DARK ENERGY OF KINETIC GRAVITY BRAIDING

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THIS TALK IS BASED ON WORK IN PROGRESS &

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IN COLLABORATION WITH

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MAIN MESSAGE

- Non-canonical scalar field which “acts” like imperfect fluid, and kinetically mixes / braids with the metric
- Manifestly stable violation of the Null Energy Condition possible
- Vanishing shift-charge corresponds to cosmological attractors similar to Ghost Condensate / “bad” k-Inflation. But here these attractors can be manifestly stable and their exact properties depend on external matter. These attractors generically evolve to de Sitter in late time asymptotic.

MORE ON THESE THEORIES & Co:
TH Cosmo Coffee, November 17

BRAIDING METRIC WITH A SCALAR FIELD

WHAT IS KINETIC GRAVITY BRAIDING?

$$S_\phi = \int \mathrm{d}^4x \sqrt{-g}\left[K\left(\phi, X\right) + G\left(\phi, X\right)\Box\phi\right]$$

where $X \equiv \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$

Minimal coupling to gravity $S_{\mathrm{tot}} = S_\phi + S_{\mathrm{EH}}$

However, derivatives of the metric are coupled

to the derivatives of the scalar, provided

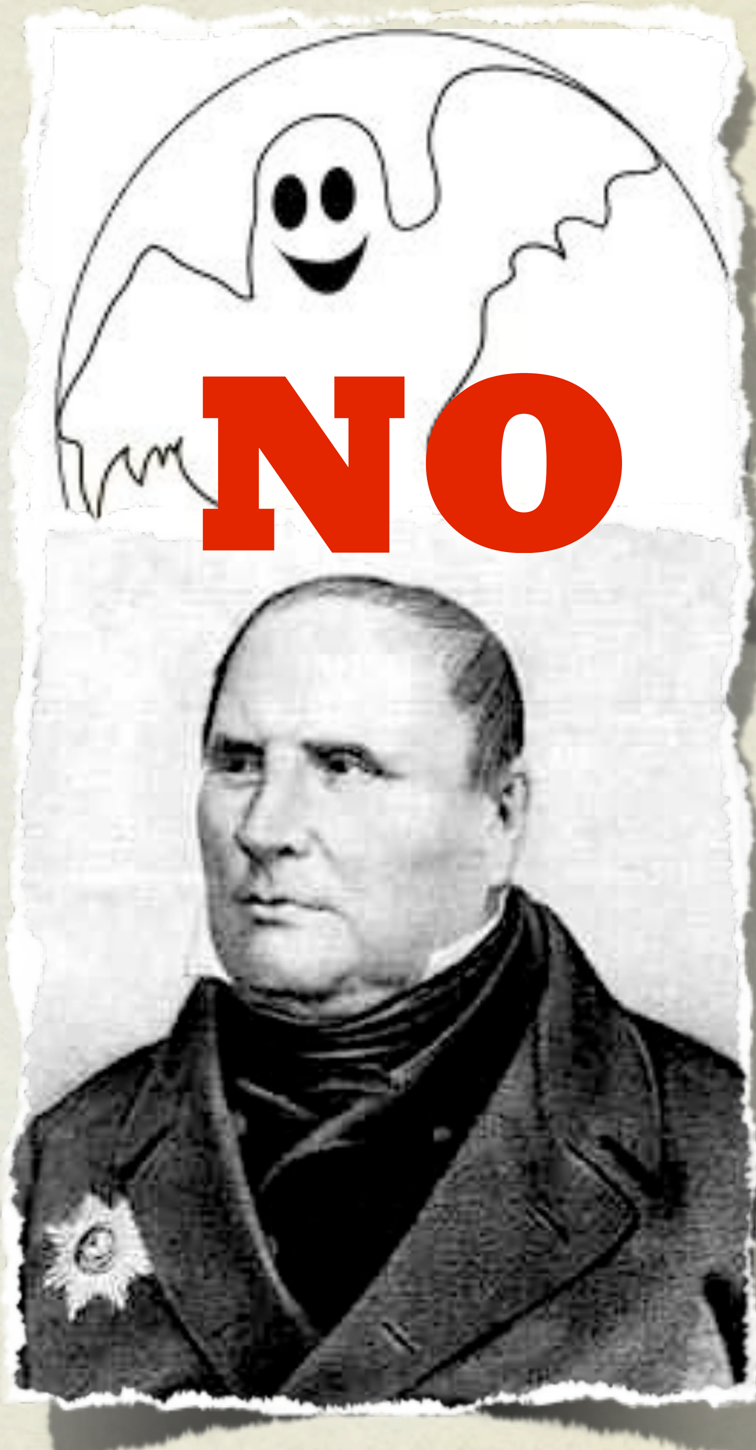
$$G_X \neq 0$$

ACTION FOR KINETIC GRAVITY BRAIDING IS SIMILAR TO EINSTEIN-HILBERT ACTION

- The second derivative (higher derivative - HD) enters the action only **linearly**

- One can eliminate the HD only by breaking the Lorentz invariant formulation of the theory. Boundary therms are required!

