Jet Angularities at the LHC

based on:

[1] SC, Fedkevych, Marzani, Reichelt, Schumann, Soyez, Theeuwes, JHEP 07 (2021) 076, 2104.06920
[2] SC, Fedkevych, Marzani, Reichelt, Eur. Phys. J. C 81 (2021) 9, 844, 2108.10024
[3] Reichelt, SC, Fedkevych, Marzani, Schumann, Soyez, announced Monday on the arXiv, 2112.09545

Simone Caletti
Milan Christmas Meeting
22 December 2021

Outline:

PART 1

- An unorthodox intro to Jets (based on arXiv:1709.06195)
- The CAESAR framework (arXiv:0407286v2)

PART 2

- Jet Angularities in Z+jet [1]
- Application: initial-gluon tagging [2]

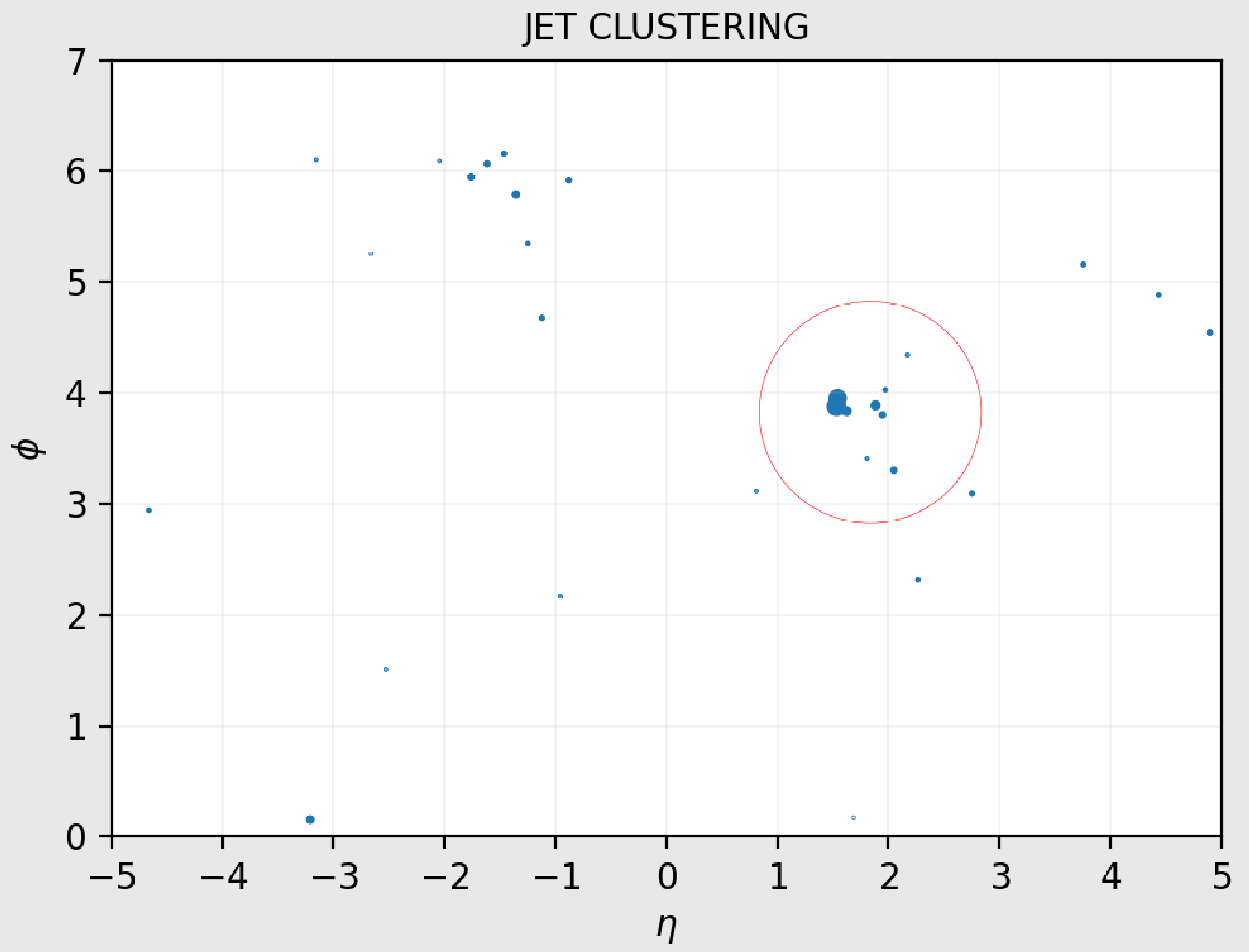
PART 3

- Transfer matrix approach to NP [3]
- Jet Angularities in dijet [3]
- Plots and comparison w/ data [3]*

*see also the CMS paper 2109.03340, Tumasyan et al.

PART 1

- An unorthodox intro to Jets (based on arXiv:1709.06195, Larkoski)
- The CAESAR framework (arXiv:0407286v2, Banfi, Salam, Zanderighi)



- Take the particles in the events as our initial list of objects.

- From this list build the *inter-particle distance* as

$$d_{ij} = \min\left(p_{T,i}^{2p}, p_{T,j}^{2p}\right) \Delta_{ij}^2$$

and the *beam distance* as

$$d_{B,i} = p_{T,i}^{2p} R^2$$

with R the jet radius.

- Iteratively find the smallest among all the two distances:

  If $d_{ij} < d_{B,i}$ then remove i and j and recombine them into a new object k which is added to the list.

  If $d_{B,i} < d_{ij}$ then i is called a *jet* and removed from the list.

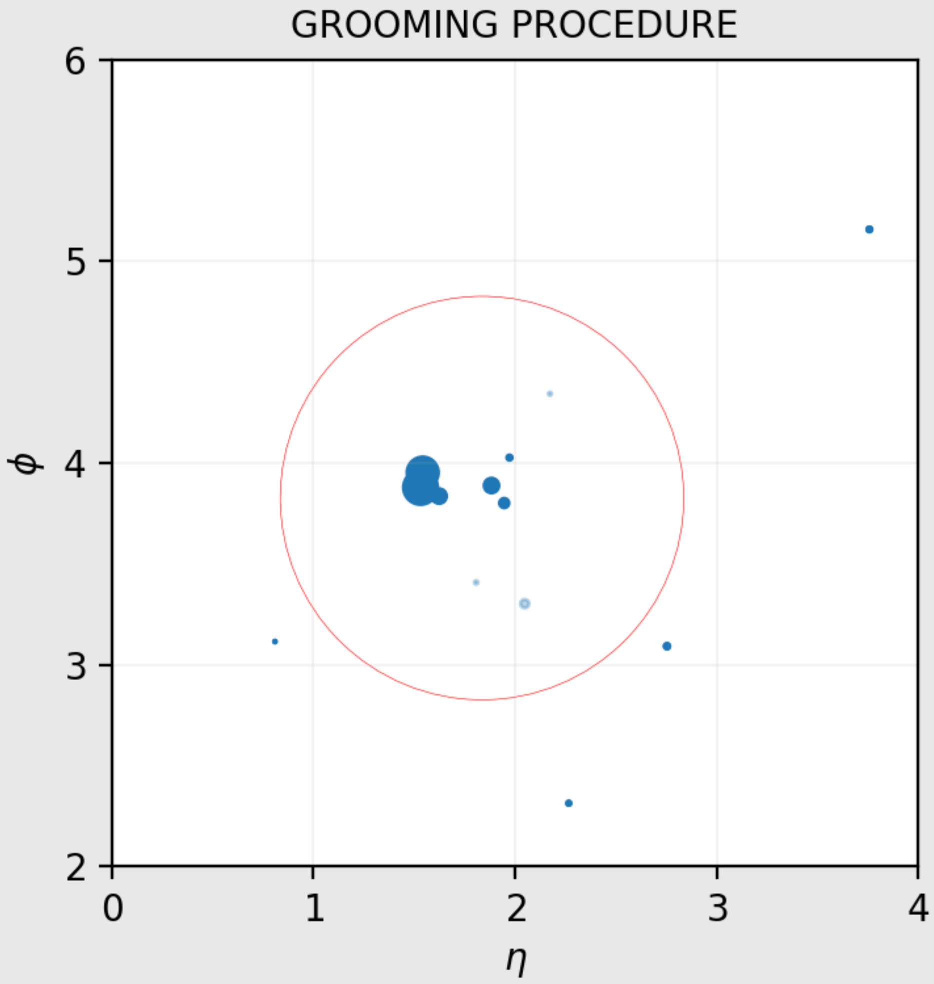
while(! list is empty)

Here we consider the SoftDrop declustering

$$\frac{\min(p_{T,i},\, p_{T,j})}{p_{T,i} + p_{T,j}} > z_{\text{cut}} \left(\frac{\Delta_{ij}}{R}\right)^{\beta}$$

1. Break the jet j into two subjets by undoing the last stage of C/A clustering and label them as j_1 and j_2.
2. If j_1 and j_2 pass the SD condition then deem j to be the final soft-drop jet.
3. Else: redefine $j = \max_{p_T}[j_1, j_2]$

while(! SD)

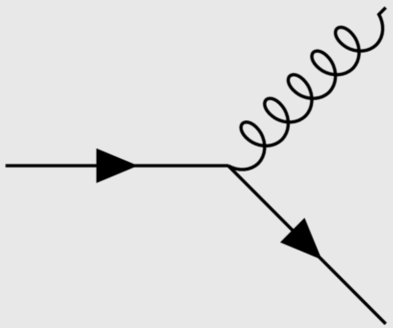


arXiv:1402.2657 (A. Larkoski, S. Marzani, G. Soyez, J. Thaler)

1. At high energies QCD is an approximately scale-invariant quantum field theory. Hence, we can consider α_S to be a constant at least in the first approximation.

2. In addition α_S is also small ($\alpha_S \ll 1$) at high energies.

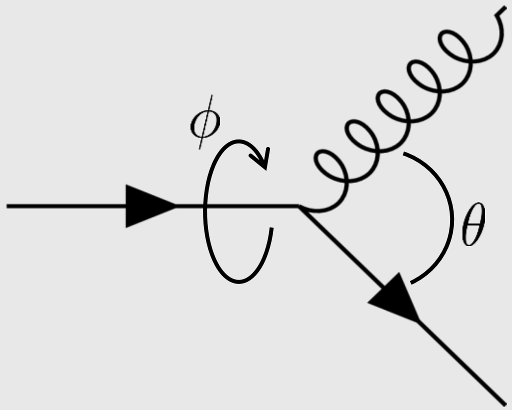
$$\mathcal{P}_{qg\leftarrow q} = \Bigg|$$



$$\Bigg|^2 = ?$$

The partons are massless thus $E_i = |\vec{p}_i|$ for $i = q,\ g$.

$$p_g = (E_g, p_x, p_y, p_z) = E_g(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$



$$p_g = E_g(1, \sin\theta, 0, \cos\theta) \qquad\qquad p_q = E_q(1, 0, 0, 1)$$

We can choose the two phase space variables as E_g and the invariant mass of the LO jet $m^2 = 2p_q \cdot p_g$

$$\text{scale invariant} \quad \Longleftrightarrow \quad P(\lambda E_g, \lambda m^2)d(\lambda E_g)d(\lambda m^2) = P(E_g, m^2)dE_g dm^2$$

$$\mathcal{P}_{qg\leftarrow q} = P(E_g, m^2) \propto \frac{dE_g}{E_g}\frac{dm^2}{m^2}$$

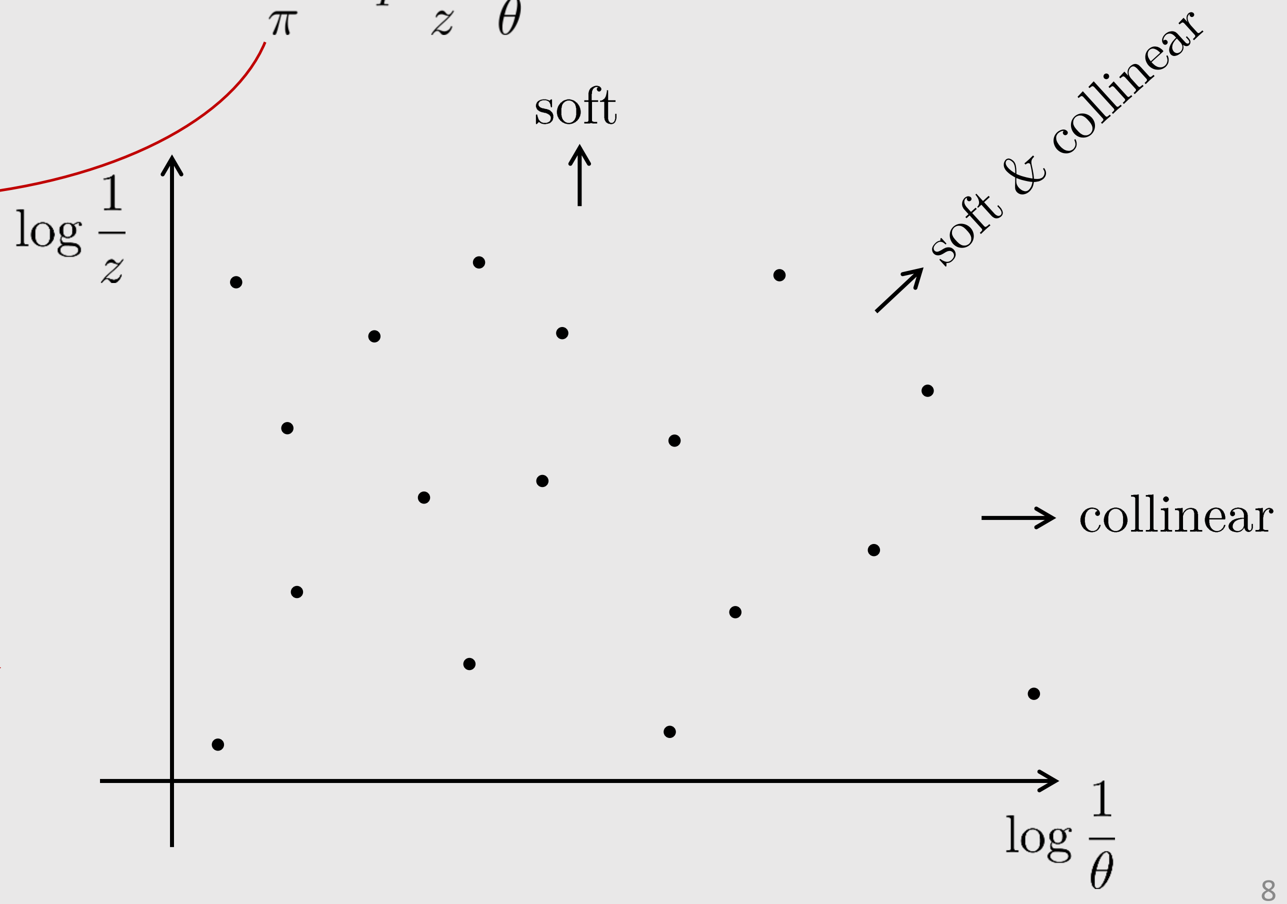
where α_S controls the strenght with which quarks and gluons interact, thus the proportionality factor is $\dfrac{\alpha_S}{\pi}C_F$

$$\mathcal{P}_{qg\leftarrow q} = P(E_g, m^2) = \frac{\alpha_S}{\pi} C_F \frac{dE_g}{E_g} \frac{dm^2}{m^2}$$

Introducing the dimensionless variable $z = \frac{E_g}{E_q + E_g}$ and using $m^2 = 2p_q \cdot p_g = 2E_q E_g (1 - \cos\theta)$

$$P(z,\theta) = dz\, d\cos\theta = \frac{\alpha_S}{\pi} C_F \frac{dz}{z} \frac{d\cos\theta}{1-\cos\theta} \xrightarrow{z \ll 1, \theta \ll 1} \frac{2\alpha_S}{\pi} C_F \frac{dz}{z} \frac{d\theta}{\theta}$$

$$\mathcal{P}_{qg\leftarrow q} = \frac{2\alpha_S}{\pi} C_F \,\mathrm{d}\left(\log\frac{1}{z}\right) \mathrm{d}\left(\log\frac{1}{\theta}\right)$$



Now we introduce the an observable such that in the soft and coll. limit can be parametrized as

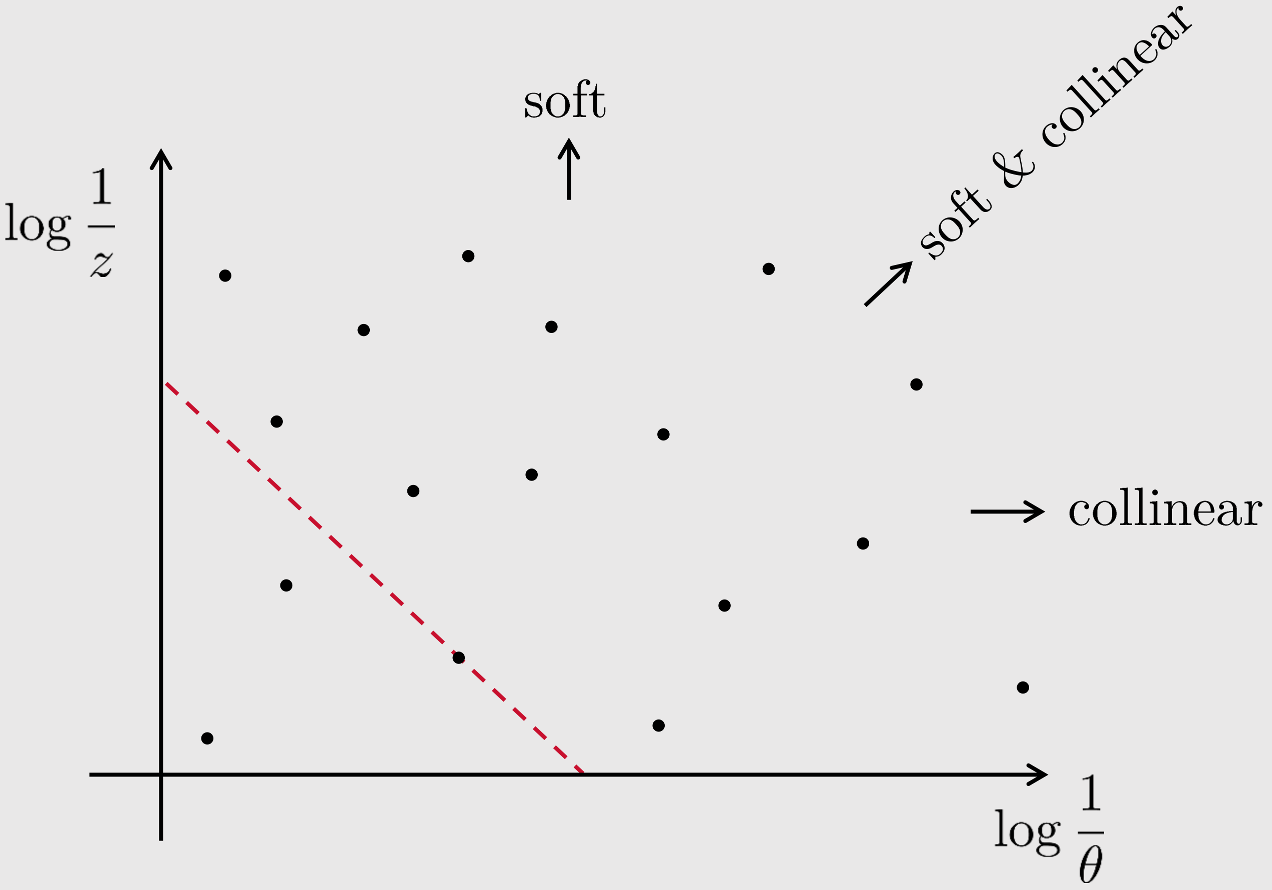
$$V(k) = d\left(\frac{k_T}{Q}\right)^a e^{-b\eta} g(\phi)$$

which is a straight line in the Lund plane.

$$\eta \sim \log\frac{1}{\theta} \qquad\qquad k_T \sim z\theta$$

$$\log\frac{1}{z} = -\left(1+\frac{b}{a}\right)\log\frac{1}{\theta} + \frac{1}{a}\log\frac{1}{v}$$

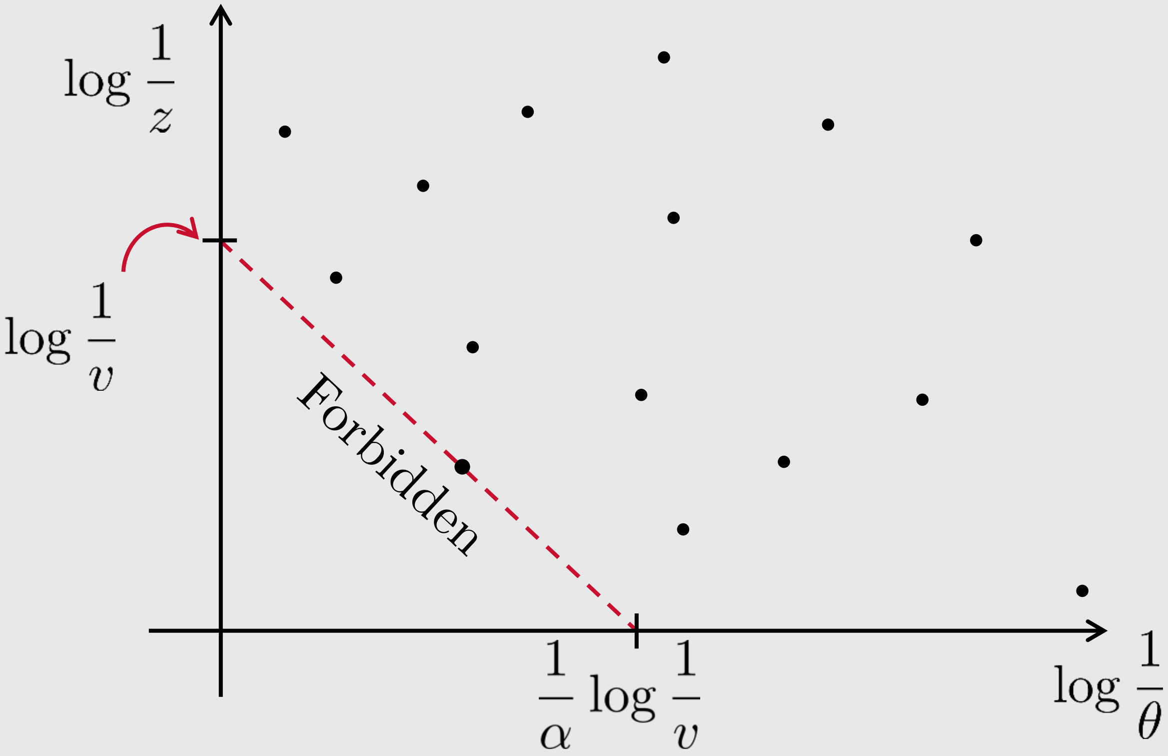
for a fixed value v of $V(k)$ (and $d = g = 1$).



ANGULARITY

$$\lambda_\alpha = \sum_{j\in\mathrm{Jet}}\left(\frac{p_{T,j}}{\sum_{j\in\mathrm{Jet}} p_{T,j}}\right)\left(\frac{\Delta_j}{R}\right)^\alpha \simeq \sum_{j\in\mathrm{Jet}} z_j\theta_j^\alpha$$

$$\Rightarrow \quad \log\frac{1}{z} = -\alpha\log\frac{1}{\theta} + \log\frac{1}{v} \quad \Rightarrow \quad \begin{cases} a = 1 \\ b = \alpha - 1 \end{cases}$$



Obs. Uniformly distributed emissions in the Lund plane means they are exponentially far apart in the real space

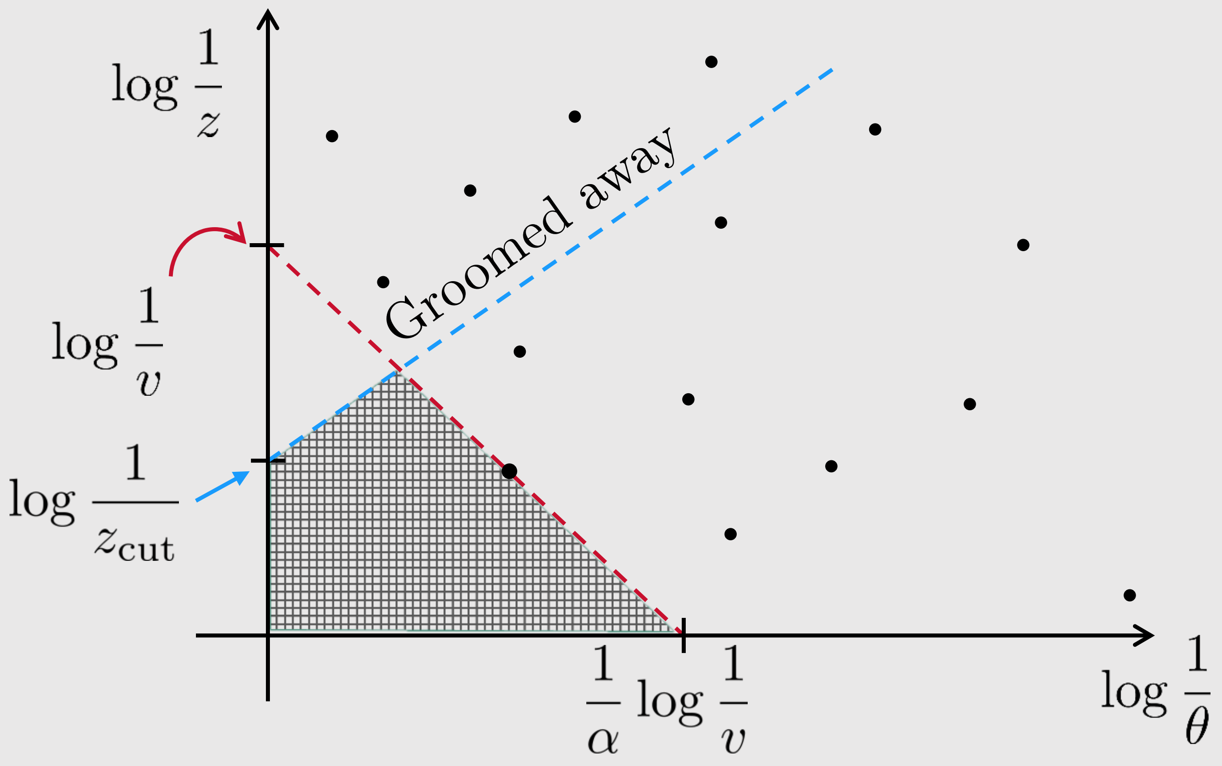
Thus one emission will dominate the others: $\lambda_\alpha = z\theta^\alpha \equiv v$

$$\text{Probability for emission} = \frac{2\alpha_S}{\pi} C_F \frac{\blacktriangle}{N}$$

where $\blacktriangle = \frac{1}{2\alpha} \log^2 v$

$$\text{Probability for no emission} = 1 - \frac{2\alpha_S}{\pi} C_F \frac{\blacktriangle}{N}$$

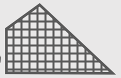
$$P(\lambda_\alpha < v) = \lim_{N\to\infty} \left(1 - \frac{2\alpha_S}{\pi} C_F \frac{\blacktriangle}{N}\right)^N = e^{-\frac{2\alpha_S}{\pi} C_F \blacktriangle}$$



If we consider also the Soft Drop condition

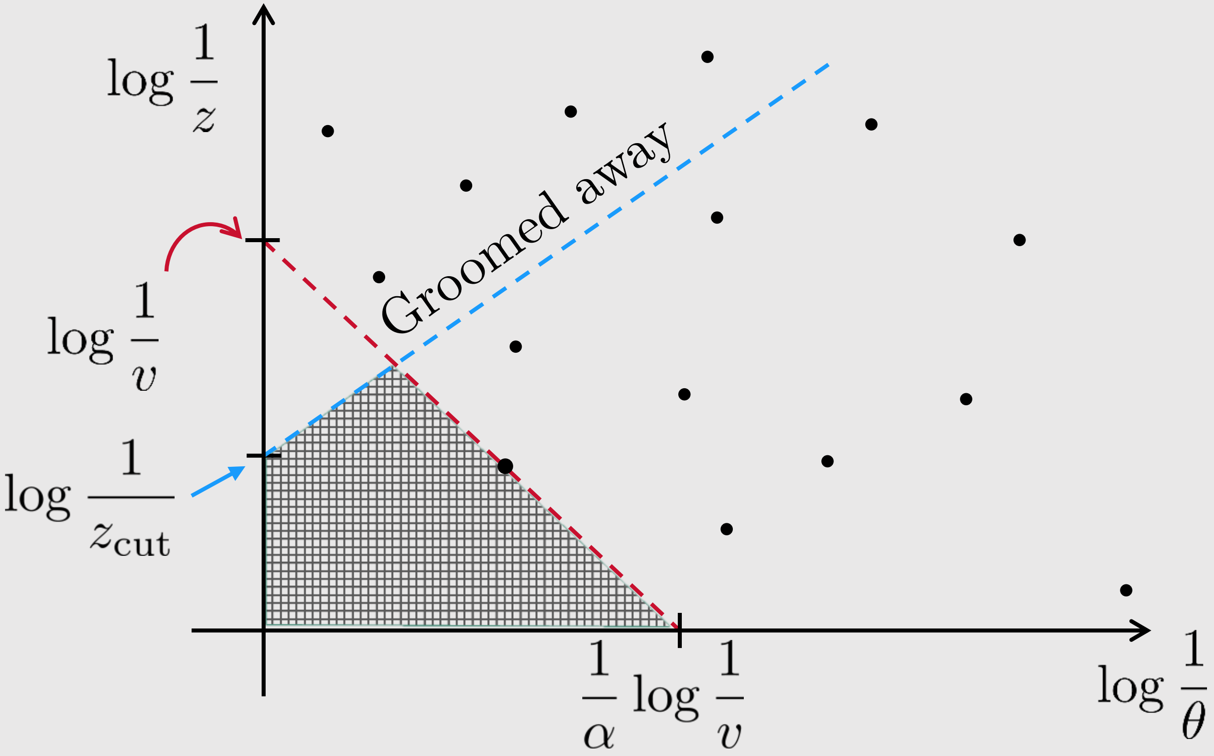
$$z > z_{\text{cut}} \theta^{\beta}$$

$$\log \frac{1}{z} < \log \frac{1}{z_{\text{cut}}} + \beta \log \frac{1}{\theta}$$

$$P(\lambda_\alpha < v) = e^{-\frac{2\alpha_S}{\pi} C_F}$$ 

What we have neglected in this naive picture?

- We achieved just the LL accuracy. We can do better!
- Non-Global Logs (NGL)
- Strong coupling runs, it is not a constant!
- Different Born configurations



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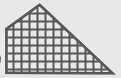
$$P(\lambda_\alpha < v) = e^{-\frac{2\alpha_S}{\pi} C_F}$$ 

$$\Sigma_{\text{res}}(v) = \sum_{\delta} \Sigma^{\delta}_{\text{res}}(v)$$

RADIATOR

$$\Sigma^{\delta}_{\text{res}}(v) = \int d\mathcal{B}_\delta \frac{d\sigma_\delta}{d\mathcal{B}_\delta} \exp\left[-\sum_{l\in\delta} R_l^{\mathcal{B}_\delta}(L)\right] \mathcal{P}^{\mathcal{B}_\delta}(L) \mathcal{S}^{\mathcal{B}_\delta}(L) \mathcal{F}^{\mathcal{B}_\delta}(L)\, \mathcal{H}^{\delta}(\mathcal{B}_\delta)$$

soft function

kinematic cuts

pdf ratio

multiple emission

PART 2

- Jet Angularities in Z+jet [1]
- Application: initial-gluon tagging [2]*

*see also arXiv:2110.09337 (SC)

Matching to NLO:

$$\Sigma_{\text{match, mult}}(\lambda_\alpha) = \sum_\delta \Sigma^\delta_{\text{match, mult}}(\lambda_\alpha)$$

1. The matched result should be correct up to NLL terms in the exponent and the expanded matched result should be correct up to and including $O\left(\alpha_S^n L^{2n-2}\right)$ terms.

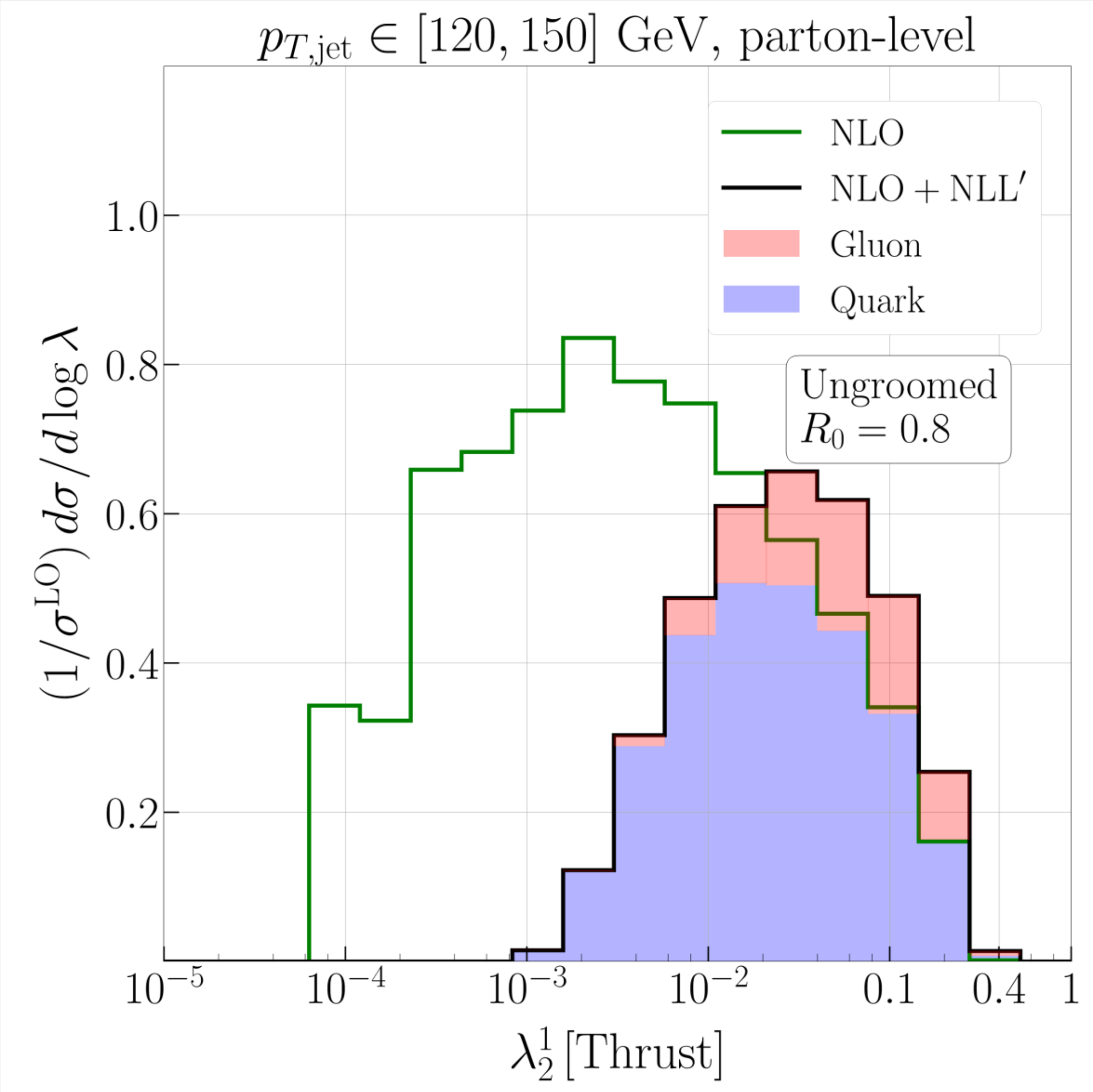
2. The expanded matched result should coincide with the fixed order result up to and including the NLO terms.

