UNIVERSITÀ DEGLI STUDI DI GENOVA

NLL small-x resummation for Higgs induced DIS

Milan Christmas meeting - 2021

Anna Rinaudo

Paper in preparation with Simone Marzani and Giovanni Ridolfi (INFN Genova) and Marco Bonvini and Federico Silvetti (INFN Roma)

21-23 December 2021

Motivation

Perturbative QCD

$$O = \sum_n \alpha_s^n \, c_n$$

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$$O = \sum_n \alpha_s^n c_n \longrightarrow c_n \supset \ln^k \left(\frac{1-x}{x} \right) \quad k = 0, \dots, n$$

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$$x \to 1$$

"Large-x" limit

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Threshold limit

$x \to 1$

"Large-x" limit

High energy limit

$x \to 0$

"Small-x" limit

Problem

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→ Spoil the convergence of the perturbative series

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↓ Theoretical prediction are no longer reliable

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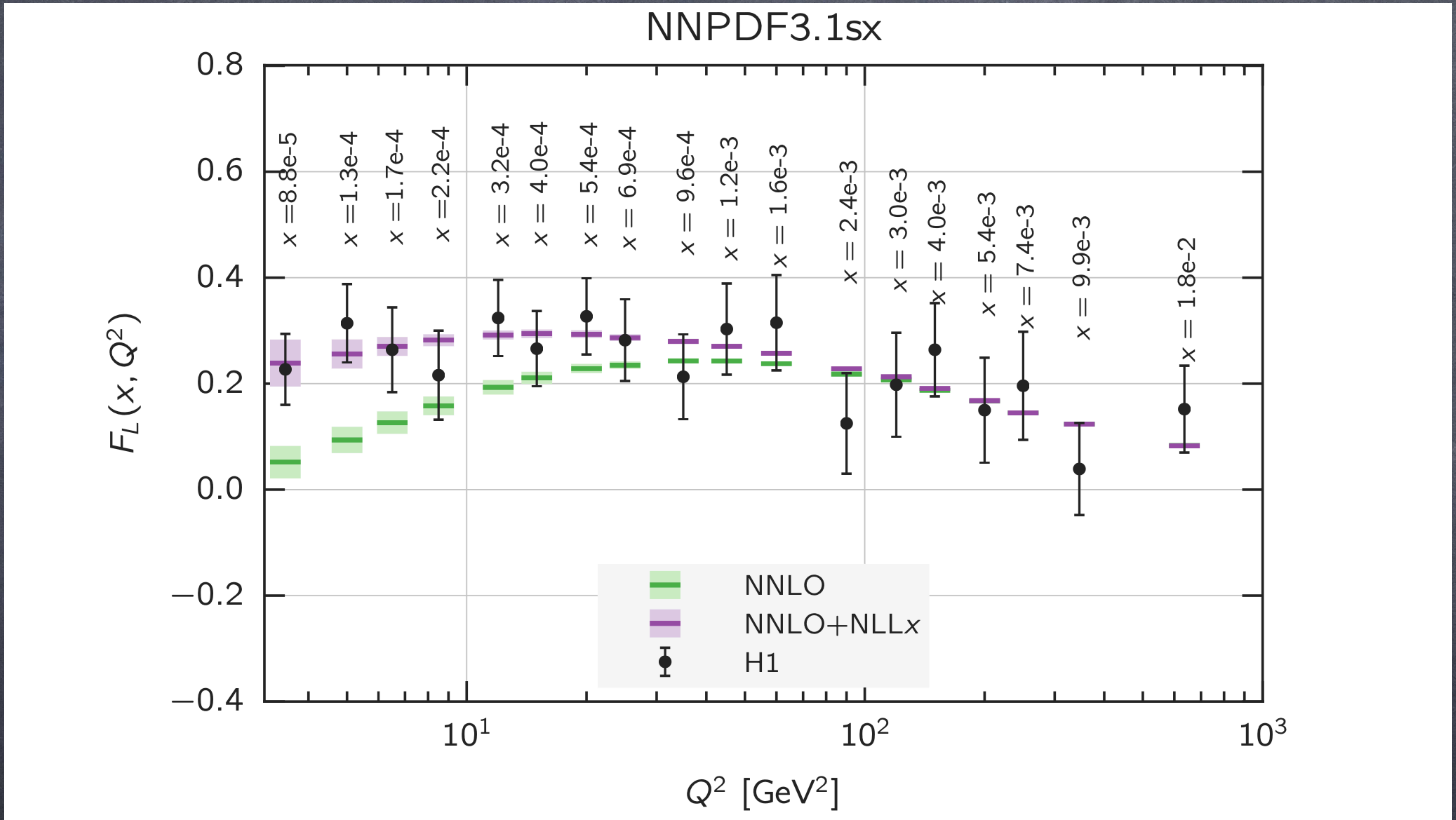
$\ln^n(x)$ ↓ LL

$\ln^{n-1}(x)$ ↓ NLL

Solution? Find out the all-order structure of these logarithmic contributions by writing them as a series and summing it

Resummation

How much does small-x resummation improve theoretical predictions?



Ball, Bertone, Bonvini, Marzani, Rojo, Rottoli (**2018**) The European Physical Journal C.

Small-x resummation:
How does it work?

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- Resummation ⟷ Factorization

We need some factorisation properties

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- Mellin Transform

$$g(N,Q^2) = \int_0^1 dx\, x^N\, g\left(x,Q^2\right) \qquad \ln^k(x) \to \frac{1}{N^{k+1}}$$

Collinear factorization theorem

$$\sigma(N, Q^2) = \sum_{i=q,g} C_i\left(N, \alpha_s(Q^2)\right) f_i\left(N, Q^2\right)$$

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Parton distribution function (PDF)

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Our goal: resum NLL logarithms in the coefficient function

High energy factorization theorem

$$\sigma(N, Q^2) = \int_0^\infty dk_\perp^2 \, \mathscr{C}\left(N, k_\perp^2, Q^2, \alpha_s\right) \, \mathscr{F}_g\left(N, k_\perp^2\right)$$

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Off- shell coefficient
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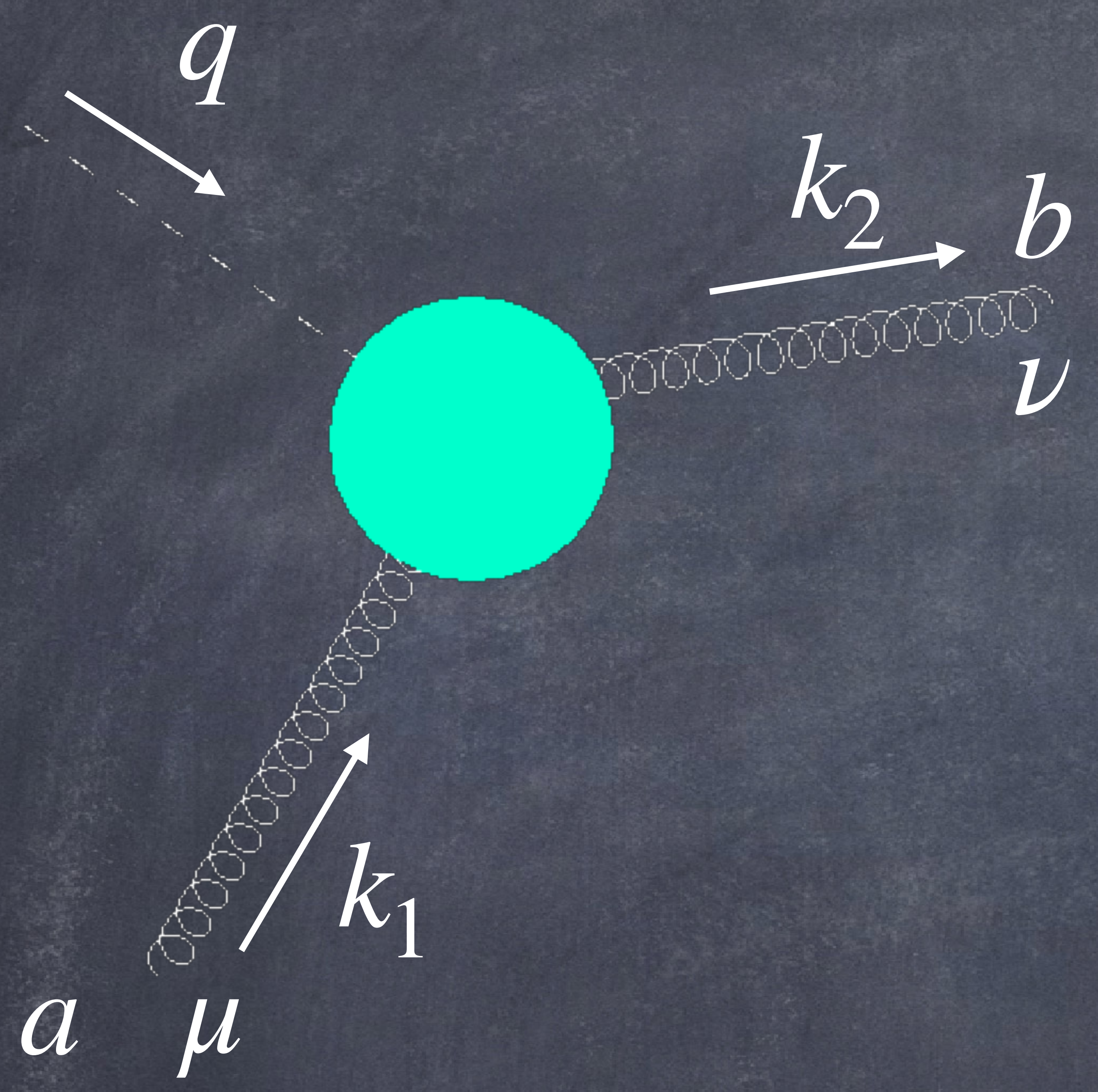
Bonvini, Marzani, and Peraro (**2016**) The European Physical Journal C.

Higgs induced DIS: What do we want to compute?

