

# Discrete R symmetries for the MSSM and beyond

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based on work with

Raby, Ratz, Ross, Schieren, Schmidt-Hoberg & Vaudrevange,  
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# Outline

- 1 Discrete R symmetries
- 2 R symmetry breakdown and the  $\mu$  term
- 3  $SU(5)$ -compatible extensions

# Motivation

- Supersymmetry, when being broken at TeV scale, is a technical solution to the hierarchy problem in SM.
- There are strong constraints on the soft SUSY breaking parameters by FCNC and CP violating processes.
- We focus on the constraints in the SUSY-preserving sector of MSSM.
- The most general effective superpotential being compatible with SM gauge symmetry in MSSM is

$$\begin{aligned}
 W = & \lambda_u H_u Q U^c + \lambda_d H_d Q D^c + \lambda_l H_d L E^c + \mu H_u H_d \\
 & + \mu' H_u L + \lambda L L E^c + \lambda' L Q D^c + \lambda'' U^c D^c D^c \\
 & + \frac{1}{M_P} (\lambda_1 Q Q Q L + \lambda_2 U^c U^c D^c E^c + \lambda_3 Q Q Q H_d + \dots)
 \end{aligned}$$

- Phenomenological constraints are

$$\text{EWSB} : \mu \sim 10^{-16} M_P,$$

$$\text{Neutrino mass} : \mu' \leq 10^{-21} M_P,$$

$$\text{Proton stability} : \lambda' \lambda'' \leq 10^{-26}, \quad \lambda' \lambda_3 \leq 10^{-10}, \\ \lambda_1 \leq 10^{-8}, \quad \lambda_2 \leq 10^{-8}.$$

- Global discrete symmetries are violated by gravity effects. So, discrete symmetries should be of gauge symmetry origin.
- $Z_2$  Matter-parity: allows the dim-5 proton decay operators.
- $Z_3$  Baryon-triality: forbids both dim-4 B violating operators and dimension-5 proton decay operators but allows dim-4 L violating operators. [Ibanez, Ross (1992)]
- $Z_6$  Proton-hexality ( $= Z_2 \otimes Z_3$ ): forbids all dim-4 B/L violating operators and dim-5 proton decay operators.  
[Dreiner, Luhn, Thormeier (2005)]
- However, all these symmetries allow the problematic  $\mu$  term.

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# Discrete R symmetries

- N=1 SUSY algebra contains a global  $U(1)_R$  symmetry, e.g. for a chiral superfield  $\Phi = \phi + \theta\psi + \theta^2 F_\phi$ :

$$\Phi(\theta) \rightarrow e^{iq\alpha} \Phi(e^{-i\alpha}\theta); \quad \phi \rightarrow e^{iq\alpha} \phi, \quad \psi \rightarrow e^{i(q-1)\alpha} \psi.$$

- Discrete subgroups of the continuous  $U(1)_R$  symmetry can arise as the discrete remnants of Lorentz group in compact dimensions in string compactifications. [Błazszyk et al (2009)]
- Discrete  $Z_N^R$  symmetries can be anomaly-free with extra matters. [Kurosawa, Maru, Yanagida (2001); Hamaguchi, Maru (2003)]
- We consider the Green-Schwarz mechanism to cancel  $Z_N^R$  anomalies. The  $Z_4^R$  symmetry is the unique one satisfying the anomaly cancellation,  $SO(10)$ -compatibility, no tree-level  $\mu$  term and all the known fermion masses.

- Suppose that the dilaton transforms as  $\delta S = \frac{i}{2}\delta_{GS}$  under an anomalous  $U(1)_A$ . Then, the  $U(1)_A$  anomalies appear from the variation of the following Green-Schwarz term,

$$\mathcal{L}_{GS} = -(\text{Im}S) \sum_{a=1}^3 k_a \frac{1}{2} \text{tr}(F_a \tilde{F}_a)$$

where  $k_a$  are the Kac-Moody levels with  $k_a = \frac{C_a}{2\pi^2\delta_{GS}}$  for the quantum  $U(1)_A$  anomalies  $C_a$ . [Green, Schwarz (1984)]

- For level-1 gauge groups, we are led to the universal anomaly conditions,

$$2\pi^2\delta_{GS} = \frac{1}{24}\text{tr}q = \frac{1}{3}\text{tr}q^3 = \text{tr}(I_G(r)q)$$

where  $\text{tr}(I_G(r)q)$  is the G-G- $U(1)_A$  mixed anomaly.

