

# Studies of Beam-Beam Effects at CEPC

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Thanks: M. Zobov(INFN/LNF), M. Migliorati (University of Rome 'La Sapienza' )

# Outline

1. Introduction
2. Simulation with CEPC Designed Parameters
3. Analysis Study of X-Z instability
4. Summary

# Crab-waist collision

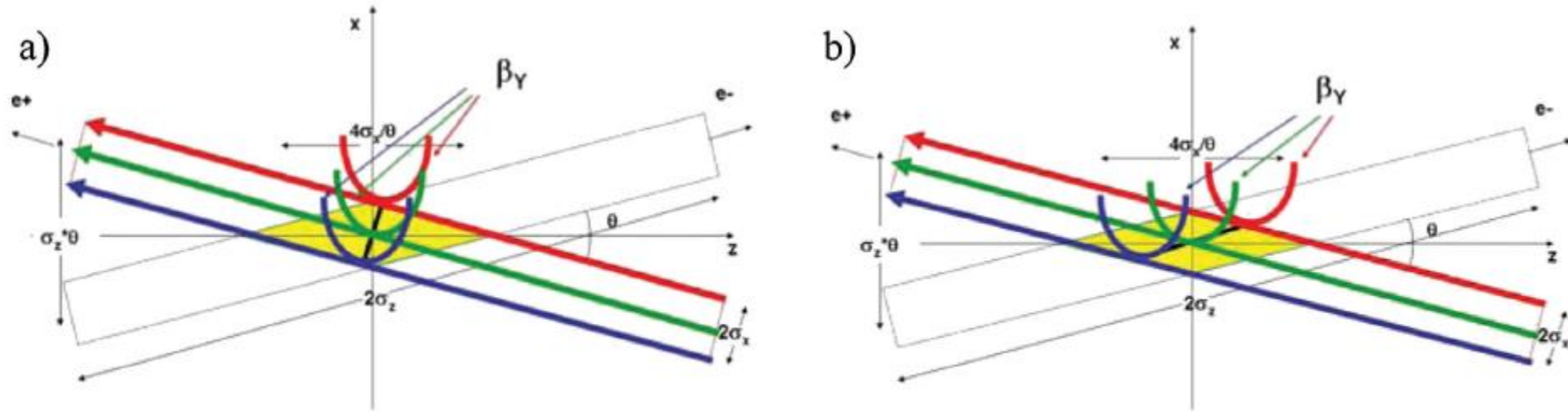


FIG. 1 (color). Crab-waist collision scheme. The color straight lines show directions of motion for particles with different horizontal deviations from the central orbit. The arrows indicate the corresponding  $\beta$  function variations along these trajectories.

$$L \propto \frac{N \xi_y}{\beta_y^*}; \quad \xi_y \propto \frac{N \sqrt{\beta_y^*/\epsilon_y}}{\sigma_z \theta}; \quad \xi_x \propto \frac{N}{(\sigma_z \theta)^2},$$

$$\varphi = \frac{\sigma_z}{\sigma_x} \tan\left(\frac{\theta}{2}\right) \approx \frac{\sigma_z}{\sigma_x} \frac{\theta}{2}.$$

$$\beta_y^* \approx \frac{\sigma_x}{\theta} \ll \sigma_z.$$

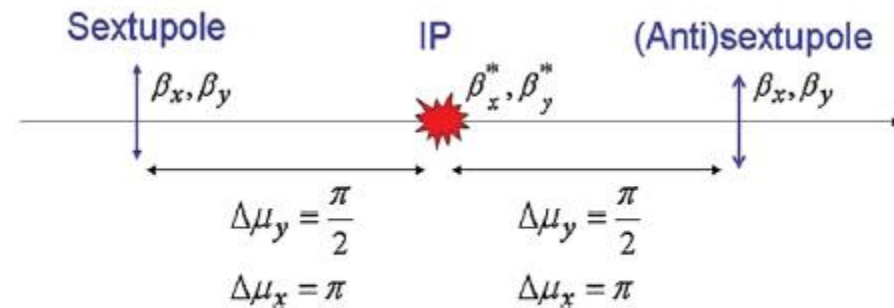
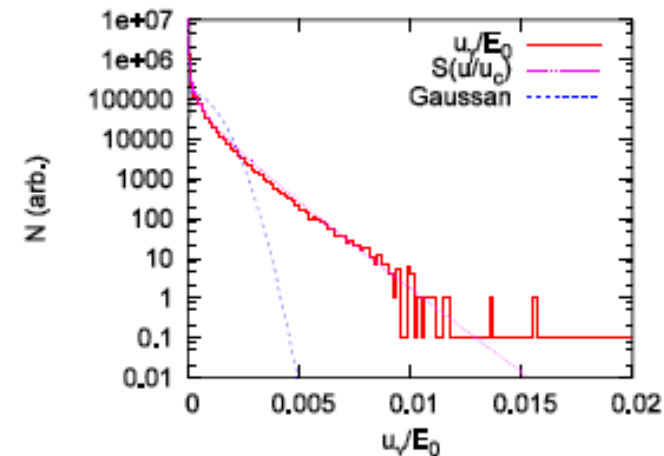
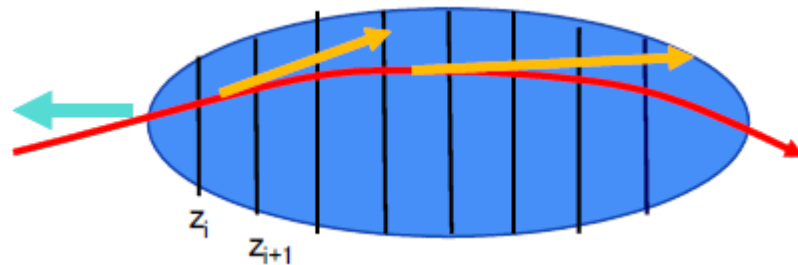


FIG. 2 (color). Crab sextupole locations.

# Beamstrahlung Effect & 3D flip-flop

- Synchrotron radiation during beam-beam interaction
- High energy photon  $\rightarrow$  Momentum acceptance  $\rightarrow$  Lifetime
- Longer bunch length and Higher energy spread
- Asymmetrical beam blowup: 3D flip-flop



# Coherent Beam-Beam Instability with a Large Crossing Angle

K. Ohmi, Int. J. Mod. Phys. A, 31, 1644014 (2016).

K. Ohmi and et al., PRL 119, 134801 (2017)

N. Kuroo et al, PHYS. REV. ACCEL. BEAMS 21, 031002 (2018)

K. Ohmi, eeFACT 2018

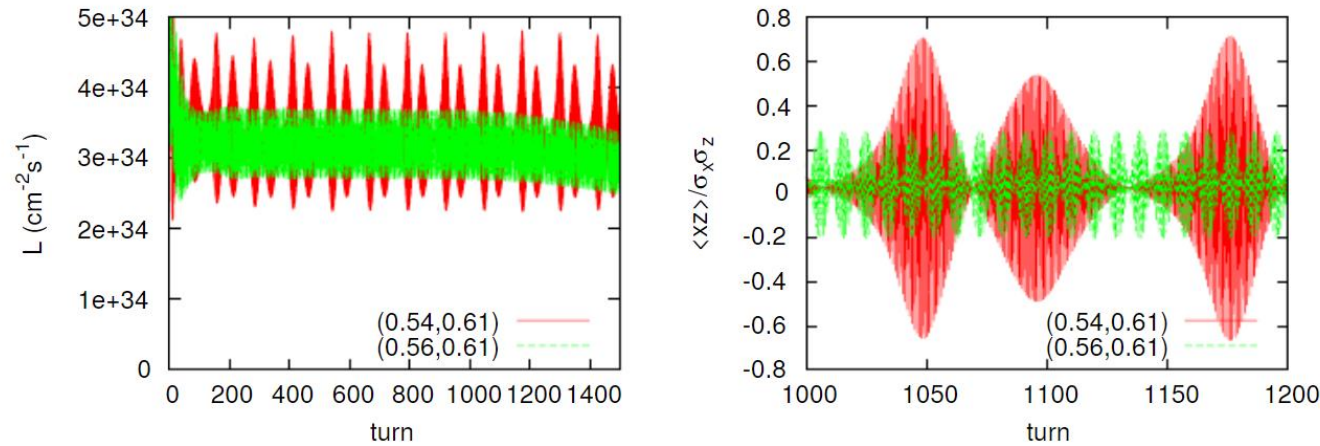


Fig. 5. Luminosity and  $\langle xz \rangle$  evolutions given by a strong-strong simulation using BBSS code.

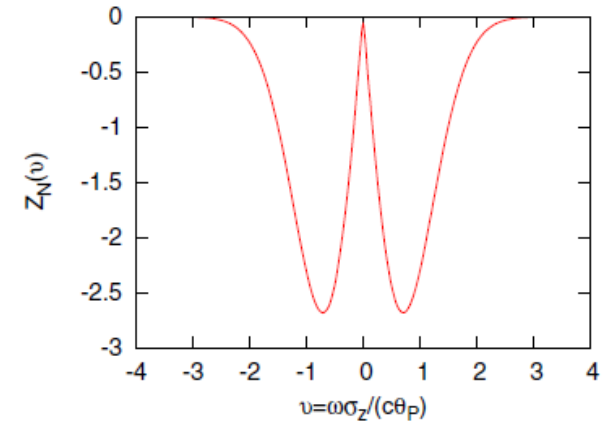


FIG. 3. Imaginary part of normalized cross impedance, where  $v = \omega \sigma_z / (c \theta_p)$ .

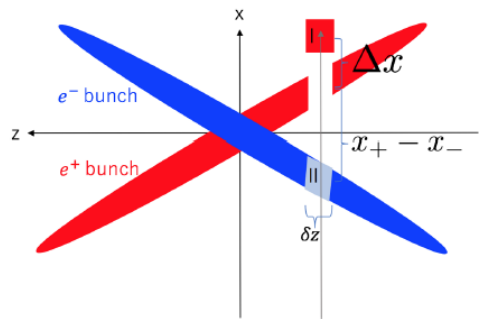


FIG. 1. Illustrative representation of the evaluation of the cross-wake force.

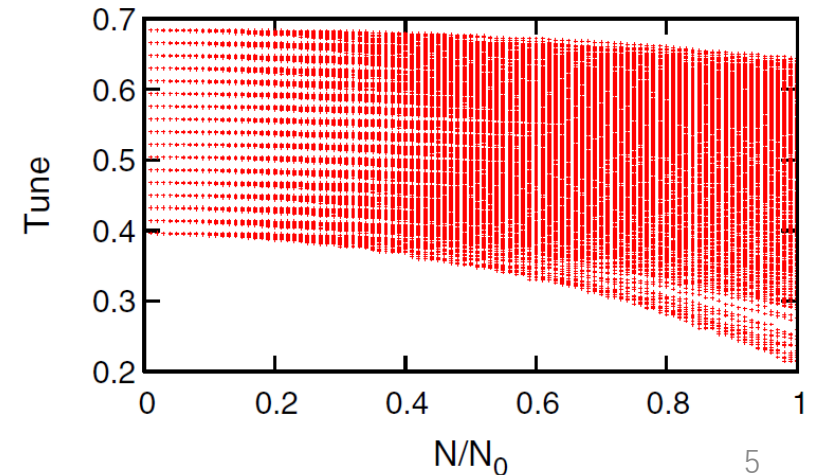
- Usual wake force gives correlation between bunch head to tail. Head-tail instability is induced by synchrotron motion

$$\Delta p_x(z) = - \int_z^\infty W(z - z') \rho_x(z') dz'$$

- **Cross wake field** gives correlation of two colliding beam by convolution of each dipole moment.

$$\Delta p_{x,\mp}(z_\mp) = - \int_{-\infty}^\infty W_x^{(\mp)}(z_\mp - z'_\pm) \rho_x^{(\pm)}(z'_\pm) dz'_\pm$$

- Cross wake force induced by the beam-beam interaction is localized at IP.