Higgs DIS



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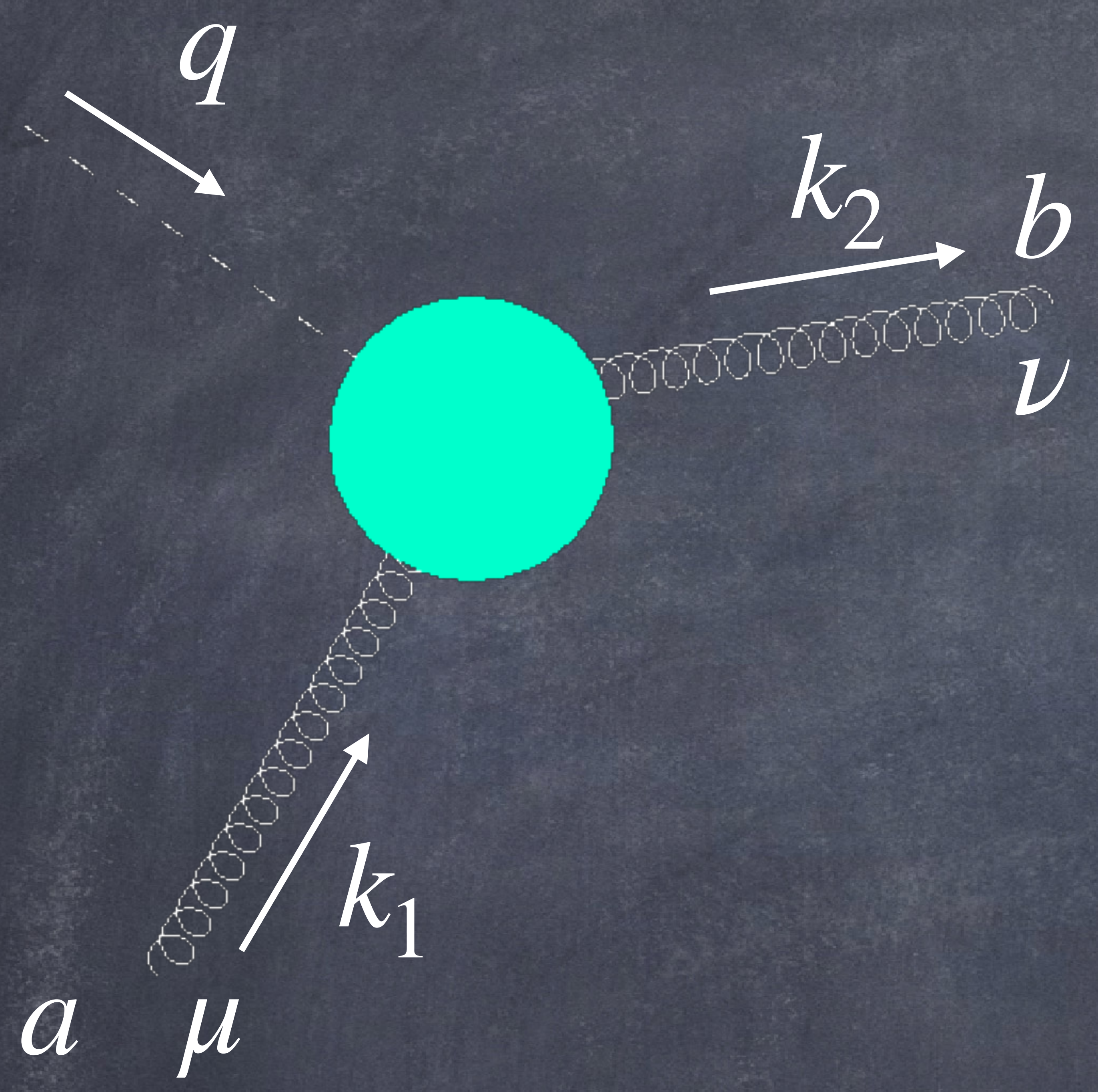
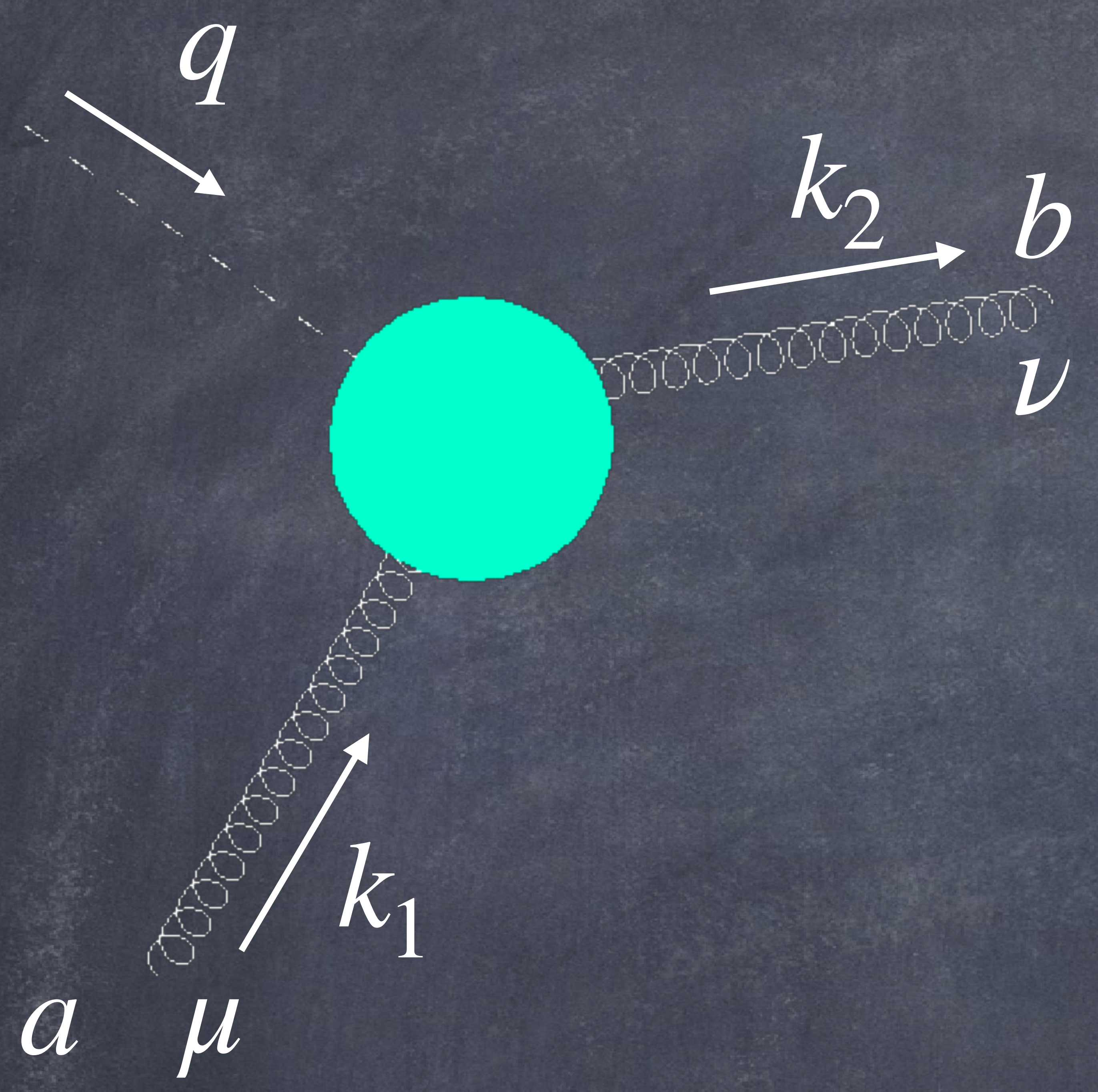


- $n_f = 0$

Higgs DIS



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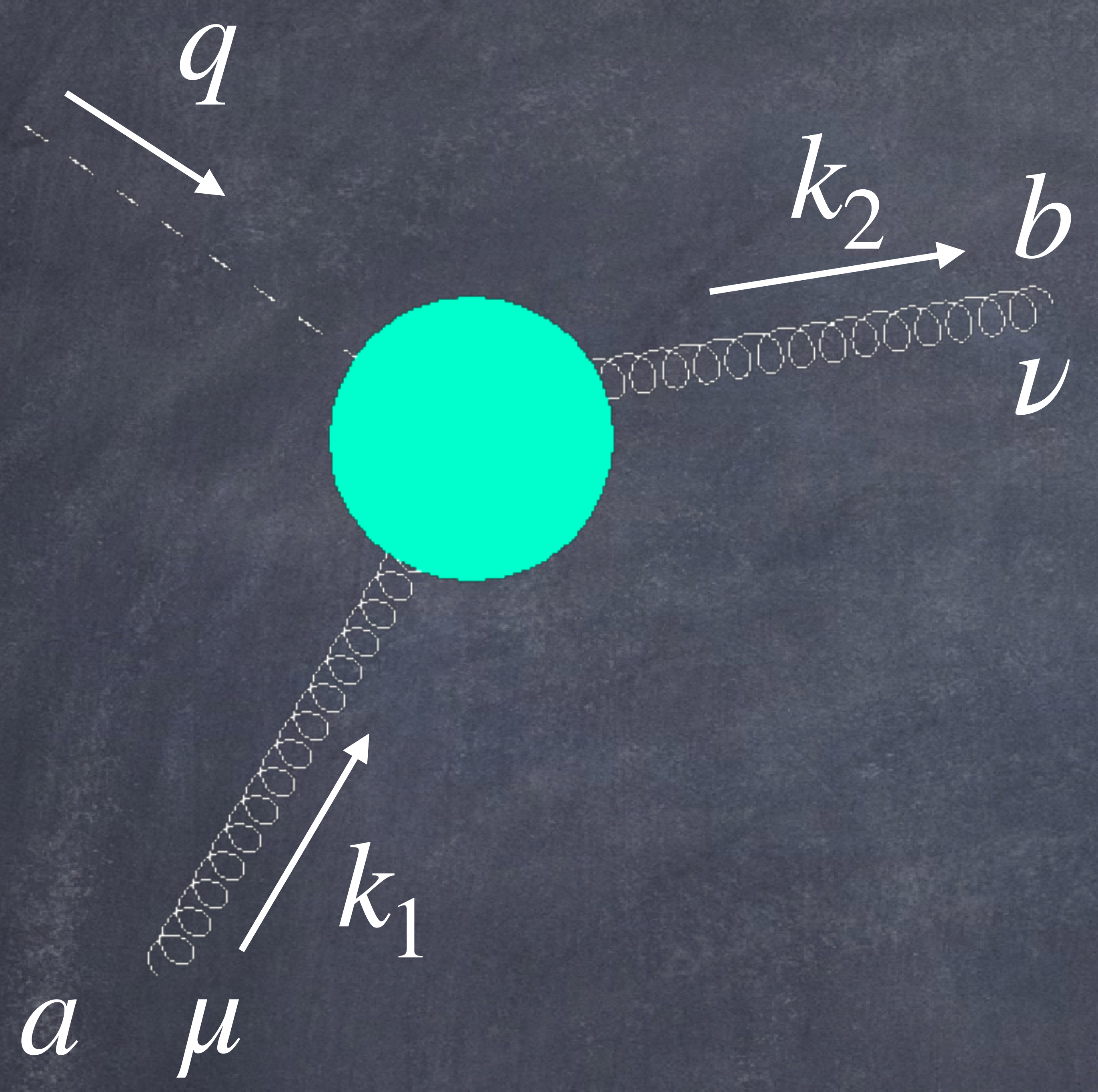
- Higgs gluon effective vertex:

$$M^{\mu\nu} = i\, c\, \delta_a^b \left[k_2^\mu k_1^\nu - g^{\mu\nu} k_1 \cdot k_2 \right]$$

Higgs DIS



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$$k_1^2 = -\vec{k}_1^{\,2}$$

Higgs DIS

We want to resum NLL terms in the coefficient function

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We have to compute the one-loop off-shell coefficient function

Key points

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1. We have to work in axial gauge: $A \cdot n = 0$

The off-shell coefficient function is free from logs if we work in axial gauge

Catani and Hautmann (**1994**) Nuclear Physics B.