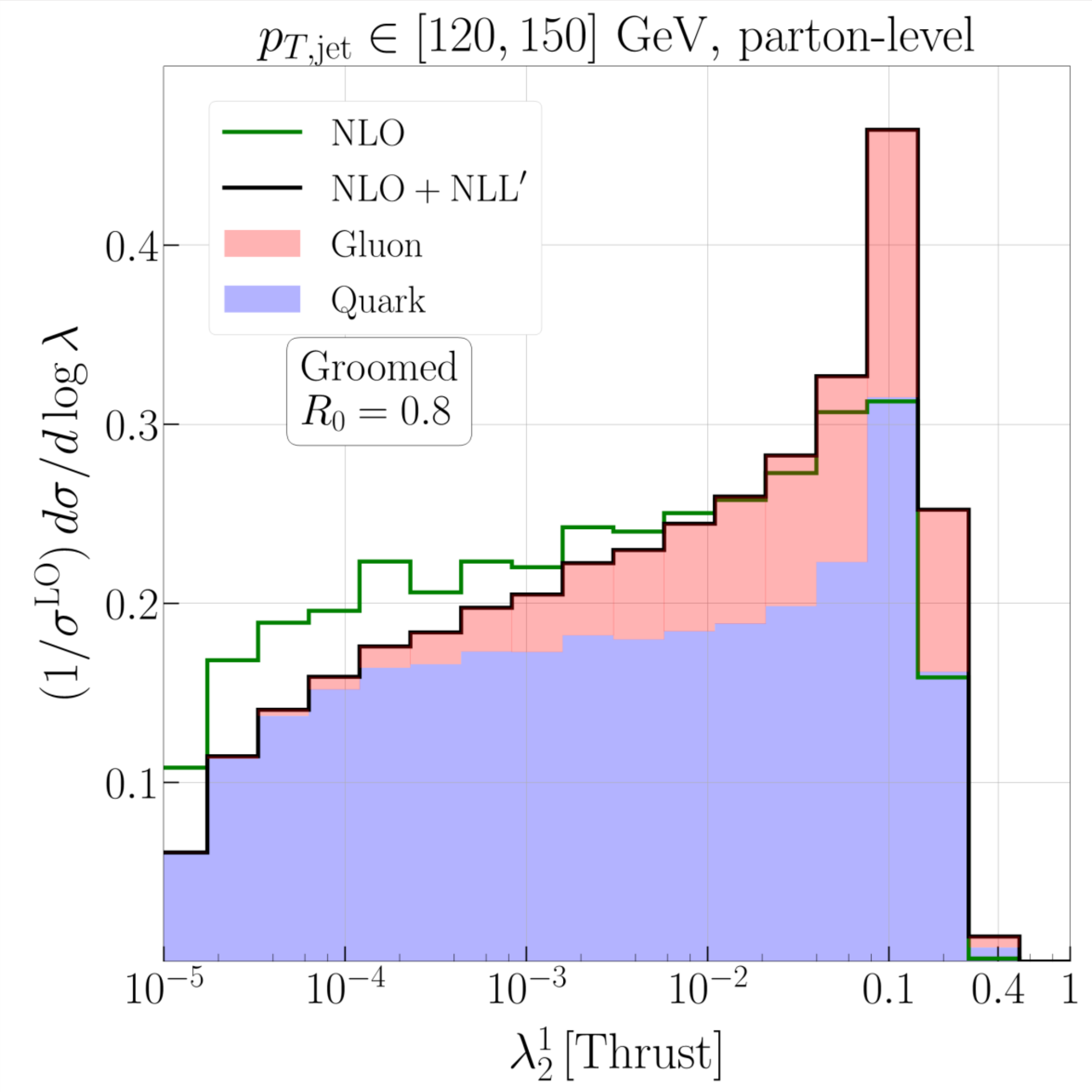
$$\Sigma^\delta_{\text{match, mult}}(\lambda_\alpha) = \Sigma^\delta_{\text{res}}(\lambda_\alpha)\left[1 + \frac{\Sigma^{\delta,(1)}_{\text{fo}}(\lambda_\alpha) - \Sigma^{\delta,(1)}_{\text{res}}(\lambda_\alpha)}{\sigma^{\delta,(0)}} + \right.$$
$$\left. + \frac{1}{\sigma^{\delta,(0)}}\left(-\bar{\Sigma}^{\delta,(2)}_{\text{fo}}(\lambda_\alpha) - \Sigma^{\delta,(2)}_{\text{res}}(\lambda_\alpha) - \Sigma^{\delta,(1)}_{\text{res}}(\lambda_\alpha)\frac{\Sigma^{\delta,(1)}_{\text{fo}}(\lambda_\alpha) - \Sigma^{\delta,(1)}_{\text{res}}(\lambda_\alpha)}{\sigma^{\delta,(0)}}\right)\right]$$

$$\begin{cases} \sigma = \Sigma(1) \\ \Sigma^{(k)} \propto \alpha^2_{\text{EW}}\alpha_{\text{S}}^{1+k} \\ \bar{\Sigma} = \sigma - \Sigma \end{cases}$$

$$\frac{\alpha_S}{2\pi} C^{\delta,(1)} \equiv \lim_{\lambda\to 0} \frac{\Sigma^{\delta,(1)}_{\text{fo}}(\lambda_\alpha) - \Sigma^{\delta,(1)}_{\text{res}}(\lambda_\alpha)}{\sigma^{\delta,(0)}}$$

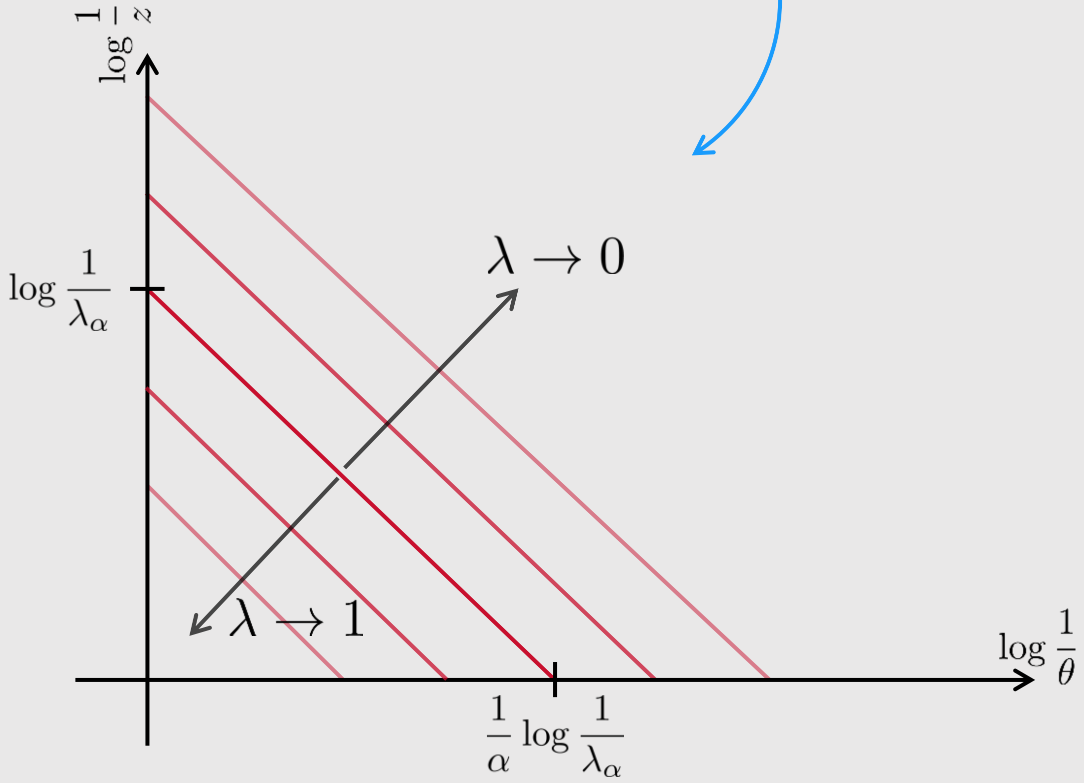
Fixed-order vs. matched (Zjet Thrust low-pT):

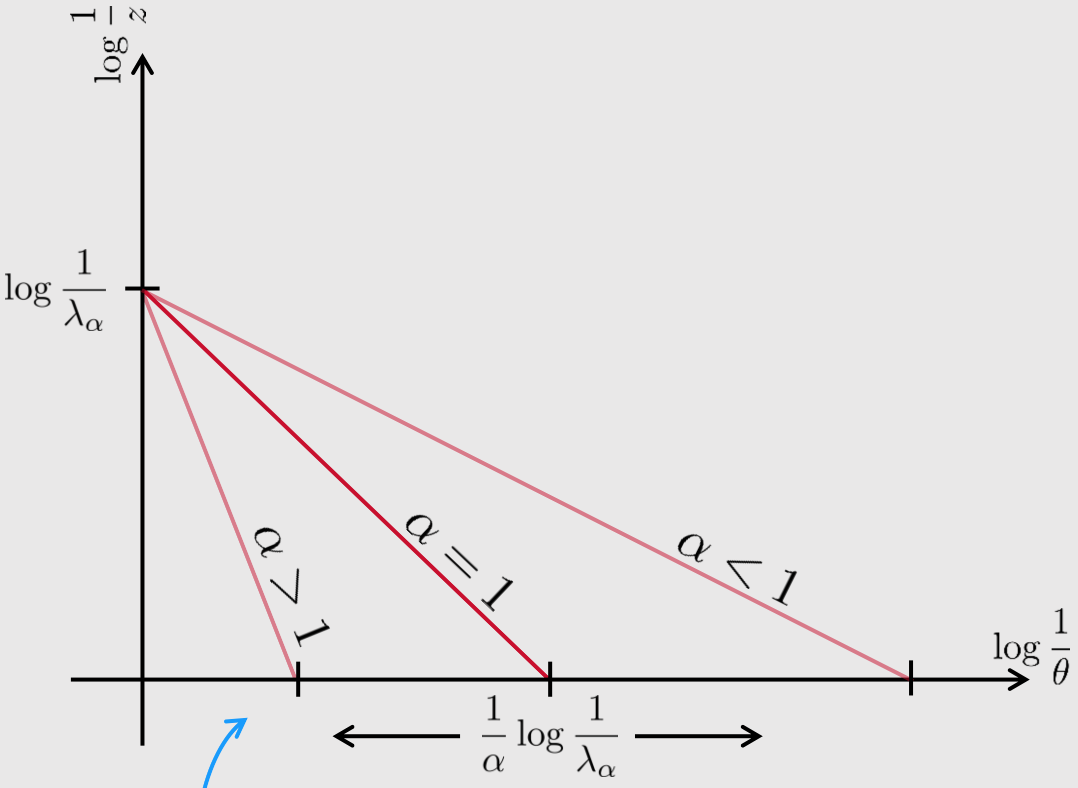




Lund plane geografy :

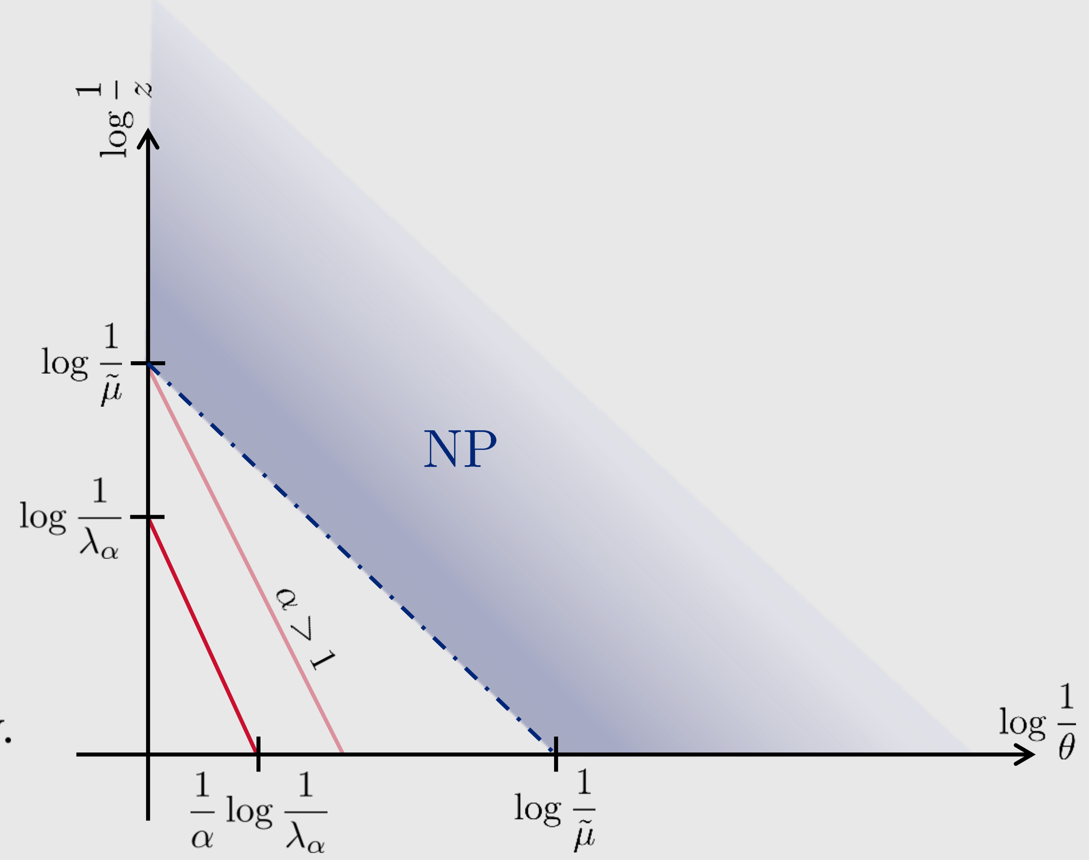
Varying λ corresponds to...





Varying α corresponds to...

Lund plane geografy:



$z\theta \gtrsim \tilde{\mu} \equiv \frac{1\,\mathrm{GeV}}{p_{T,\mathrm{Jet}}R}$ Perturbative region

Which corresponds to $\log\frac{1}{z} \lesssim \log\frac{1}{\tilde{\mu}} - \log\frac{1}{\theta}$

We can push $\lambda \to 0$ until we run into the NP boundary.

In the ungroomed case which end-point of the angularity line do that for first depends on α. For $\alpha > 1$:

$$\log\frac{1}{\lambda_\alpha} = \log\frac{1}{\tilde{\mu}} \quad \Rightarrow \quad \lambda_\alpha \simeq \tilde{\mu}$$

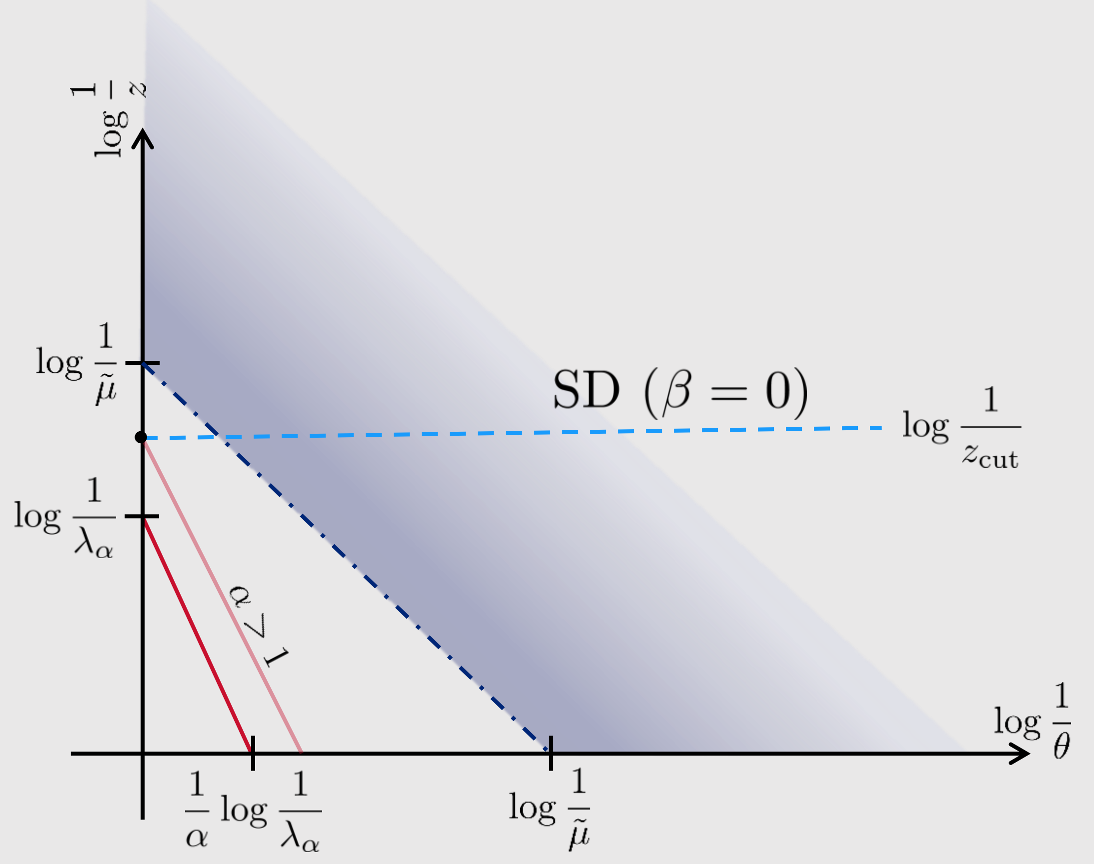
Instead for $\alpha < 1$:

$$\frac{1}{\alpha}\log\frac{1}{\lambda_\alpha} \simeq \log\frac{1}{\tilde{\mu}} \quad \Rightarrow \quad \lambda_\alpha \simeq \tilde{\mu}^\alpha$$

Lund plane geografy:

$$\log \frac{1}{z} < \log \frac{1}{z_{\text{cut}}} + \beta \log \frac{1}{\theta}$$ SoftDrop condition

$$\log \frac{1}{z} < \log \frac{1}{\lambda_\alpha} - \alpha \log \frac{1}{\theta}$$ Angularity veto



If we add the SoftDrop condition we have a transition point before the angularity line run into the NP boundary.

$$\log \frac{1}{z_{\text{cut}}} = \log \frac{1}{\lambda_\alpha} \quad \Rightarrow \quad \lambda_\alpha = z_{\text{cut}}$$

Lund plane geografy:

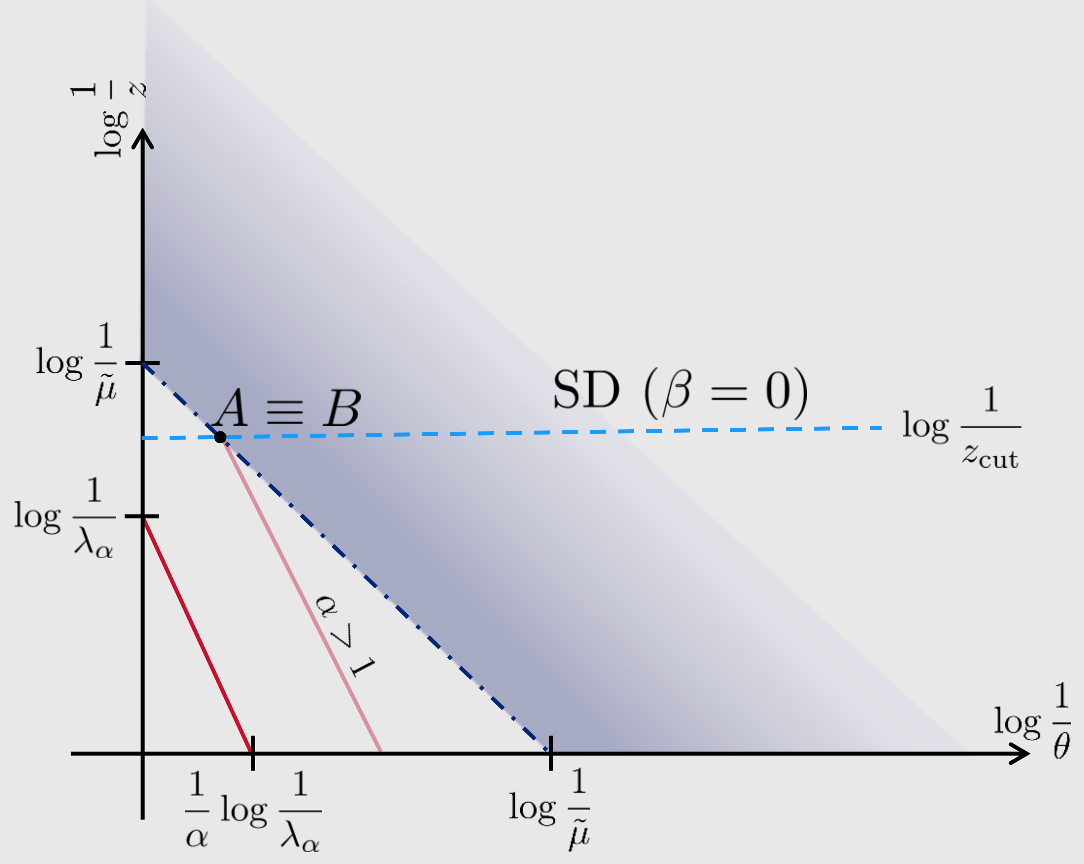
$$\log\frac{1}{z} < \log\frac{1}{z_{\text{cut}}} + \beta\log\frac{1}{\theta} \qquad \text{SoftDrop condition}$$

$$\log\frac{1}{z} \lesssim \log\frac{1}{\tilde{\mu}} - \log\frac{1}{\theta} \qquad \text{Perturbative region}$$

$$\log\frac{1}{z} < \log\frac{1}{\lambda_\alpha} - \alpha\log\frac{1}{\theta} \qquad \text{Angularity veto}$$

$$x_A = y_A = \frac{1}{\beta+1}\log\frac{z_{\text{cut}}}{\tilde{\mu}}$$

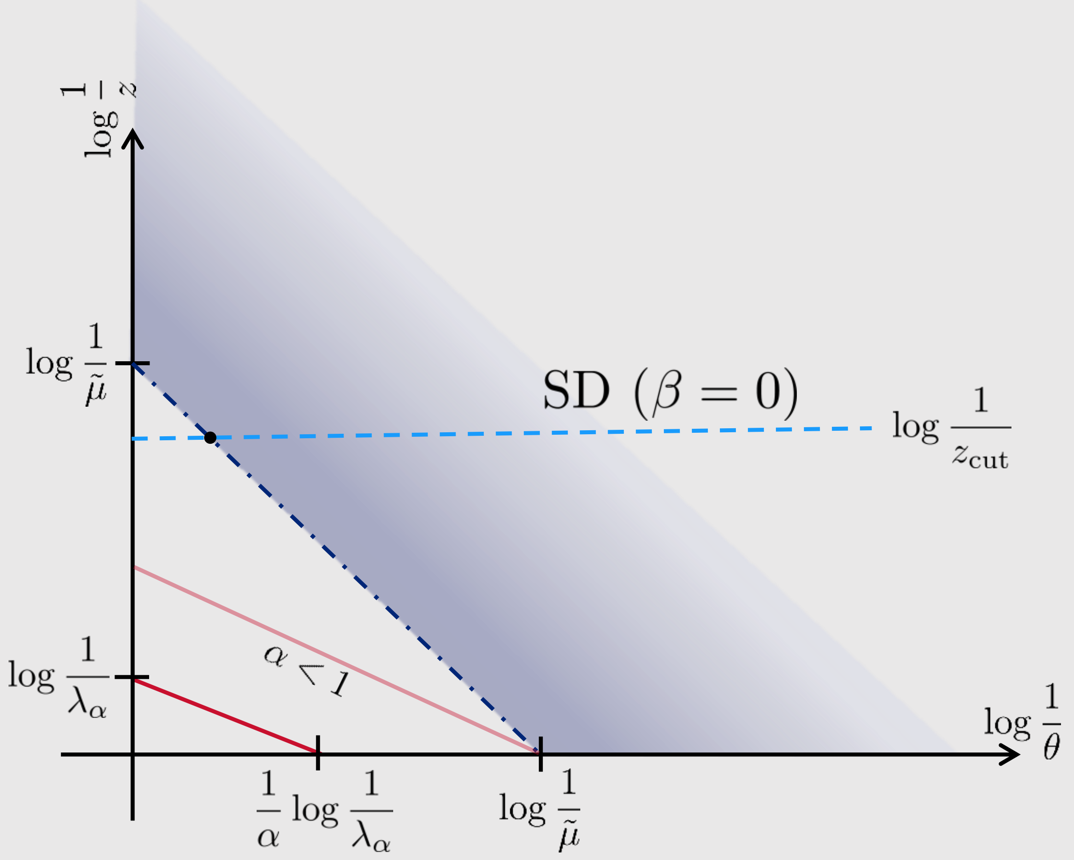
$$x_B = y_B = \frac{1}{\alpha-1}\log\frac{\tilde{\mu}}{\lambda_\alpha}$$



Now, considering also the SoftDrop condition the $\alpha < 1$ and the $\alpha > 1$ situations become different. For $\alpha > 1$ we have:

$$\lambda_\alpha \gtrsim \tilde{\mu}\left(\frac{\tilde{\mu}}{z_{\text{cut}}}\right)^{\frac{\alpha-1}{\beta+1}}$$

Lund plane geografy:



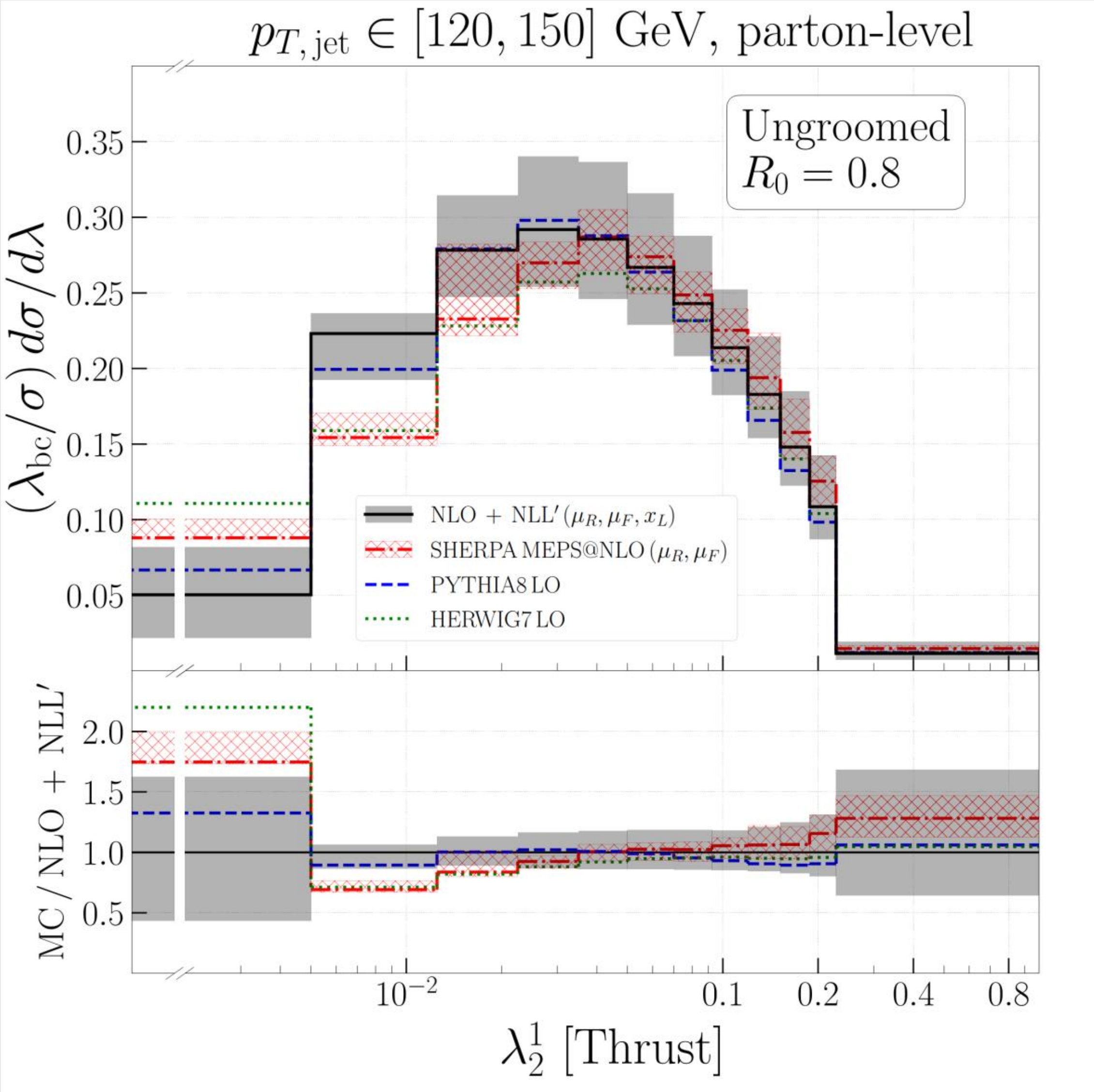
For the $\alpha < 1$ case the SoftDrop condition doesn't affect the intersection point with the NP boundary.