# Why have we started with the longitudinal impedance?

1. In the collision scheme with Crab Waist and Large Piwinski Angle the luminosity and tune shifts **strongly depend on the bunch length**

$$L \propto \frac{N\xi_y}{\beta_y^*}, \quad \xi_y \propto \frac{N\sqrt{\beta_y/\epsilon_y}}{\sigma_z\theta}, \quad \xi_x \propto \frac{N}{(\sigma_z\theta)^2}$$

2. For the future circular colliders with extreme beam parameters in collision several new effects become important such as beamstrahlung, coherent X-Z instability and 3D flip-flop. **The longitudinal beam dynamics plays an essential role for these effects**

# Interplay between beam-beam interaction, beamstrahlung and longitudinal impedance

## X-Z Instability

1. Tune shift of stable tune areas due to the impedance related synchrotron frequency reduction
2. Reduction of sizes of the stable tune areas
3. Smaller beam blowup presumably due to the synchrotron frequency spread induced by the impedance

## In Stable Areas

D.Leshenok and et al. PHYS. REV. ACCEL. BEAMS 23, 101003 (2020)

1. Longer bunch length
2. Smaller energy spread than that due to beamstrahlung alone
3. Eventual damping of the microwave instability due to longer bunches and overall higher energy spread

# Code model

CODE: IBB

K. Hirata et al., PA 40, 205-228 (1993)

K. Hirata, PRL, 74, 2228 (1995)

Y. Zhang et al., PRST-AB, 8, 074402 (2005)

K. Ohmi, IPAC16

Y. Zhang et al., PRAB 23, 104402, (2020)

- Linear Arc Map with SR radiation
- Horizontal crossing angle: Lorentz boost map
- Bunch slice number is about 10 times Piwinski angle
- Slice-Slice collision: Synchro-beam mapping and PIC
- Synchrotron radiation during collision
- Longitudinal wake potential is calculated in frequency domain before IP each turn



Try to answer:  
if the machine parameter is reasonable

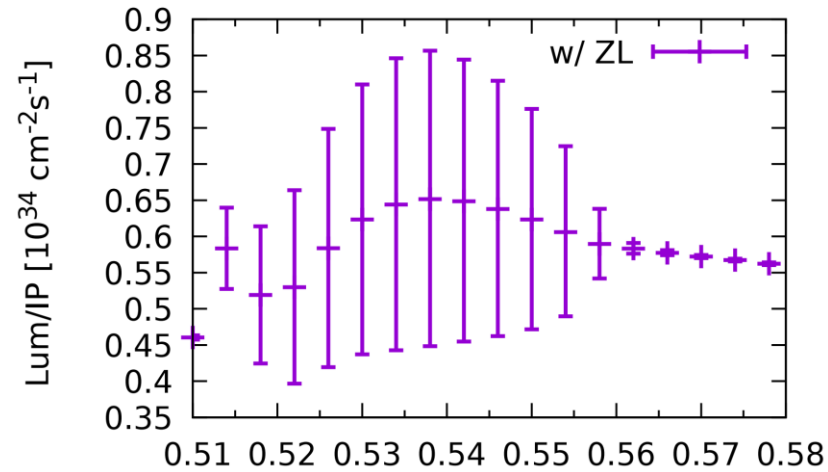
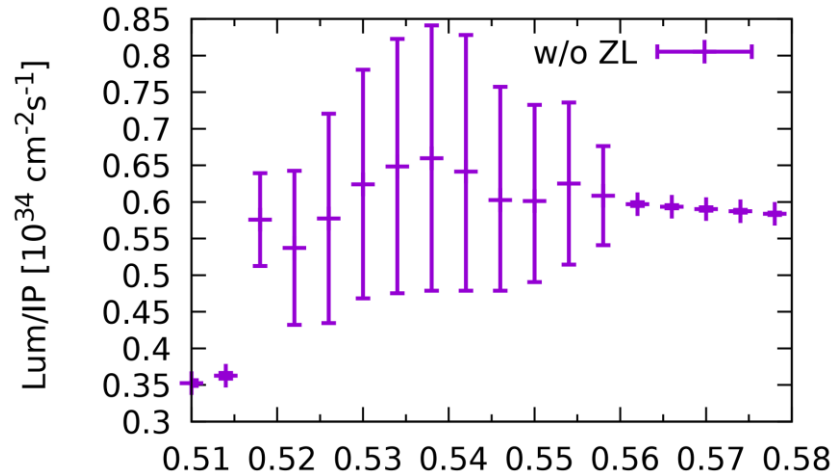
Limit of bunch population by beam-beam interaction:

- Beamstrahlung lifetime ( $t_{\text{tbar/Higgs}}$ )
- If X-Z instability is suppressed ( $t_{\text{tbar/Higgs/W/Z}}$ )
- If asymmetric bunch current collision is stable
- If there exist large enough stable working point space

# ttbar

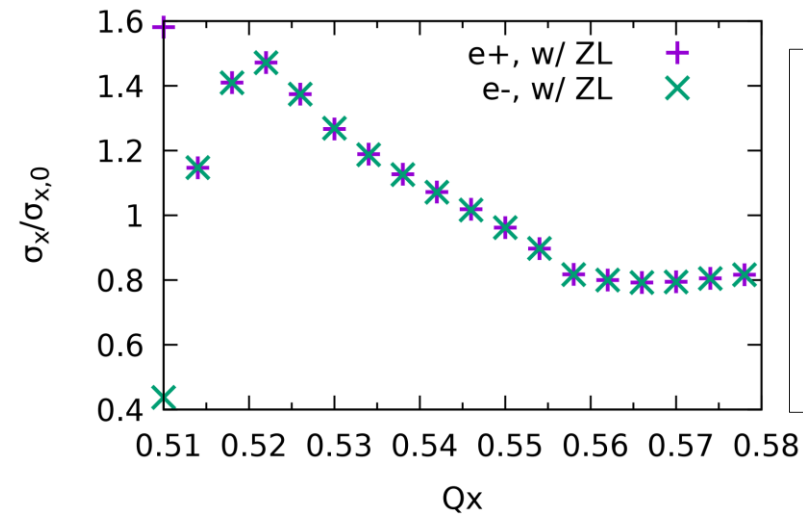
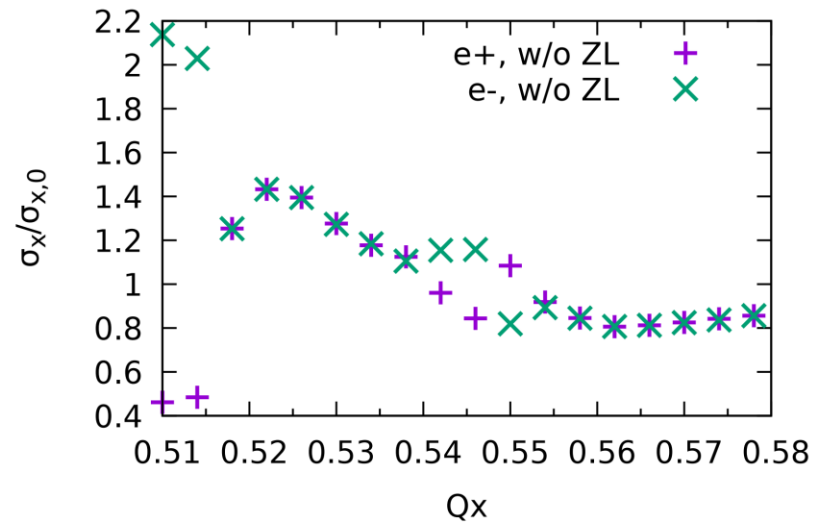
- Very strong SR damping (longitudinal damping time: 20 turns)
  - Instability may be suppressed?
- Medium Piwinski Angle  $\sim 1$ , BEPCII/KEKB: 0.5, CEPC-Z > 20
  - Beam-beam resonance could not be well suppressed by Crab-Waist
- Horizontal beam-beam parameter: 0.07, still very high
  
- It is very different from H/W/Z

# Luminosity and horizontal beam size versus $Q_x$ (ttbar, CW=0.5)



- Design Lum=0.5e34
- Stable width of  $v_x \sim 0.01$

D. Shatilov, 143<sup>rd</sup>  
FCCee Optics Meeting



Excited coherent modes are associated with synchrotron satellites of half-integer resonance:

$$2\nu_x - 2m\nu_s = n$$

Since the wavelength cannot be less than  $2L_i$ , we get the condition:

$$m \leq \phi$$

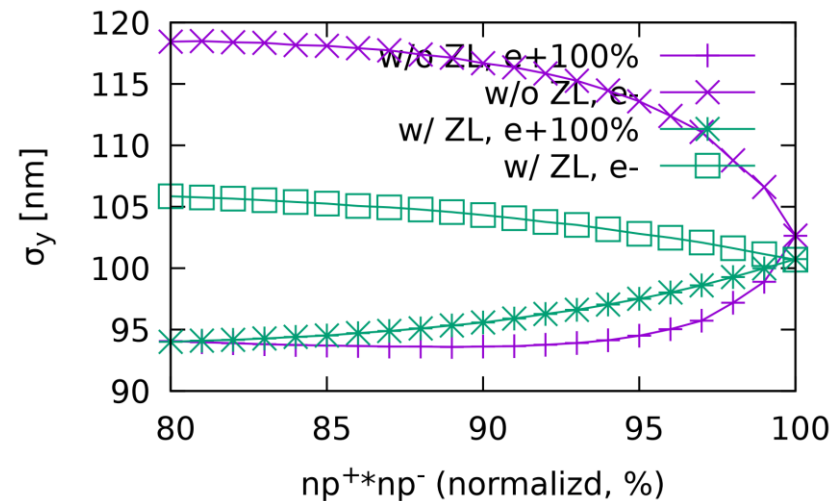
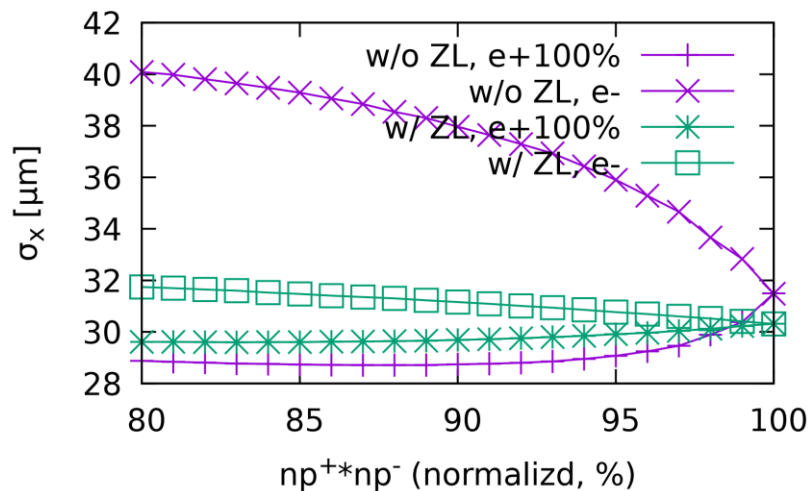
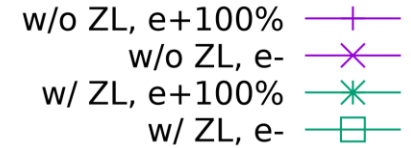
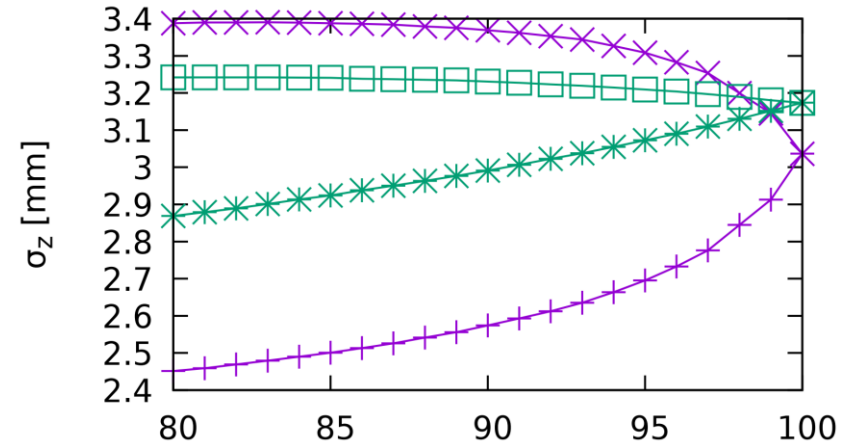
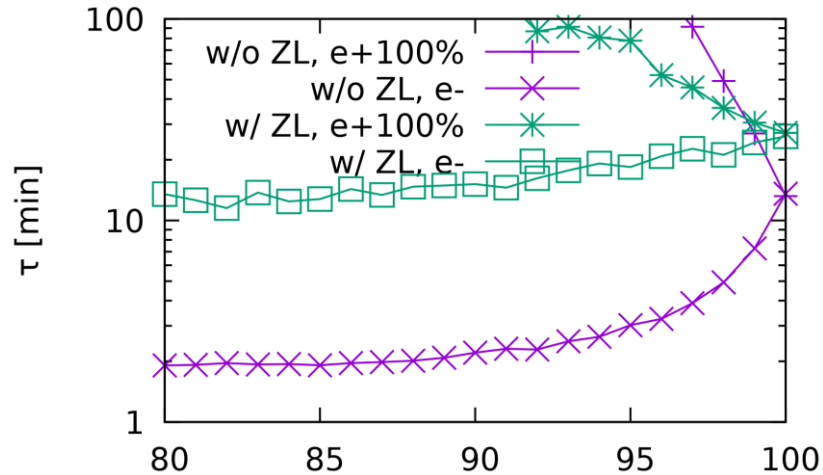
If  $\phi$  is not too large, we can solve the problem by choosing

