Zero-jettiness resummation for top-quark pair production at the LHC

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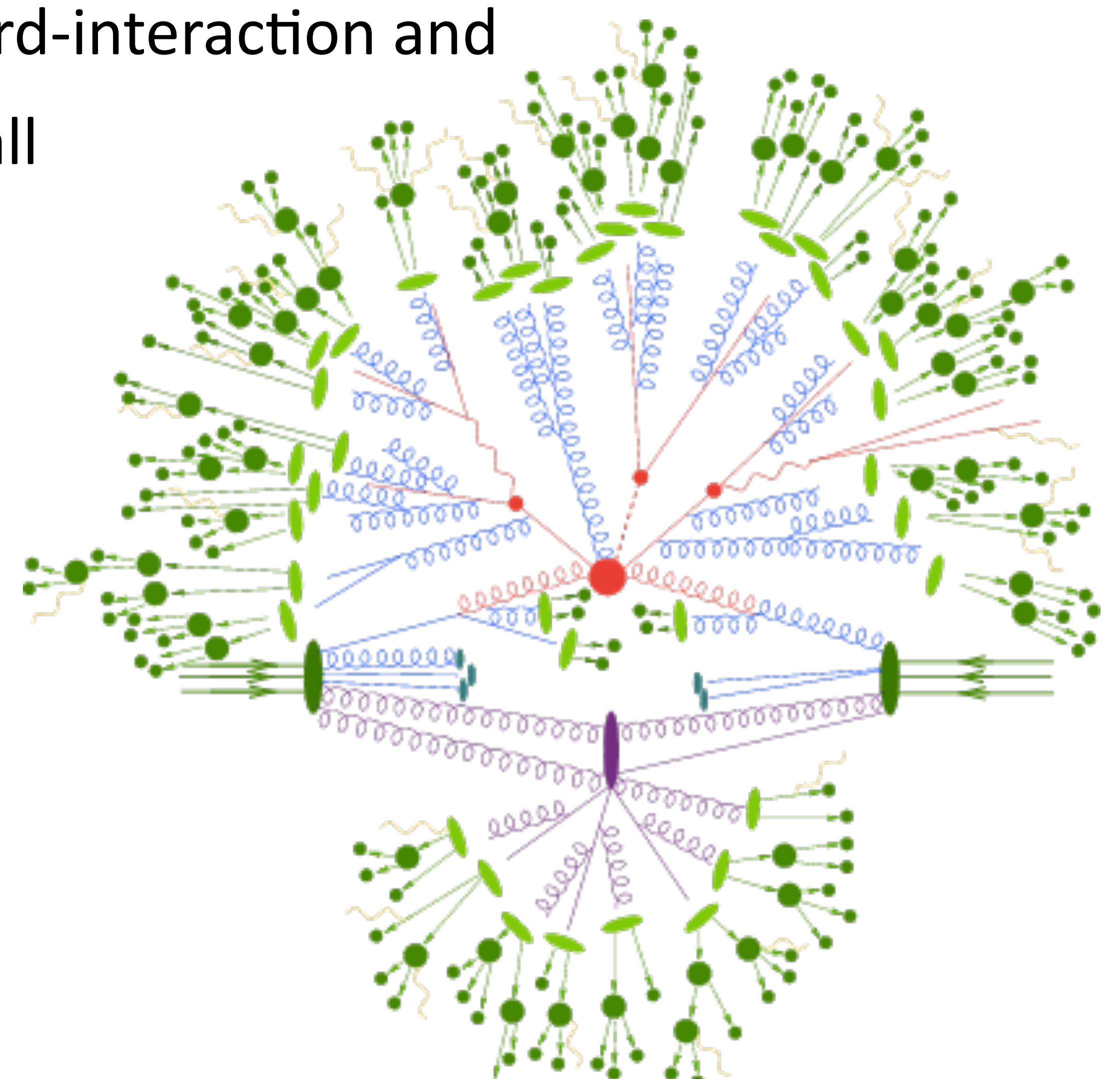
Overview of the talk

- Motivation/Introduction
- N-Jettiness
- Factorization & resummation of 0-Jettiness for top-quark pair production at the LHC
- Results
- Outlook

Based on [arXiv:2111.03632], S. Alioli, AB, M.A. Lim

Motivation

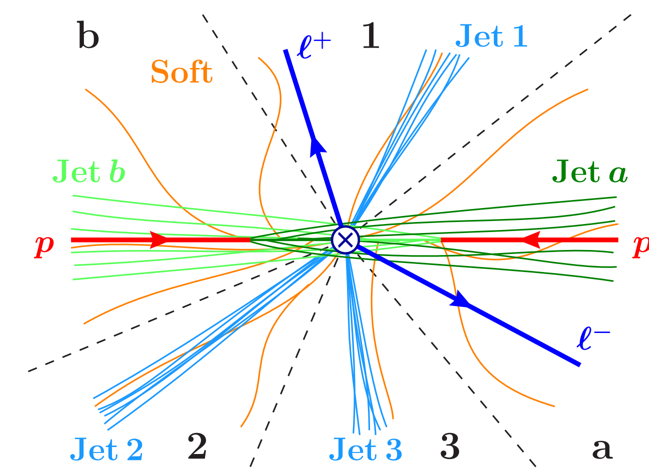
- MC event generators are essential tools for particle physics phenomenology
- They provide realistic simulations: first principles QFT calculations are combined with parton showers and hadronization modelling
- They start from a perturbative description of the hard-interaction and predict the evolution of the event down to very small (nonperturbative) scales $\mathcal{O}(1)$ GeV
- State-of-the-art is the inclusion of partonic NNLO corrections. Several methods are available for colour-singlet processes (UNNLOPS, MiNNLOPS, GENEVA)



N-Jettiness and Factorization

- N-jettiness resolution variables: given an M-particle phase space point with $M \geq N$

$$\mathcal{T}_N(\Phi_M) = \sum_k \min\{\hat{q}_a \cdot p_k, \hat{q}_b \cdot p_k, \hat{q}_1 \cdot p_k, \ldots, \hat{q}_N \cdot p_k\}$$



- The limit $\mathcal{T}_N \to 0$ describes a N-jet event where the unresolved emissions be either soft or collinear to the final state jets or initial state beams

- Color singlet final state, relevant variable is 0-jettiness aka “beam thrust”

$$\mathcal{T}_0 = \sum_k |\vec{p}_{kT}|\, e^{-|\eta_k - Y|}$$

- Cross section factorizes in the limit $\mathcal{T}_0 \to 0$ [Stewart, Tackmann,Waalewijn `09,`10], three different scales arise

$$\mu_H = Q, \quad \mu_B = \sqrt{Q\mathcal{T}_0}, \quad \mu_S = \mathcal{T}_0$$

$$\frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0 d\mathcal{T}_0} = \sum_{ij} \underbrace{H_{ij}^{\gamma\gamma}(Q^2, t, \mu_H)}_{\text{NNLO}} U_H(\mu_H, \mu) \left\{ \left[\underbrace{B_i(t_a, x_a, \mu_B)}_{\text{NNLO}} \otimes U_B(\mu_B, \mu) \right] \right.$$
$$\left. \times \left[B_j(t_b, x_b, \mu_B) \otimes U_B(\mu_B, \mu) \right] \right\} \otimes \left[\underbrace{S(\mu_s)}_{\text{NNLO}} \otimes U_S(\mu_S, \mu) \right]$$