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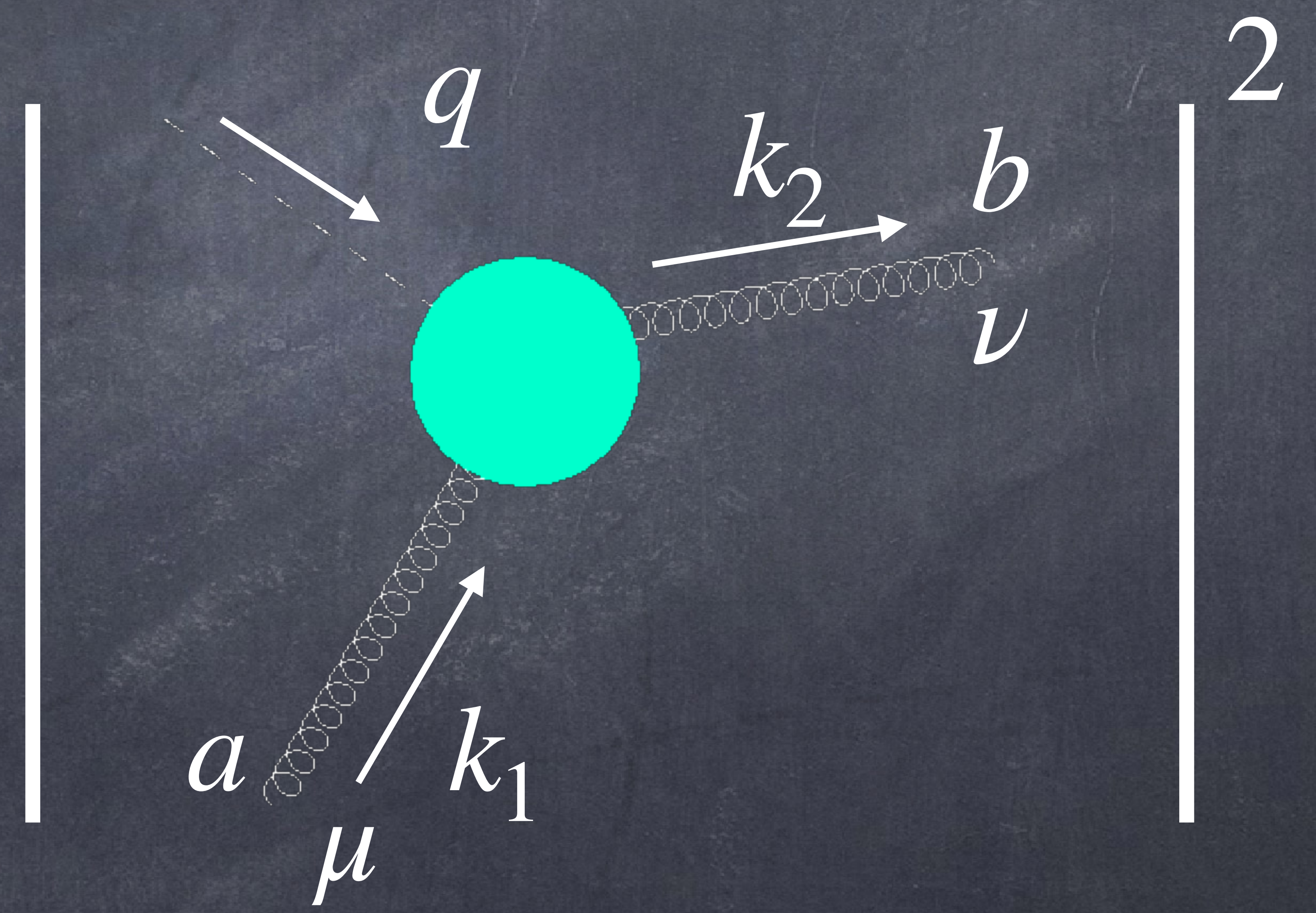
2. We have to define the sum over polarisation of an off-shell gluon



Axial gauge

- Growing number of terms due to gauge choice

$$\Pi^{\mu\nu}_{a,b}(k,n) = i\,\delta_{a,b}\left[\frac{-g^{\mu\nu}}{k^2} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k^2\,k\cdot n}\right]$$

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- Non covariant loop integrals

$$I = \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2(k-k_1)^2(k-k_2)^2(k\cdot n)}$$

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$$I_n = \int \frac{d^dk}{(2\pi)^d} \frac{f\left(k \cdot n\right)}{D_1\, D_2\, \ldots\, D_n}$$

Non covariant loop integrals

$$I_n = \int \frac{d^d k}{(2\pi)^d} \frac{f(k \cdot n)}{\underbrace{D_1 D_2 \, \ldots \, D_n}_{\text{Covariant denominators}}}$$

Covariant denominators

Non covariant loop integrals

Non-covariant part

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Covariant denominators

Non covariant loop integrals

Non-covariant part

$$\frac{1}{(k\cdot n)} \to \frac{k\cdot n}{(k\cdot n)^2+\delta^2}$$

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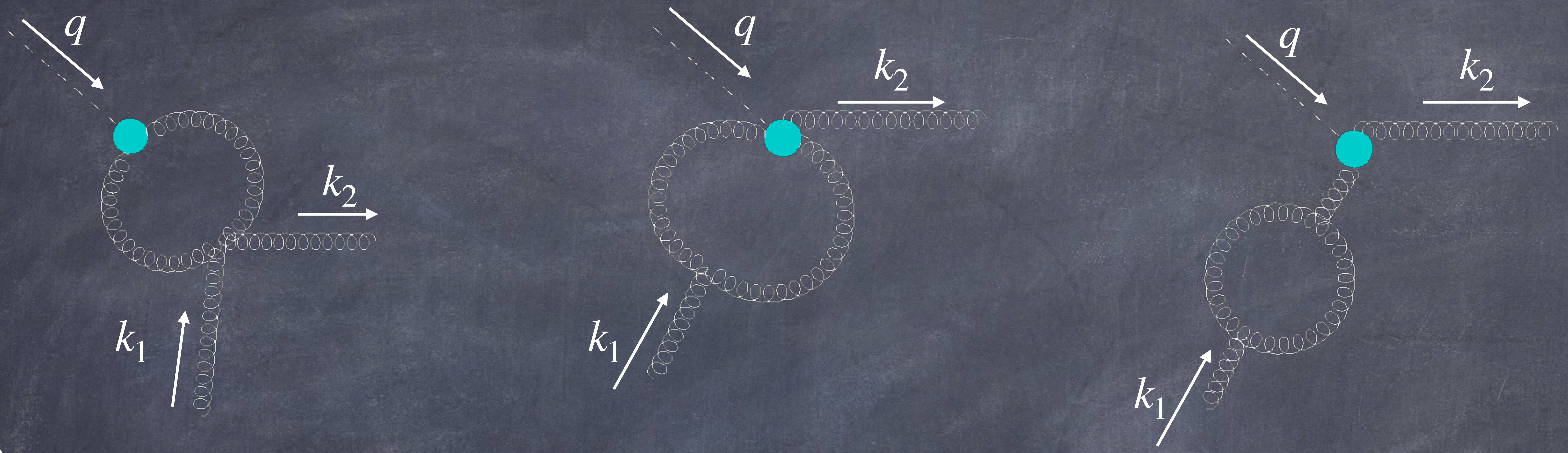
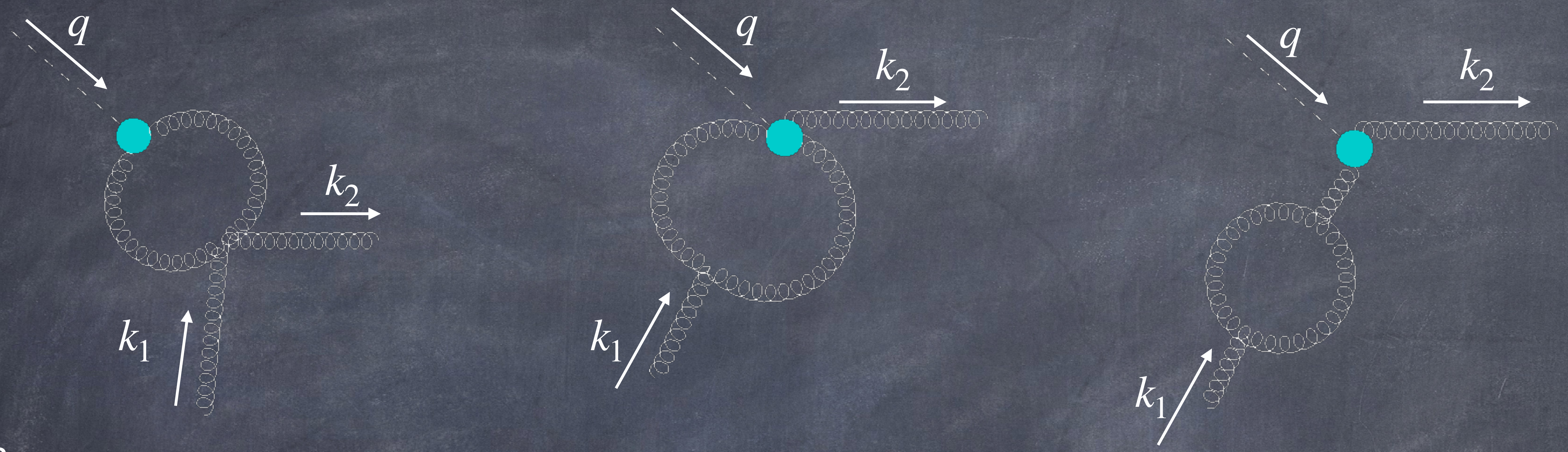
Covariant denominators

Non covariant loop integrals



$k_1^2 \neq 0$

Non covariant loop integrals



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$$I_2 = \int \frac{d^d k}{(2\pi)^d} \frac{f\left(k \cdot n\right)}{k^2 (k-l)^2}$$

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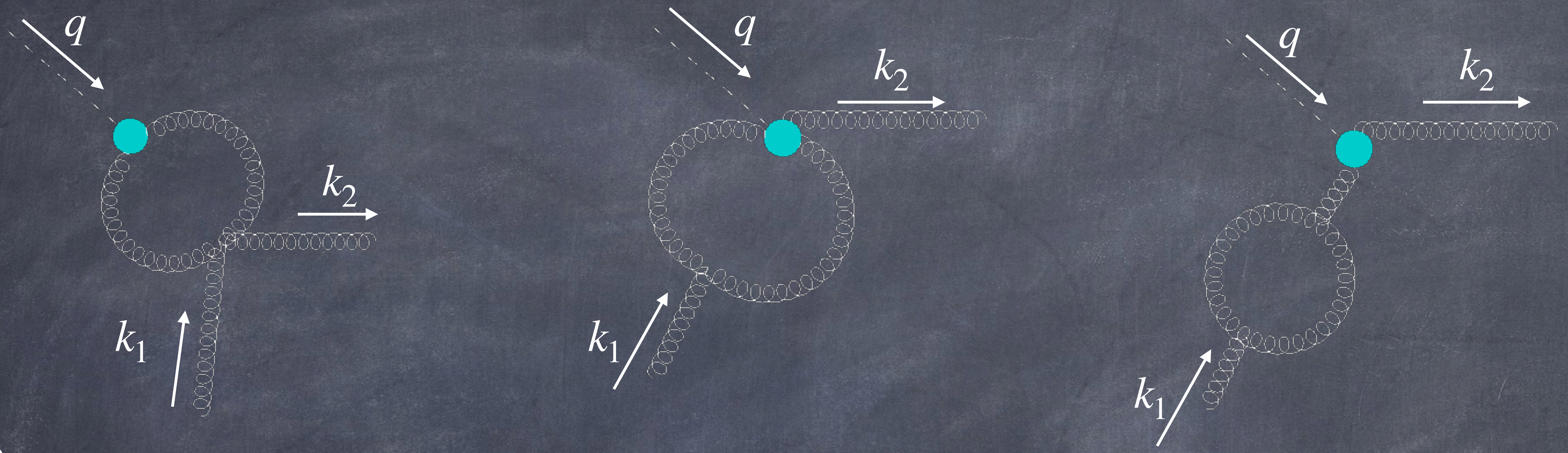

