

Intrinsic charm in the proton



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Milan, Christmas meeting 2021

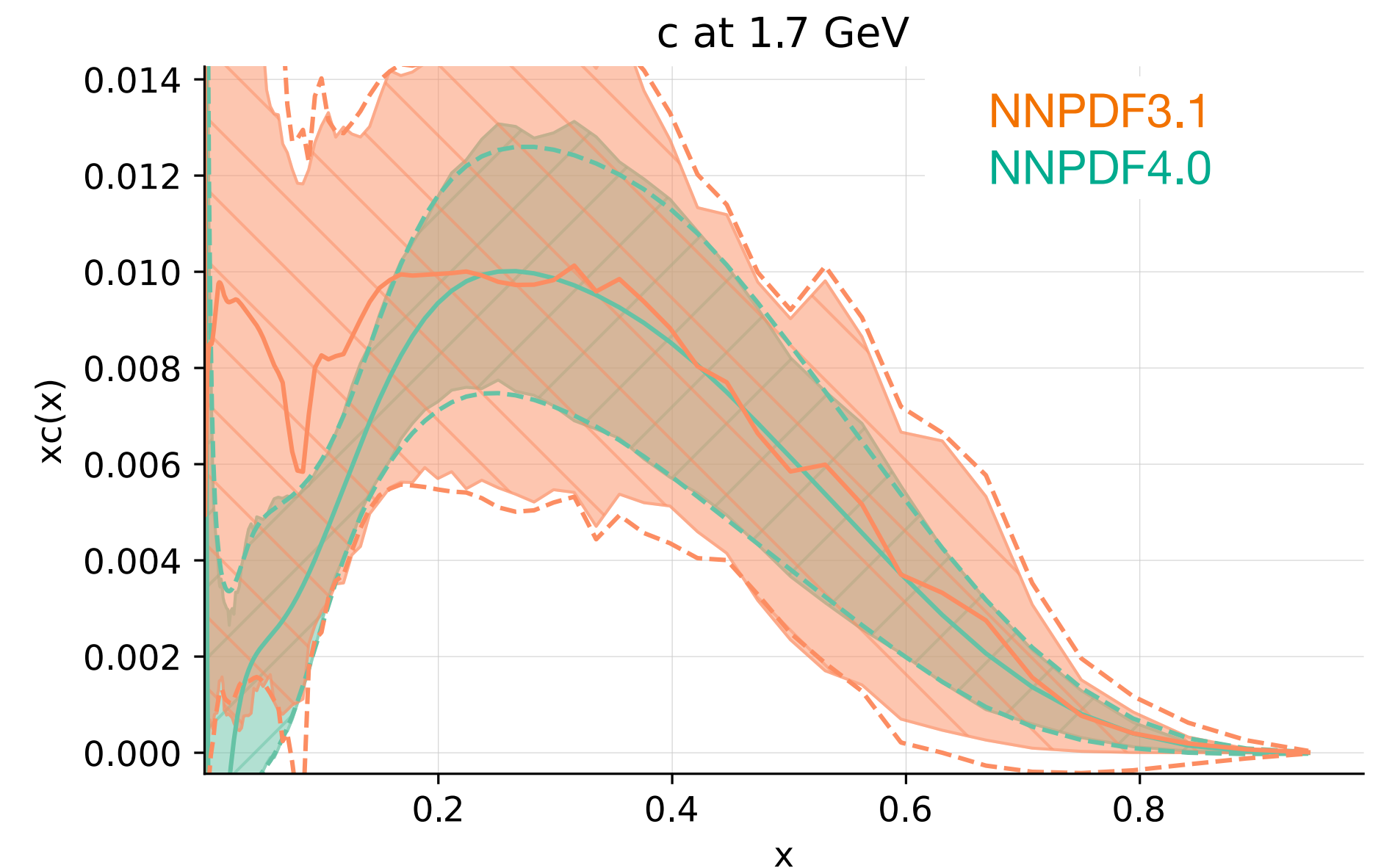
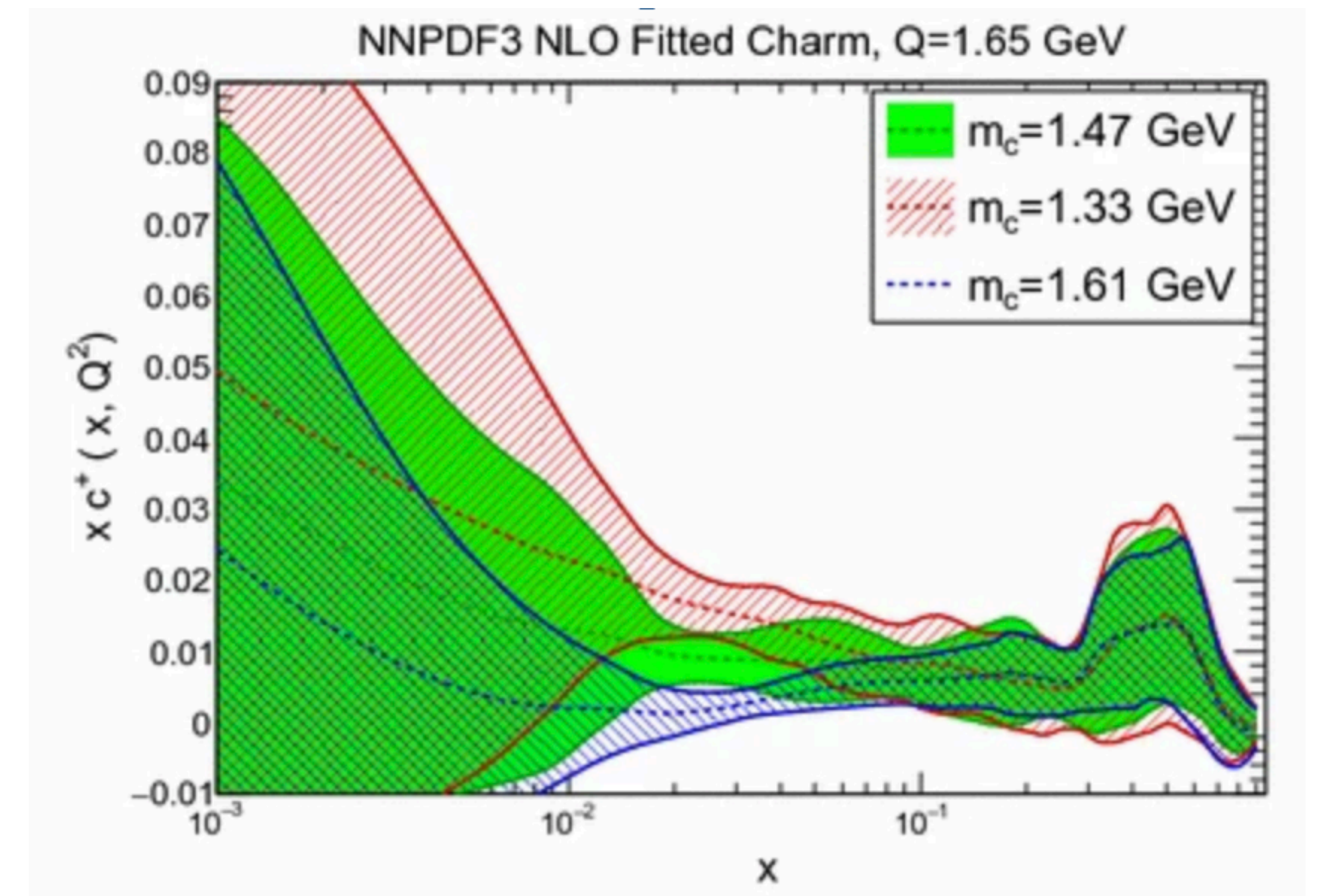
Nikhef

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Introduction & Motivation

Do heavy quarks contribute to the proton wave function at low scales?

- Will focus on the charm PDF, as the *natural candidate* to answer this question
- Will present results based on a work:
NNPDF collaboration “Charm in the proton”
(in preparation)
- Results are based on the (recently released) NNPDF4.0

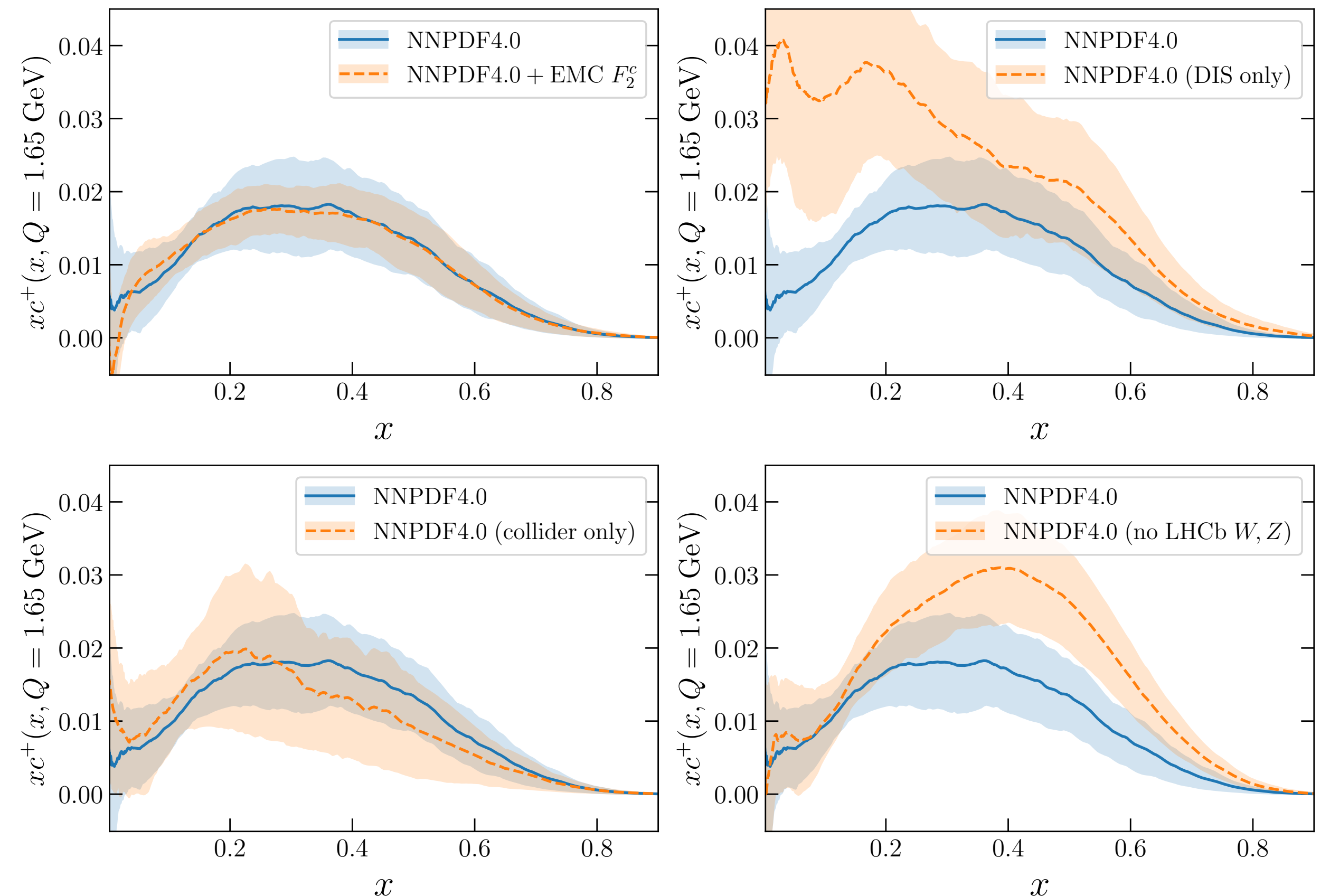


The charm pdf in 4 FNS

NNPDF 4.0 baseline

$$xc^+(x, Q_0, \Theta) = \left(x^{\alpha_\Sigma}(1-x)^{\beta_\Sigma} NN_\Sigma(x, \Theta) - x^{\alpha_{T15}}(1-x)^{\beta_{T15}} NN_{T15}(x, \Theta) \right) / 4$$

- The charm PDF is parametrised as an independent combination at scale $Q_0 = 1.65 \text{ GeV}$, and $n_f = 4$
- c^+ at the fitting scale exhibits a non vanishing peak in the *high x* region and vanishes at *low-x*
- $\bar{c} = c$
- Constrains are coming mainly from collider data
- NNPDF 4.0 is consistent with EMC data.



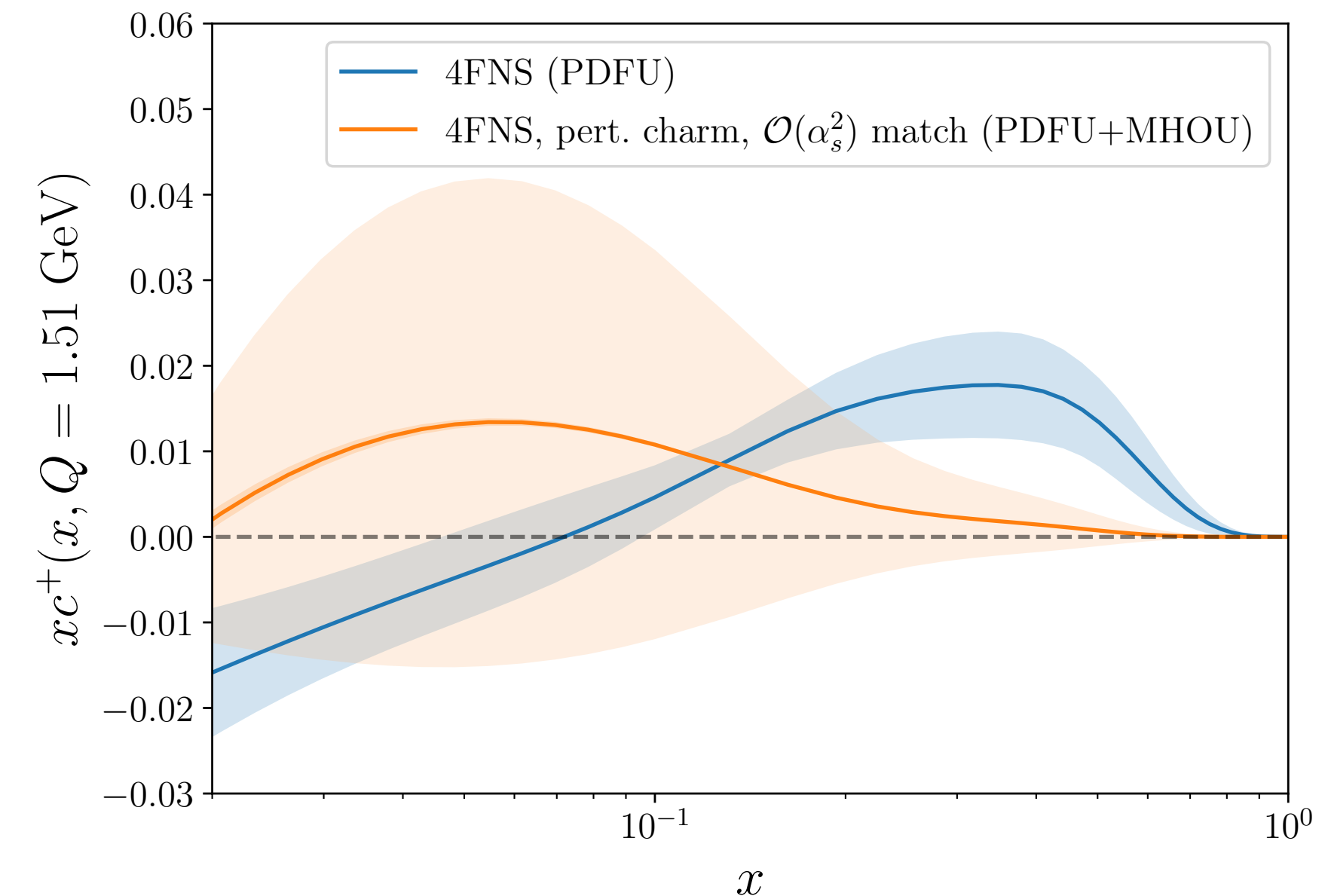
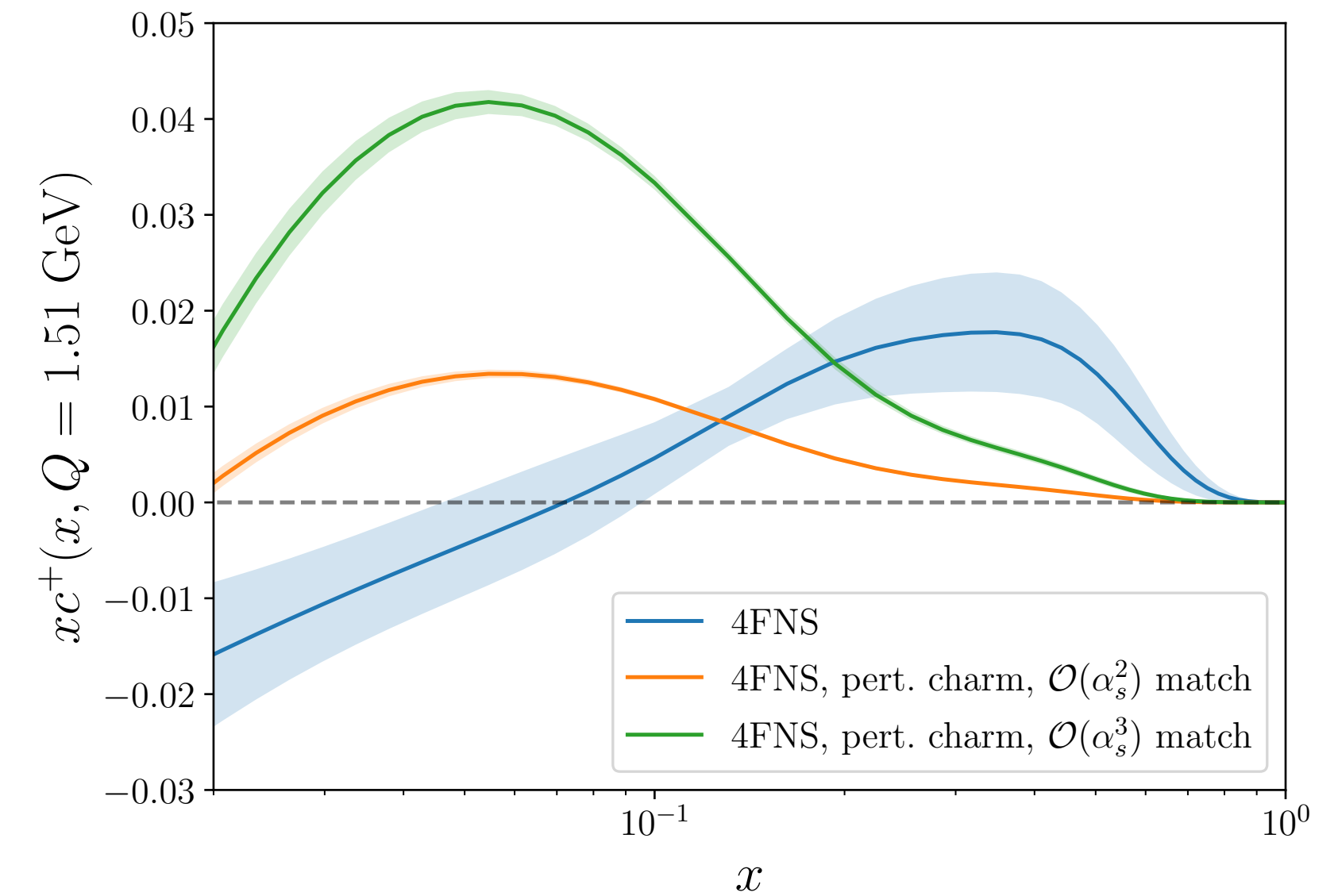
Why do we need intrinsic charm?

