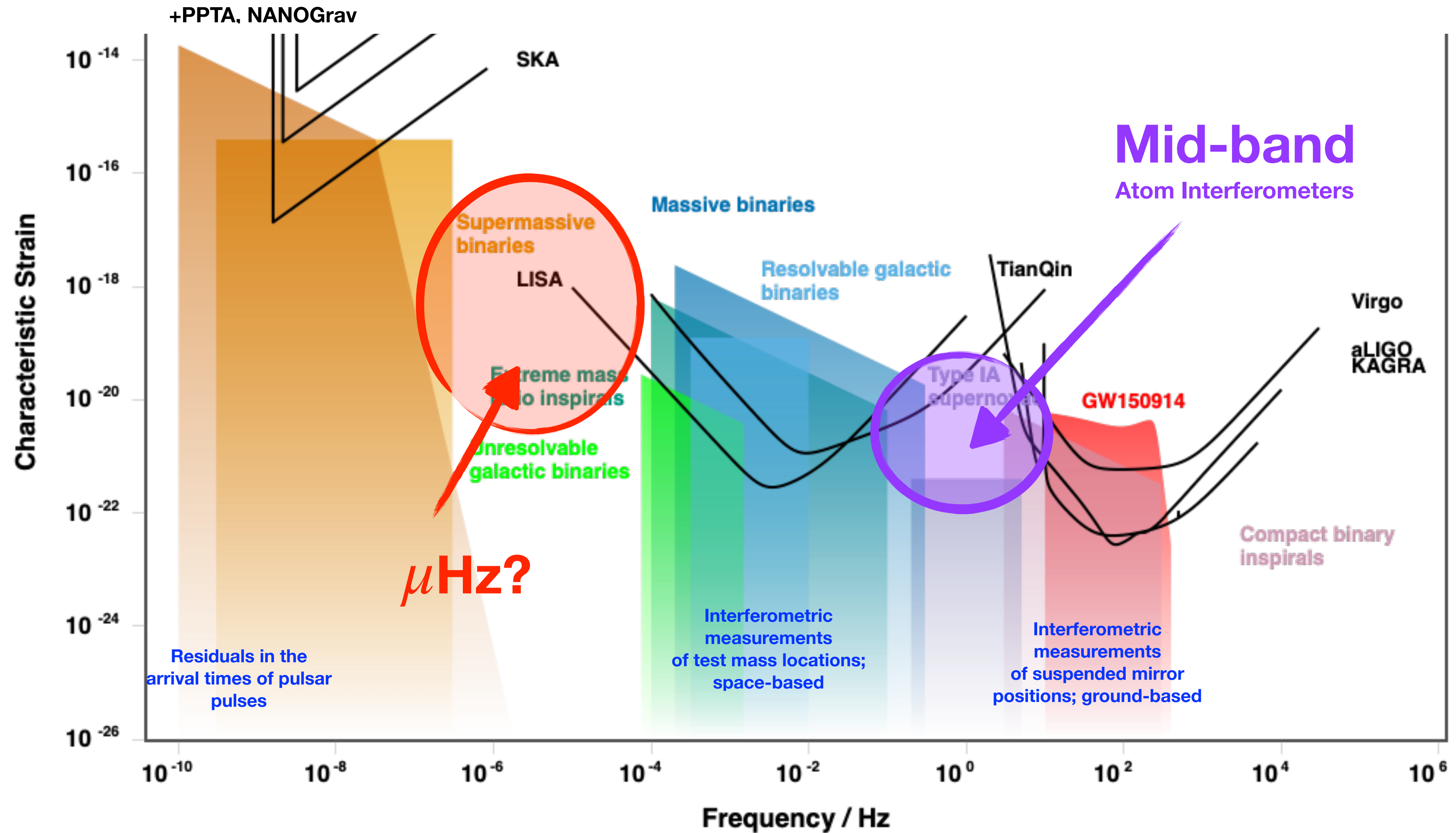


Gravitational Wave Detection between NANOGRAV and LISA

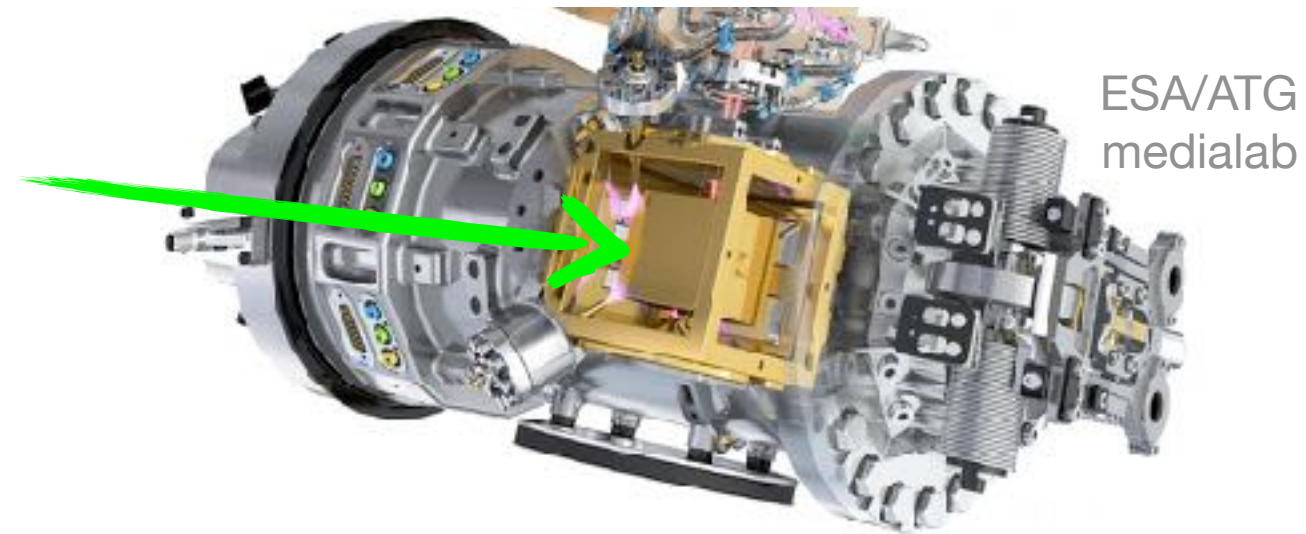
**Surjeet Rajendran
The Johns Hopkins University**

The Gravitational Wave Spectrum



Local Test Mass based GW Detection 101

Satellites with drag-free test masses (TM)



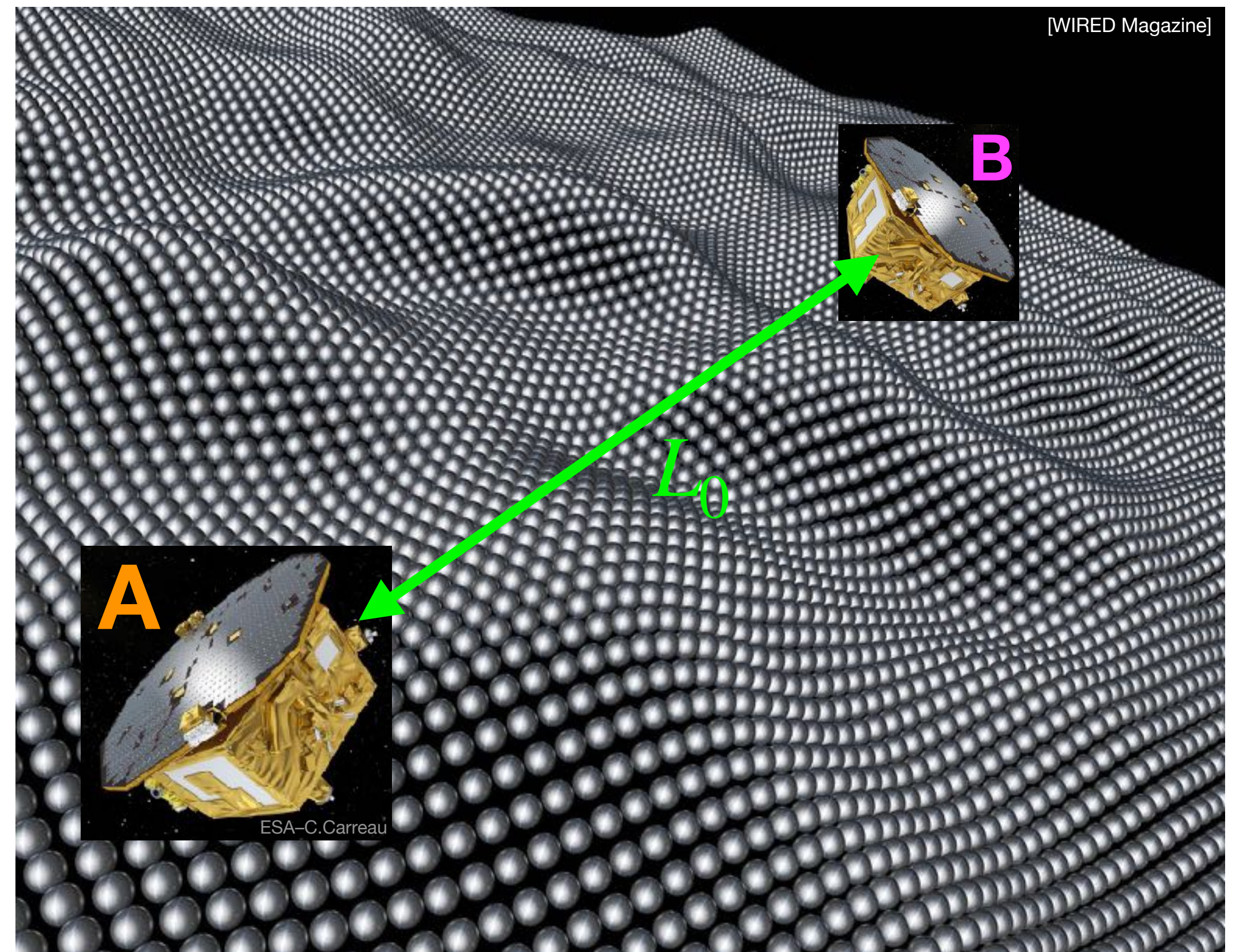
Light travel time (= proper distance) between test masses is modulated by GW

Emitter (A) sends pulse at $t_A = t_0$;
receiver (B) gets pulse at $t_B = t_0 + \Delta t$:

$$\Delta t = L_0 \left(1 - \frac{h_0}{2} \text{sinc}(\omega_{gw} L_0 / 2) \cos[\omega_{gw}(t_0 + L_0 / 2)] \right) + \mathcal{O}(h_0^2)$$

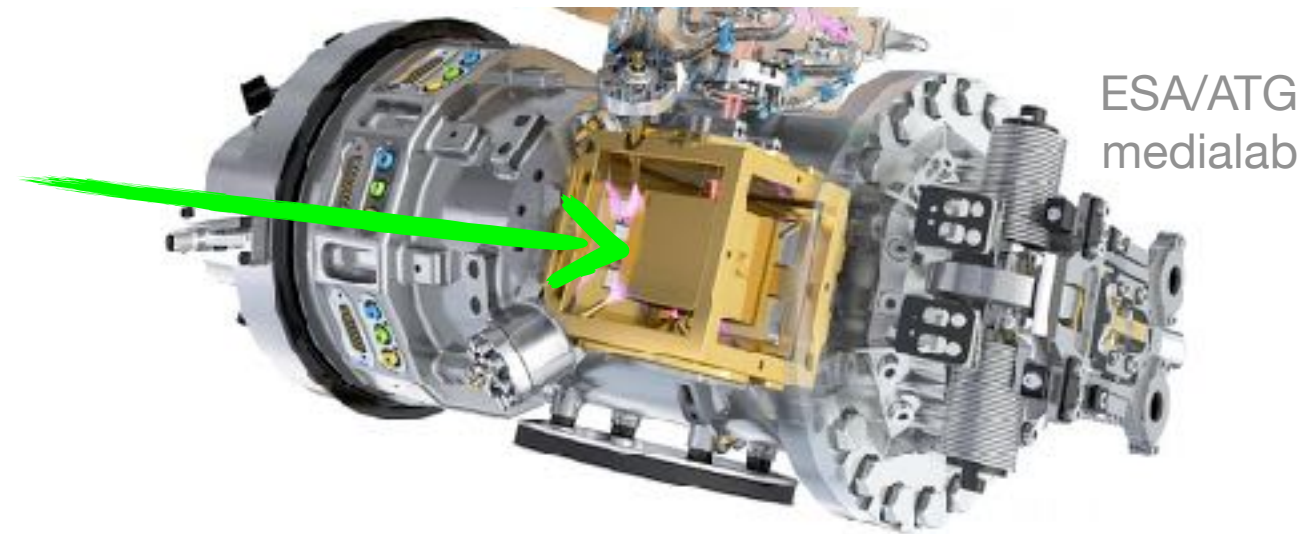
$$\longrightarrow L_0 \left(1 - \frac{h_0}{2} \cos[\omega_{gw} t_0] \right) + \mathcal{O}(h_0^2) \quad [\omega_{gw} L_0 \ll 1]$$

← GW strain amplitude



Local Test Mass based GW Detection 101

Satellites with drag-free test masses (TM)



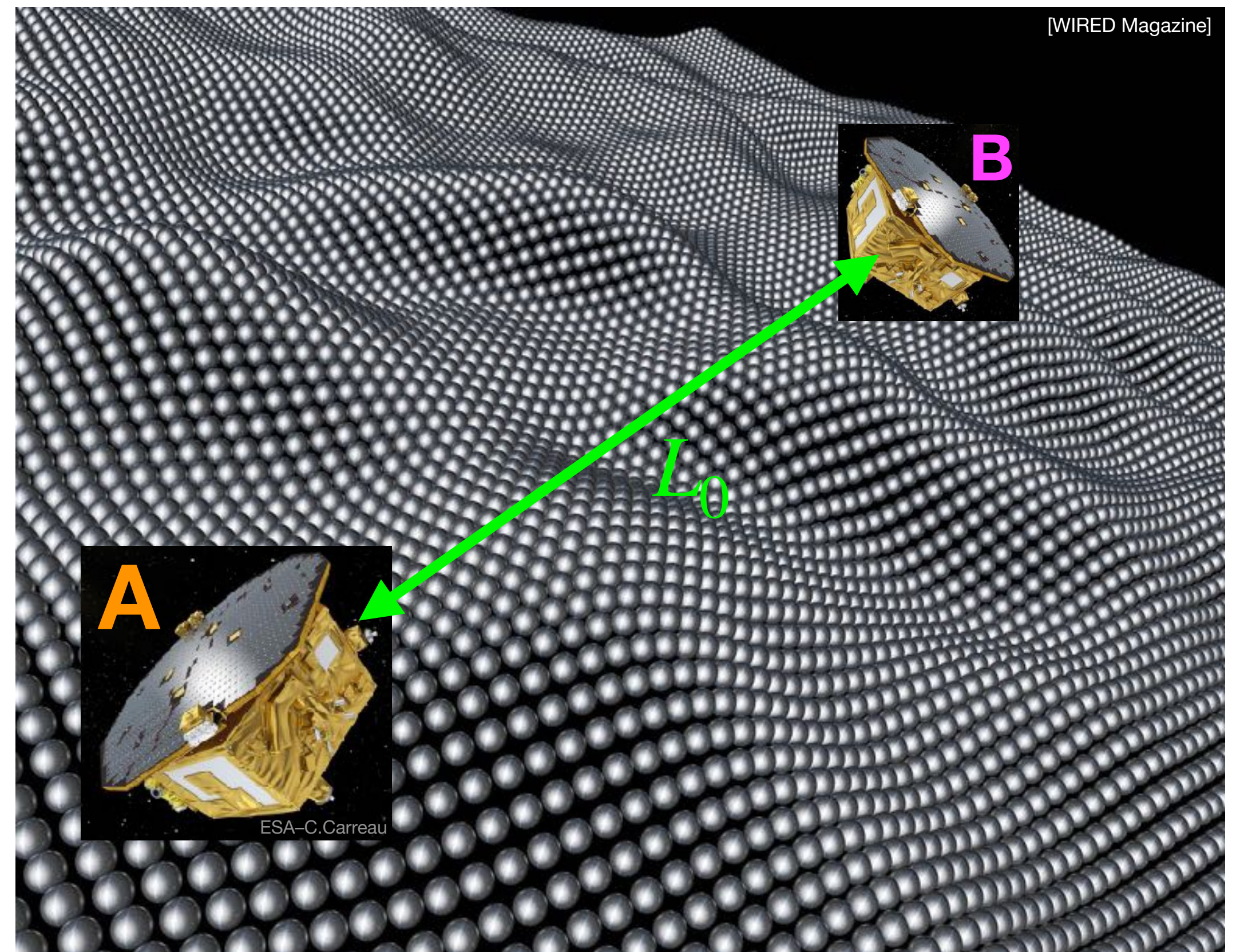
Light travel time (= proper distance) between test masses is modulated by GW

Emitter (A) sends pulse at $t_A = t_0$;
receiver (B) gets pulse at $t_B = t_0 + \Delta t$:

$$\Delta t = L_0 \left(1 - \frac{h_0}{2} \text{sinc}(\omega_{gw} L_0 / 2) \cos[\omega_{gw}(t_0 + L_0 / 2)] \right) + \mathcal{O}(h_0^2)$$

$$\longrightarrow L_0 \left(1 - \frac{h_0}{2} \cos[\omega_{gw} t_0] \right) + \mathcal{O}(h_0^2) \quad [\omega_{gw} L_0 \ll 1]$$