$$\nu_x > \nu_x^* = 0.5 + \phi\nu_s$$

- 3D flip-flop is suppressed considering longitudinal impedance
- Dynamic effect squeeze horizontal beam size

# ttbar: Asymmetrical Collision

- Beam lifetime is increased due to longitudinal impedance
- ~10% asymmetry of bunch population could be accepted for stability

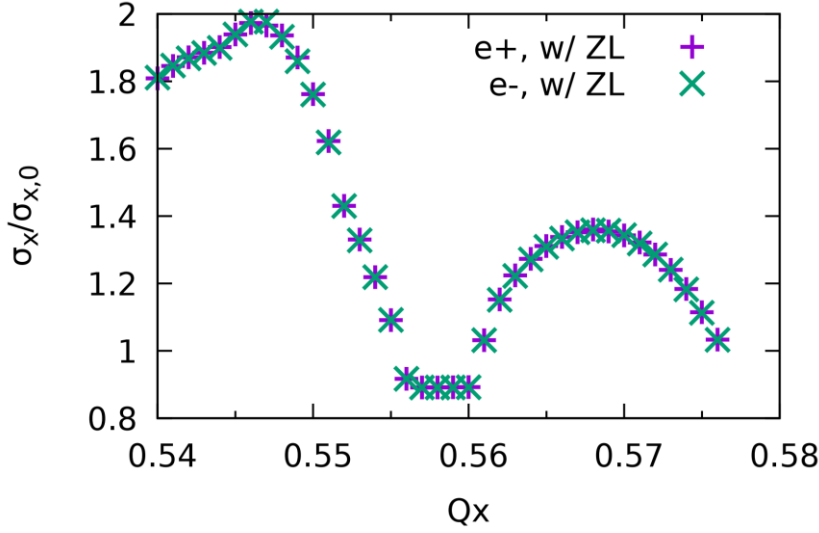
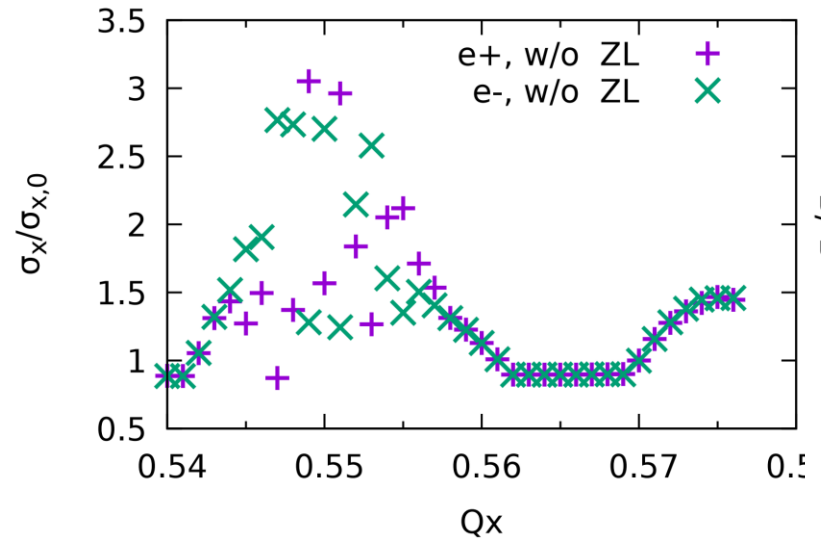
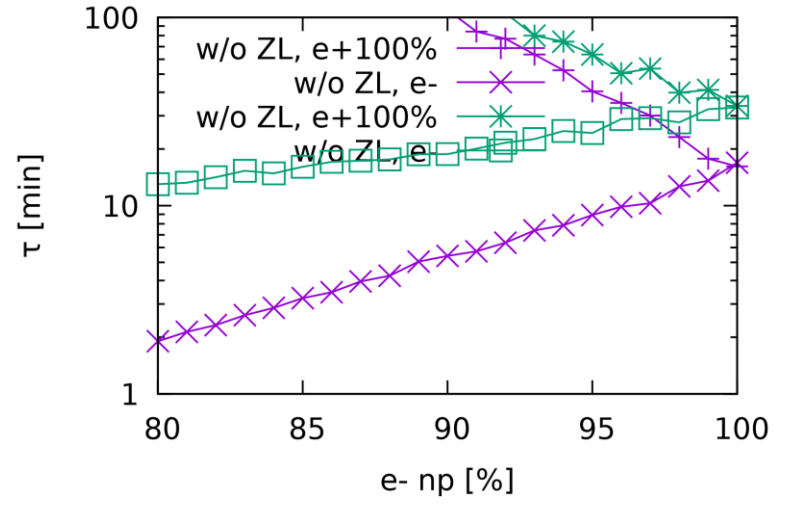
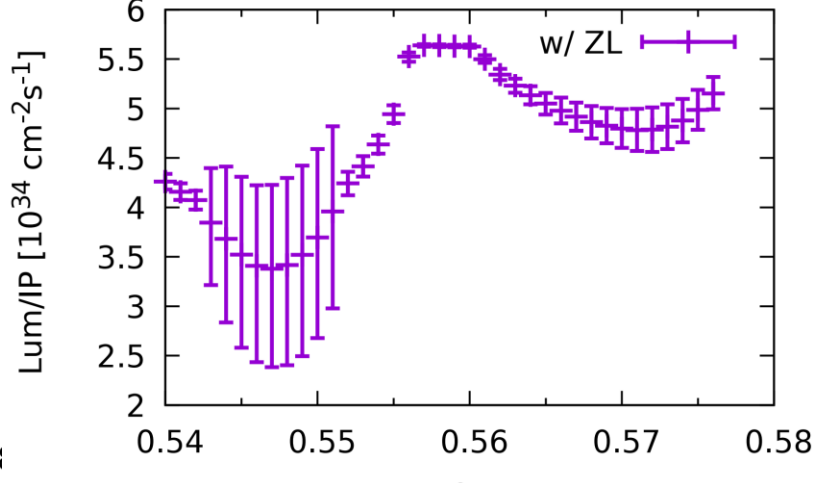
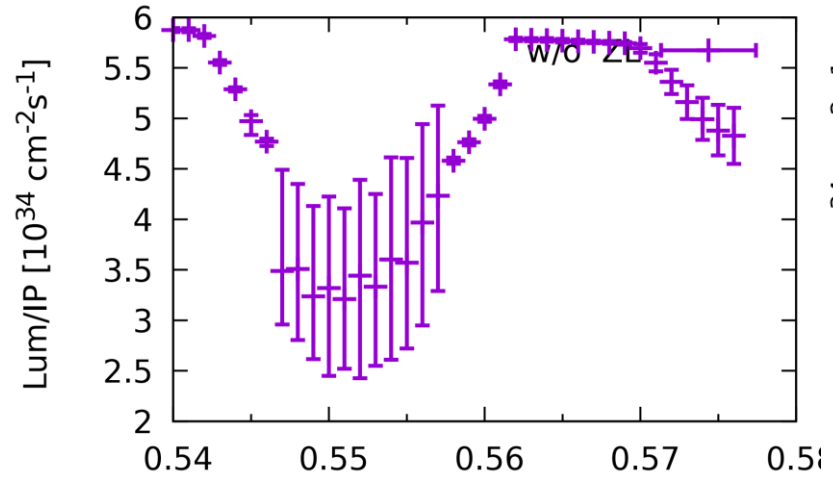


# Higgs

- Large Piwinski Angle  $\sim 6$  (CW=0.8)
- Beamstrahlung Lifetime
- X-Z instability

