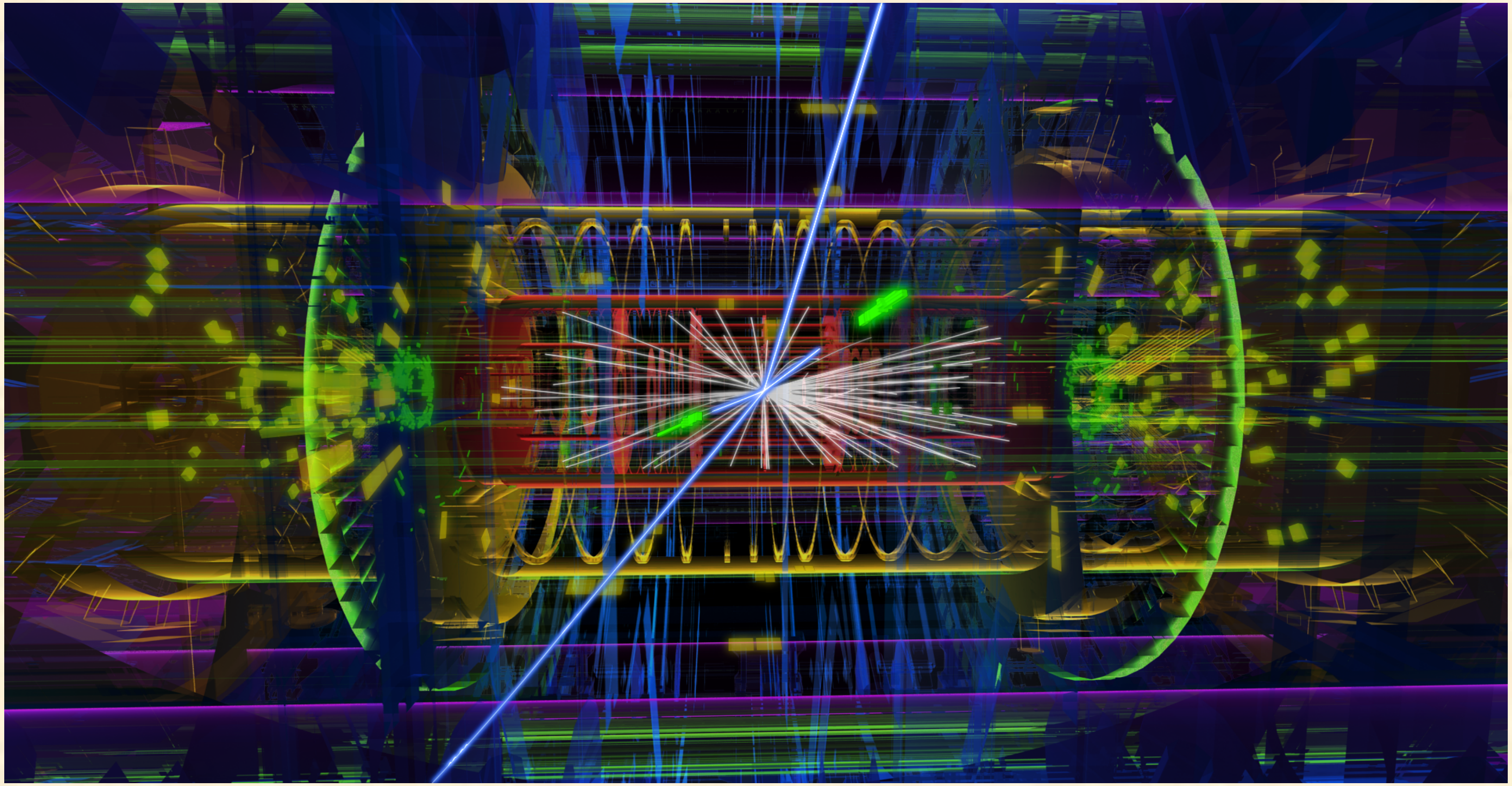
The particles are "very small"

$$R = \Sigma_{react}\cdot L \approx 10^{-12}b\cdot L \approx some\quad 1000\ H$$

During collider run we had in Run 1 …
1400 bunches circulating,
with 800 Mio proton collisions per second in the experiments
and collected only 450 Higgs particles in three years.

... and why all that ??