- Despite of the HD in the action still equations of motion are of the 2nd order:
  NO new degrees of freedom -
  NO Ostrogradsky's ghosts



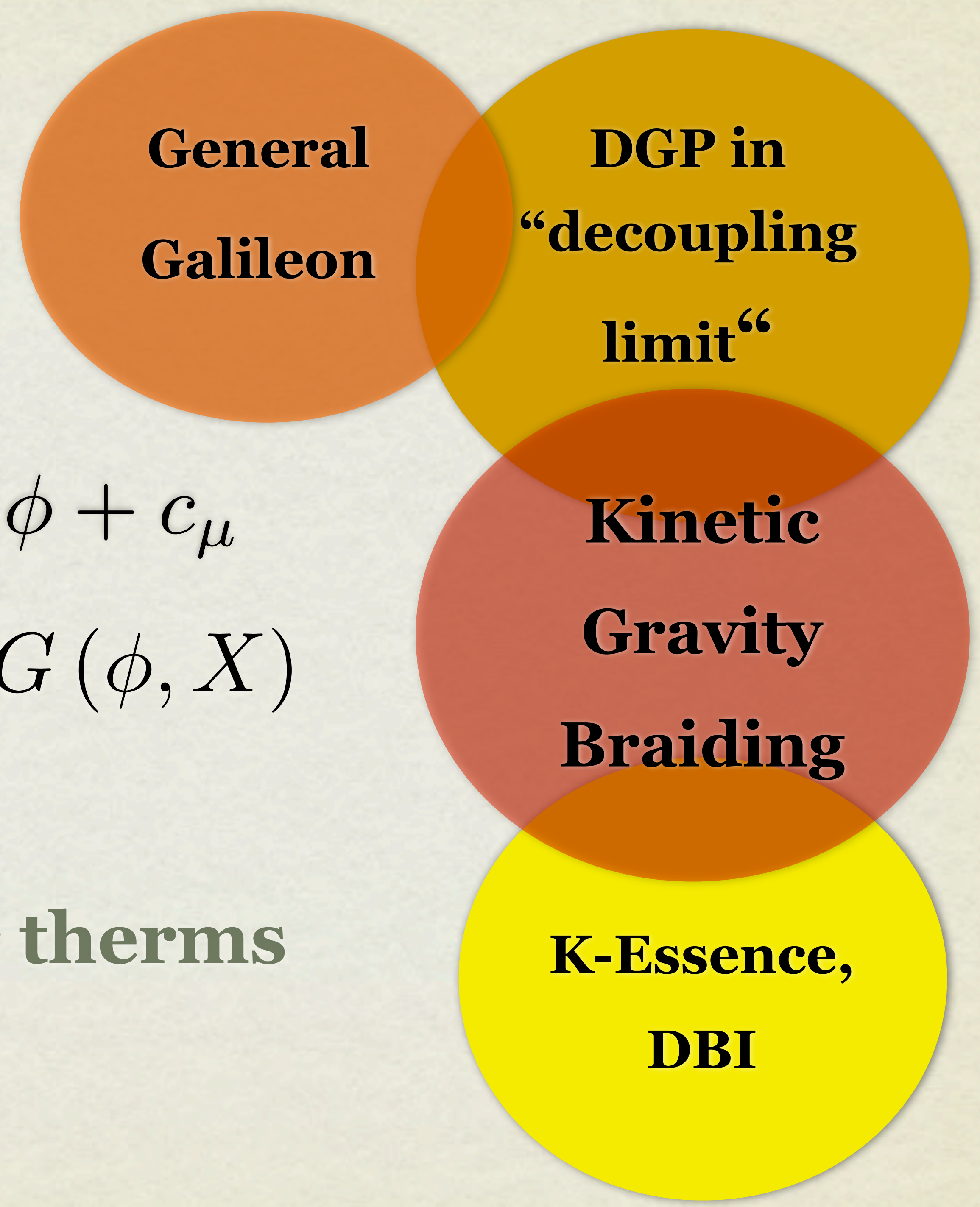
KINETIC GRAVITY BRAIDING≈ GALILEON, BUT

- Does **not require** the Galileon symmetry

$$\phi \longrightarrow \phi + c \quad \text{and} \quad \partial_\mu \phi \rightarrow \partial_\mu \phi + c_\mu$$