- Design Lum=5e34
- Stable width of  $Q_x \sim 0.004$  considering ZL

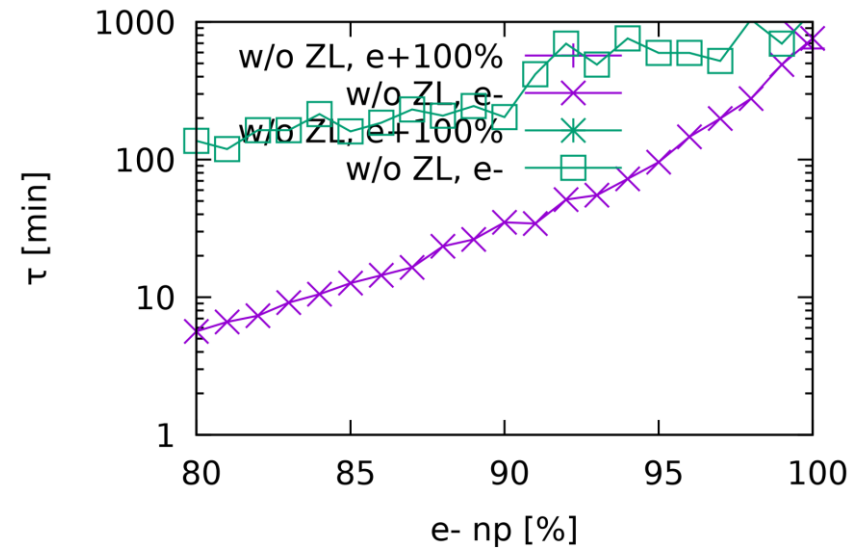
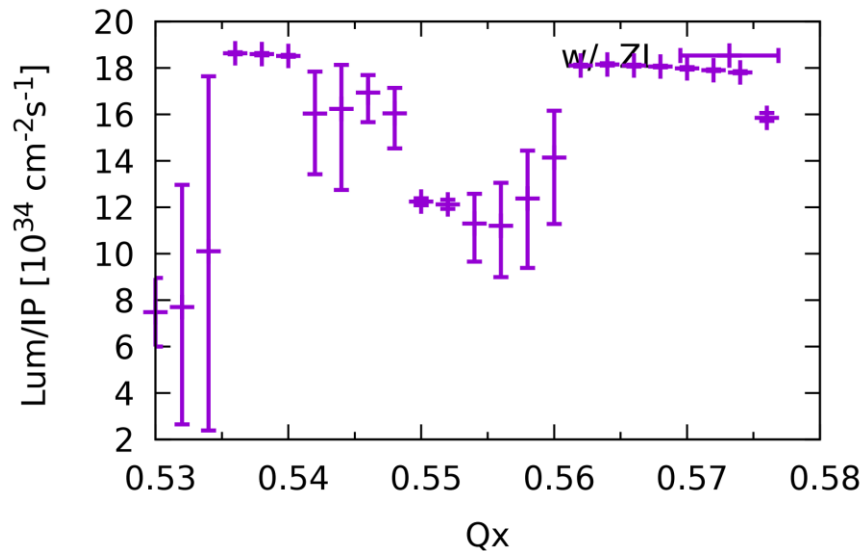
- Beam lifetime is increased due to longitudinal impedance
- 10% asymmetry of bunch population could be accepted.



# W

- Piwinski angle is similar to that of Higgs(CW=0.8)
- Beamstrahlung lifetime is not an issue
- X-Z instability

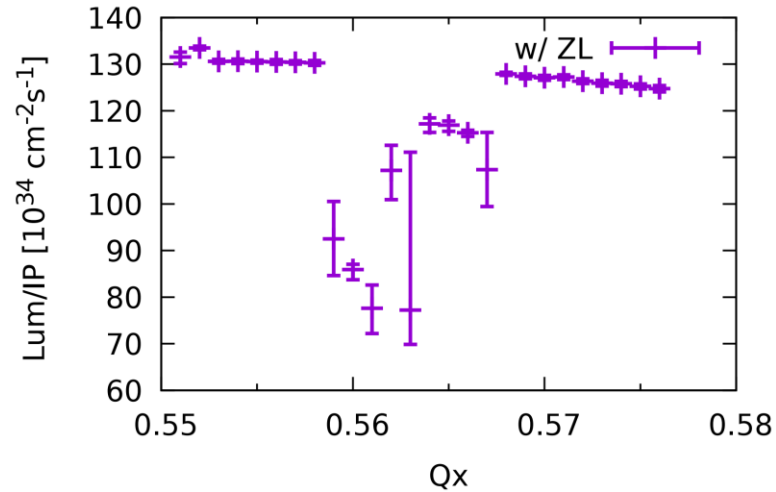
- Design Lum=16e34
- The optimized nux is near 0.57 w/ and w/o longitudinal impedance.
- Stable tune width  $\sim 0.007$  considering ZL
- Beam lifetime is increased due to longitudinal impedance
- 10% asymmetry of bunch population could be accepted.



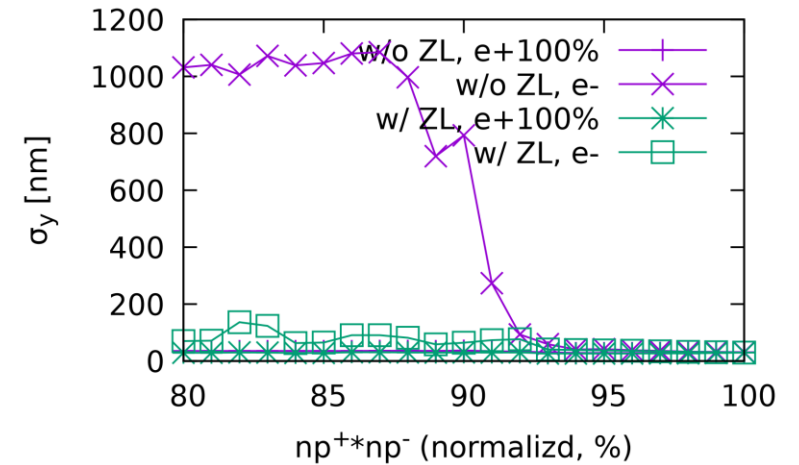
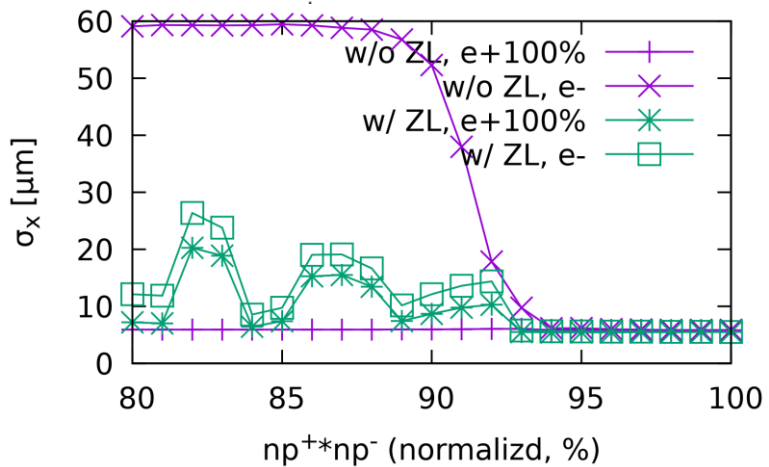
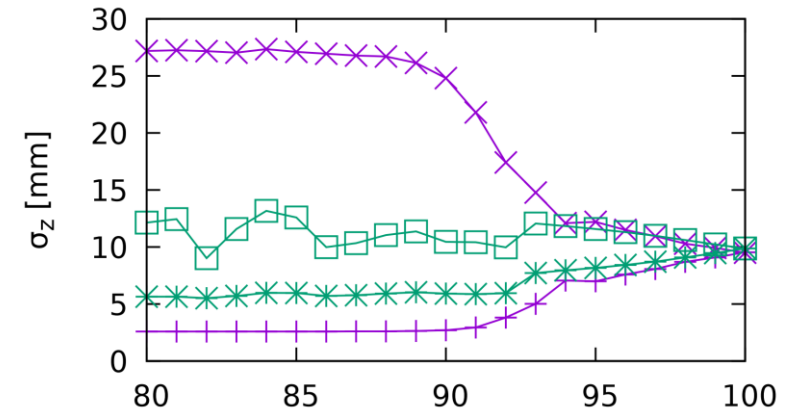
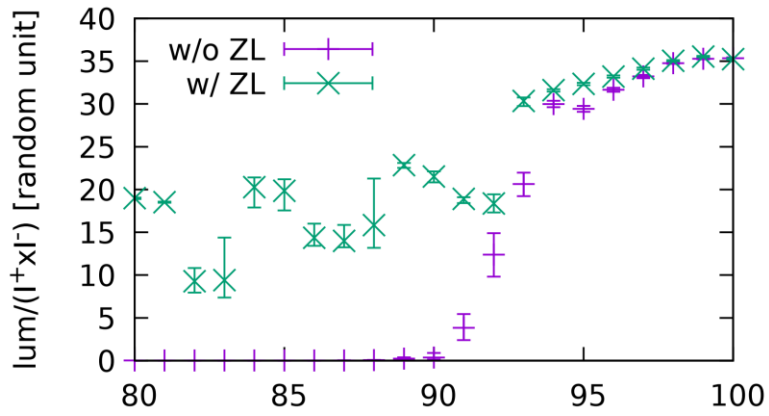
# Z

- Piwinski Angle: 25 (CW=1)
- FODO CELL: 60/60
- X-Z instability

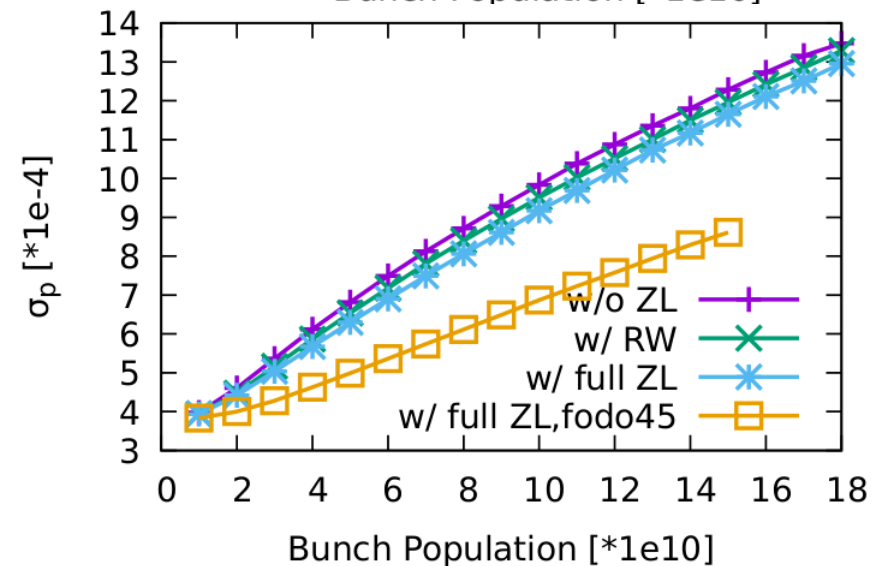
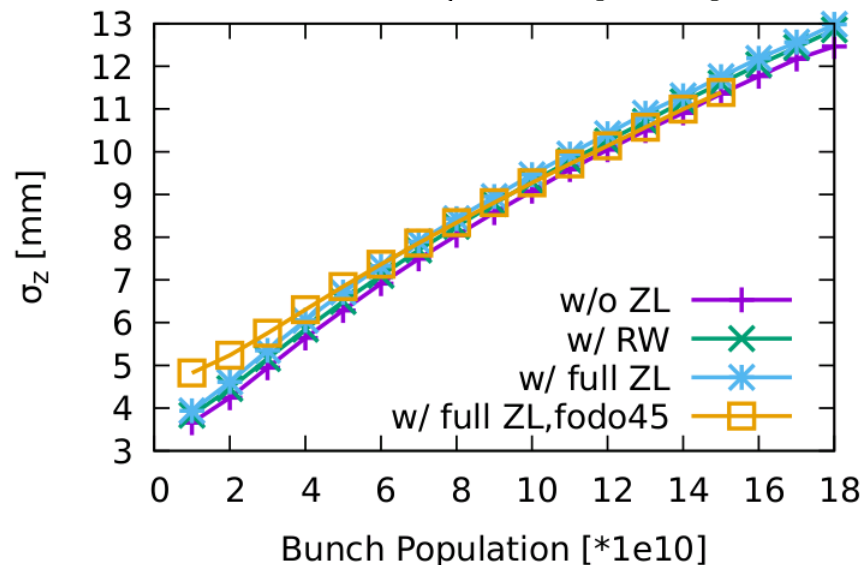
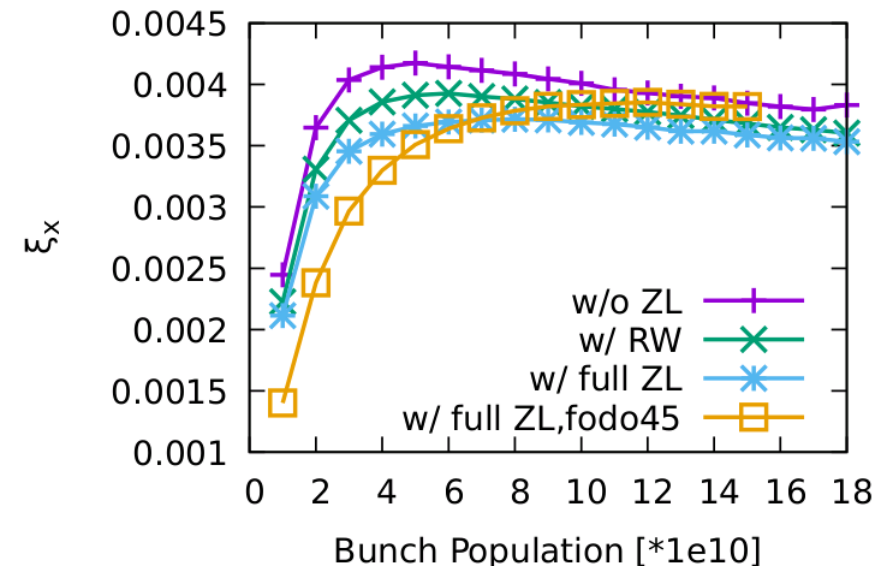
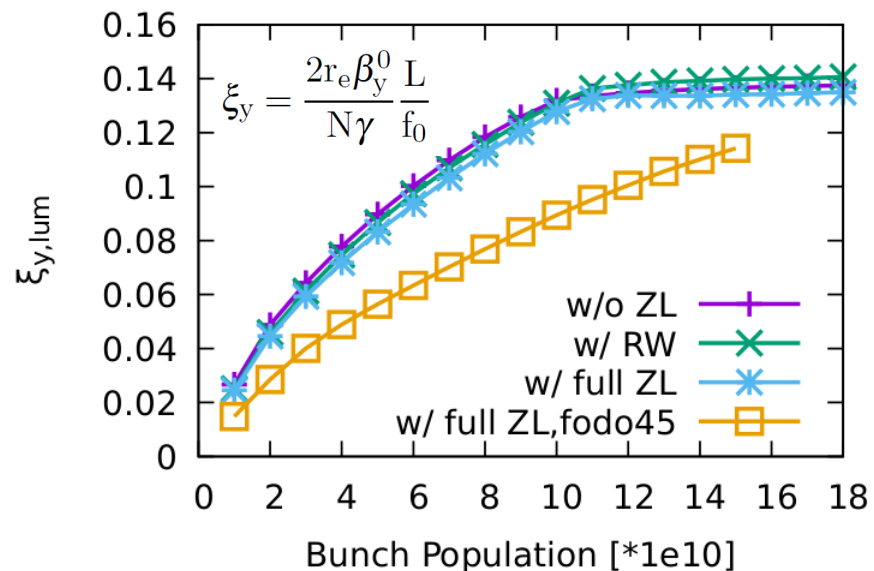
- Design Lum=115e34
- Stable width of  $\nu_x \sim 0.01$