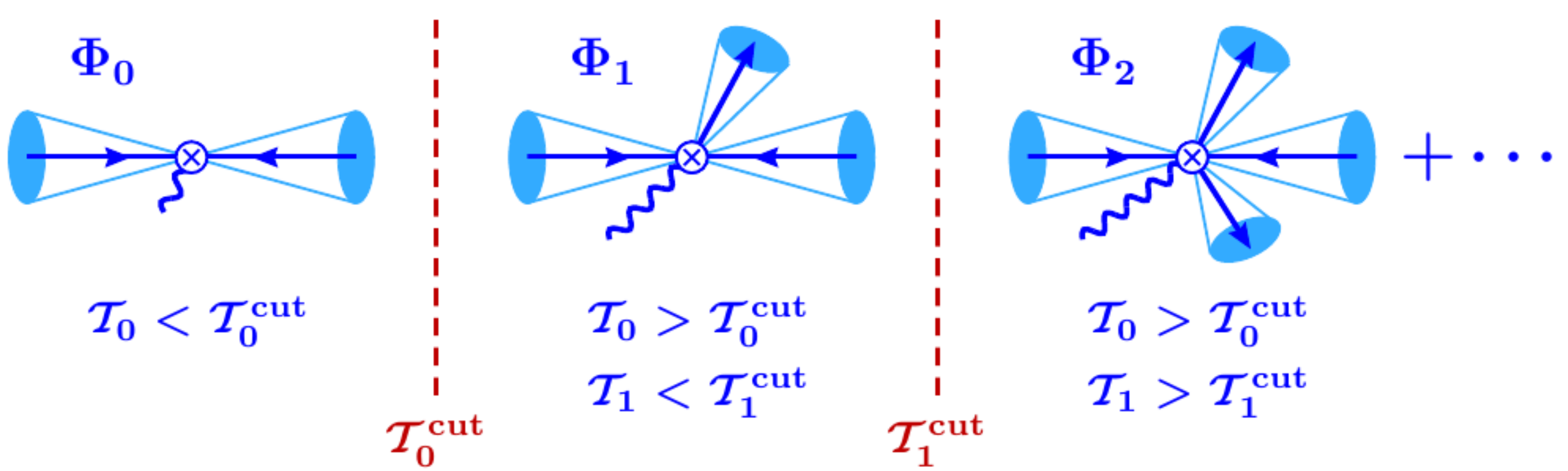
Monte Carlo implementation

- GENEVA [Alioli,Bauer,Berggren,Tackmann, Walsh `15], [Alioli,Bauer,Tackmann,Guns `16], [Alioli,Broggio,Lim, Kallweit,Rottoli `19],[Alioli,Broggio,Gavardi,Lim,Nagar,Napoletano,Kallweit,Rottoli `20-`21] combines 3 theoretical tools that are important for QCD predictions into a single framework
  - fully differential fixed-order calculations, up to NNLO via 0-jettiness or q_T subtraction
  - up to NNLL` resummation for 0-jettiness in SCET or N^{3}LL for q_T via RadISH for colour singlet processes
  - shower and hadronize events (PYTHIA8)
- IR-finite definition of events based on resolution parameters $\mathcal{T}_0^{\mathrm{cut}}$ and $\mathcal{T}_1^{\mathrm{cut}}$

$$\Phi_0 \text{ events:} \quad \frac{d\sigma_0^{\mathrm{MC}}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}),$$

$$\Phi_1 \text{ events:} \quad \frac{d\sigma_1^{\mathrm{MC}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}}),$$

$$\Phi_2 \text{ events:} \quad \frac{d\sigma_{\geq 2}^{\mathrm{MC}}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}})$$



- When we take $\mathcal{T}_N^{\mathrm{cut}} \to 0$, large logarithms of $\mathcal{T}_N^{\mathrm{cut}}$, $\mathcal{T}_N$ appear and need to be resummed
- Including the higher-order resummation will improve the accuracy of the predictions across the whole spectrum

0-jettiness resummation for $t\bar{t}$ production

- Top-quark properties are very interesting, interplay with the Higgs sector

- It is desirable to have a NNLO+PS calculation. Extrapolation from fiducial to inclusive phase space is done using NLO event generators, see for example [Behring, Czakon, Mitov, Papanastasiou, Poncelet `19])

- Recently, NNLO+PS for $t\bar{t}$ production available via MINNLOPS formalism [Mazzitelli, Monni, Nason, Re, Wiesemann, Zanderighi `20,`21]

- Including higher-order resummation can improve the description of observables (this is the case of the GENEVA generator)

0-jettiness resummation for $t\bar{t}$ production

- To reach NNLO+PS accuracy in GENEVA
  - NLO calculations for $t\bar{t}$ and $t\bar{t}$+jet
  - Resummed calculation at NNLL` in the resolution variable $\mathcal{T}_0$
  - q_T resummation via SCET (NNLL in 1307.2464) or direct QCD [1408.4564], [1806.01601] NNLL' ingredients (soft functions) in [1901.04005], [Angeles-Martinez, Czakon, Sapeta 1809.01459] but they are not publicly available
  - 0-jettines resummation is used for colour-singlet in GENEVA, must be extended for $t\bar{t}$ production
  - Definition of 0-jettiness has to be adapted with top-quarks in the final state, we choose to treat them like EW particles and exclude them from the sum over radiation
  - We first need to develop the resummation framework

Factorization

We derived a factorization formula (see 2111.03632 Appendix A) using SCET+HQET in the region where $M_{t\bar{t}} \sim m_t \sim \sqrt{\hat{s}}$ are all hard scales. In case of boosted regime $M_{t\bar{t}} \gg m_t$ situation similar to [Fleming, Hoang,Mantry,Stewart `07][Bachu,Hoang,Mateu,Pathak,Stewart `21]

Hard functions (color matrices)

$$\frac{d\sigma}{d\Phi_0 d\tau_B} = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \int dt_a\, dt_b \boxed{B_i(t_a, z_a, \mu)\, B_j(t_b, z_b, \mu)}\, \mathrm{Tr}\left[\boxed{\mathbf{H}_{ij}(\Phi_0, \mu)}\, \boxed{\mathbf{S}_{ij}\left(M\tau_B - \frac{t_a + t_b}{M}, \Phi_0, \mu\right)}\right]$$

Beam functions [Stewart, Tackmann, Waalewijn, [1002.2213], known up to N^{3}LO

Soft functions (color matrices)

It is convenient to transform the soft and beam functions in Laplace space to solve the RG equations, the factorization formula is turn into a product of (matrix) functions