High Light of the HEP-Year 2012 / 13 naturally the HIGGS

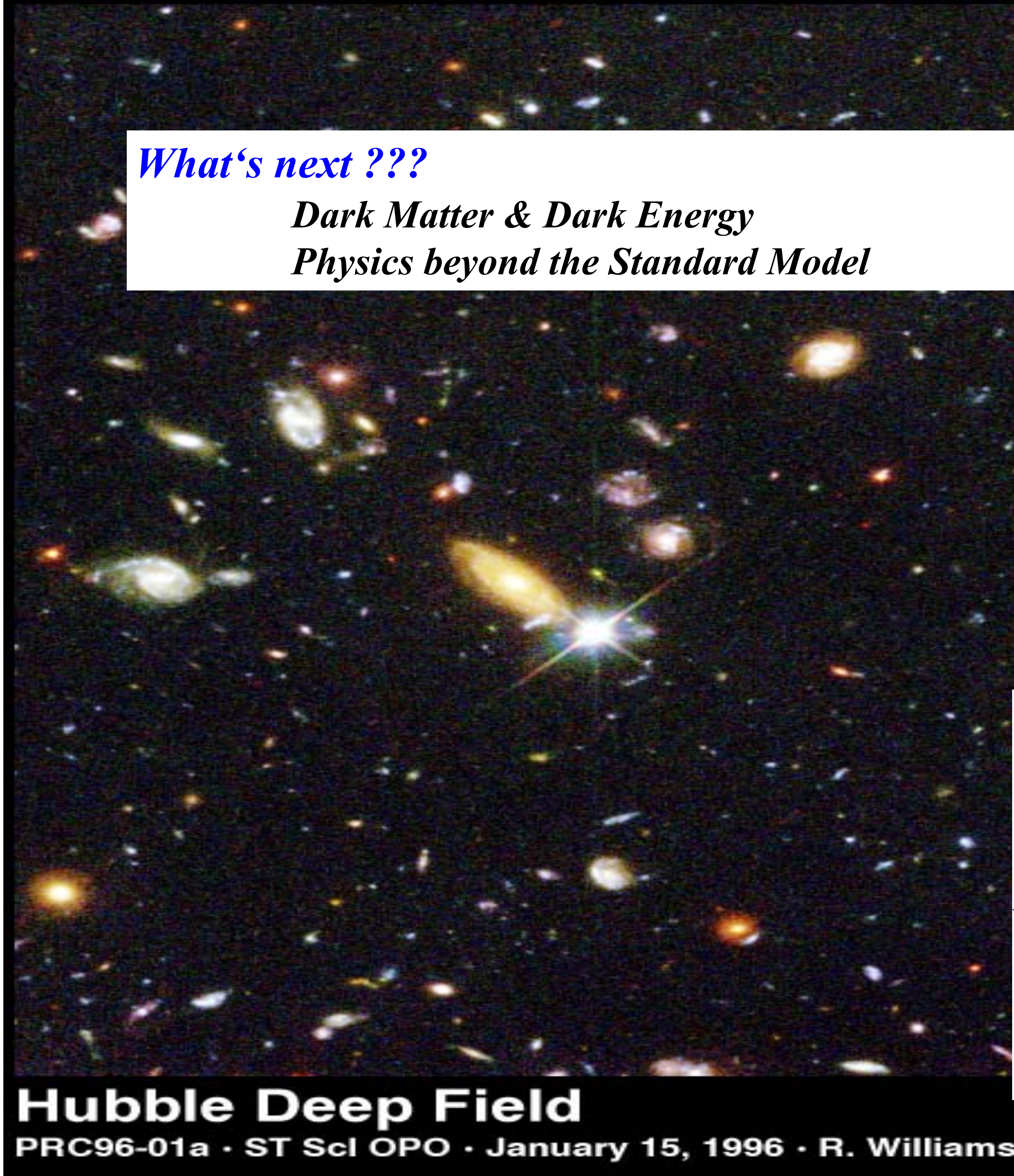


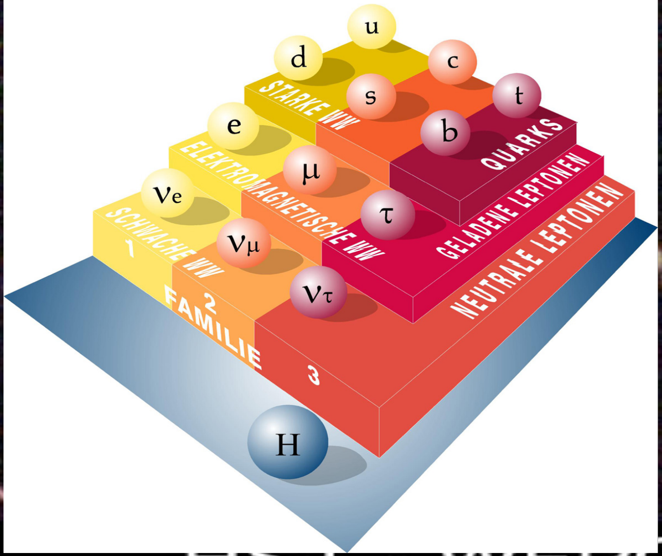
ATLAS event display: Higgs => two electrons & two muons

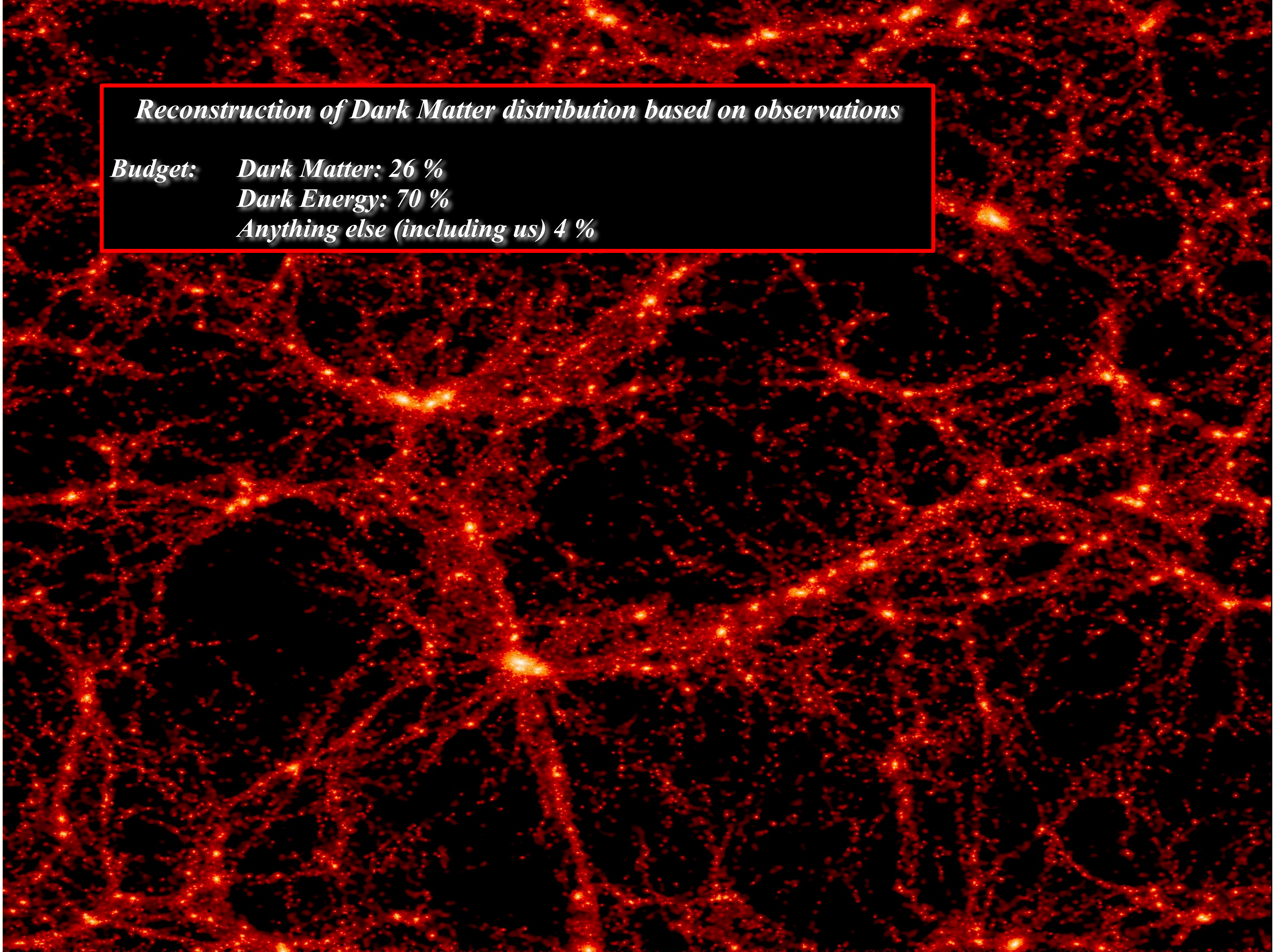
$$E = m_0c^2 = m_{e1} + m_{e2} + m_{\mu 1} + m_{\mu 2} = 125.4\ GeV$$

What‘s next ???