- General functions $K(\phi, X)$ and $G(\phi, X)$

- Minimal coupling to gravity **in the action, no higher order therms**



EXPANSIONS IN GRADIENT TERMS

- K-Essence, DBI etc

$$K(\phi, X) \sim X\left(1 + c_1(\phi)X + c_2(\phi)X^2 + ...\right)$$

- Kinetic Gravity Braiding – integrate the canonical kinetic energy by parts

$$G(\phi, X)\Box\phi \sim -\phi\Box\phi\left(1 + \tilde{c}_1(\phi)X + \tilde{c}_2(\phi)X^2 + ...\right)$$

EQUATION OF MOTION

- "Shift-Charge"/ Particle Current: J^μ
- New Equivalent Lagrangian: $\mathcal{P}$
- Equation of motion: $$\boxed{\nabla_\mu J^\mu = \mathcal{P}_\phi}$$

Both Einstein equations and the scalar equation of motion contain second derivatives of the metric and of the scalar field: kinetic mixing / braiding

IMPERFECT FLUID FOR TIMELIKE GRADIENTS

- Four velocity $u_\mu \equiv \frac{\nabla_\mu \phi}{\sqrt{2X}}$ projector: $\perp_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$
- Chemical potential / mass per particle: $m = \sqrt{2X}$
- "Diffusion"/"energy transport" coefficient: $\kappa \equiv 2XG_X$
- Total Pressure: $\mathcal{P} = P - \kappa \dot{m}$
- Energy Flow: $q_\mu = -\kappa \perp_\mu^\nu \nabla_\nu m = m \perp_{\mu\nu} J^\nu$
- Energy Momentum Tensor:

$$T_{\mu\nu} = \epsilon u_\mu u_\nu - \perp_{\mu\nu} \mathcal{P} + u_\mu q_\nu + u_\nu q_\mu$$

ENERGY CONSERVATION IN COMOVING VOLUME

Energy conservation: $u_\nu \nabla_\mu T^{\mu\nu} = 0$

$$\mathrm{d}E = -\mathcal{P}\mathrm{dV} + m\mathrm{d}\mathcal{N}_{\mathrm{dif}}$$

Euler relation: $$\epsilon = mJ - P$$

Momentum conservation:

$$\perp_{\mu\nu} \nabla_\lambda T^{\lambda\nu} = 0$$

ATTRACTORS

Euler relation: $\epsilon = mJ - P$

for no particles: $J_* = 0$

$$\epsilon = -P$$

EXAMPLE: GHOST CONDENSATE / K-INFLATION WITH DIFFERENT STRUCTURE OF THE KINETIC TERM

- Lagrangian $\mathcal{L} = X\left(-1 + \mu\Box\phi\right)$
- shift-charge density $J = \dot{\phi}(3\mu H\dot{\phi} - 1)$
- equation of motion $\dot{J} + 3HJ = 0$