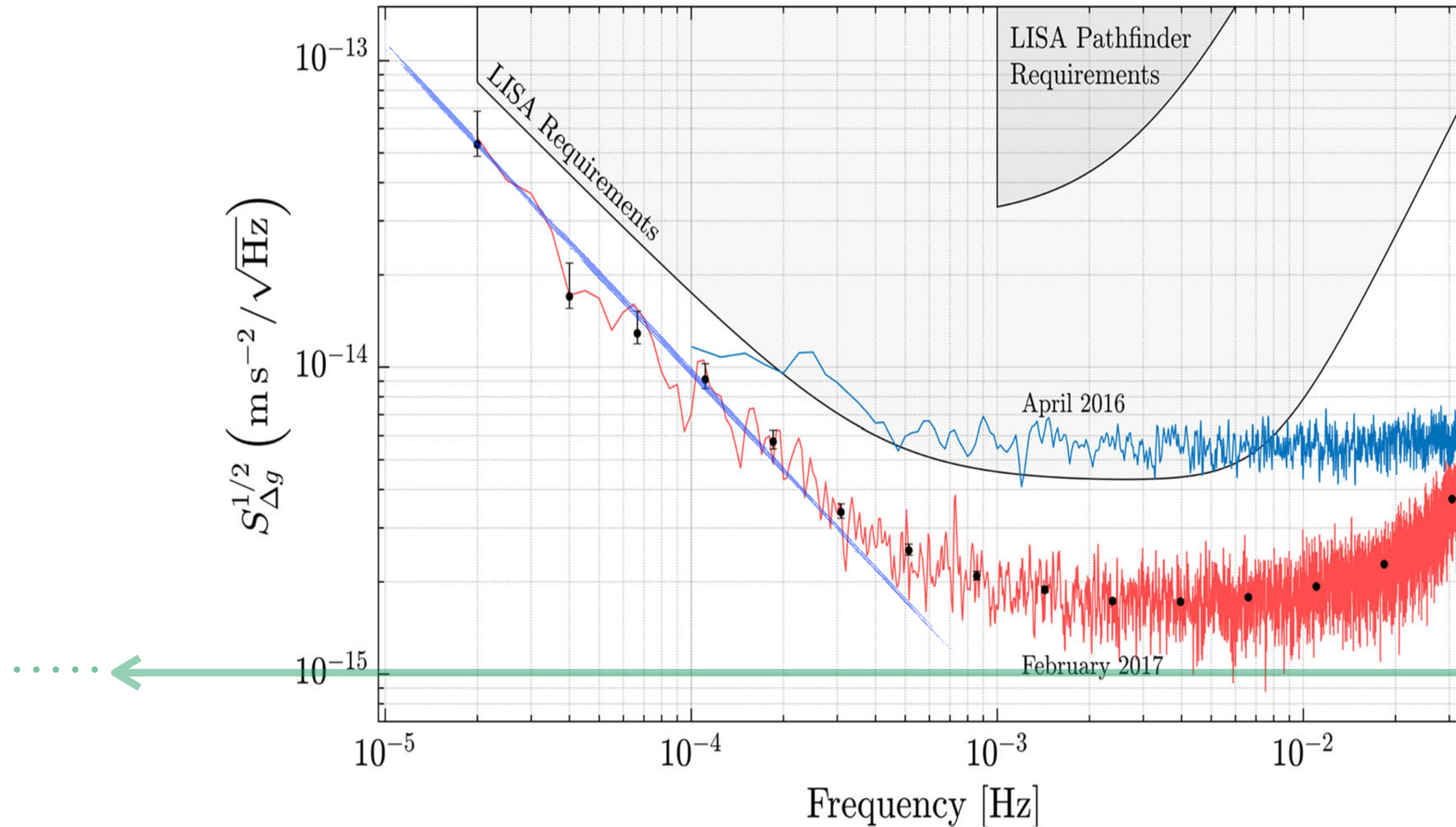
← GW strain amplitude



How drag free?

LISA Pathfinder Results

LISA Pathfinder. Phys. Rev. Lett. **120**, 061101 (2018)



LISA Pathfinder

- Residual gas
- Charging
- TM actuator noise
- Laser intensity noise
- etc.

Mission Concept

Suppress acceleration noise by having very large test mass

$$a = \frac{F}{M}$$

How do we launch?



Mission Concept

Suppress acceleration noise by having very large test mass

$$a = \frac{F}{M}$$

How do we launch?

Natural Objects?



Planets?

Stable Center of Mass
Unstable Surface: Seismics/
atmospheric effects

Mission Concept

Suppress acceleration noise by having very large test mass

$$a = \frac{F}{M}$$

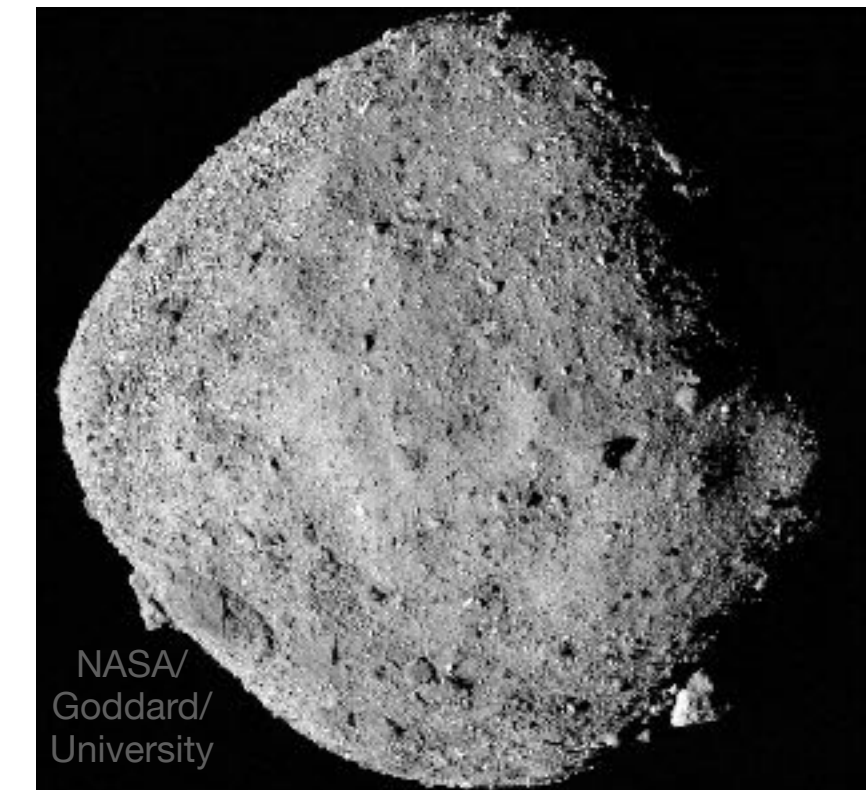
How do we launch?

Natural Objects?



Planets?

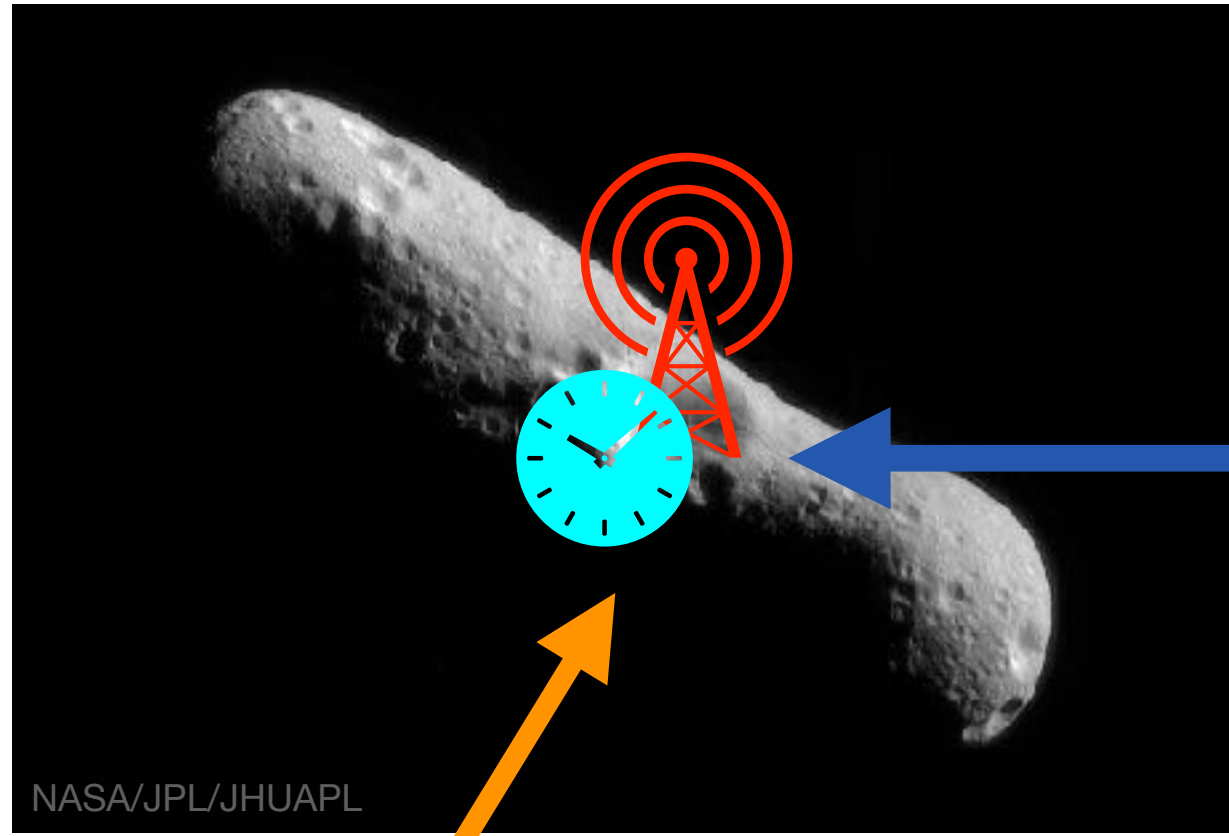
Stable Center of Mass
Unstable Surface: Seismics/
atmospheric effects



Asteroids?