Dark Matter & Dark Energy
Physics beyond the Standard Model

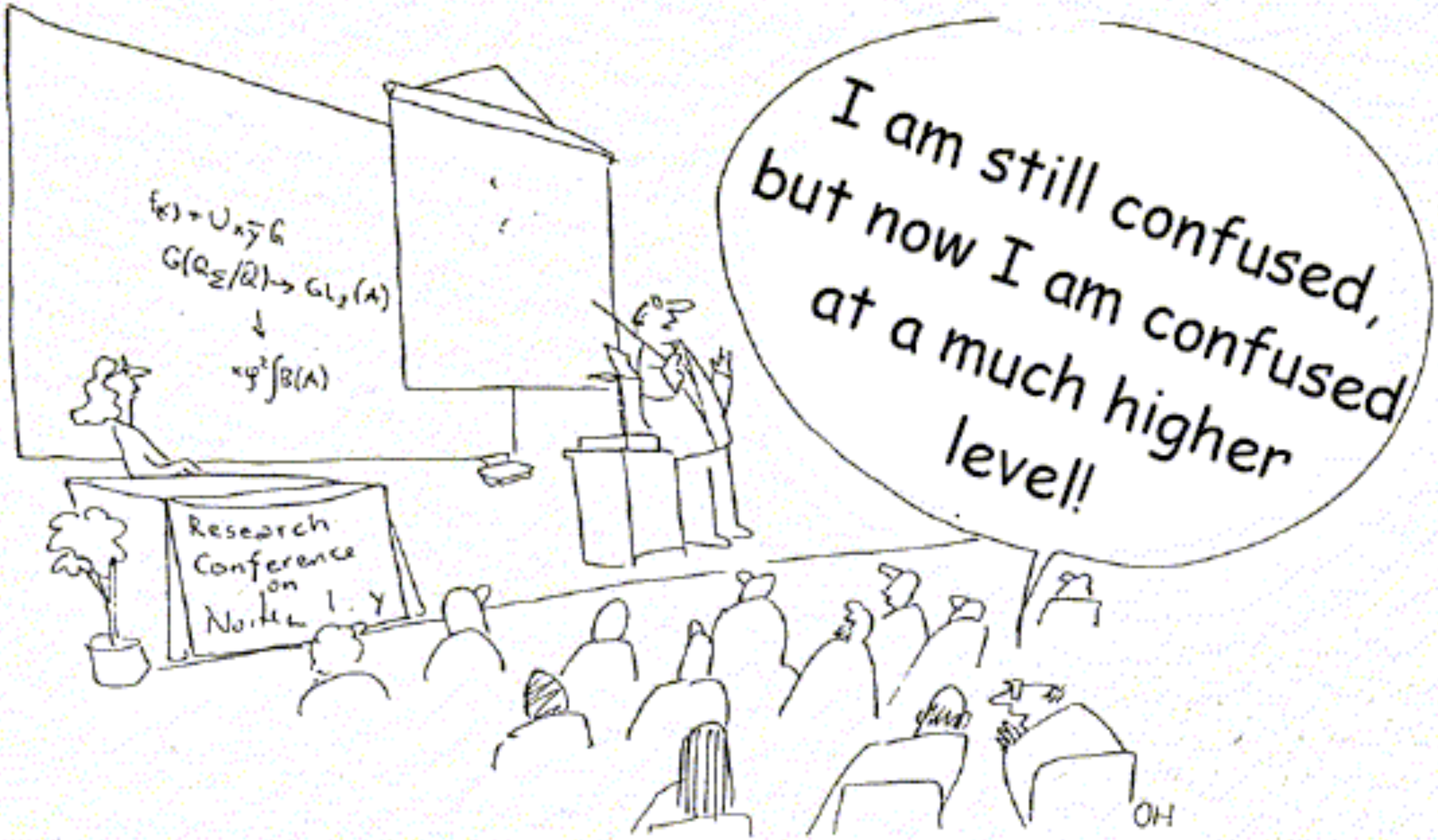






Reconstruction of Dark Matter distribution based on observations

Budget: *Dark Matter: 26 %*
Dark Energy: 70 %
Anything else (including us) 4 %

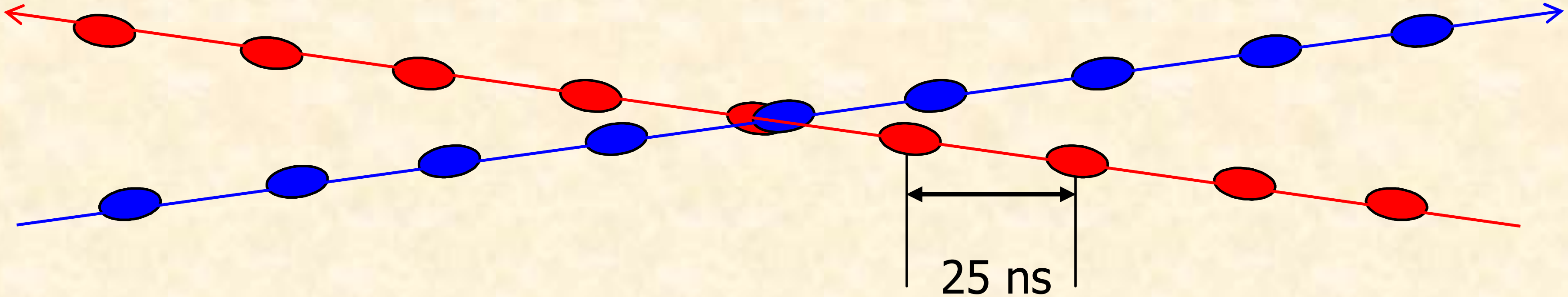

$f(x) + U_{x\bar{y}} G$
$G(Q_\Sigma/Q) \to GL_2(A)$
$\downarrow$
$\times\varphi^2 \int B(A)$
Research Conference on Noite l. y
I am still confused, but now I am confused at a much higher level!
OH