The fully perturbative scenario

- **Perturbative charm** functional form is fully determined by the DGLAP evolution and the initial boundary conditions.
- In this case PDF uncertainties are clearly not the dominant source of uncertainties. Needs to estimate MHOUs and mass dependence.

However...

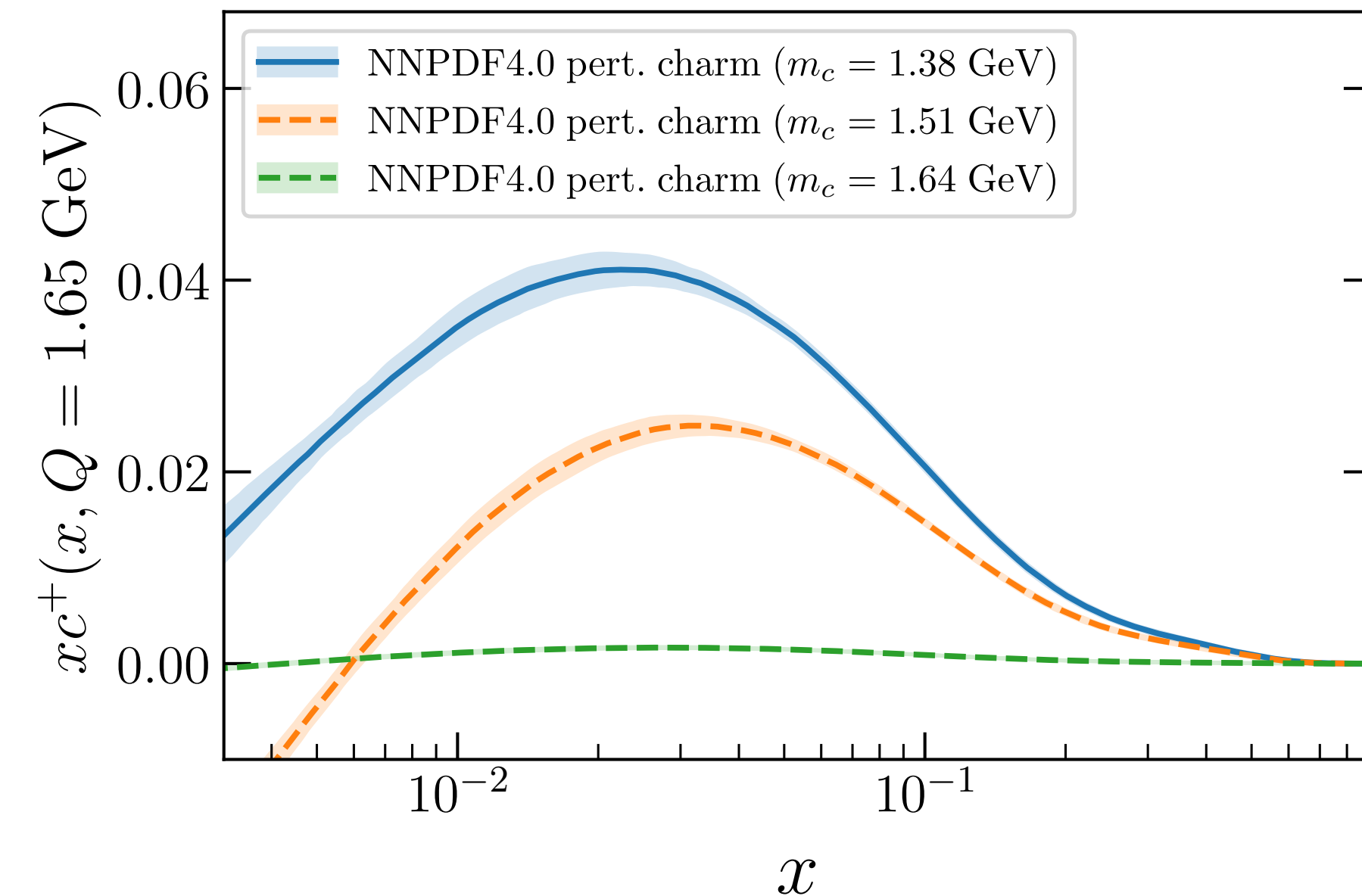
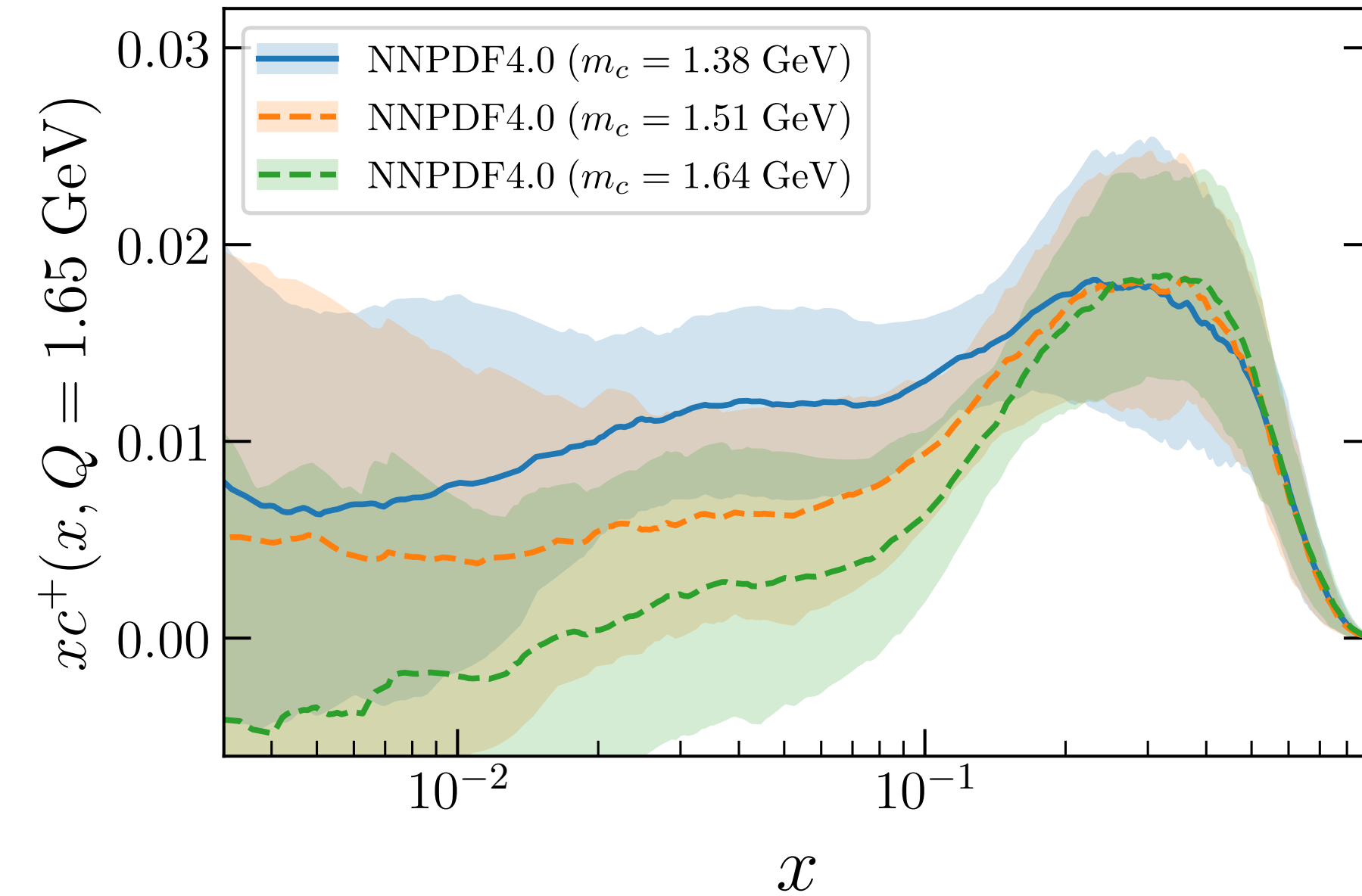
- The post fit χ^2 favours a fitted charm scenario compared to a purely perturbative charm.
- Fully perturbative charm is not compatible with the fitted one.



Why do we need intrinsic charm?

Mass dependence

- NNPDF4.0 fit is carried out using heavy quark pole mass
- Charm mass is varied in the range:
$$m_c = 1.51 \pm 0.13 \text{ GeV}$$
- The fitted charm is much more stable upon mass variation especially in the high x region compared to the perturbative one.



The Intrinsic charm scenario

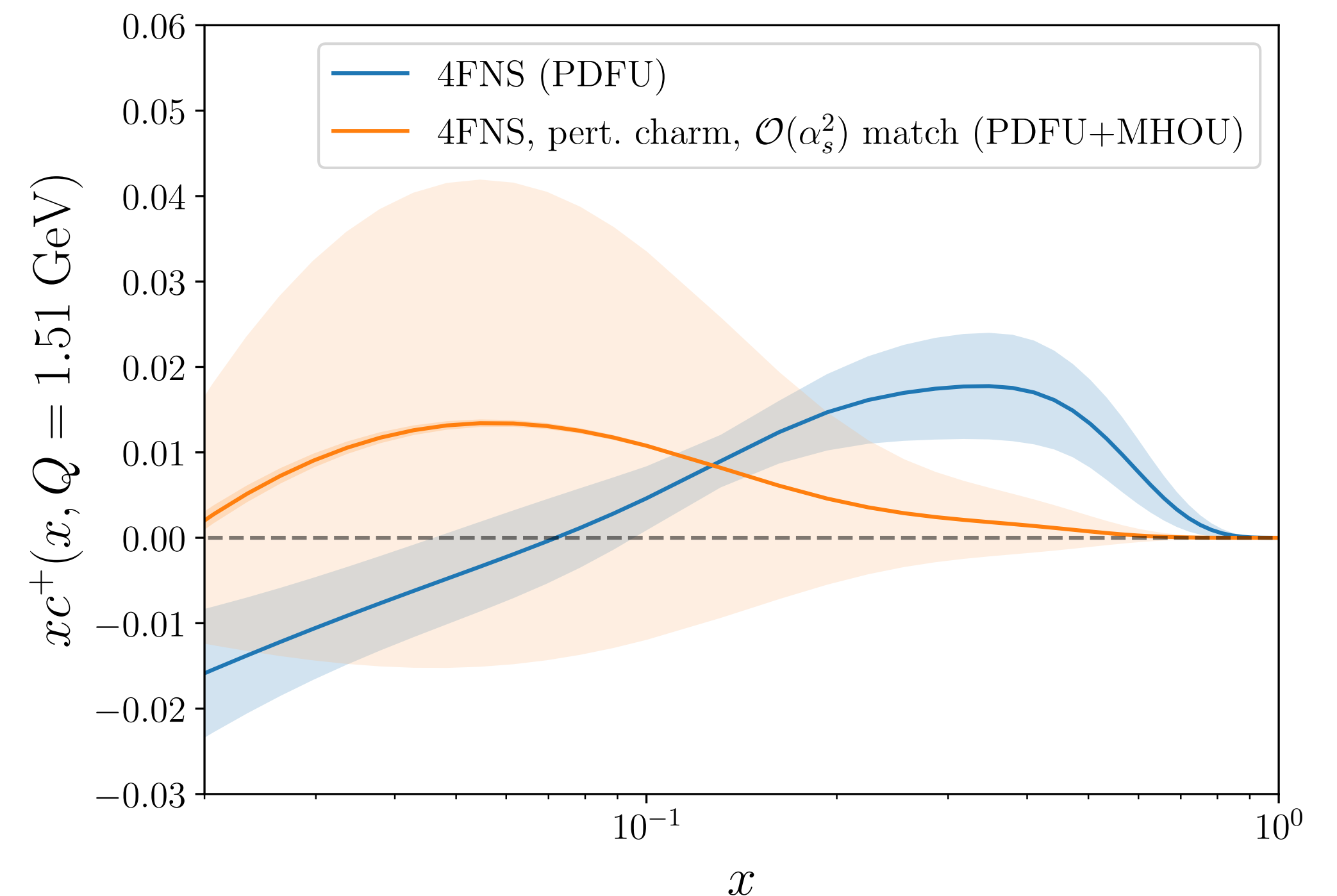
Perturbative vs Fitted

- **Perturbative charm** functional form is fully determined by the DGLAP evolution and the initial boundary conditions.
- Below the charm mass scale the perturbative charm is vanishing by definition
- **Fitted charm in 4FNS** contains both the intrinsic and the perturbative components.

how can we separate them?