- For a non-R discrete  $Z_N$ , the universal anomaly conditions are

$$2\pi^2 \delta_{GS} = \frac{1}{24} \text{tr} q = \text{tr}(l_G(r)q) \pmod{\eta}$$

where  $\eta = N(N/2)$  for odd(even)  $N$  and  $q$  is the  $Z_N$  charge.

- For a discrete  $Z_N^R$ , we have G-G- $Z_N^R$  anomalies containing the gaugino contribution with R-charge  $\alpha$ ,

$$A_{G-G-Z_N^R} = \text{tr}(l_G(r)q) + l(\text{adj}) \cdot \alpha \pmod{\eta}.$$

So, the anomaly universality becomes different than the case of non-R discrete symmetries.

[Chamseddine, Dreiner (1996); K.-Y.Choi, HML (2009)]



# Unique $Z_4^R$ for the MSSM

[Babu, Gogoladze, Wang (2003); Lee et al (2010)]

- In MSSM with R-charges,  $q_{10_i}$  for  $10_i$  and  $q_{\bar{5}_i}$  for  $\bar{5}_i$ , the mixing  $SU(3)_C$  and  $SU(2)_L$  anomalies are

$$A_{3-3-Z_N^R} = \frac{1}{2} \sum (3q_{10_i} + q_{\bar{5}_i} - 4\alpha) + 3\alpha,$$

$$A_{2-2-Z_N^R} = \frac{1}{2} \sum (3q_{10_i} + q_{\bar{5}_i} - 4\alpha) + 2\alpha + \frac{1}{2}(q_H + q_{\bar{H}} - 2\alpha).$$

- The anomaly cancellation with  $A_{3-3-Z_N^R} = A_{2-2-Z_N^R} \pmod{\eta}$  leads to the condition for the R-charges of Higgs doublets,

$$q_H + q_{\bar{H}} = 4\alpha \pmod{2\eta}.$$

$\Rightarrow$  The  $\mu$  term is forbidden because  $H\bar{H}$  has R-charge  $2\alpha$ .  
[cf. A non-R  $Z_N$  with  $\alpha = 0$  does not forbid the  $\mu$  term.]

- For  $Z_N^R$  commuting with  $SO(10)$ ,  $q_{10_i} = q_{\bar{5}_i} = q$ : the quark and charged lepton masses and the Weinberg operator  $(LH_u)^2$  for neutrino masses yields

$$2q + q_H = 2\alpha \pmod{N},$$

$$2q + q_{\bar{H}} = 2\alpha \pmod{N},$$

$$2q + 2q_H = 2\alpha \pmod{N}.$$

- The solution is  $q_H = q_{\bar{H}} = 0 \pmod{N}$  and  $2q = 2\alpha \pmod{N}$ . Then, with  $\alpha = 1$ , the  $Z_4^R$  symmetry is the unique one forbidding the  $\mu$  term.
- The  $Z_4^R$  symmetry also forbids all the dangerous B/L violating terms in MSSM.
- The  $Z_4^R$  symmetry is embeddable in string compactifications.

[Blaszczyk et al (2009)]

## $U(1)_Y$ -mixed and pure $Z_N^R$ anomalies

- When hypercharge  $Y$  is normalized by the underlying GUT, the  $U(1)_Y - U(1)_Y - Z_N^R$  anomaly is

$$A_{1-1-Z_N^R} = \frac{3}{5} \left\{ 2 \left( \frac{1}{2} \right)^2 [q_H + q_{\bar{H}} - 2] \right. \\ \left. + 3(q_{\bar{5}} - 1) \left[ 2 \left( \frac{1}{2} \right)^2 + 3 \left( \frac{1}{3} \right)^2 \right] \right. \\ \left. + 3(q_{10} - 1) \left[ 6 \left( \frac{1}{6} \right)^2 + 3 \left( \frac{2}{3} \right)^2 + (1)^2 \right] \right\}.$$

- The  $U(1)_Y$  anomaly coefficient is not invariant under shifting charges by multiples of  $N$  so there must be a charge assignment satisfying the anomaly universality.
- For the case of  $Z_4^R$ ,  $A_{3-3-Z_4^R} = A_{2-2-Z_4^R} = 1 \pmod{2}$ . Thus, the anomaly universality is satisfied for  $q_H = q_{\bar{H}} = -4$ .