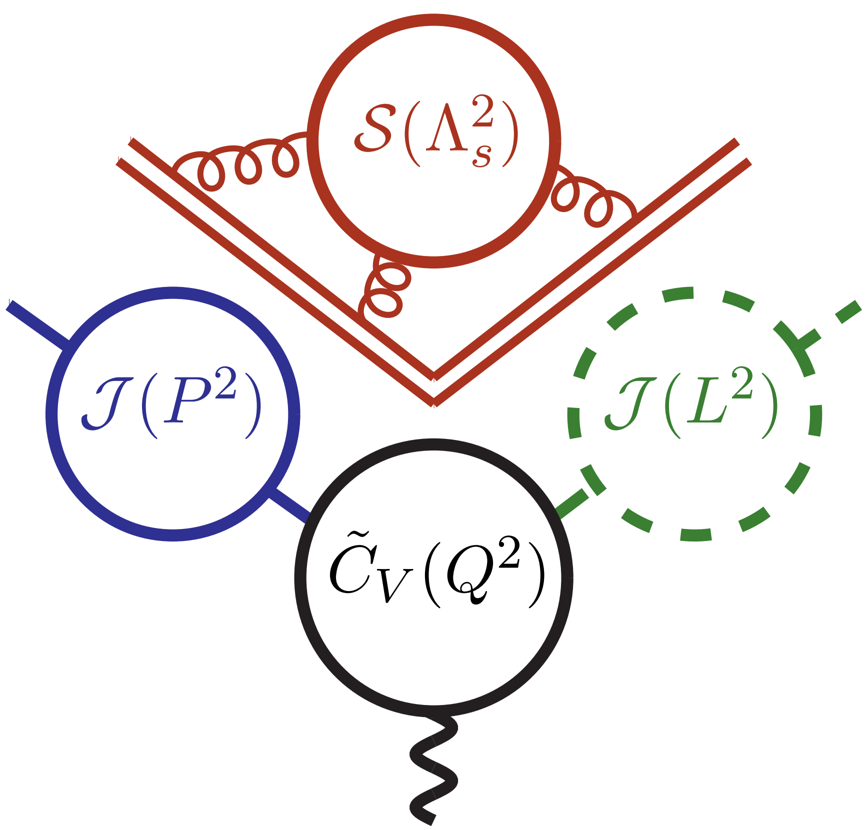
$$\mathscr{L}\left[\frac{d\sigma}{d\Phi_0 d\tau_B}\right] = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \tilde{B}_i\left(\ln\frac{M\kappa}{\mu^2}, z_a\right) \tilde{B}_j\left(\ln\frac{M\kappa}{\mu^2}, z_b\right) \mathrm{Tr}\left[\mathbf{H}_{ij}\left(\ln\frac{M^2}{\mu^2}, \Phi_0\right) \tilde{\mathbf{S}}_{ij}\left(\ln\frac{\mu^2}{\kappa^2}, \Phi_0\right)\right]$$

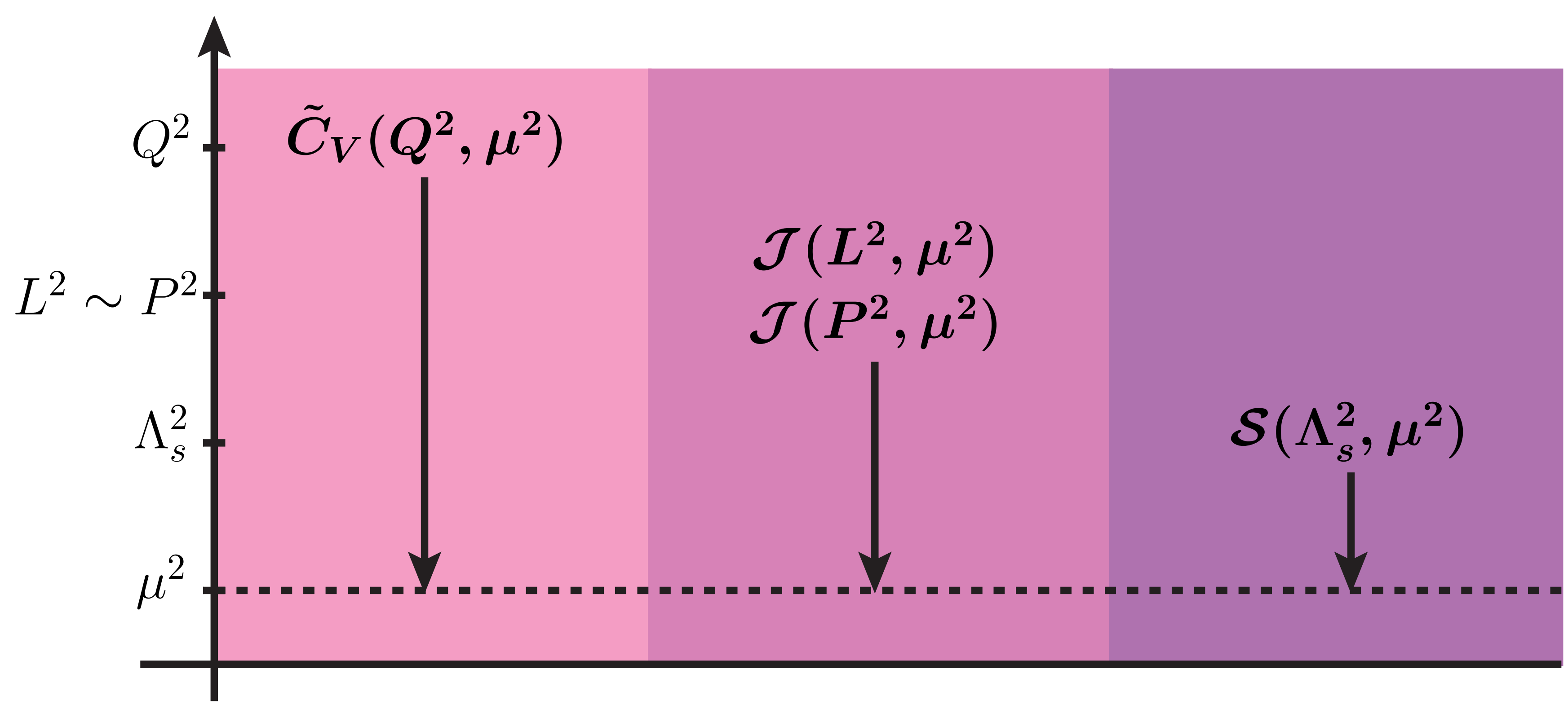
Resummation

- Resummation program in EFT schematically

$$L \equiv \ln\left(\frac{\text{“hard” scale}}{\text{“soft” scale}}\right)$$

  - separation of scales (factorization formula)
  - evaluate each single scale factor in fixed order perturbation theory at a scale for which it is free of large logs
  - use Renormalization Group (RG) equations to evolve the factors to a common scale





Hard functions

The hard functions arise from matching the full theory onto the EFT, they can be extracted from colour decomposed loop amplitudes. At NLO it was first computed in [Ahrens, Ferroglia, Neubert, Pecjak, Yang, 1003.5827]. They satisfy the RG equations

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{\Gamma}_H(M,\beta_t,\theta,\mu)\,\mathbf{H}(M,\beta_t,\theta,\mu) + \mathbf{H}(M,\beta_t,\theta,\mu)\,\mathbf{\Gamma}_H^\dagger(M,\beta_t,\theta,\mu)$$

Solution:

$$\mathbf{H}(M,\beta_t,\theta,\mu) = \mathbf{U}(M,\beta_t,\theta,\mu_h,\mu)\mathbf{H}(M,\beta_t,\theta,\mu_h)\mathbf{U}^\dagger(M,\beta_t,\theta,\mu_h,\mu)$$

$$\boxed{\mathbf{U}(M,\beta_t,\theta,\mu_h,\mu) = \exp\left[2S(\mu_h,\mu) - a_\Gamma(\mu_h,\mu)\left(\ln\frac{M^2}{\mu_h^2} - i\pi\right)\right]\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu)}$$