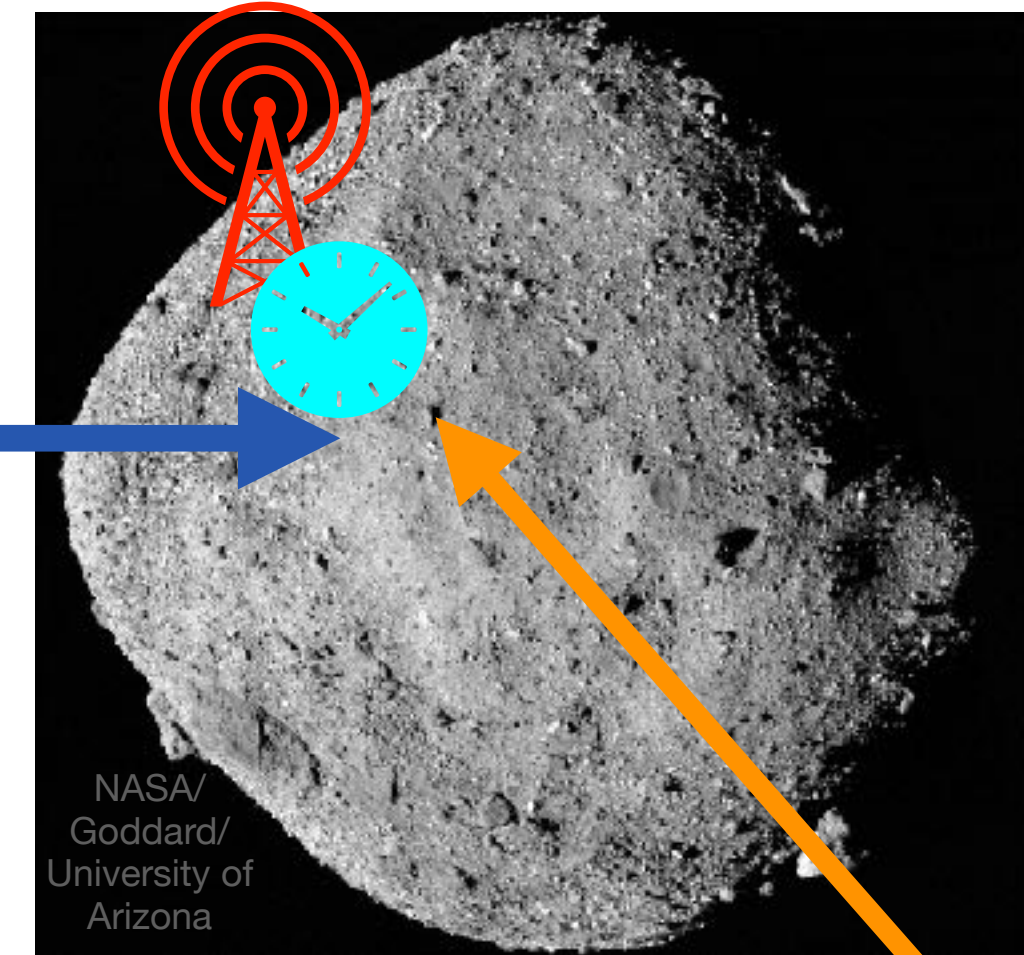
Few km scale rocks

Mission Concept



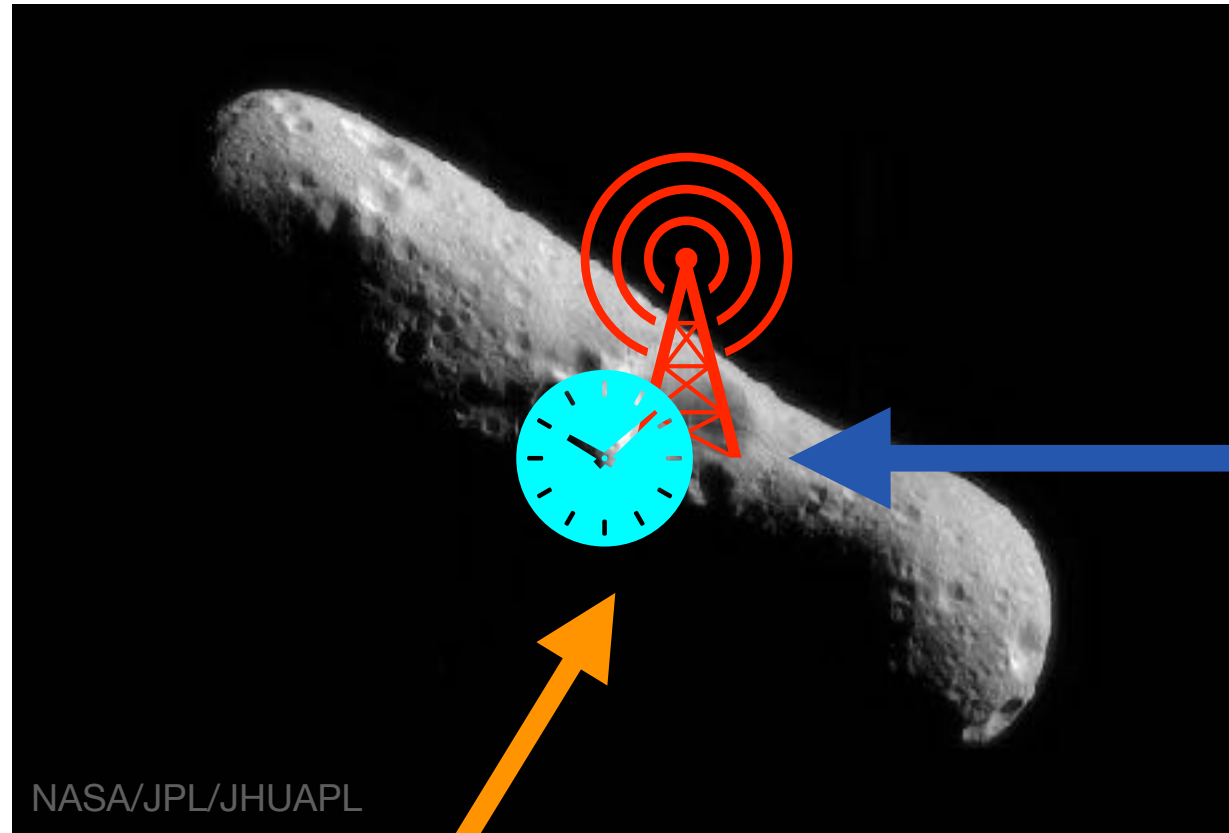
Deployed base station

Radio/Laser Range



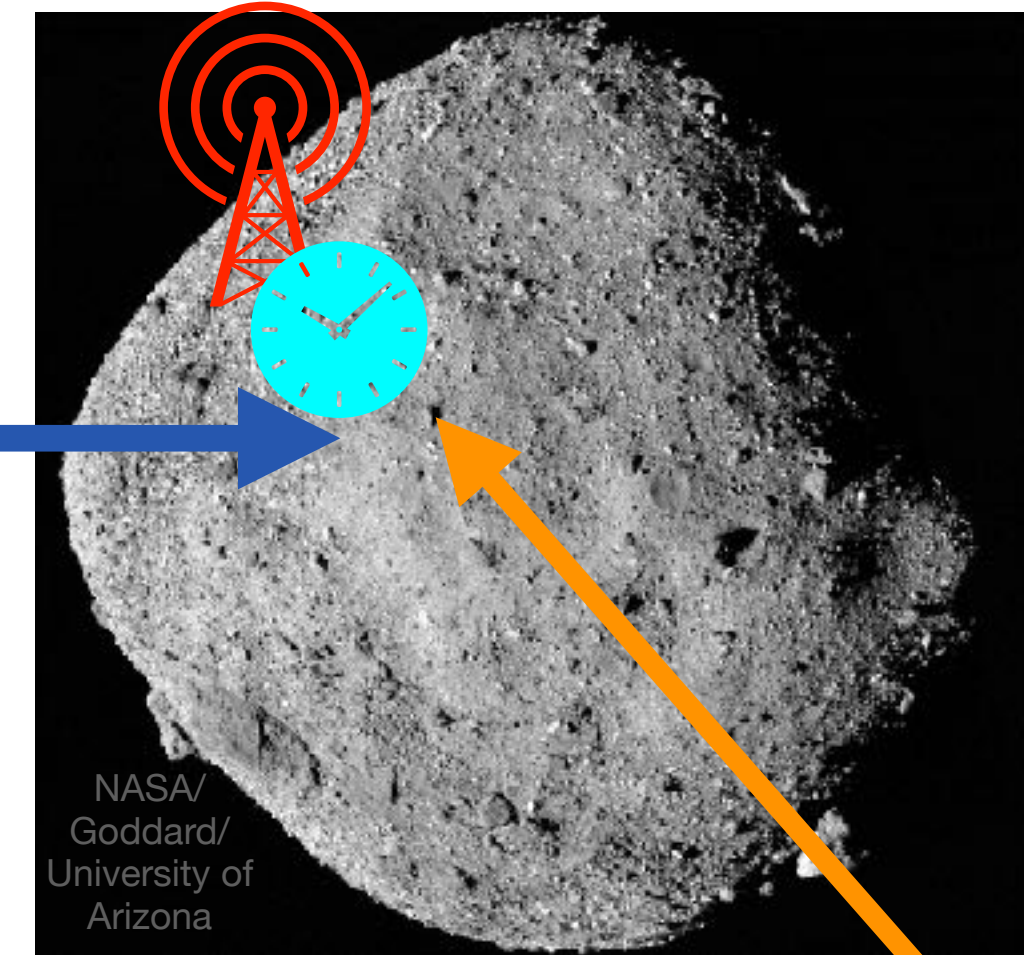
Deployed base station

Mission Concept



Deployed
base station

Radio/Laser Range

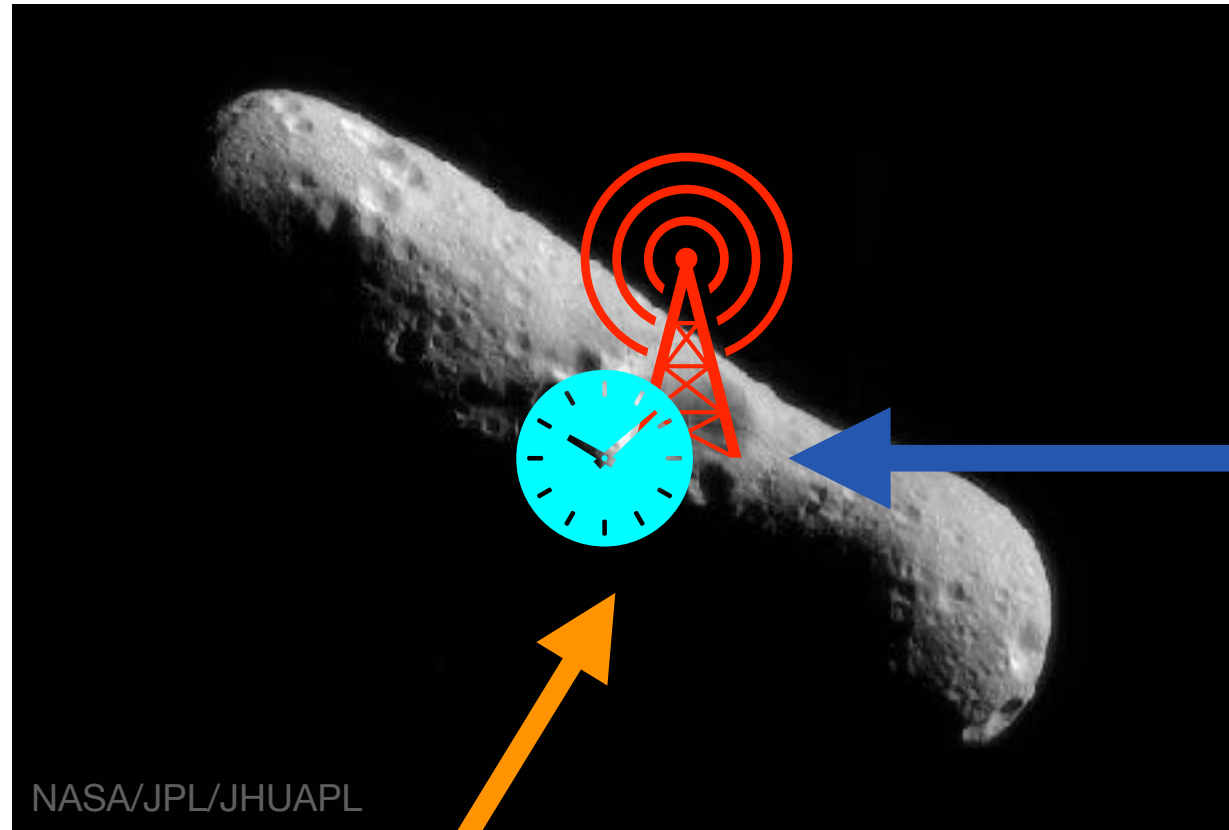


Deployed
base station

Stability?

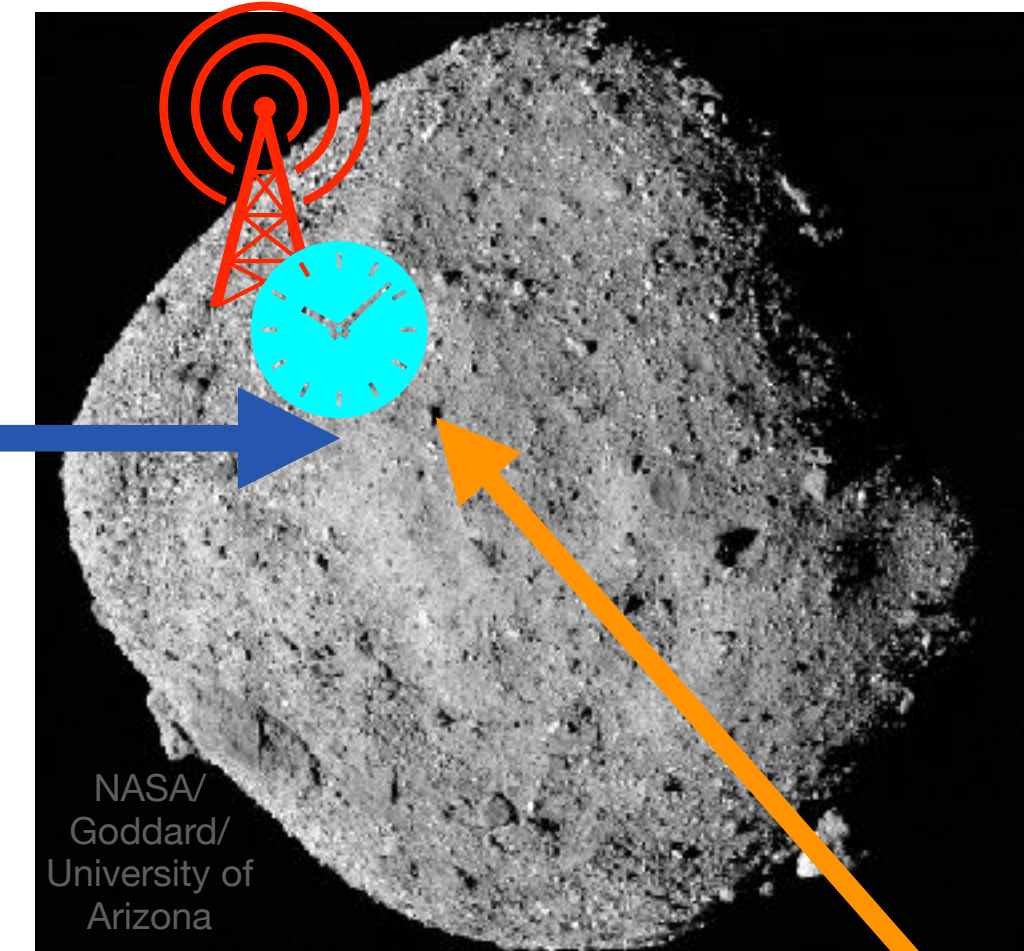
$$h \sim 10^{-17} - 10^{-18}, L \sim 1 \text{ AU} \implies \delta x \sim hL \sim 0.1 \mu\text{m}$$

Mission Concept



Deployed
base station

Radio/Laser Range



Deployed
base station

Stability?

$$h \sim 10^{-17} - 10^{-18}, L \sim 1 \text{ AU} \implies \delta x \sim hL \sim 0.1 \mu\text{m}$$

Land on Asteroids?

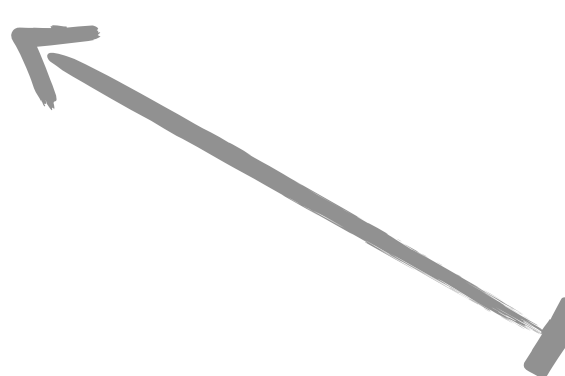
Do we have good enough atomic clocks?

Is the asteroid surface/center of mass stable enough?

Noise sources

- Gravitational pull of large bodies (planets, moons) - ephemeris and $G_N M_{\text{obj}}$ known
- Solar intensity fluctuations (CoM + torques)
- Solar wind fluctuations (CoM + torques)
- Thermal cycling
- Noise at rotational period
- Gravity Gradient Noise from other $\sim 10^6$ asteroids in Main Belt
- Seismics
- Charging
- Magnetic forces and torques
- Collisions
- Tidal deformation
- etc...

- Clock noise
- Link (shot/thermal) noise



M.A.Fedderke., P. W. Graham, and S. Rajendran.
Phys. Rev. D **103**, 103017 (2021) [2011.13833].

Noise sources

- Gravitational pull of large bodies (planets, moons) - ephemeris and $G_N M_{\text{obj}}$ known

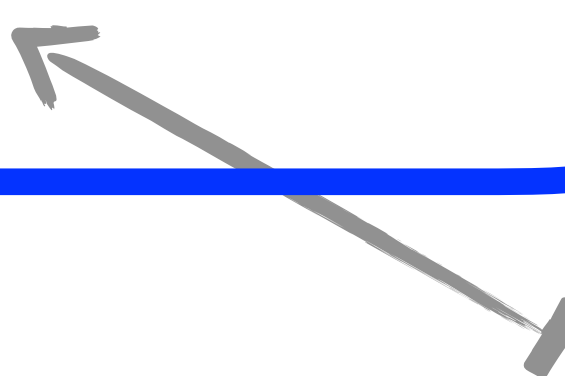
- Solar intensity fluctuations (CoM + torques)
- Solar wind fluctuations (CoM + torques)
- Thermal cycling
- Noise at rotational period
- Gravity Gradient Noise from other $\sim 10^6$ asteroids in Main Belt
- Seismics

- Charging
- Magnetic forces and torques
- Collisions
- Tidal deformation
- etc...

- Clock noise
- Link (shot/thermal) noise

Dominant Noise Sources

M.A.Fedderke., P. W. Graham, and S. Rajendran.
Phys. Rev. D **103**, 103017 (2021) [2011.13833].



Noise sources

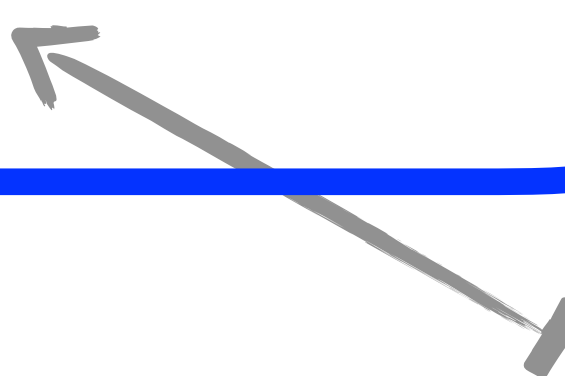
- Gravitational pull of large bodies (planets, moons) - ephemeris and $G_N M_{\text{obj}}$ known

- Solar intensity fluctuations (CoM + torques)
- Solar wind fluctuations (CoM + torques)
- Thermal cycling
- Noise at rotational period
- Gravity Gradient Noise from other $\sim 10^6$ asteroids in Main Belt
- Seismics

- Charging
- Magnetic forces and torques
- Collisions
- Tidal deformation
- etc...

Dominant Noise Sources

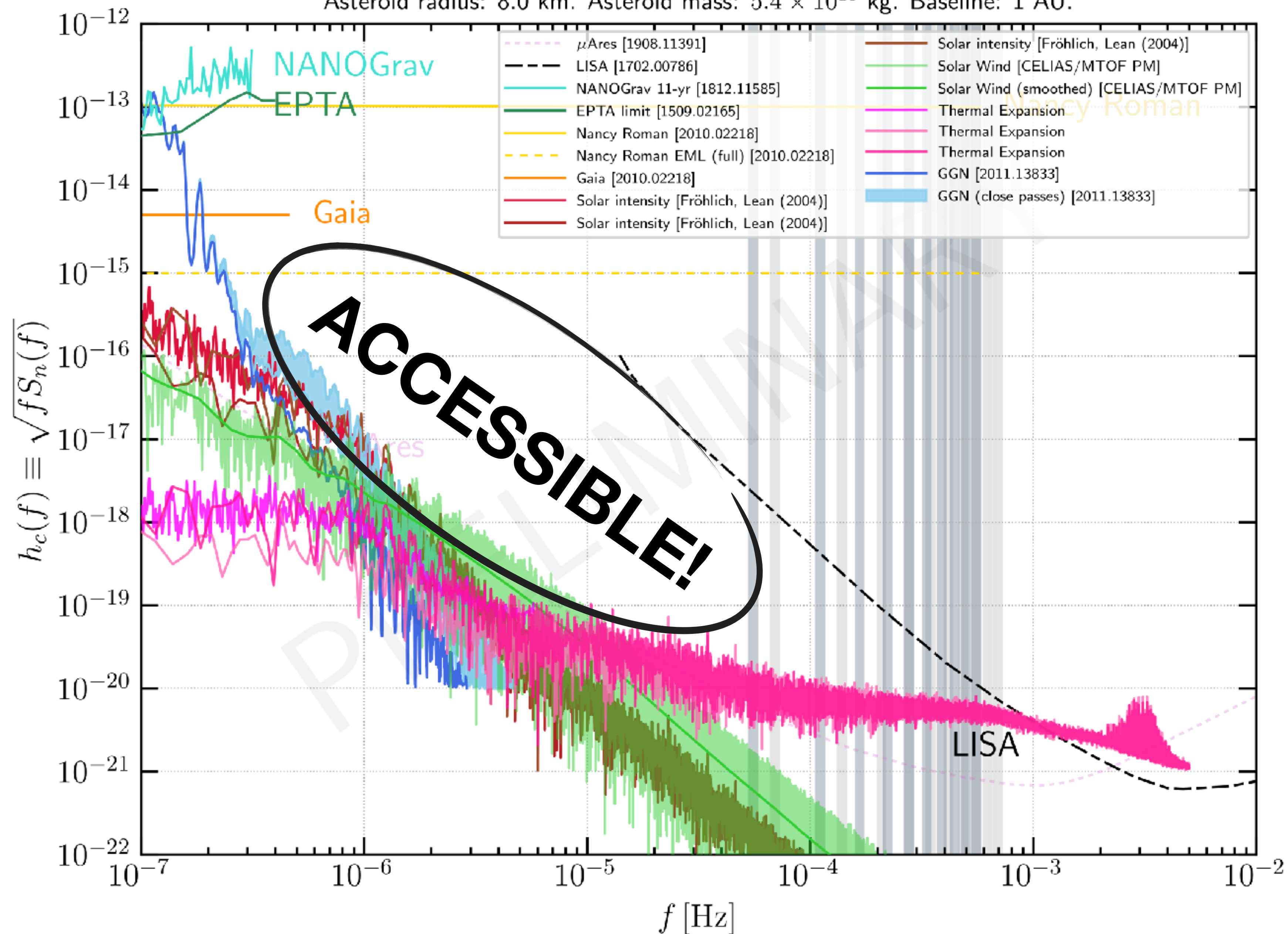
M.A.Fedderke., P. W. Graham, and S. Rajendran.
Phys. Rev. D **103**, 103017 (2021) [2011.13833].



- Clock noise
- Link (shot/thermal) noise

Link Noise Sources

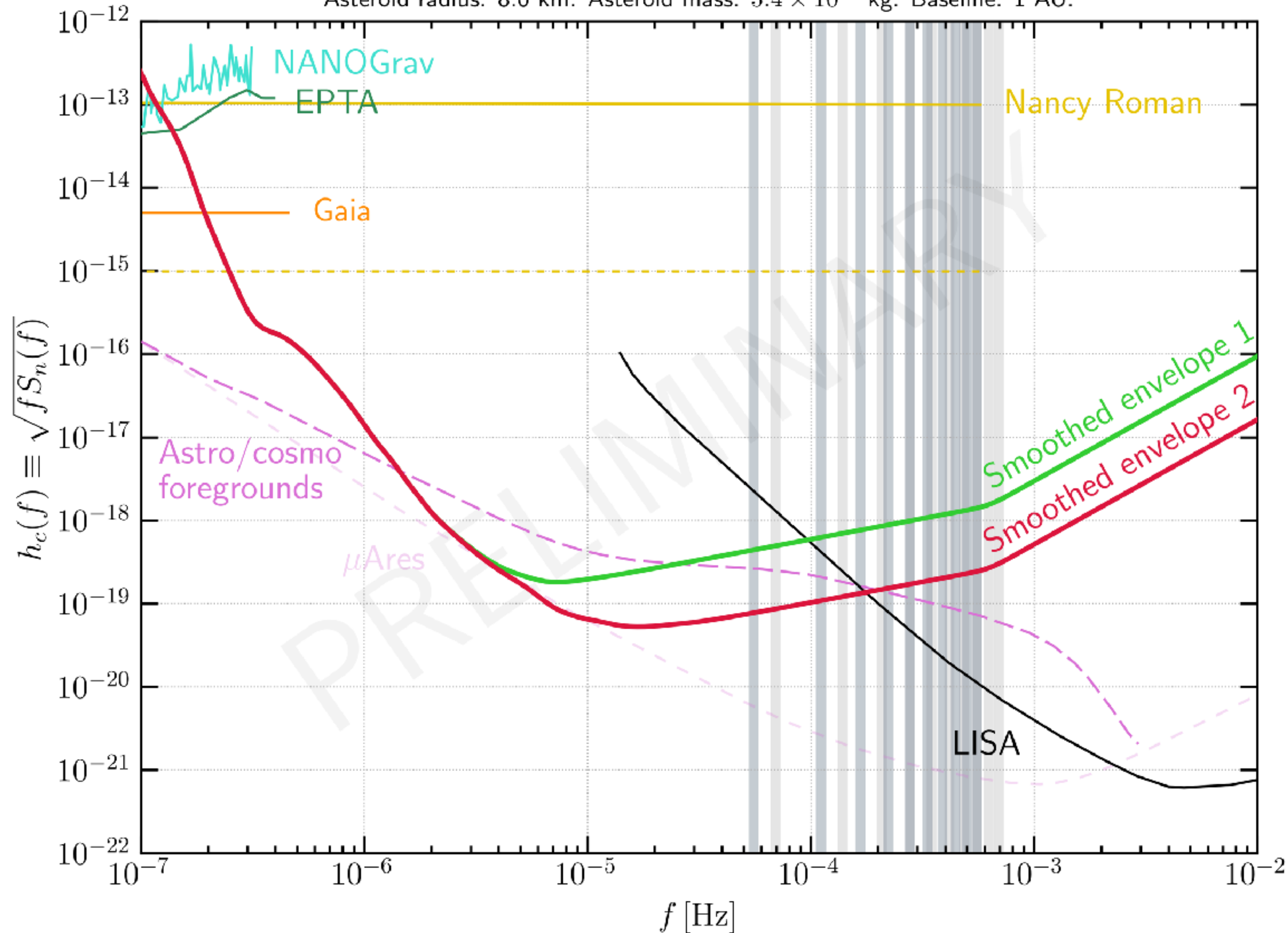
Asteroid radius: 8.0 km. Asteroid mass: 5.4×10^{15} kg. Baseline: 1 AU.