Booooooom

LHC Operation:
Machine Protection & Safety

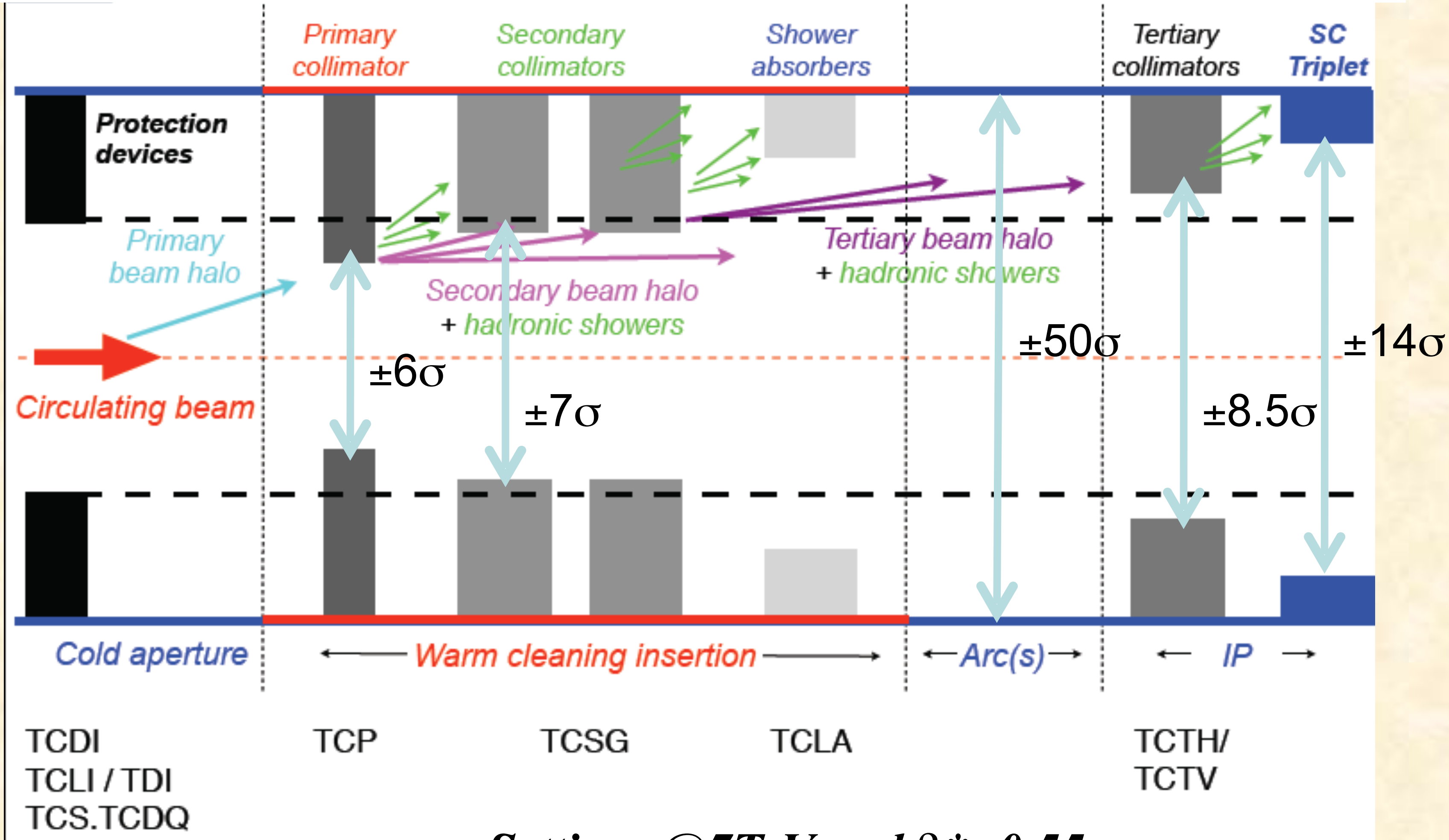
Energy stored in magnet system	10	GJ
Energy stored in one main dipole circuit	1.1	GJ
Energy stored in one beam	362	MJ

Enough to melt 500 kg of copper



$2\cdot10^{12}$ $4\cdot10^{12}$ $8\cdot10^{12}$ $6\cdot10^{12}$ 450 GeV p Strahl

LHC Aperture and Collimation



Settings @7TeV and β*=0.55 m

Beam size (σ) = 300 µm (@arc)

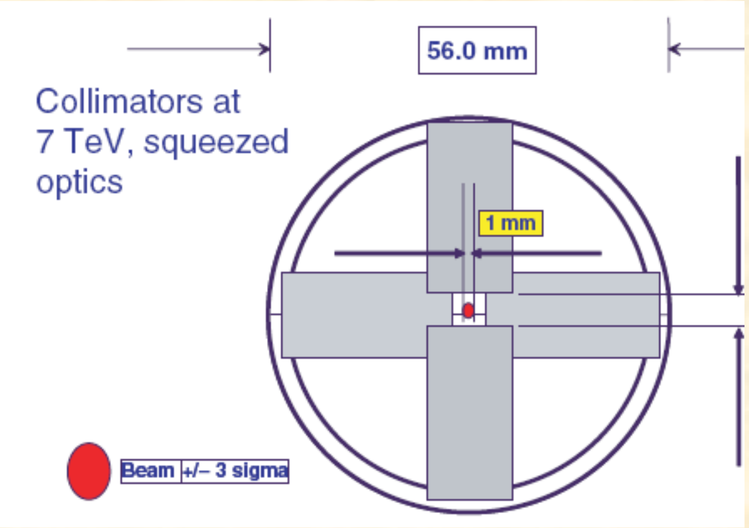
Beam size (σ) = 17 µm (@IR1, IR5)