$$J \propto a^{-3}$$

INFLATION BRINGS THE SCALAR TO ATTRACTOR

$$J_* = 0$$

NONTRIVIAL ATTRACTOR

No Particles: $\boxed{J_* = 0}$

$$\dot{\phi}_* = (3\mu H_*)^{-1}$$

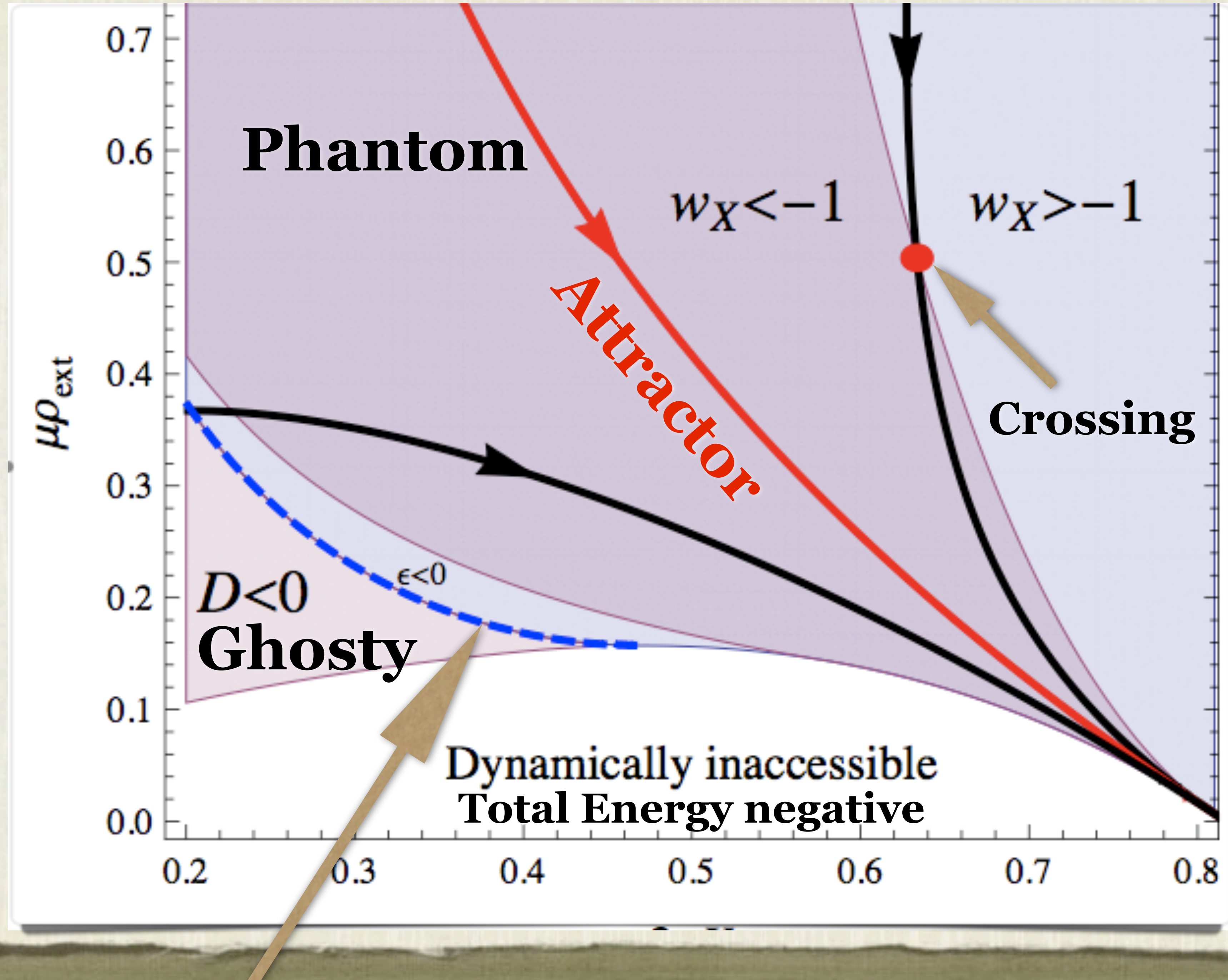
$$H_*^2 = \frac{1}{6}\rho_{\text{ext}}\left(1 + \sqrt{1 + \frac{2}{3}\left(\mu\rho_{\text{ext}}\right)^{-2}}\right)$$