Need to work out the 3 FNS charm PDF



A brief digression on DGLAP

Solving DGLAP with EKO



$$\frac{d}{d\alpha_s} \tilde{\mathbf{f}}(\mu_F^2) = -\frac{\gamma(a_s)}{\beta(a_s)} \cdot \tilde{\mathbf{f}}(a_s)$$

- The formal solution of DGLAP can be written as in Mellin-space:

$$\tilde{\mathbf{f}}(a_s) = \tilde{\mathbf{E}}(a_s \leftarrow a_s^0) \cdot \tilde{\mathbf{f}}(a_s^0)$$

$$\tilde{\mathbf{E}}(a_s \leftarrow a_s^0) = \mathcal{P} \exp \left[-\int_{a_s^0}^{a_s} \frac{\gamma(a'_s)}{\beta(a'_s)} da'_s \right]$$

- In x-space:

$$\mathbf{f}(x_k, a_s) = \mathbf{E}_{k,j}(a_s \leftarrow a_s^0) \mathbf{f}(x_j, a_s^0)$$

→ The **EKO**, a rank 4 tensor

- Evolution is performed in **Intrinsic** Evolution basis: $\text{span}\{g, \Sigma, V, V_3, T_3, V_8, T_8, c^+, c^-, b^+, b^-, t^+, t^-\}$
- Solution is available at: LO, NLO, NNLO
- EKO implements various solution methods: *Exact, Truncated, Expanded*
see: [documentation](#)

From 4FNS to 3FNS

The matching conditions

At the heavy quark mass threshold the number of active flavour changes:

$$\tilde{\mathbf{f}}^{(n_f+1)}(Q_1^2) = \tilde{\mathbf{E}}^{(n_f+1)}(Q_1^2 \leftarrow \mu_h^2) \mathbf{R}^{(n_f)} \tilde{\mathbf{A}}^{(n_f)}(\mu_h^2) \tilde{\mathbf{E}}^{(n_f)}(\mu_h^2 \leftarrow Q_0^2) \tilde{\mathbf{f}}^{(n_f)}(Q_0^2)$$

Rotation from *intrinsic*^(nf) basis to *intrinsic*^(nf+1)

The **Operator Matrix Elements**:

- In FFNS there are no difference between forward and backward evolution (just swap the integration bounds)
- The OME depends on $\alpha_s(\mu_h^2)$ and on $\log(\mu_h^2/m_h^2)$
- OME are available up to N3LO (*NLO for heavy quark entries*)

$$\mathbf{A}^{(n_f)}(\mu_h^2) = \mathbf{I} + a_s^{(n_f)}(\mu_h^2) \mathbf{A}^{(n_f),(1)} + a_s^{(n_f),2}(\mu_h^2) \mathbf{A}^{(n_f),(2)} + a_s^{(n_f),3}(\mu_h^2) \mathbf{A}^{(n_f),(3)} + \mathcal{O}(\alpha_s^4)$$

$$\begin{pmatrix} \tilde{V}_{(n_f)} \\ \tilde{h}^- \end{pmatrix}^{n_f+1}(\mu_h^2) = \tilde{\mathbf{A}}_{NS,h^-}^{(n_f)}(\mu_h^2) \begin{pmatrix} \tilde{V}_{(n_f)} \\ \tilde{h}^- \end{pmatrix}^{n_f}(\mu_h^2)$$

$$\begin{pmatrix} \tilde{g} \\ \tilde{\Sigma}_{(n_f)} \\ \tilde{h}^+ \end{pmatrix}^{n_f+1}(\mu_h^2) = \tilde{\mathbf{A}}_{S,h^+}^{(n_f)}(\mu_h^2) \begin{pmatrix} \tilde{g} \\ \tilde{\Sigma}_{(n_f)} \\ \tilde{h}^+ \end{pmatrix}^{n_f}(\mu_h^2)$$

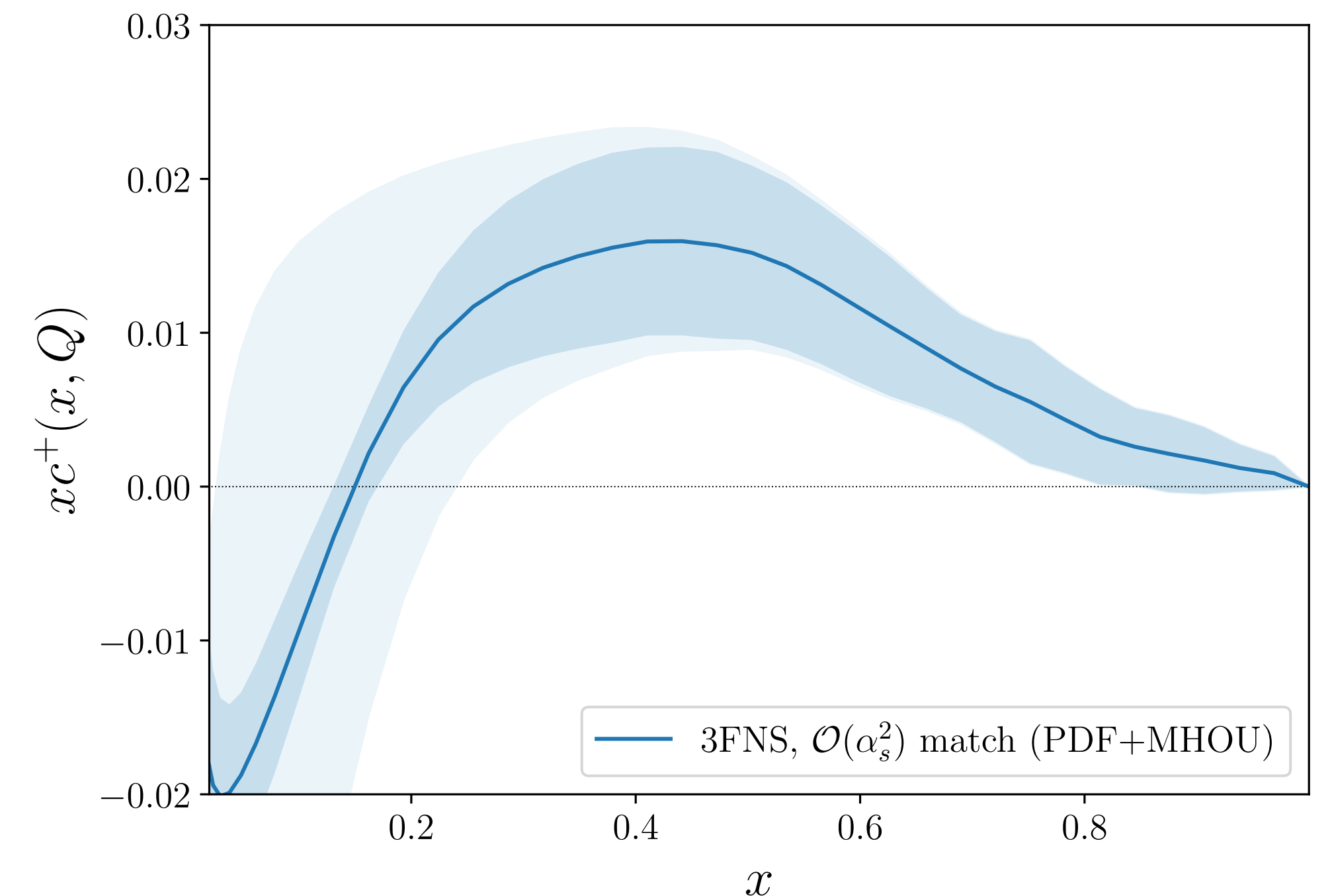
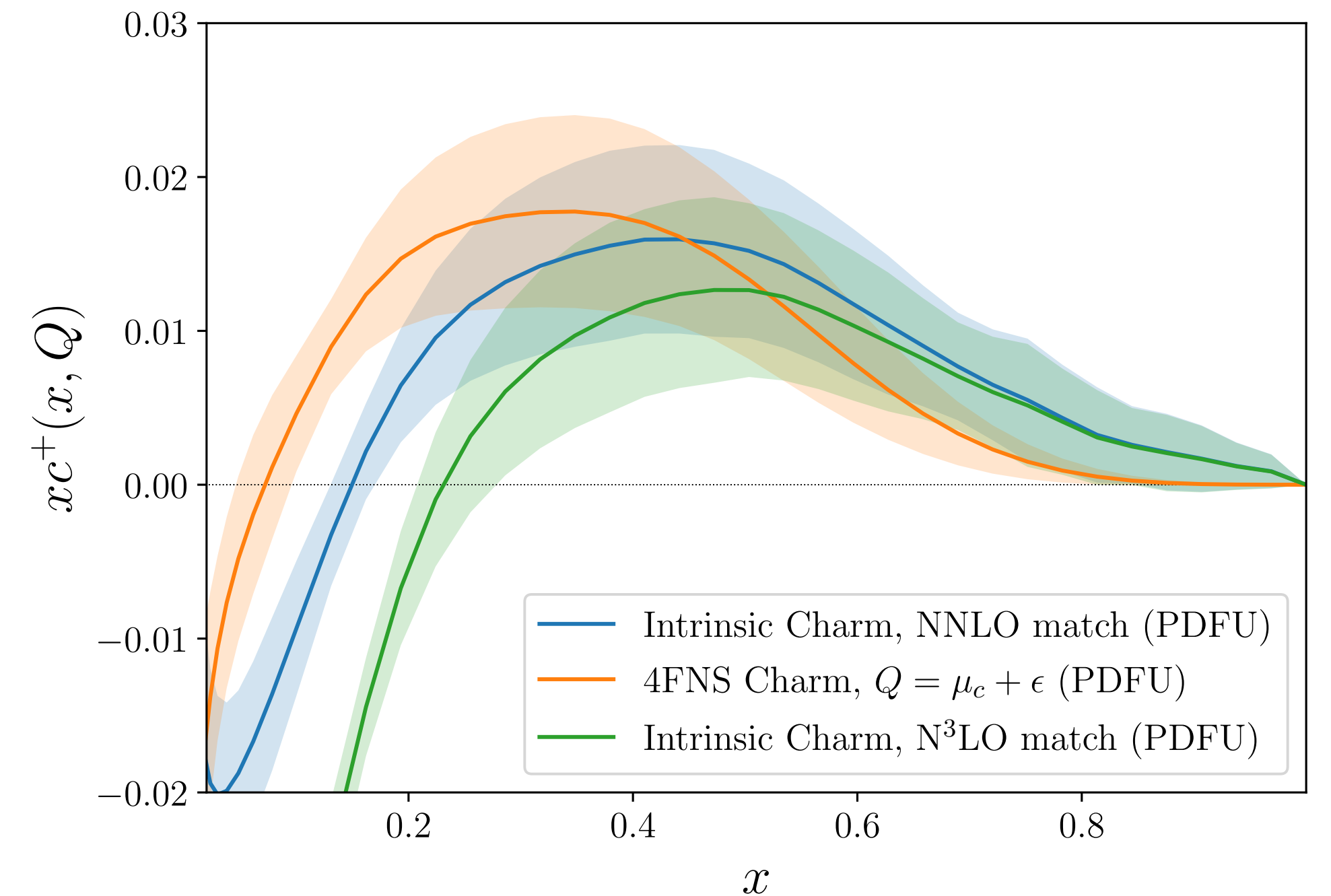
The Intrinsic charm

Charm in 3FNS

- Starting from the fitting scale we evolve the NNPDF4.0 baseline to $Q = m_c$.
- When passing the heavy quark threshold we need to apply the basis rotation and then invert the OMEs $\tilde{A}_{i,j}$.
- The remaining part of the charm pdf is the **Intrinsic component**, which is scale independent

In 3FNS:

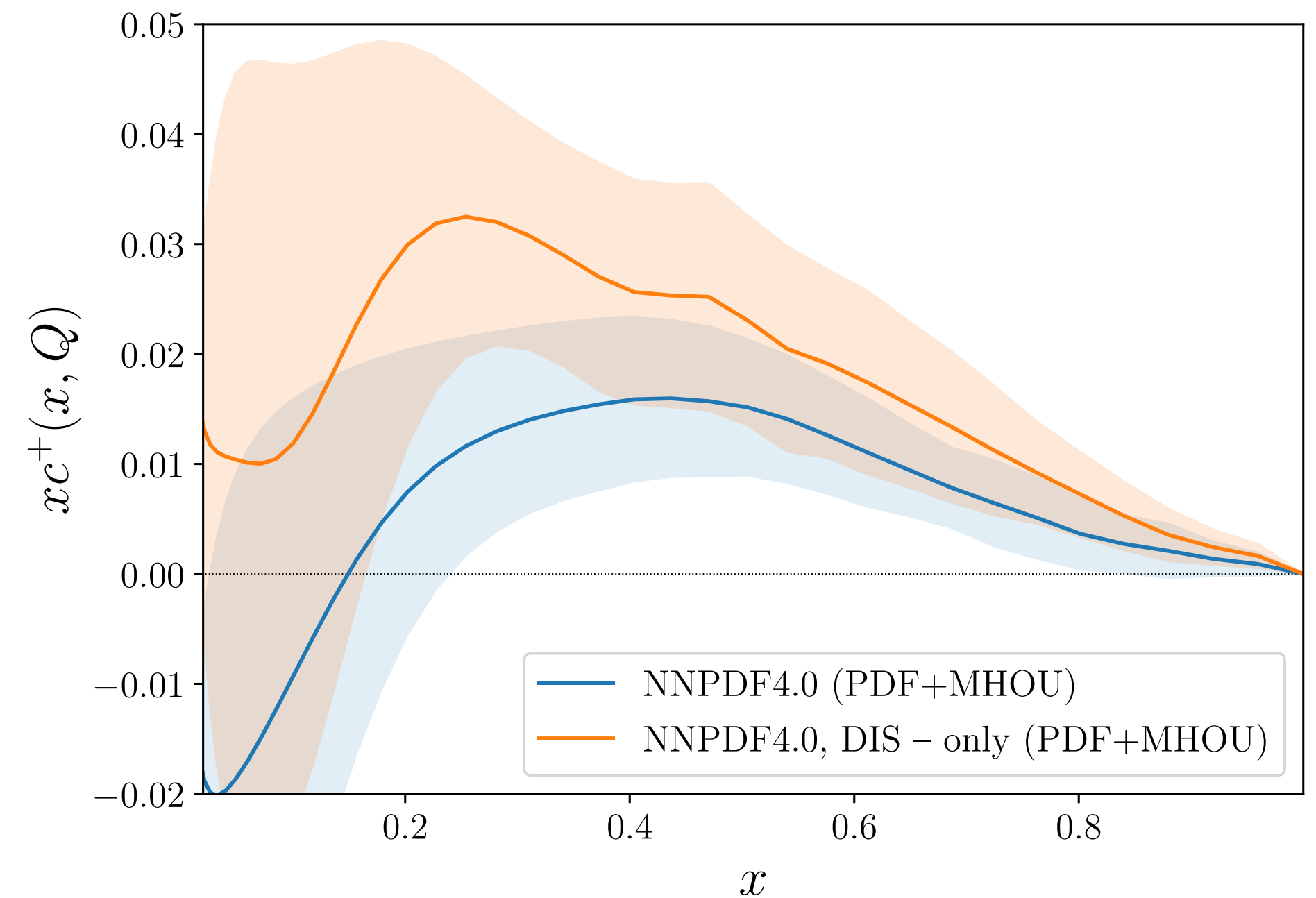
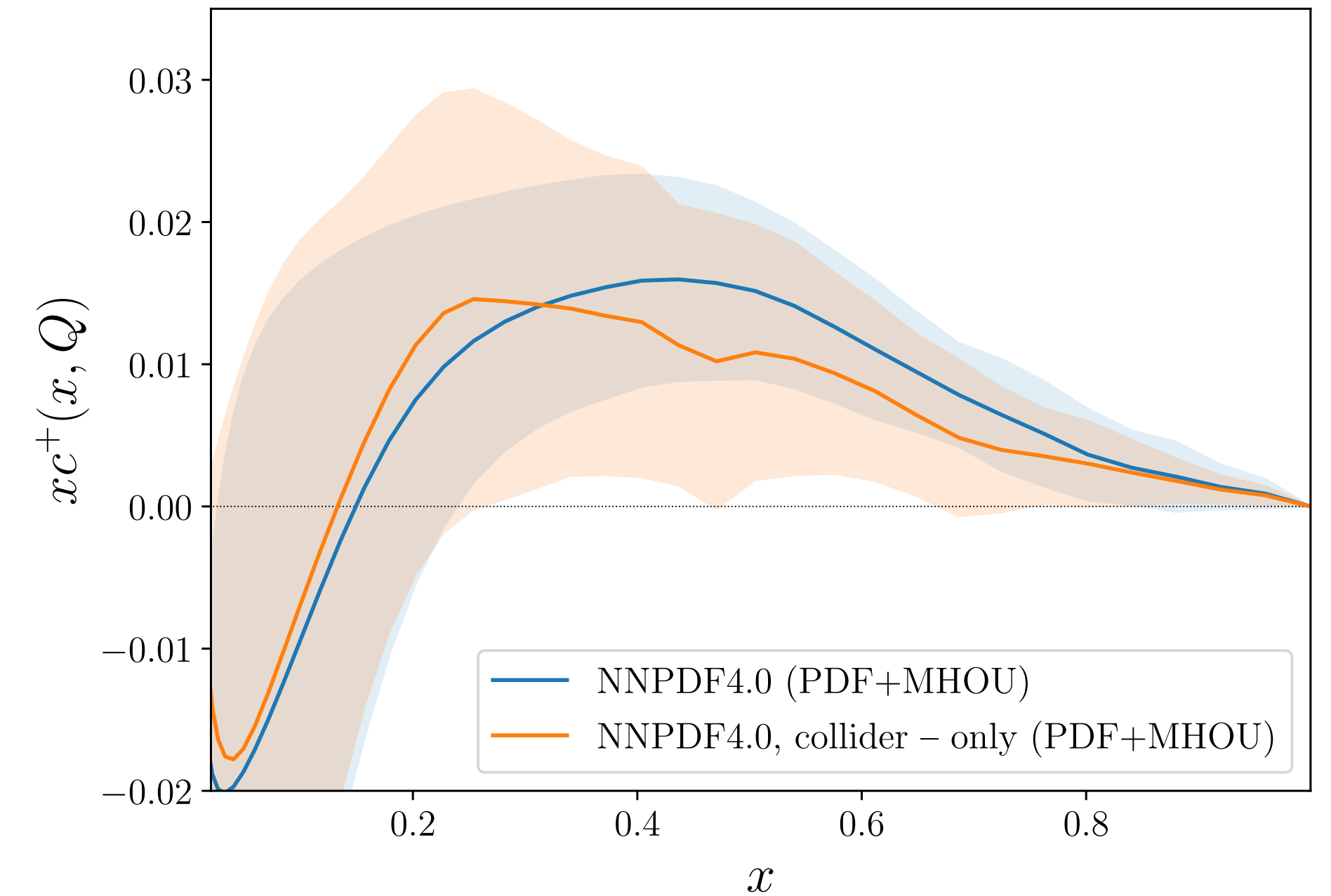
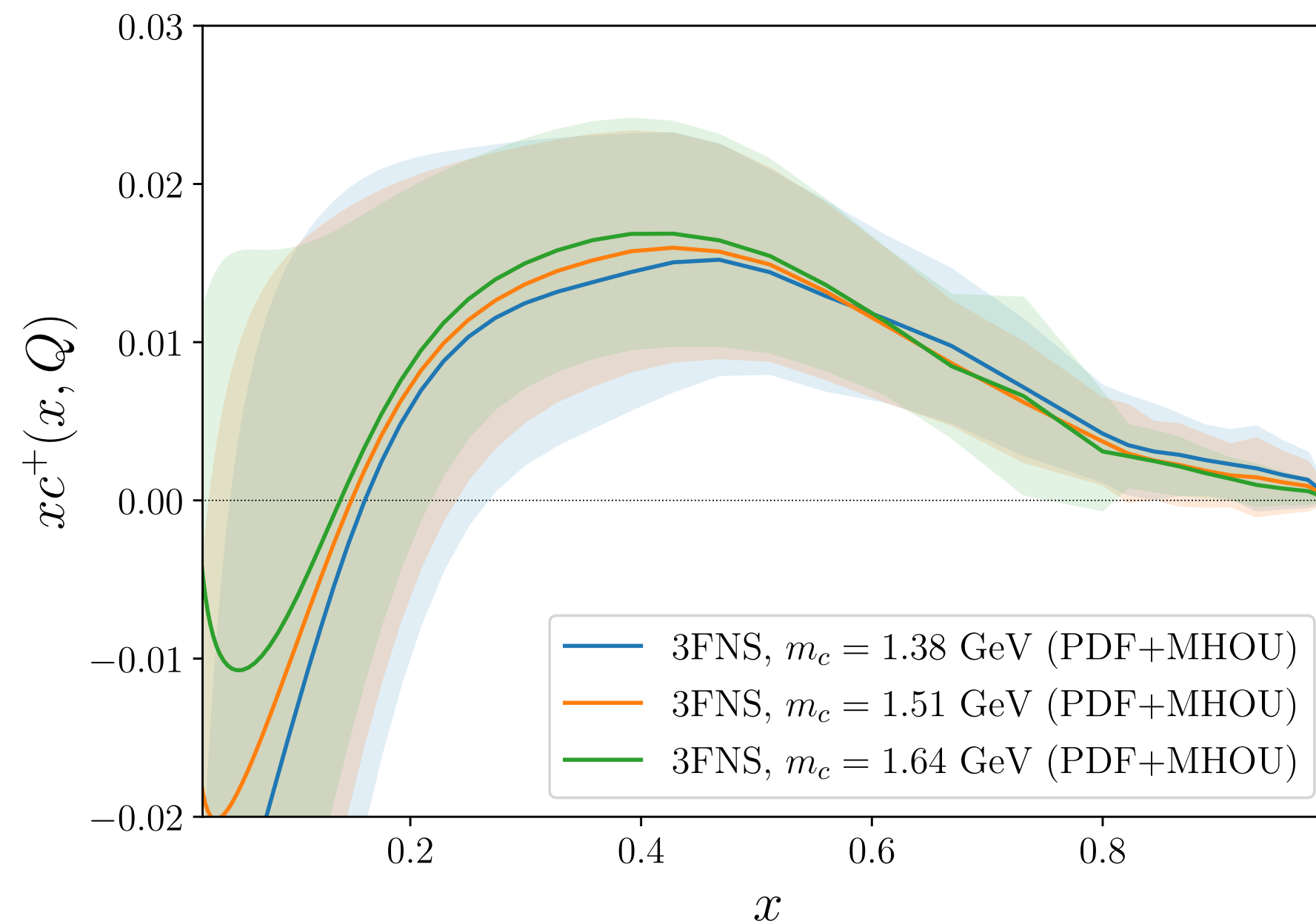
- The *valence-like* peak is still present.
- For $x \leq 0.2$ the perturbative uncertainties are quite large
- The carried momentum fraction is within 1%