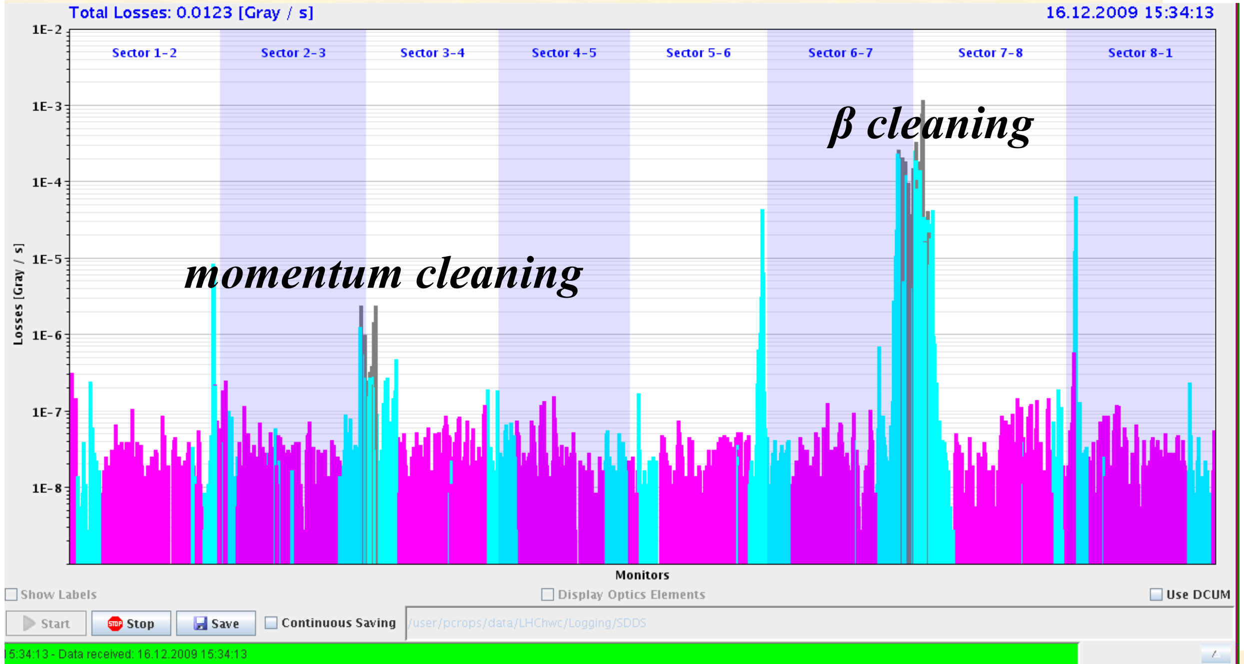
S. Redaelli, OP WG on Checkout, 08-11-2007

LHC Operation:
Machine Protection & Safety

... Komponenten des Machine Protection Systems :

beam loss monitors
QPS
permit server
orbit control
power supply control
collimators
online on beam check of all (?)
hardware components
a fast dump
the gaussian beam profile

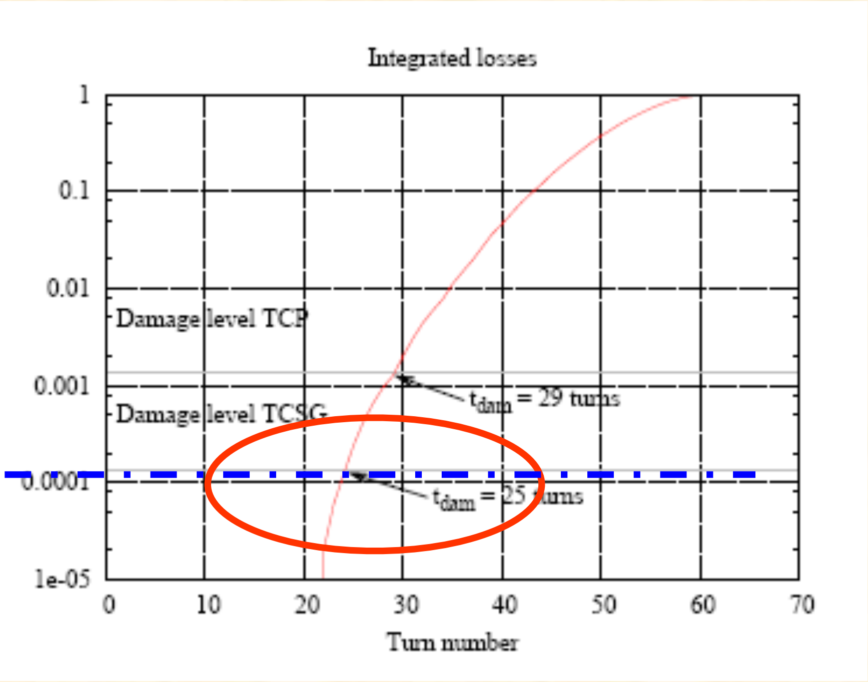



Total Losses: 0.0123 [Gray / s]

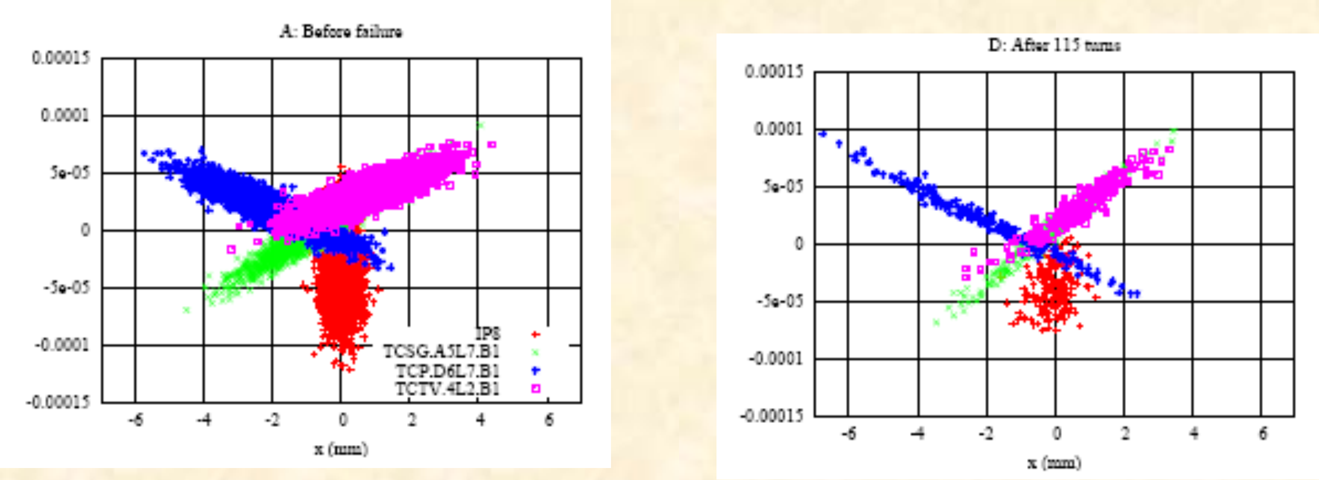
16.12.2009 15:34:13

15:34:13 - Data received: 16.12.2009 15:34:13

/user/pcrops/data/LHCbwc/Logging/SDDS



LHC Operation: Machine Protection & Safety



What will happen in
case of Hardware Failure

Phase space deformation in case of failure of RQ4.LR7
(A. Gómez)