- The  $(Z_N^R)^3$  anomaly does not yield model independent constraints as it depends on the charge normalization non-linearly. [Banks, Dine(1992); Araki et al(2008)]
- On the other hand, the grav-grav- $Z_N^R$  anomaly is of potential interest for phenomenology, because additional light singlets may be necessary:

$$A_{g-g-Z_N^R} = -21 + 8 + 3 + 1 + 3[10(q_{10} - 1) + 5(q_{\bar{5}} - 1)] + 2(q_H + q_{\bar{H}} - 2) = 24A_{G-G-Z_N^R} \text{ mod } \eta.$$

- For the case of  $Z_4^R$ , this becomes  $-9 - 4 = 24 \text{ mod } 2$ , which is not satisfied. So, there must be additional SM singlet fermions with R-charges such that the sum is odd. In the presence of GS mechanism, the dilatino cancels the grav-grav- $Z_4^R$  anomaly.

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# R symmetry breakdown and the $\mu$ term

- The  $\mu$  term is generated by

$$\int d^4\theta X^\dagger H \bar{H} + \text{h.c.} \quad \text{or} \quad \int d^2\theta Y H \bar{H} + \text{h.c.}$$

where  $X, Y$  are singlets having R-charges, 0 and 2, respectively. A nonzero F-term of  $X$  or a scalar vev of  $Y$  breaks  $Z_4^R$ , leaving the unbroken  $Z_2$ , which corresponds to R-parity in MSSM.

- In our case, the  $Z_4^R$  symmetry breaks down through a nonzero superpotential vev coming from non-perturbative effects. Identifying  $Y$  with  $\langle W \rangle$ , the  $\mu$  term is of order  $\frac{\langle W \rangle}{M^2}$  with  $M$  being the messenger scale by Giudice-Masiero mechanism.

[Giudice, Masiero (1988)]

- When the Higgs doublets come from the Wilson lines of extra dimensions, the higher dimensional gauge invariance leads to Kähler potential,

$$K = K_0 + |H|^2 + |\bar{H}|^2 + (H\bar{H} + \text{h.c.}) + \dots$$

In supergravity, the holomorphic coupling  $H\bar{H}$  in Kähler potential, leads to the  $\mu$  term,  $\langle W \rangle H\bar{H}$ .

- When the gaugino condensate in hidden sector exists, it can couple to the Higgs bilinear in the R-covariant form,

$$W_{np} = e^{-bS} (AH\bar{H} + \kappa QQQL)$$

where  $b$  is constant and  $A, \kappa$  are constants built of some field VEVs.

- For  $\frac{\langle W \rangle}{M_P^2} \sim Ae^{-bS}$ , the  $\mu$  term is of order the gravitino mass while the coefficients of the dimension-5 proton decay operators are as small as  $10^{-15}/M_P$ .

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## $SU(5)$ -compatible extensions

- For the  $Z_N^R$  being compatible with  $SU(5)$ , the anomaly universality, leads to more  $Z_N^R$ 's with  $N$  being the divisor of 24.
- Allowing Yukawa couplings and Weinberg operators and no R-parity violating couplings leads to  $N = 4, 6, 8, 12, 24$ . All but  $Z_6^R$  are anomalous, requiring a Green-Schwarz mechanism.
- For  $Z_8^R$ , the NMSSM is possible by including a singlet  $N$ :

$$W = \lambda N H \bar{H} + \kappa N^3.$$

- For higher order  $Z_N^R$ 's than  $Z_8^R$ , the  $\mu$  term is generated by an intermediate-scale vev of an singlet  $S$ :

$$W = \frac{1}{M_P} (\lambda S^2 H \bar{H} + \kappa S^4).$$

- In the case of  $Z_{12}^R$ , the axion coming from the singlet  $S$  solves the strong CP problem too.

# Conclusion

- We have obtained the unique  $Z_4^R$  symmetry being compatible with  $SO(10)$  in MSSM, solving the problems of the  $\mu$  term and the dimension-5 proton decay operators.
- In the case of  $Z_4^R$ , the  $\mu$  term of order the gravitino mass is generated by Giudice-Masiero mechanism.
- The  $Z_4^R$  symmetry is of possible origin from the isometry of internal dimensions in string compactifications and the supergravity coupling for the  $\mu$  term is due to the higher dimensional gauge invariance of the Higgs doublets.
- There are more interesting  $Z_N^R$  models, which are compatible with  $SU(5)$  and are beyond the MSSM by having light singlets or the axion.