3FNS stability and accuracy

Mass dependence and dataset variation

- Intrinsic charm is stable upon mass variation
- Scale independency
- Always vanishing for $x \leq 0.2$
- MHOU coming from NNLO-N3LO matching



The Intrinsic charm

Comparison with Models

- BHPS model: [\[Phy. Letter B \(1980\) 451-455\]](#)

$$p \rightarrow uudc\bar{c}$$

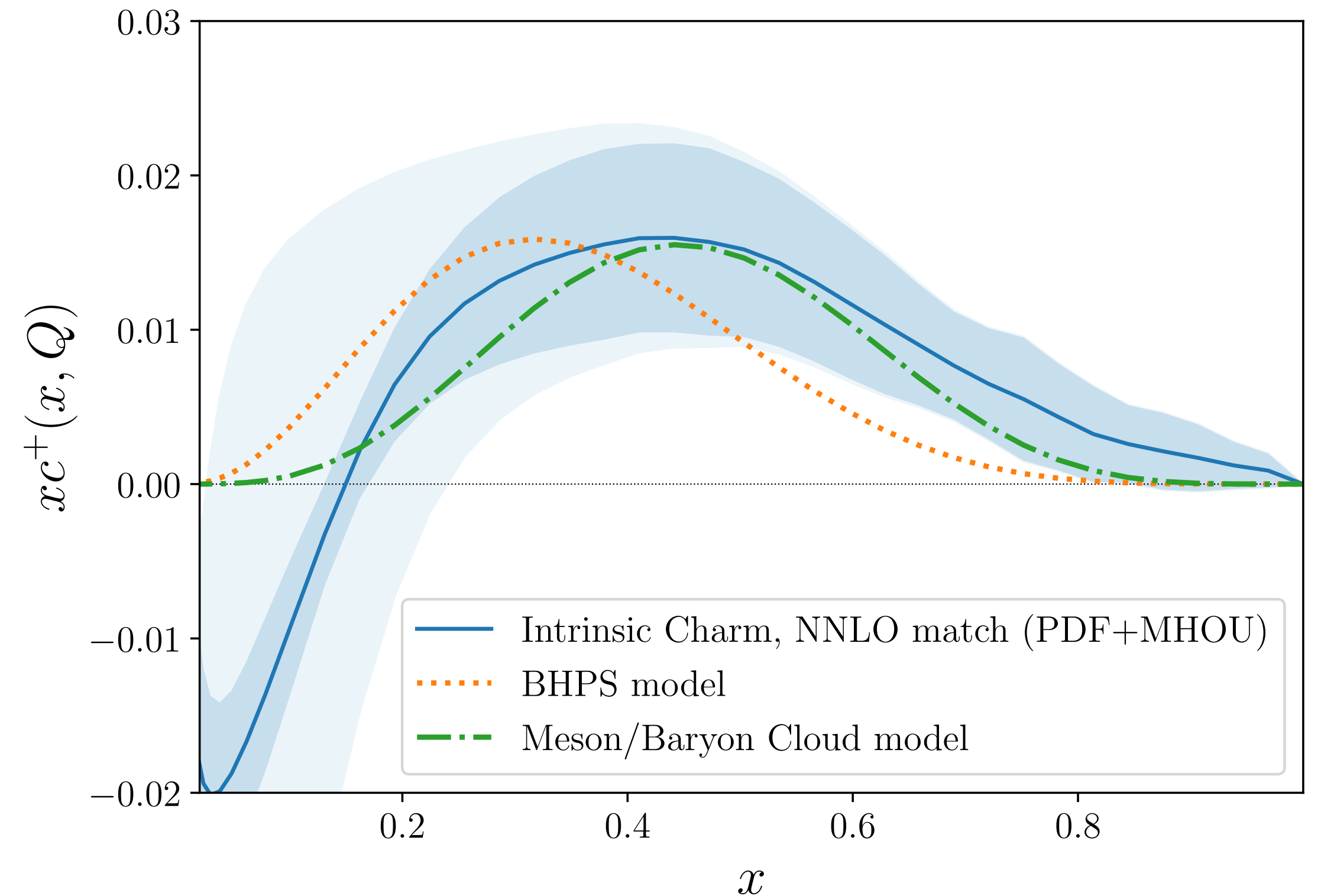
$$xc^+ = \frac{1}{2}Nx^3\left[\frac{1}{3}(1-x)(1+10x+x^2) + 2x(1+x^2)\ln(x)\right]$$

- Meson Baryon model: [\[arxiv:1311.1578\]](#)

$$p \rightarrow \Lambda_c^+ + \bar{D}_0$$

$$xc^+ = \frac{N}{B(\alpha+2, \beta+1)} x^{(1+\alpha)}(1-x)^\beta$$

- $\bar{c} = c$ by assumption in BHPS, not true in M/B models.
- Work in the limit $m_c \gg m_p$

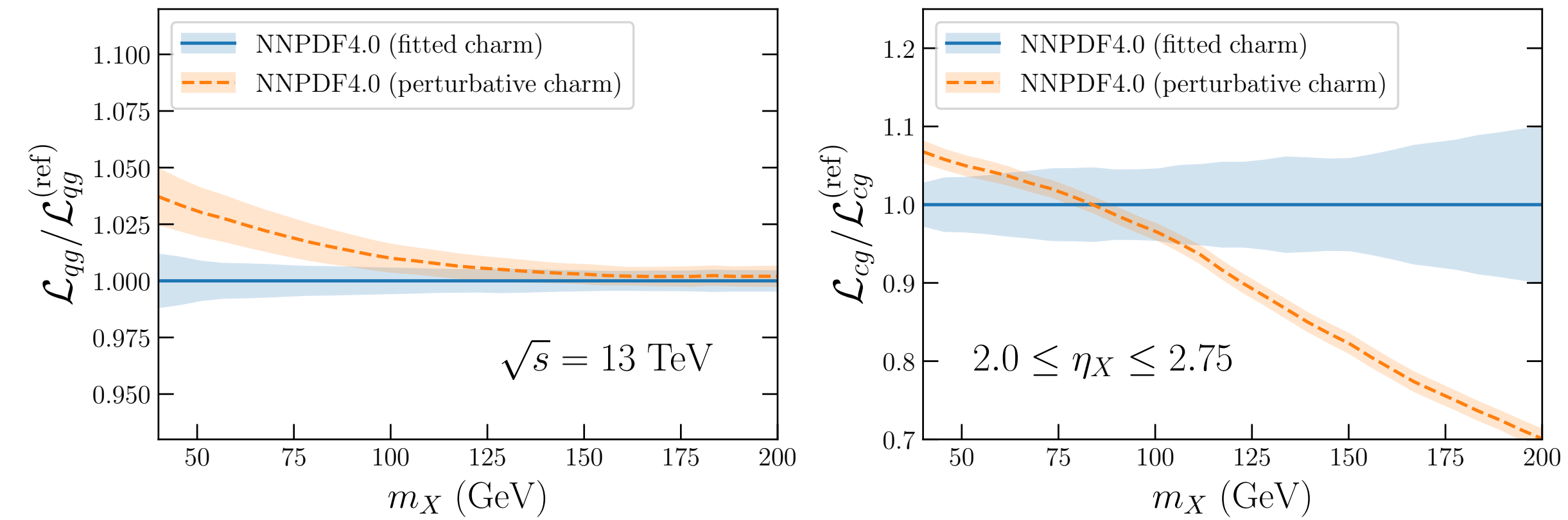


Impact on LHC observables

Partonic luminosities

To see where the differences between fitted and perturbative charm can be evident you can look at partonic lumi of: $pp \rightarrow X$

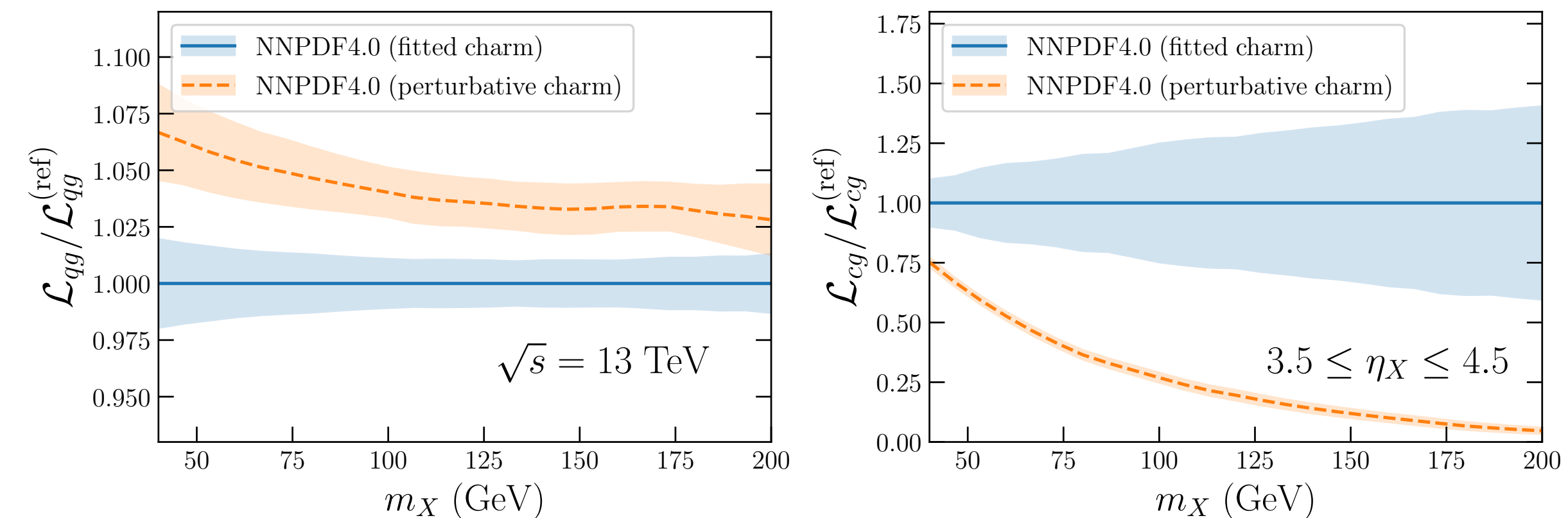
Central region LHCb



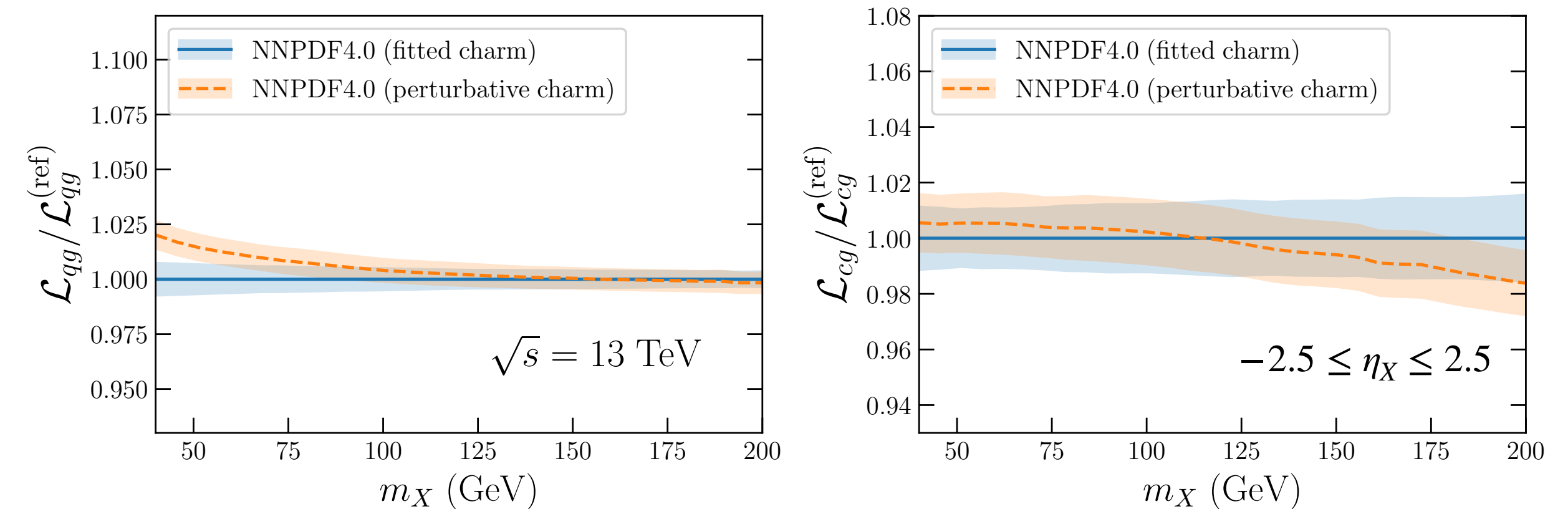
$$\mathcal{L}_{ab} = \frac{1}{s} \int_{\frac{m_X^2}{s}}^1 \frac{dx}{x} f_a(x, m_X^2) f_b(x, m_X^2) \theta(y_X - y_{\min}) \theta(y_{\max} - y_X)$$

$$\mathcal{L}_{cg} = \mathcal{L}_{cg} + \mathcal{L}_{\bar{c}g} \quad \mathcal{L}_{qg} = \sum_{i=1}^{n_f} \mathcal{L}_{q_i g} + \mathcal{L}_{\bar{q}_i g}$$