Short Summary of the studies:

quench in sc. arc dipoles: τ_{loss} ***=20 - 30 ms***

BLM system reacts in time, QPS is not fast enough

quench in sc. arc quadrupoles: τ_{loss} ***=200 ms***

BLM & QPS react in time

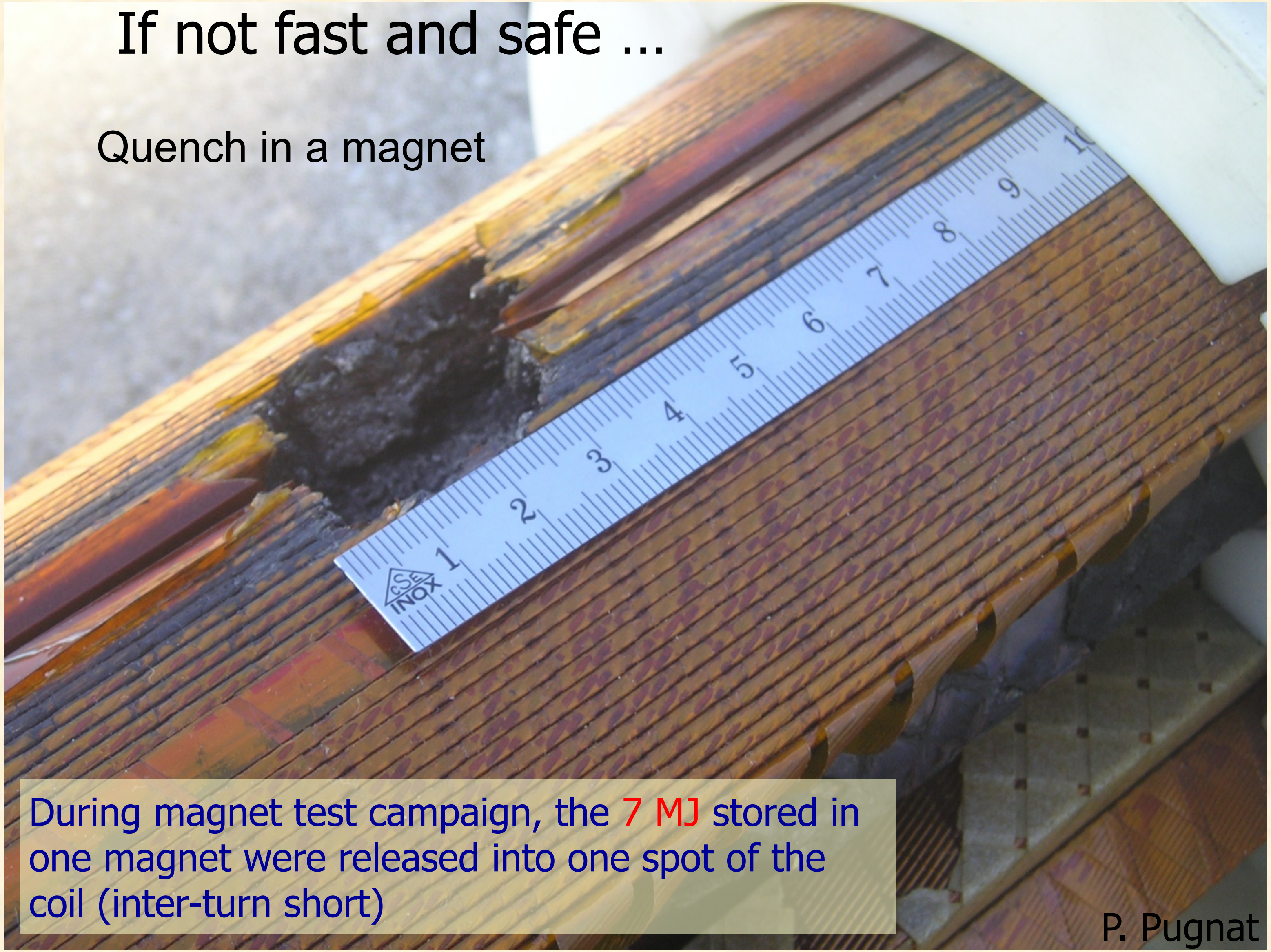
failure of nc. quadrupoles: τ_{det} ***= 6 ms***