STABLE

$$\dot{\phi}_* = 0$$

$$H_*^2 = \frac{1}{3}\rho_{\text{ext}}$$

GHOSTY



Phase portrait for scalar field & dust

DARK ENERGY

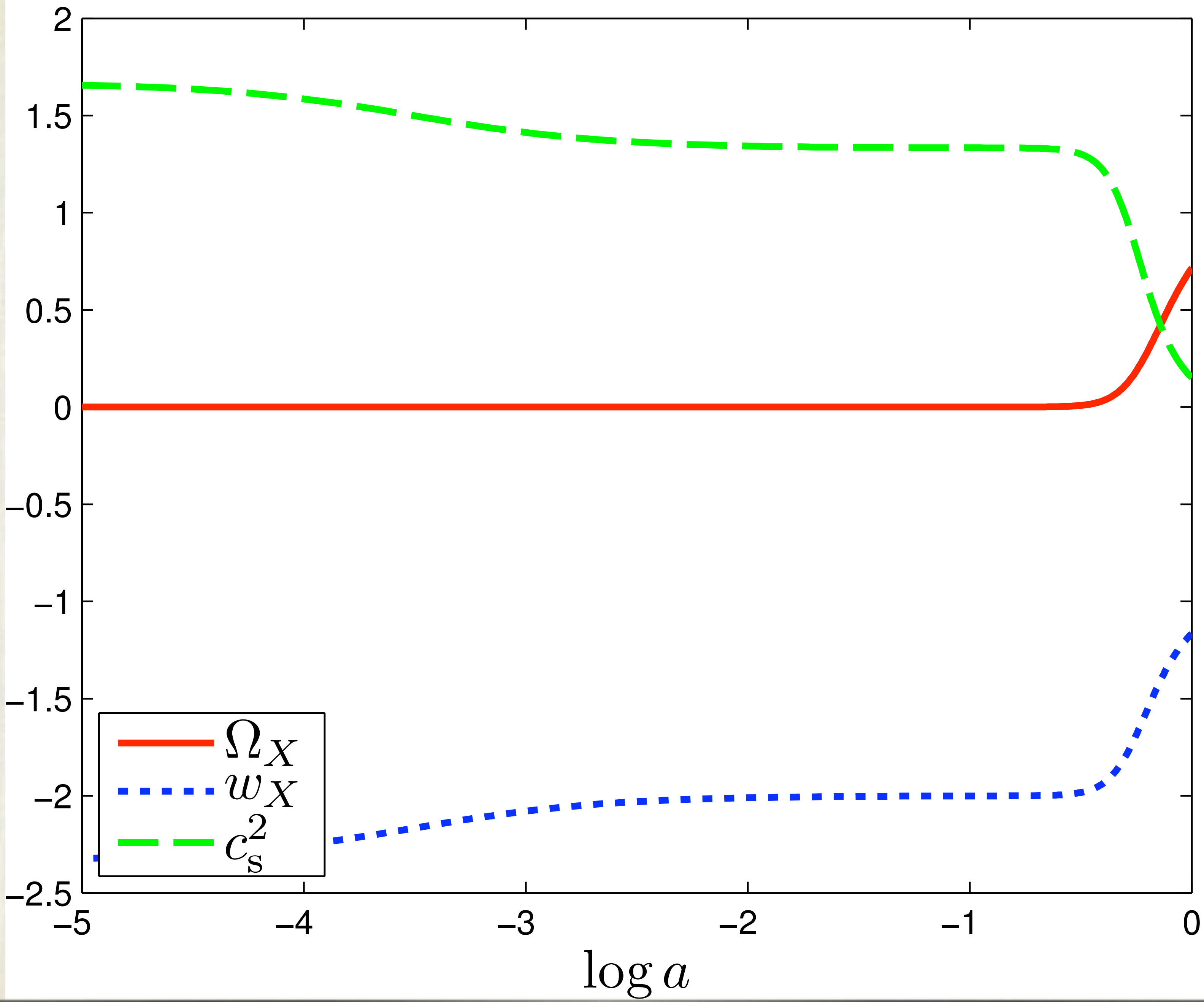
today $\sqrt{\frac{3}{2}}\mu\rho_{\rm ext} \ll 1$ ➜ $H_*^2 \simeq \frac{1}{6}\sqrt{\frac{2}{3}}\mu^{-1}$

$\Lambda_* \simeq \frac{1}{2}\sqrt{\frac{2}{3}}\mu^{-1} \simeq 3\rho_{\rm CDM}$ ➜ $\sqrt{\frac{3}{2}}\rho_{\rm CDM}\mu \simeq \frac{1}{6} \ll 1$

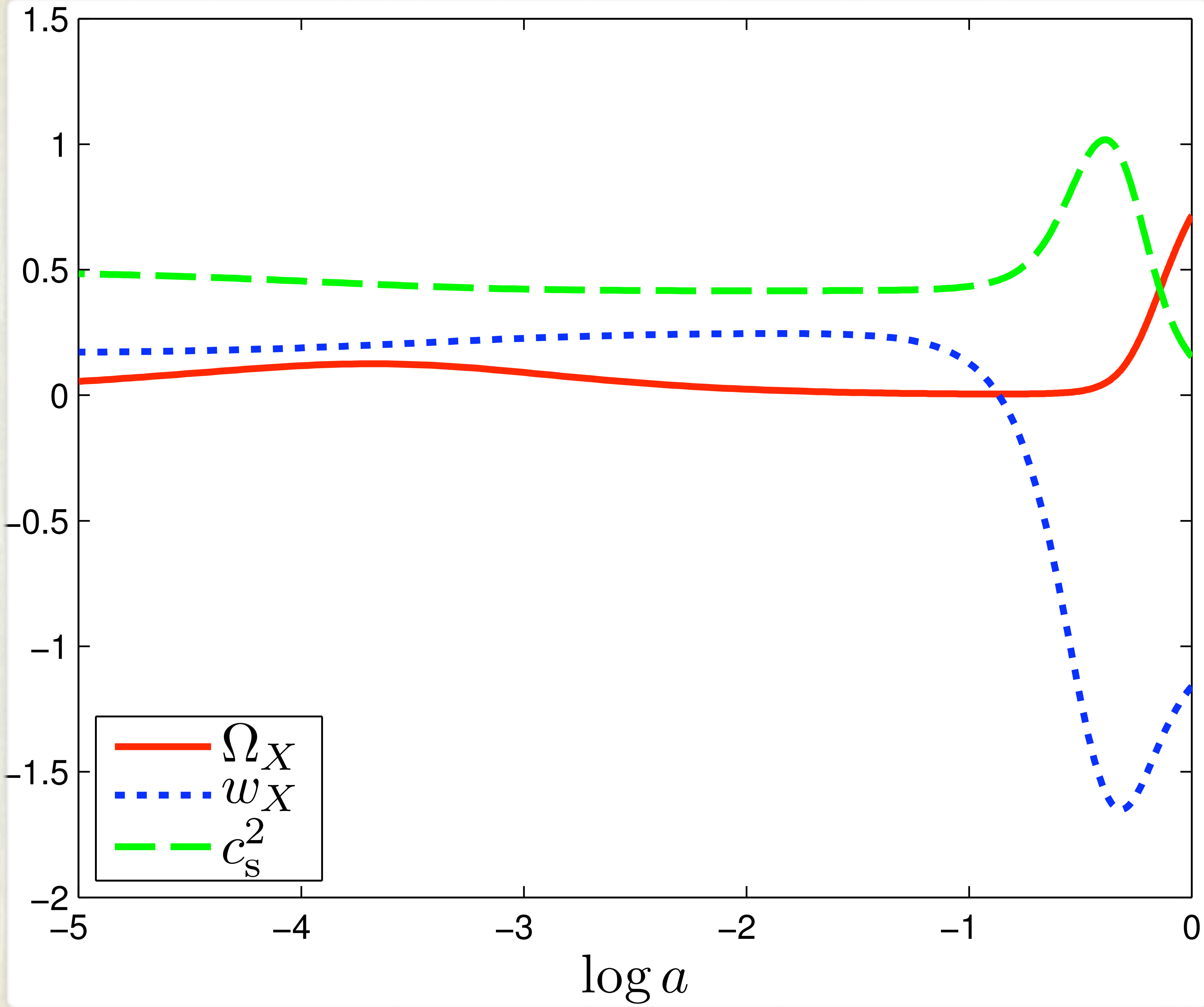
$$\text{Mass Scale} \sim \mu^{-1/3} \sim (H_0^2 M_{\rm Pl})^{1/3} \sim 10^{-13}\text{eV}$$