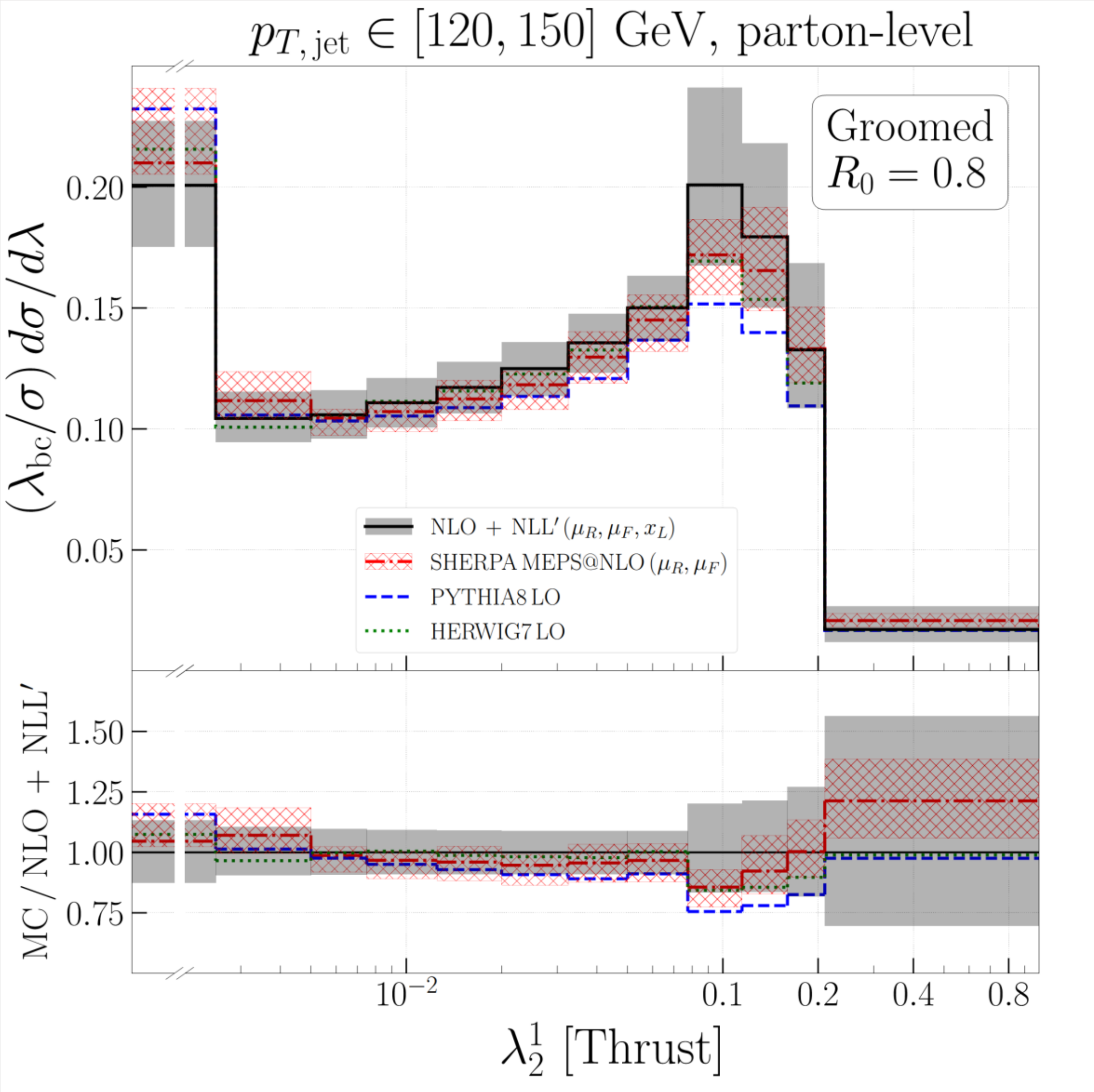
Summarizing, we expect (resummed) perturbative to provide a good description of the physical process for

Ungroomed jets or SD jets with $\alpha \leq 1$: $\quad \lambda_\alpha \gtrsim \tilde{\mu}^{\,\min[\alpha,1]}$

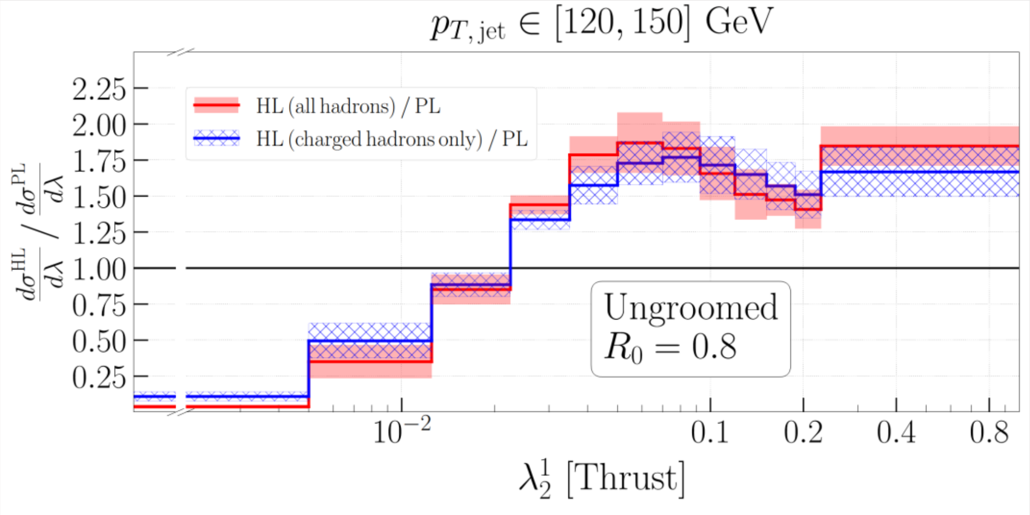
SD jets with $\alpha > 1$: $\quad \lambda_\alpha \gtrsim \tilde{\mu}\left(\frac{\tilde{\mu}}{z_{\rm cut}}\right)^{\frac{\alpha-1}{\beta+1}}$

NLO + NLL′ (Zjet Thrust low-pT):

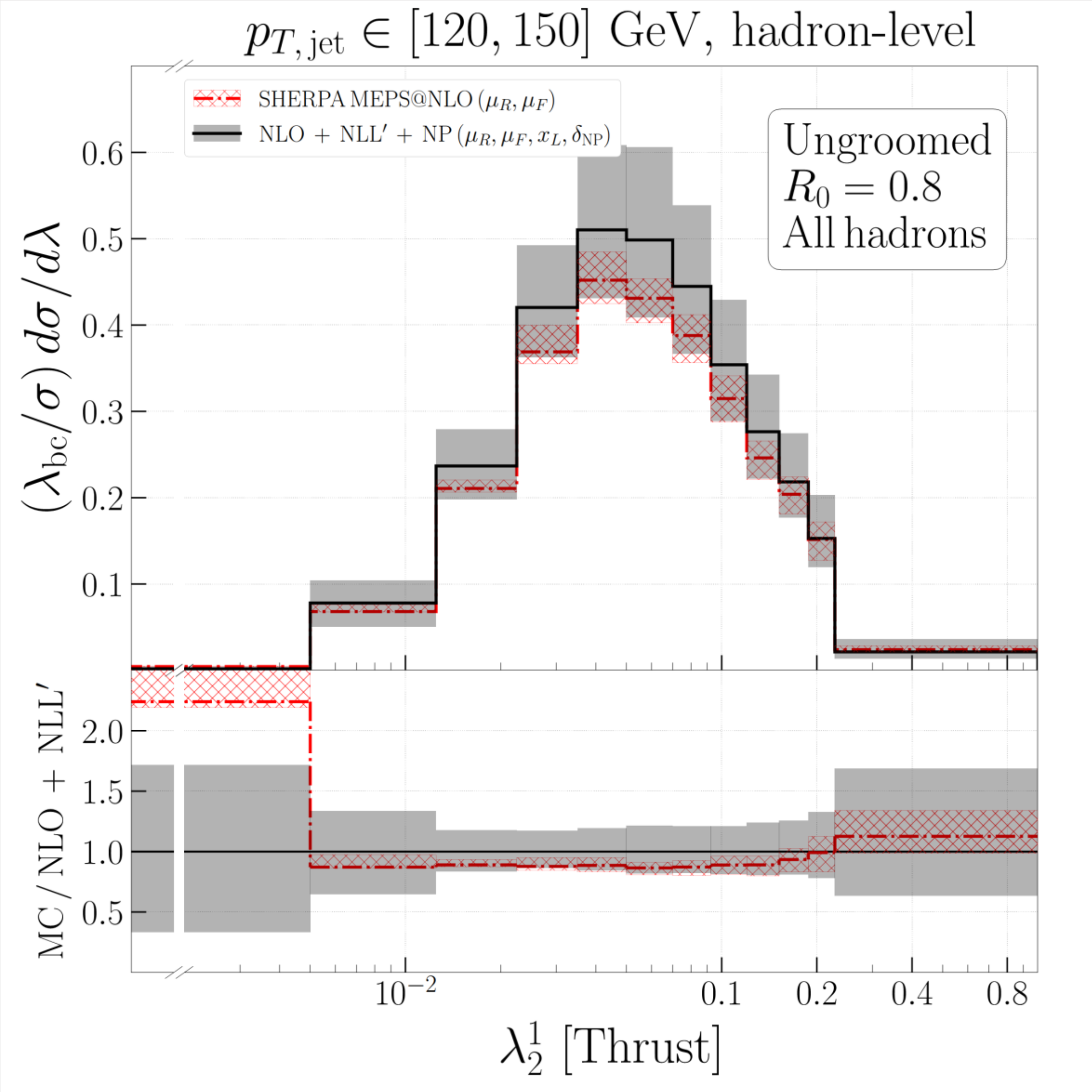




NLO + NLL′+ NP(HL/PL):



To consider NP effect we multiply the NLO+NLL' prediction on a per-bin basis by the corresponding central value of the $\left(\frac{d\sigma^{HL}}{d\lambda}\right) / \left(\frac{\sigma^{PL}}{d\lambda}\right)$ ratio.



Initial-gluon tagging:

What people sometimes think we mean

Ill defined

↓

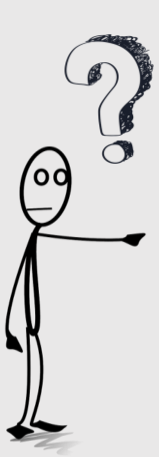
Quark as noun

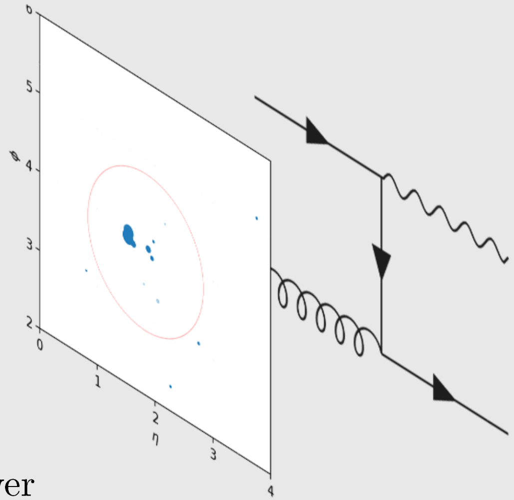
Quark as adjective

Well defined

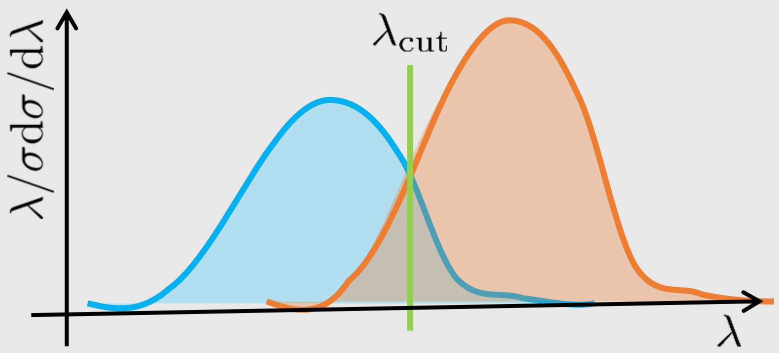
↑

What we mean





- A quark parton
- A Born-level quark parton
- The initiating quark parton in a final state shower
- An eikonal line with baryon number 1/3 and carrying triplet color charge
- A quark operator appearing in a hard matrix element in the context of a factorization theorem
- A parton-level jet that has been quak-tagged using an IRC-safe flavored jet algorithm
- A phase space region (as defined by an unambiguous hadronic fiducial cross section measurement) that yields an enriched sample of quarks (as interpreted by some suitable, though fundamentally ambiguous, criterion)



from arXiv:1704.03878 (P. Gras et al.)

Initial-gluon tagging:

$$\varepsilon_k = \frac{\Sigma_{ij}(\lambda_{\rm cut})}{\Sigma_{ij}(1)} = \frac{1}{\sigma_{ij}} \int_0^{\lambda_{\rm cut}} \frac{d\sigma_{ij}}{d\lambda} d\lambda$$

with $\quad i\, j \to Z\, k$

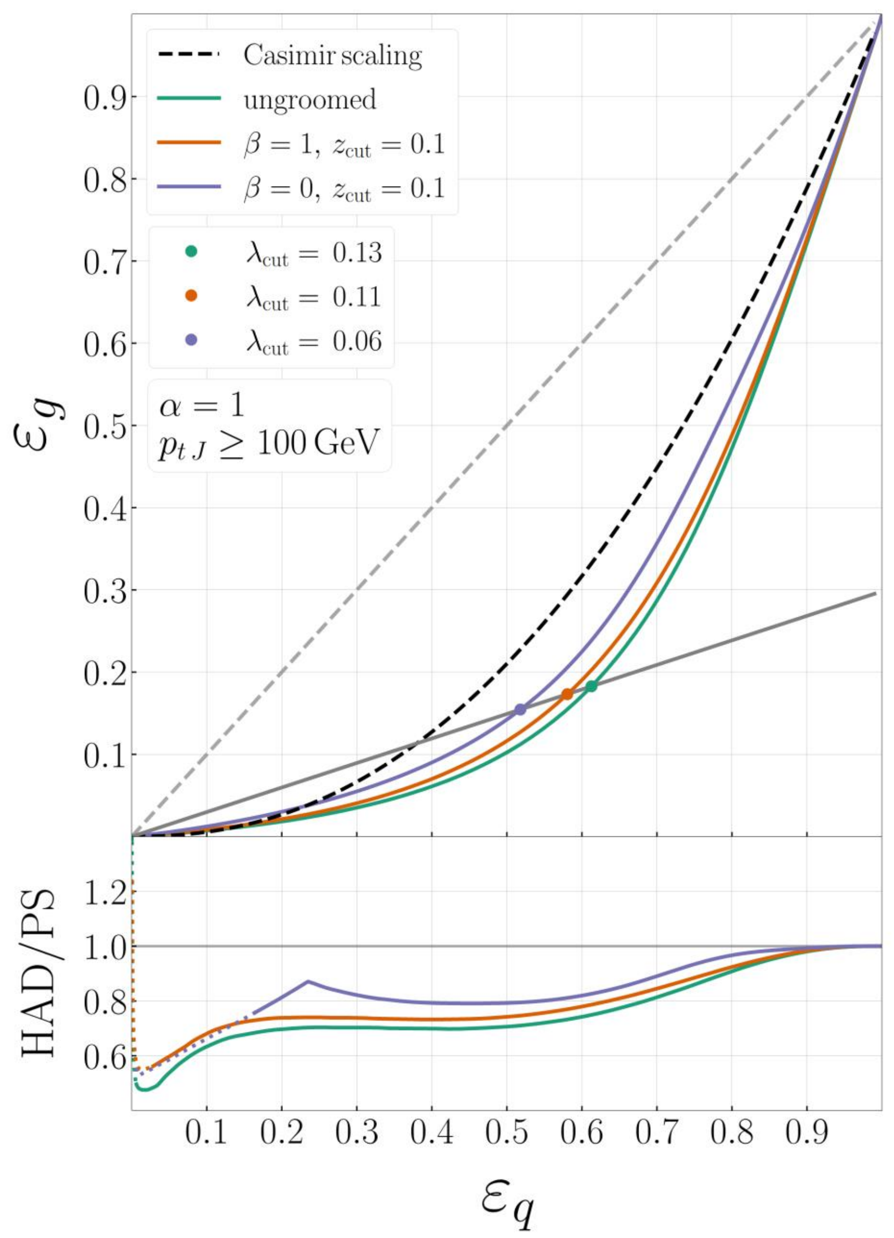
Q-jet true positive rate

$$\tilde{f}_g = \frac{\varepsilon_q \sigma_{qg}}{\varepsilon_g \sigma_{qq} + \varepsilon_q \sigma_{qg}} = \frac{\varepsilon_q f_g}{\varepsilon_g (1 - f_g) + \varepsilon_q f_g}$$

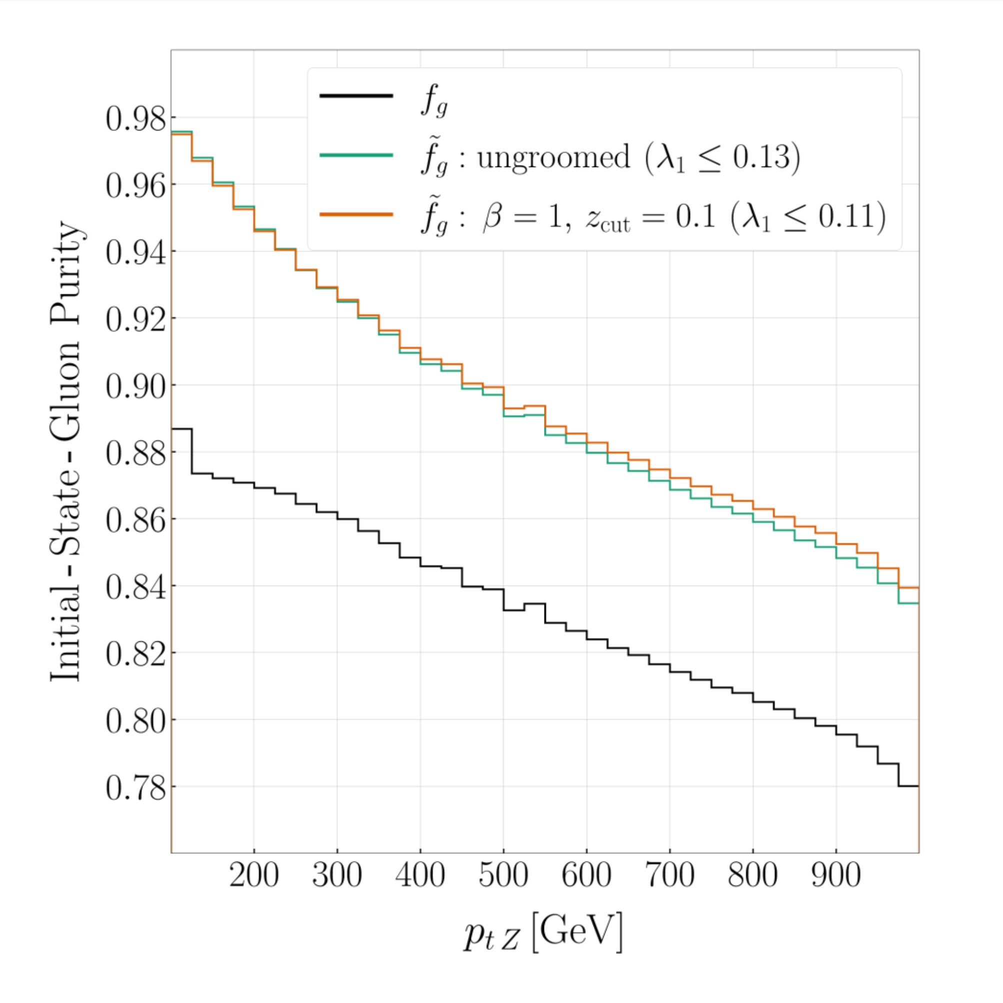
(enhanced) purity

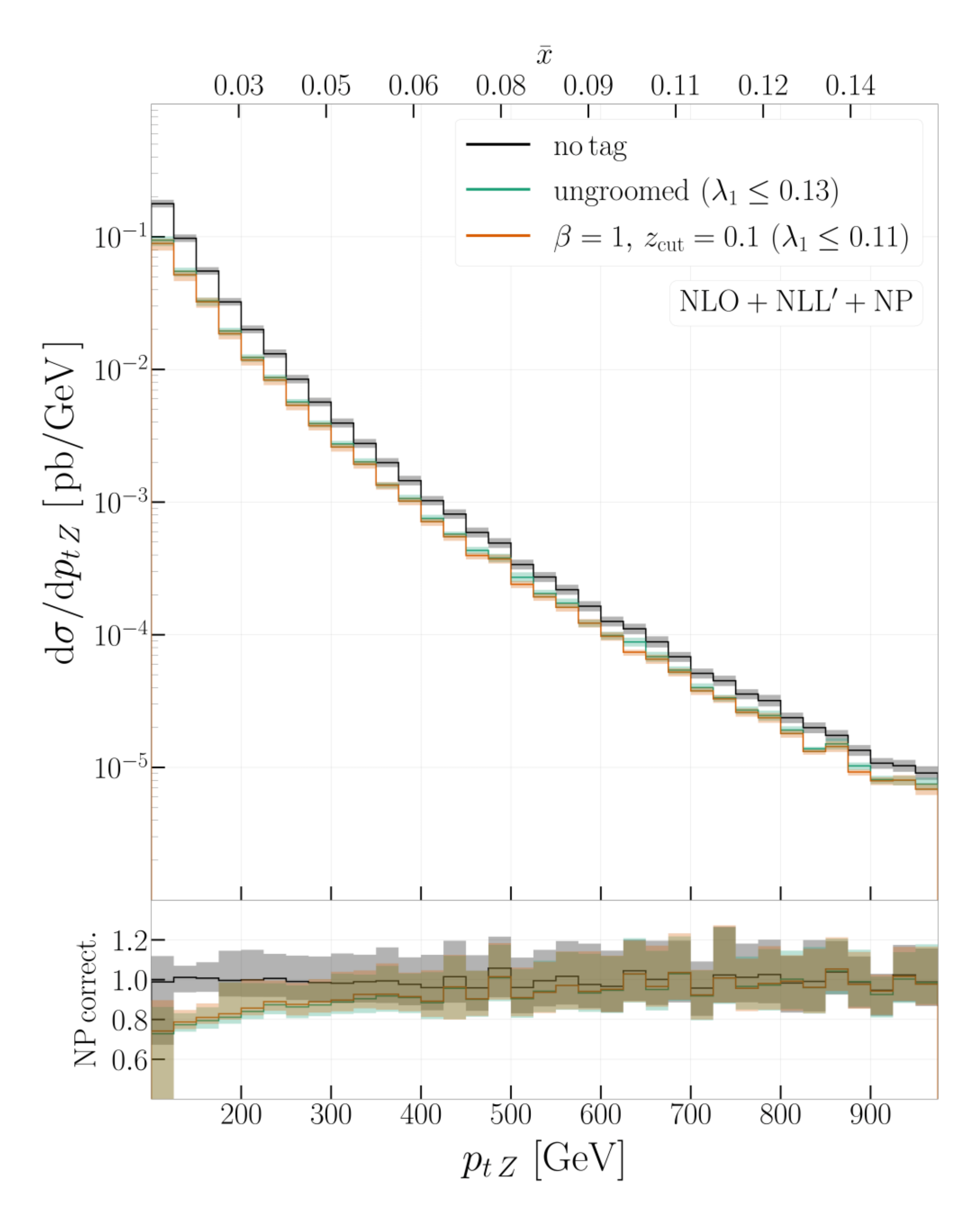
Q-jet false positive rate

$$\varepsilon_g = \frac{f_g (1 - \tilde{f}_g)}{\tilde{f}_g (1 - f_g)} \varepsilon_q$$



Initial-gluon tagging:

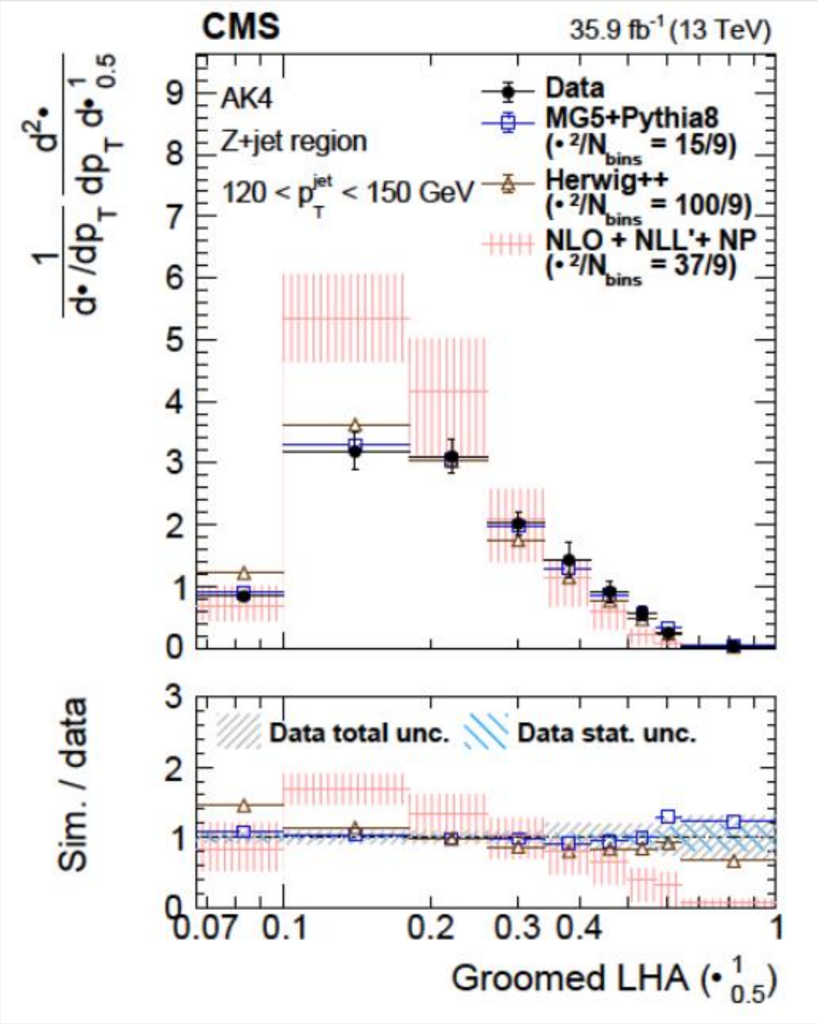




PART 3

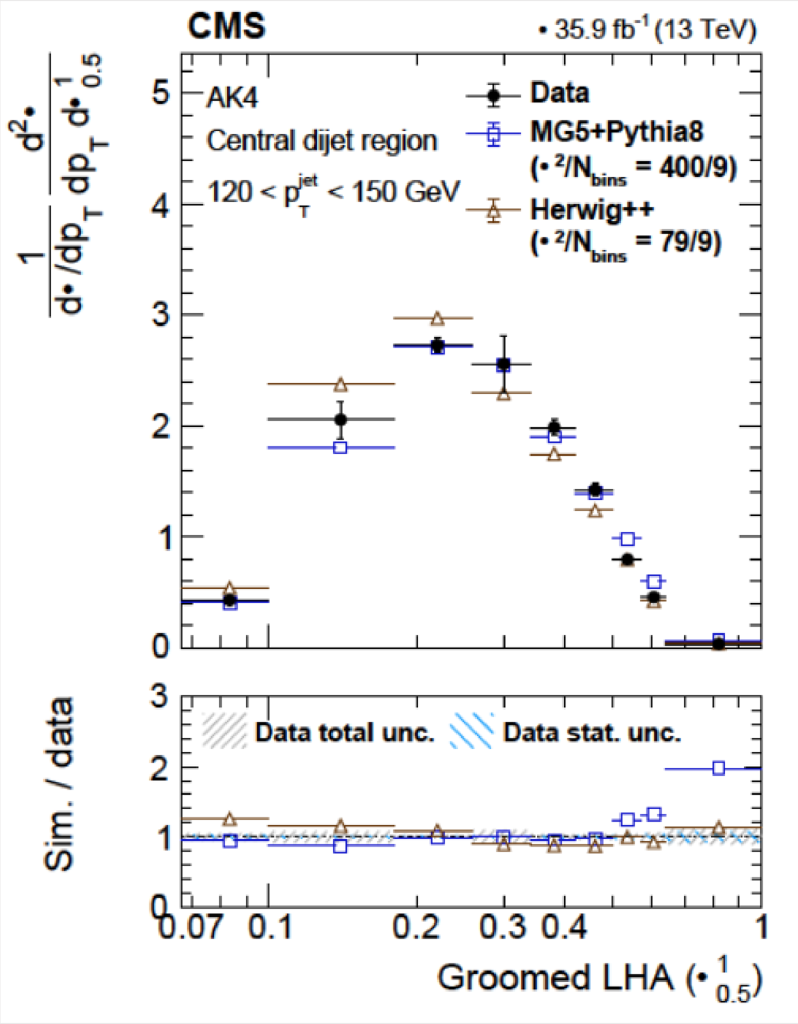
- Transfer matrix approach to NP [3]
- Jet Angularities in dijet [3]
- Plots and comparison w/ data [3]*

Fig. 10 of the CMS paper*:



Worse level of agreement between data and NLO+NLL'+NP prediction for the LHA…

…and why not the same predictions for dijet?



*arXiv:2109.03340 (Tumasyan et al.)

Transfer matrix:

The idea is to improve the simple HL/PL method to extract NP corrections. Consider $\mathcal{P}$ the partonic configuration which is the result of a scattering process and consider $\mathcal{H}$ a map which accounts for hadronisation and UE corrections. We define the transfer operator as the conditional probability to measure a HL set of observables $\vec{v}_h$ evaluated on $\mathcal{H}(\mathcal{P})$, given that the PL obsevables were $\vec{v}_p$.

$$\mathcal{T}(\vec{v}_h|\vec{v}_p) = \frac{\int d\mathcal{P} \frac{d\sigma}{d\mathcal{P}} \delta^{(m)}\left(\vec{v}_p - \vec{V}(\mathcal{P})\right) \delta^{(n)}\left(\vec{v}_h - \vec{V}(\mathcal{H}(\mathcal{P}))\right)}{\int d\mathcal{P} \frac{d\sigma}{d\mathcal{P}} \delta^{(m)}\left(\vec{v}_p - \vec{V}(\mathcal{P})\right)} \longrightarrow \mathcal{T}_{hp} = \frac{\int d\mathcal{P} \frac{d\sigma}{d\mathcal{P}} \Theta_p(\mathcal{P}) \Theta_h(\mathcal{H}(\mathcal{P}))}{\int d\mathcal{P} \frac{d\sigma}{d\mathcal{P}} \Theta_p(\mathcal{P})}$$

$$\frac{d^m \sigma^{\mathrm{HL}}}{dv_{h,1} \dots dv_{h,m}} = \int d^m \vec{v}_p \mathcal{T}(\vec{v}_h|\vec{v}_p) \frac{d^m \sigma^{\mathrm{PL}}}{dv_{p,1} \dots dv_{p,m}} \longrightarrow \Delta\sigma_h^{\mathrm{HL}} = \sum_p \mathcal{T}_{hp} \Delta\sigma_p^{\mathrm{PL}}$$

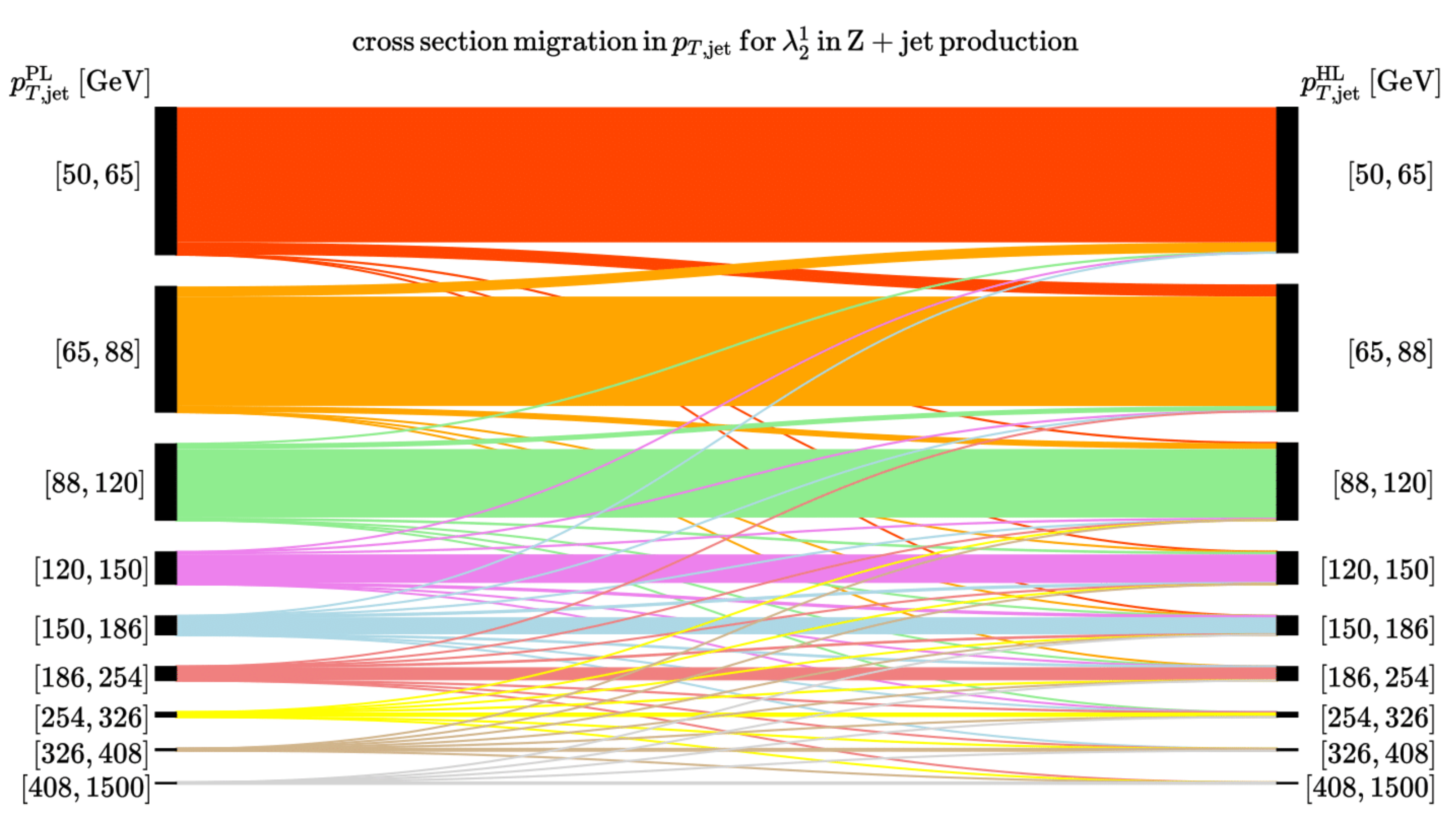
with

$$\Theta_p(\mathcal{P}) = \prod_{i=1}^{m} \theta(V_i(\mathcal{P}) - v_{p,i}^{\min}) \theta(v_{p,i}^{\max} - V_i((P)))$$

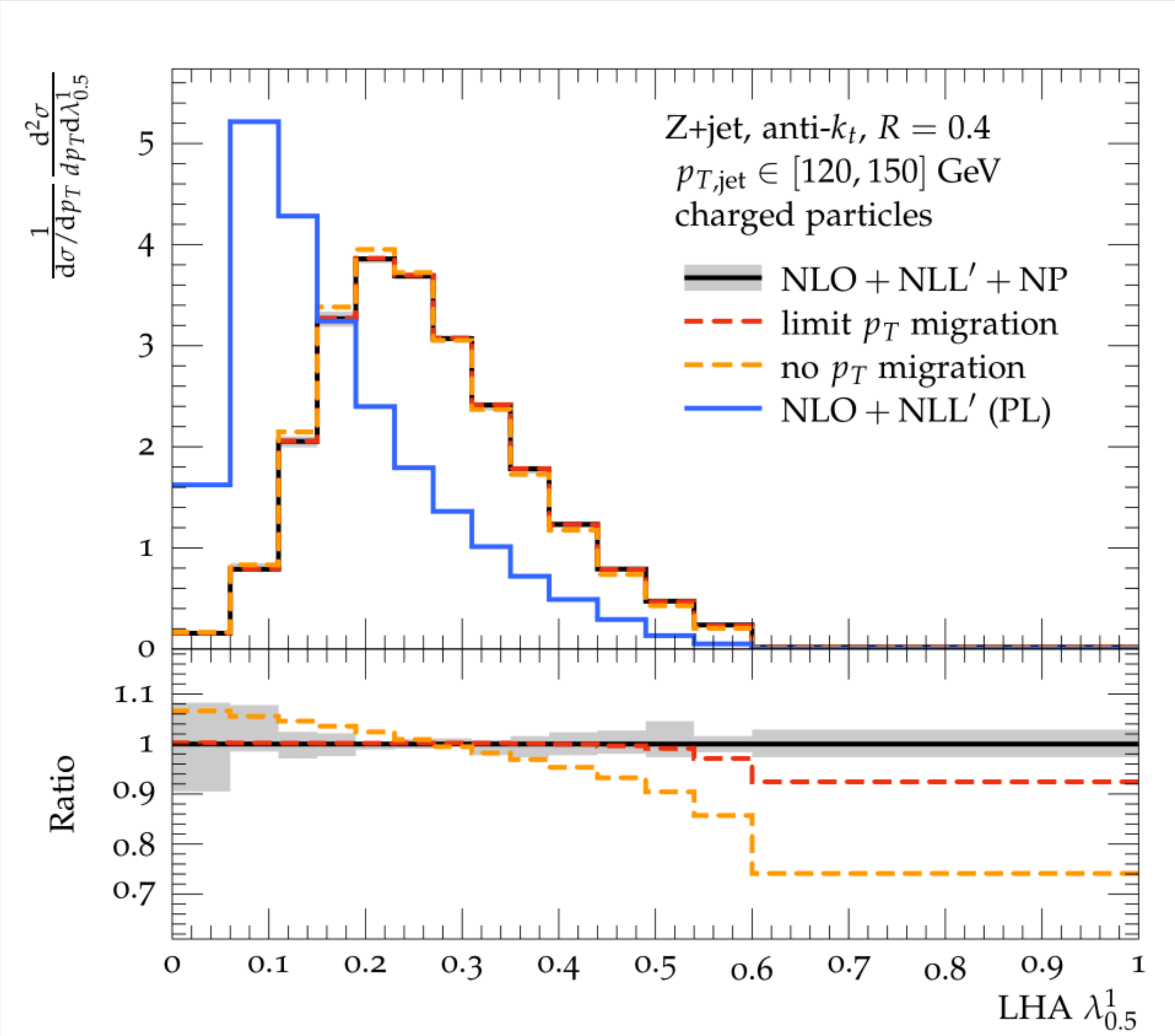
$$\Theta_h(\mathcal{H}(\mathcal{P})) = \prod_{i=1}^{m} \theta(V_i(\mathcal{H}(\mathcal{P})) - v_{h,i}^{\min}) \theta\left(v_{h,i}^{\max} - V_i(\mathcal{H}((P)))\right)$$

Transfer matrix:

The idea is to improve the simple HL/PL method to extract NP corrections. Consider $\mathcal{P}$ the partonic configuration which is the result of a scattering process and consider $\mathcal{H}$ a map which accounts for hadronisation and UE corrections. We define the transfer operator as the conditional probability to measure a HL set of observables $\vec{v}_h$ evaluated on $\mathcal{H}(\mathcal{P})$, given that the PL obsevables were $\vec{v}_p$.