**ASTEROIDS
ARE
EXCELLENT
TEST MASSES
IN THE
 μ Hz BAND!**

Projections

Asteroid radius: 8.0 km. Asteroid mass: 5.4×10^{15} kg. Baseline: 1 AU.



Asteroids are excellent test masses for a GW detector in the μHz band

~ m scale laser/radio ranging between on-asteroid base stations equipped with transmit/receive capability and atomic clocks gets excellent sensitivity

Strongly motivates:

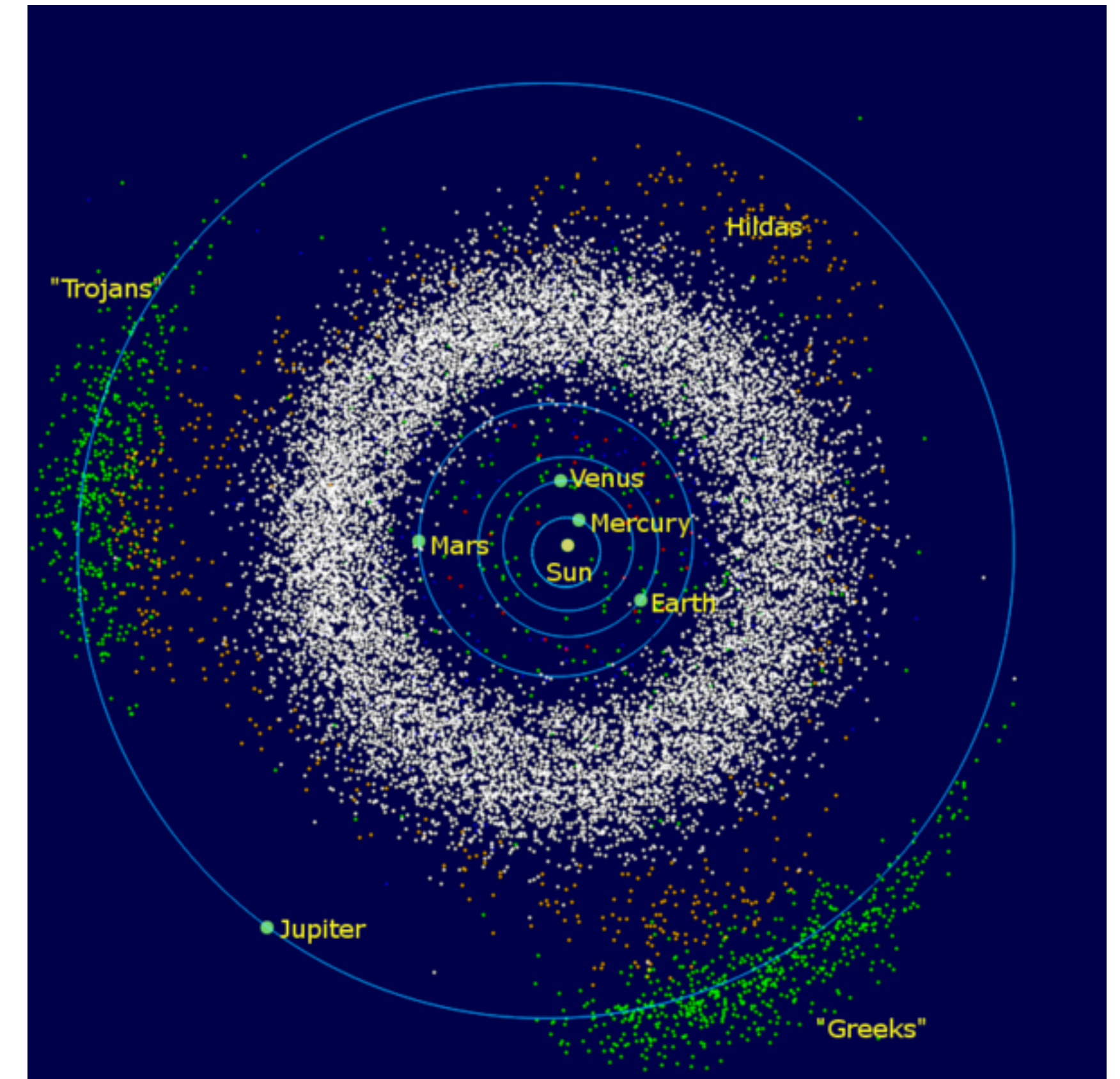
- a detailed technical design study
- in-situ seismic / plastic deformation monitoring of asteroids in upcoming missions
- space-qualifying cold-atom atomic clocks

Astrometry

(10 nHz - 1 μ Hz)

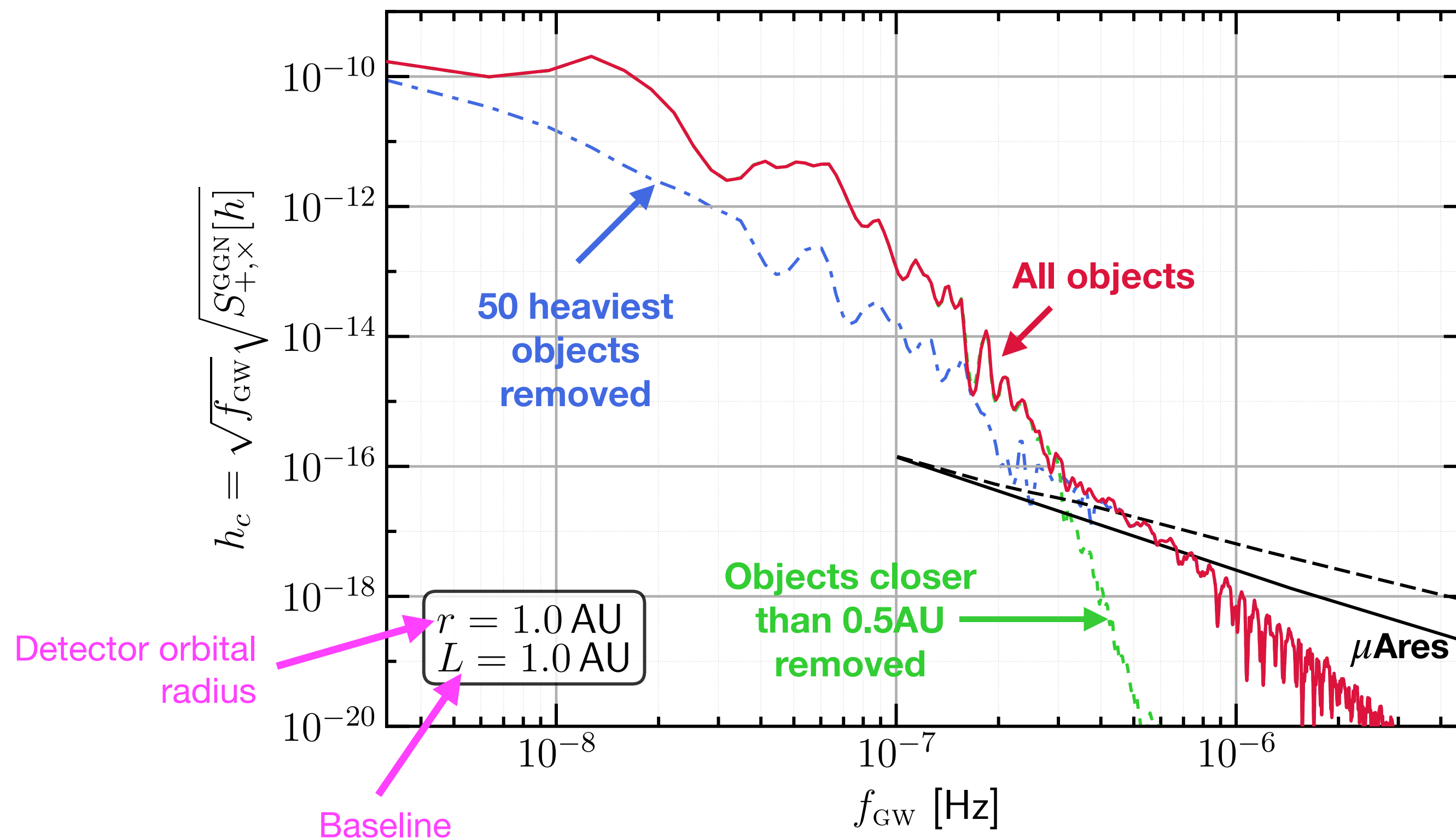
Gravity Gradient Noise below μHz

- The Sun, Planets (and Pluto), Moons
 - Relatively few
 - Masses (or $G_N M$) and locations known
 - **Not** noise (model out)
- The Inner Solar System asteroids
 - $\mathcal{O}(10^6)$ objects
 - *Generally*, masses poorly / indirectly determined
 - Locations are known to some extent
 - **NOT** reasonable to assume that one can successfully model these out
 - Asteroid gravity gradient noise (GGN)!



Gravity Gradient Simulation

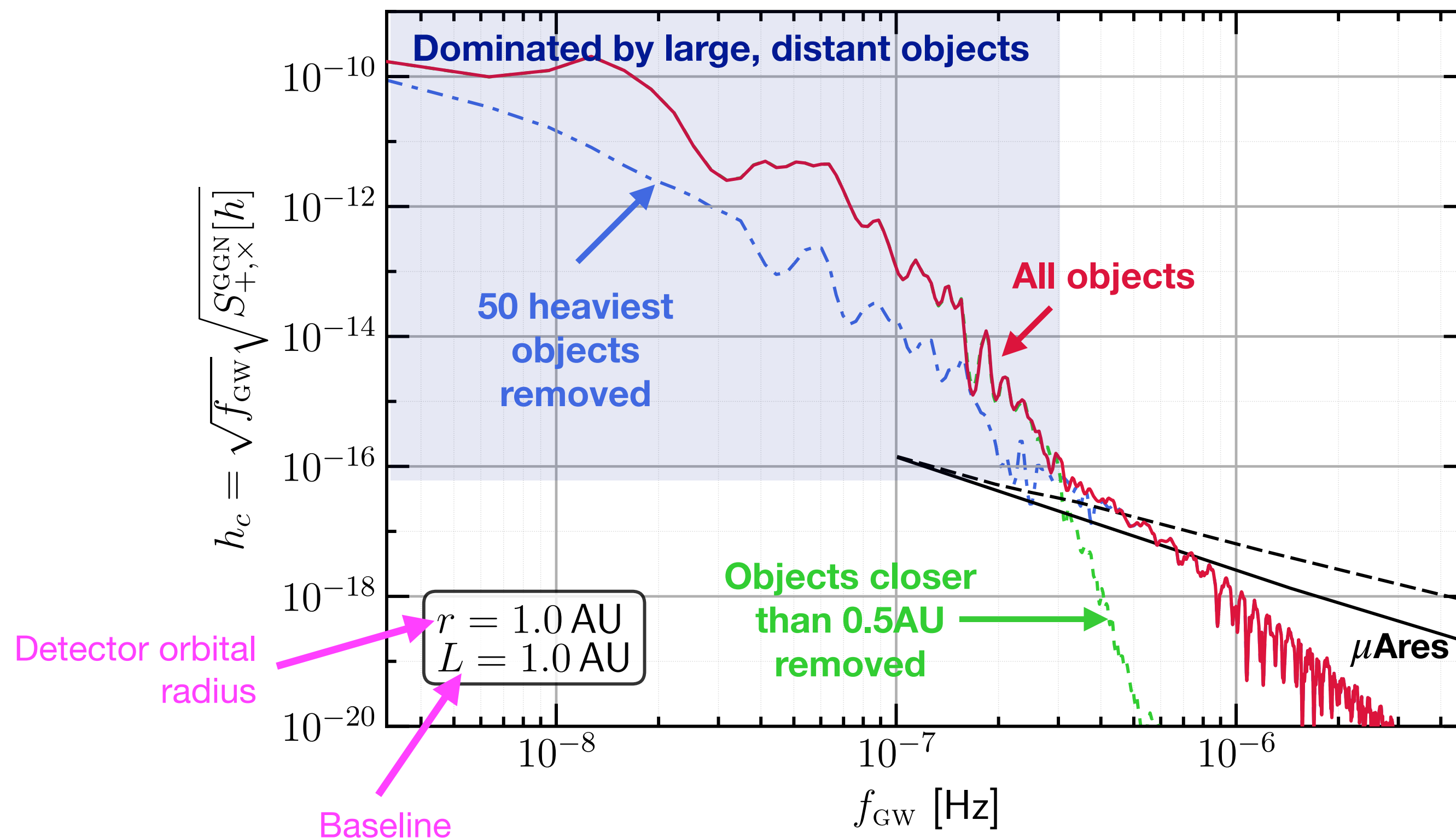
- Use NASA JPL Small-Body Database
- 10-year mission simulation
- Detectors on circular orbit @ 1AU; asteroids on elliptical orbits (not N -body)
- Test mass accelerations from asteroids \longrightarrow “noise” power spectrum (details) \longrightarrow strain sensitivity.



- Problematic for **ANY** local-test mass-based GW detector with Inner Solar System baselines, up to frequencies $\sim (\text{few}) \times 10^{-7} \text{ Hz}$
- Removing ~ 50 heavy distant objects does not change this conclusion
- At higher frequency, noise drops off. But only $\sim 1/6$ of objects in database used in simulation: missing diameters for smaller, closer passing objects...

Gravity Gradient Simulation

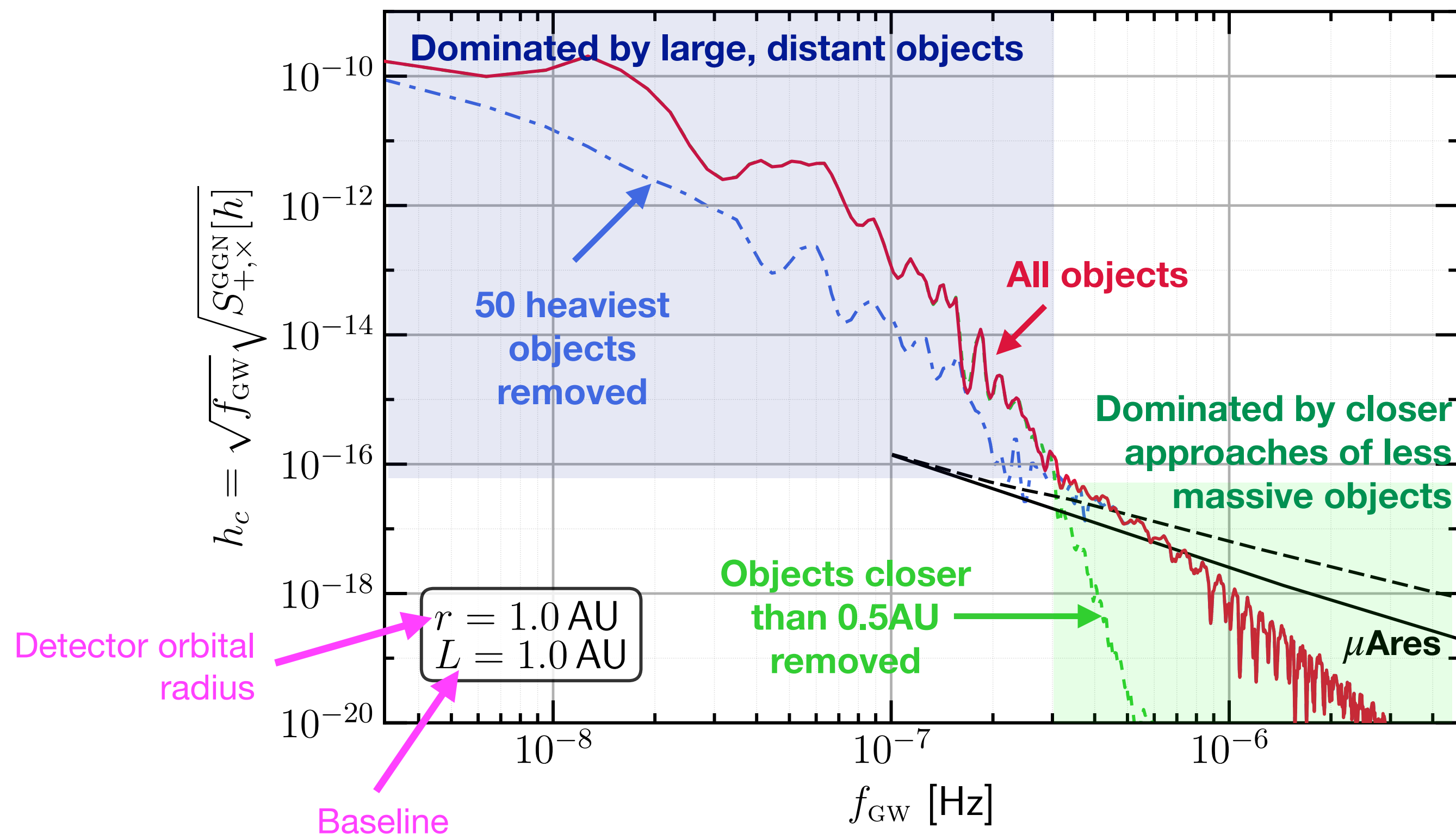
- Use NASA JPL Small-Body Database
- 10-year mission simulation
- Detectors on circular orbit @ 1AU; asteroids on elliptical orbits (not N -body)
- Test mass accelerations from asteroids \longrightarrow “noise” power spectrum (details) \longrightarrow strain sensitivity.