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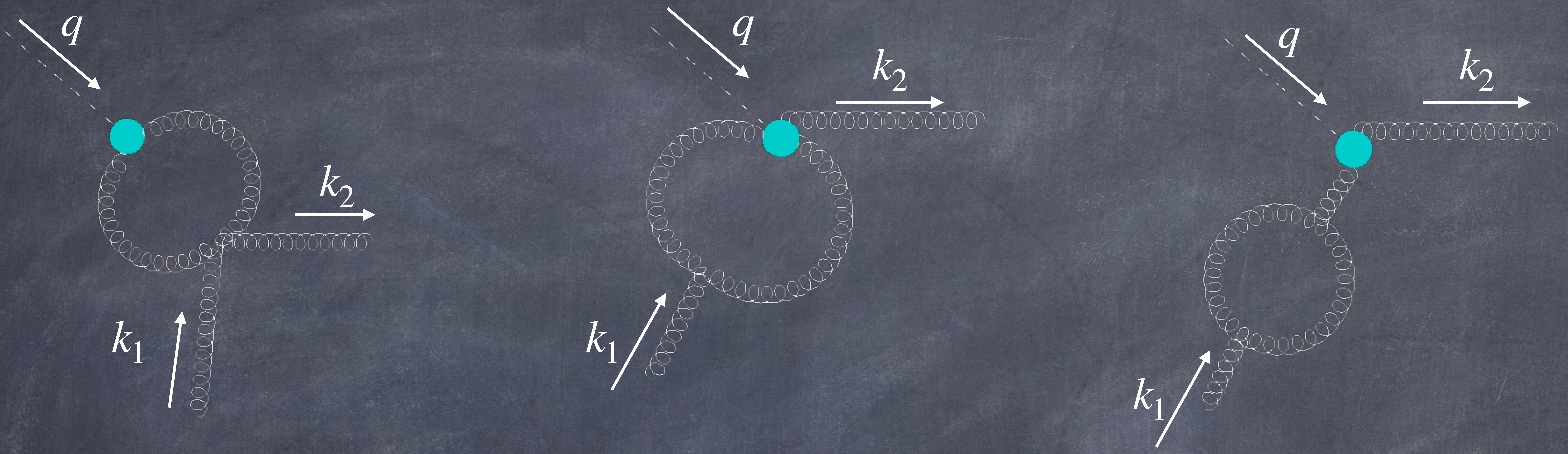
$$I_2 = \int \frac{d^d k}{(2\pi)^d} \frac{f\left(k \cdot n\right)}{k^2 (k-l)^2} = \frac{i}{16\pi^2} \left(\frac{4\pi}{-l^2}\right)^{\epsilon} \frac{\Gamma(1+\epsilon)}{\epsilon} \int_0^1 dz\, f\left(l_+ z\right) z^{-\epsilon} (1-z)^{-\epsilon}$$

$$l_+ = l \cdot n$$

Non covariant loop integrals



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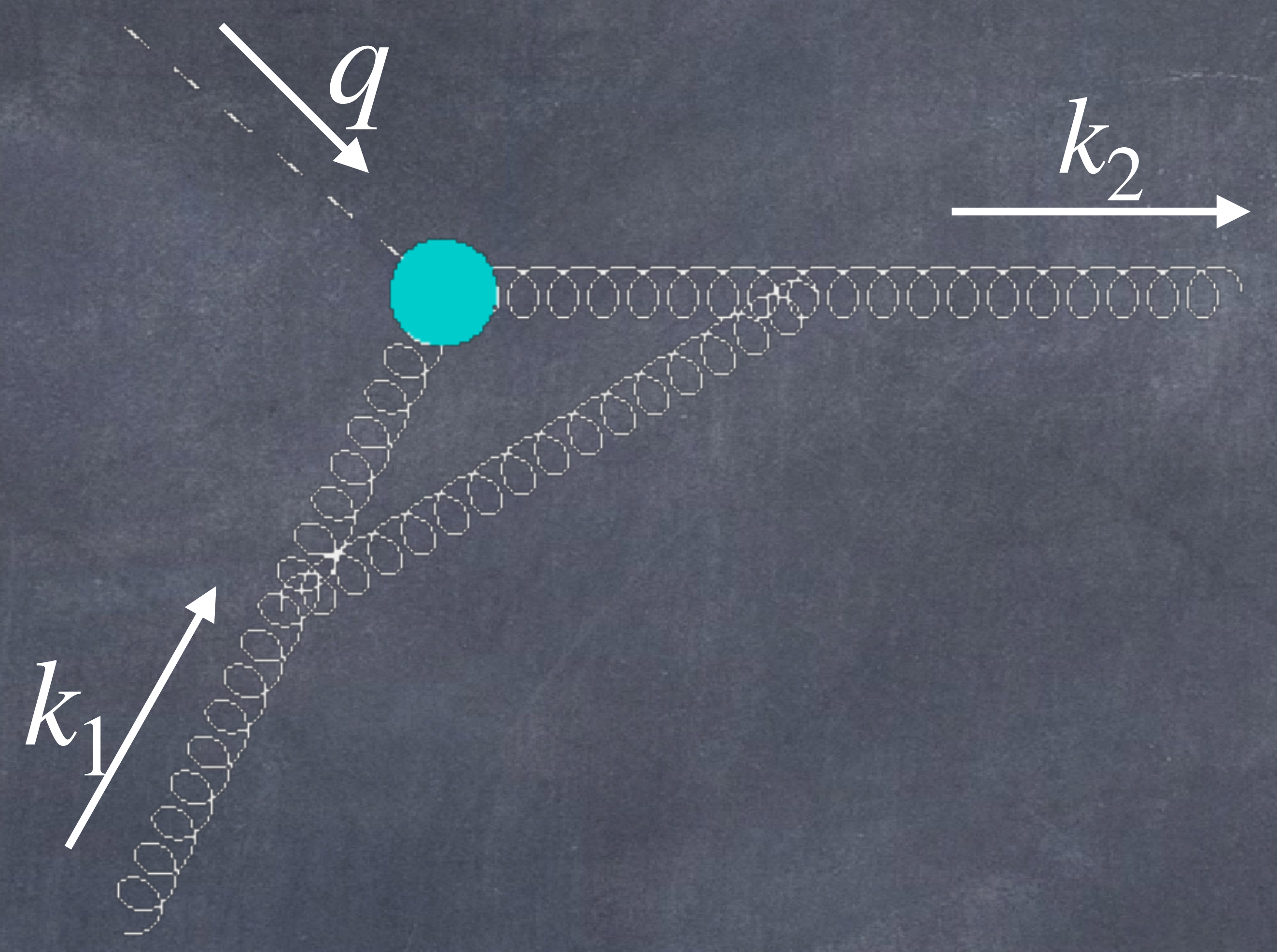

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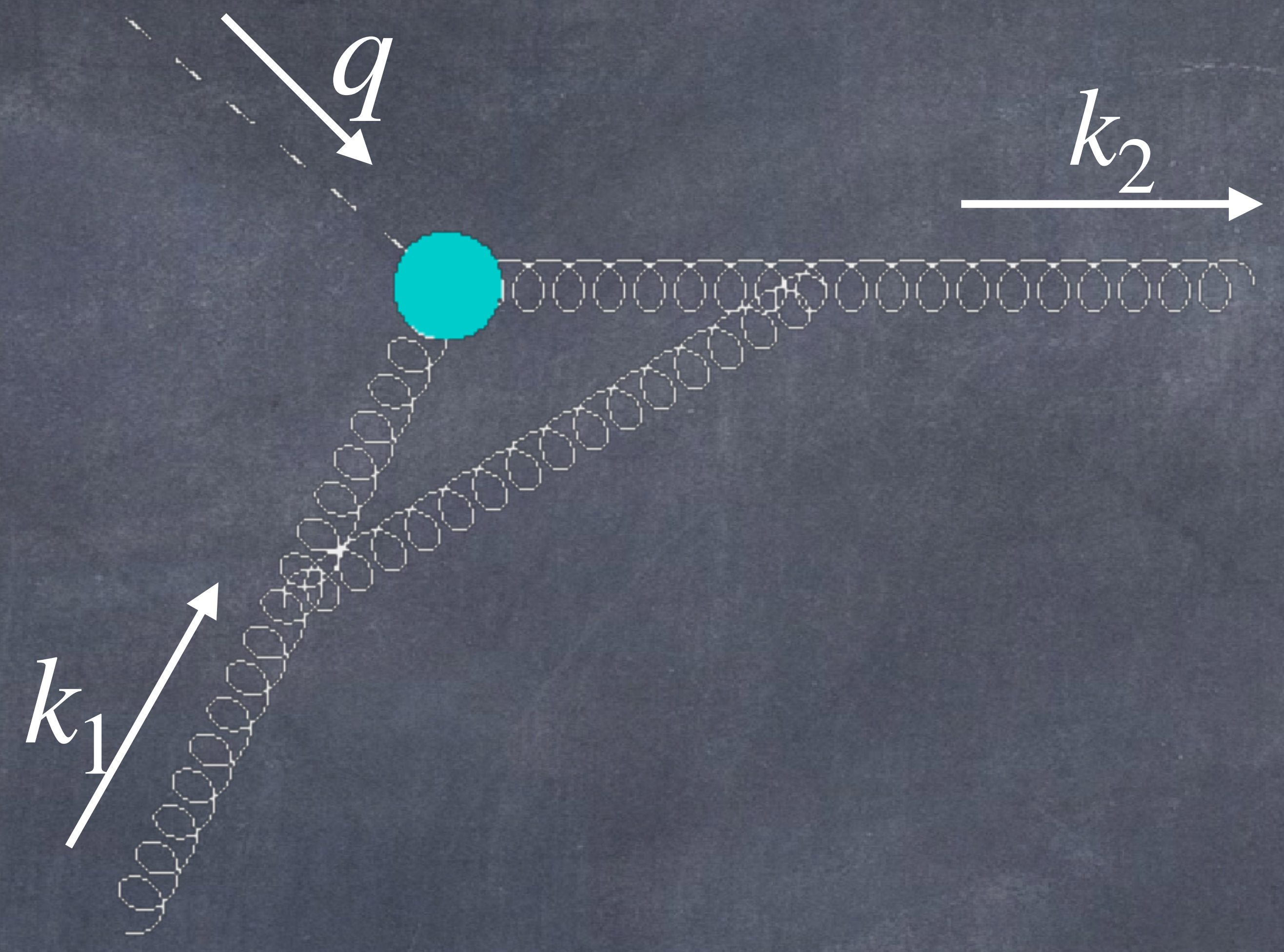