Transfer matrix:

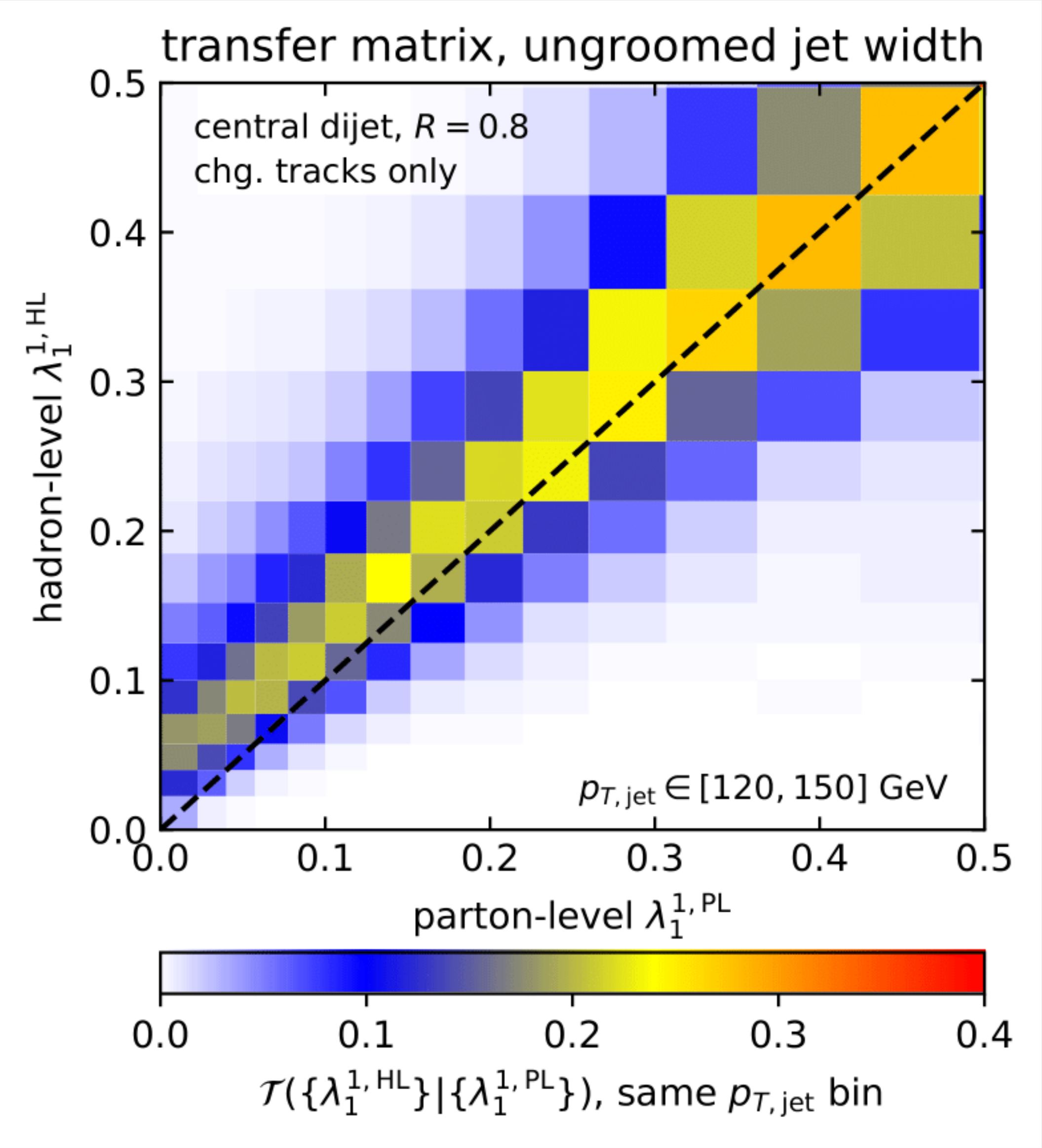


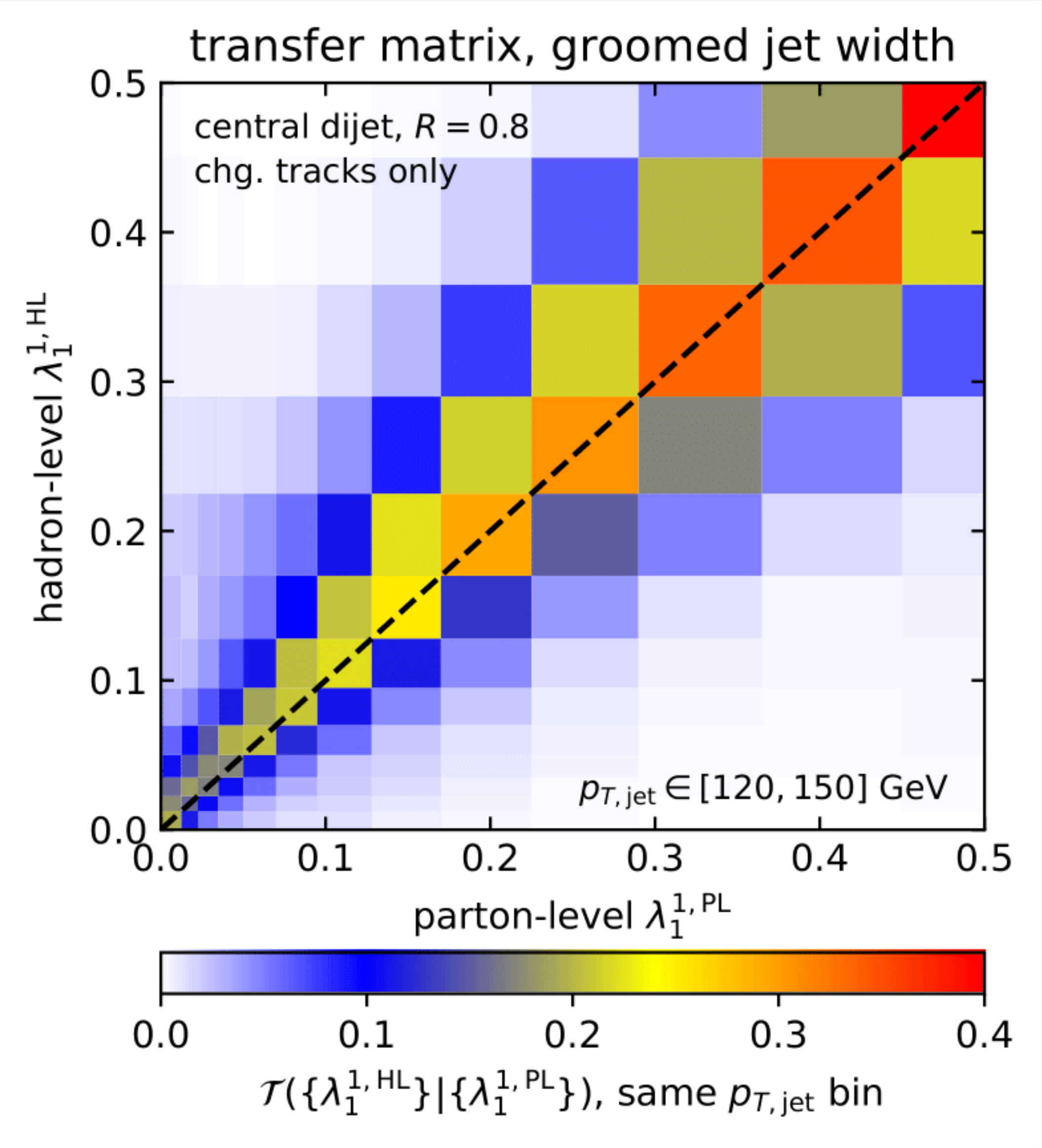
no pT = considering migration between different observable bins in this pT bin

limit pT migration = including also migration from the neighbouring pT bins

NLO+NLL'+NP: NP here means The full transfer matrix approach

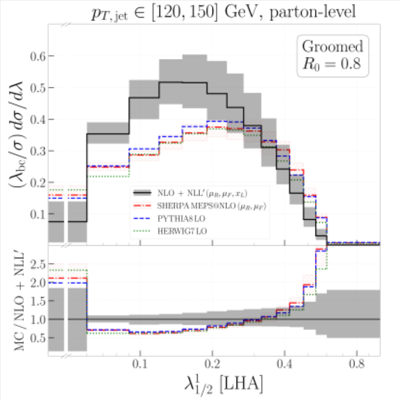
Transfer matrix:





Improved Zjet LHA:

NLO+NLL'
distribution



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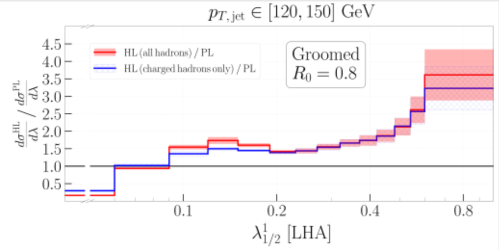

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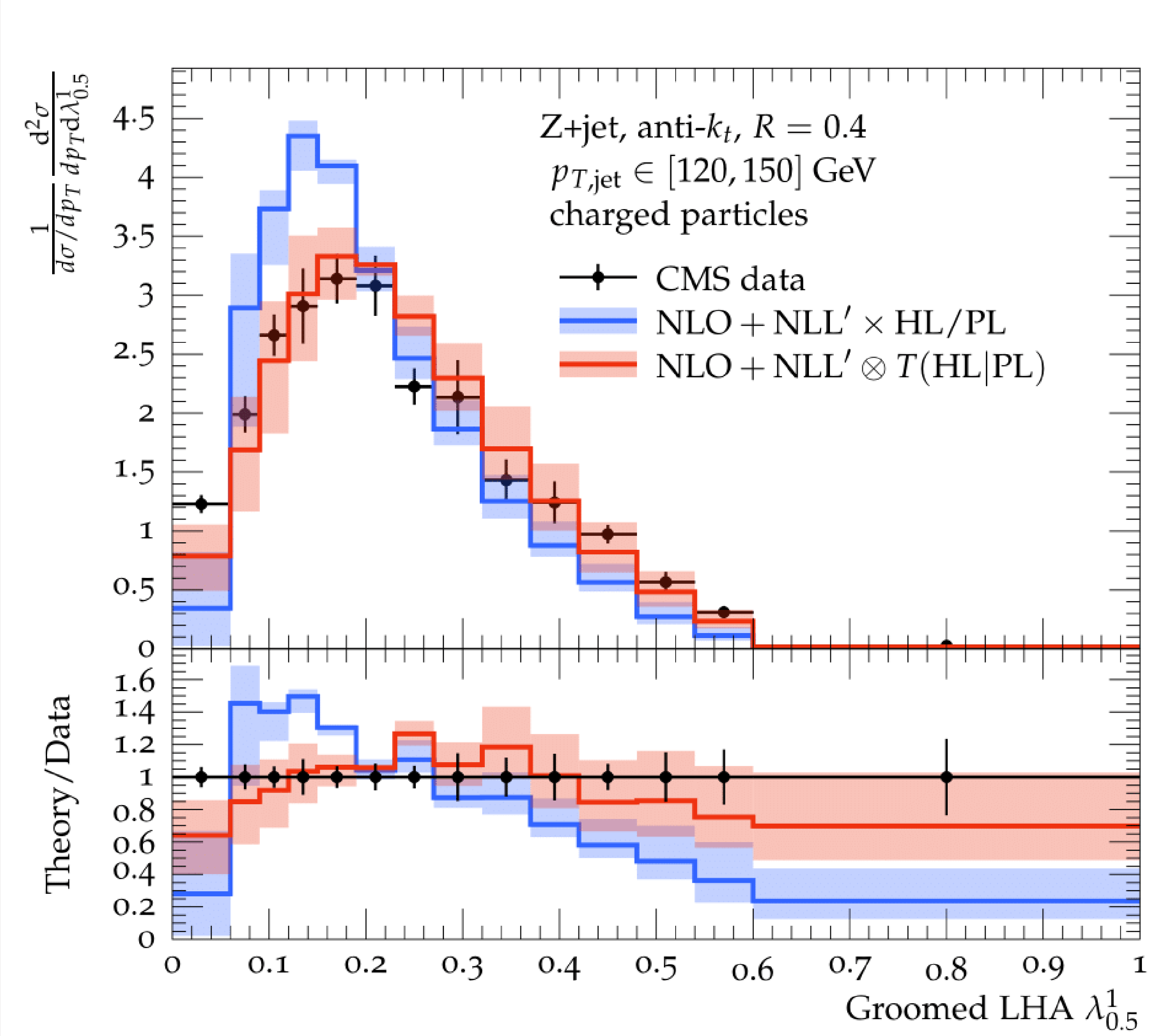
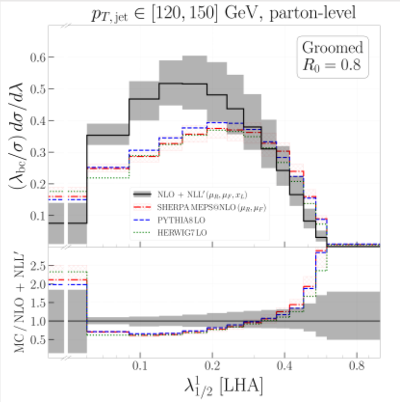


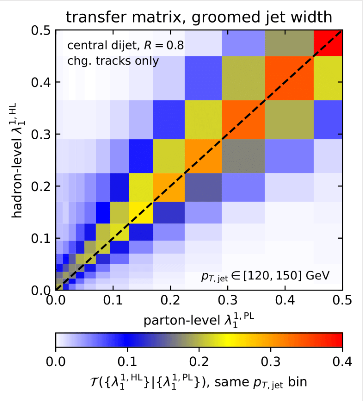
per-bin
multiplication

Improved Zjet LHA:

NLO+NLL'
distribution

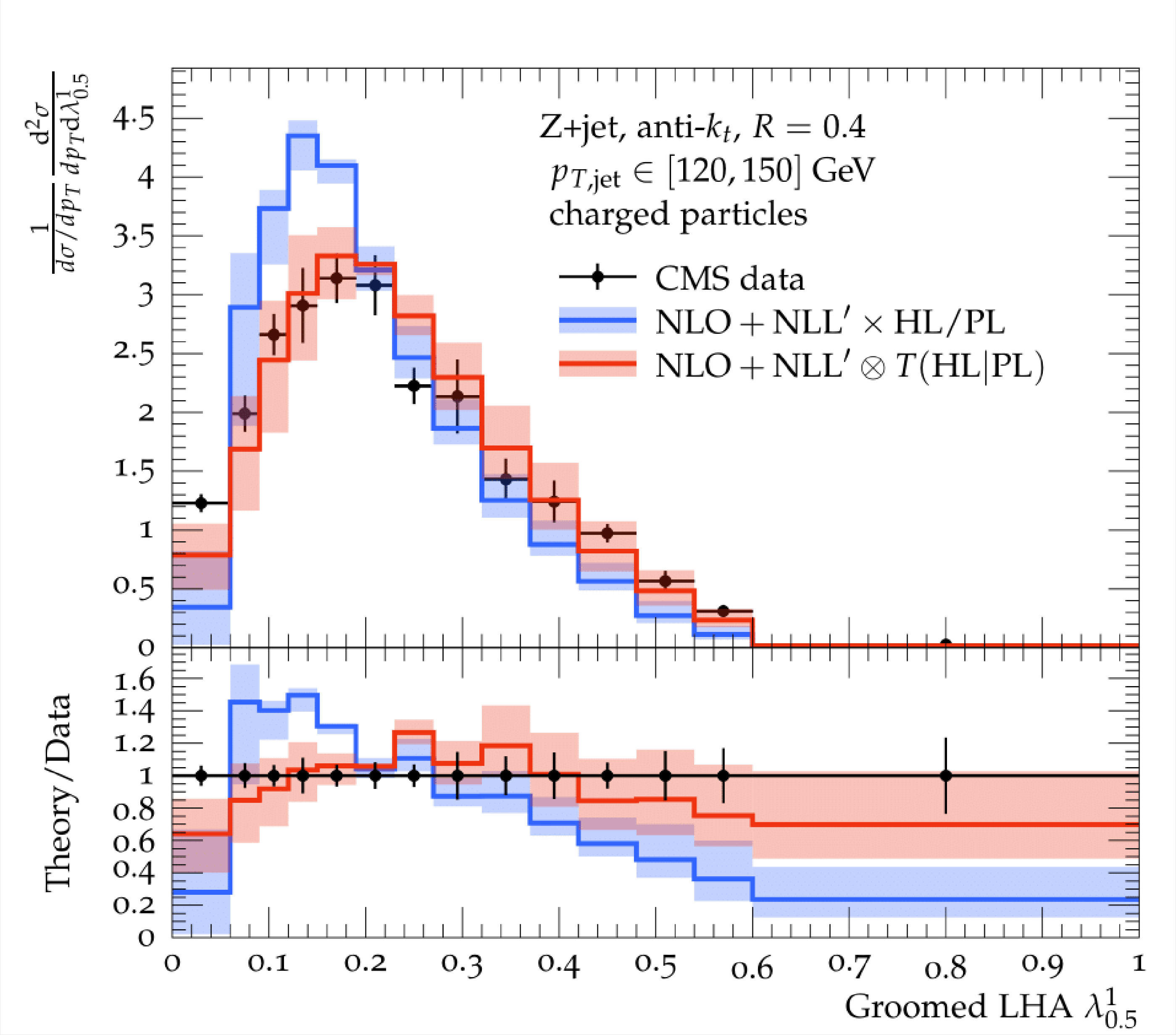


$\otimes$

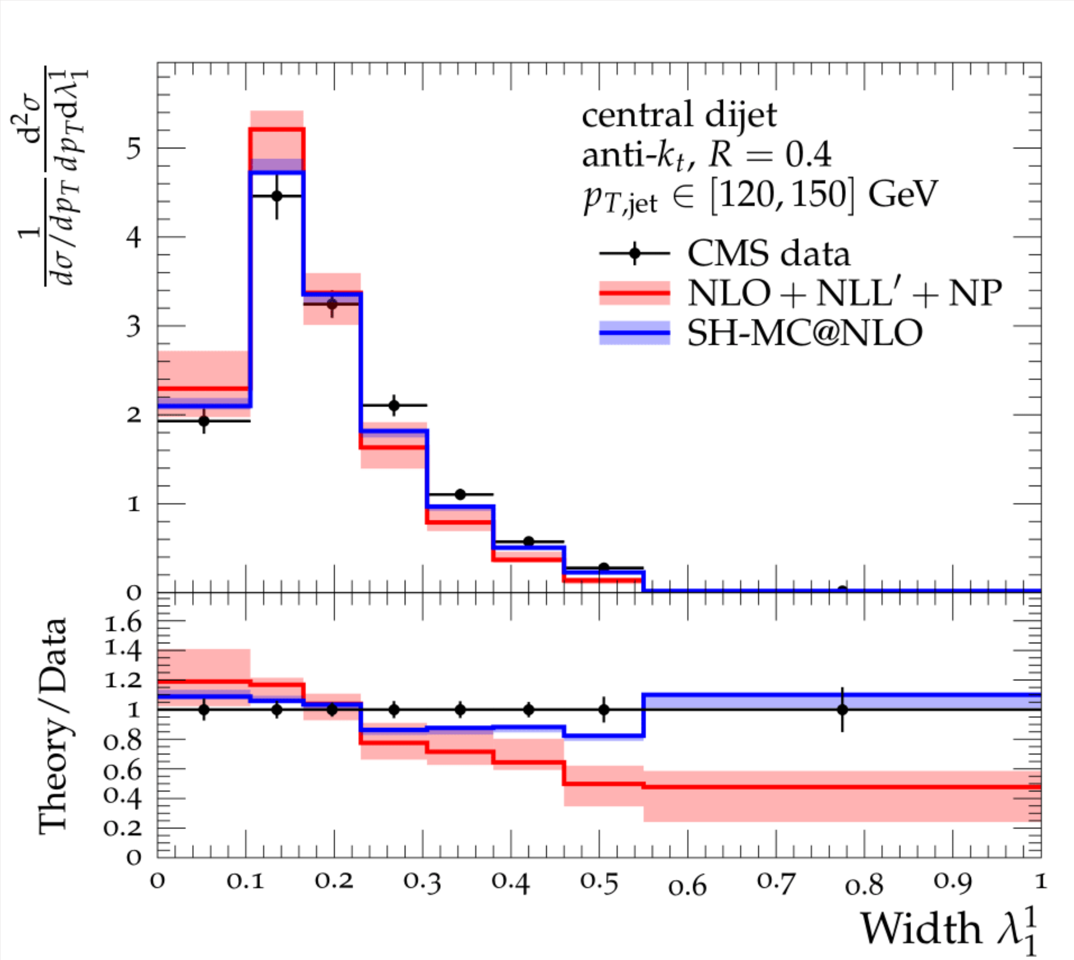


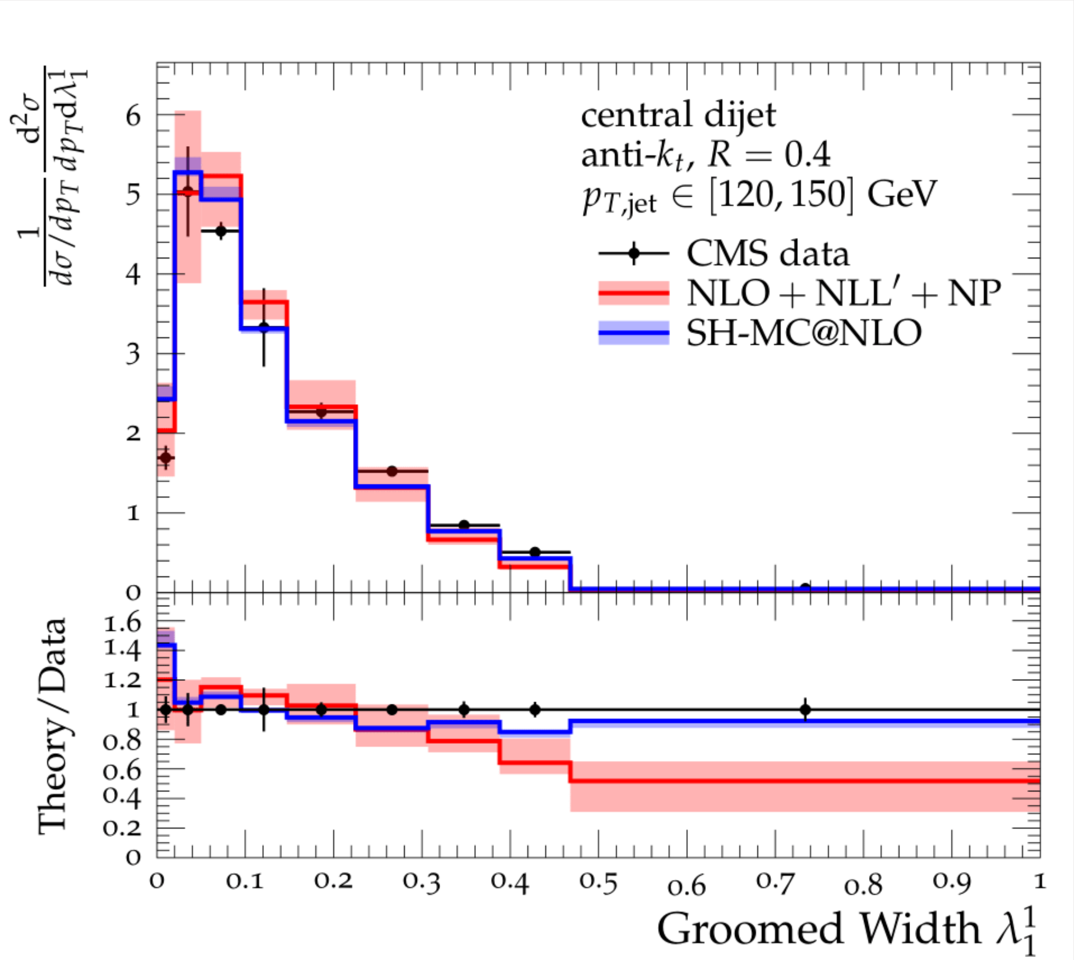
Transfer matrix

=



Dijet plots:





All the plots on https://www.theorie.physik.uni-goettingen.de/~sschuma/JetAngularities/

Conclusions and Outlook:

- Jet Angularity distributions (groomed and ungroomed $\otimes$ several p_T bins $\otimes R = 0.4$ and 0.8) at NLO+NLL'+NP are available and we found a fair agreement with CMS data. On the other hand, resummed calculations suffer from rather large theoretical uncertainities, dominated by variation of the resummation scale $\Rightarrow$ NNLL accuracy.

- The transfer matrix approach for dealing with NP corrections clearly gives an improved description of the data. It would be interesting to compare the transfer matrix approach to results obtained using first-principle field-theoretical arguments.

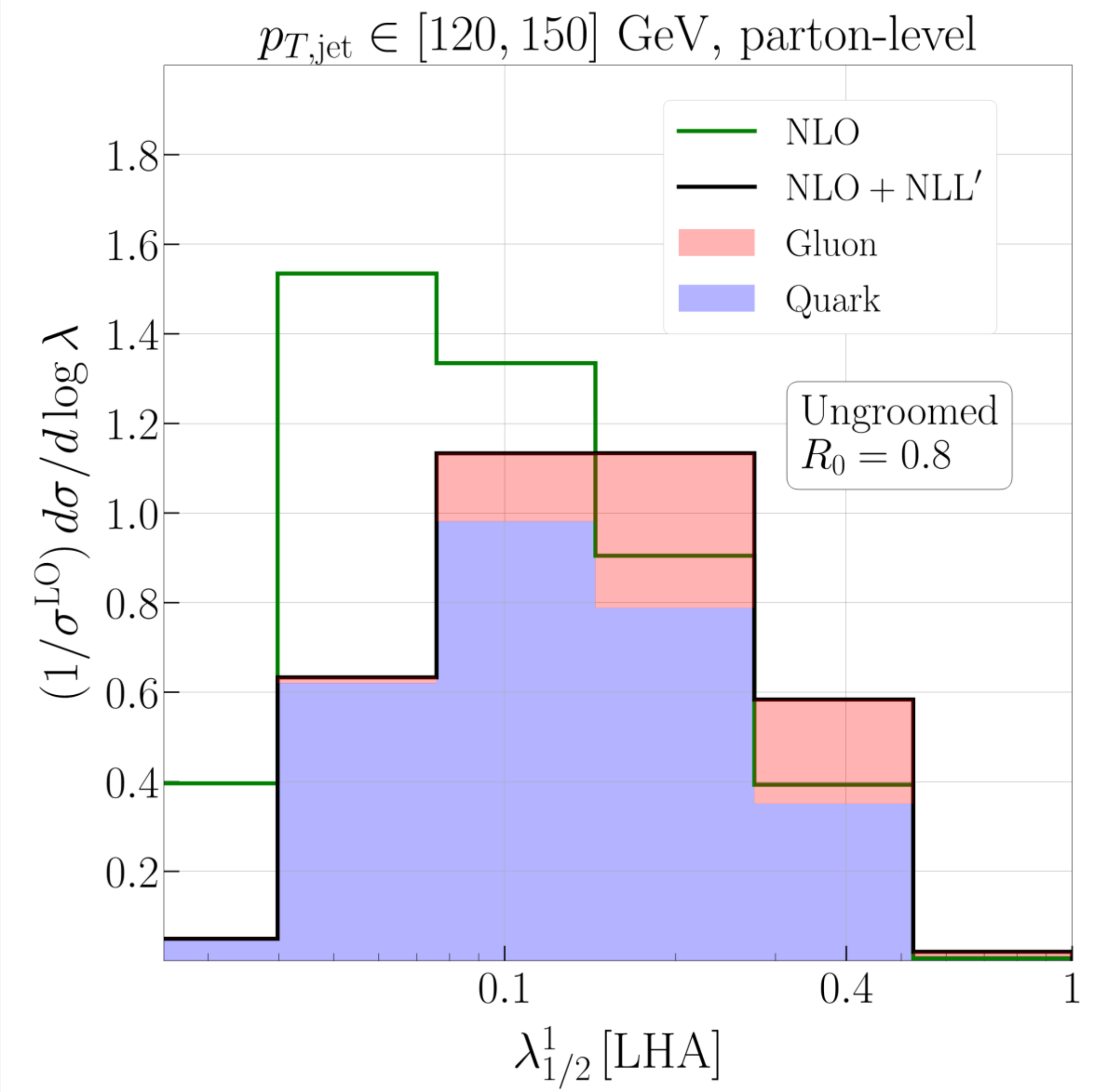
- Jet Angularities can be successfully used as q/g tagger in Z+jet events in order to significantly enhance the gluon-initiated contributions. What is the impact of this type of observable on PDF fits? We can also improve our tagger with the transfer matrix.

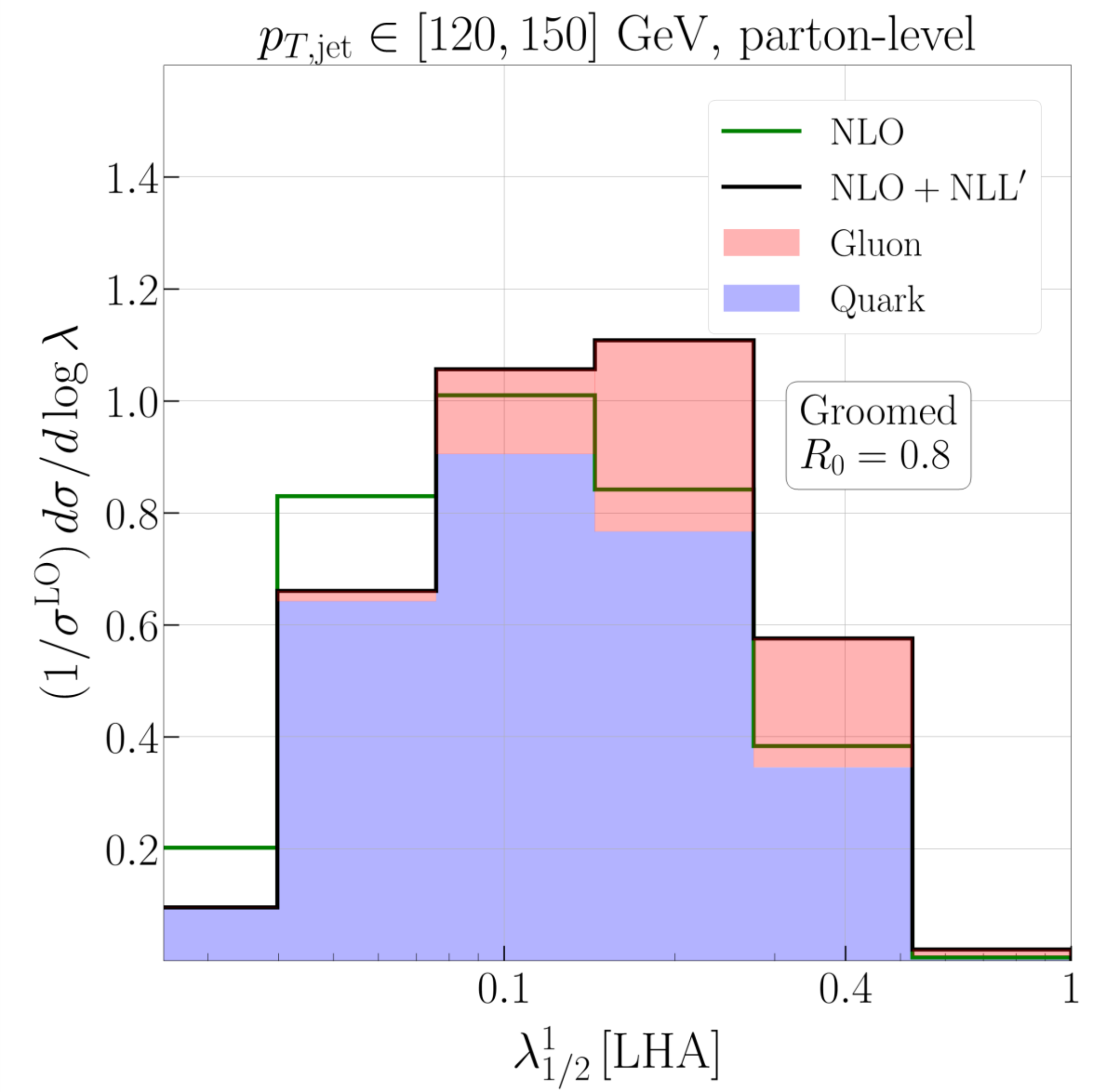
Thank You

Backup Slides

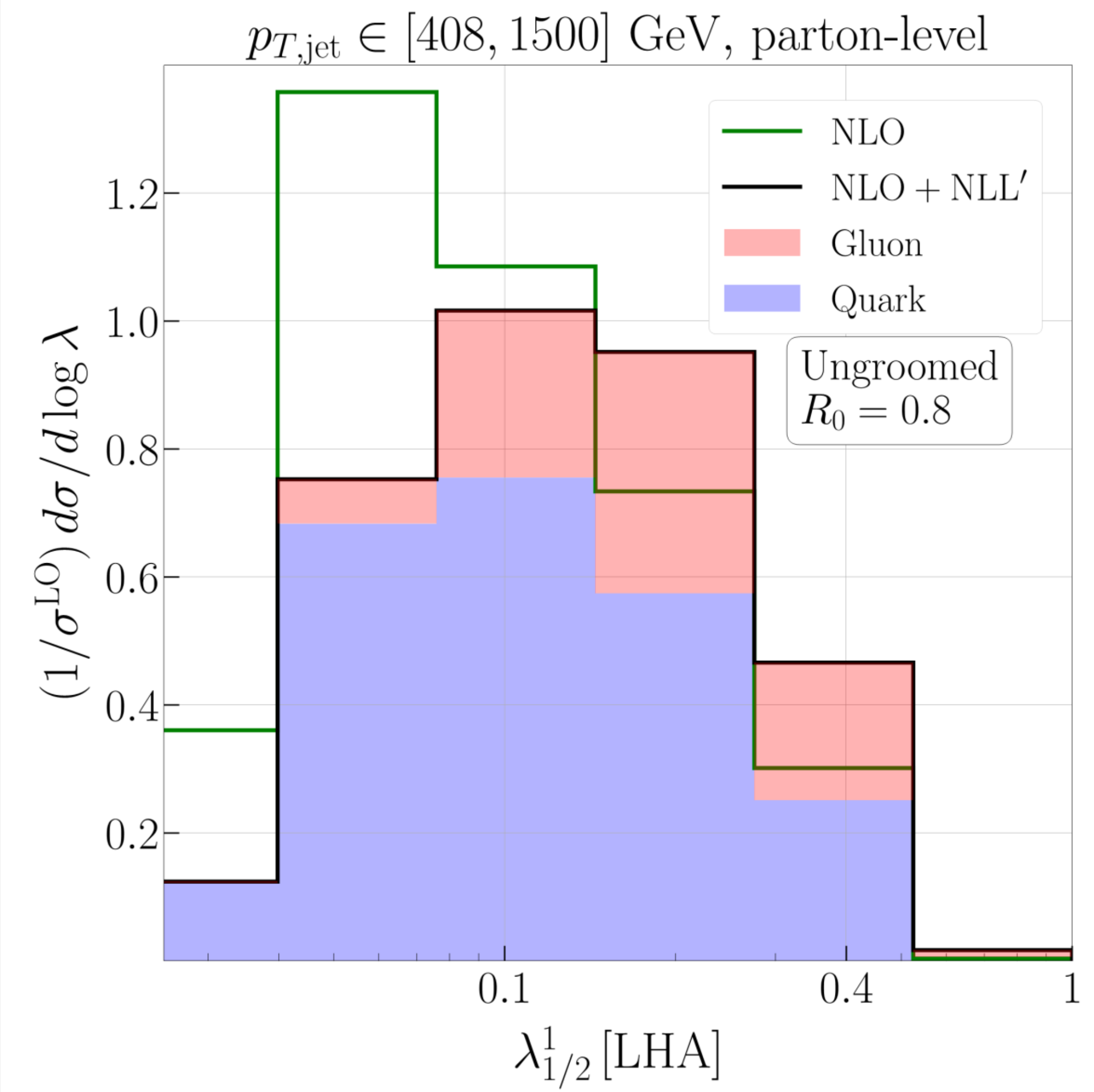
- Fixed-order vs Matched
- NP corrections (HL/PL)
- NLO + NLL' + HL/PL
- Discarded slides