- Problematic for **ANY** local-test mass-based GW detector with Inner Solar System baselines, up to frequencies $\sim (\text{few}) \times 10^{-7} \text{ Hz}$
- Removing ~ 50 heavy distant objects does not change this conclusion
- At higher frequency, noise drops off. But only $\sim 1/6$ of objects in database used in simulation: missing diameters for smaller, closer passing objects...

Gravity Gradient Simulation

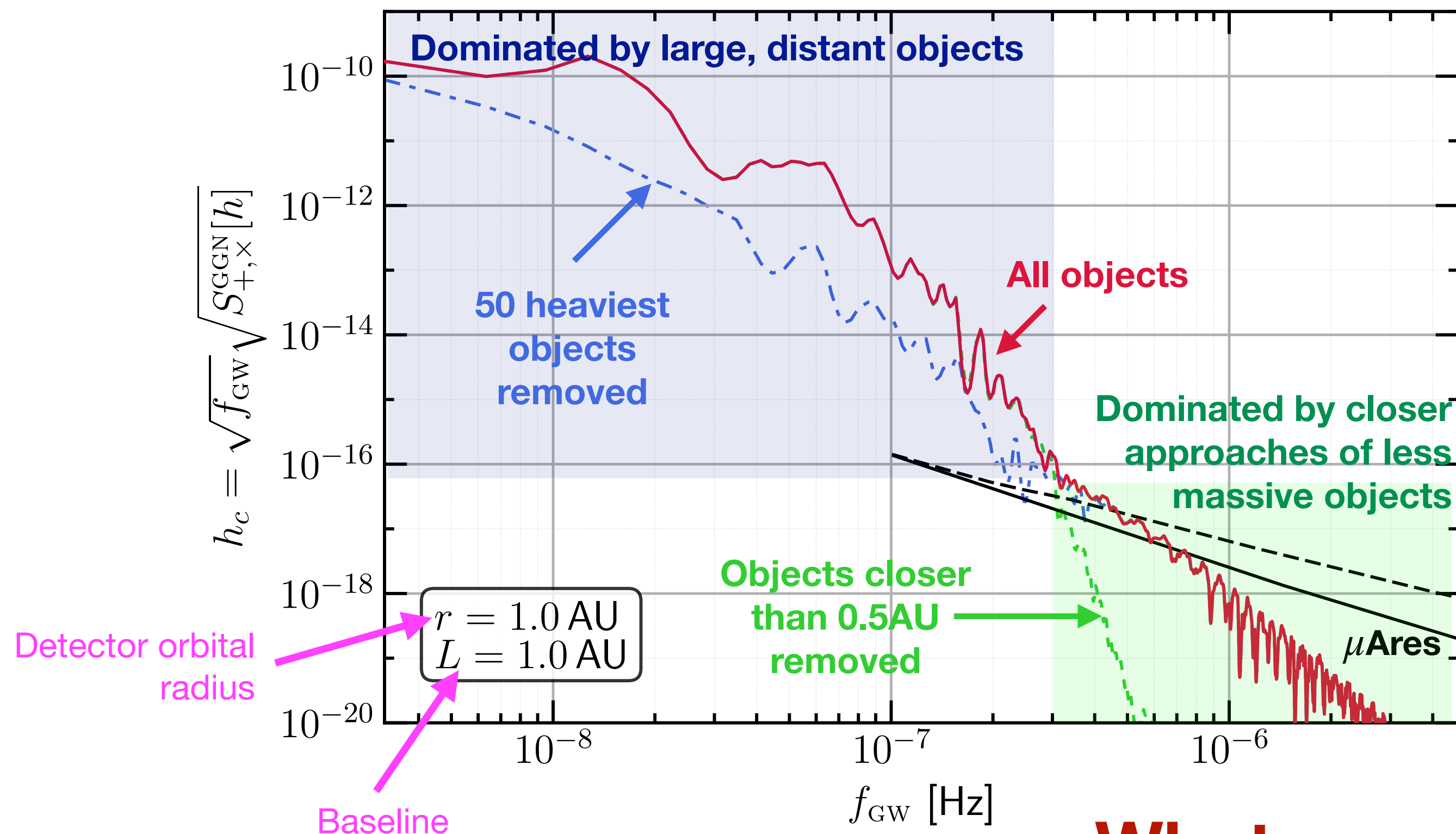
- Use NASA JPL Small-Body Database
- 10-year mission simulation
- Detectors on circular orbit @ 1AU; asteroids on elliptical orbits (not N -body)
- Test mass accelerations from asteroids \longrightarrow “noise” power spectrum (details) \longrightarrow strain sensitivity.



- Problematic for **ANY** local-test mass-based GW detector with Inner Solar System baselines, up to frequencies $\sim (\text{few}) \times 10^{-7} \text{ Hz}$
- Removing ~ 50 heavy distant objects does not change this conclusion
- At higher frequency, noise drops off. But only $\sim 1/6$ of objects in database used in simulation: missing diameters for smaller, closer passing objects...

Gravity Gradient Simulation

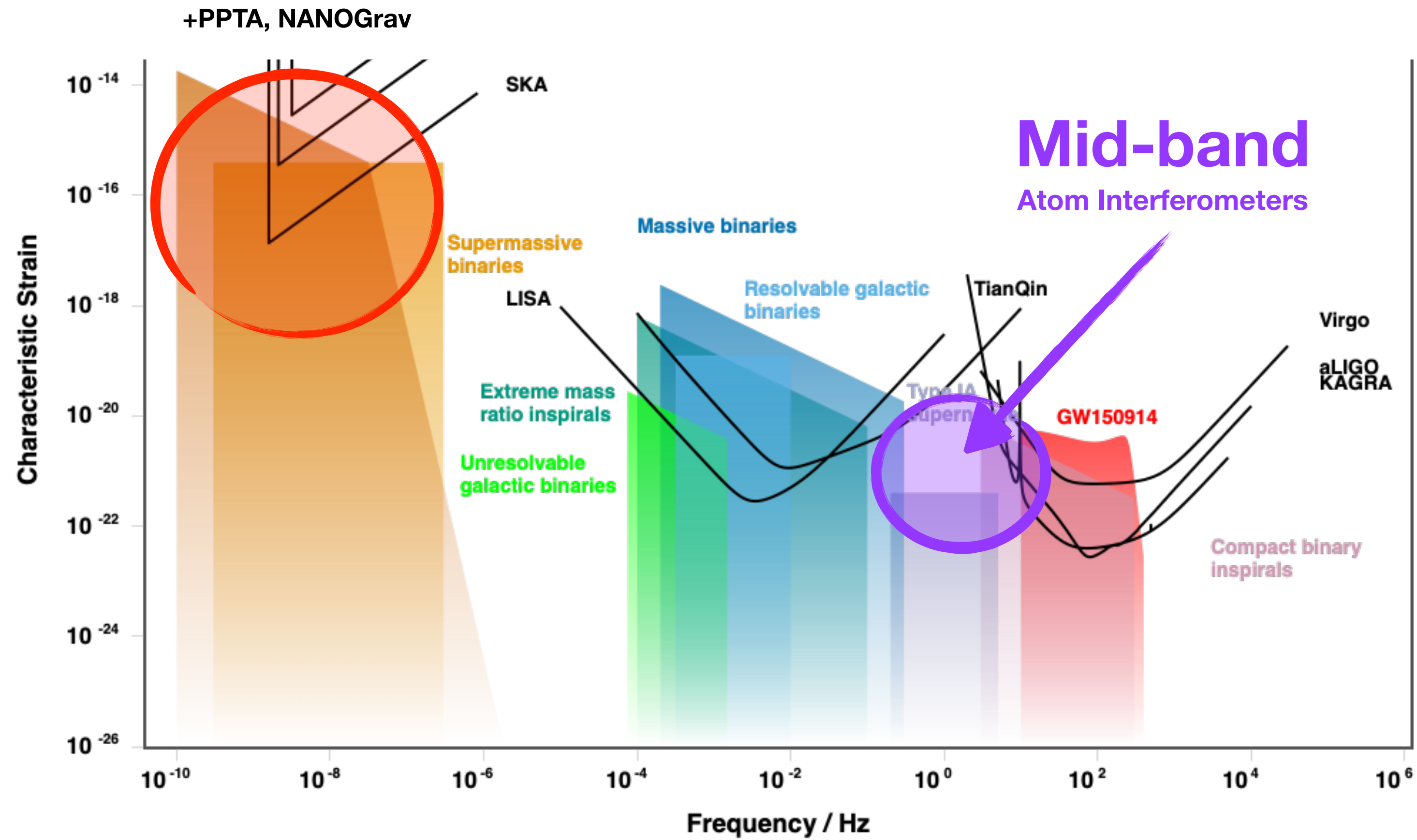
- Use NASA JPL Small-Body Database
- 10-year mission simulation
- Detectors on circular orbit @ 1AU; asteroids on elliptical orbits (not N -body)
- Test mass accelerations from asteroids \longrightarrow “noise” power spectrum (details) \longrightarrow strain sensitivity.



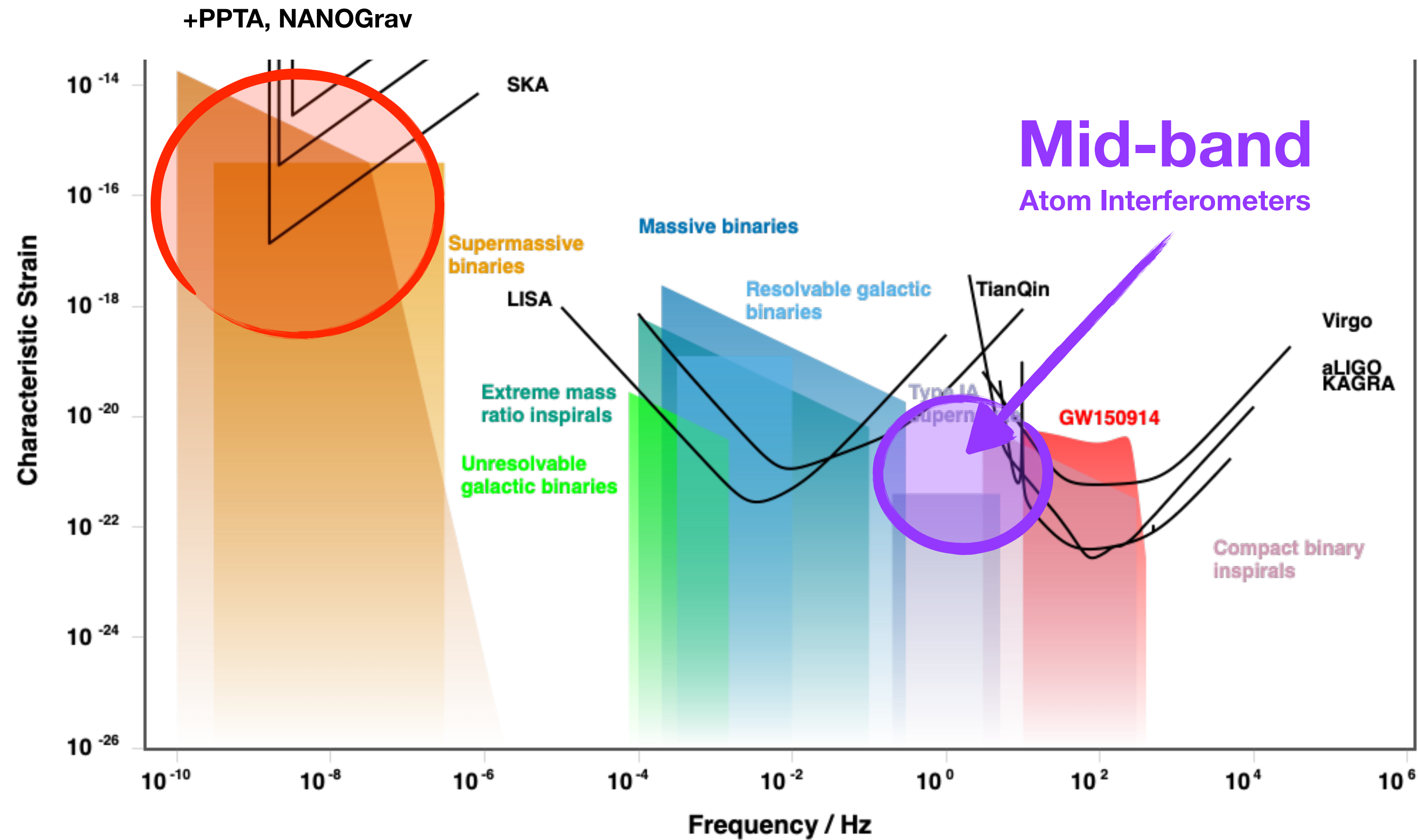
What can we do?

- Problematic for **ANY** local-test mass-based GW detector with Inner Solar System baselines, up to frequencies $\sim (\text{few}) \times 10^{-7} \text{ Hz}$
- Removing ~ 50 heavy distant objects does not change this conclusion
- At higher frequency, noise drops off. But only $\sim 1/6$ of objects in database used in simulation: missing diameters for smaller, closer passing objects...