Forward region LHCb



Central region ATLAS-CMS

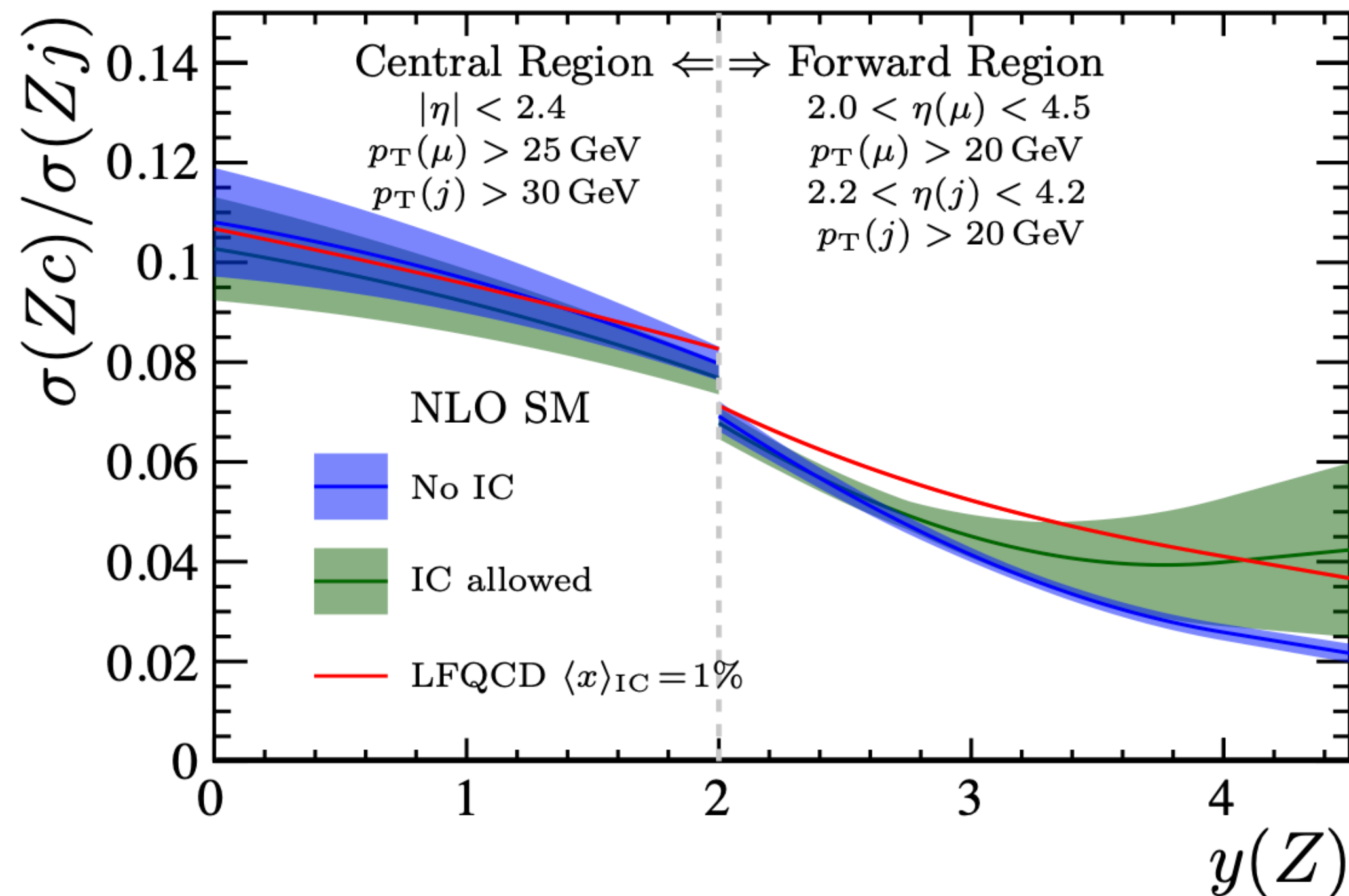
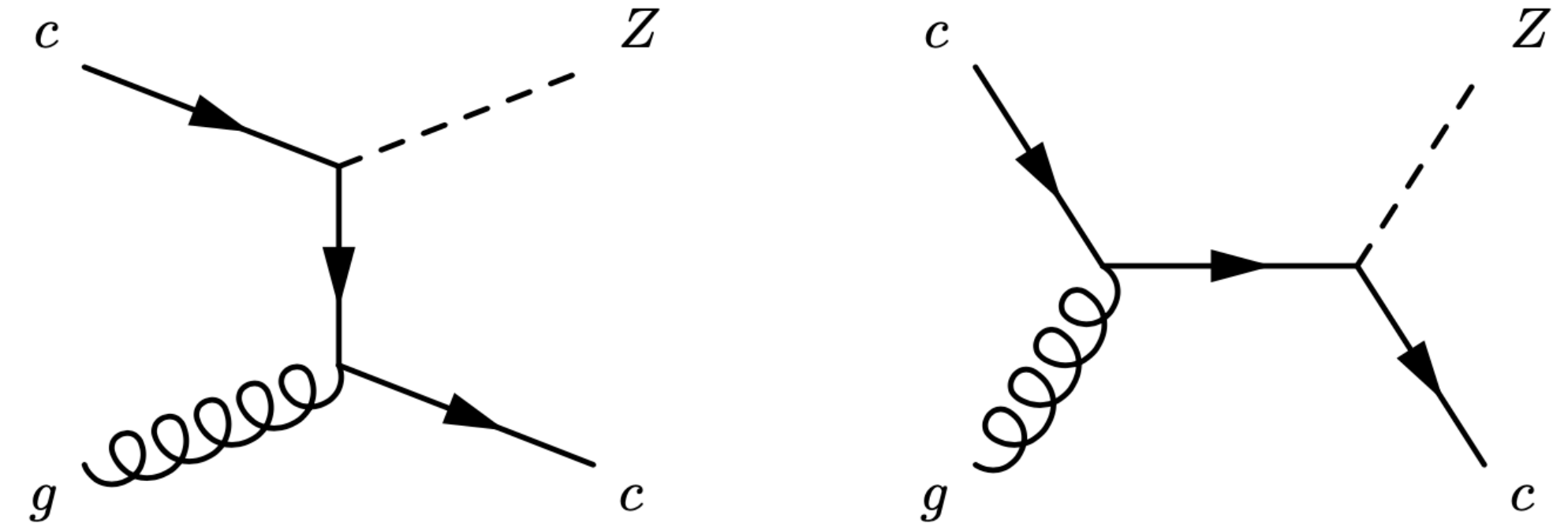


Impact on LHC observables

Z+charm production @ LHCb

We validate our observation of Intrinsic charm evaluating the prediction for:

Z + c production at LHCb [\[arxiv:2109.08084\]](https://arxiv.org/abs/2109.08084)



$$R_j^c(y_Z) = \frac{\sigma_{Zc}}{\sigma_{Zj}}$$

$y(Z)$	\mathcal{R}_j^c (%)	
2.00–2.75	6.84 ± 0.54	± 0.51
2.75–3.50	4.05 ± 0.32	± 0.31
3.50–4.50	4.80 ± 0.50	± 0.39
2.00–4.50	4.98 ± 0.25	± 0.35

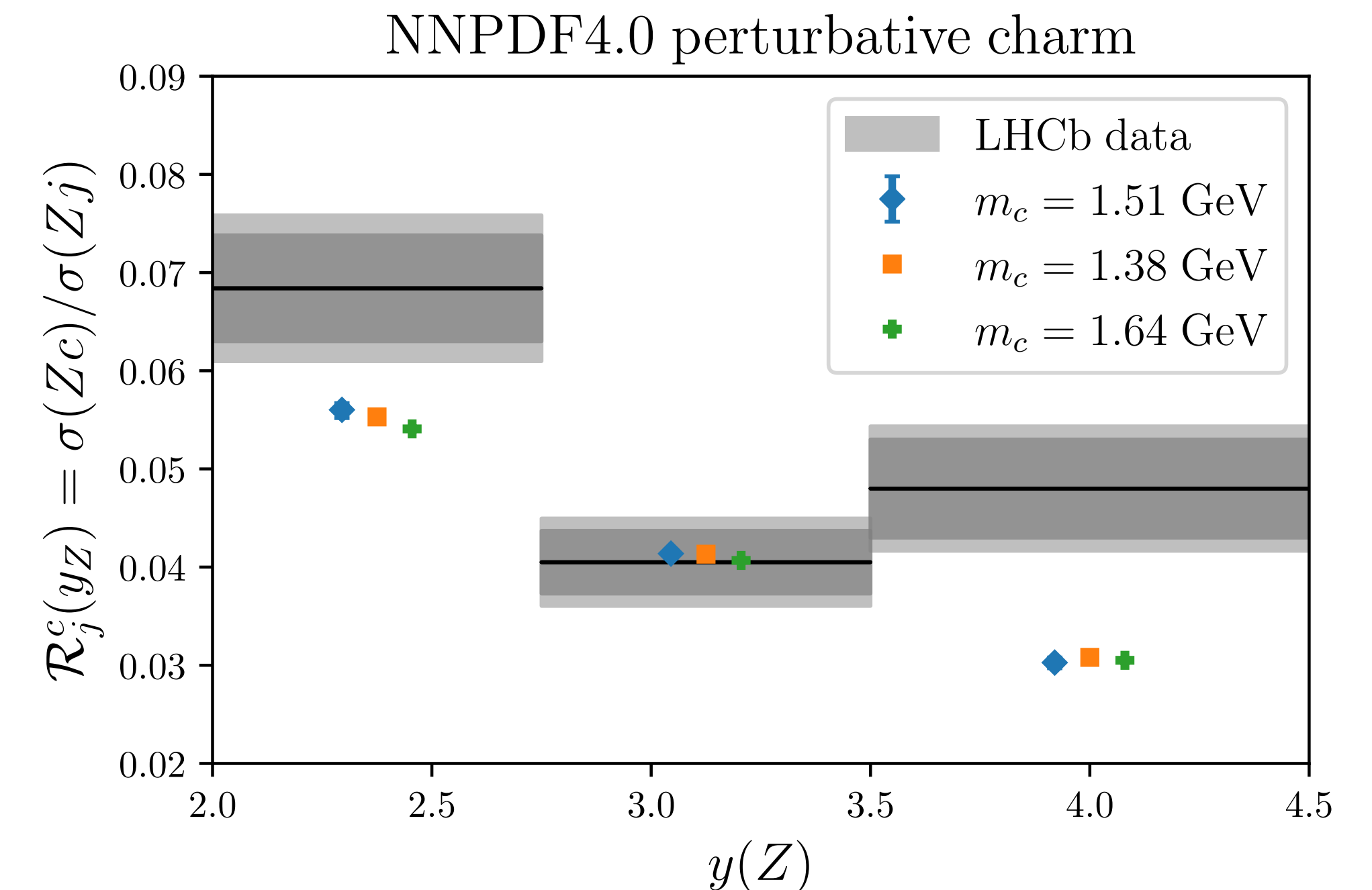
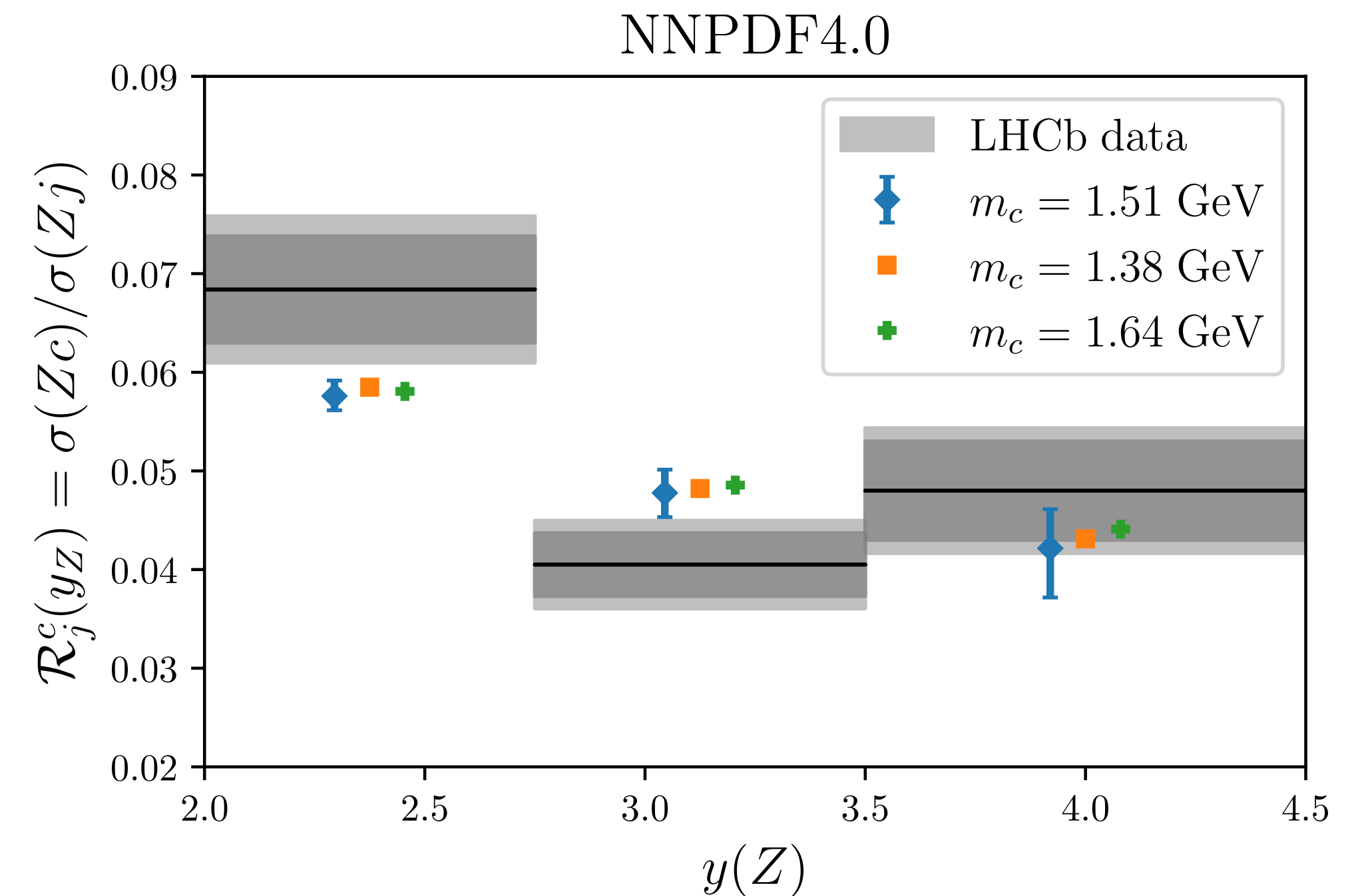
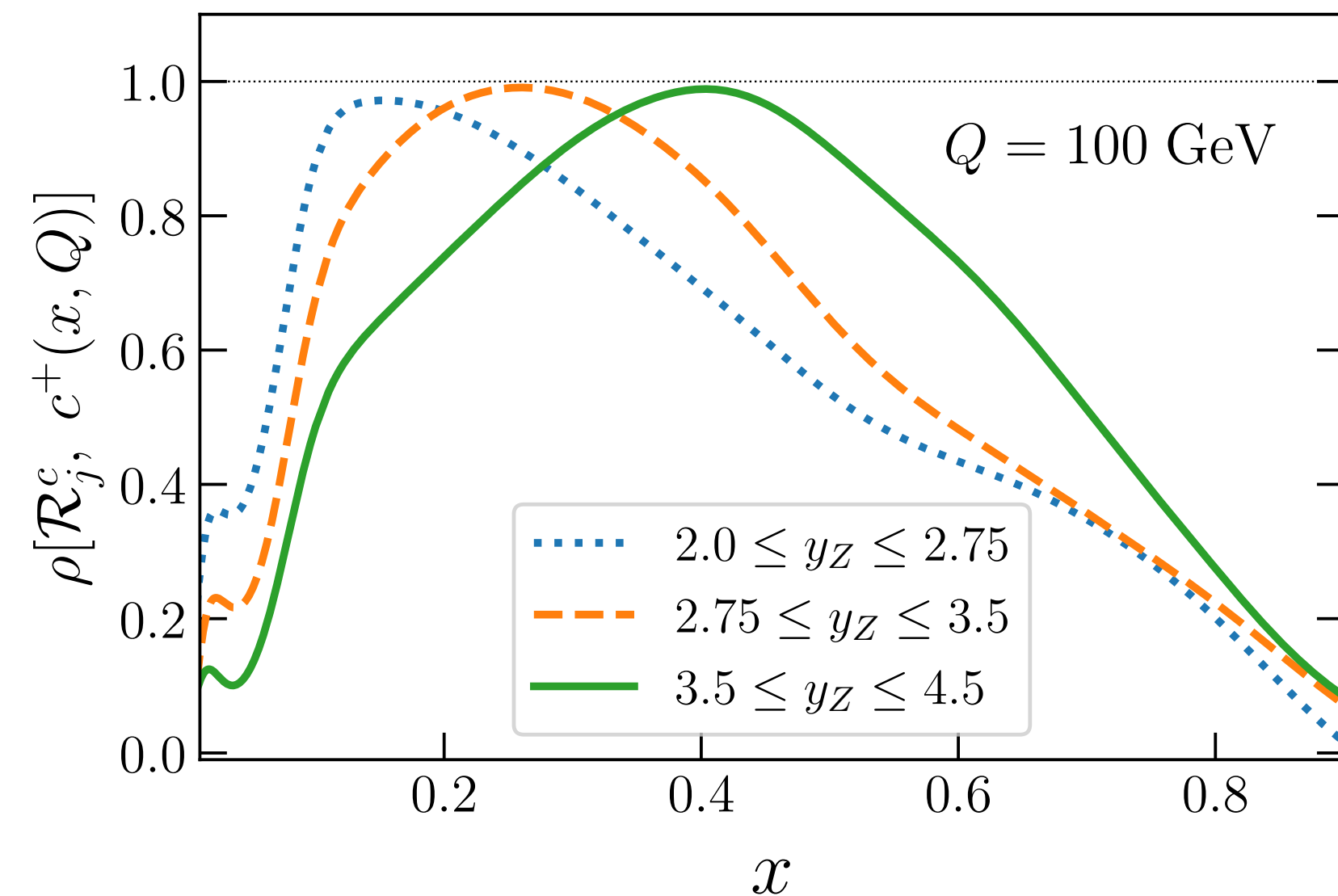
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Impact on LHC observables

Z+charm production @ LHCb

Our in-house prediction, with *Powheg* @ NLO+PS [[arxiv: 1009.5594](https://arxiv.org/abs/1009.5594)]

- Better agreement is found with the NNPDF4.0 baseline especially in the forward region
- Predictions are also stable upon charm mass variation
- NNLO corrections not taken into account yet
- High correlation with the charm PDF and LHCb observable:



The Intrinsic charm

Our current best estimation

To achieve the best sensitivity on the intrinsic charm, we add the LHCb results to the NNPDF4.0 baseline.