Only 5% asymmetry of bunch population could be accepted.



# Evolution of Parameters during Injection





# X-Z instability tune scan with and without beam coupling impedance (CEPC CDR)

By including the impedance stable areas become narrower and are shifted in frequency

After the horizontal beta function reduction from 0.2 m down to 0.15 m

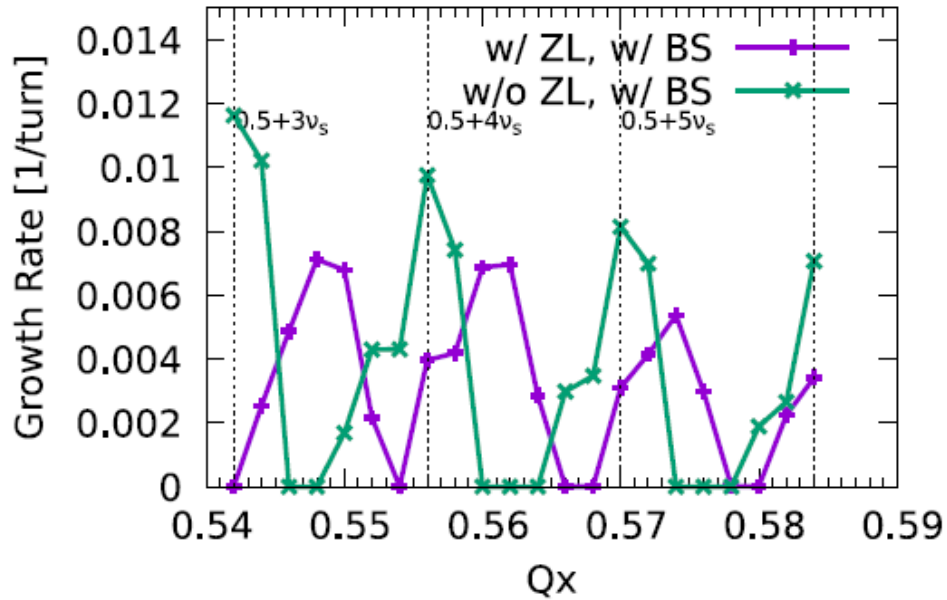
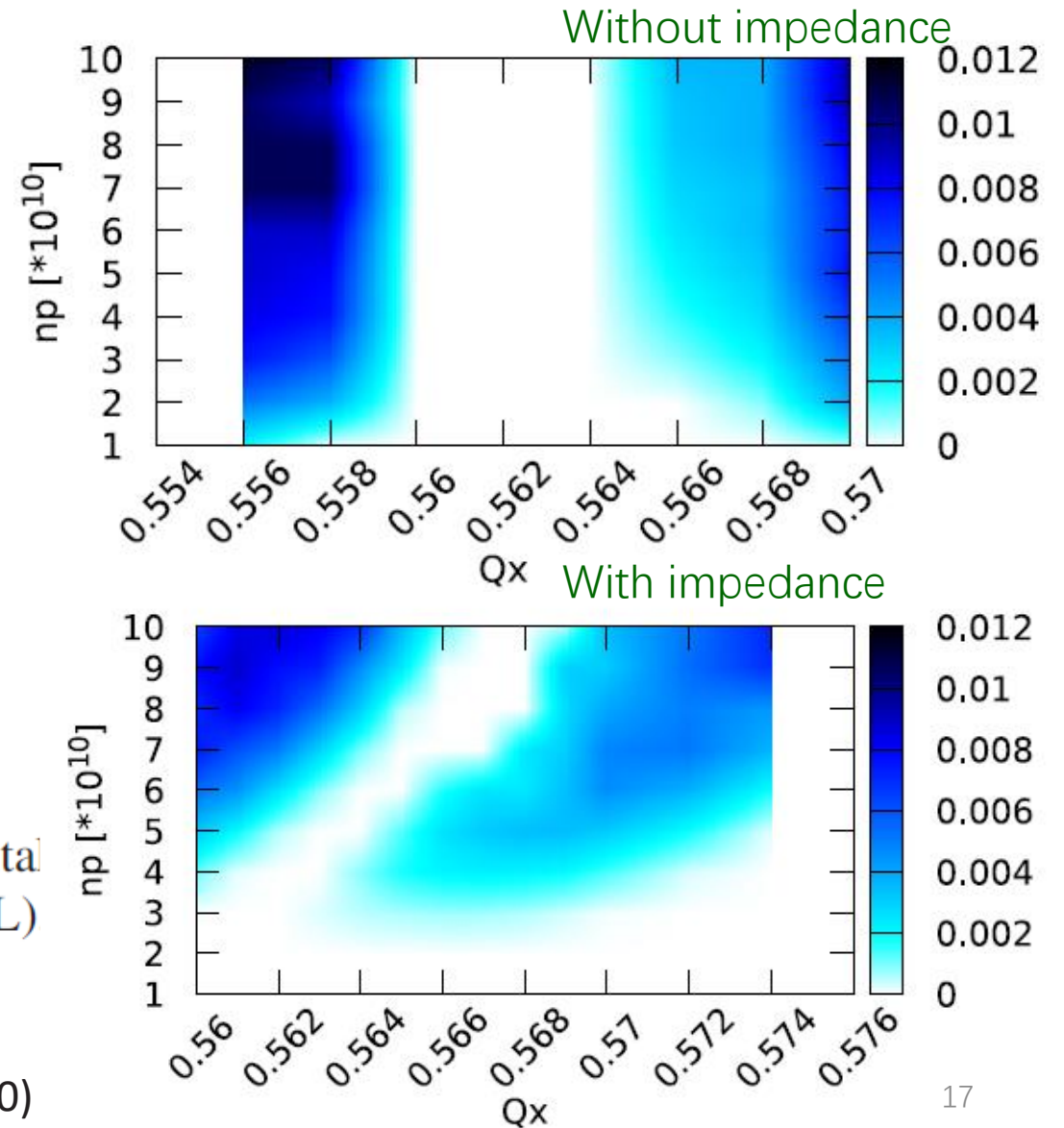


FIG. 3. The horizontal beam size growth rate versus horizontal tune with and without longitudinal coupling impedance (ZL) Beamstrahlung (BS) effect is turned on.



# Potential well distortion with Longitudinal Impedance

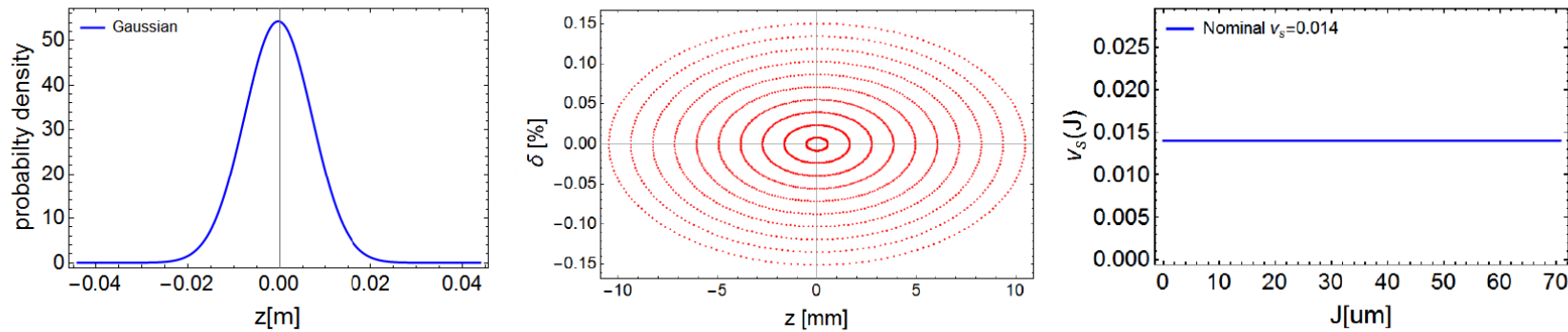


Figure: Longitudinal beam dynamics without longitudinal impedance

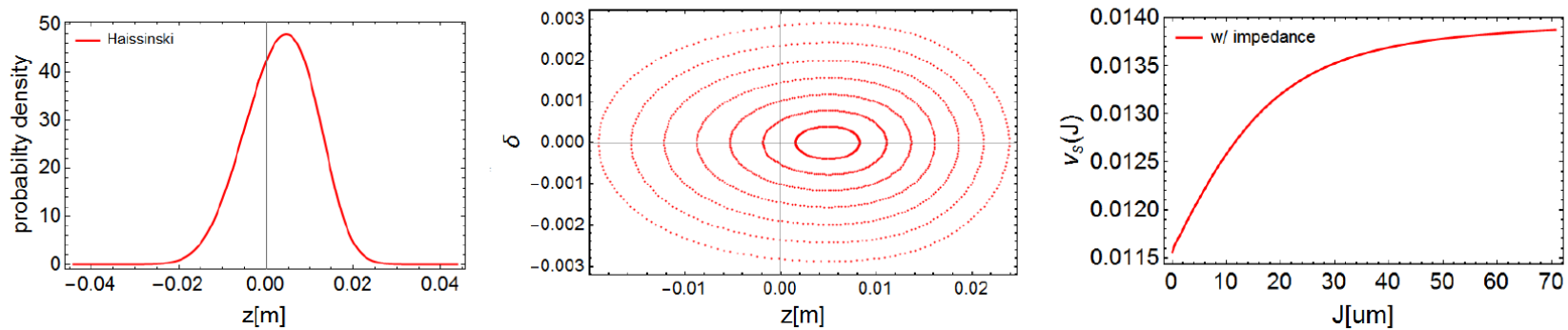


Figure: Longitudinal beam dynamics with longitudinal impedance

# Analysis without PWD

- Azimuthal and Radial Mode Expansion:

$$x(J, \phi; t) = \sum_{l,k} x_{kl}(t) \sqrt{\frac{k!}{(|l|+k)!}} \hat{J}^{|l|/2} L_k^{(|l|)}(\hat{J}) e^{i l \phi}$$

$$p_x(J, \phi; t) = \sum_{l,k} p_{kl}(t) \sqrt{\frac{k!}{(|l|+k)!}} \hat{J}^{|l|/2} L_k^{(|l|)}(\hat{J}) e^{i l \phi}$$

- Synchrotron-betatron motion of dipole moment vector  $(x_{kl}, p_{kl})$  in Arc

$$M_0 = e^{-2\pi i l \nu_z} \delta_{kk'} \delta_{ll'} \begin{pmatrix} \cos \mu_x & \sin \mu_x \\ -\sin \mu_x & \cos \mu_x \end{pmatrix}.$$

For example,  
 $k_{\max} = 80, l_{\max} = 8$

- Momentum Kick due to localized Wake force (beam-beam)

$$M_W = \begin{pmatrix} 1 & 0 \\ -2M_{klk'l'} & 1 \end{pmatrix} \quad \Delta p_{kl}(t) = -2 \sum_{k'l'} M_{klk'l'} x_{k'l'}(t).$$

$$M_{k\ell, k'\ell'} = \pm \frac{\beta_x}{2} i^{l-l'-1} \int_{-\infty}^{\infty} d\omega Z(\omega) g_{kl}(\omega) g_{k'l'}(\omega),$$

where  $g_{kl}(\omega) = \frac{1}{\sqrt{2\pi k!(|l|+k)!}} \left(\frac{\omega\sigma}{\sqrt{2}c}\right)^{|l|+2k} e^{-\omega^2\sigma^2/2c^2}.$

- Stability analysis of  $M_0 M_W$

# Analysis with PWD

- Azimuthal Mode Expansion

$$x(J, \phi) = \sum_{l=-\infty}^{\infty} x_l(J) e^{il\phi}, \quad p_x(J, \phi) = \sum_{l=-\infty}^{\infty} p_l(J) e^{il\phi}$$

- Synchro-betatron motion in Arc

$$\begin{pmatrix} x_l(J) \\ p_l(J) \end{pmatrix} = e^{-2\pi i l \nu_s(J)} \begin{pmatrix} \cos \mu_x & \sin \mu_x \\ -\sin \mu_x & \cos \mu_x \end{pmatrix} \begin{pmatrix} x_l(J) \\ p_l(J) \end{pmatrix} \equiv M_0 \begin{pmatrix} x_l(J) \\ p_l(J) \end{pmatrix}$$

- Action Discretization

we truncate  $l$  at  $\pm l_{\max}$ , and discretize  $J$  at  $J_1, J_2, \dots, J_{n_J}$ .

For example,

$$\begin{pmatrix} x_l(J_i) \\ p_l(J_i) \end{pmatrix} = e^{-2\pi i l \nu_s(J_i)} \begin{pmatrix} \cos \mu_x & \sin \mu_x \\ -\sin \mu_x & \cos \mu_x \end{pmatrix} \begin{pmatrix} x_l(J_i) \\ p_l(J_i) \end{pmatrix} \equiv M_0 \begin{pmatrix} x_l(J_i) \\ p_l(J_i) \end{pmatrix}$$

$n_J = 40$ ,  $l_{\max} = 8$

- Momentum Kick due to localized Wake force (beam-beam)

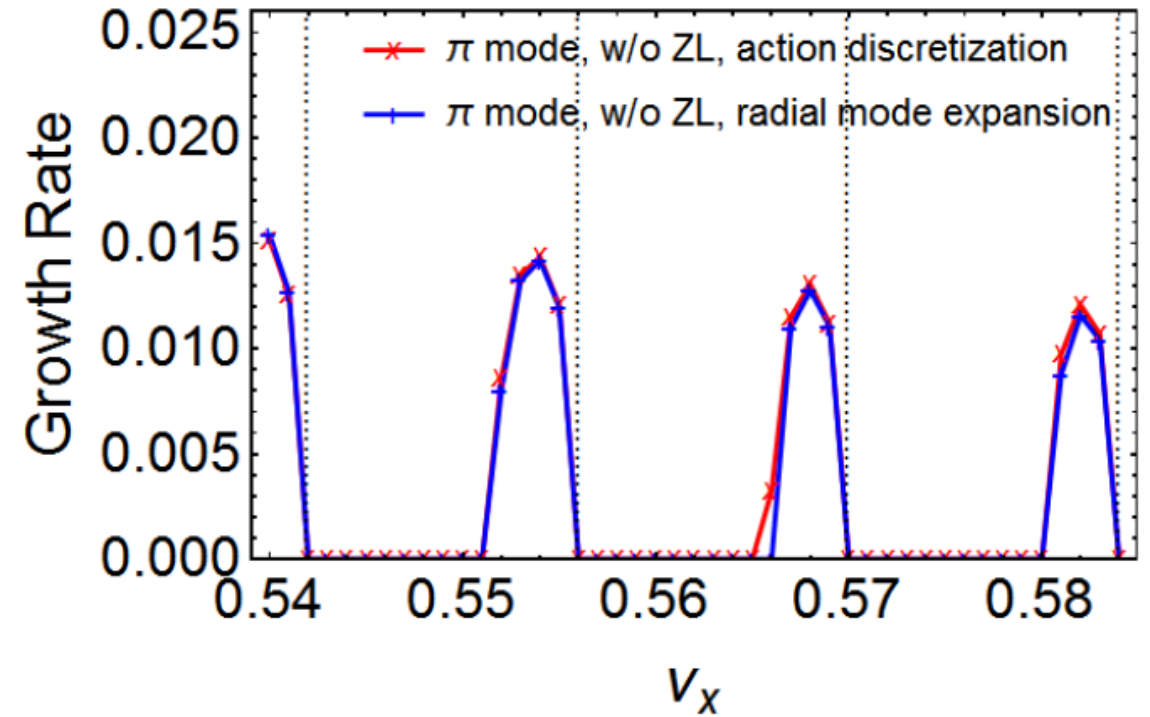
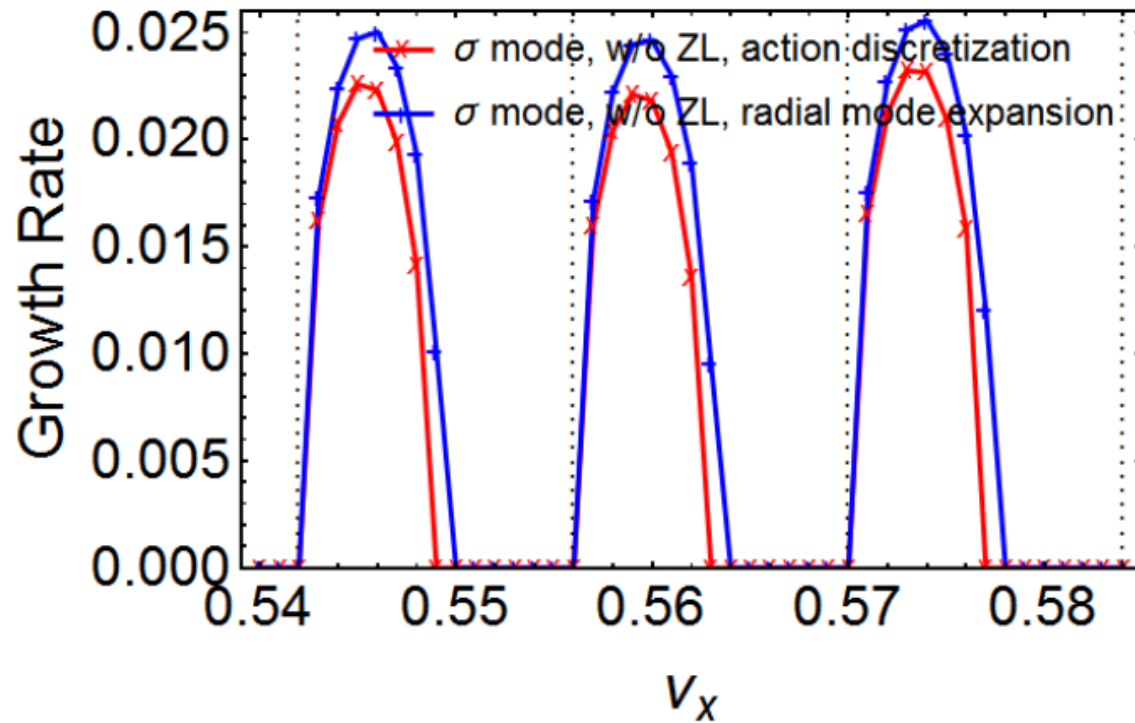
$$M_W = \begin{pmatrix} 1 & 0 \\ \beta_x M_{lil'i'} & 1 \end{pmatrix}.$$

$$\begin{aligned} \Delta p_l(J_i) &= \mp \frac{\beta_x}{2\pi} \sum_{l'} \sum_{i'} \Delta J_{i'} W_{ll'}(J_i, J_{i'}) \psi(J_{i'}) x_{l'}(J_{i'}) \\ &\equiv \beta_x M_{lil'i'} x_{l'}(J_{i'}). \end{aligned}$$