Length: **100 km**

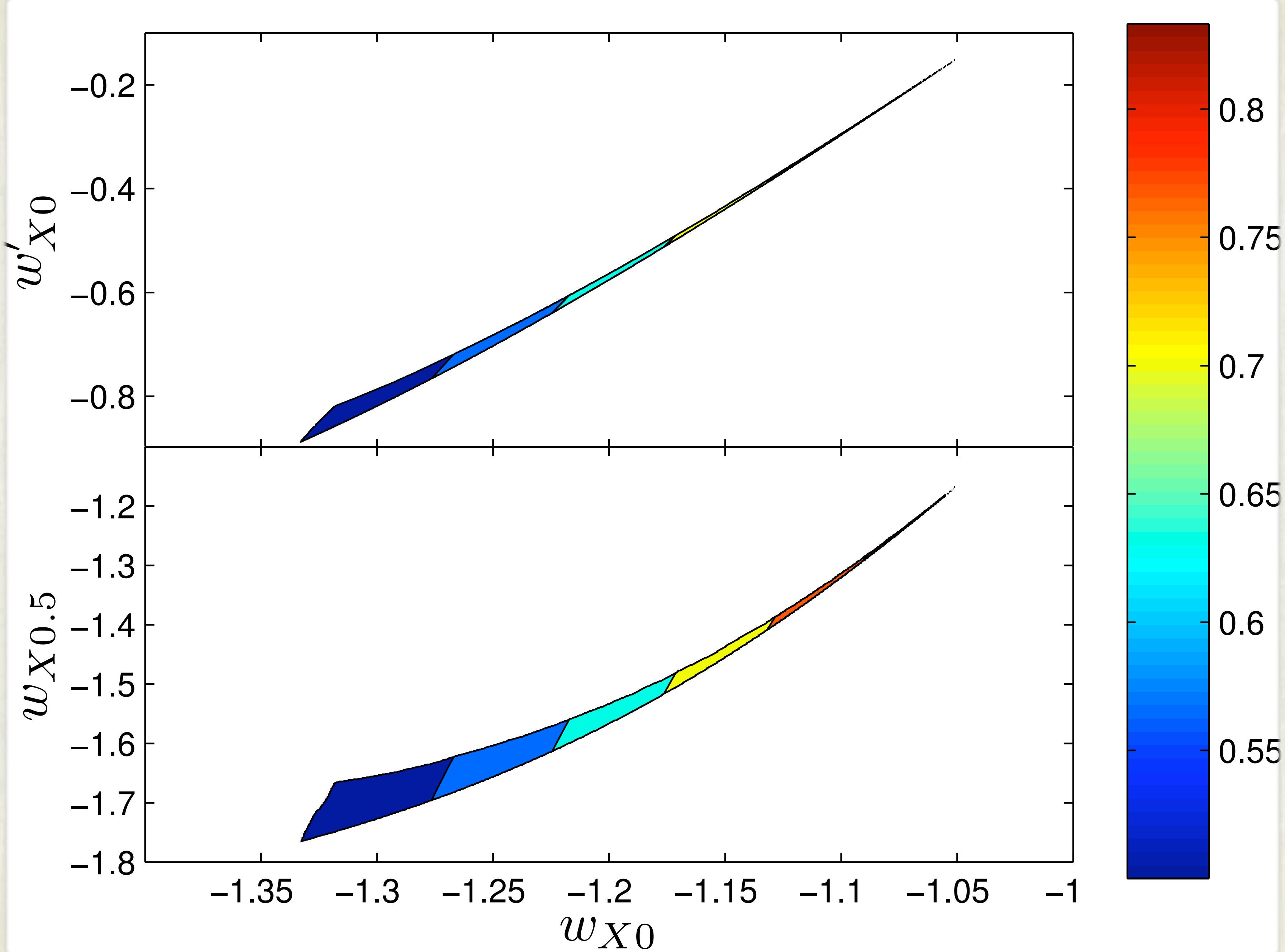
In Quintessence the size of the universe