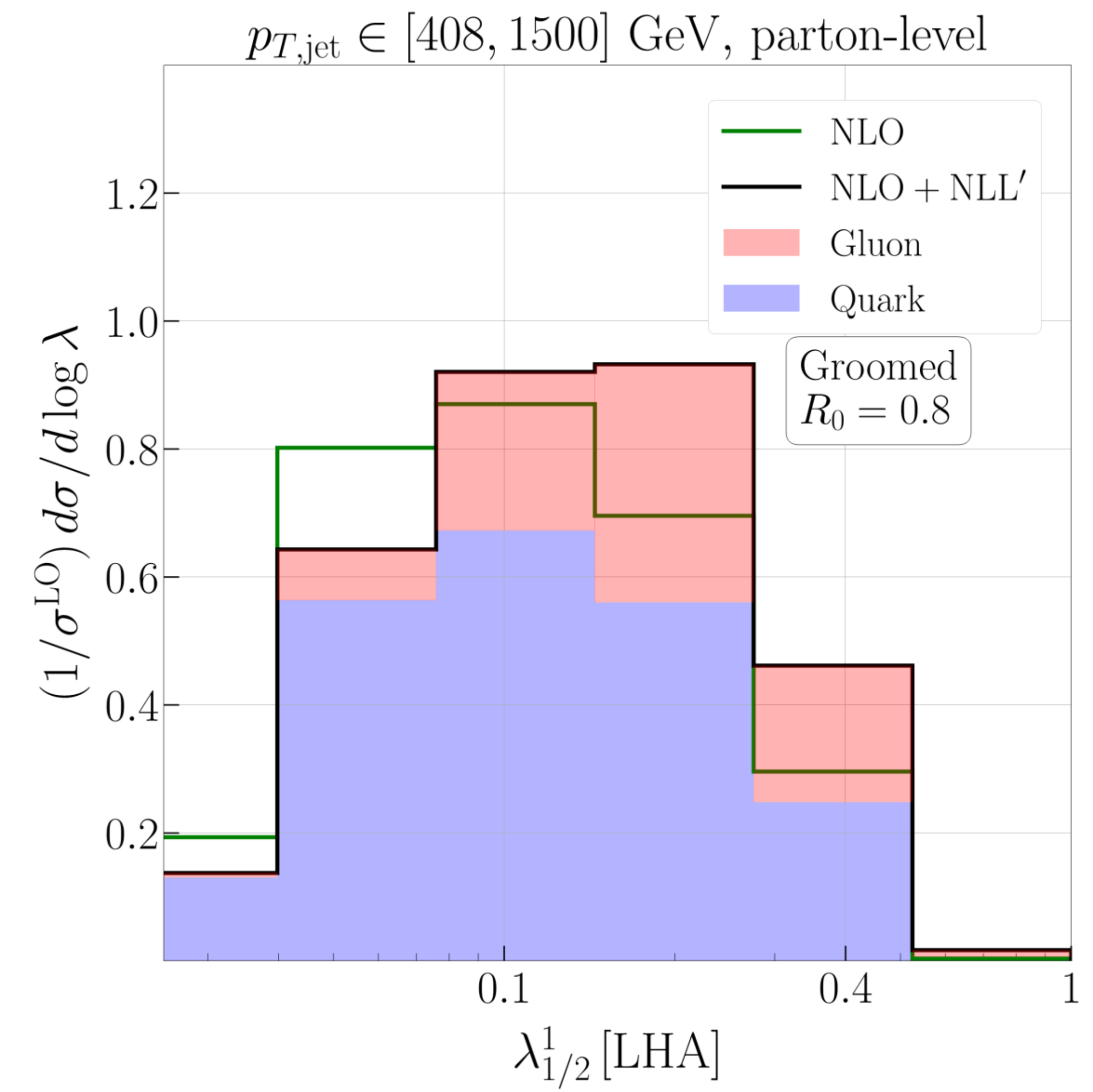
Fixed-order vs. matched (Zjet LHA low-pT):



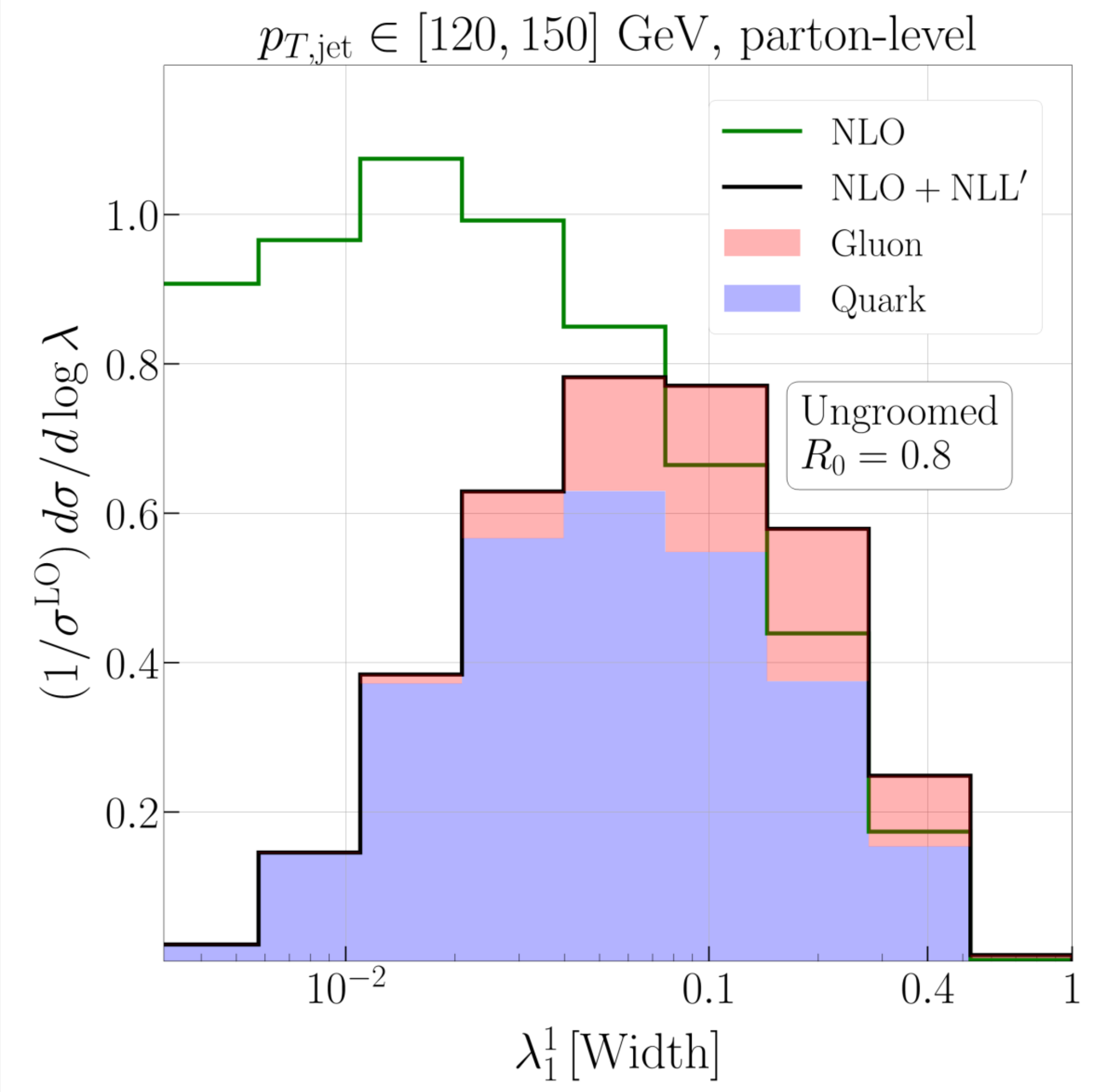


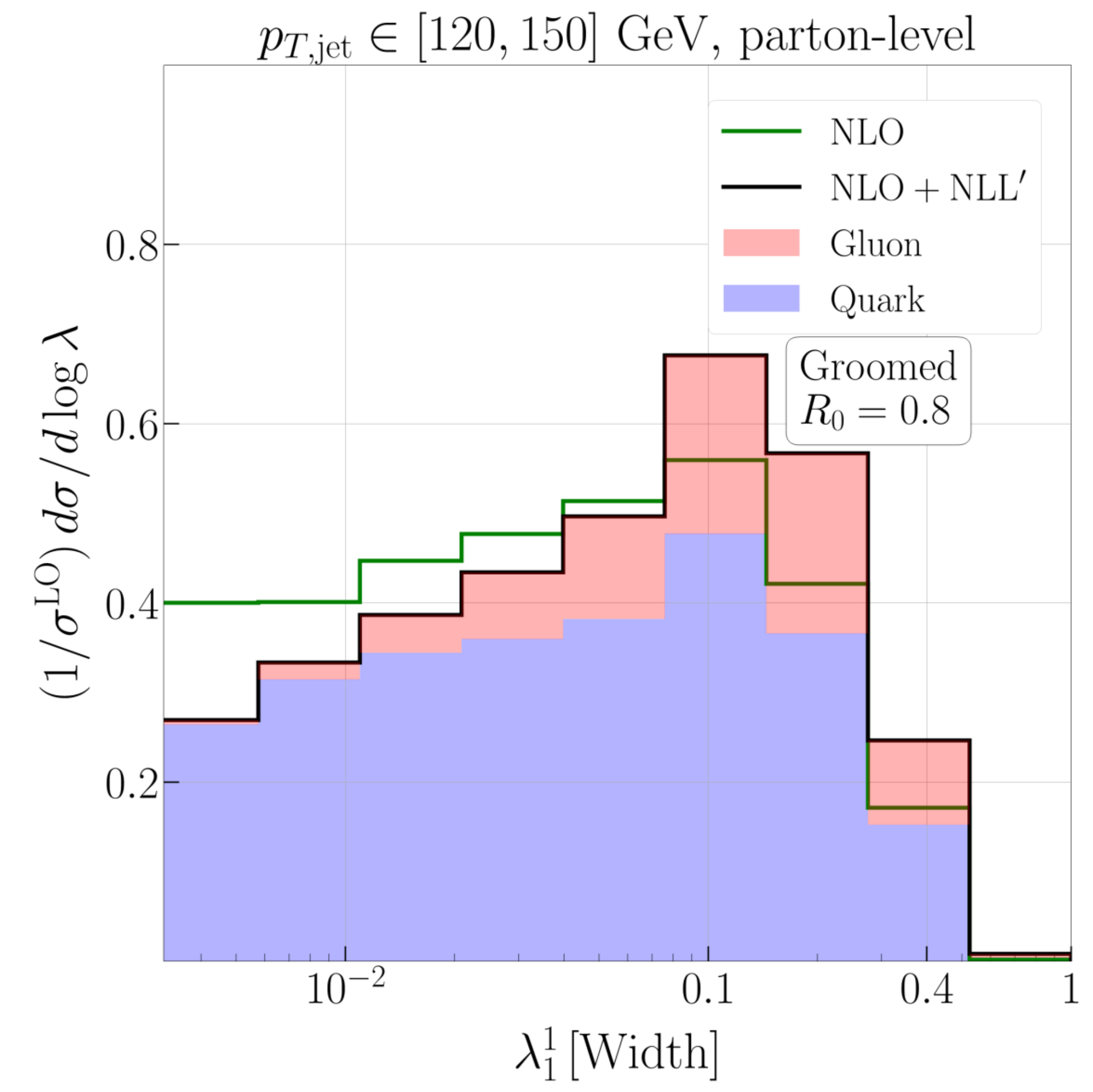
Fixed-order vs. matched (Zjet LHA high-pT):



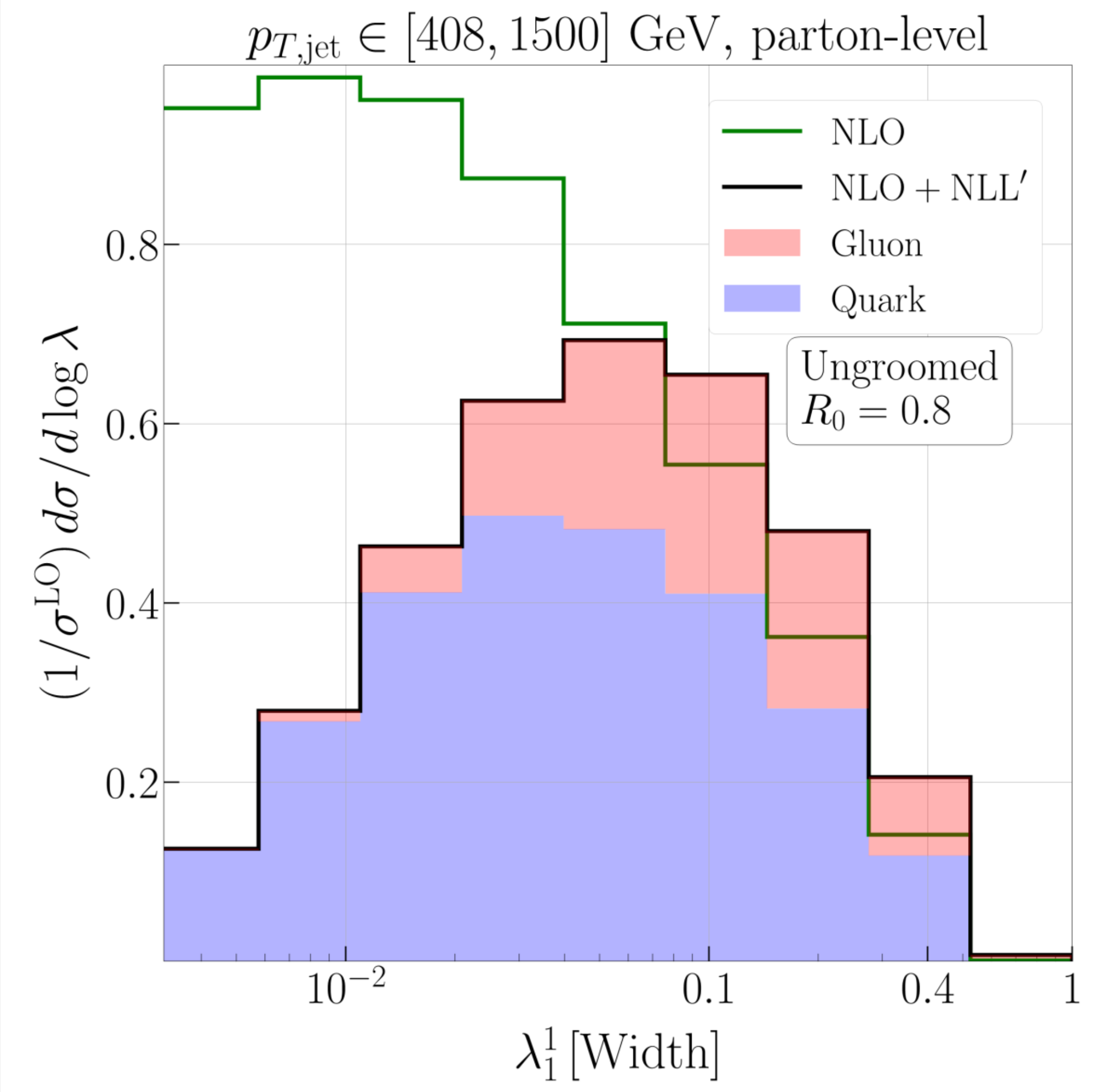


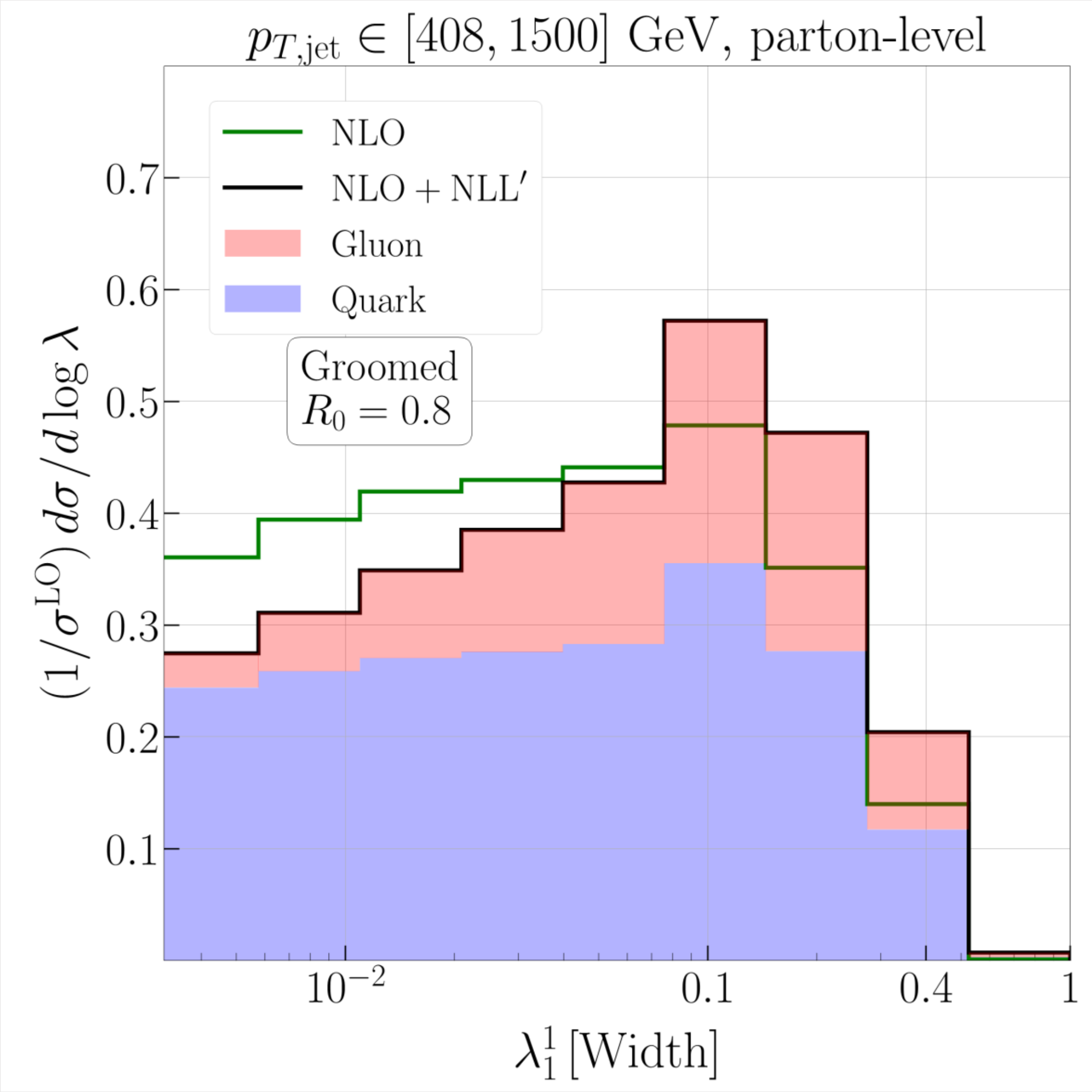
Fixed-order vs. matched (Zjet Width low-pT):



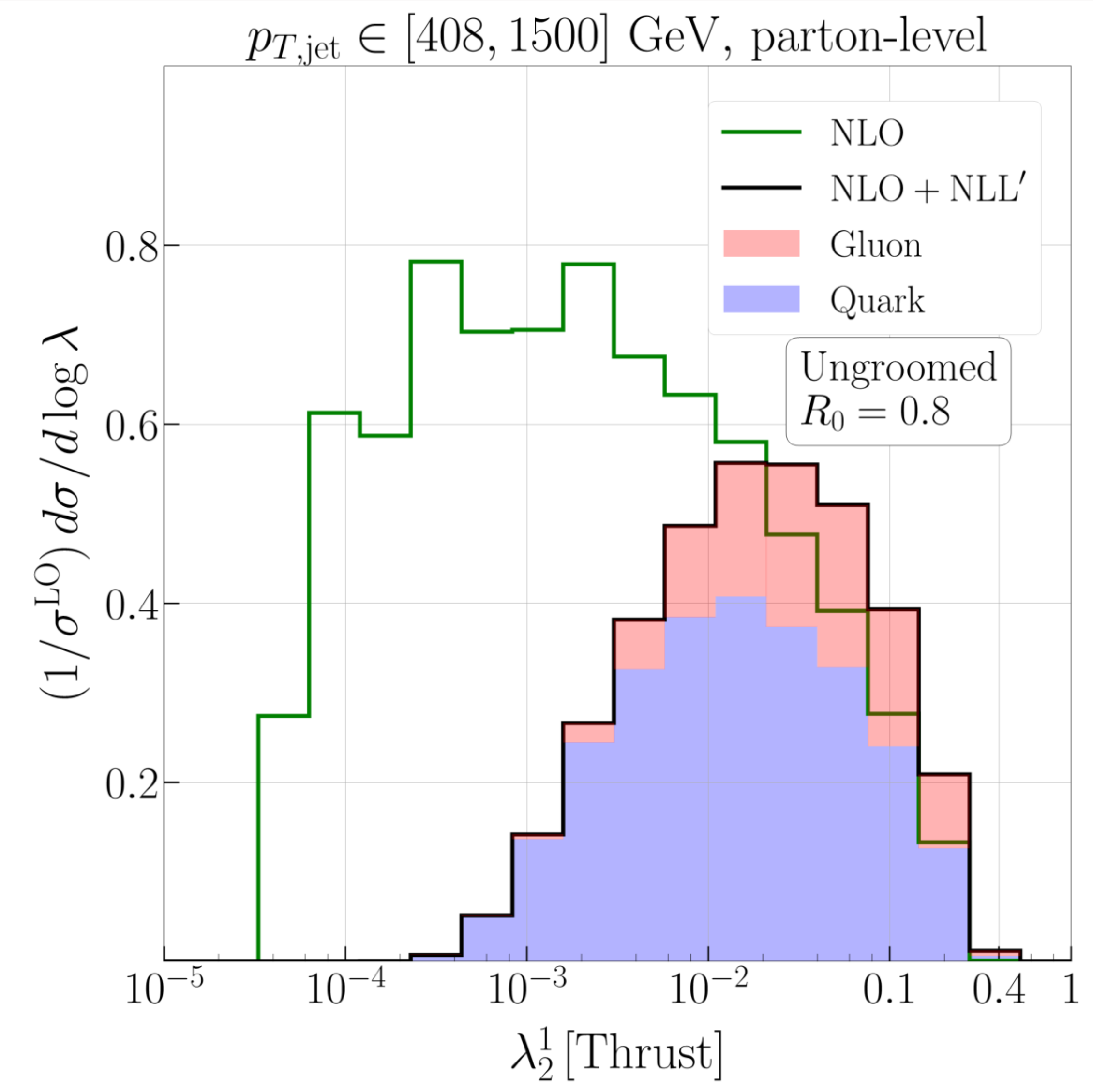


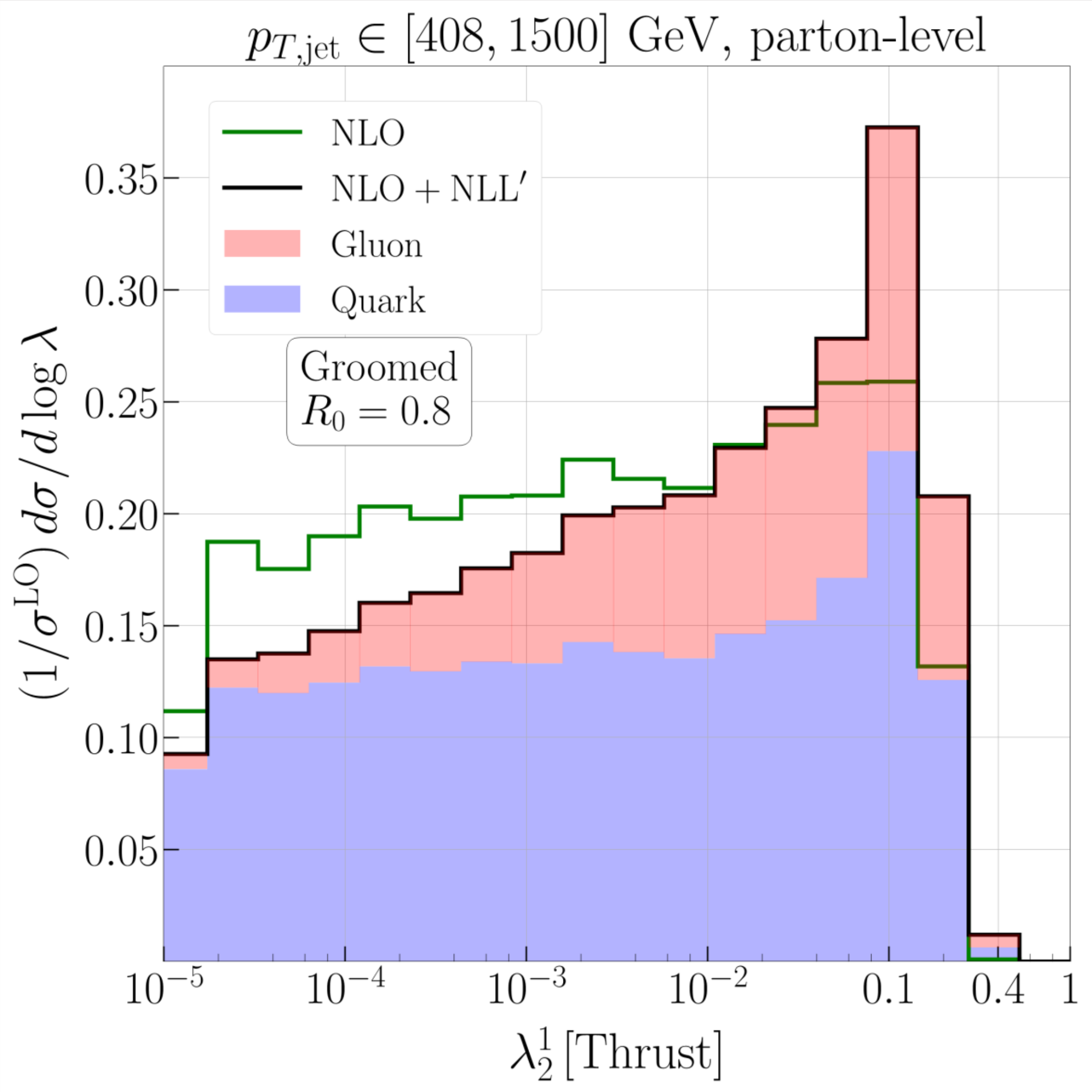
Fixed-order vs. matched (Zjet Width high-pT):



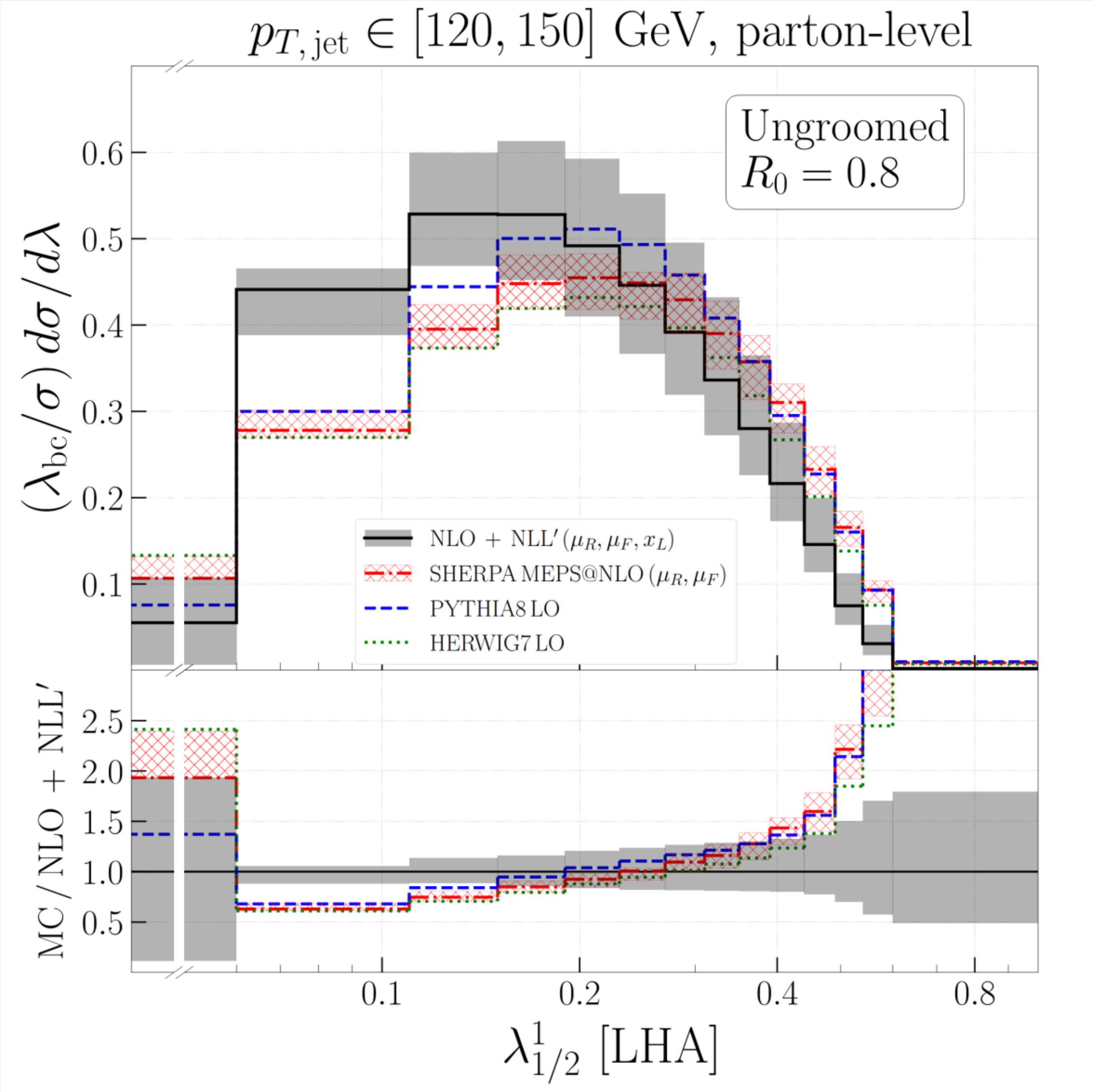


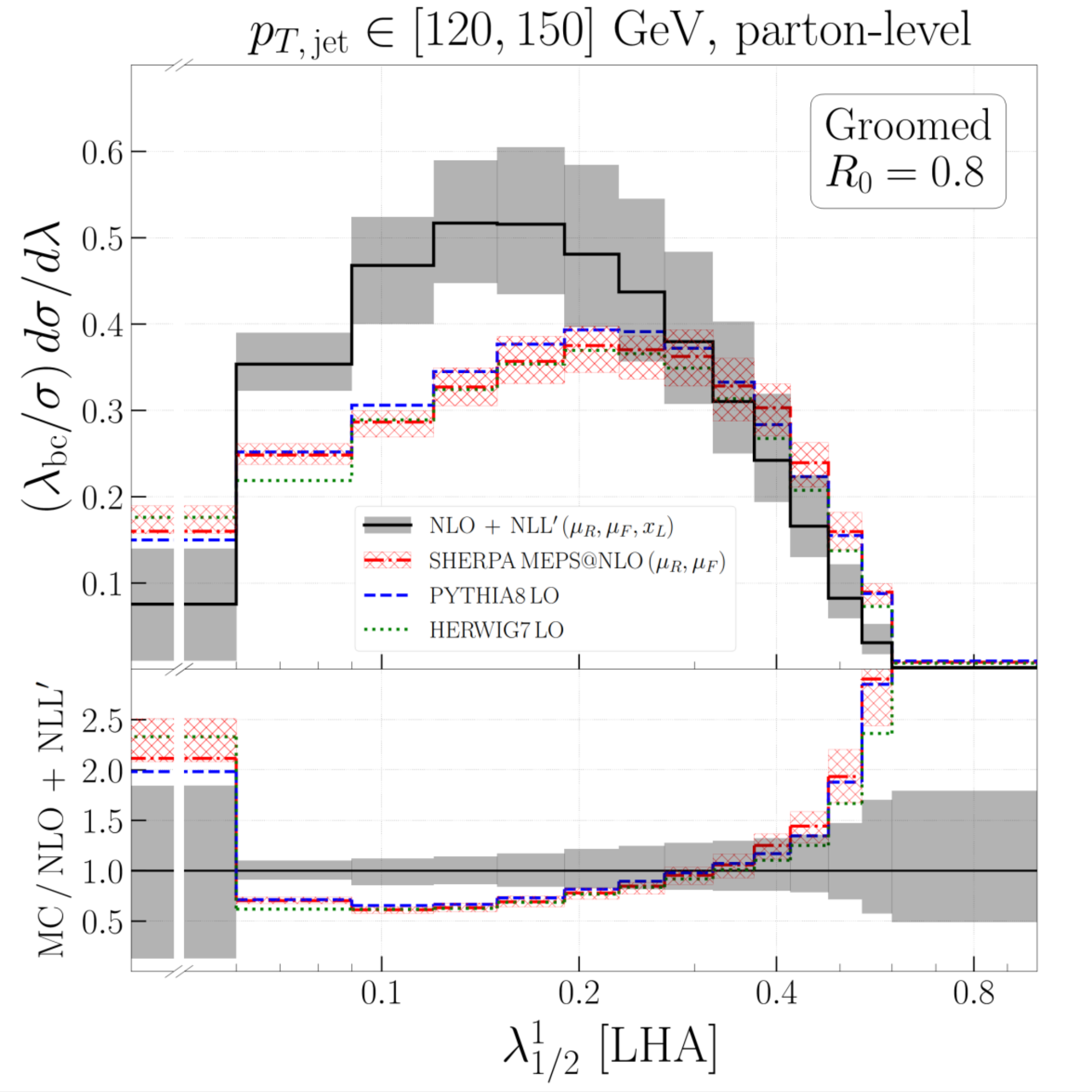
Fixed-order vs. matched (Zjet Thrust high-pT):