We have split the anomalous dimension into a cusp (diagonal in colour space) and non-cusp (not diagonal) part

$$\mathbf{\Gamma}_H(M,\beta_t,\theta,\mu) = \Gamma_{\text{cusp}}(\alpha_s)\left(\ln\frac{M^2}{\mu^2} - i\pi\right) + \boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha_s)$$ [Ferroglia, Neubert, Pecjak, Yang,`09]

$$\mathbf{u}(M,\beta_t,\theta,\mu_h,\mu) = \mathcal{P}\exp\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu)}\frac{\mathrm{d}\alpha}{\beta(\alpha)}\boldsymbol{\gamma}^h(M,\beta_t,\theta,\alpha)$$

We evaluate the matrix exponential **u** as a series expansion in α_s [Buchalla,Buras,Lautenbacher `96]

Soft functions

We computed the soft functions matrices at NLO which were unknown for this observable

$$\mathbf{S}^{(1)}_{\text{bare},\,ij}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu) = \sum_{\alpha,\,\beta} \boldsymbol{w}_{ij}^{\alpha\beta}\, \hat{\mathcal{I}}_{\alpha\beta}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu)$$

$$\hat{\mathcal{I}}_{\alpha\beta}(k_a^+, k_b^+, \beta_t, \theta, \epsilon, \mu) = -\frac{2(\mu^2 e^{\gamma_E})^\epsilon}{\pi^{1-\epsilon}} \int \mathrm{d}^d k \frac{v_\alpha \cdot v_\beta}{v_\alpha \cdot k\, v_\beta \cdot k}\, \delta(k^2)\Theta(k^0)$$
$$\times \left[\delta(k_a^+ - k\cdot n_a)\Theta(k\cdot n_b - k\cdot n_a)\,\delta(k_b^+) + \delta(k_b^+ - k\cdot n_b)\Theta(k\cdot n_a - k\cdot n_b)\,\delta(k_a^+)\right]$$

One can average over the two hemisphere momenta, the soft functions satisfies the RG equation in Laplace space

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) = \left[\Gamma_{\text{cusp}} L - \boldsymbol{\gamma}^{s\dagger}\right]\tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu) + \tilde{\mathbf{S}}_B(L, \beta_t, \theta, \mu)\left[\Gamma_{\text{cusp}} L - \boldsymbol{\gamma}^{s}\right]$$

Solution in momentum space, where we used the consistency relation among anomalous dimensions $\boldsymbol{\gamma}^s = \boldsymbol{\gamma}^h + \gamma^B\,\mathbf{1}$

$$\mathbf{S}_B(l^+, \beta_t, \theta, \mu) = \exp\left[4S(\mu_s, \mu) + 2a_{\gamma^B}(\mu_s, \mu)\right]$$
$$\times\ \mathbf{u}^\dagger(\beta_t, \theta, \mu, \mu_s)\,\tilde{\mathbf{S}}_B(\partial_{\eta_s}, \beta_t, \theta, \mu_s)\,\mathbf{u}(\beta_t, \theta, \mu, \mu_s)\,\frac{1}{l^+}\left(\frac{l^+}{\mu_s}\right)^{2\eta_s}\frac{e^{-2\gamma_E \eta_s}}{\Gamma(2\eta_s)}$$

$$\eta_s \equiv -2a_\Gamma(\mu_s, \mu)$$

Beam functions

The beam functions are given by convolutions of perturbative kernels with the standard PDFs $f_i(x,\mu)$

$$B_i(t,z,\mu) = \sum_j \int_z^1 \frac{d\xi}{\xi} I_{ij}(t, z/\xi, \mu) f_j(\xi,\mu)$$

I_{ij} kernels are known up to N^{3}LO, process independent

RG equation in Laplace space is given by

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{B}_i(L_c,z,\mu) = \Big[-2\,\Gamma_{\text{cusp}}(\alpha_s)\,L_c + \gamma_i^B(\alpha_s)\Big]\tilde{B}_i(L_c,z,\mu)$$

with solution in momentum space

$$B(t,z,\mu) = \exp\left[-4S(\mu_B,\mu) - a_{\gamma^B}(\mu_B,\mu)\right]\tilde{B}(\partial_{\eta_B},z,\mu_B)\frac{1}{t}\left(\frac{t}{\mu_B^2}\right)^{\eta_B}\frac{e^{-\gamma_E\eta_B}}{\Gamma(\eta_B)}$$

where $\eta_B \equiv 2a_\Gamma(\mu_B,\mu)$ and the collinear log is given by $L_c = \ln\left(M\kappa/\mu^2\right)$

Resummed result for the cross section

We can combine the solutions for the hard, soft and beam functions to obtain

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0 \mathrm{d}\tau_B} = U(\mu_h, \mu_B, \mu_s, L_h, L_s)$$
$$\times \operatorname{Tr}\left\{ \mathbf{u}(\beta_t, \theta, \mu_h, \mu_s)\, \mathbf{H}(M, \beta_t, \theta, \mu_h)\, \mathbf{u}^\dagger(\beta_t, \theta, \mu_h, \mu_s)\, \tilde{\mathbf{S}}_B(\partial_{\eta_s} + L_s, \beta_t, \theta, \mu_s) \right\}$$
$$\times \tilde{B}_a(\partial_{\eta_B} + L_B, z_a, \mu_B) \tilde{B}_b(\partial_{\eta'_B} + L_B, z_b, \mu_B) \frac{1}{\tau_B^{1-\eta_{\text{tot}}}} \frac{e^{-\gamma_E \eta_{\text{tot}}}}{\Gamma(\eta_{\text{tot}})}$$

where

$$U(\mu_h, \mu_B, \mu_s, L_h, L_s) =$$
$$\exp\left[4S(\mu_h, \mu_B) + 4S(\mu_s, \mu_B) + 2a_{\gamma^B}(\mu_s, \mu_B) - 2a_\Gamma(\mu_h, \mu_B)\, L_h - 2a_\Gamma(\mu_s, \mu_B)\, L_s \right]$$