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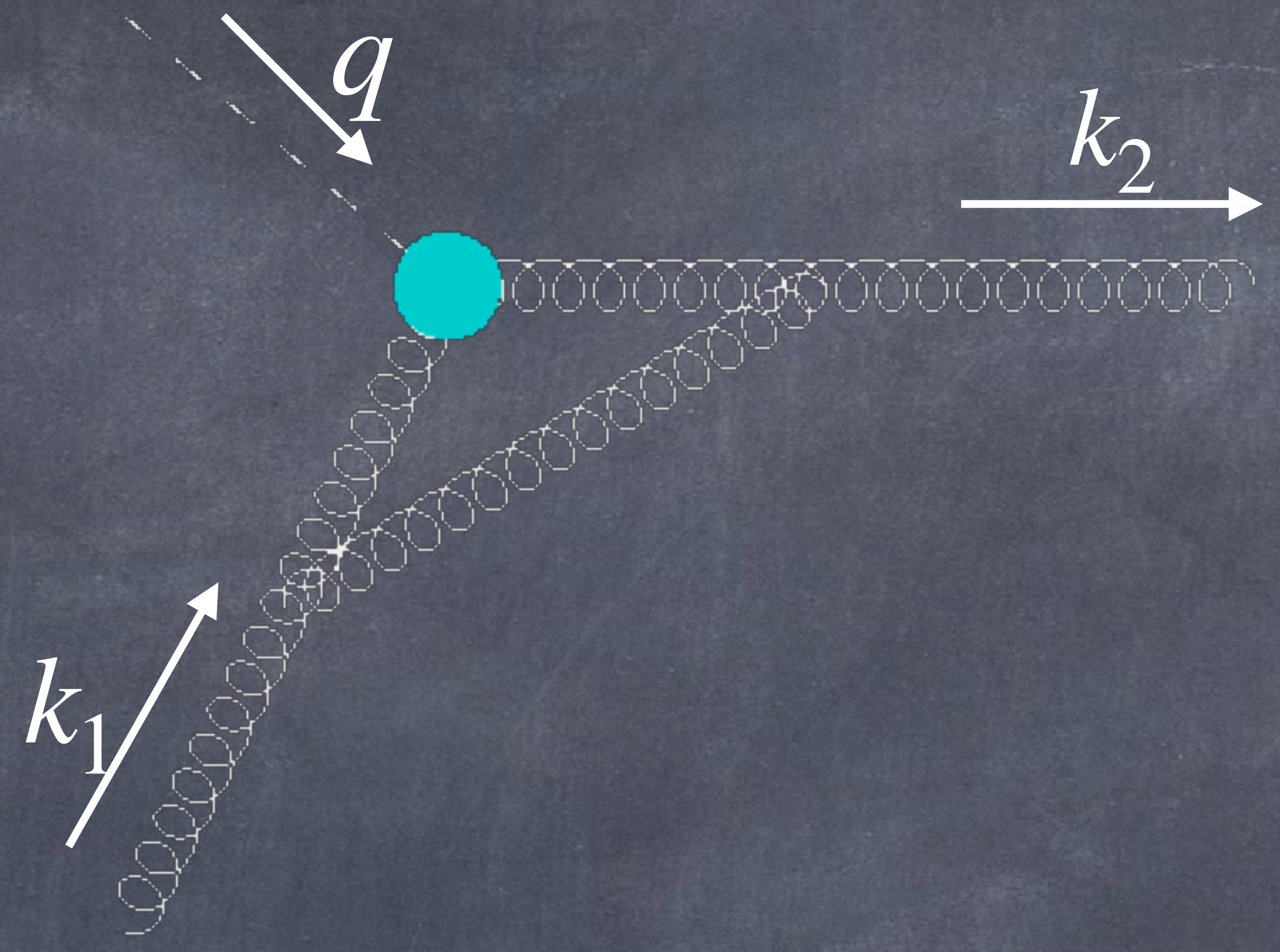

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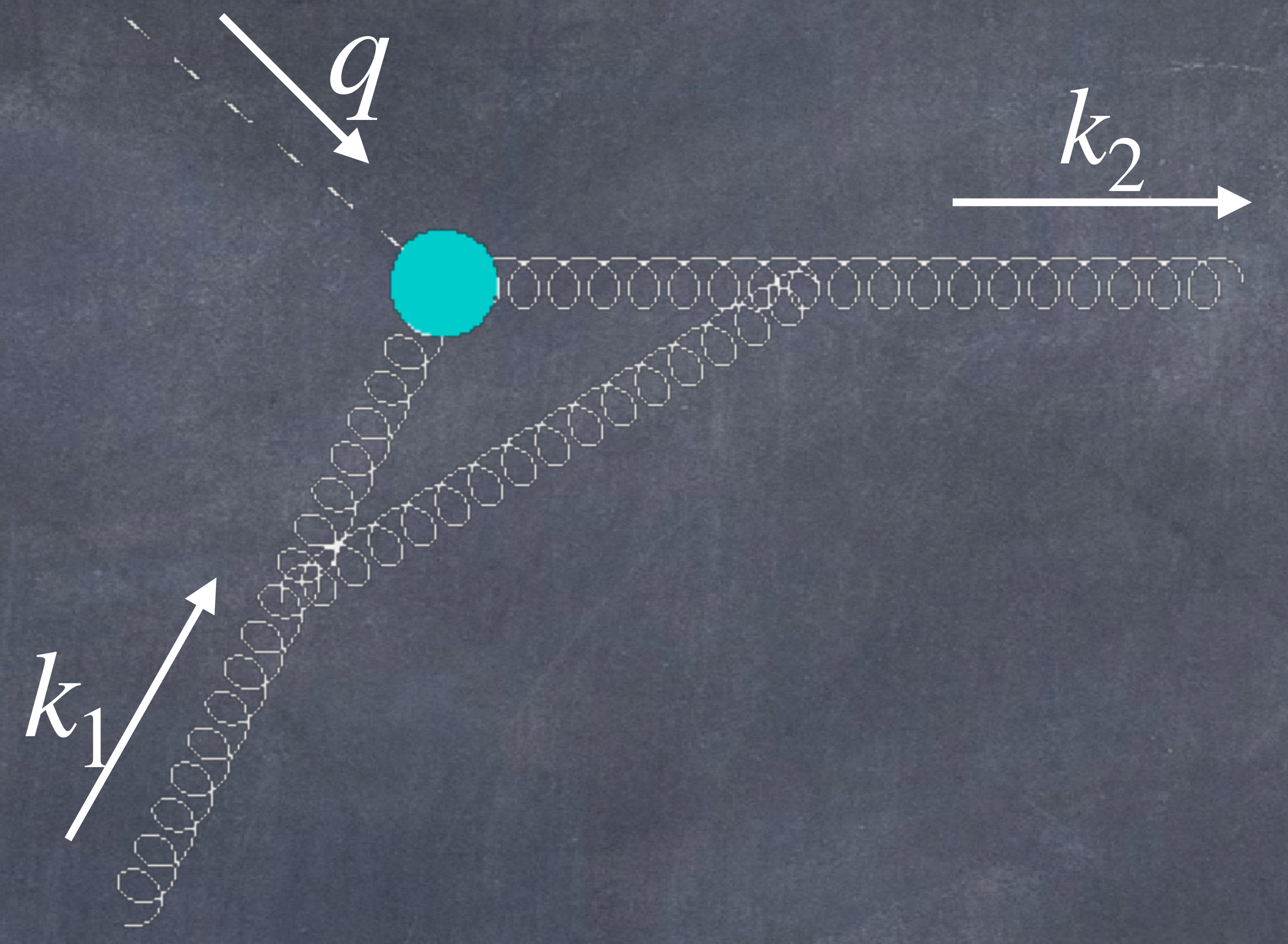
$$\Bigg[(x)^{\epsilon}\int_0^x dz\, f\left(k_{1\,+}z\right)\, z^{-\epsilon}\,(b-z)^{-1-\epsilon}\,{}_2F_1\left(1+\epsilon,-\epsilon;1-\epsilon;\frac{b-x}{x}\frac{z}{b-z}\right)+$$

$$(1-x)^{\epsilon}\int_x^1 dy f\left(k_{1\,+}y\right)(b-y)^{-1-\epsilon}(1-y)^{-\epsilon}\,{}_2F_1\left(1+\epsilon,-\epsilon;1-\epsilon;\frac{(1-y)(b-x)}{(b-y)(1-x)}\right)\Bigg]$$

Non covariant loop integrals



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Sum over polarisation of an off-shell gluon

$$k_1^\mu = k^\mu + k_t^\mu$$
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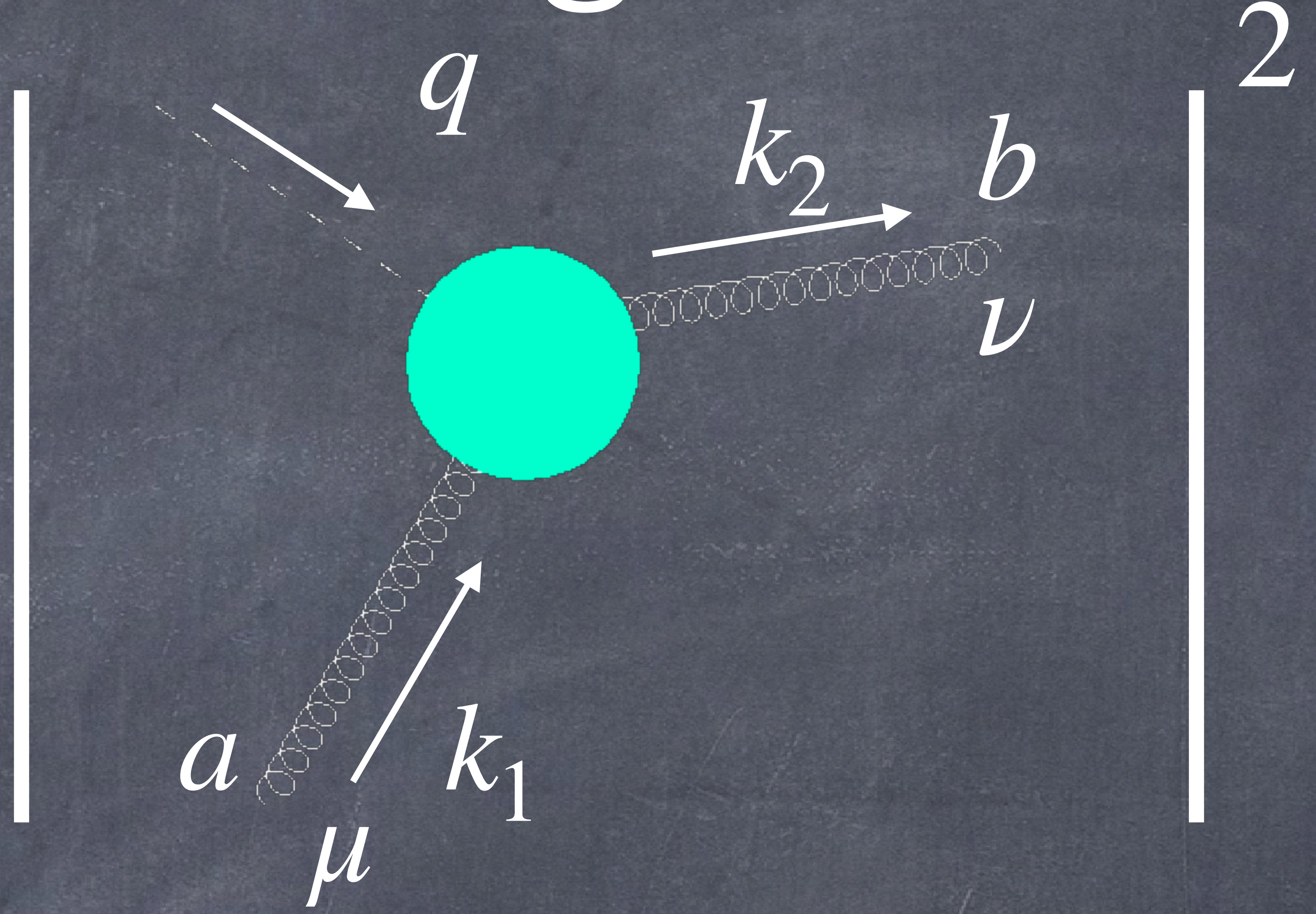