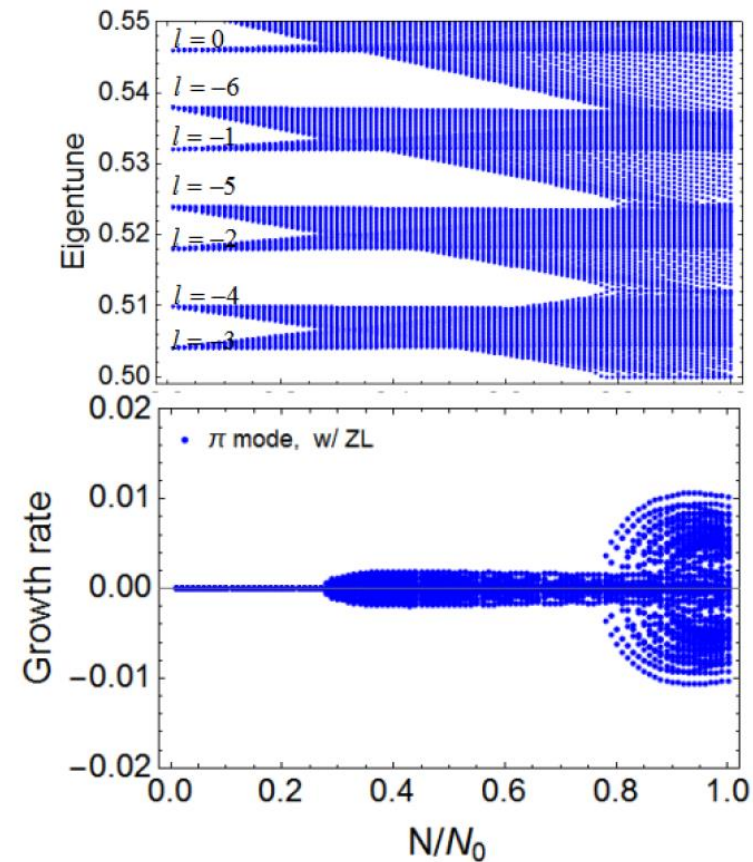
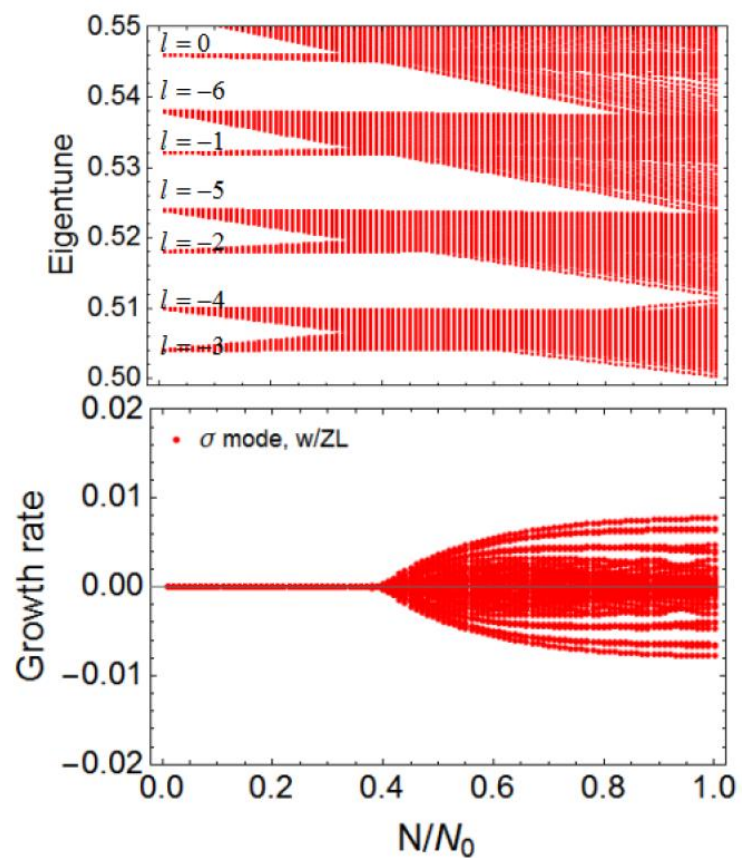
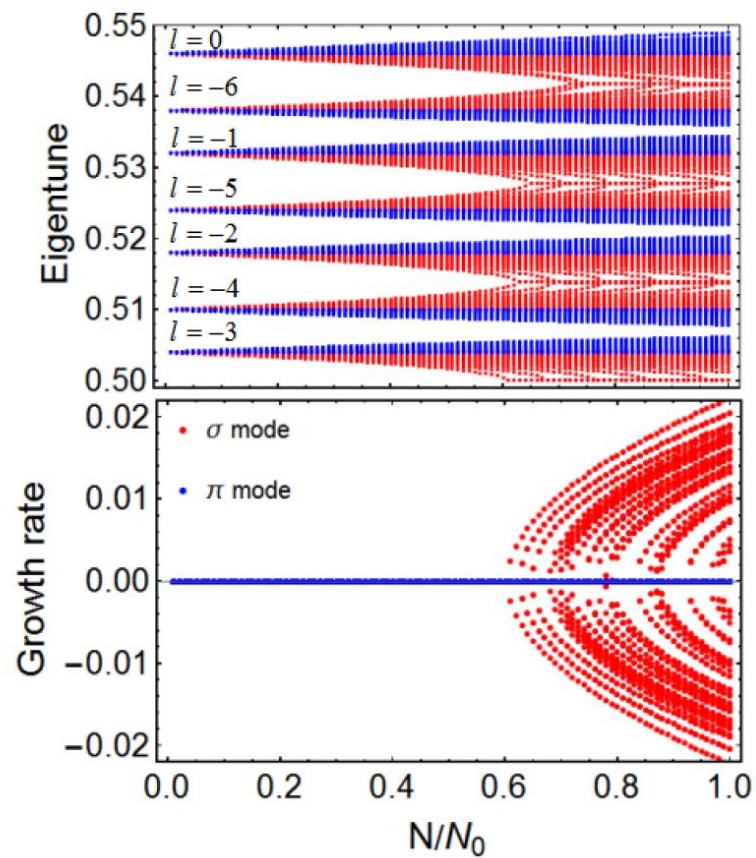
- Stability analysis of  $M_0 M_W$

$$W_{ll'}(J_i, J_{i'}) = \sum_j \sum_{j'} e^{-il\phi_j + il'\phi_{j'}} W_x(z(J_i, \phi_j) - z(J_{i'}, \phi_{j'})) \Delta\phi_j \Delta\phi_{j'}.$$

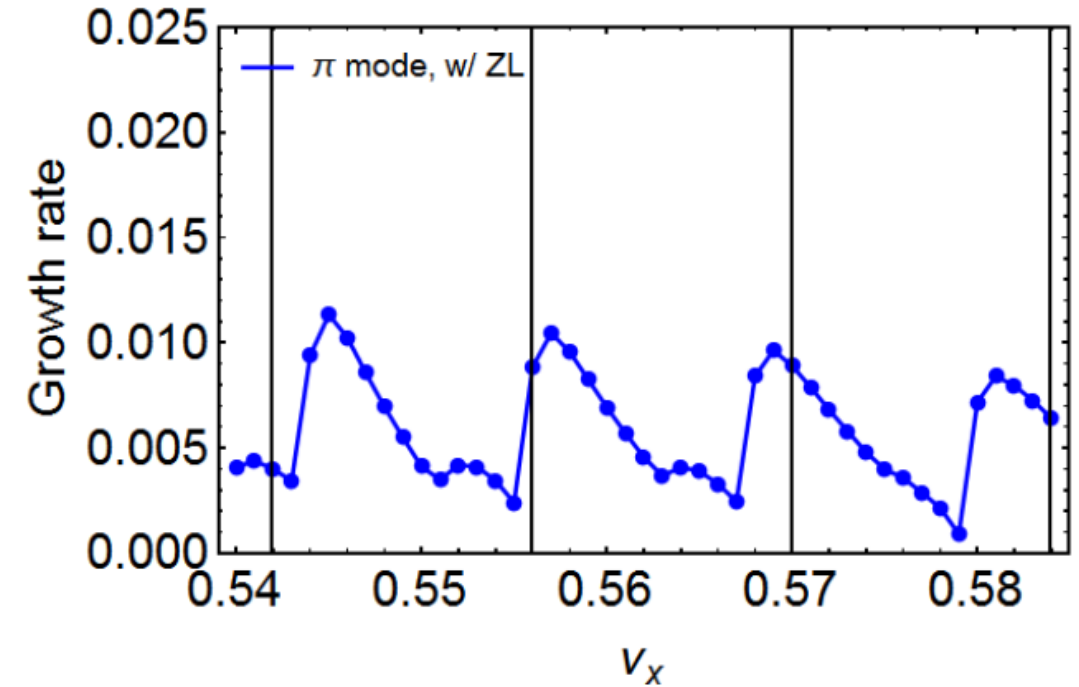
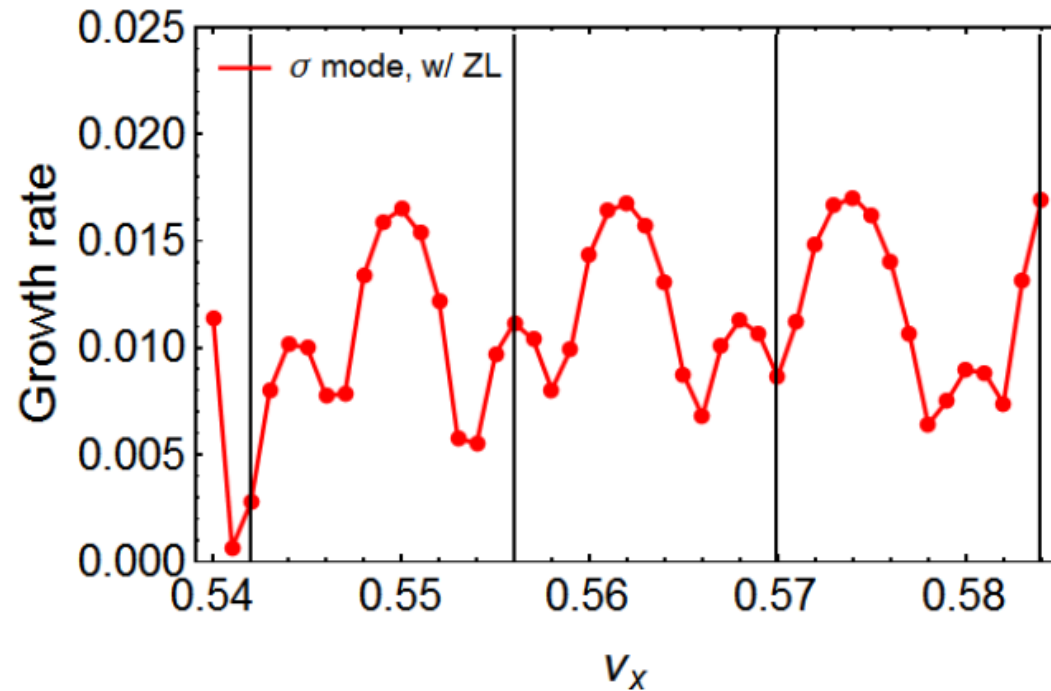
# Crosscheck between two analysis methods