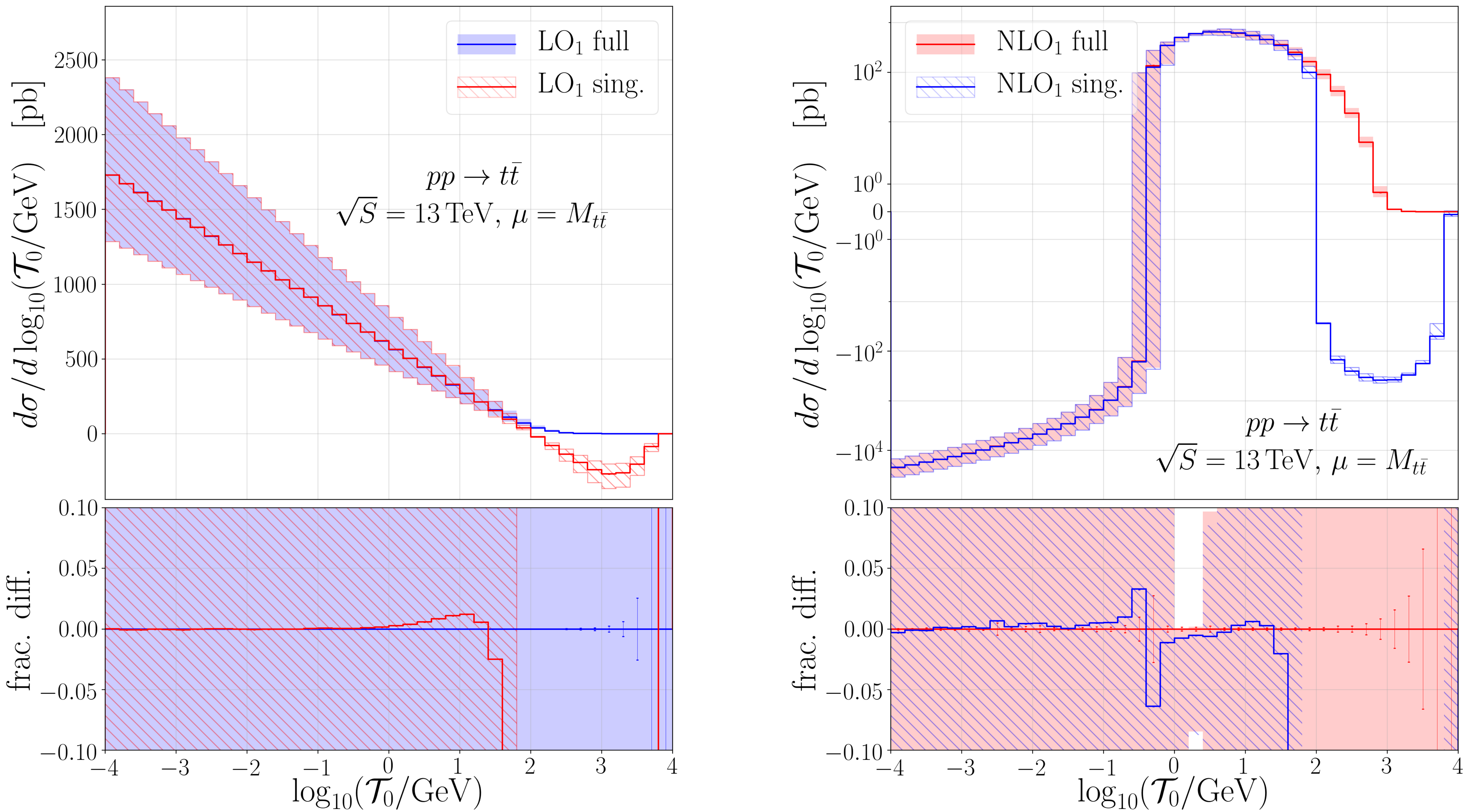
and $L_s = \ln(M^2/\mu_s^2)$, $L_h = \ln(M^2/\mu_h^2)$, $L_B = \ln(M^2/\mu_B^2)$ and $\eta_{\text{tot}} = 2\eta_S + \eta_B + \eta_{B'}$

Resummed result for the cross section

- We have
  - hard functions at NLO
  - soft functions at NLO, by knowing the two-loop soft anomalous dimensions we can solve the RG equations order by order and obtain all the NNLO logarithmic contributions, we only miss $\delta(\mathcal{T}_0)$ terms at NNLO
  - beam functions at NNLO (for initial states with quarks and gluons)
  - two-loop anomalous dimensions
- We can resum to NNLL. We are missing $\delta(\mathcal{T}_0)$ terms (NNLO hard functions and NNLO soft). If we include everything we know we obtain a $\mathrm{NNLL}'_{\mathrm{a}}$ result
- We construct an approximate (N)NLO formula which reproduces the fixed-order behaviour of the spectrum (for $\mathcal{T}_0 > 0$)