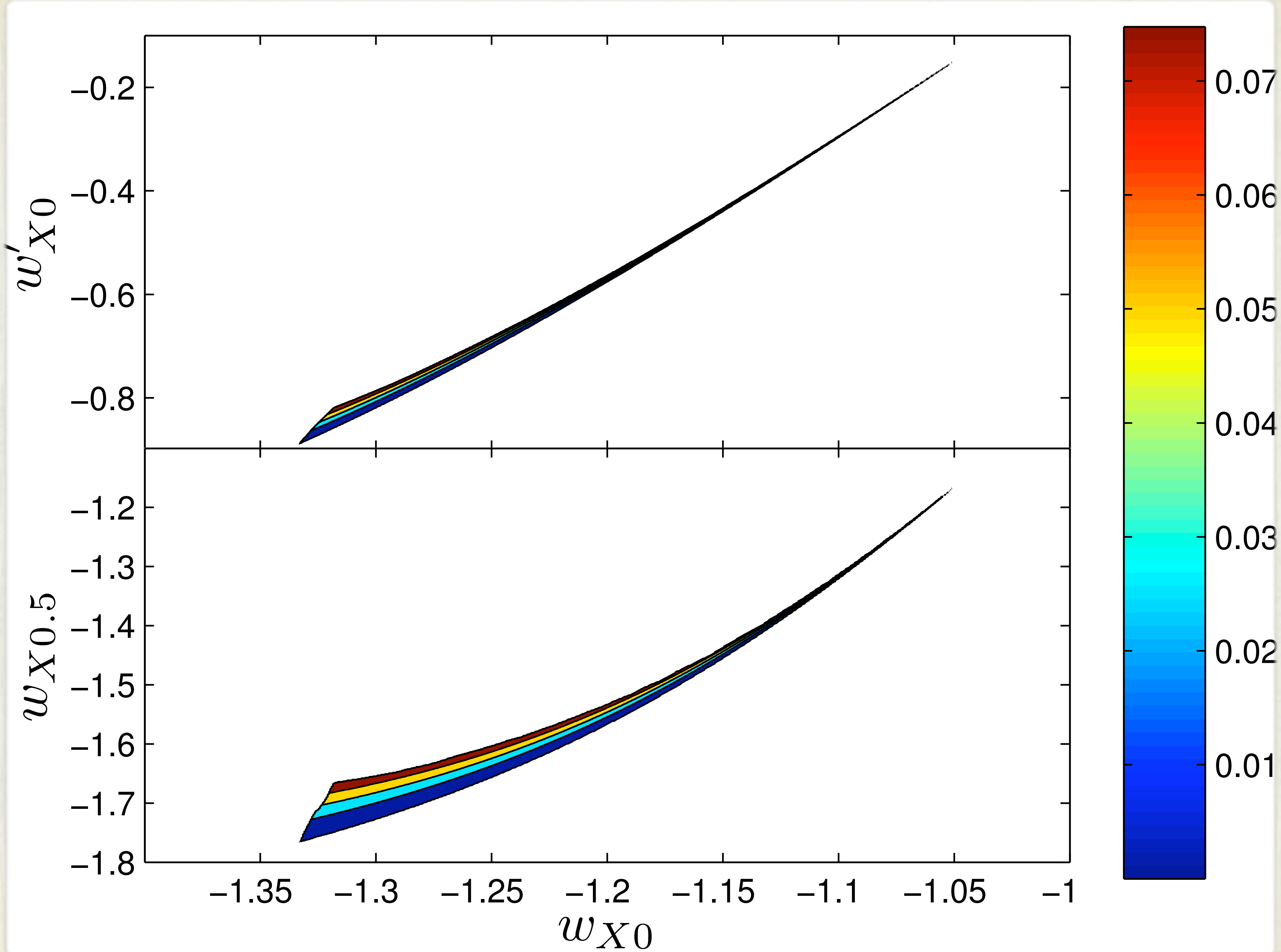
Evolution of dark energy properties in the Friedmann universe also containing dust and radiation. The scalar evolves on its attractor throughout the presented period. During matter domination $w_X = -2$, while $w_X = -7/3$ during radiation domination. The sound speed is superluminal when the scalar energy density is subdominant, becoming subluminal when $\Omega_X \approx 0.1$ and $w_X \approx -1.4$