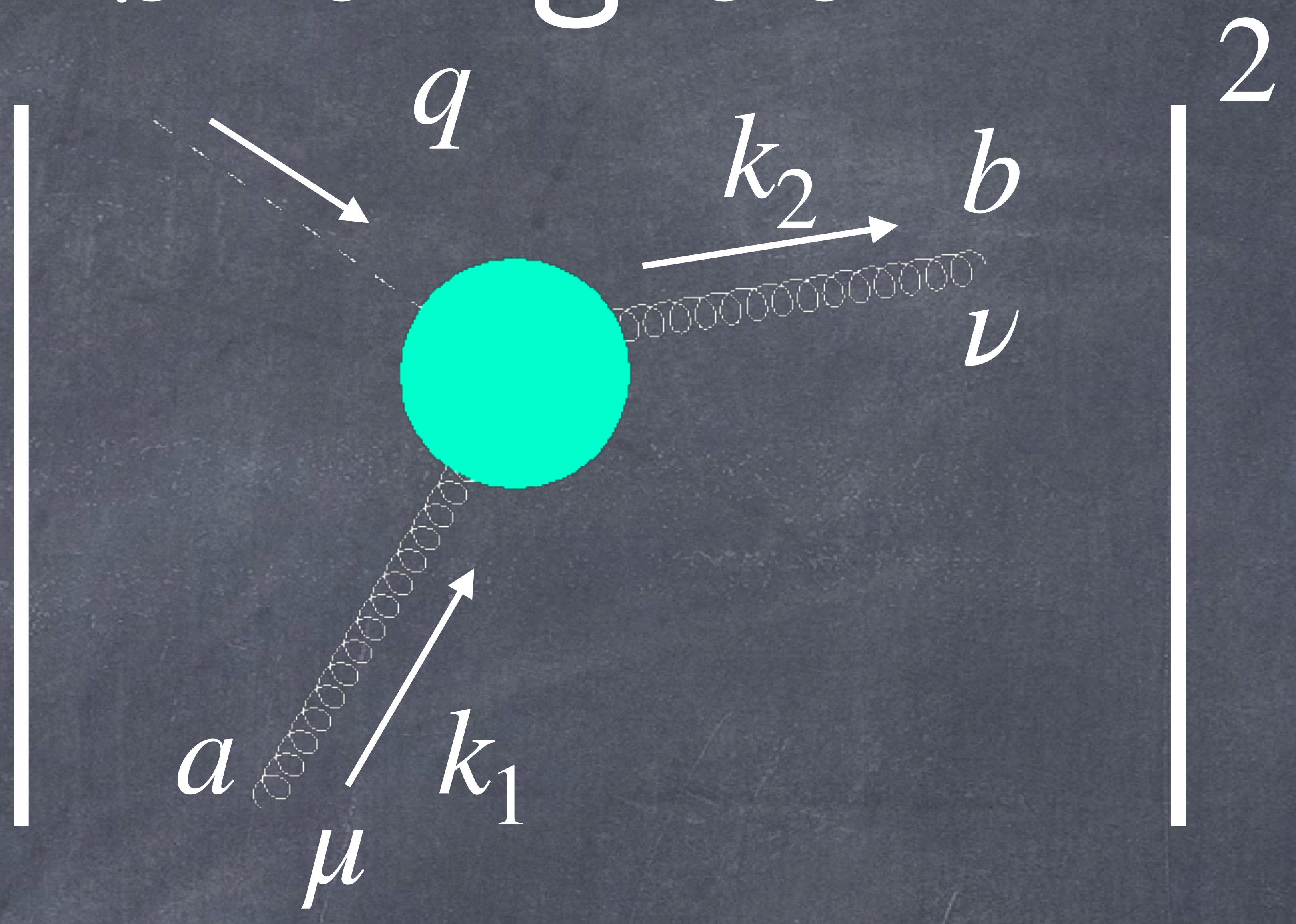
Sum over polarisation of an off-shell gluon

$$k_1^\mu = k^\mu + k_t^\mu$$
$$k_1^2 = k_t^2$$

$$d_{CH}^{\mu\nu} = (d-2)\frac{k_t^\mu k_t^\nu}{\vec{k}_t^{\,2}}$$

$$\lim_{\vec{k}_t^{\,2}\to 0} \langle d_{CH}^{\mu\nu} \rangle = g_\perp^{\mu\nu}$$

Works at tree level



Catani and Hautmann (**1994**) Nuclear Physics B.

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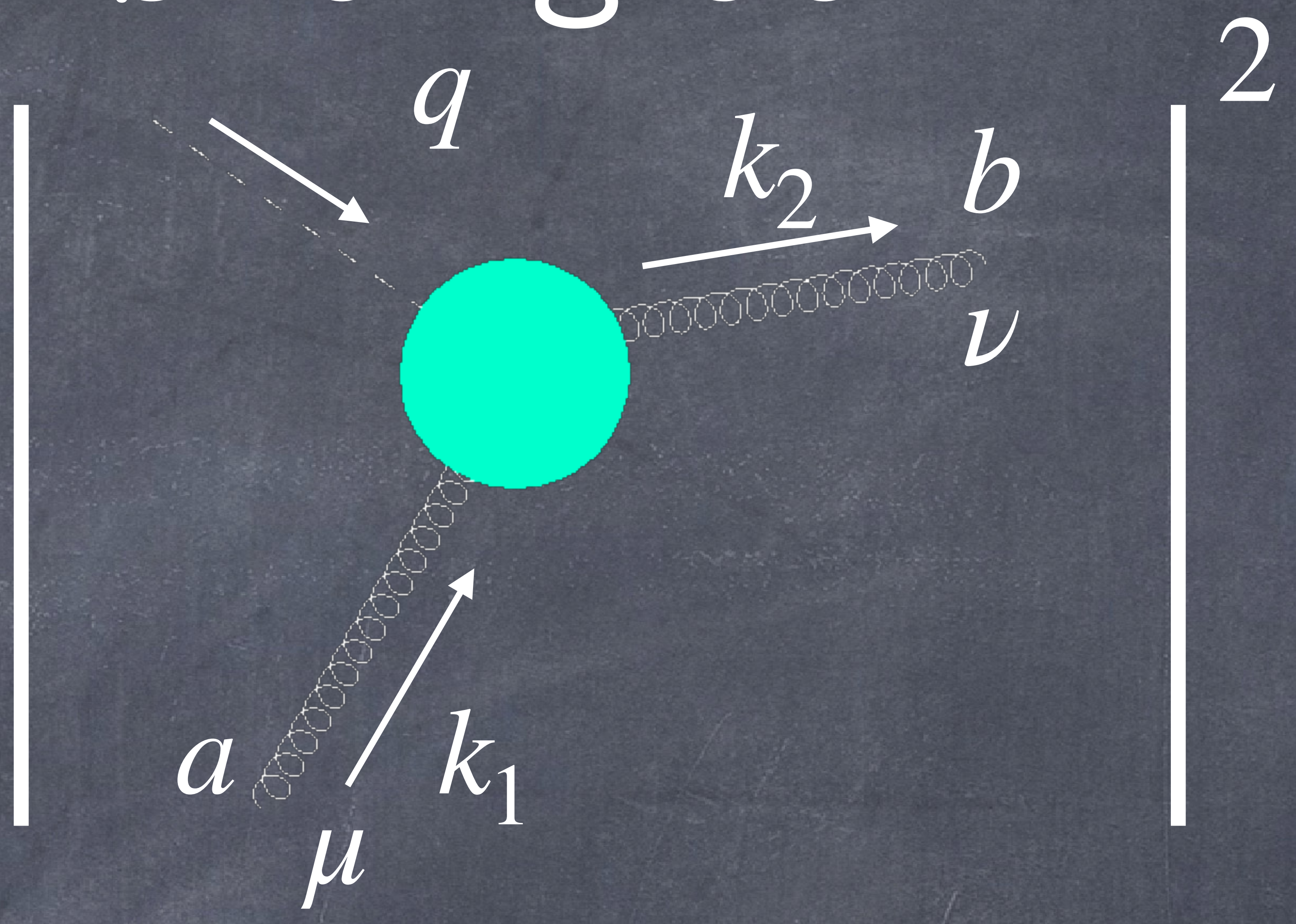