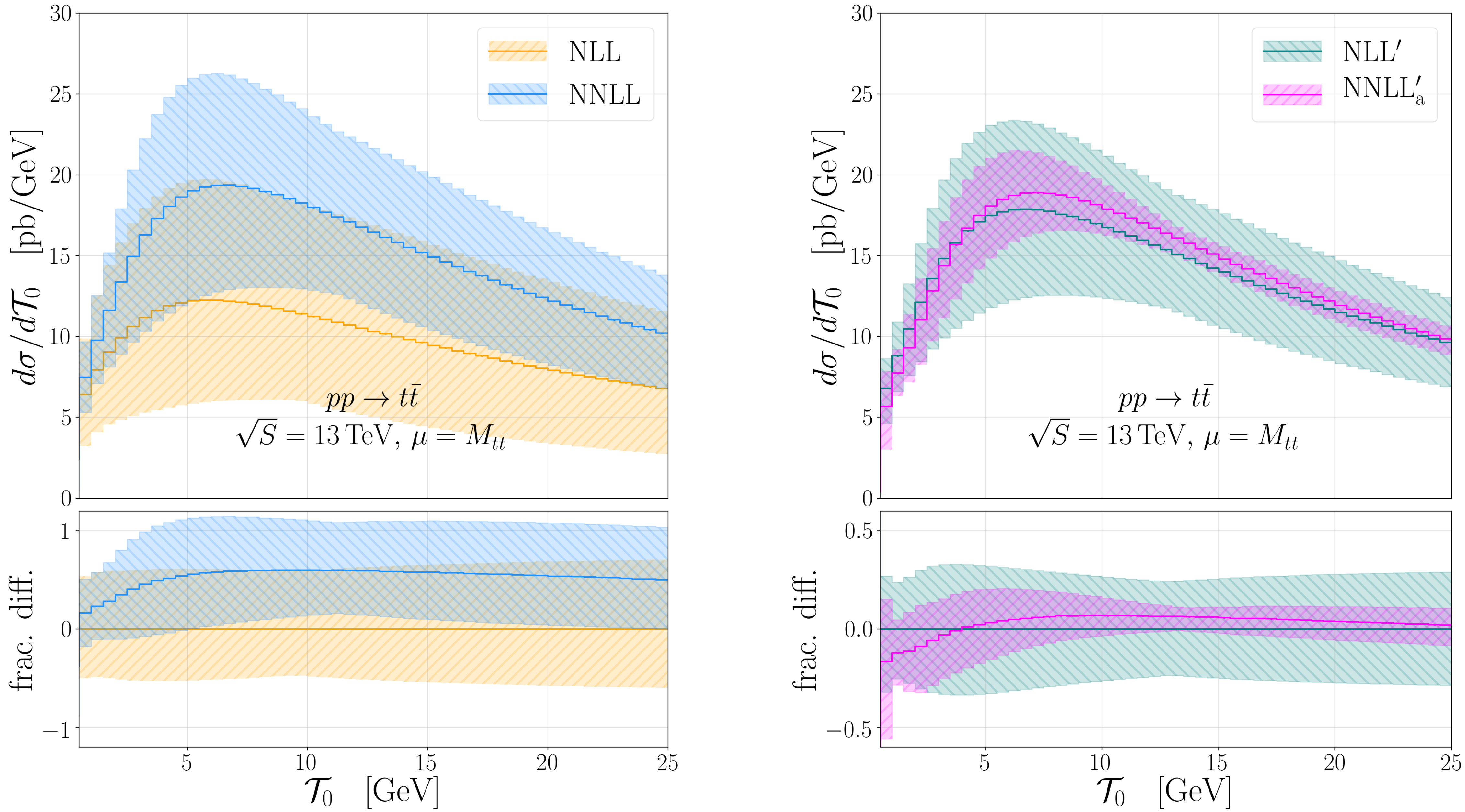
Resummed results

Fixed-order comparisons, approximate NLO and approximate NNLO vs LO_1 and NLO_1



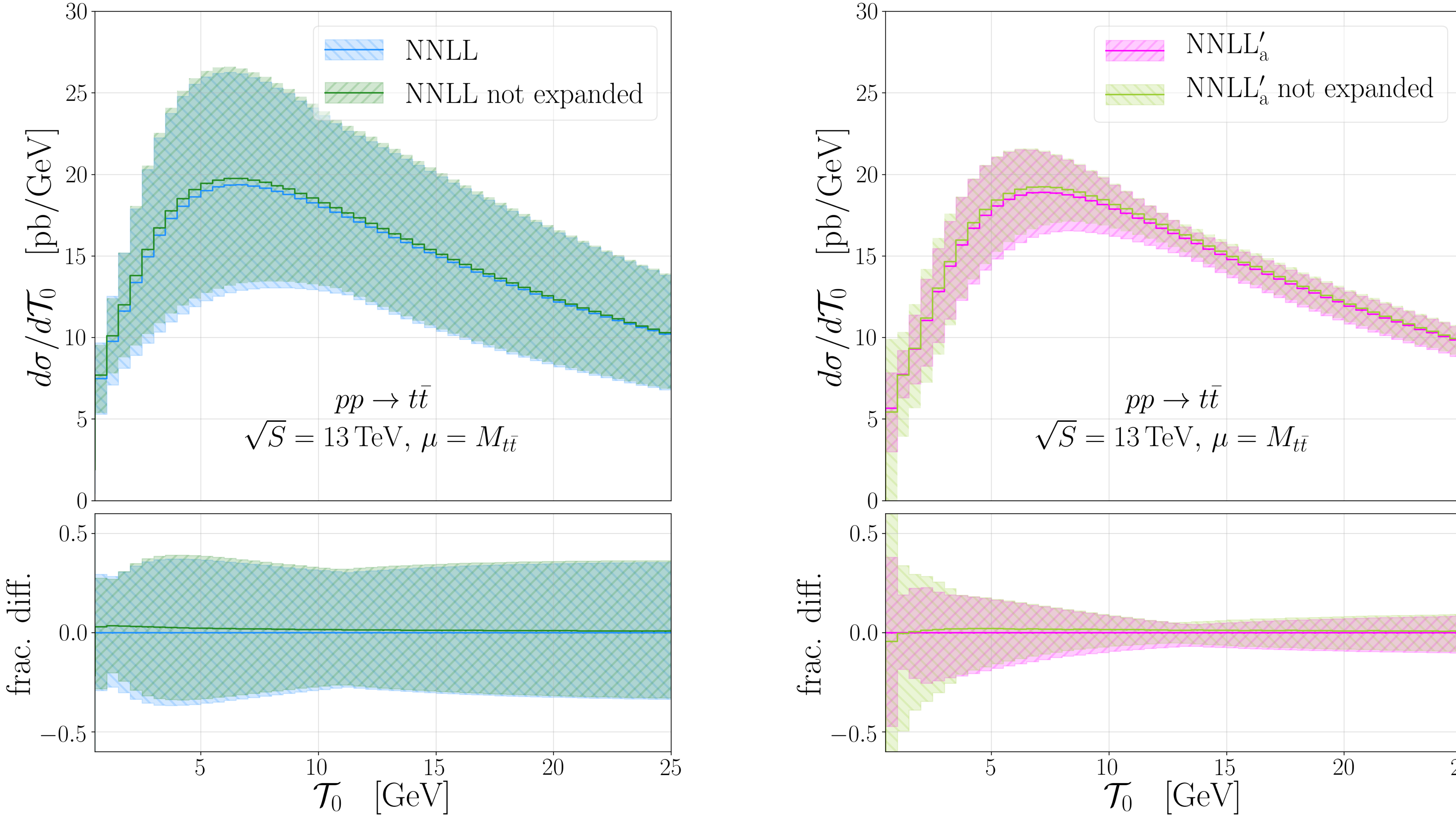
Resummed results

$\mathrm{NNLL}'_{\mathrm{a}}$ is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales



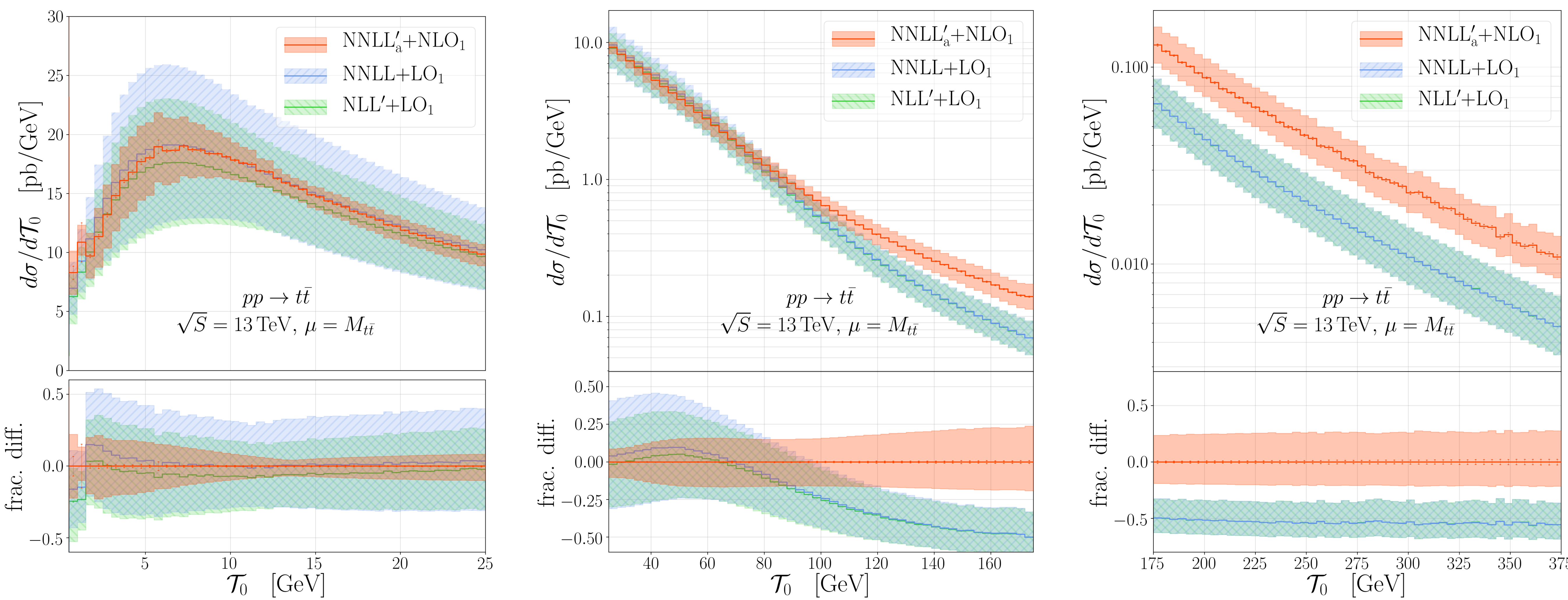
Resummed results

The evolution matrix **u** is evaluated in α_s expansion, we can choose to expand or not expand U, the difference is quite small



Matched results to fixed-order

$$\frac{\mathrm{d}\sigma^{\mathrm{match}}}{\mathrm{d}\mathcal{T}_0} = \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0} + \frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\mathcal{T}_0} - \left[\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0}\right]_{\mathrm{FO}}$$



Outlook

- Calculate and extract all the missing ingredients to reach NNLL$'$ accuracy for the top-quark pair production process (hard and soft functions). Implement in GENEVA event generator