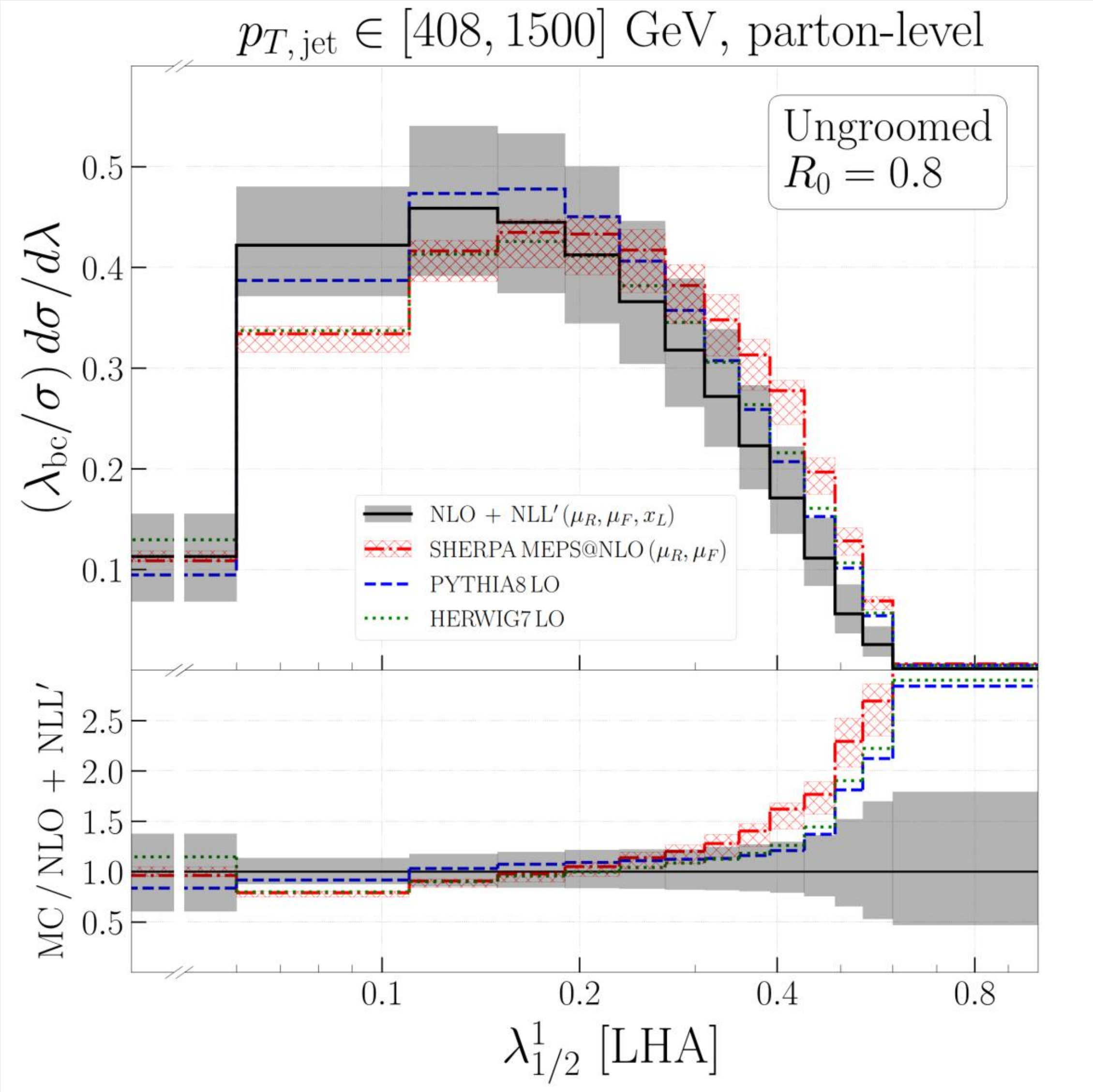


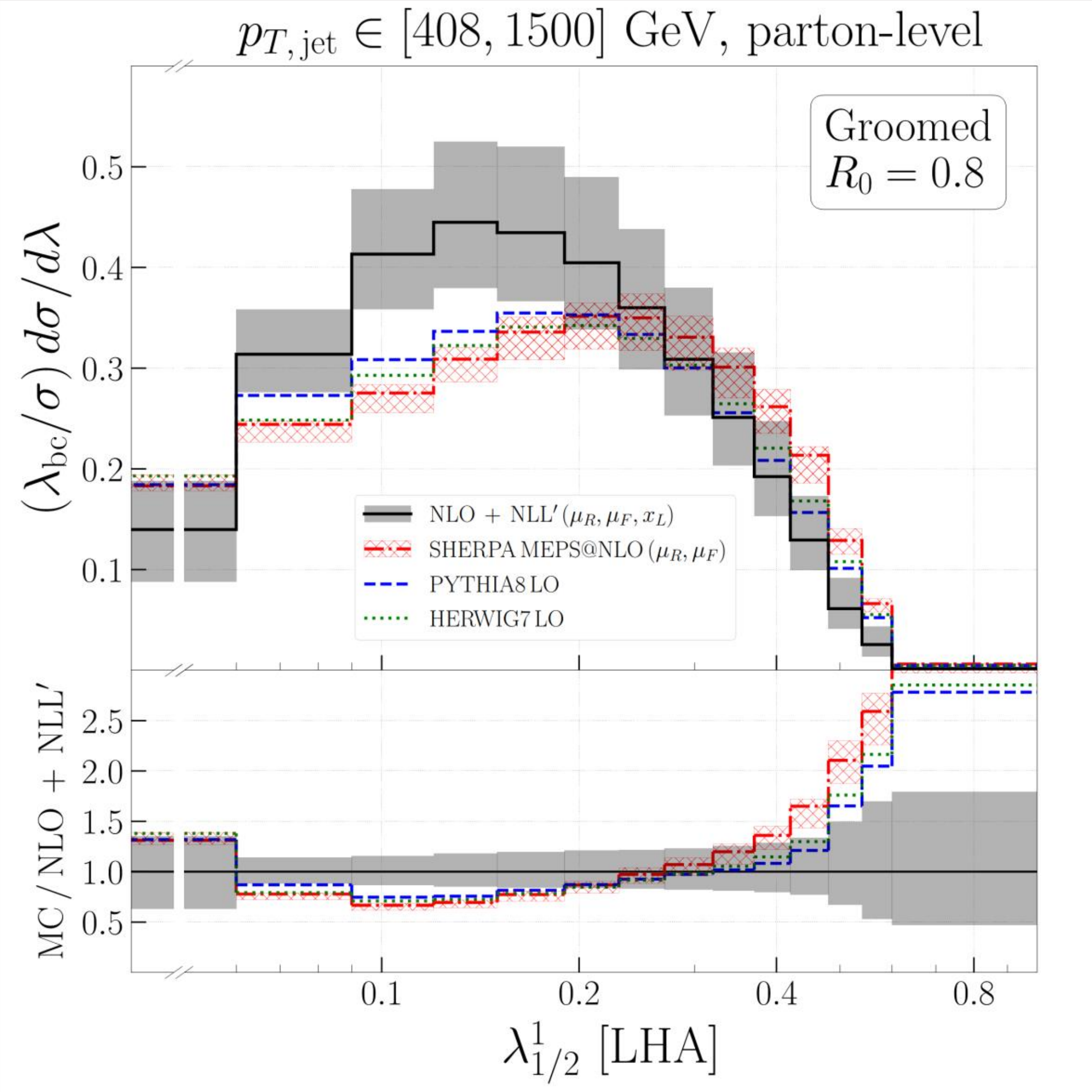
NLO + NLL′ (Zjet LHA low-pT):



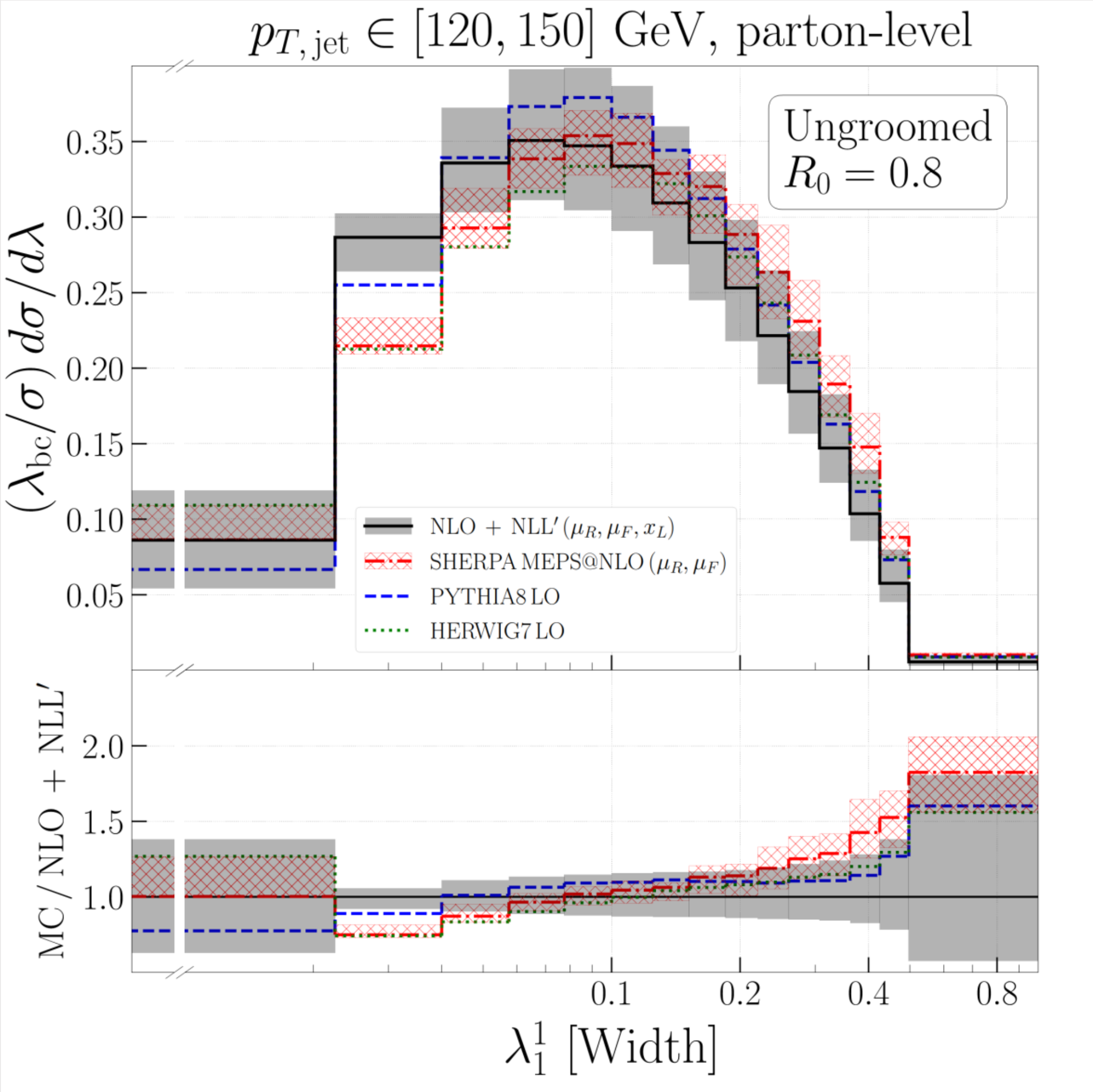


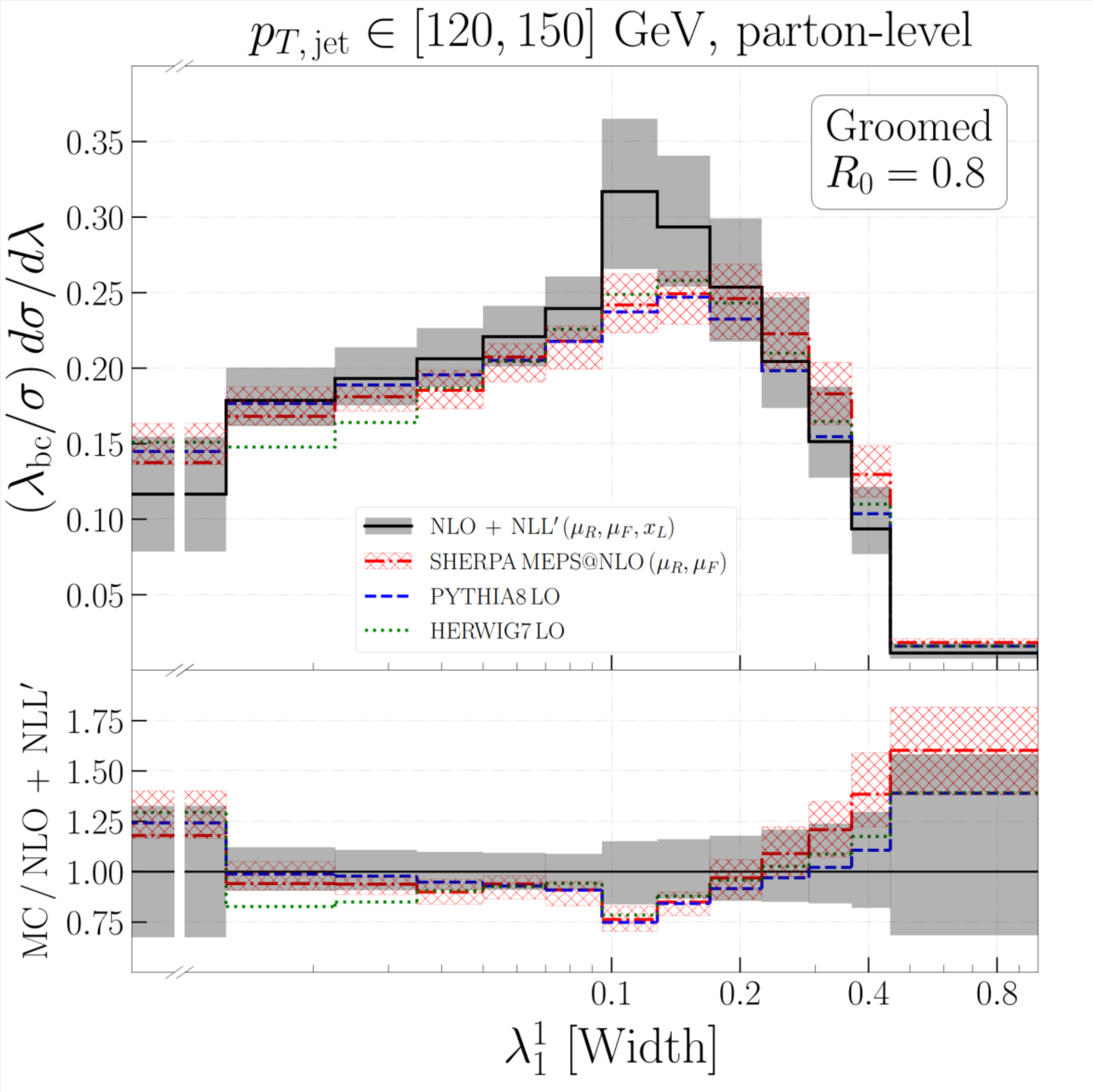
NLO + NLL′ (Zjet LHA high-pT):



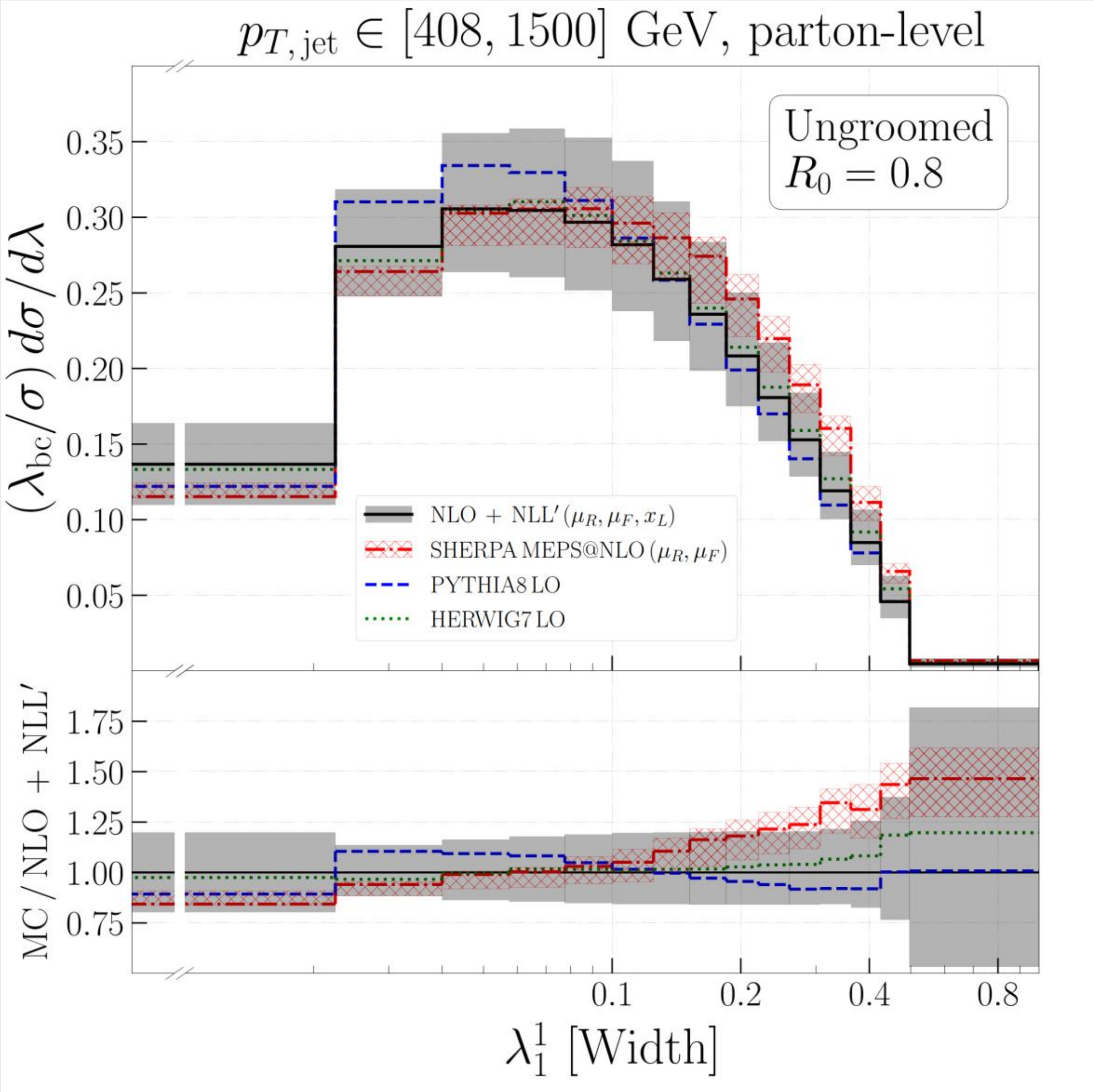


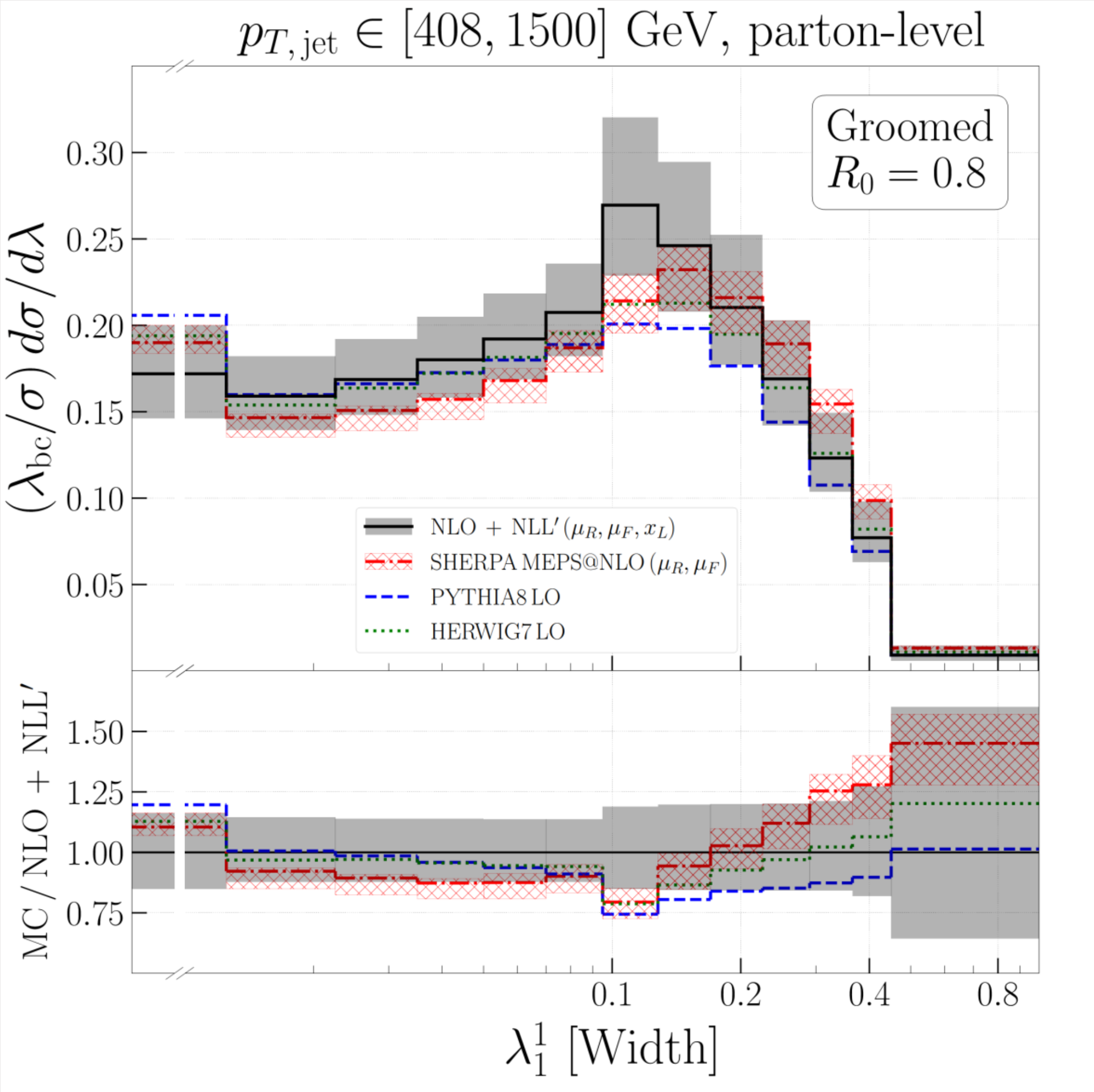
NLO + NLL′ (Zjet Width low-pT):



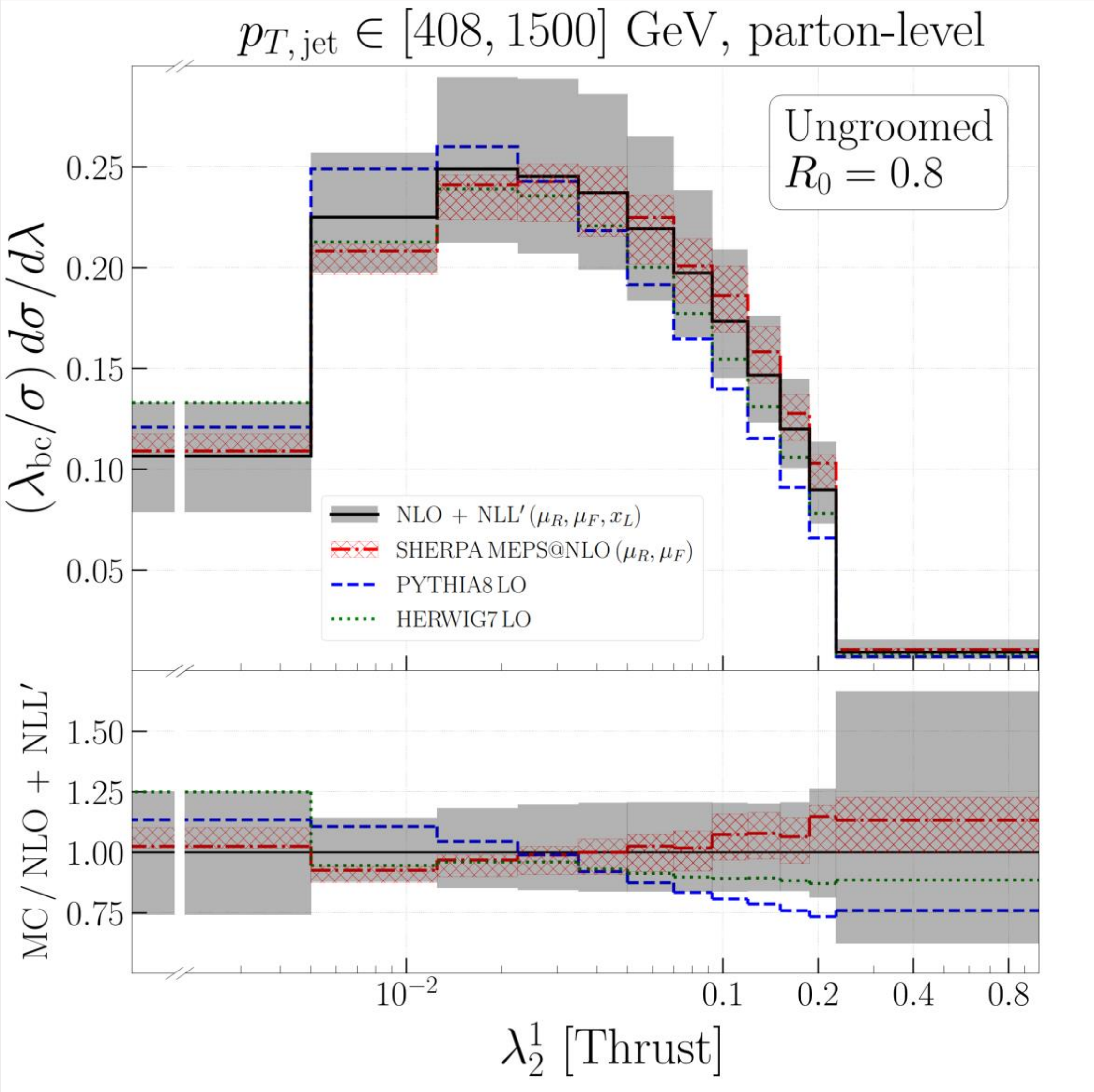


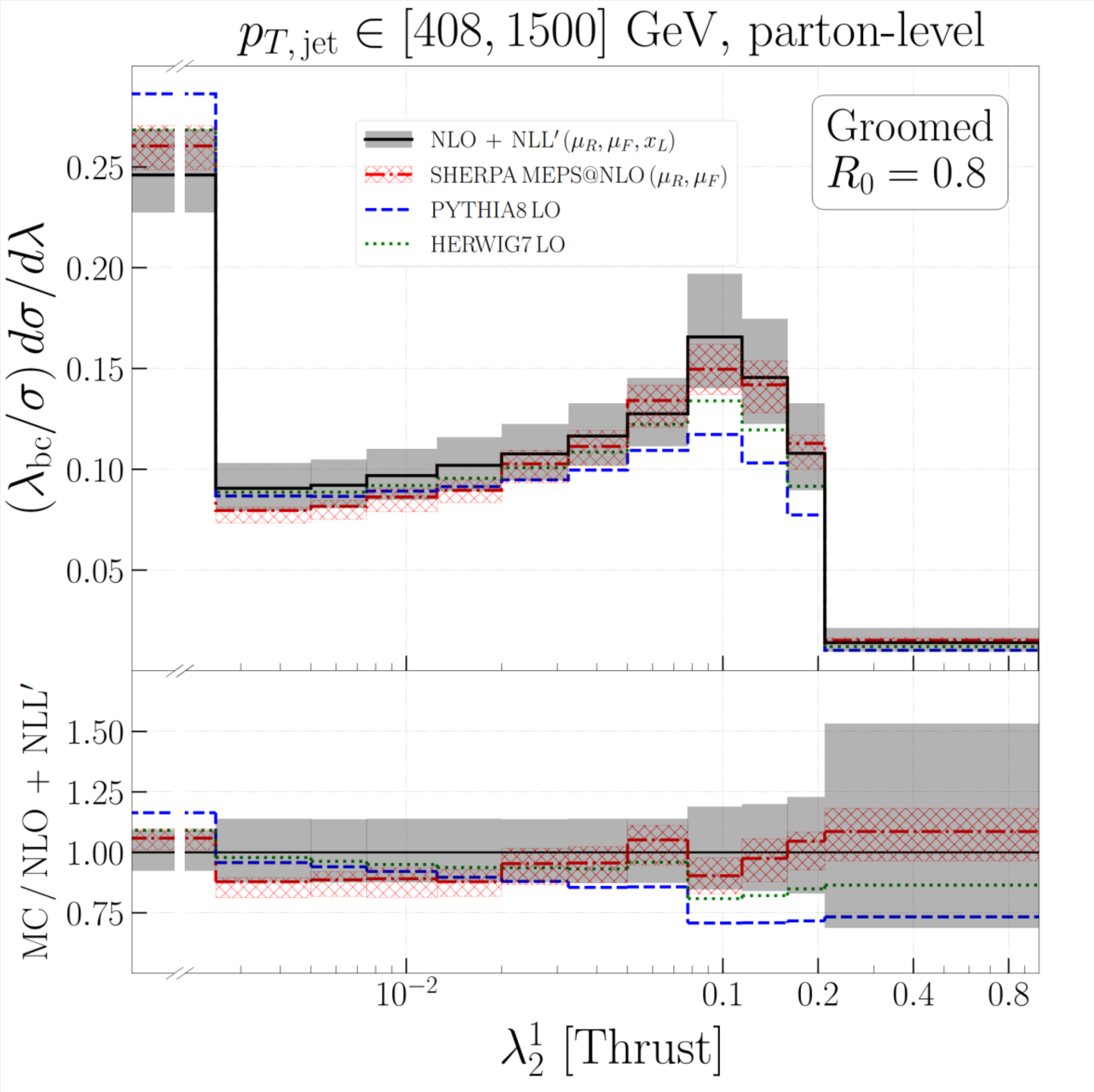
NLO + NLL′ (Zjet Width high-pT):



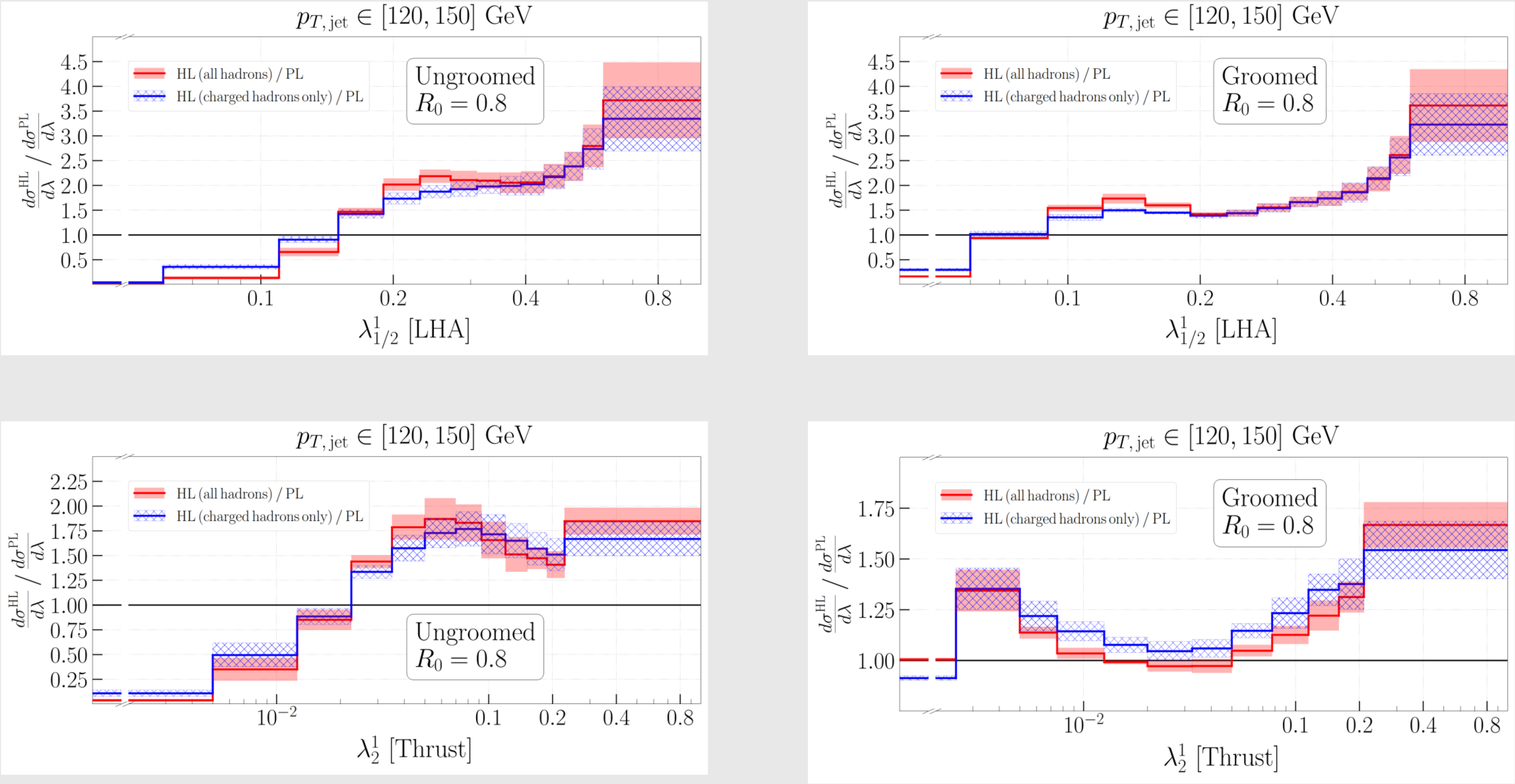


NLO + NLL′ (Zjet Thrust high-pT):