Works at tree level

Have to be modified when we study one-loop amplitudes

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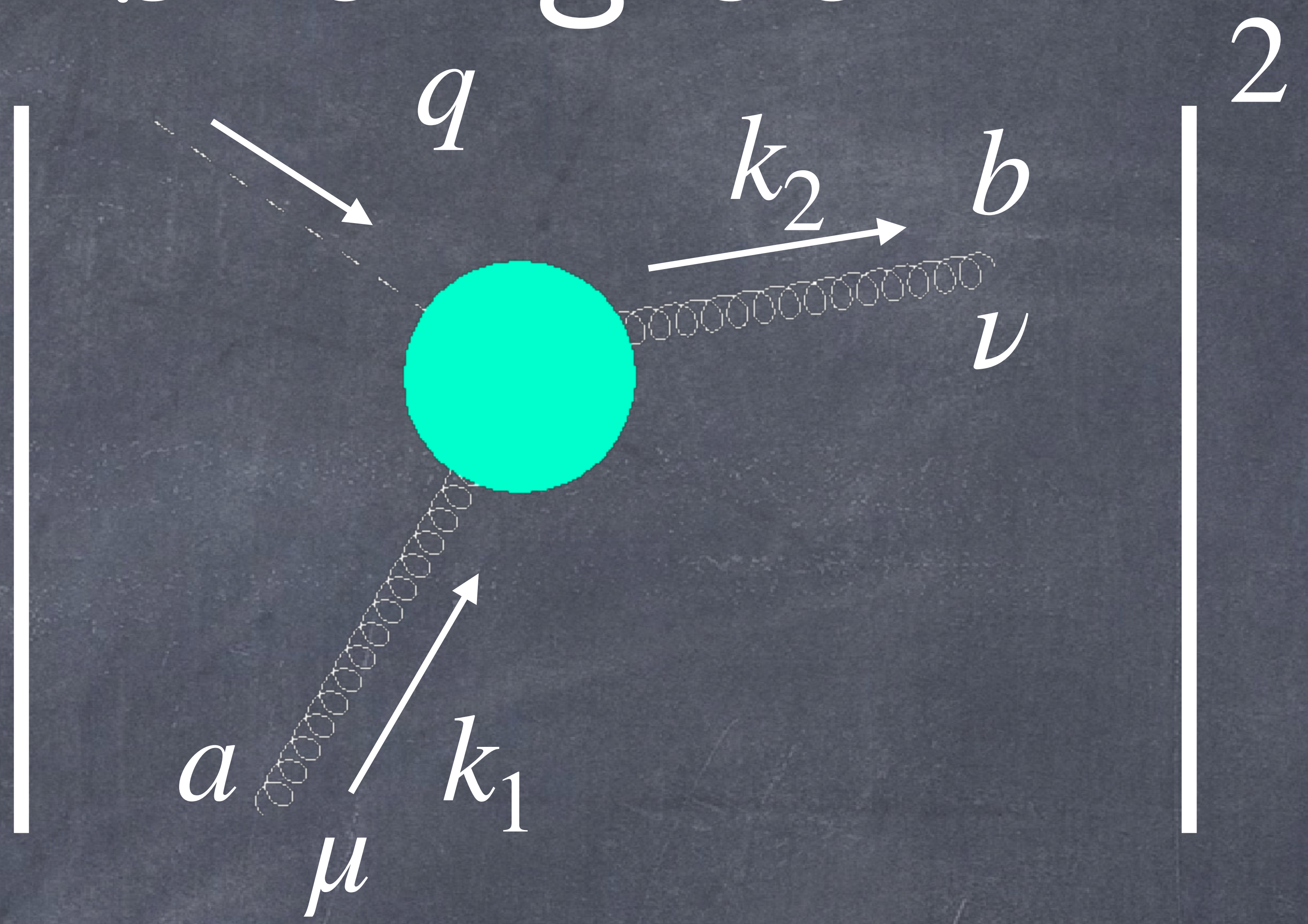

Works at tree level

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We are testing different definitions of the sum over polarisation of an off-shell gluon

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Sum over polarisation of an off-shell gluon

Possible ways to define $d^{\mu\nu}$:

- Most general two indices symmetric tensor that satisfies $A \cdot n = 0$
- Numerator of the propagator in light-cone gauge
- Gluon tensor from squared amplitude

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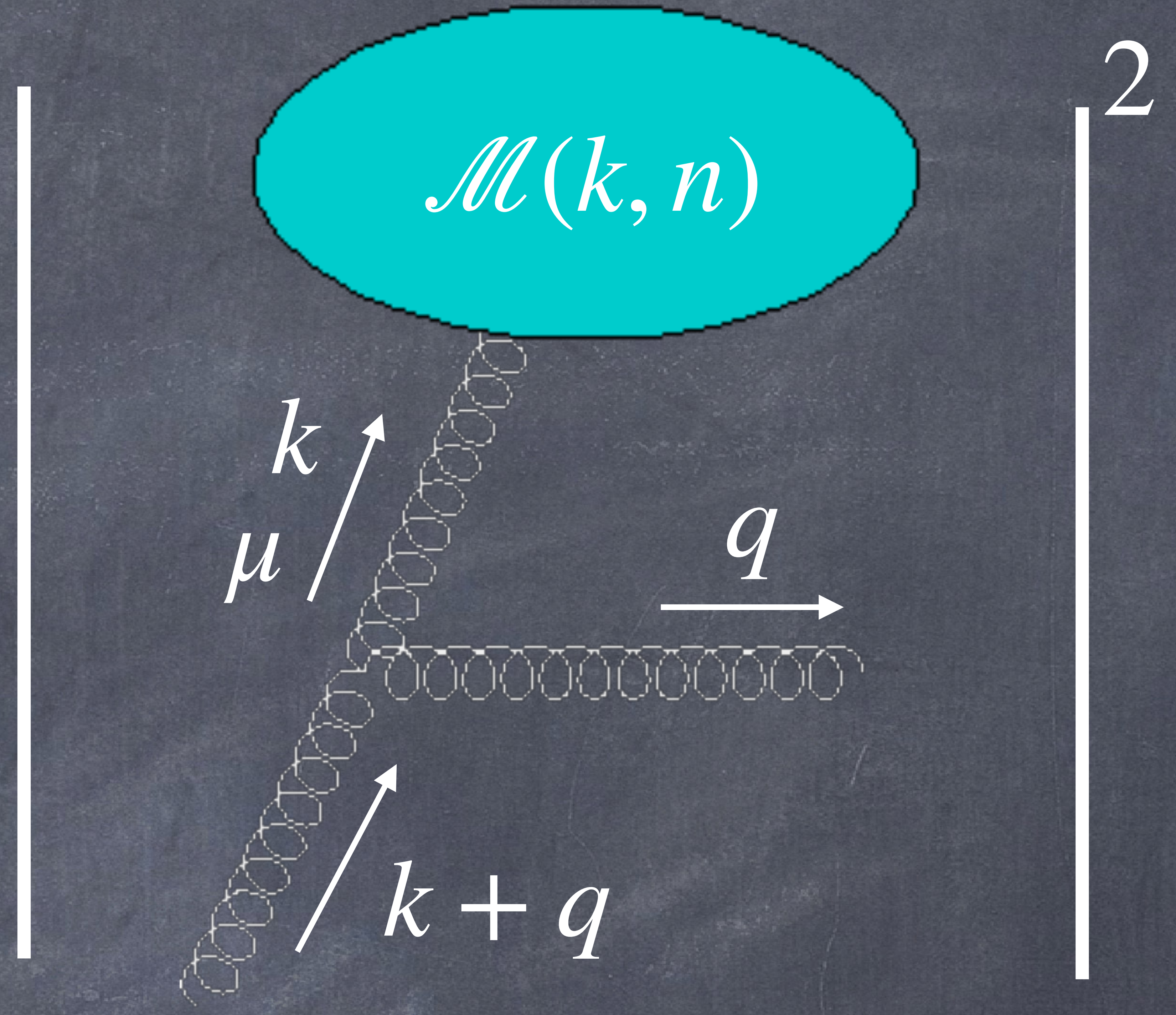
Gluon tensor from squared amplitude

$$\sum_{\text{pol}} |\mathscr{M}(k)|^2 = \mathscr{M}_\mu(k,n)\,\mathscr{M}^*_\nu(k,n)\,d^{\mu\nu}_{\text{off}}(k,n)$$



Gluon tensor from squared amplitude

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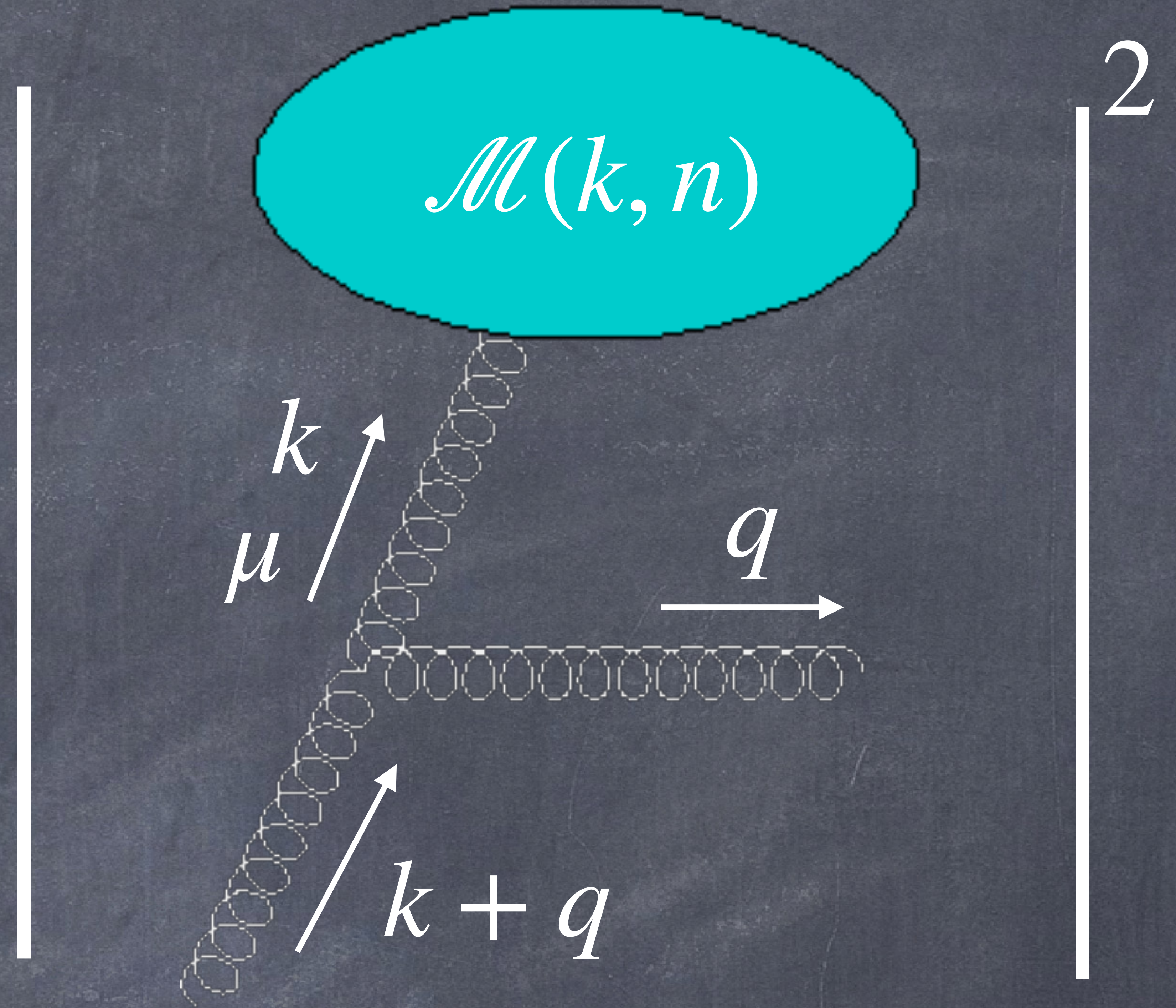
$$d^{\mu\nu}_{\text{off}}(k,n) = \Delta^{\mu\rho}(k,n)\Delta^{\nu\sigma}(k,n)T_{\rho\sigma}(k,n)$$