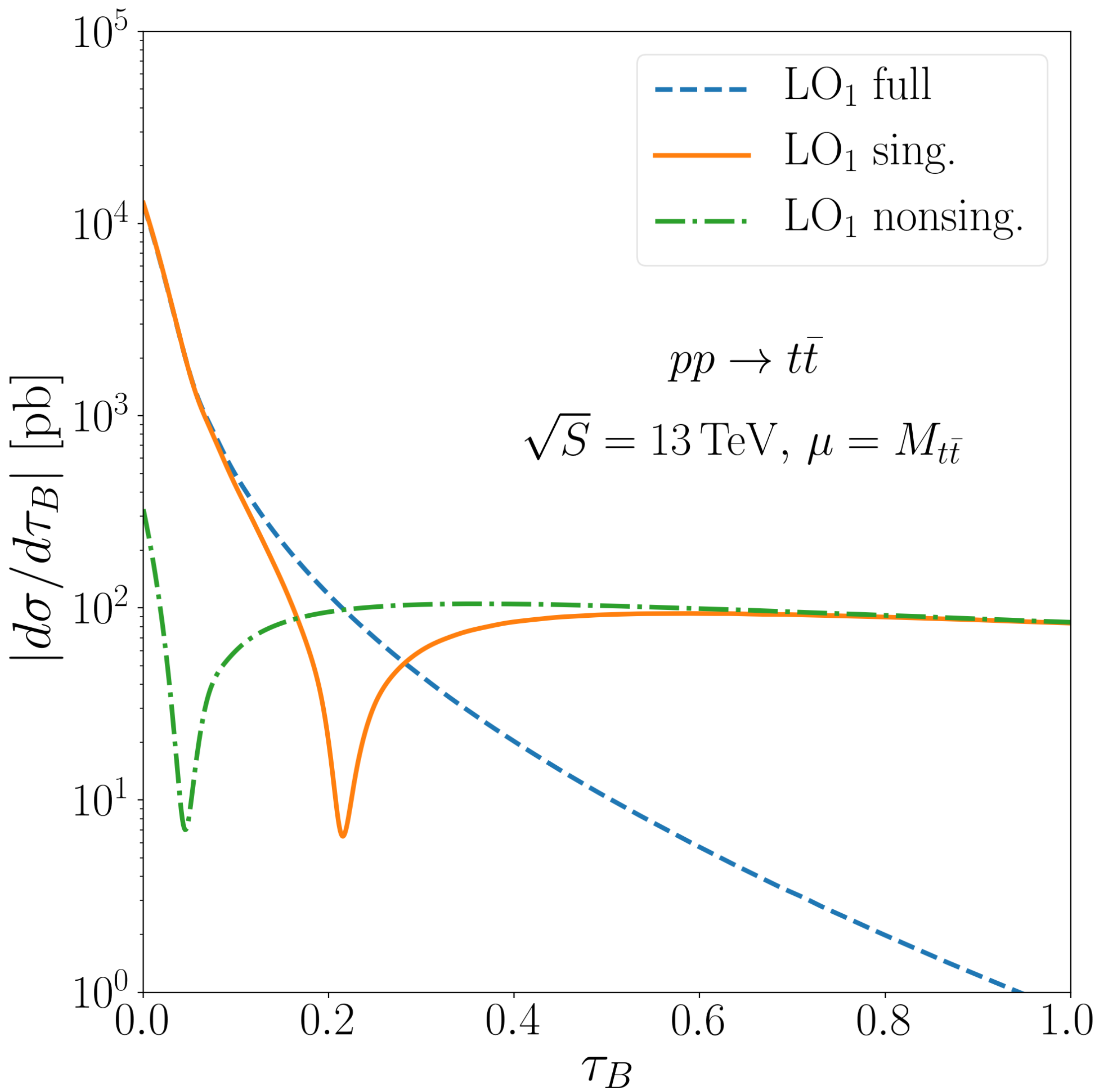
- Study the boosted top regime $m_t \ll M_{t\bar{t}}$ at the LHC

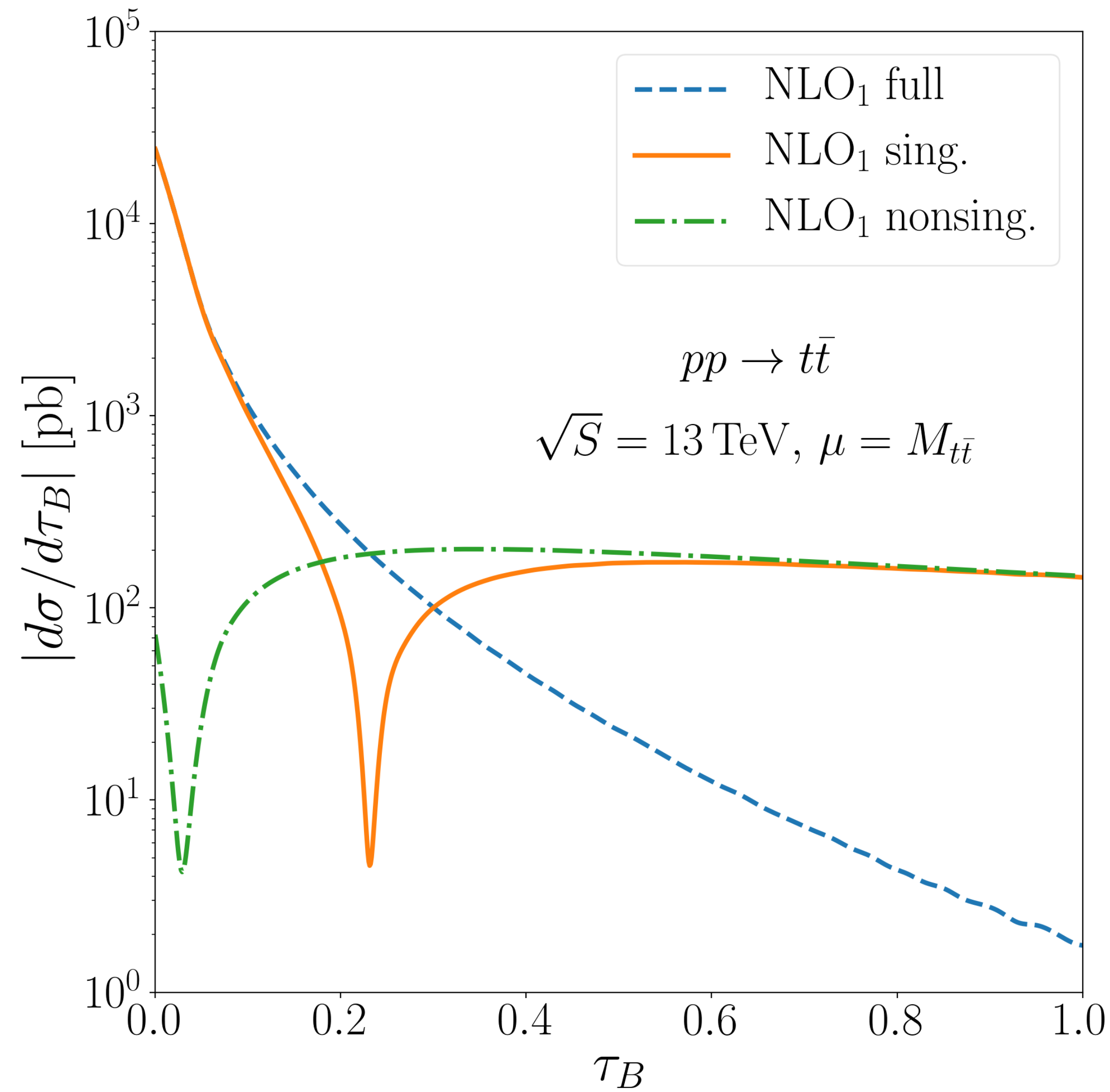
- Extend top-quark pair to study associated production of a top-pair and a heavy boson $t\bar{t}V$ ($V = H, W^{\pm}, Z$) [AB,Ferroglia,Pecjak,Signer, Yang `15], [AB,Ferroglia,Pecjak,Ossola `16],[AB,Ferroglia,Pecjak,Yang `16], [AB,Ferroglia,Pecjak,Ossola,Sameshima `17],[AB,Ferroglia,Frederix, Pagani,Pecjak,Tsinikos `19]

Thank you!