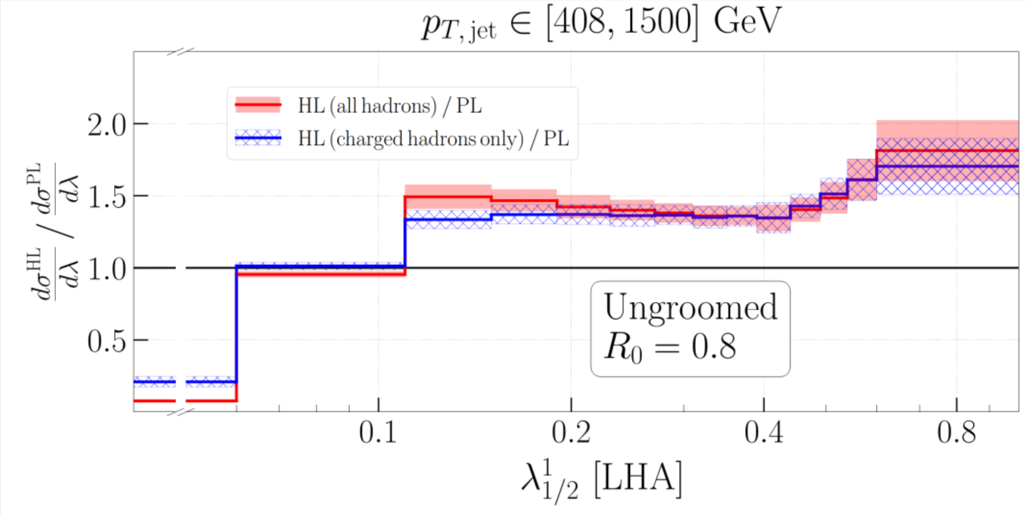


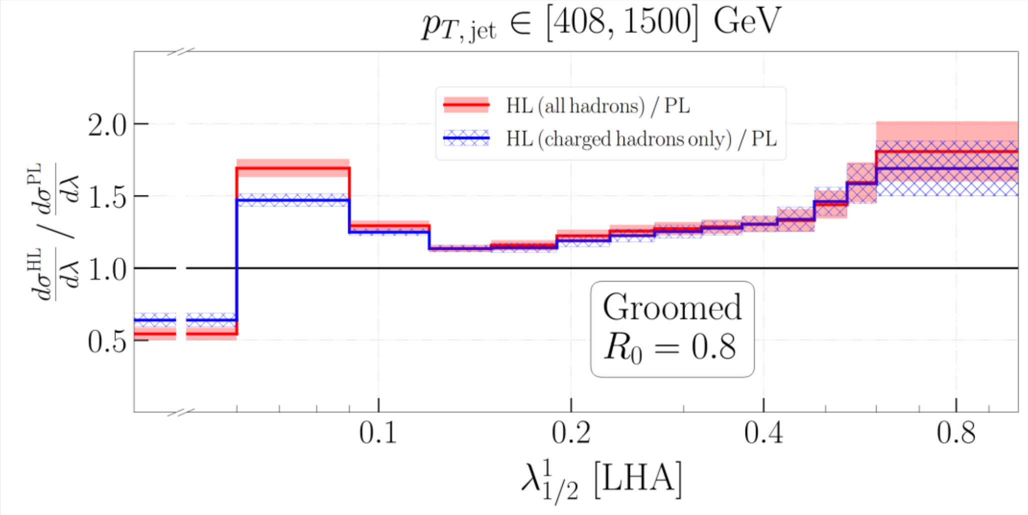


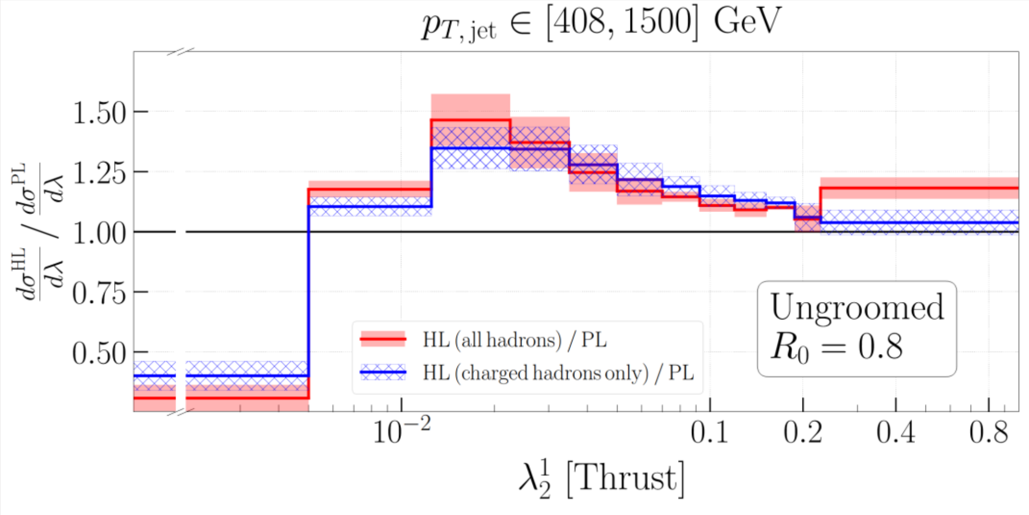
NP corrections from MC:

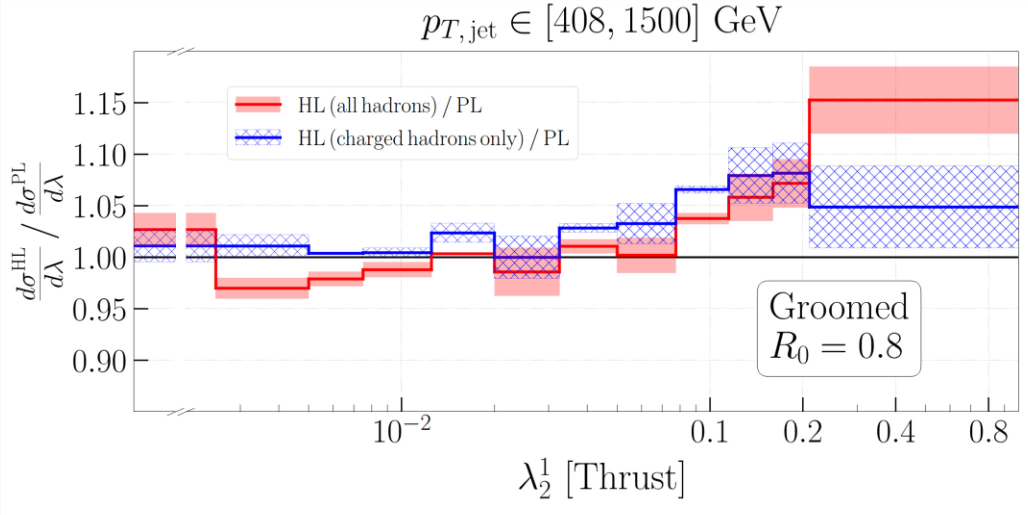


NP corrections from MC:

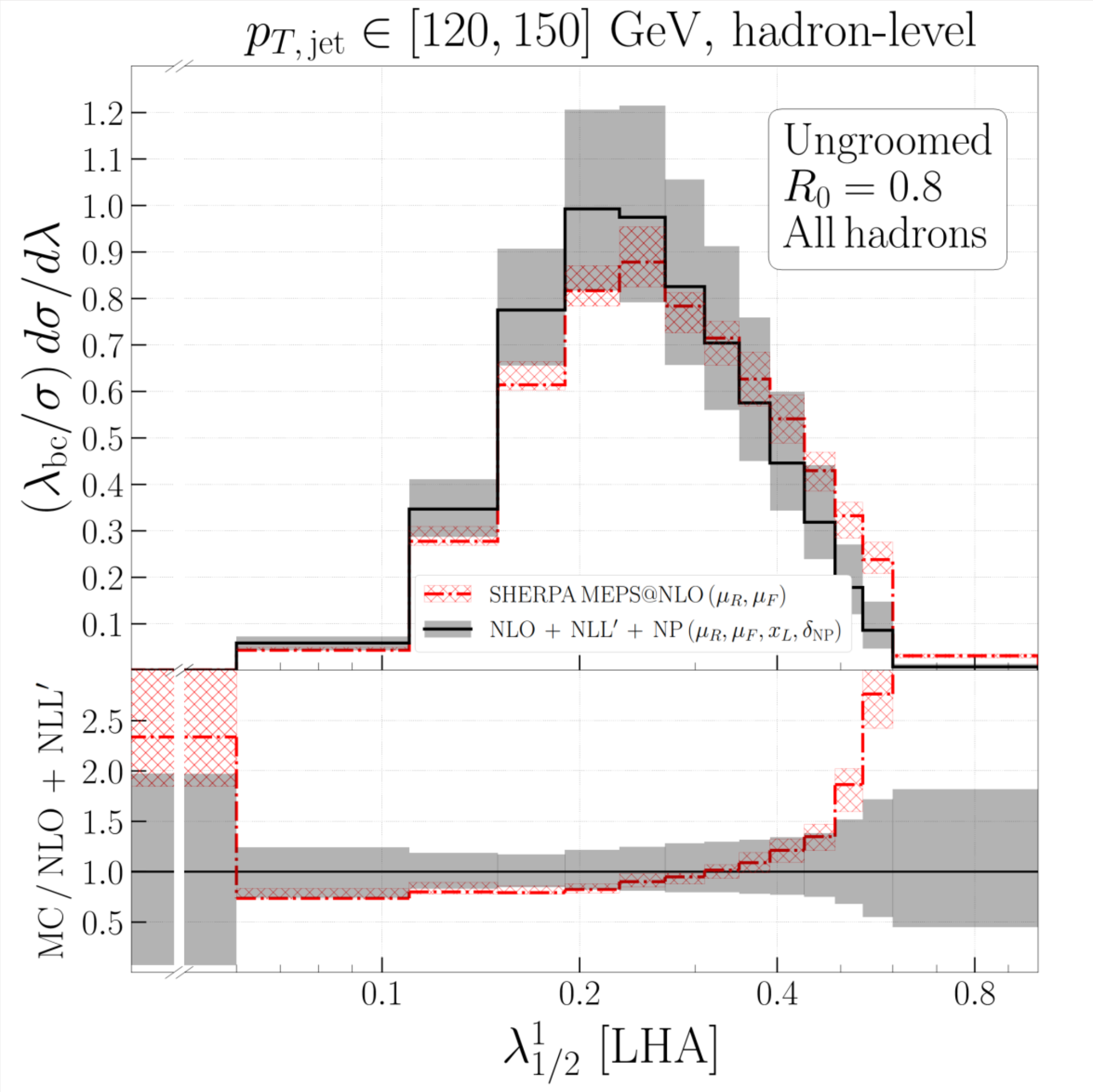


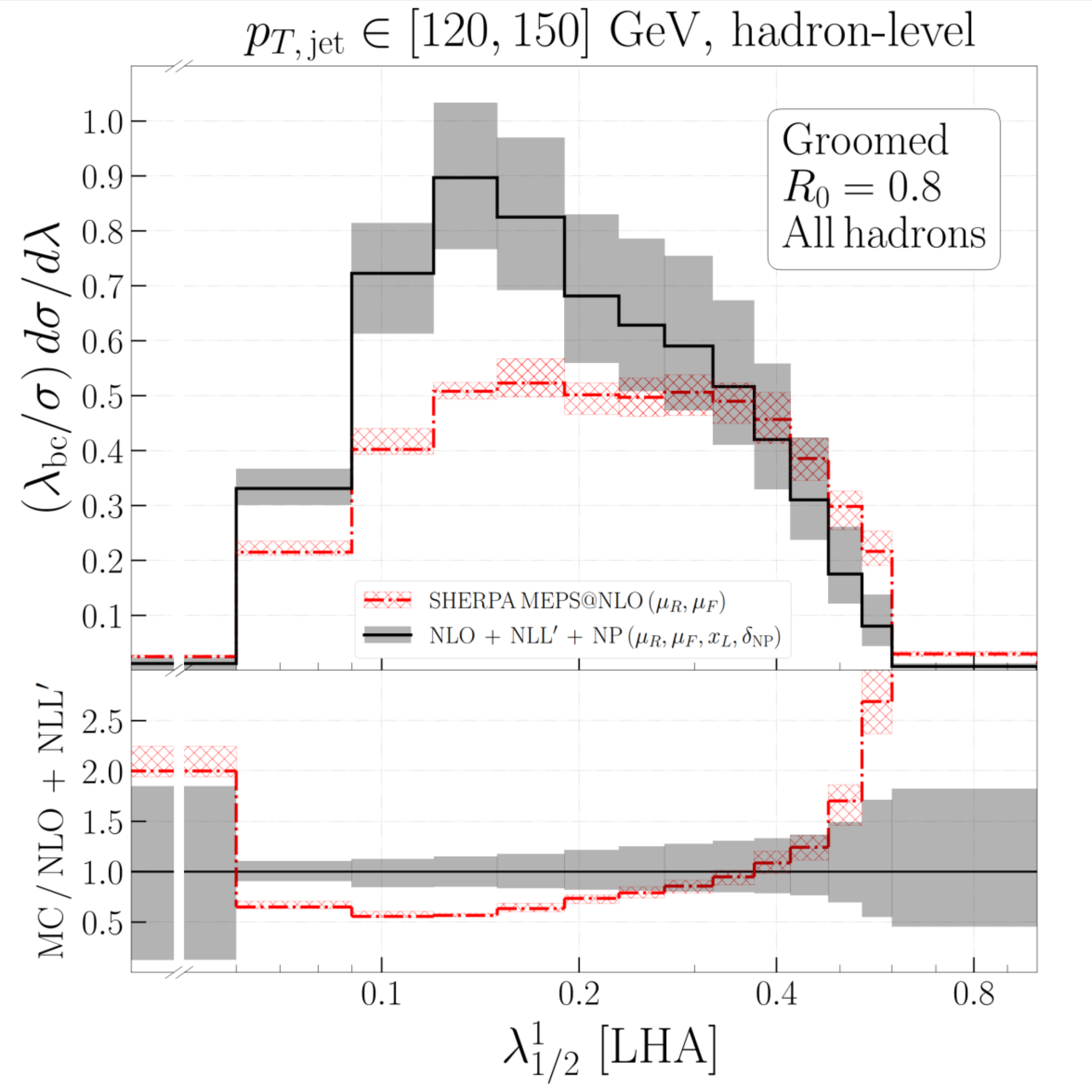




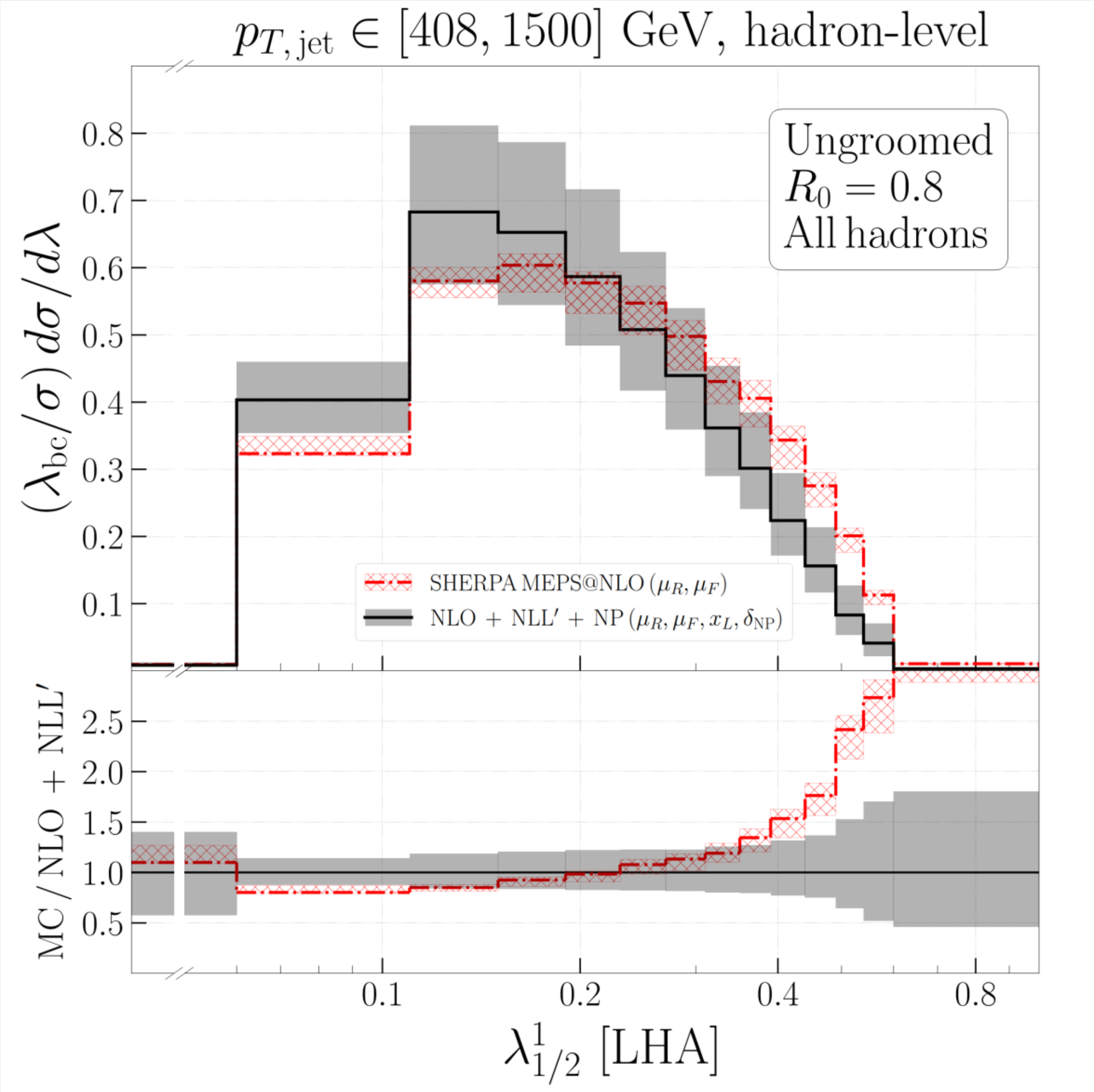


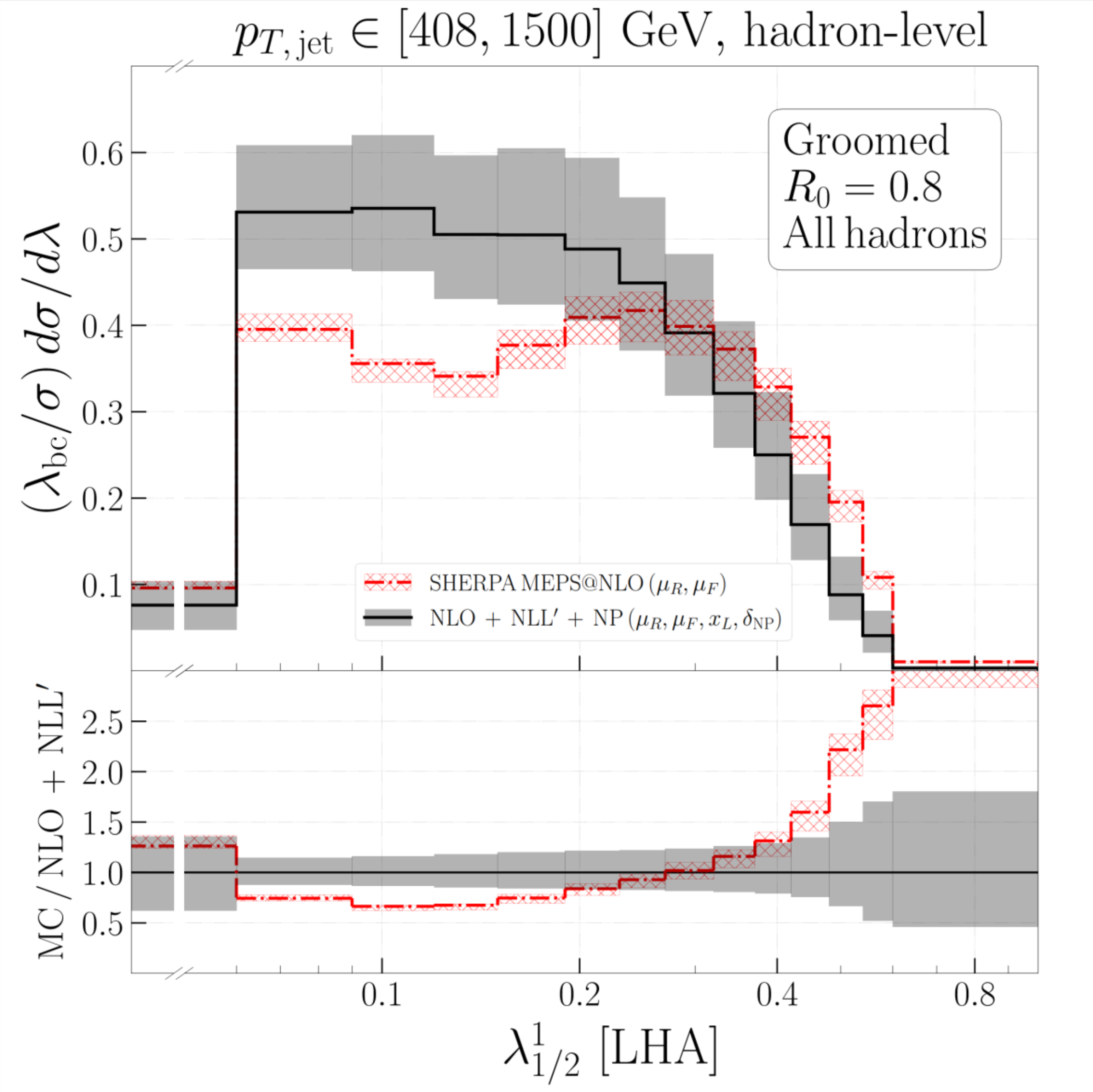
NLO + NLL′+ NP(HL/PL):



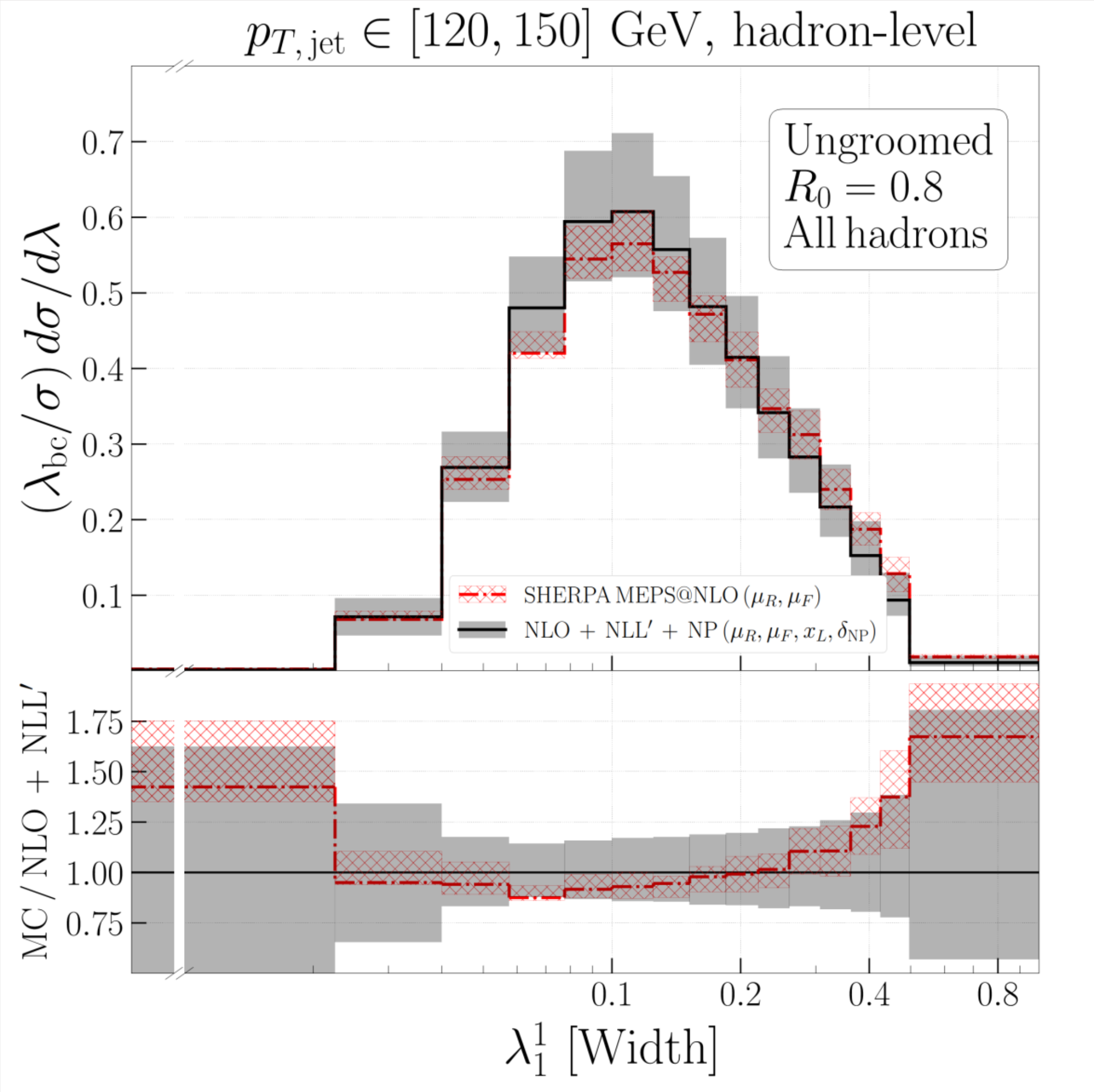


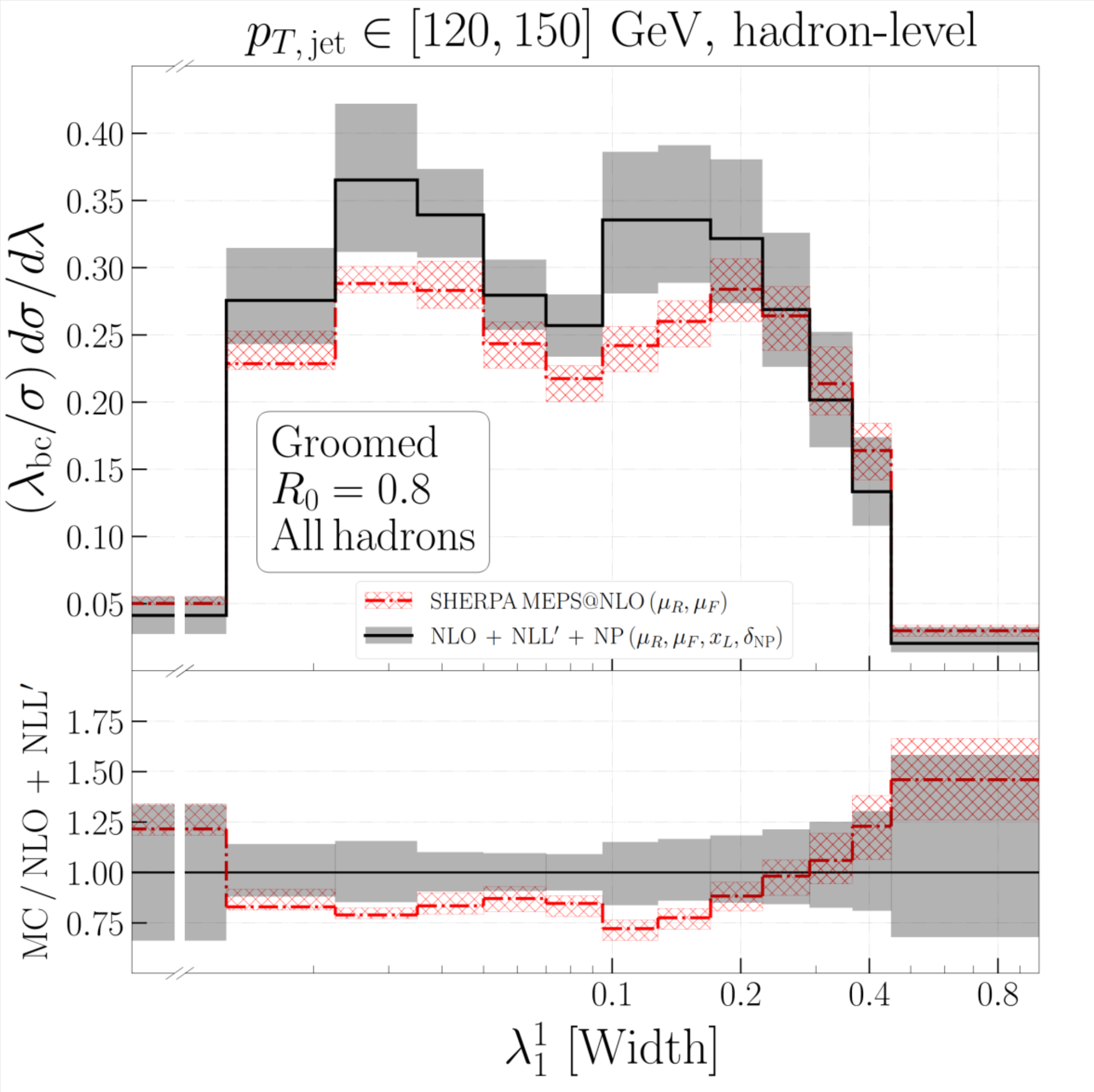
NLO + NLL′+ NP(HL/PL):



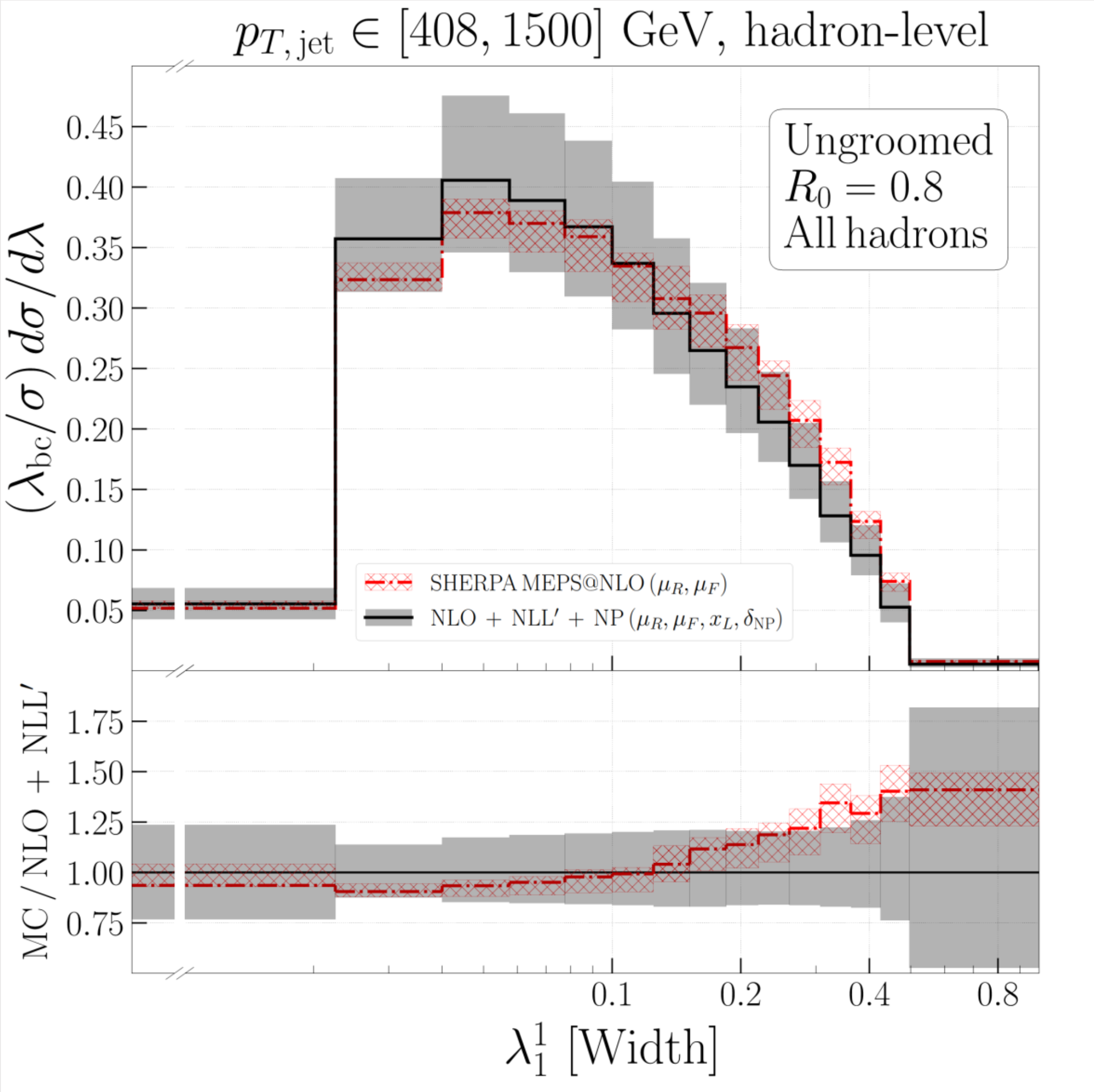


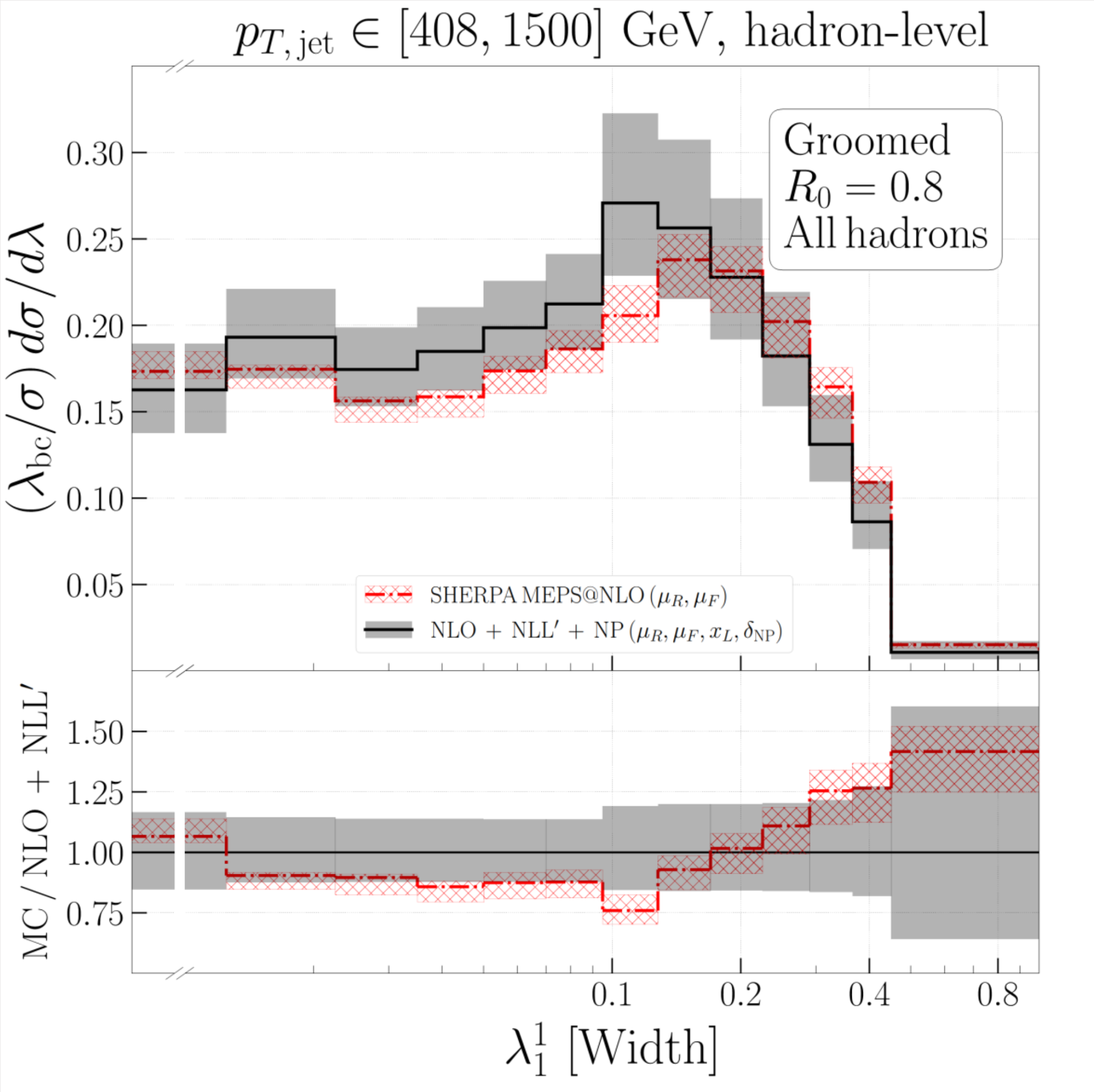
NLO + NLL′+NP (HL/PL):