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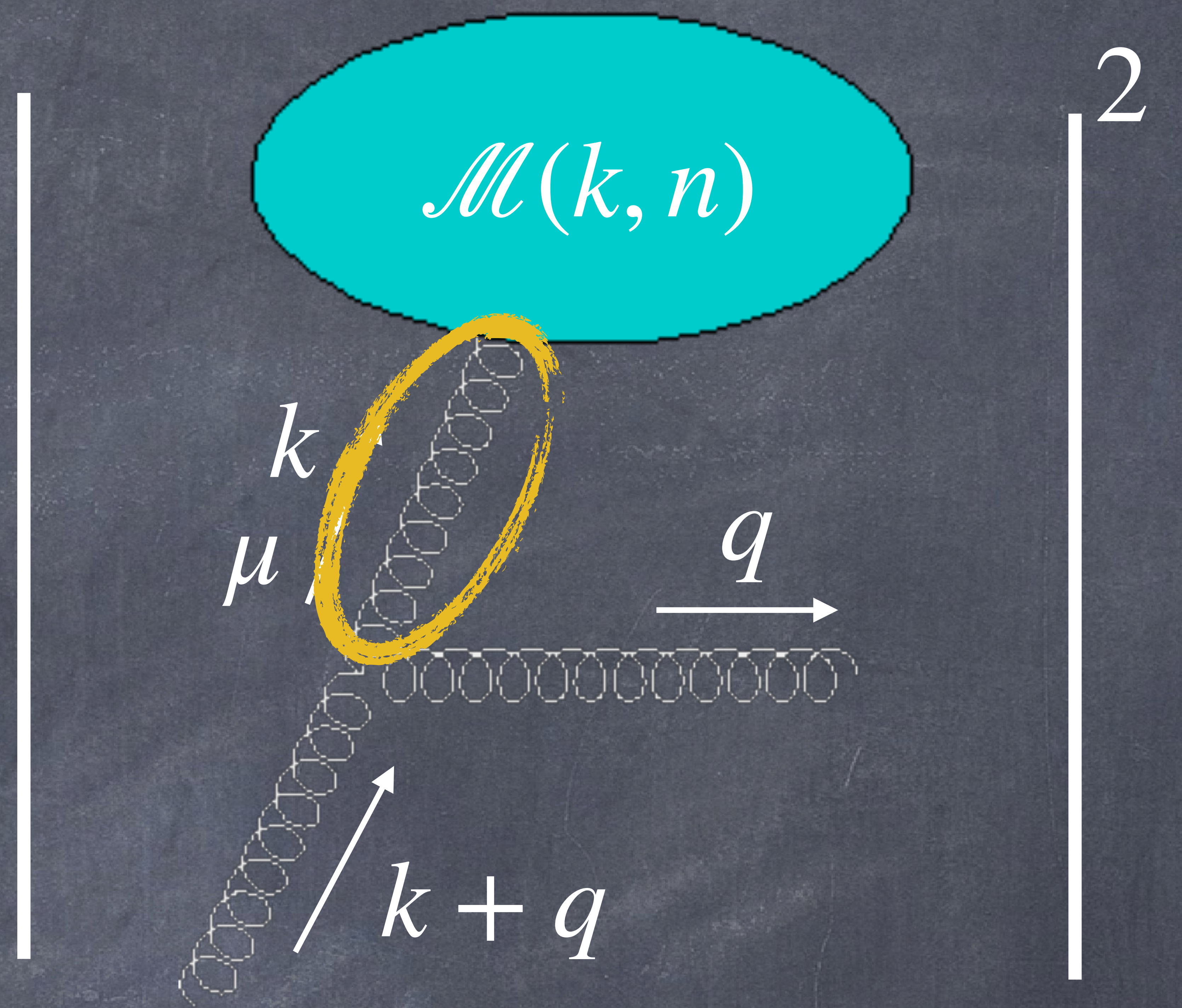
Gluon propagator



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Gluon propagator

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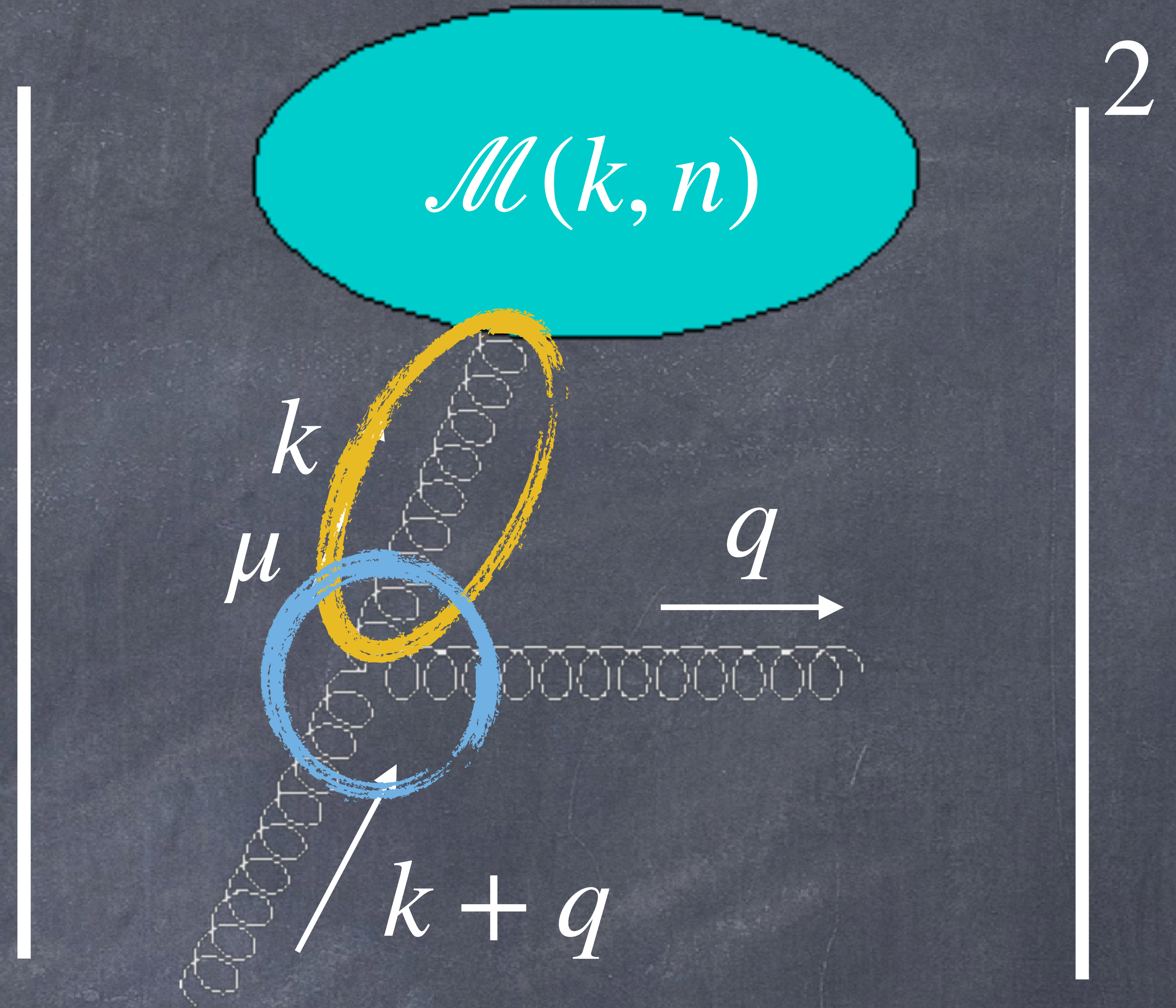

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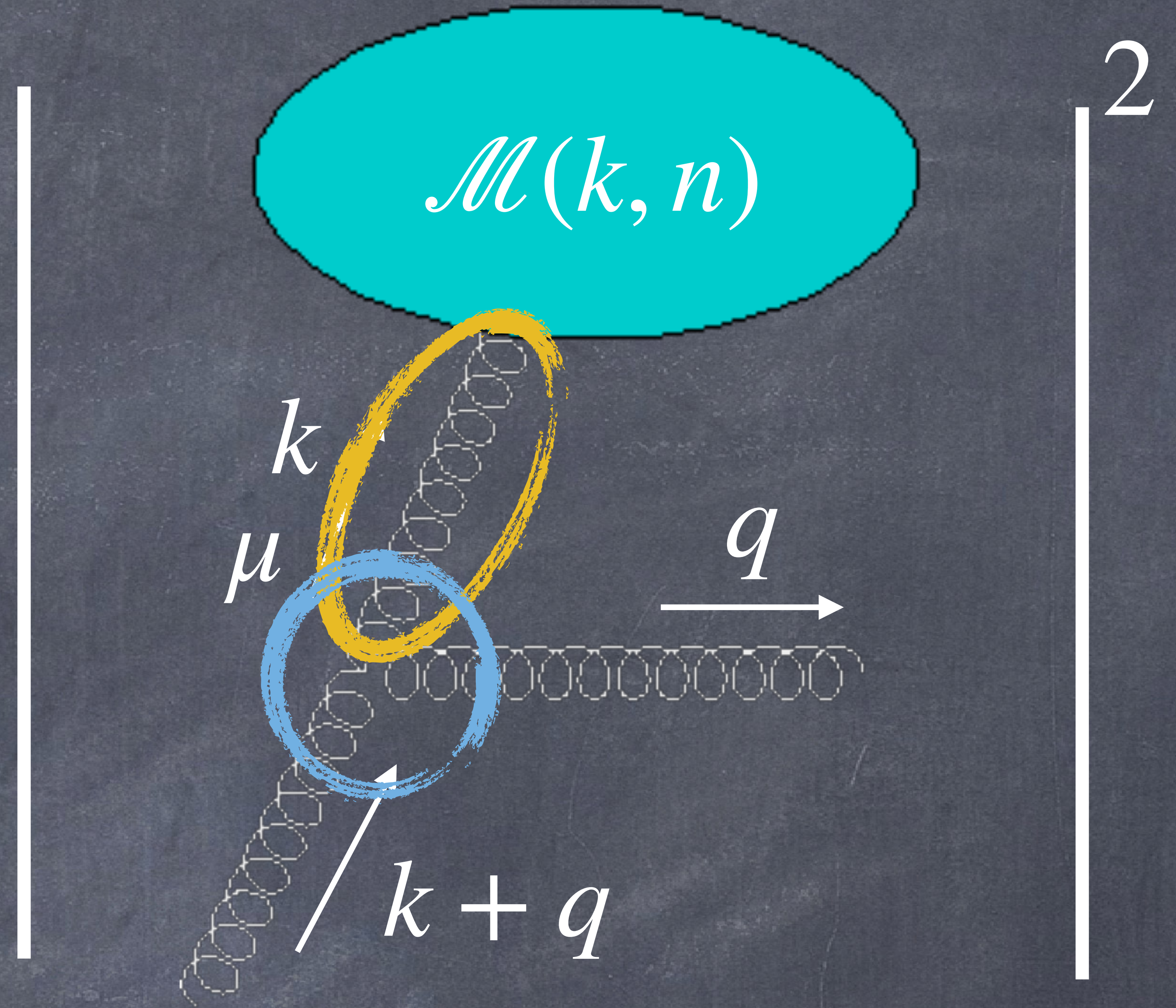

$$d^{\mu\nu}_{\text{off}}(k,n) = -\frac{A}{k^4}\left(-g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k\cdot n} + \frac{2D}{A}\frac{k^2}{(k\cdot n)^2}n^\mu n^\nu\right)$$

Where are we?

$$C_g(N, \alpha_s) = \int_0^\infty dk_\perp^2 \, \mathscr{C}\left(N, k_\perp^2, Q^2, \alpha_s\right) \, \mathscr{U}\left(N, k_\perp^2, Q^2\right)$$

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