Backup slides

Singular vs Nonsingular contributions





Resummed results

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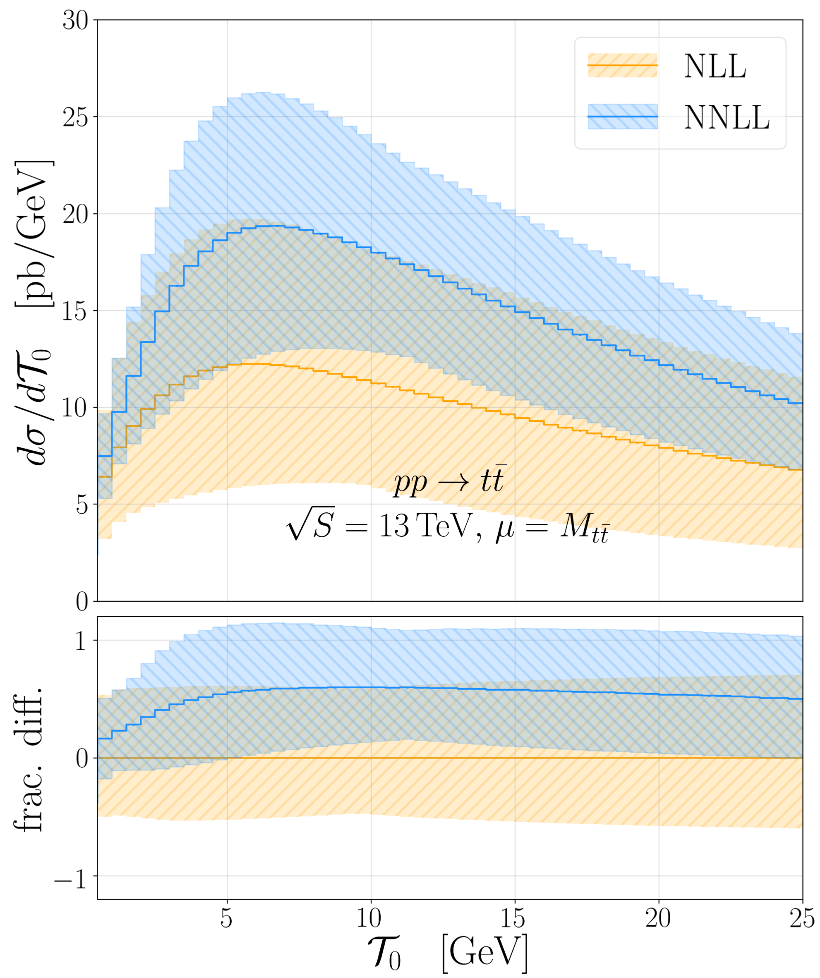

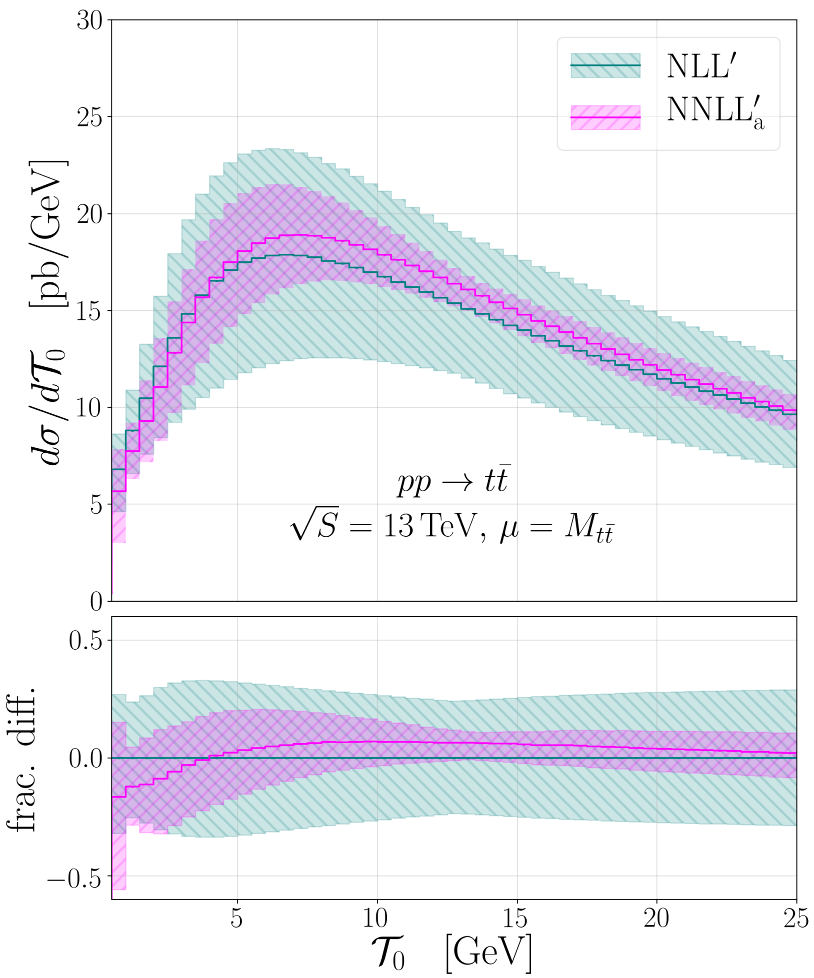

$$
\begin{aligned}
\mu_H &= \mu_{\text{NS}}\,,\\
\mu_S(\mathcal{T}_0) &= \mu_{\text{NS}}\ f_{\text{run}}(\mathcal{T}_0/M)\,,\\
\mu_B(\mathcal{T}_0) &= \mu_{\text{NS}}\ \sqrt{f_{\text{run}}(\mathcal{T}_0/M)}
\end{aligned}
$$

$$
f_{\text{run}}(y) = \begin{cases}
y_0\big[1 + (y/y_0)^2/4\big] & y \le 2y_0\,,\\
y & 2y_0 \le y \le y_1\,,\\
y + \frac{(2-y_2-y_3)(y-y_1)^2}{2(y_2-y_1)(y_3-y_1)} & y_1 \le y \le y_2\,,\\
1 - \frac{(2-y_1-y_2)(y-y_3)^2}{2(y_3-y_1)(y_3-y_2)} & y_2 \le y \le y_3\,,\\
1 & y_3 \le y\,.
\end{cases}
$$

$$y_0 = 1.0\,\text{GeV}/M\,, \quad \{y_1, y_2, y_3\} = \{0.1, 0.175, 0.25\}$$