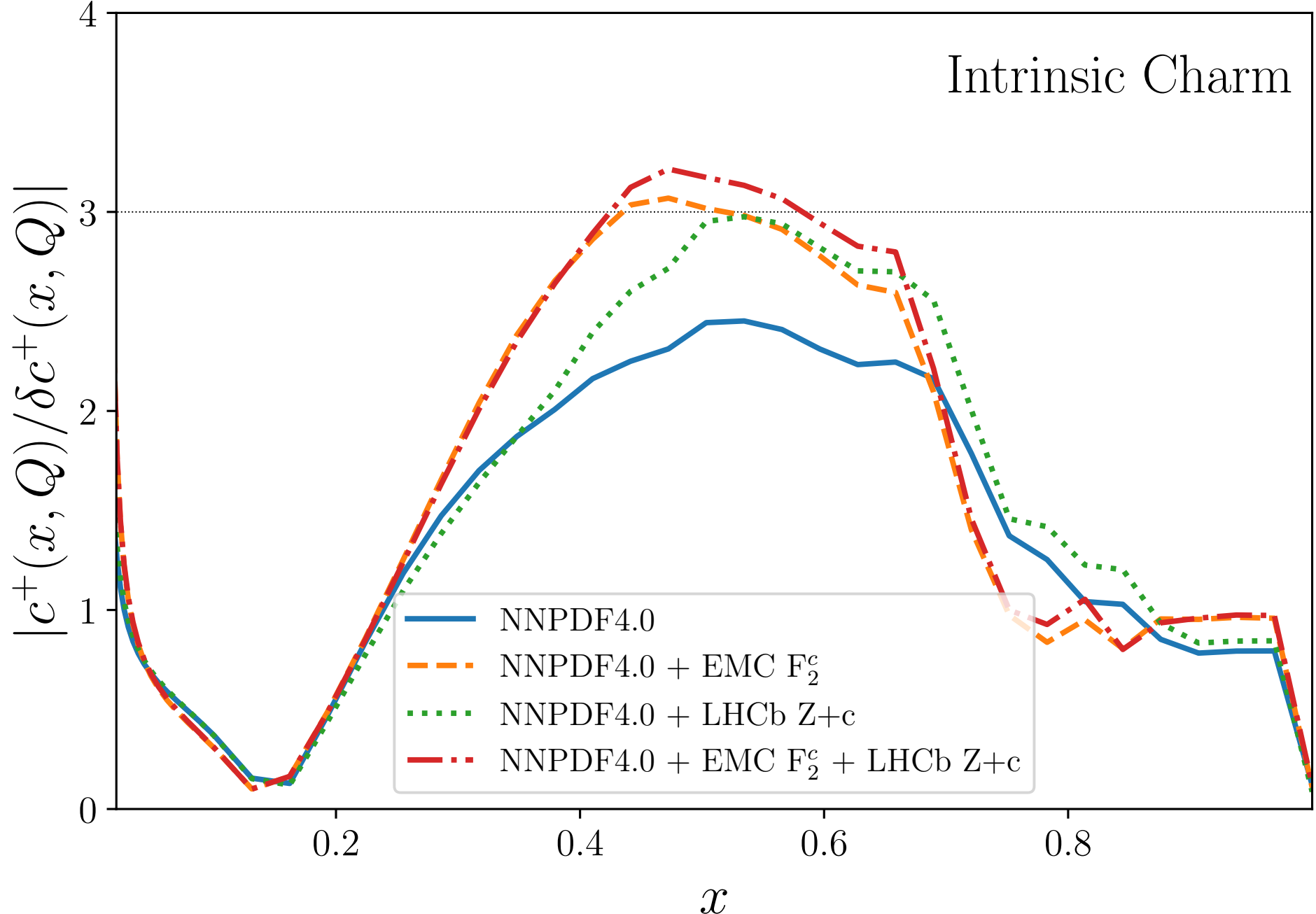
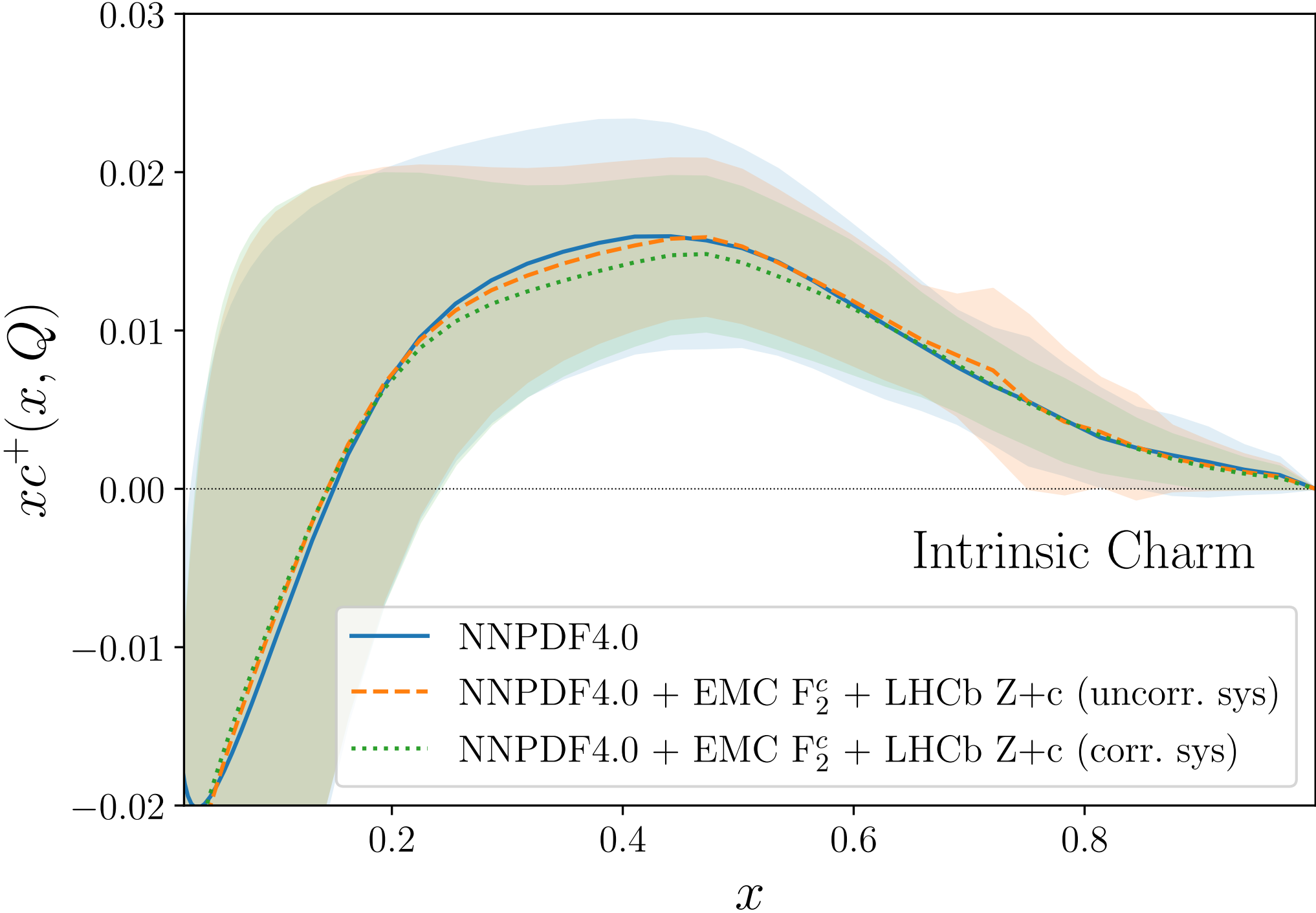
We compute:

- **local significance:**

- **momentum fraction:**

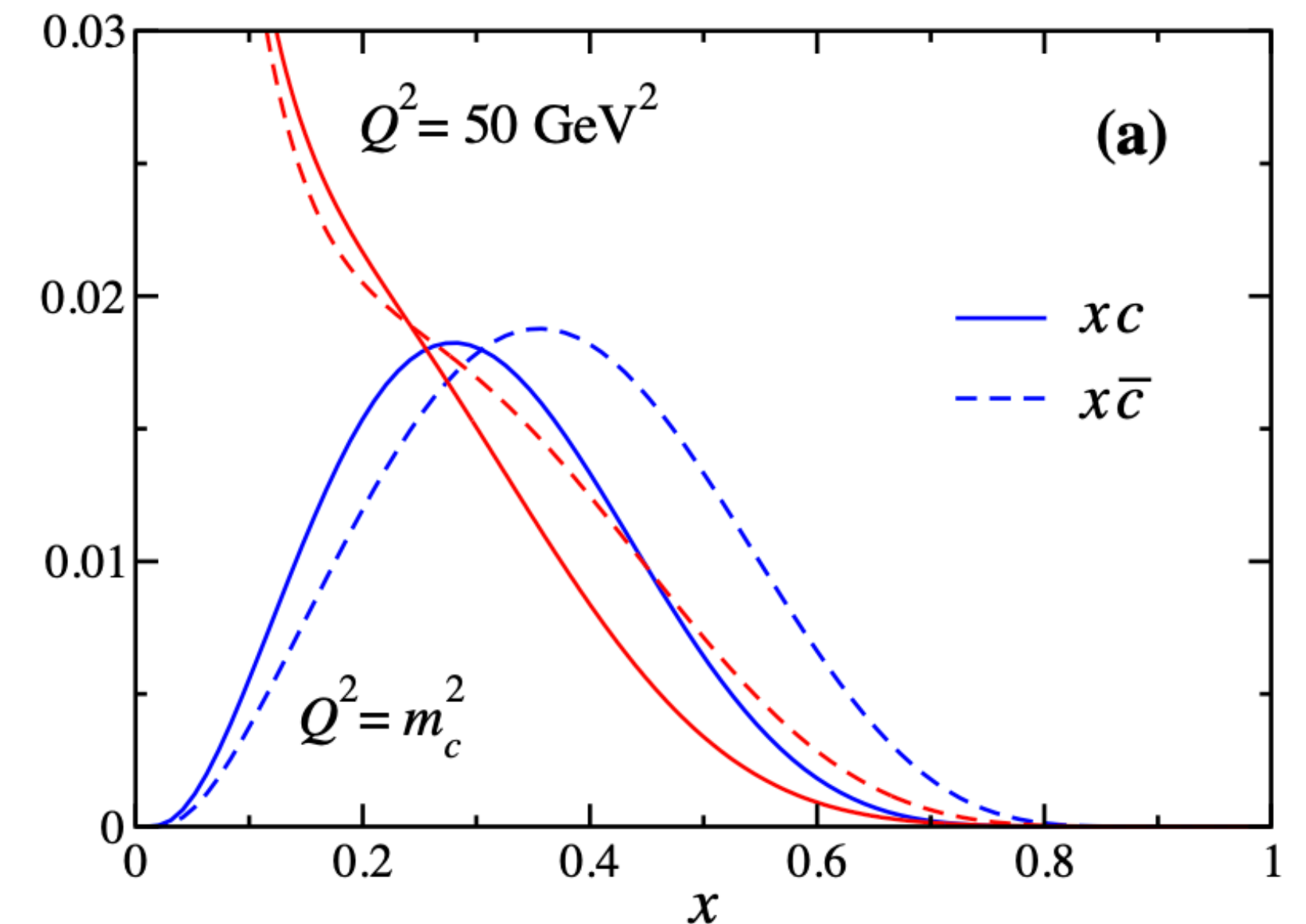
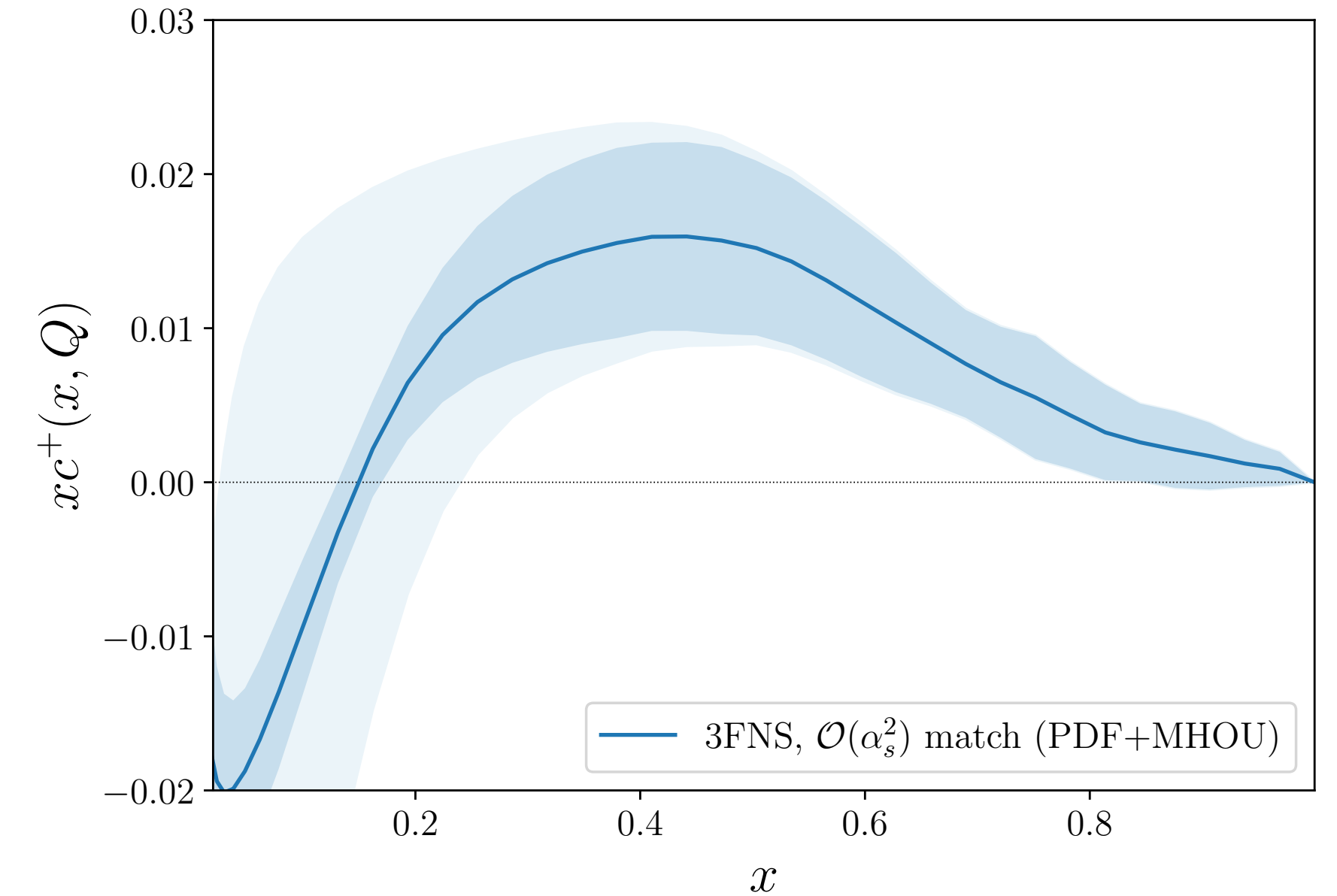
$$[c] = \int_0^1 xc^+(x, Q^2) dx$$

m_c	Dataset	$[c](Q)$ (%)
1.51 GeV	Baseline	$0.62 \pm 0.28_{\text{pdf}} \pm 0.54_{\text{mhou}}$
1.51 GeV	Baseline+EMC	$0.60 \pm 0.18_{\text{pdf}} \pm 0.54_{\text{mhou}}$
1.51 GeV	Baseline+EMC+LHBCb Zc	$0.60 \pm 0.17_{\text{pdf}} \pm 0.59_{\text{mhou}}$
1.38 GeV	Baseline	$0.47 \pm 0.27_{\text{pdf}} \pm 0.62_{\text{mhou}}$
1.64 GeV	Baseline	$0.77 \pm 0.28_{\text{pdf}} \pm 0.48_{\text{mhou}}$



Summary and outlook

- Evidence of a non zero intrinsic charm c^+ in $n_f = 3$, carrying a momentum fraction total within 1%.
- Our intrinsic charm is in agreement with most recent LHC results.
- Impact of the N3LO matching conditions is relevant. Need to properly quantify MHOUs and possibly move towards N3LO pdf fit.
- Is it worth to try a fit with independent c^- parametrisation as suggested by the Meson/Baryon models?