Distant Proof Masses



Distant Proof Masses

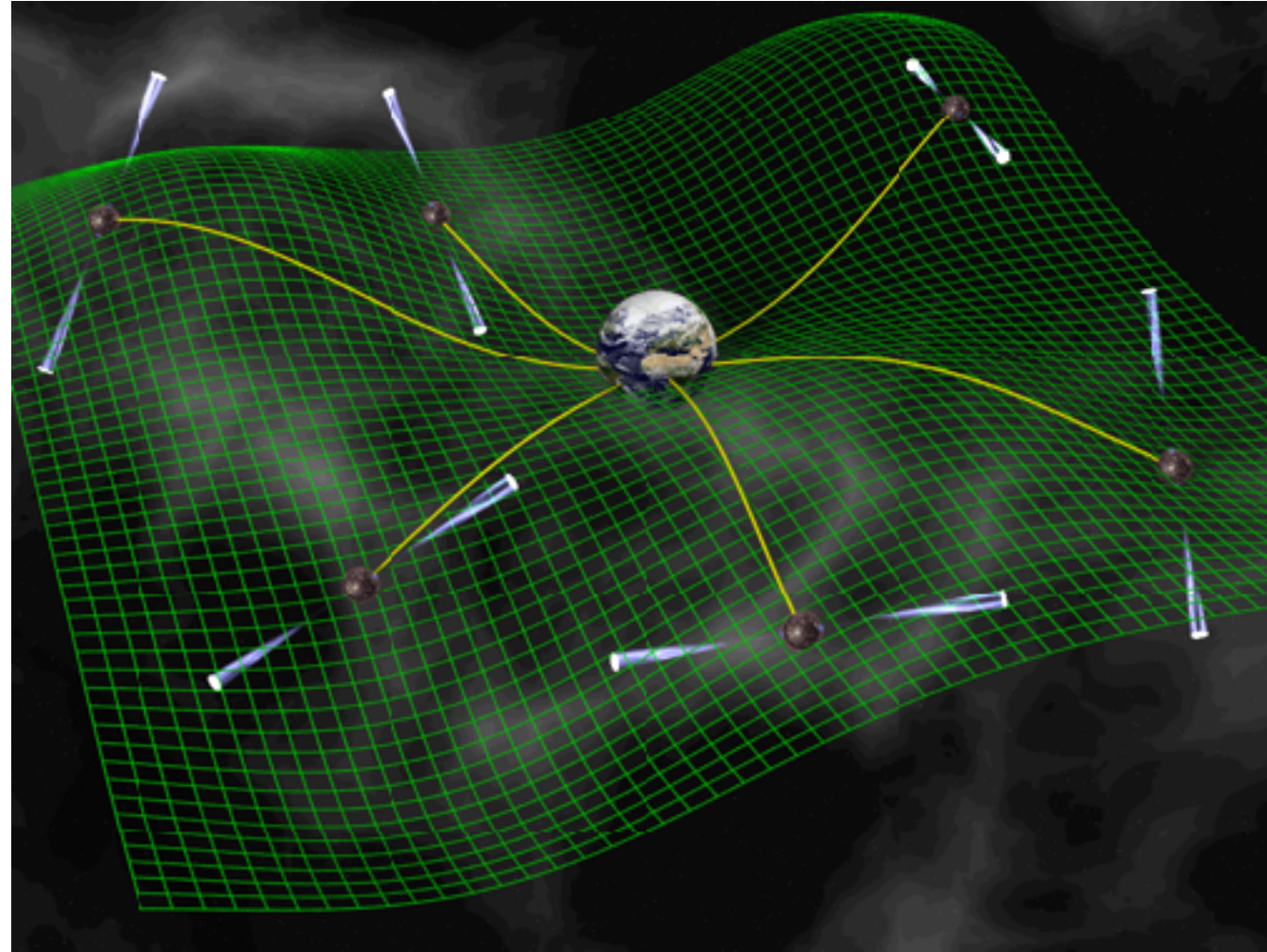


$$\delta x \sim hL \implies \delta x \sim h\lambda_{GW}$$

Drastic Signal Boost - without increasing local noise

PTAs limited by shot noise

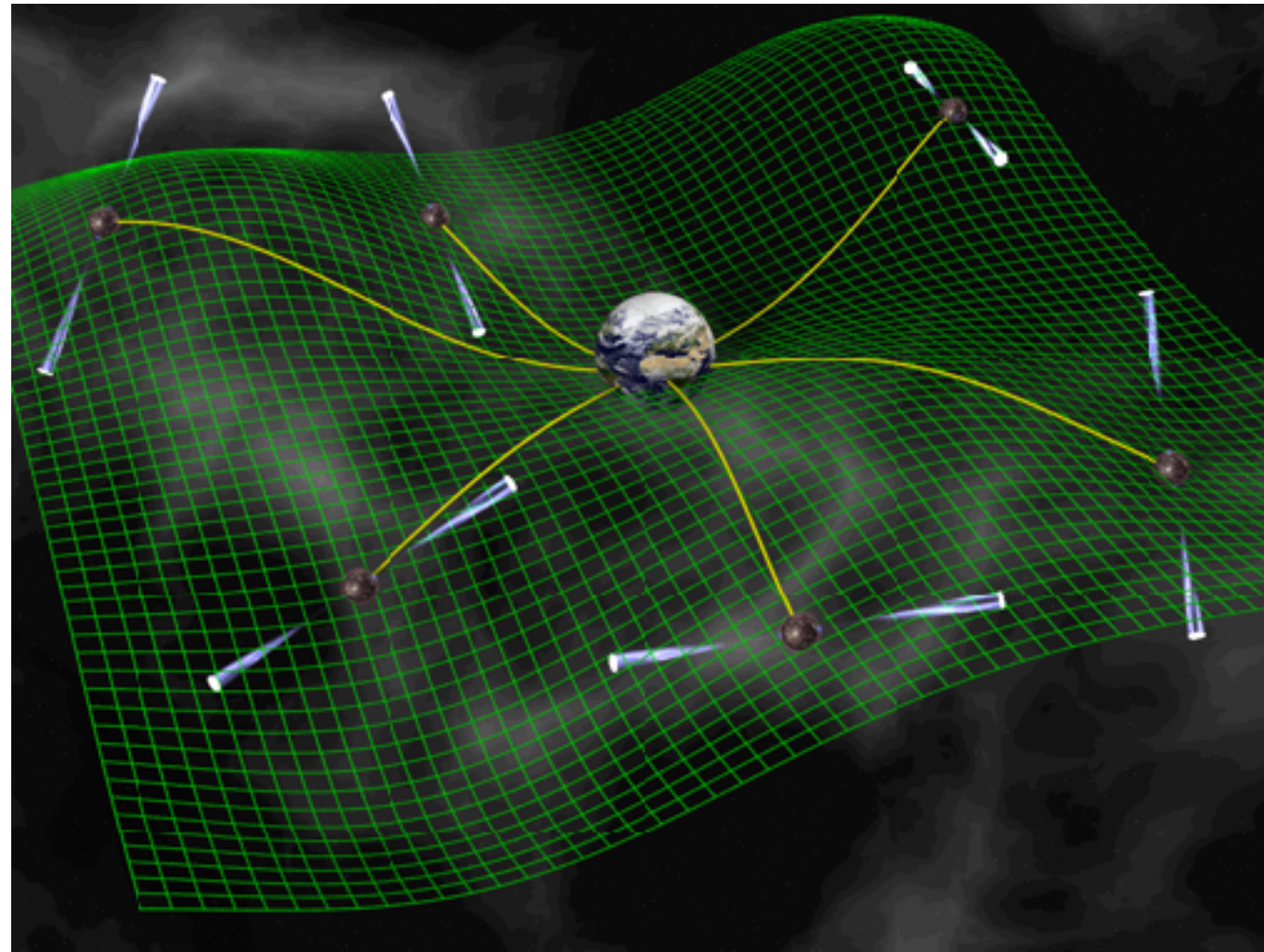
Distant Proof Masses



Pulsar Timing Arrays: Detect gravitational waves by looking for modulation in the arrival times of pulsar signals

Works because pulsars are known to be stable clocks

Distant Proof Masses

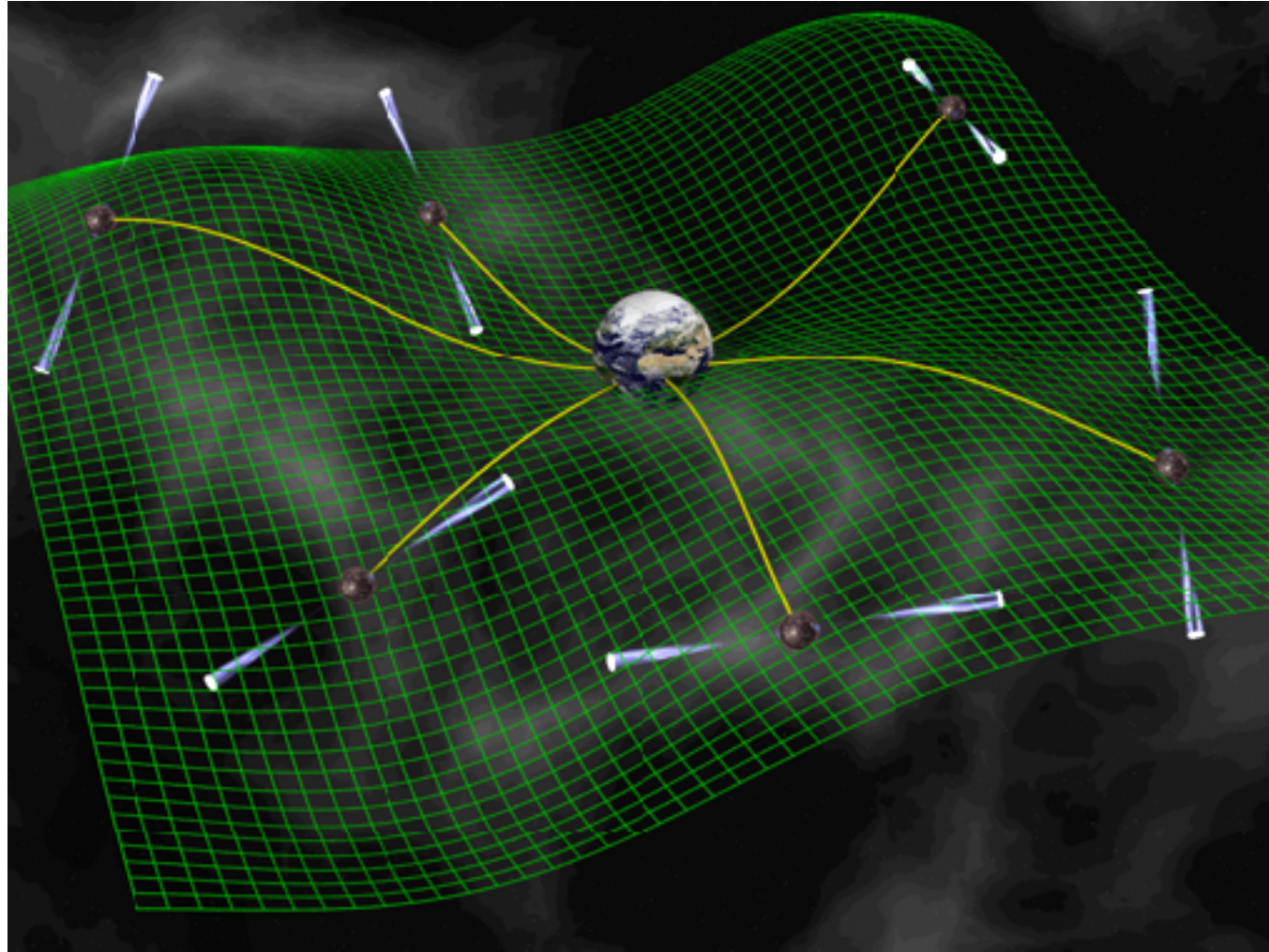


Pulsar Timing Arrays: Detect gravitational waves by looking for modulation in the arrival times of pulsar signals

Works because pulsars are known to be stable clocks

What are other stable aspects of distant astrophysical objects?

Distant Proof Masses



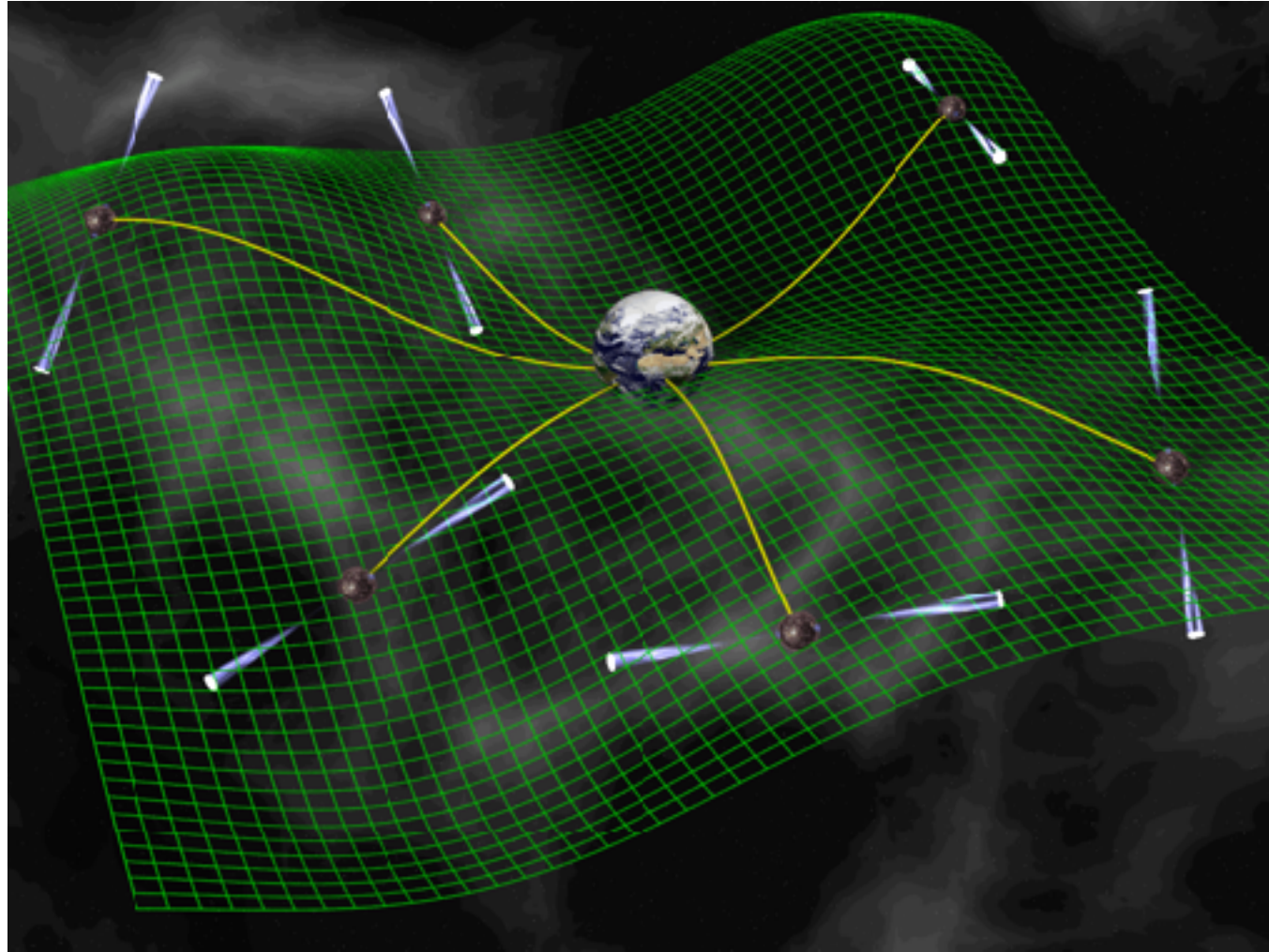
Pulsar Timing Arrays: Detect gravitational waves by looking for modulation in the arrival times of pulsar signals

Works because pulsars are known to be stable clocks

What are other stable aspects of distant astrophysical objects?

Center of mass positions of stars?

Distant Proof Masses



Pulsar Timing Arrays: Detect gravitational waves by looking for modulation in the arrival times of pulsar signals

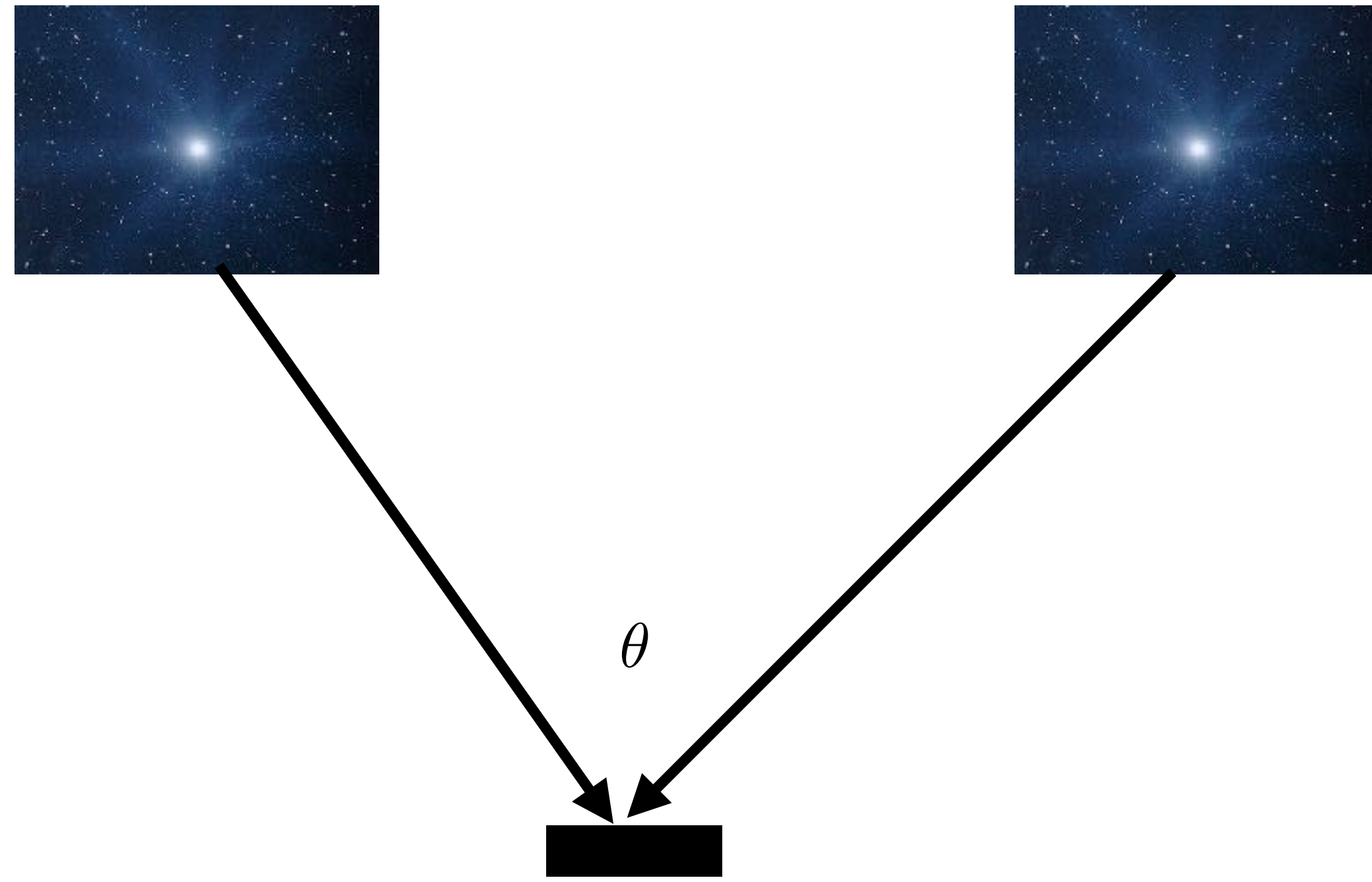
Works because pulsars are known to be stable clocks

What are other stable aspects of distant astrophysical objects?

Center of mass positions of stars?

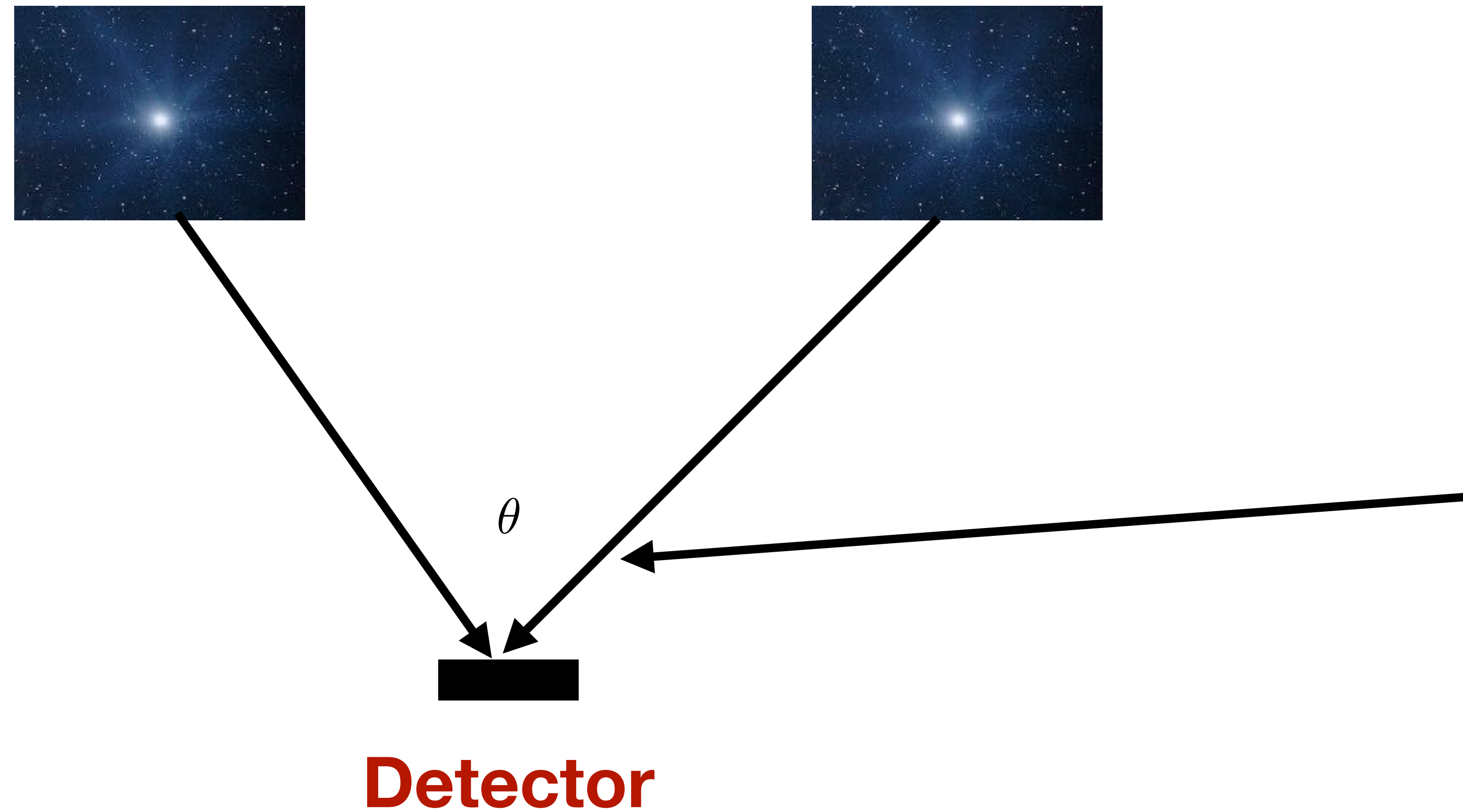
Measure center of mass position via angle measurement?