τ_{damage} ***= 6.4 ms***

→ FMCM installed

failure of nc. dipole: τ_{damage} ***= 2 ms***

Energy stored in the magnets: quench



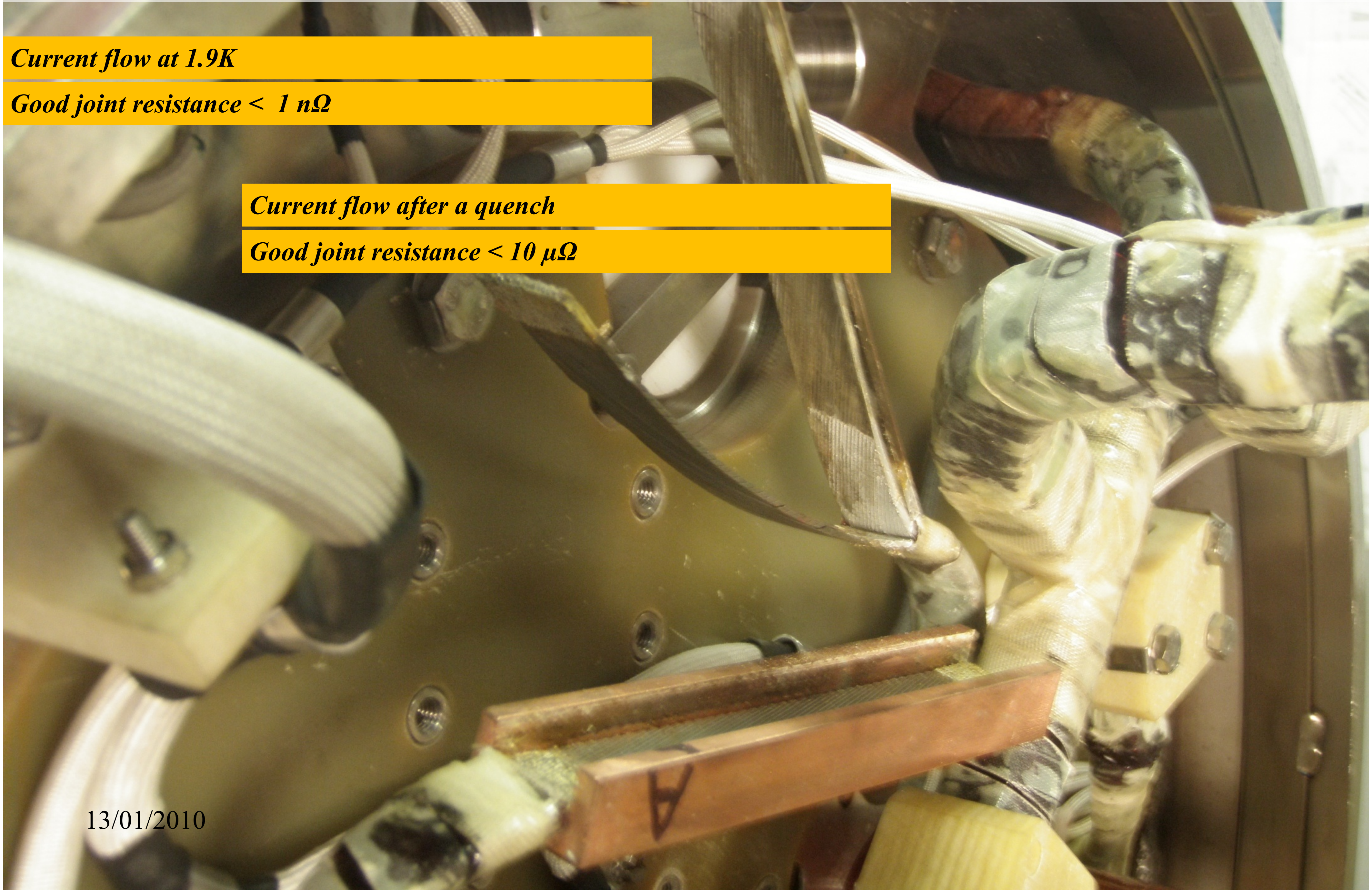
If not fast and safe …

Quench in a magnet

During magnet test campaign, the 7 MJ stored in one magnet were released into one spot of the coil (inter-turn short)

P. Pugnat

LHC Interconnects
(… splices / … Loetverbindungen / … silent killers)