From [\[arxiv:1311.1578\]](https://arxiv.org/abs/1311.1578)

*“My drawing was not a picture of a hat. It was a picture of a
boa constrictor digesting an elephant.”*



Thank you!

Backup slides

From 4FNS to 3FNS

The matching conditions

$$\mathbf{A}^{(n_f)}(\mu_h^2) = \mathbf{I} + a_s^{(n_f)}(\mu_h^2)\mathbf{A}^{(n_f),(1)} + a_s^{(n_f),2}(\mu_h^2)\mathbf{A}^{(n_f),(2)} + a_s^{(n_f),3}(\mu_h^2)\mathbf{A}^{(n_f),(3)} + \mathcal{O}(\alpha_s^4)$$

NLO

$$\mathbf{A}_{S,h^+}^{(n_f),(1)} = \begin{pmatrix} A_{gg,H}^{S,(1)} & 0 & A_{gH}^{S,(1)} \\ 0 & 0 & 0 \\ A_{Hg}^{S,(1)} & 0 & A_{HH}^{(1)} \end{pmatrix}$$

NNLO

$$\mathbf{A}_{S,h^+}^{(n_f),(2)} = \begin{pmatrix} A_{gg,H}^{S,(2)} & A_{gq,H}^{S,(2)} & 0 \\ 0 & A_{qq,H}^{ns,(2)} & 0 \\ A_{Hg}^{S,(2)} & A_{Hq}^{ps,(2)} & 0 \end{pmatrix}$$

N3LO

$$\mathbf{A}_{S,h^+}^{(n_f),(3)} = \begin{pmatrix} A_{gg,H}^{S,(3)} & A_{gq,H}^{S,(3)} & 0 \\ A_{qg,H}^{S,(3)} & A_{qq,H}^{ns,(3)} + A_{qq,H}^{ps,(3)} & 0 \\ A_{Hg}^{S,(3)} & A_{Hq}^{ps,(3)} & 0 \end{pmatrix}$$

$$\mathbf{A}_{nsv,h^-}^{(n_f),(1)} = \begin{pmatrix} 0 & 0 \\ 0 & A_{HH}^{(1)} \end{pmatrix}$$

$$\mathbf{A}_{nsv,h^-}^{(n_f),(2)} = \begin{pmatrix} A_{qq,H}^{ns,(2)} & 0 \\ 0 & 0 \end{pmatrix}$$

$$\mathbf{A}_{nsv,h^-}^{(n_f),(3)} = \begin{pmatrix} A_{qq,H}^{ns,(3)} & 0 \\ 0 & 0 \end{pmatrix}$$

Inversion can be computed exactly or expanding in α_s :

$$\mathbf{A}_{exp}^{-1}(\mu_h^2) = \mathbf{I} - a_s(\mu_h^2)\mathbf{A}^{(1)} + a_s^2(\mu_h^2)\left[\mathbf{A}^{(2)} - (\mathbf{A}^{(1)})^2\right] + a_s^3(\mu_h^2)\left[-\mathbf{A}^{(3)} + 2\mathbf{A}^{(1)}\mathbf{A}^{(2)} - (\mathbf{A}^{(1)})^3\right] + \mathcal{O}(a_s^4)$$

3FNS stability and accuracy

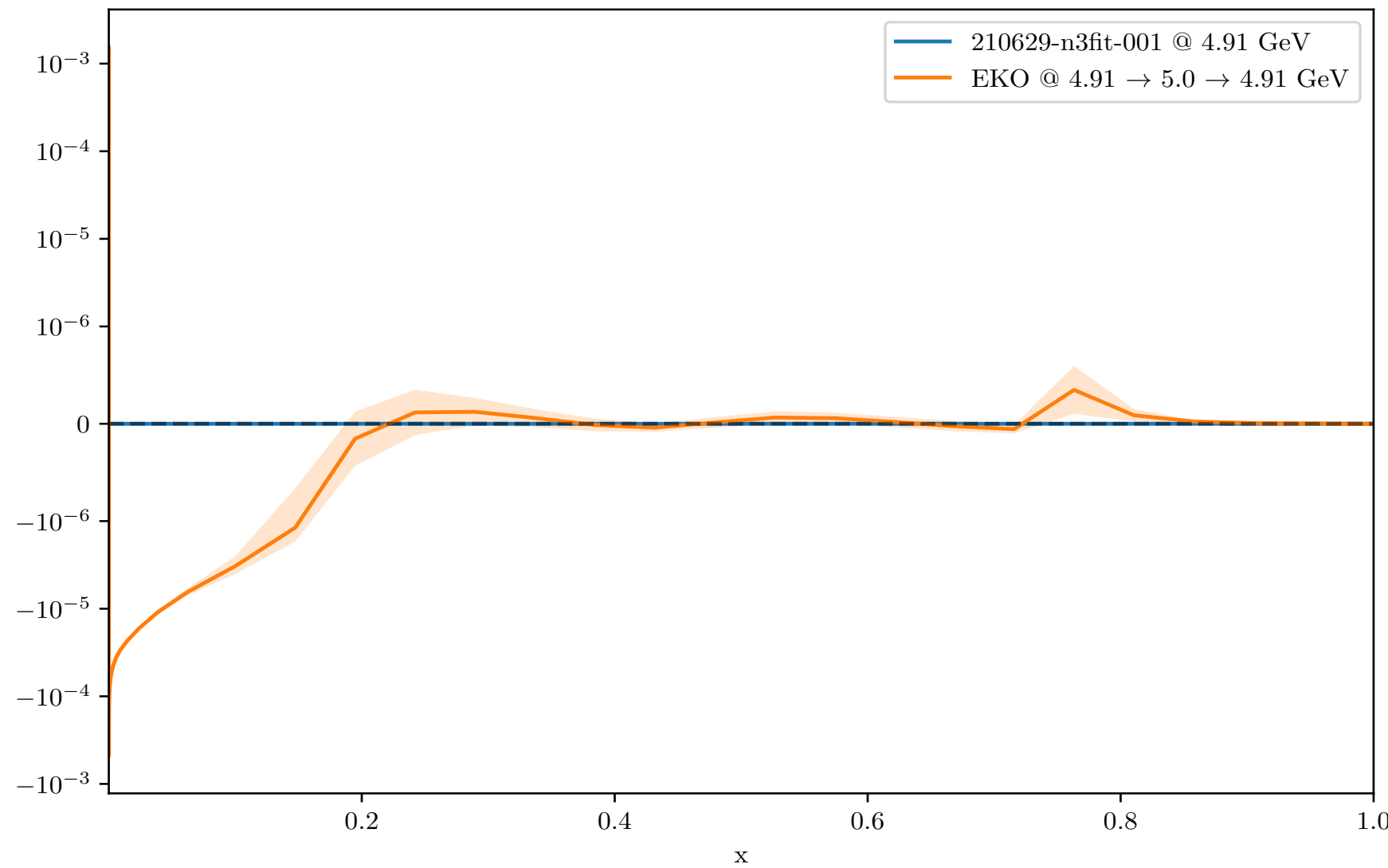
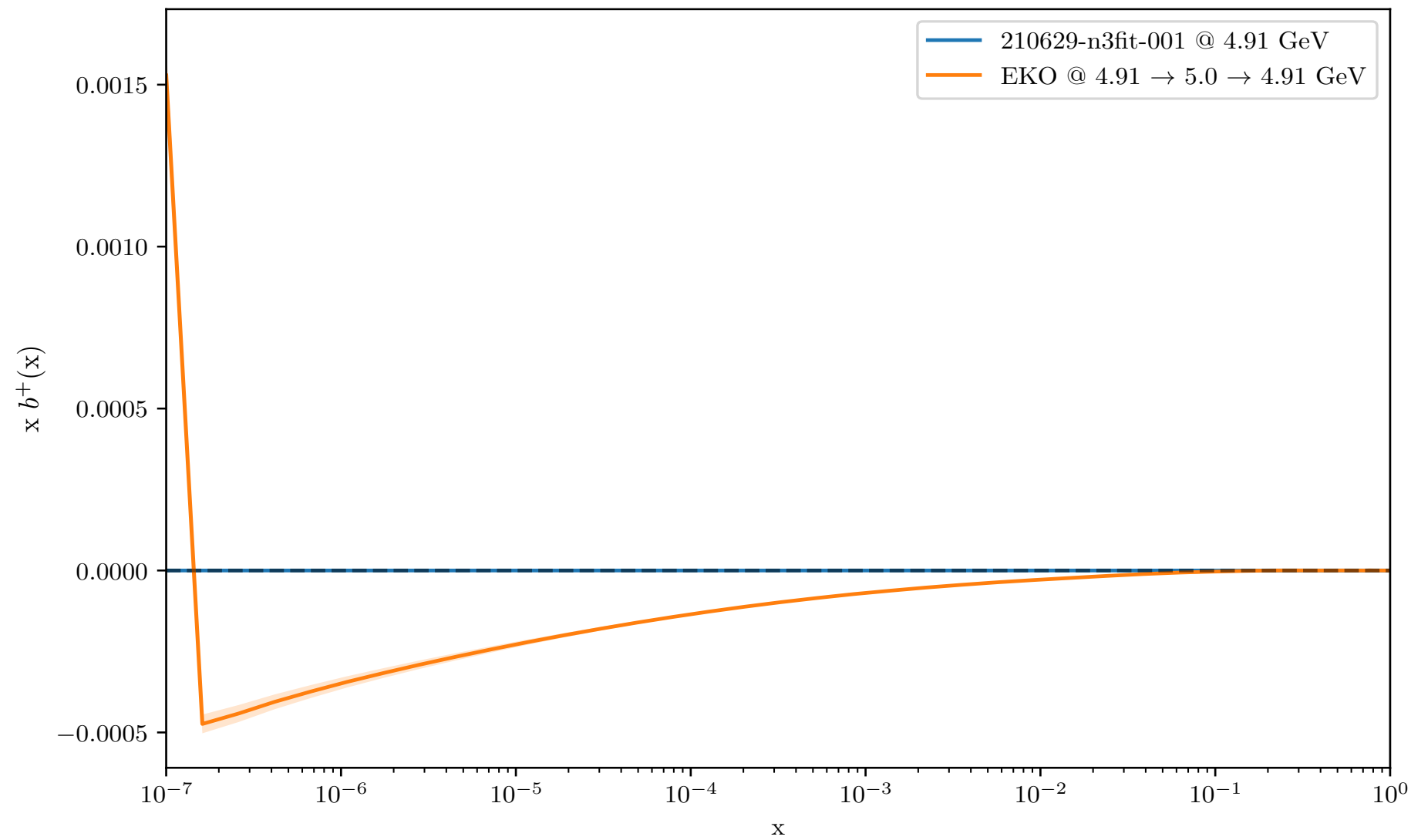
The EKO closure test

$$\tilde{\mathbf{f}}(Q^0) = \tilde{\mathbf{E}}(Q^0 \leftarrow Q) \cdot \tilde{\mathbf{E}}(Q \leftarrow Q^0) \cdot \tilde{\mathbf{f}}(Q^0) ??$$

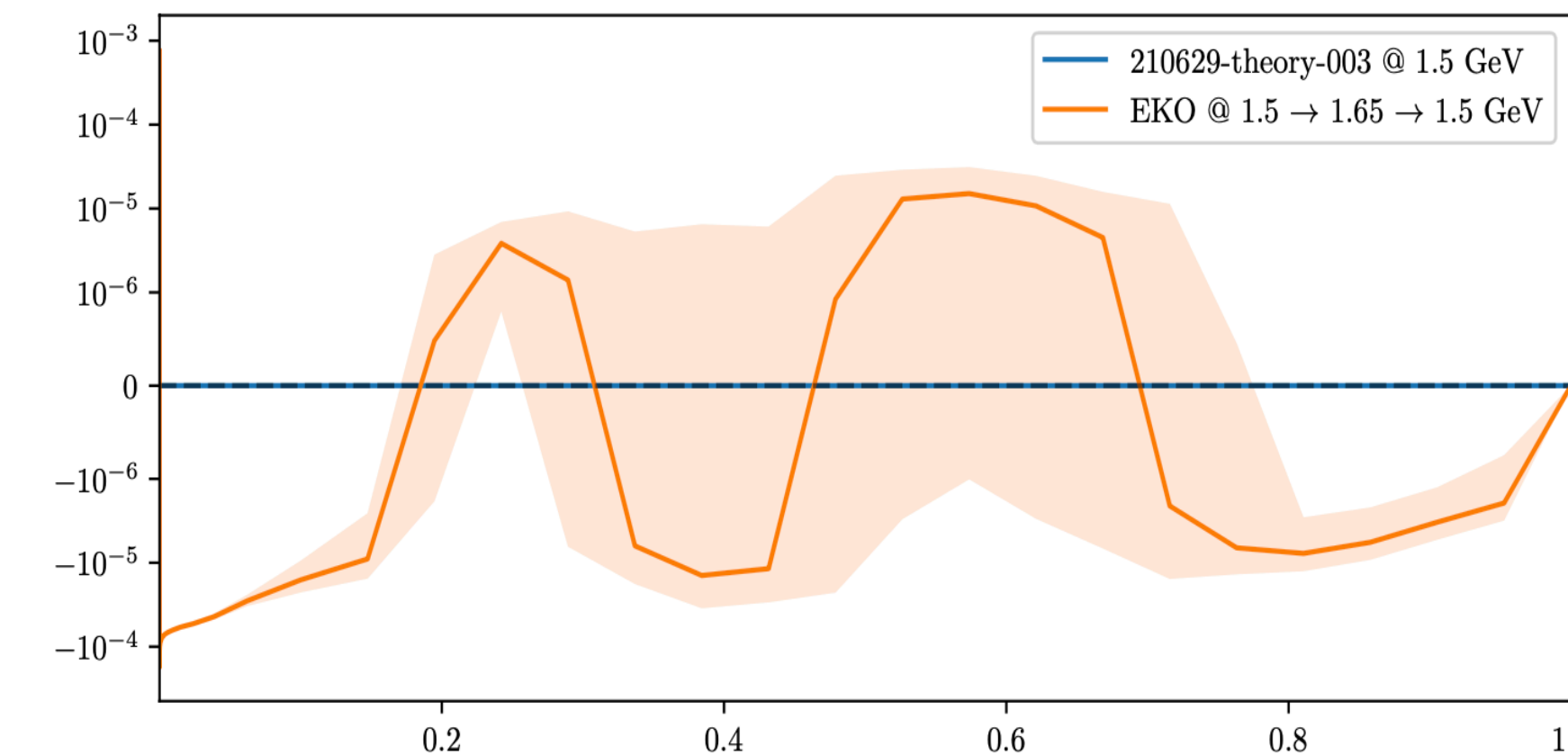
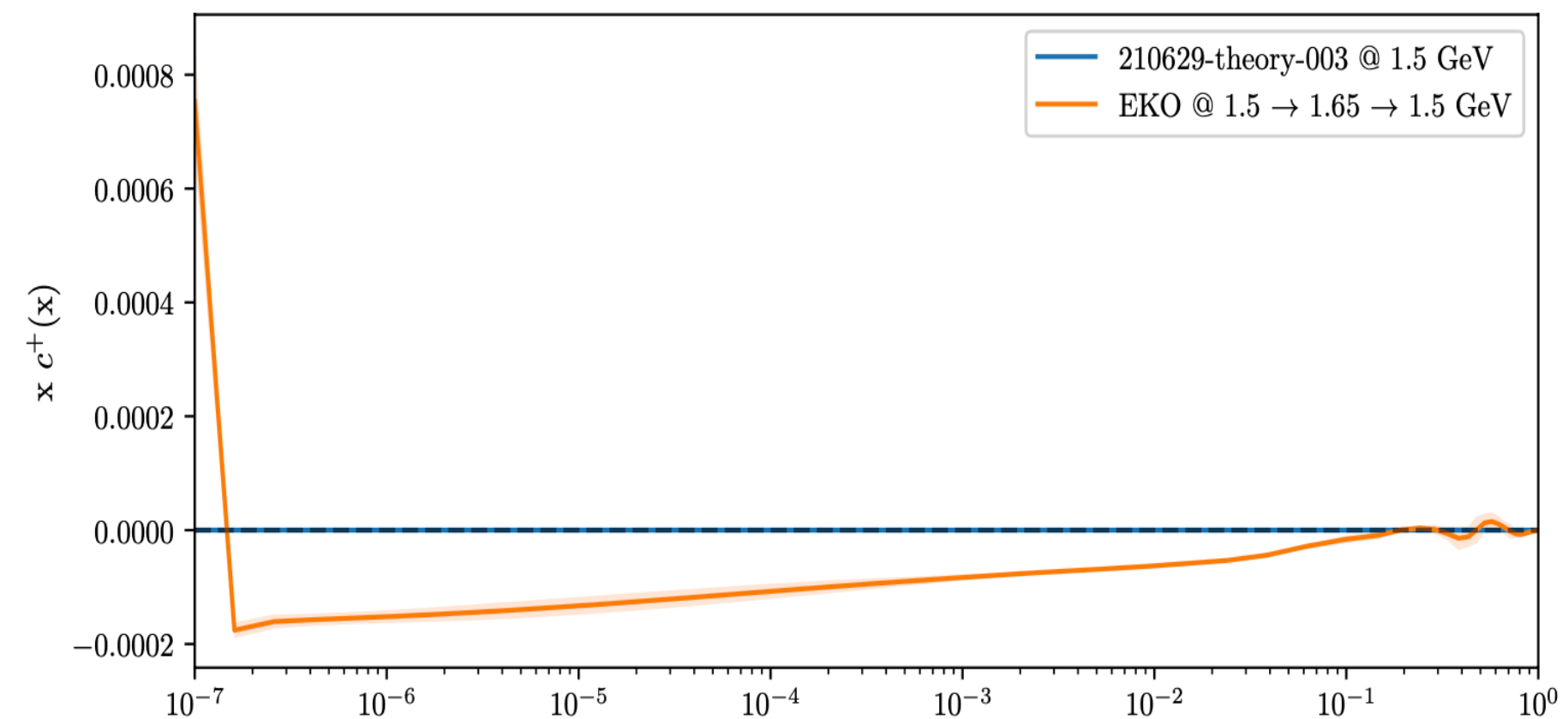
Low x