# Eigen-Mode Analysis w/o and w/ ZL

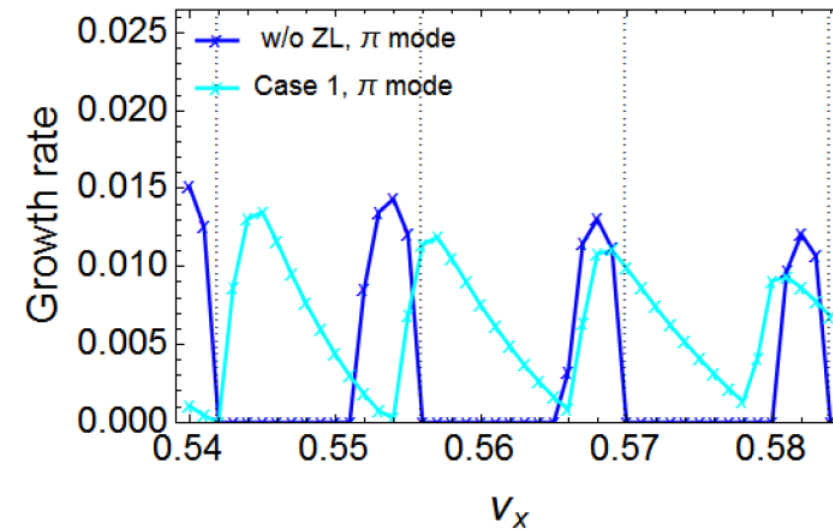
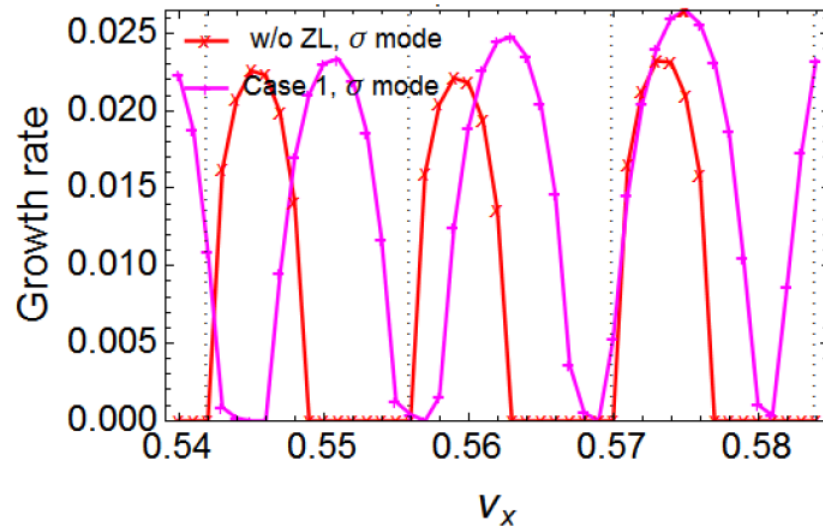


# Tune Scan w/ ZL



- Gap between neighboring peaks reduces from 0.014 to  $\sim 0.011$
- New resonance structures appear

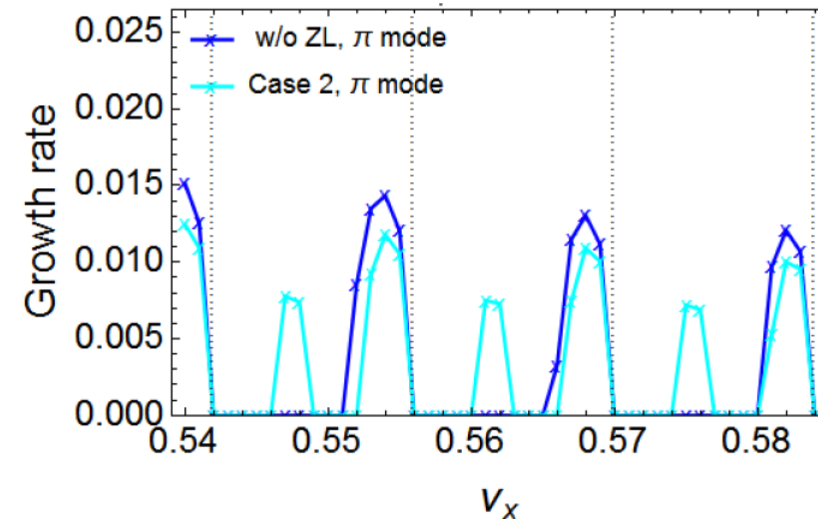
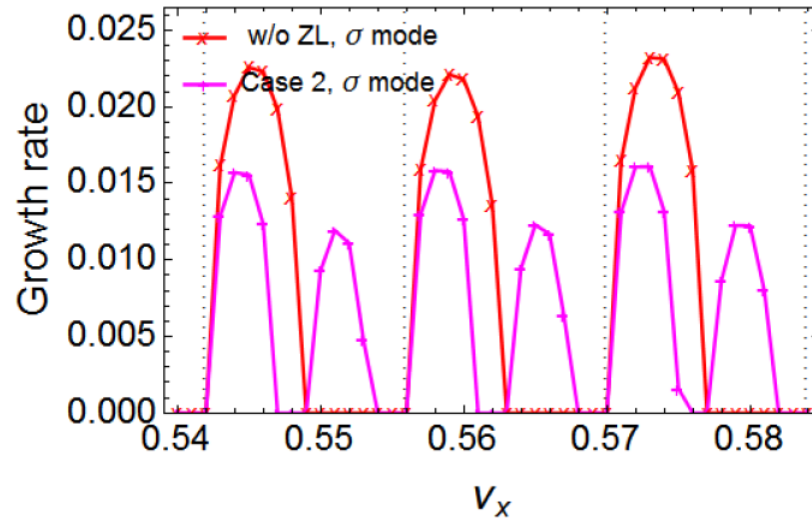
# Effects of Incoherent Synchrotron Tune Shift



- Phase space distortion is not included w/ ZL
- Only incoherent  $\nu_s$  is considered
- Instability peak is shifted and gap between peaks is squeezed
- No resonance structure between peaks

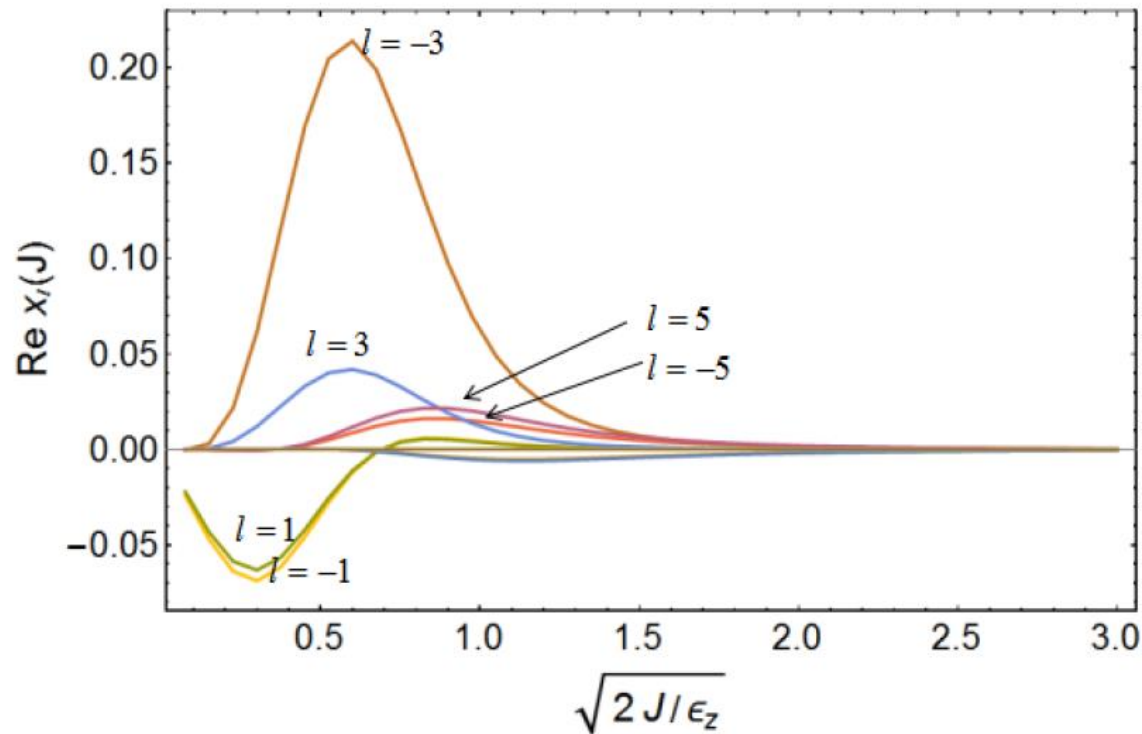


# Effects of Phase Space Distortion

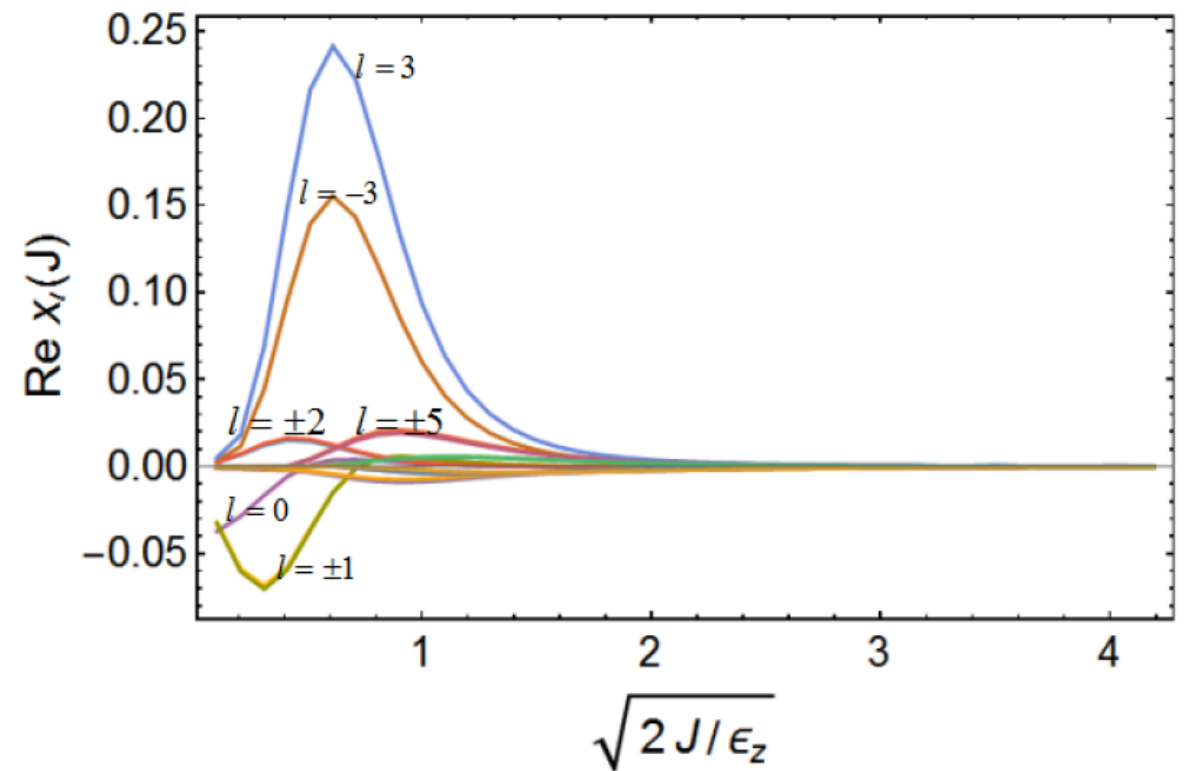


- Phase space distortion is considered w/ ZL
- Synchrotron tune  $\nu_s$  is kept constant (0.014)
- gap between peaks is unchanged
- new unstable tune appear

# Eigen Vector Analysis w/o and w/ ZL



**W/O ZL**, excited modes exist only for  $l = \pm 1, \pm 3, \pm 5 \dots$  and not for  $l = 0, \pm 2, \pm 4 \dots$ .



**W/ ZL** but only consider distortion of longitudinal phase space and keep  $\nu_s(J)$  unchanged. All parity modes for  $l = 0, \pm 1, \pm 2, \pm 3 \dots$  are excited and could be mixed.

# Summary

- Beamstrahlung lifetime is increased by ZL, while X-Z instability is enhanced
- The X-Z instability is well suppressed at W/Z for newest machine parameters.
- The stable working point space for CEPC-Higgs is about  $Q_x \sim 0.004$  when considering ZL.
- 10% asymmetry of bunch population could be accepted for  $t\bar{t}$ /Higgs/W. But only 5% asymmetry for Z.
- New analysis method has been developed to understand the combined effect (BB+ZL)