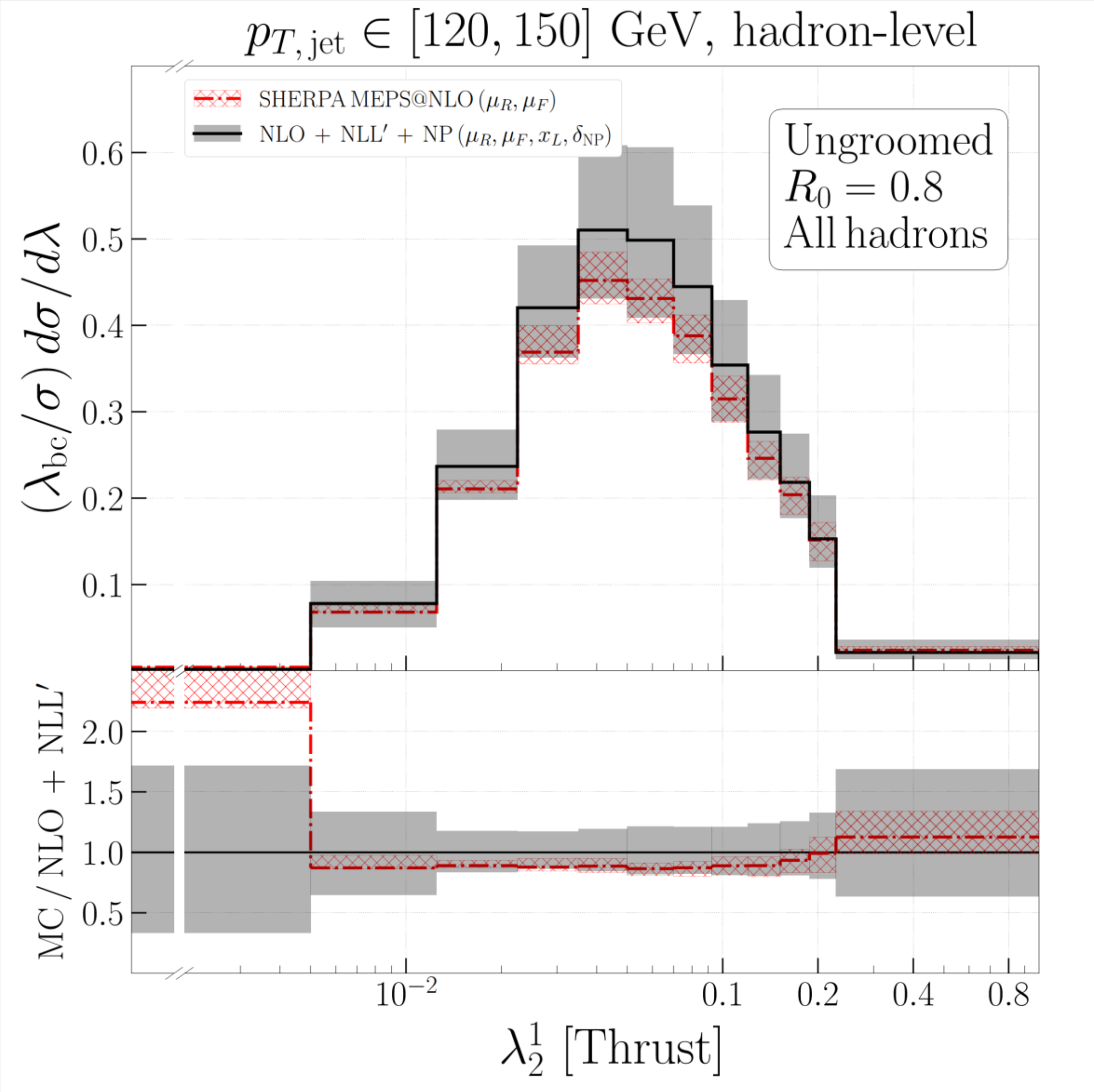


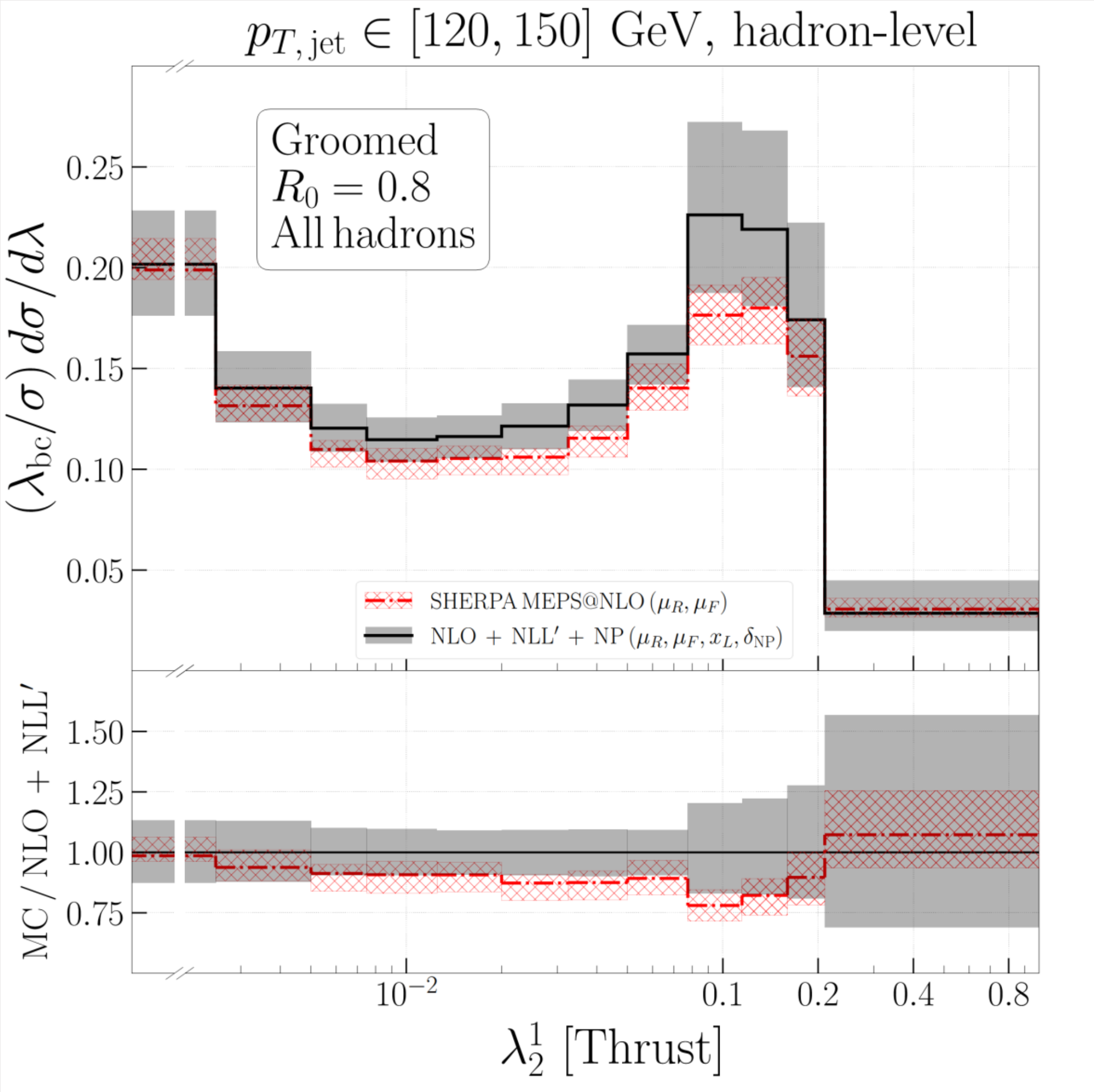
NLO + NLL′+NP (HL/PL):



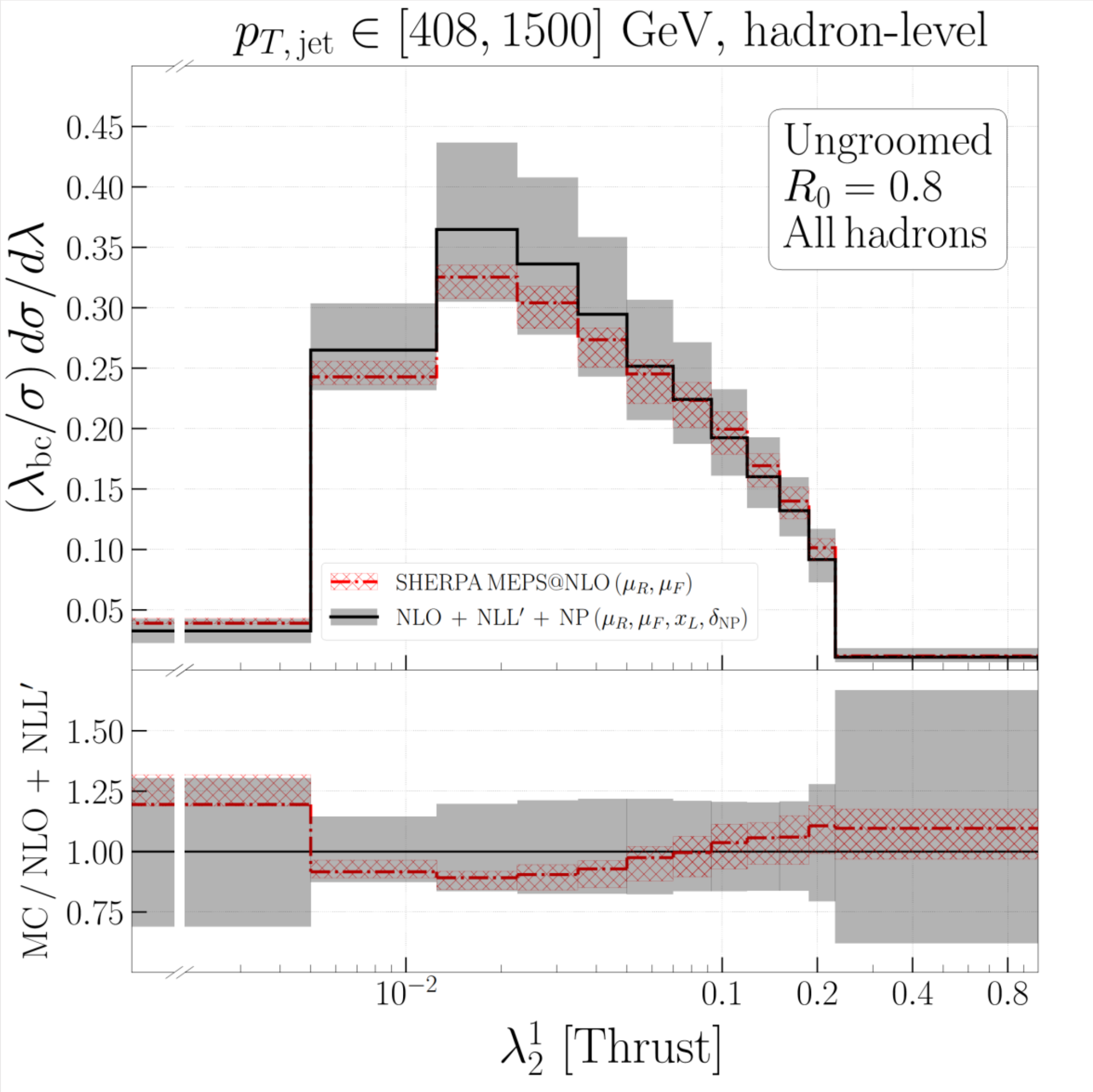


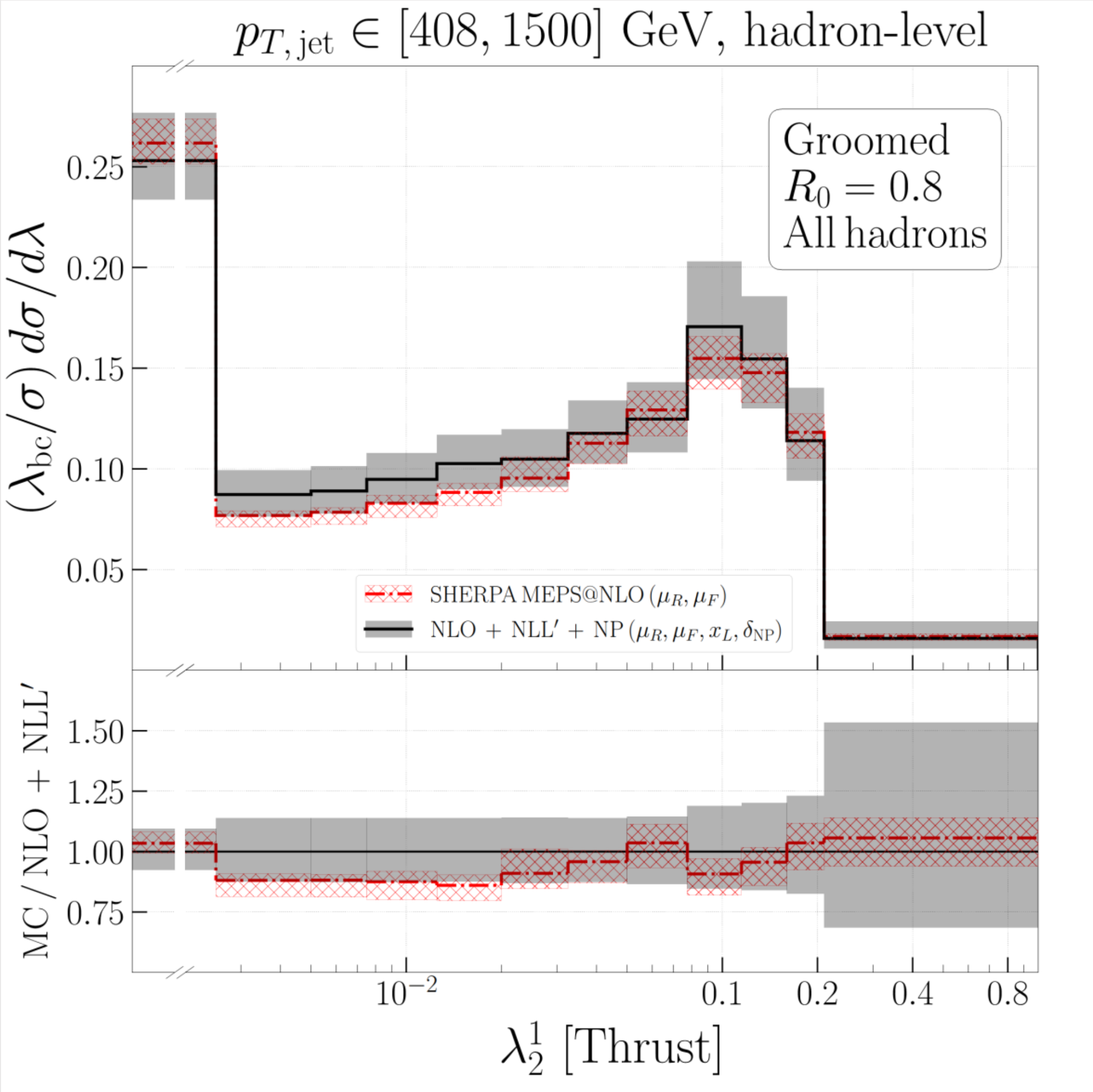
NLO + NLL′+ NP(HL/PL):





NLO + NLL′+ NP(HL/PL):





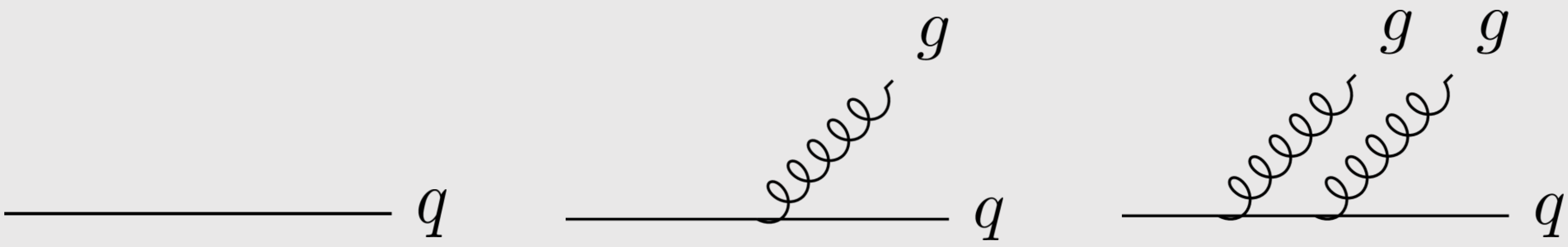
$$\mathcal{P}_{qg\leftarrow q} = P(E_g, m^2) = \frac{\alpha_S}{\pi} C_F \frac{dE_g}{E_g} \frac{dm^2}{m^2}$$

Introducing the dimensionless variable $z = \frac{E_g}{E_q+E_g}$ and using $m^2 = 2p_q \cdot p_g = 2E_q E_g(1-\cos\theta)$

$$P(z,\theta) = dz d\cos\theta = \frac{\alpha_S}{\pi} C_F \frac{dz}{z} \frac{d\cos\theta}{1-\cos\theta} \xrightarrow{z\ll 1, \theta \ll 1} \frac{2\alpha_S}{\pi} C_F \frac{dz}{z} \frac{d\theta}{\theta}$$

Obs. Note the probability is divergent in the soft and/or collinear limit.

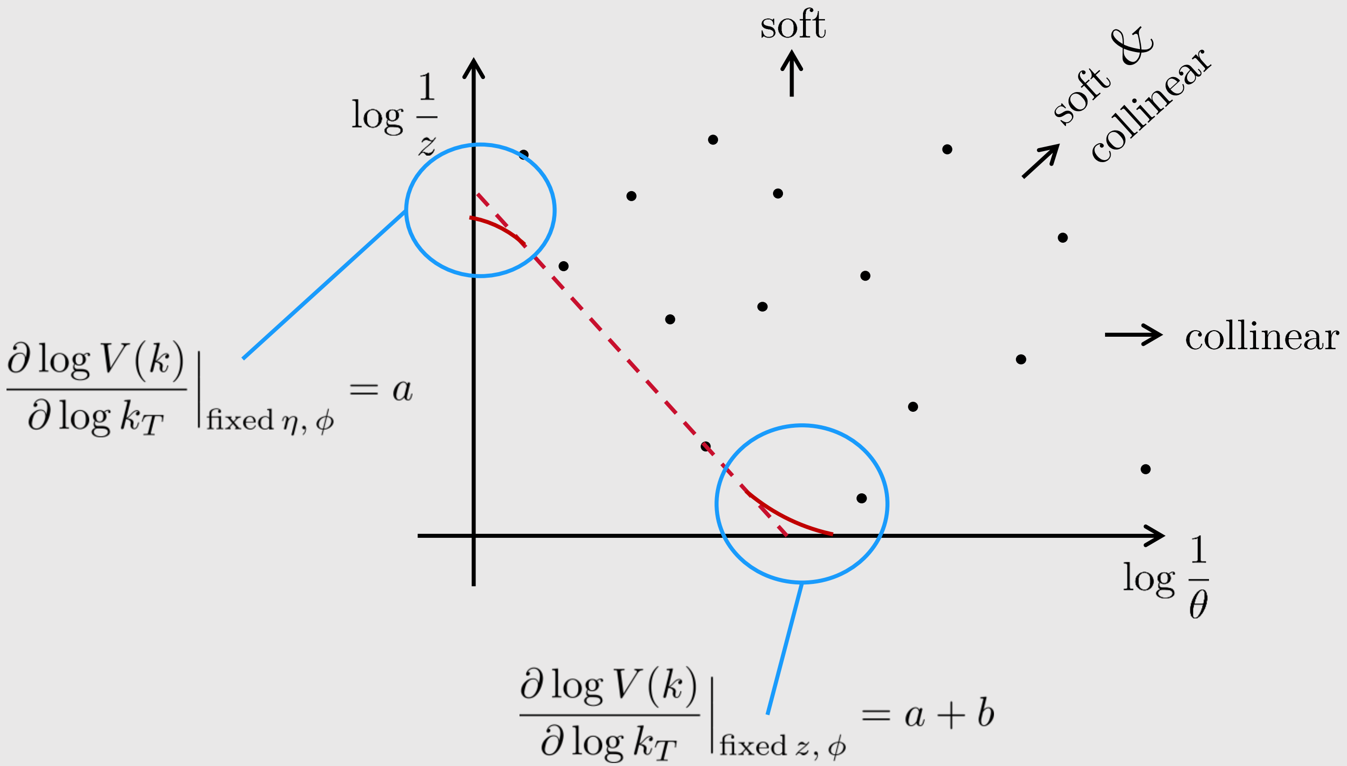
There is no measurement we can perform to distinguish between:

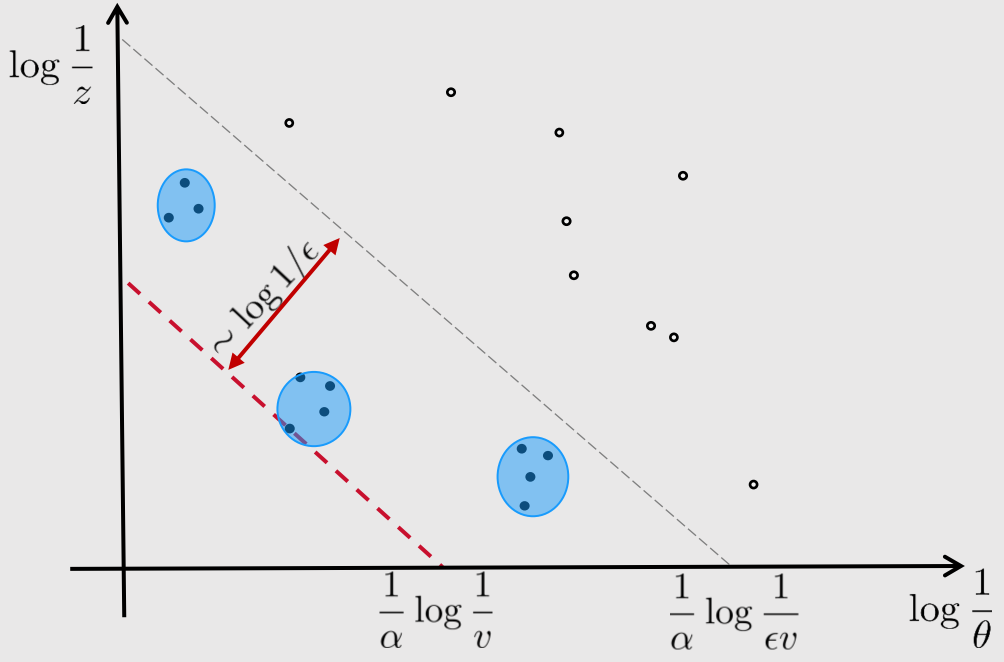


and so on...

Every one these configurations of soft and/or collinear gluons and a quark have a divergent probability. So how do we proceed?

KLN th.





$$\Sigma_{\text{res}}(v) = \sum_{\delta} \Sigma^{\delta}_{\text{res}}(v)$$

RADIATOR

$$\Sigma^{\delta}_{\text{res}}(v) = \int d\mathcal{B}_\delta \frac{d\sigma_\delta}{d\mathcal{B}_\delta} \exp\left[-\sum_{l\in\delta} R_l^{\mathcal{B}_\delta}(L)\right] \mathcal{P}^{\mathcal{B}_\delta}(L) \mathcal{S}^{\mathcal{B}_\delta}(L) \mathcal{F}^{\mathcal{B}_\delta}(L) \mathcal{H}^{\delta}(\mathcal{B}_\delta)$$

pdf ratio; soft function; multiple emission; kinematic cuts

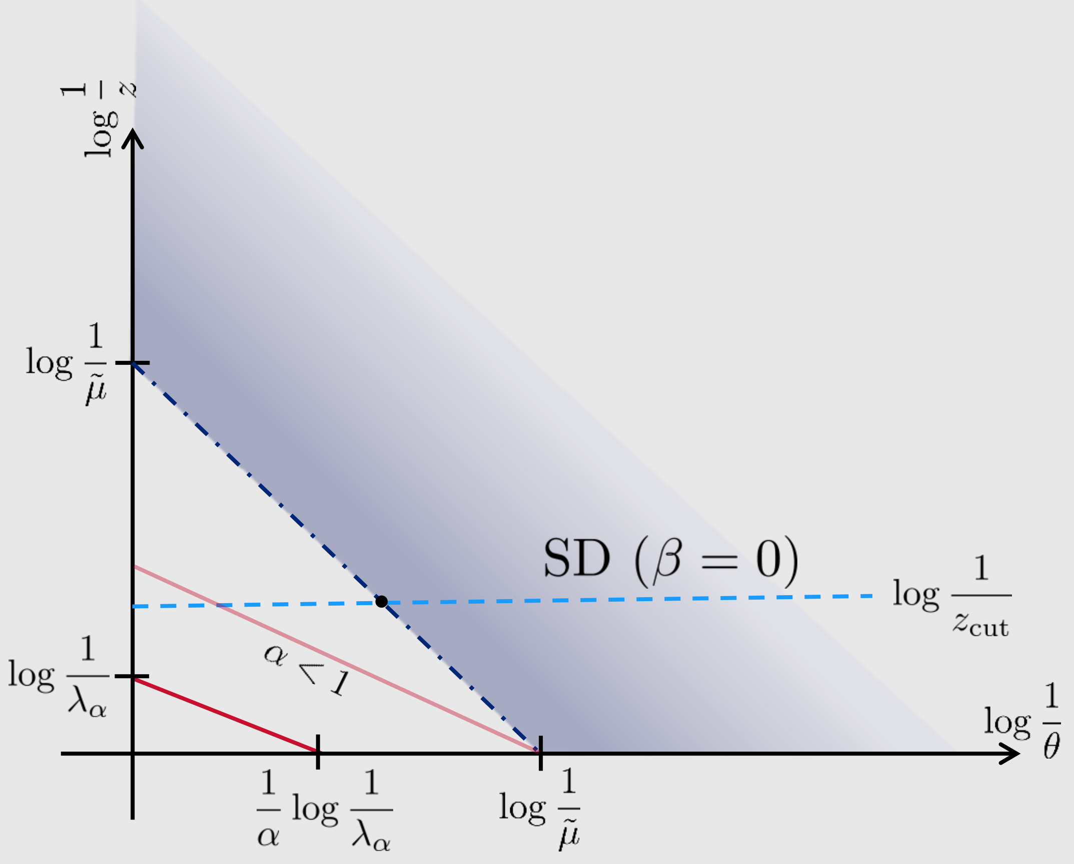
Lund plane geografy:

However it is possible that for big enough $z_{\rm cut}$ we hit SD transition point before reaching the NP boundary also for the $\alpha < 1$ case.

$$\log\frac{1}{z_{\rm cut}} \lesssim \left[\log\frac{1}{\lambda_\alpha} - \cancel{\alpha\log\frac{1}{\theta}}\right]_{\text{at the NP boundary}}$$

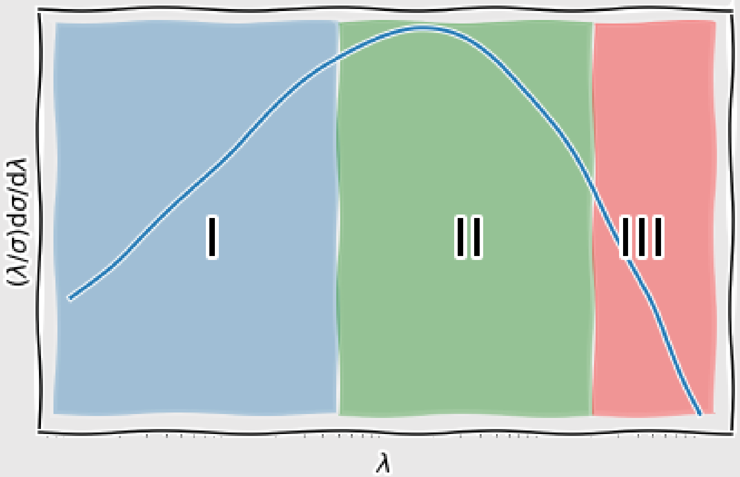
$$\lesssim \alpha\log\frac{1}{\tilde{\mu}} \quad \Rightarrow \quad z_{\rm cut} \gtrsim \tilde{\mu}^\alpha$$

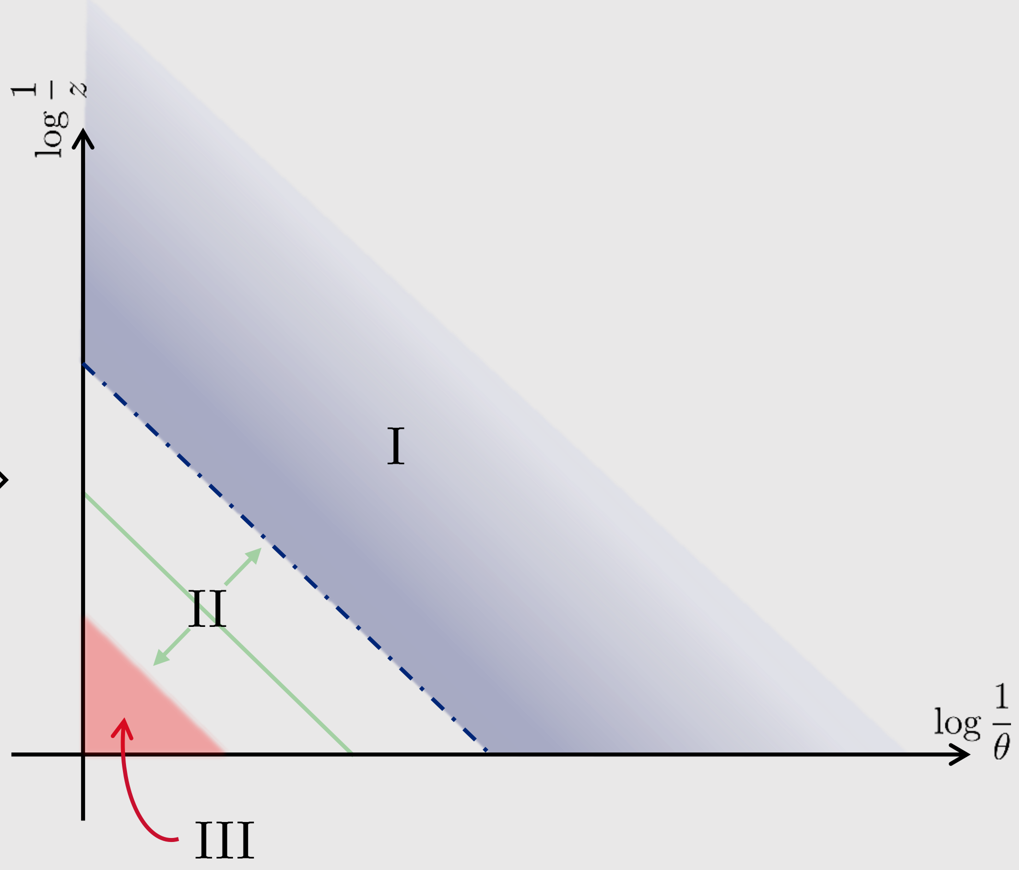
$$z_{\rm cut} \gtrsim \left(\frac{1\text{GeV}}{100\text{GeV}\cdot 0.8}\right)^{1/2} \simeq 0.11$$



In this case we hit the SD transition point before the NP region but this does not affect the intersection point with the NP boundary.

Lund plane geografy:





In addition from our LL picture we have that:

$$p(\lambda_\alpha) \simeq \frac{\mathrm{d}}{\mathrm{d}\log\lambda_\alpha} e^{-\frac{2\cdot 0.118}{\pi} C_i \frac{\log^2 \lambda_\alpha}{2\alpha}} = -\frac{0.075}{\alpha} C_i \log\lambda_\alpha \cdot e^{-0.075\cdot C_i \frac{\log^2\lambda_\alpha}{2\alpha}}$$

α	$\lambda_\alpha^{\text{max, quark}}$	$\lambda_\alpha^{\text{max, gluon}}$
0.5	0.107	0.225
1	0.042	0.122
2	0.011	0.051