High x

$Q_0 = m_b$



$Q_0 = m_c$



Numerical accuracy is:

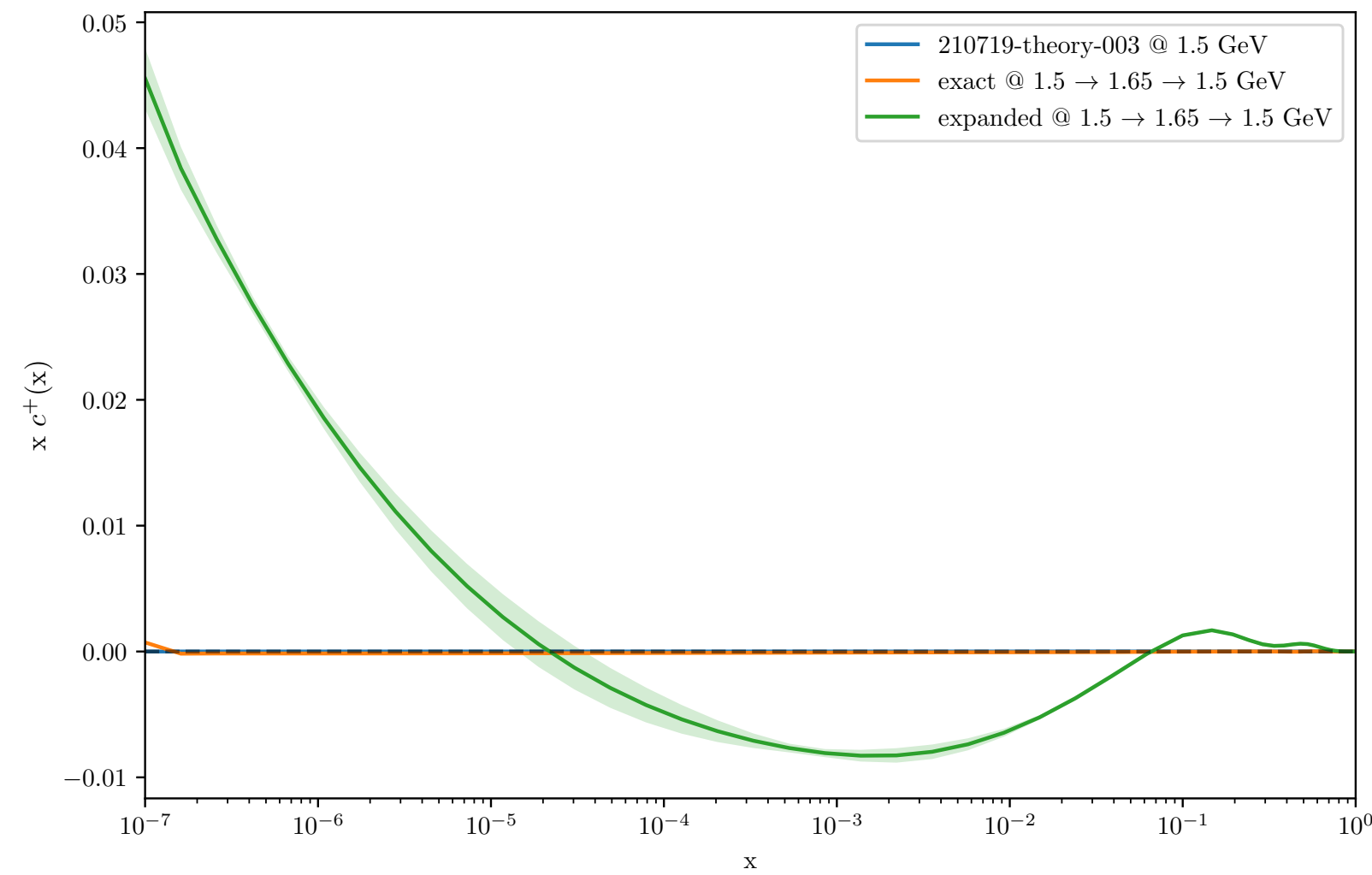
$\approx 2 \cdot 10^{-4}$ at *low x*

$\approx 10^{-5}$ for *high x* region

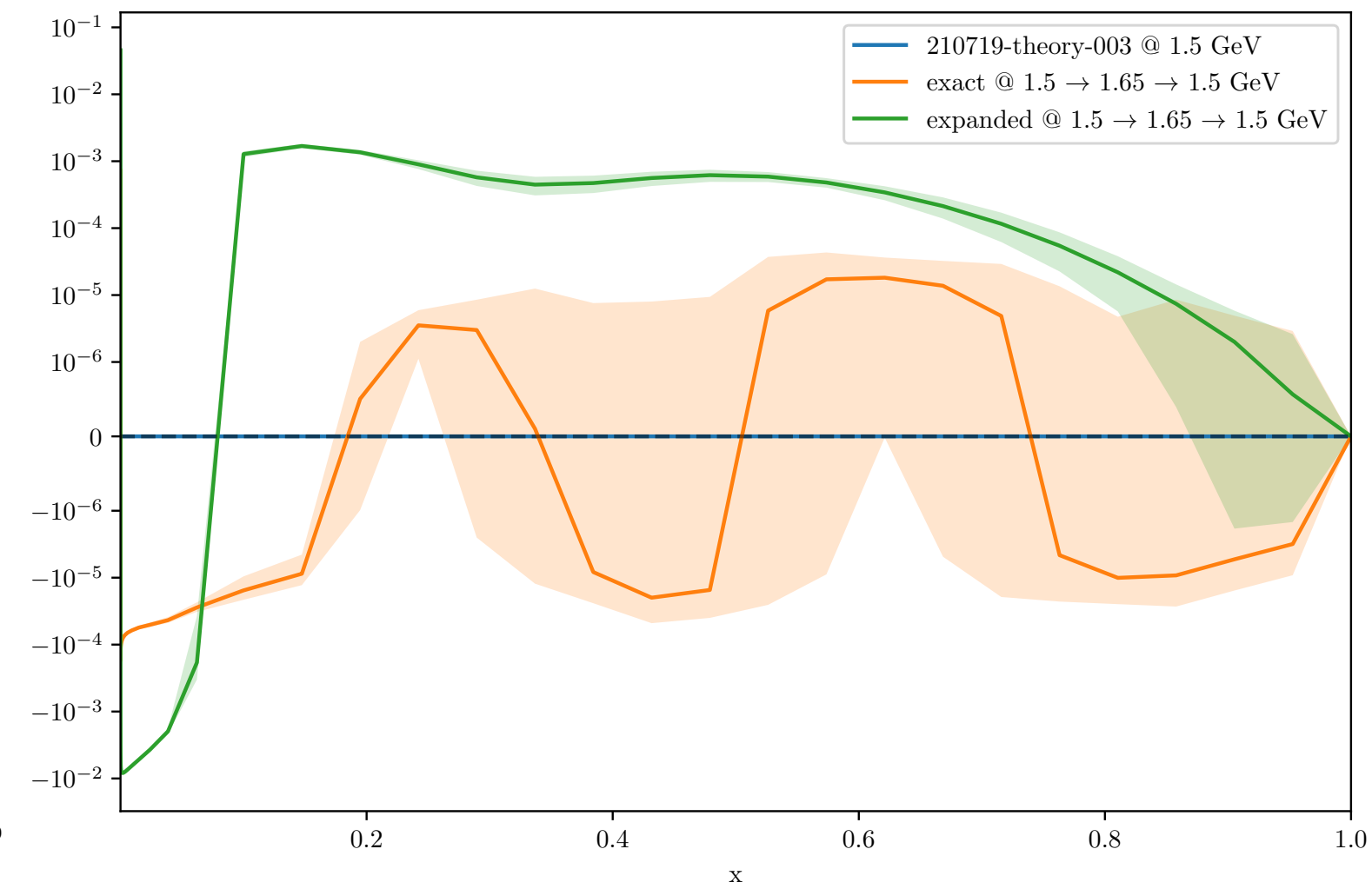
3FNS stability and accuracy

Exact vs Expanded inversion

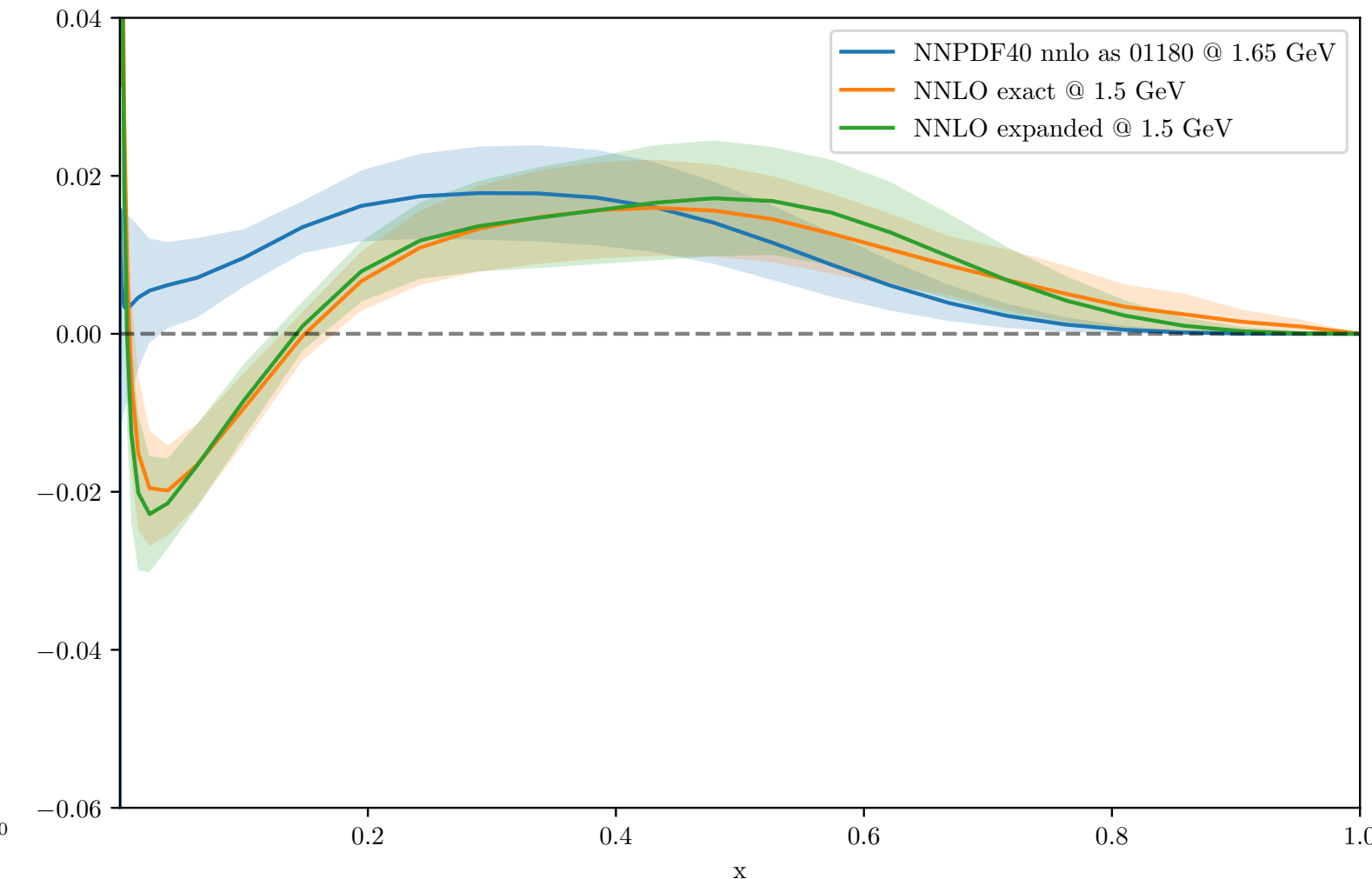
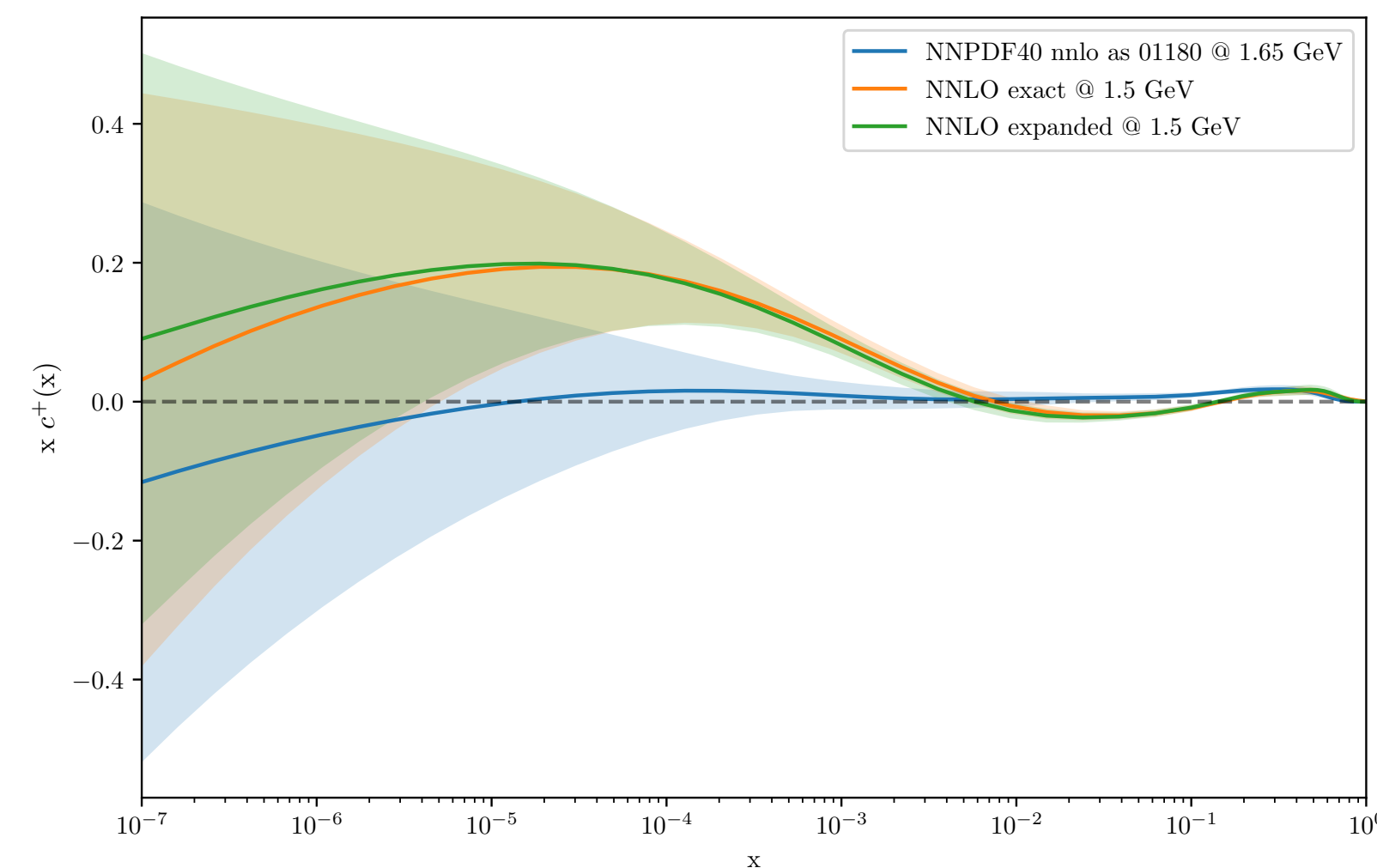
• [210719-theory-003](#) = NNPDF4.0 pch



Low x



High x



- **Exact:** the matching matrices are inverted exactly in N-space, and then integrated entry by entry
- **Expanded:** the matching matrices are inverted through a perturbative expansion in α_s before the Mellin inversion:

The Intrinsic charm

Low x