Current flow at 1.9K

Good joint resistance < 1 nΩ

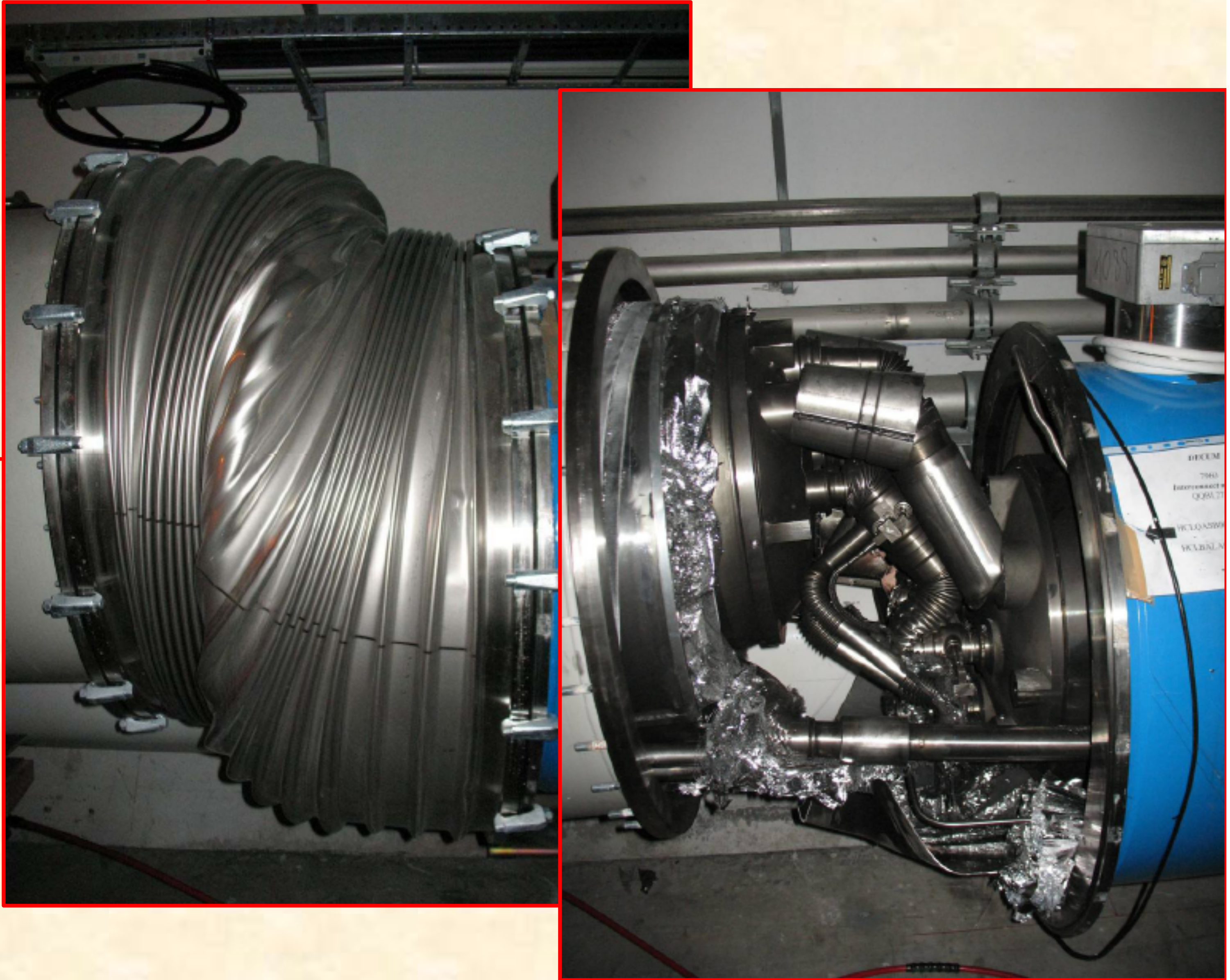
Current flow after a quench

Good joint resistance < 10 μΩ

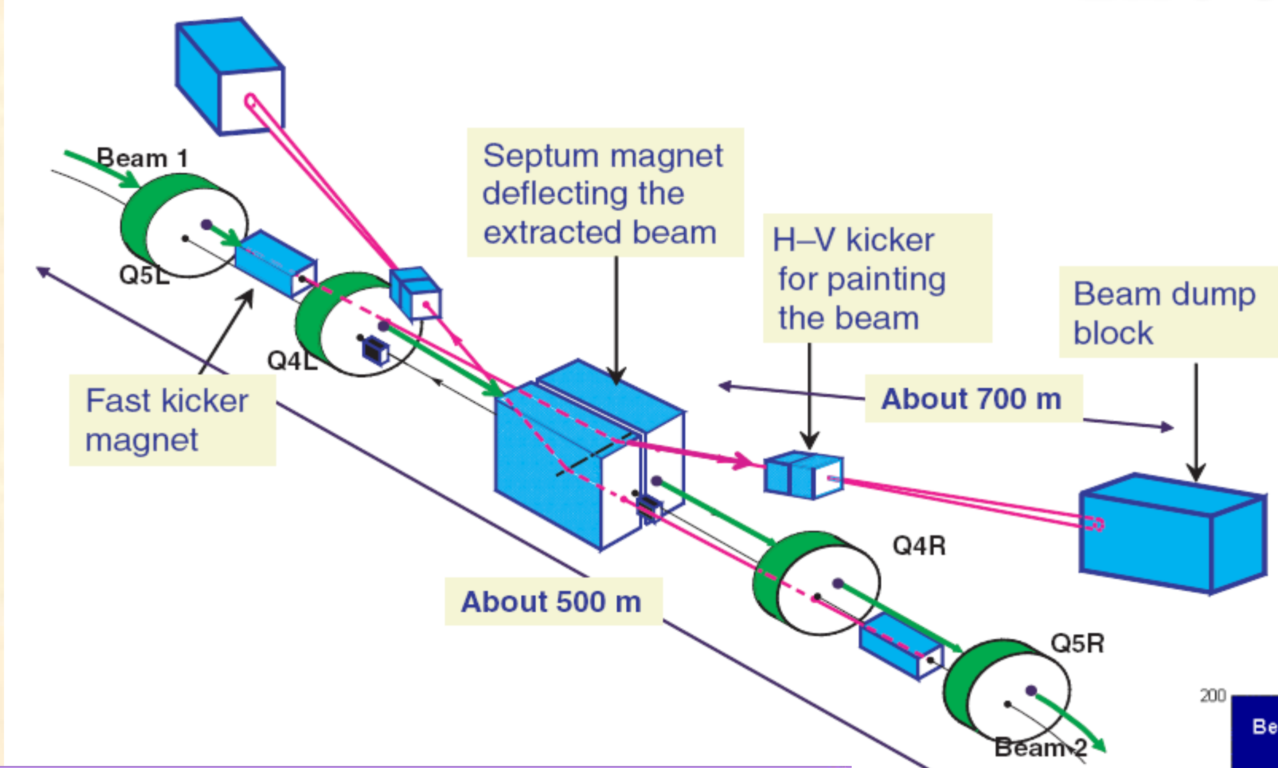
13/01/2010

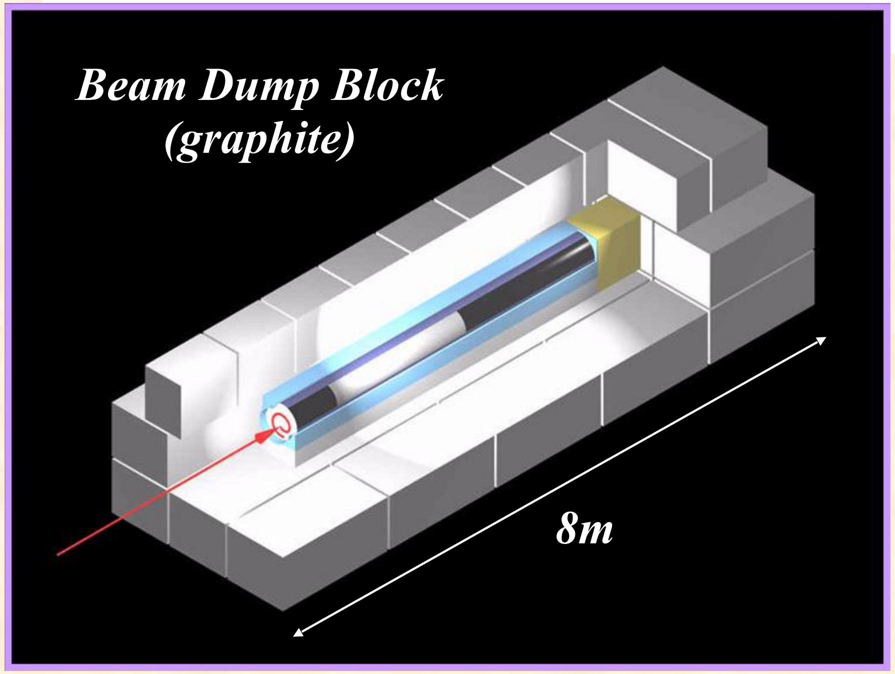
Quench in a bus bar (19 Sep 2008)

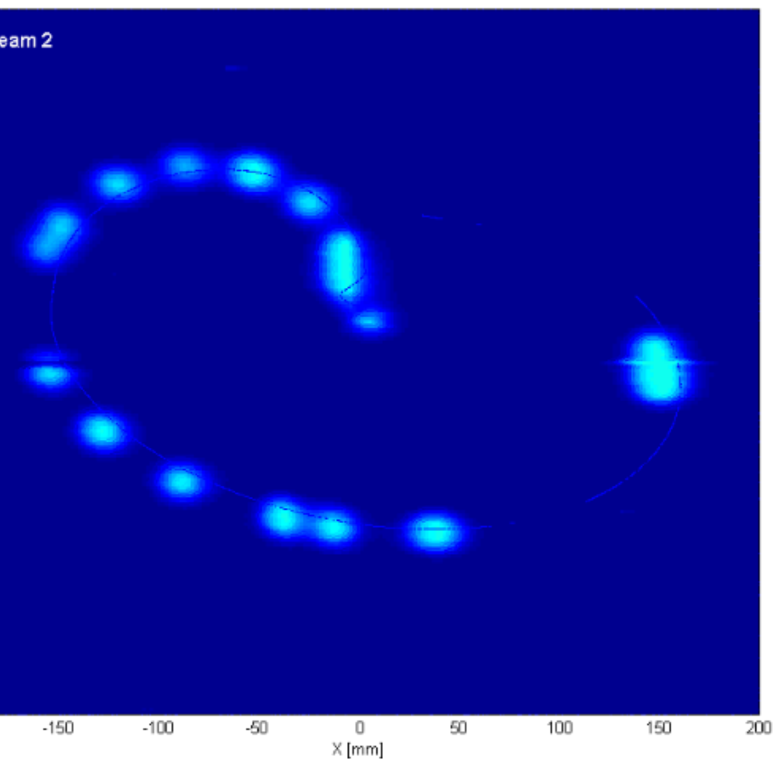
Electrical arc between C24 and Q24



LHC Operation: Dump System







Physics Beyond the Standard Model (BSM)

Example: Dark Matter

Corbelli & Salucci (2000);
Bergstrom (2000)

The outer region of galaxies rotate faster than expected from visible matter

$$\frac{mv}{r} = \frac{m^* M * G}{r^2}$$

$$v_{circ} = \sqrt{\frac{M(r) * G}{r}}$$

Dark matter would explain this

Other observations exist … (grav. lens effects)
but all through gravity

What is it?

(One explanation is super-symmetry)