Evolution of DE properties in the Friedmann universe which also contains dust and radiation. The energy density in the scalar is J-dominated (off attractor) until a transition during the matter domination epoch. This allows the scalar to increase its contribution to the total energy budget throughout radiation domination ($w_X = 1/6$) and provide an early DE peaked at matter-radiation equality, from whence it begins to decline with $w_X = 1/4$. The transition to the attractor behaviour is rapid. The equation of state crosses $w_X = -1$ and the scalar energy density begins to grow. The final stages of evolution are on the attractor and are similar to those presented in previous figure.



$0.1 < \Omega_{\rm m} < 0.5$ and $\Omega_{X{\rm eq}} < 0.1$. The shading contours correspond to the energy density of DE today Ω_{X0}. Two parameterisations of DE behaviour are shown: w_X and w'_X evaluated today, and w_X evaluated today and at $z = 1/2$. The requirement that the energy density in DE at matter-radiation equality be small, $\Omega_X^{\rm eq} < 0.1$ forces the value of the shift charge to be small today $Q_0 < 10^{-2}$. This means that in the most recent history, the evolution has effectively been on attractor or very close to it and the permitted value of w_X is very restricted and determined to all intents and purposes by Ω_X^0.



The shading representing the contribution of DE to energy density at matter-radiation equality. We choose to cut the parameters such that the contribution to this early DE at that time is no larger than 10%. It can clearly be seen that values of w_X closer to -1 are obtained when the shift charge is larger, but this leads to more early DE, eventually disagreeing with current constraints

FAR OFF ATTRACTOR

w_X	$\Omega_X \approx 0$	$\Omega_X \approx 1$
MD:	$1/4$	$\frac{1}{5}\left(1-\frac{1}{5}\Omega_{\rm ext}\right)$
RD:	$1/6$	$\frac{1}{5}\left(1-\frac{2}{3}\Omega_{\rm ext}\right)$
QdS:	$\frac{1}{2}\left(1-\frac{1}{2}(1+w_{\rm ext})\right)$	$\frac{1}{5}\left(1-(w_{\rm ext}-\frac{1}{5})\Omega_{\rm ext}\right)$
c_s^2		
MD:	$5/12$	$\frac{23}{75}-\frac{12\Omega_{\rm ext}}{125}$
RD:	$1/2$	$\frac{23}{75}-\frac{56\Omega_{\rm ext}}{375}$
QdS:	$\frac{1}{6}+\frac{1}{4}(1+w_{\rm ext})$	$\frac{23}{75}+\frac{4\Omega_{\rm ext}}{125}(2+5(1+w_{\rm ext}))$

Table 2: Values of the equation-of-state parameter and the speed of sound for the off-attractor scalar dominated by the shift-current J. MD and RD refer to the domination of the external fluid by matter or radiation, respectively, QdS is the inflationary quasi-de-Sitter phase.

ON ATTRACTOR

w_X		
	$\Omega_X \approx 0$	$\Omega_X \approx 1$
MD:	-2	$-1 - \Omega_{\rm ext}/2$
RD:	$-7/3$	$-1 - \frac{2}{3}\Omega_{\rm ext}$
QdS:	$-1 - (1 + w_{\rm ext})$	$-1 - \frac{1}{2}(1 + w_{\rm ext})\Omega_{\rm ext}$
$c_{\rm s}^2$		
MD:	$4/3$	$\frac{5}{12}\Omega_{\rm ext}$
RD:	$5/3$	$\frac{1}{2}\Omega_{\rm ext}$
QdS:	$\frac{1}{3} + (1 + w_{\rm ext})$	$\frac{\Omega_{\rm ext}}{6}\,(2 + 3(1 + w_{\rm ext}))$

Table 1: Limiting values of the equation of state parameter and the speed of sound for the scalar evolving on its attractor. MD and RD refer to the domination of the external fluid by matter or radiation, respectively, QdS is the inflationary quasi-de-Sitter phase.

THANKS A LOT FOR YOUR ATTENTION!