Astrometry



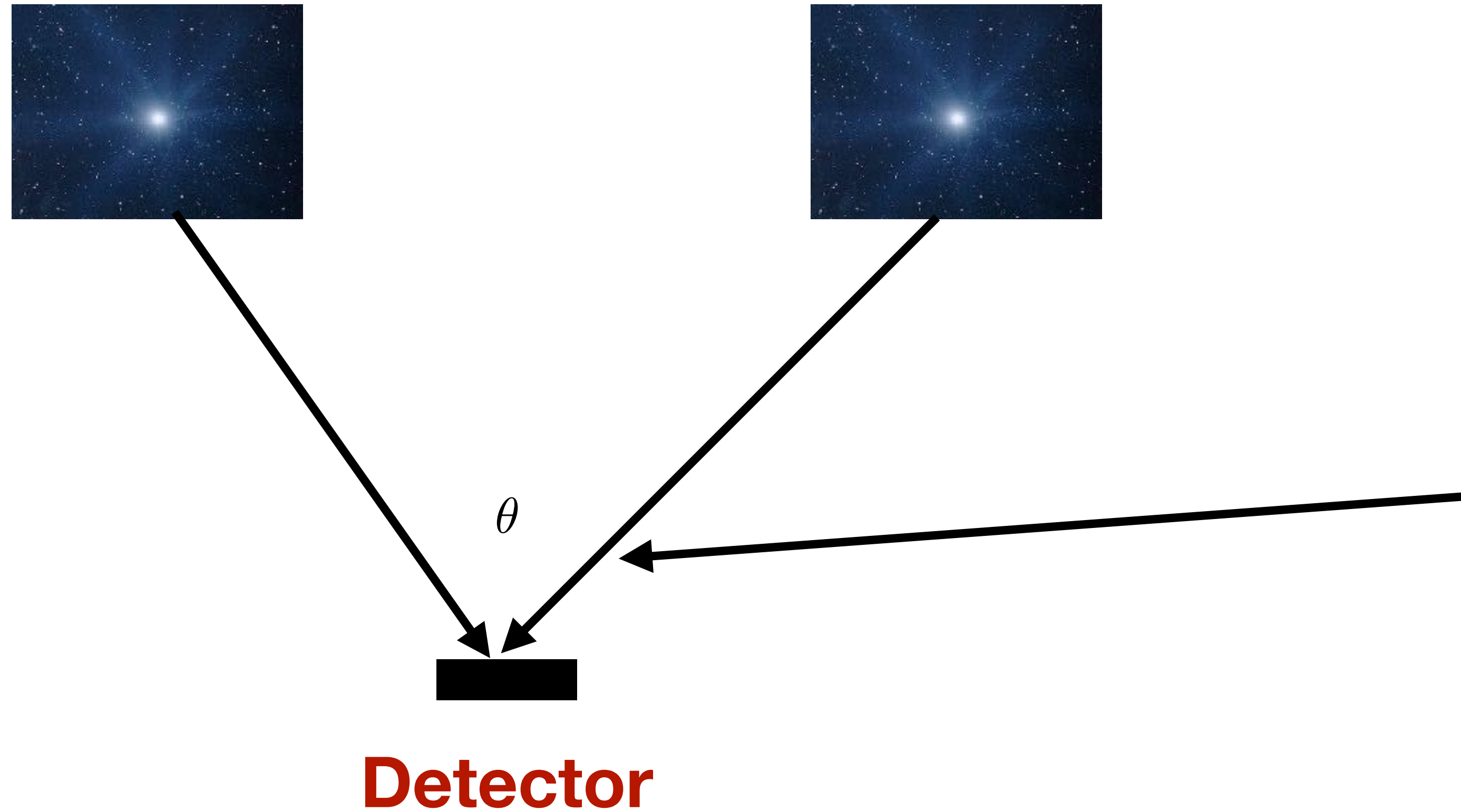
Detector

Astrometry



Gravitational wave bends the path taken by light near the detector

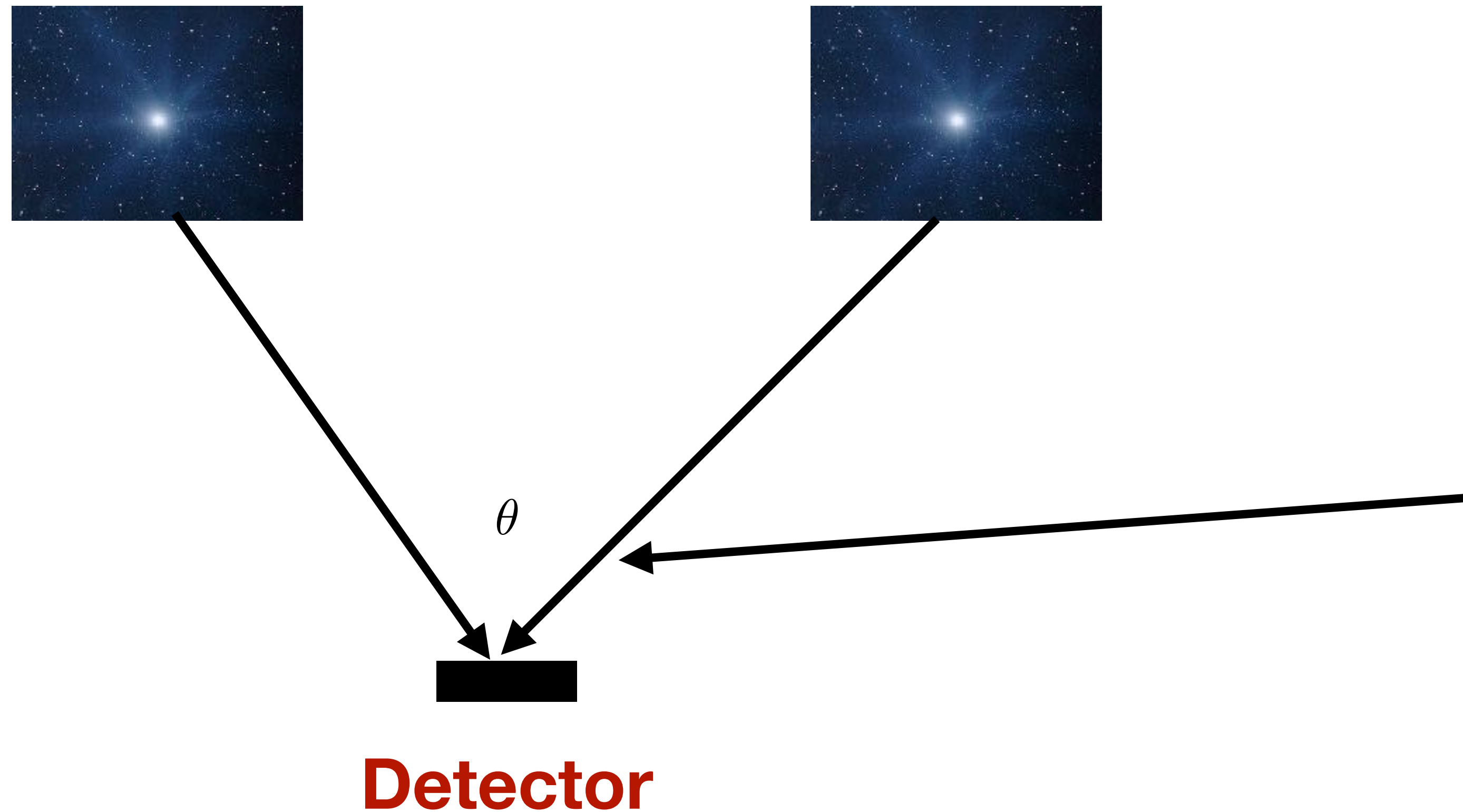
Astrometry



Gravitational wave bends the path taken by light near the detector

$$\theta \rightarrow \theta + \delta\theta \approx \theta + h$$

Astrometry

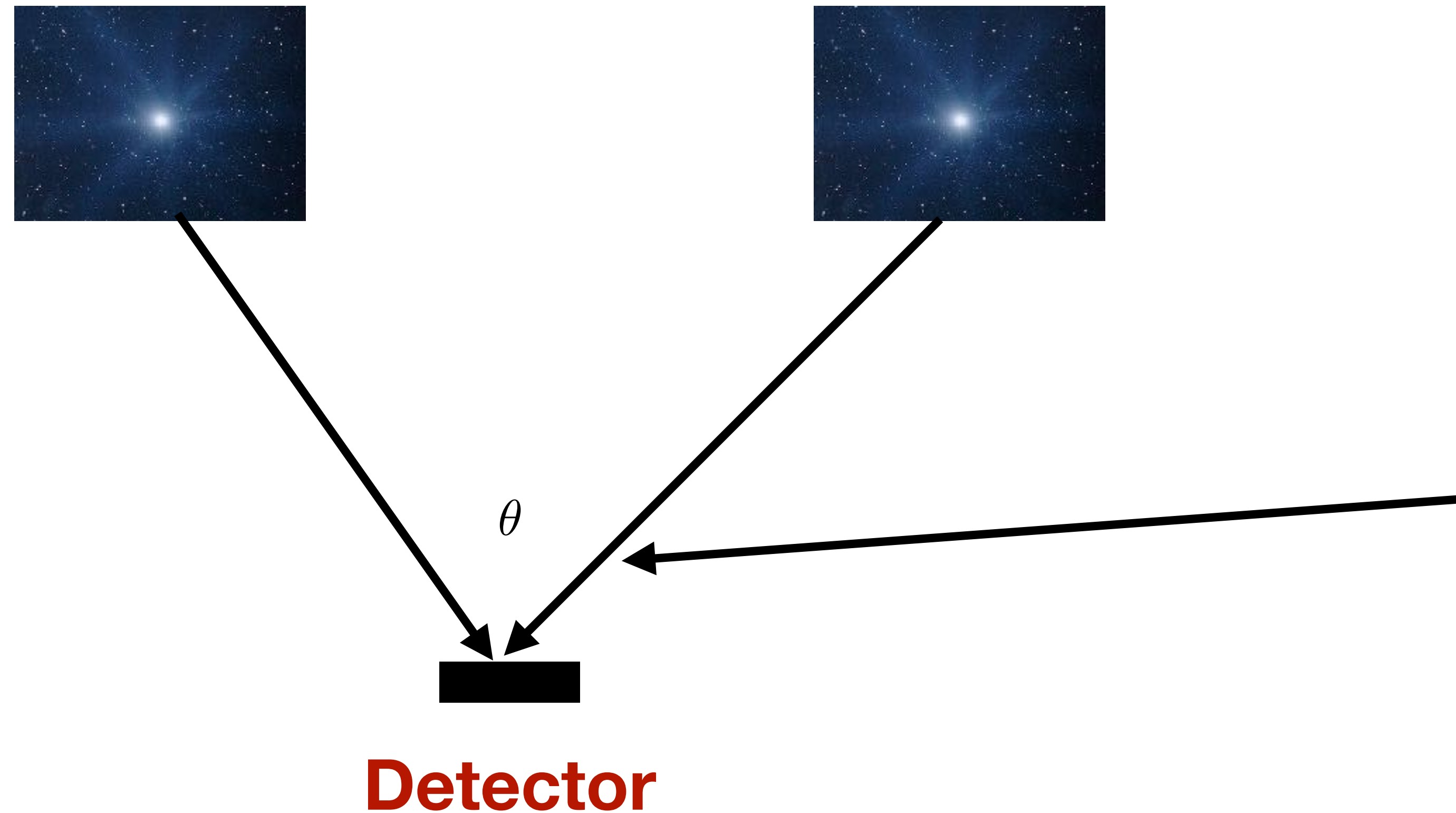


Gravitational wave bends the path taken by light near the detector

$$\theta \rightarrow \theta + \delta\theta \approx \theta + h$$

For interesting sources, need to measure $h \sim 10^{-15} - 10^{-16}$

Astrometry



Gravitational wave bends the path taken by light near the detector

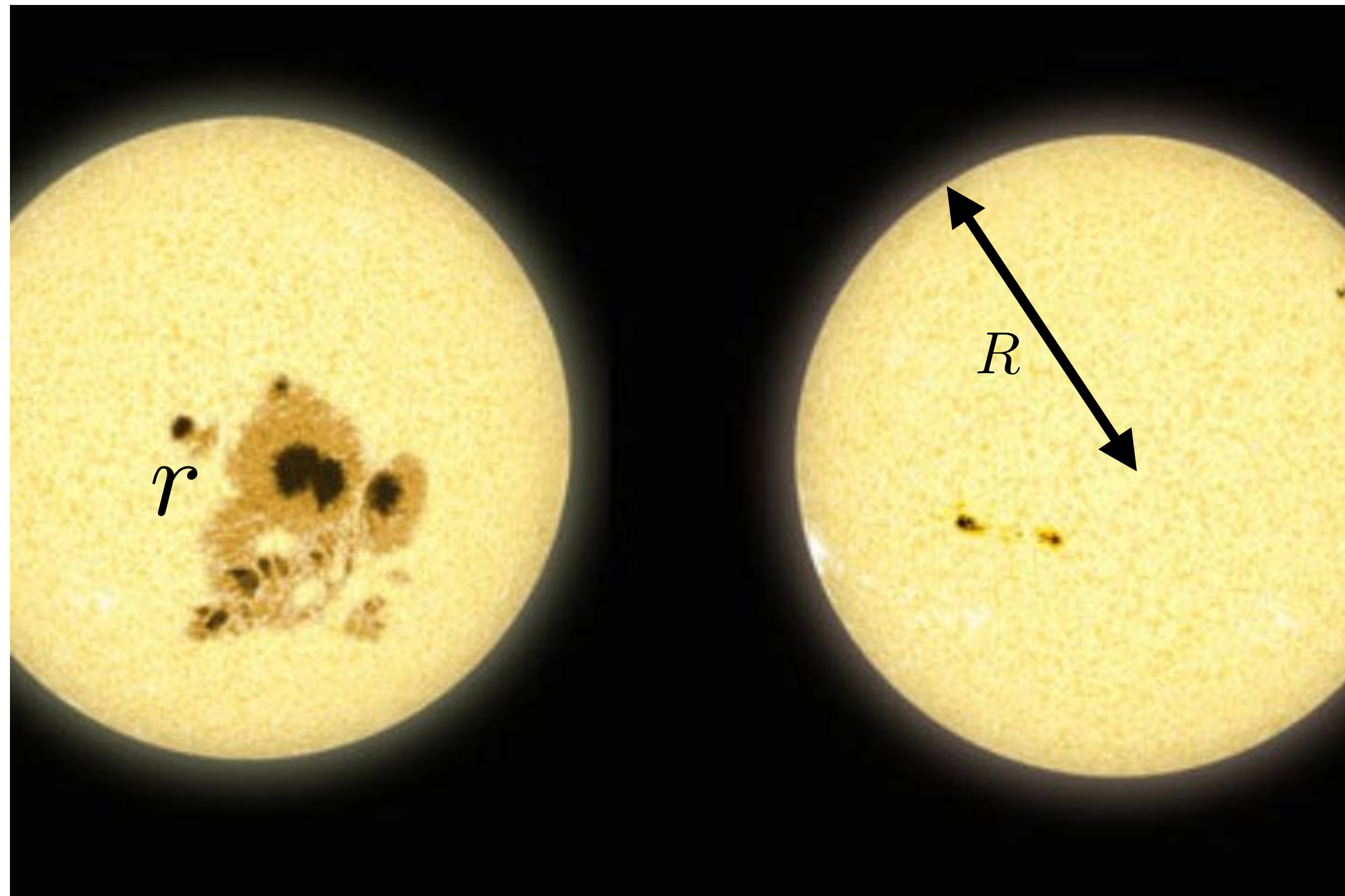
$$\theta \rightarrow \theta + \delta\theta \approx \theta + h$$

For interesting sources, need to measure $h \sim 10^{-15} - 10^{-16}$

How stable? How do we measure?

Stability

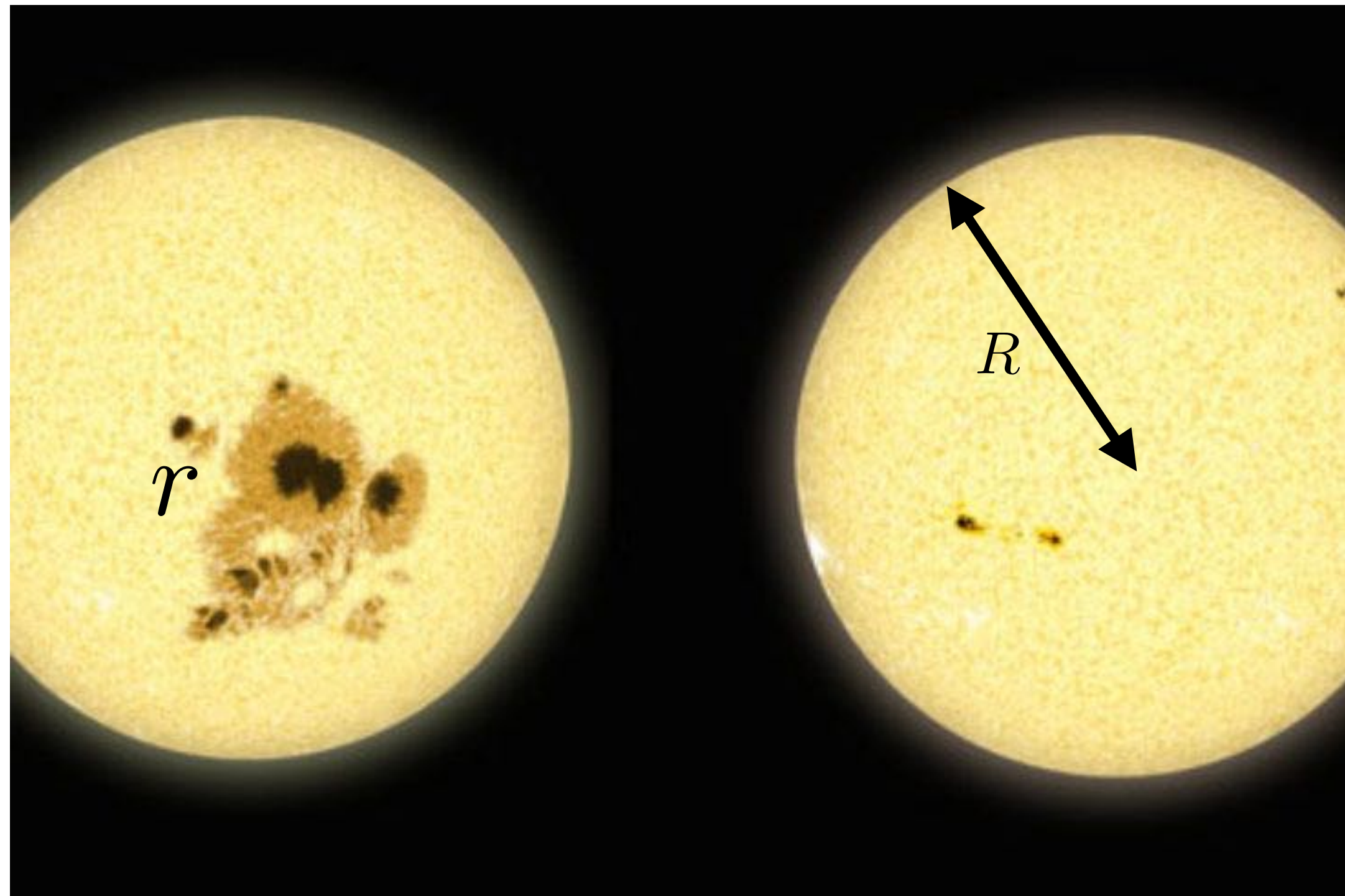
**Measuring position via light from object
I.e. photometric center instead of center of mass**



**Motion of star spot leads to wobble of
photometric center**

Stability

**Measuring position via light from object
i.e. photometric center instead of center of mass**



**Motion of star spot leads to wobble of
photometric center**

$$\delta\theta \sim \left(\frac{r}{R}\right)^2 \frac{R}{L}$$

r : spot size

R : stellar radius

L : distance to star

**Smaller $R \Rightarrow$ Larger spot size
Longer $L \Rightarrow$ Larger spot size**

Stability

$$\delta\theta \sim \left(\frac{r}{R}\right)^2 \frac{R}{L}$$

White Dwarfs?

Stability

$$\delta\theta \sim \left(\frac{r}{R}\right)^2 \frac{R}{L}$$

White Dwarfs?

Known non-magnetic white dwarfs with intensity stability better than 0.01 percent (Kepler/K2) (Landolt Standards)

Stability

$$\delta\theta \sim \left(\frac{r}{R}\right)^2 \frac{R}{L}$$

White Dwarfs?

**Known non-magnetic white dwarfs with intensity stability better than 0.01 percent (Kepler/K2)
(Landolt Standards)**

For $h \sim 10^{-16}$ @ 0.01 percent instability, $L \sim \text{kpc}$

Stability

$$\delta\theta \sim \left(\frac{r}{R}\right)^2 \frac{R}{L}$$

White Dwarfs?

Known non-magnetic white dwarfs with intensity stability better than 0.01 percent (Kepler/K2) (Landolt Standards)

For $h \sim 10^{-16}$ @ 0.01 percent instability, $L \sim \text{kpc}$

Planets?

> 10 nHz, Red Giant phase vaporizes minor bodies

Stability

$$\delta\theta \sim \left(\frac{r}{R}\right)^2 \frac{R}{L}$$

White Dwarfs?

Known non-magnetic white dwarfs with intensity stability better than 0.01 percent (Kepler/K2) (Landolt Standards)

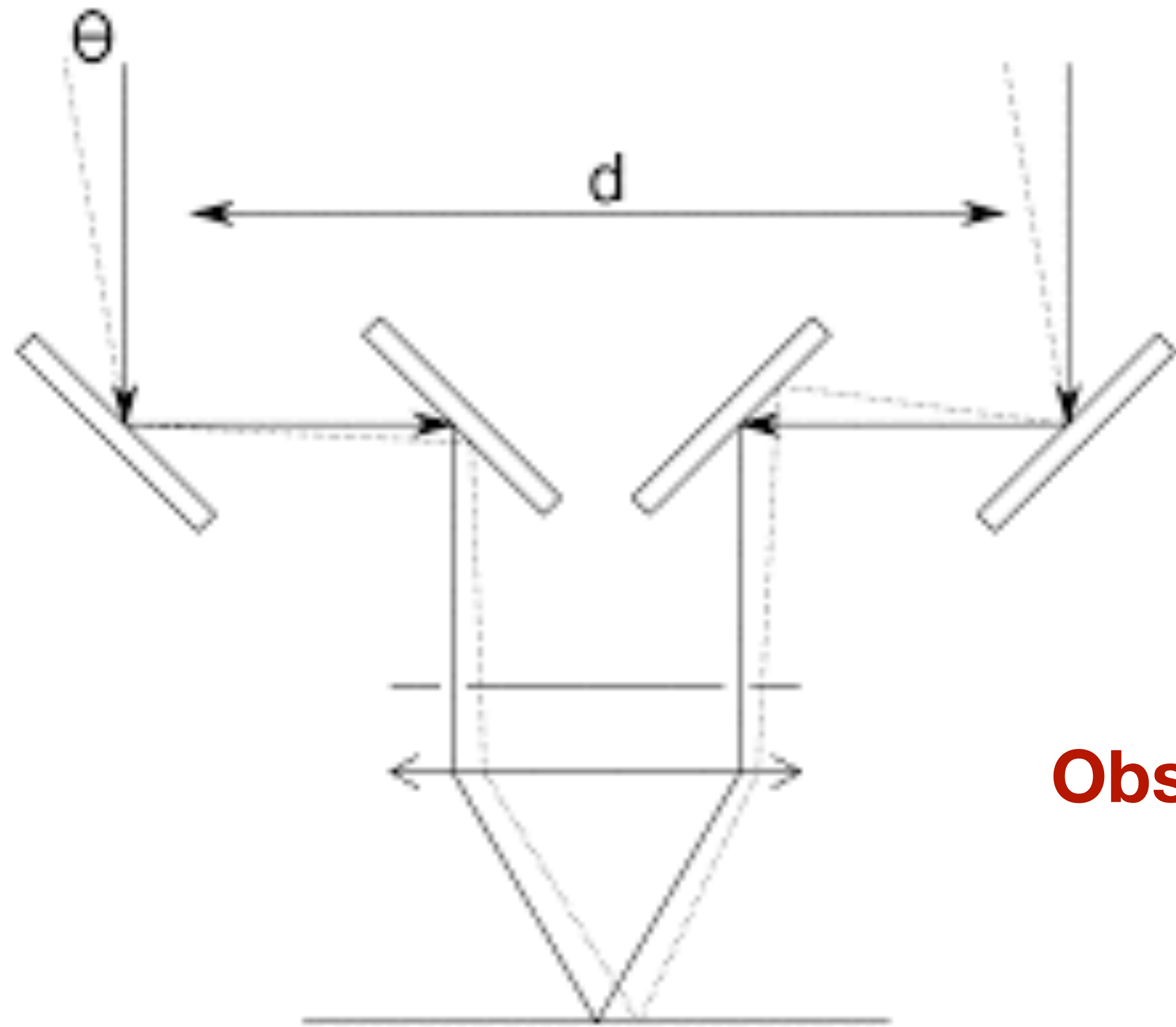
For $h \sim 10^{-16}$ @ 0.01 percent instability, $L \sim \text{kpc}$

Planets?

> 10 nHz, Red Giant phase vaporizes minor bodies

Astrometry of white dwarfs?

Stellar Imager



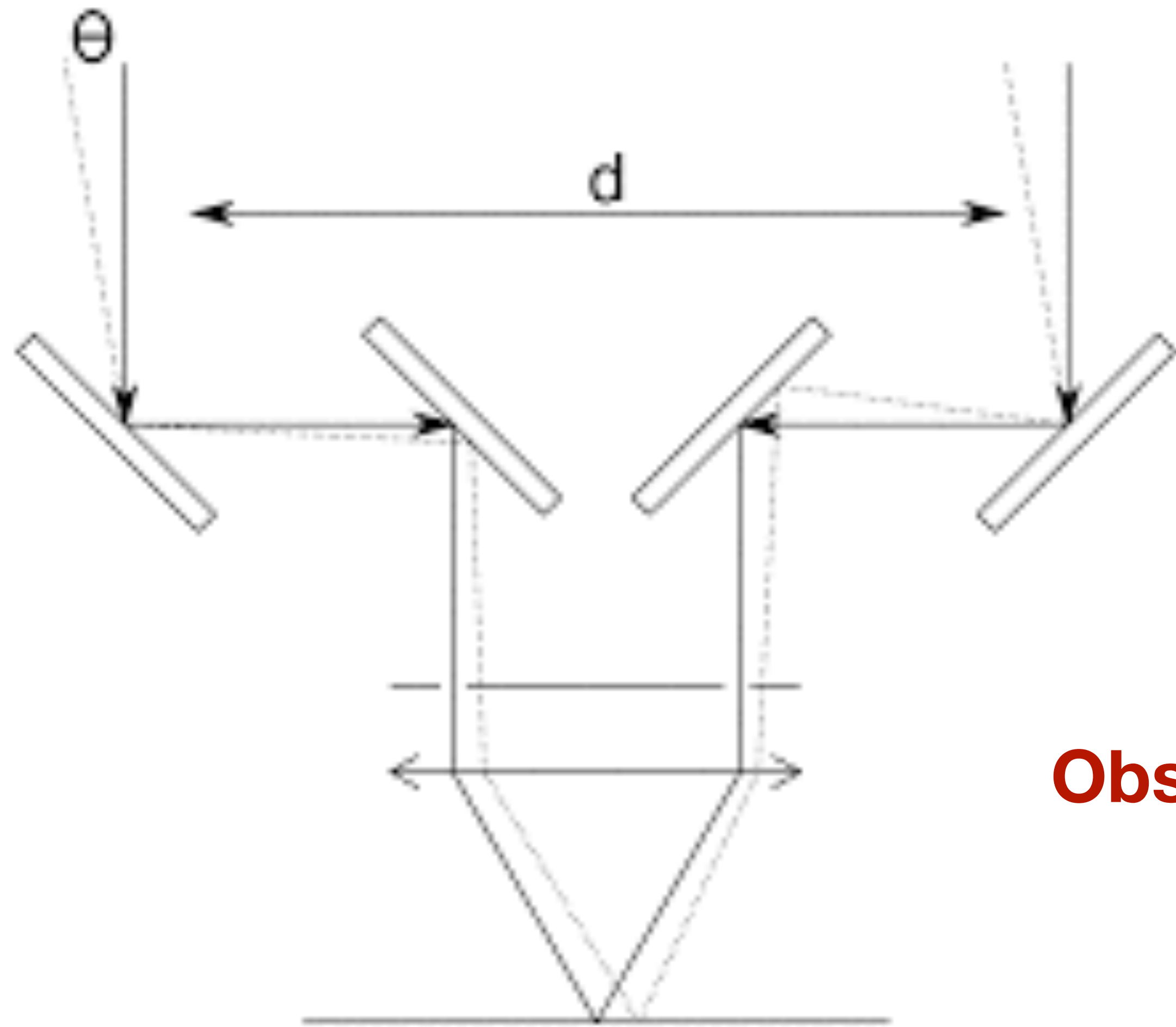
$$\delta\theta \sim \frac{\lambda}{d} \frac{1}{\sqrt{N_\gamma}}$$

$$\lambda \sim 120 \text{ nm}, d \sim 1 \text{ km} \implies h \sim 10^{-16}$$

(For white dwarf @ kpc)

Observe small number of stars for long periods

Stellar Imager



$$\delta\theta \sim \frac{\lambda}{d} \frac{1}{\sqrt{N_\gamma}}$$

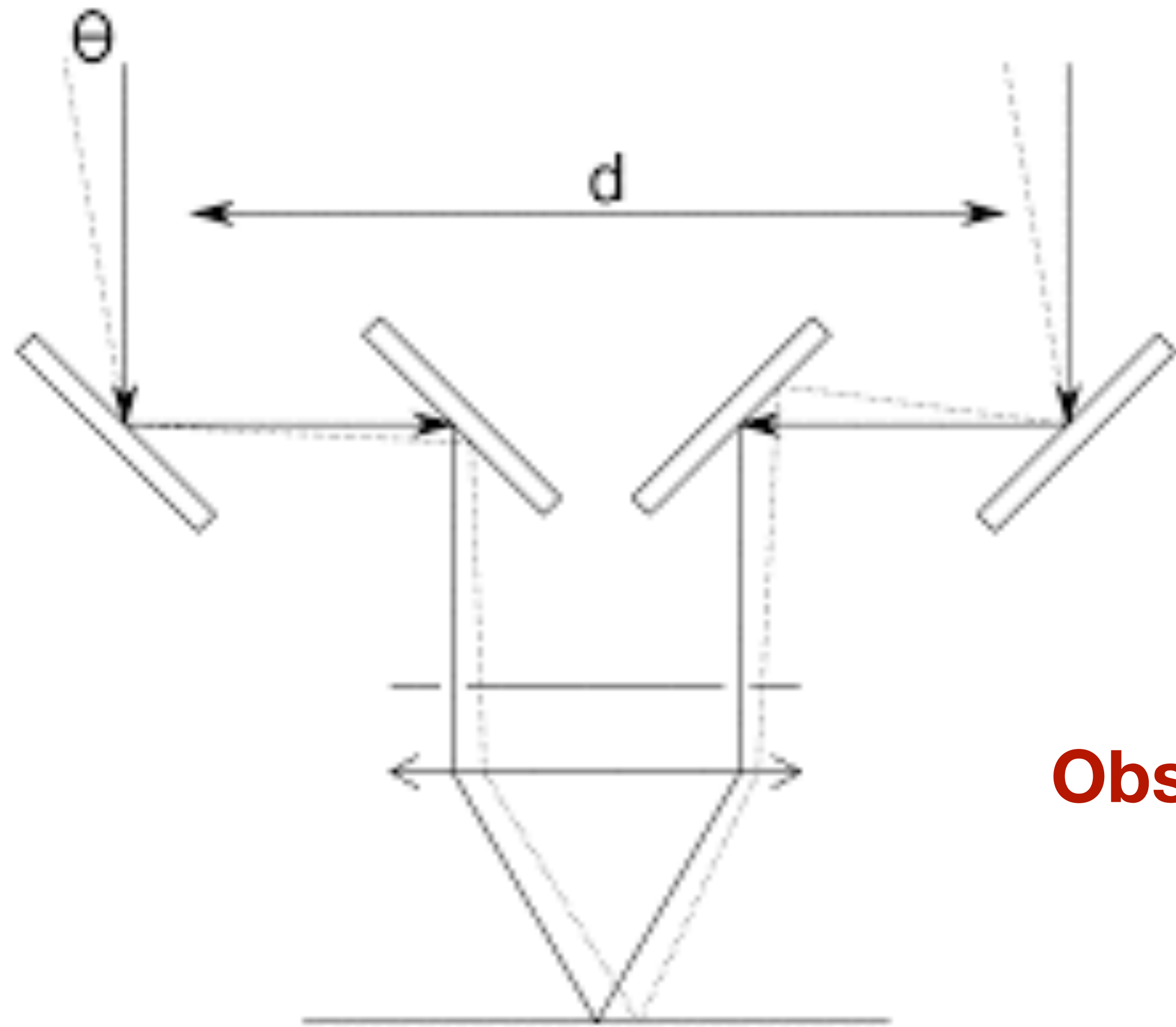
$$\lambda \sim 120 \text{ nm}, d \sim 1 \text{ km} \implies h \sim 10^{-16}$$

(For white dwarf @ kpc)

Observe small number of stars for long periods

Better measurements of intensity fluctuations could make numbers better

Stellar Imager



$$\delta\theta \sim \frac{\lambda}{d} \frac{1}{\sqrt{N_\gamma}}$$

$$\lambda \sim 120 \text{ nm}, d \sim 1 \text{ km} \implies h \sim 10^{-16}$$

(For white dwarf @ kpc)

Observe small number of stars for long periods

Better measurements of intensity fluctuations could make numbers better

Adds to Science Case of such missions!

Conclusions

The Gravitational Wave Spectrum

Gravitational wave astronomy is here to stay!

Strong science case to probe entire gravitational wave spectrum

Strong case for using asteroids as test masses in the μHz frequency range. Motivates asteroid seismology measurements and development of space-qualified atomic clocks

Gravity gradient noise from asteroids below μHz motivates astrometry of photometrically stable non-magnetic white dwarfs. Adds to the science case of interferometry missions like Stellar Imager