Detecting High-Frequency Gravitational Waves with Microwave Cavities

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based on: *arxiv:2112.11465*

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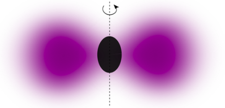
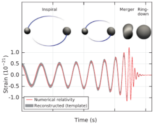
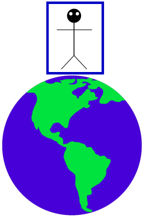
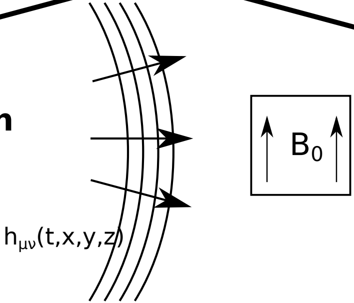
High Frequency GW Sources

Proper detector frame



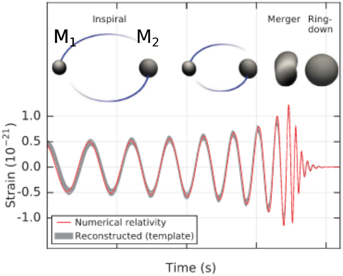
GW detection with cavities



[[1602.03837, kipac.stanford.edu]]

Sources

Mergers of sub-solar mass objects



[1602.03837]

innermost stable circular orbit (ISCO)

$$r_{\rm ISCO} = 0.02\,{\rm m}\,\frac{M_b}{10^{-6}\,M_\odot}$$

$$f_{\rm ISCO} = 1.1\,{\rm GHz}\left(\frac{10^{-6}\,M_\odot}{M_b}\right)$$

$$\omega_g \simeq 14\text{ GHz}\times\frac{10^{-6}M_\odot}{M_b}\left(\frac{r_{\rm ISCO}}{r_b}\right)^{\frac{3}{2}}$$

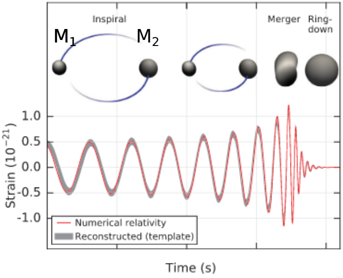
$M_1 = M_b = M_2$

$$\mathcal{N}_{\rm cyc} = 10^{-3}\left(\frac{10^5}{Q}\right)\left(\frac{10^{-6}M_\odot}{M_b}\right)^{\frac{5}{3}}\left(\frac{1\,{\rm GHz}}{\omega_g}\right)^{\frac{5}{3}}$$

with $Q = \frac{f}{\Delta f}$.

$$\mathcal{N}_{\rm cyc} \geq Q \Rightarrow M_b \leq 10^{-11}\,M_\odot\left(\frac{10^5}{Q}\right)^{\frac{6}{5}}\left(\frac{1\,{\rm GHz}}{\omega_g}\right)$$

Mergers of sub-solar mass objects



[1602.03837]

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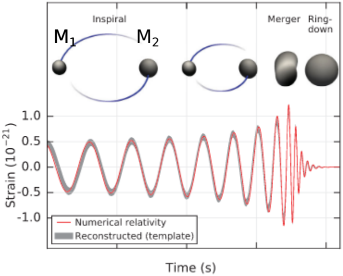
$$\omega_g \simeq 14\text{ GHz} \times \frac{10^{-6}M_\odot}{M_b}\left(\frac{r_{\mathrm{ISCO}}}{r_b}\right)^{\frac{3}{2}}$$

$M_1 = M_b = M_2$

Sub solar mass objects:

- primordial black holes (PBHs) [Hawking 71]
- boson and fermion stars [Palenzuela 07,Giudice 16,Palenzuela 17,Helfer 18]
- gravitino stars [Narain 06] and gravistars [Mazur 04]
- dark matter blobs [Diamond 21]

Mergers of sub-solar mass objects



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$M_1 = M_b = M_2$

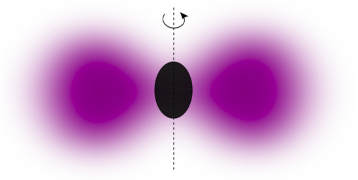
expected strain

$$h_0 \sim 10^{-29} \times \left(\frac{1\text{ pc}}{D}\right)\left(\frac{M_b}{10^{-11}\,M_\odot}\right)^{5/3}\left(\frac{\omega_g}{1\text{ GHz}}\right)^{2/3}$$

D is distance to Binary merger. If PBHs are 100% DM

$$D = 7.2 \times 10^{-4}\,{\rm pc}\left(\frac{M_b}{10^{-11}\,M_\odot}\right)^{\frac{1}{3}} \quad \Rightarrow \quad \text{best case scenario} \;\Rightarrow h_0 \sim 10^{-26}$$

Boson clouds from PBH superradiance



[kipac.stanford.edu]

- Annihilation of bosons

$$m_a \sim \mu\text{eV} \times (10^{-4}\, M_\odot / M_{\text{PBH}})$$
$$\omega_g = 2m_a \sim \text{GHz} \times (m_a/\mu\text{eV})$$

- GW waveform is monochromatic and coherent over very long timescales

expected strain [Arvanitaki et al.'12]

$$h_0 \sim 10^{-27} \times \left(\frac{10\,\text{kpc}}{D}\right)\left(\frac{M_{\text{PBH}}}{10^{-4}\, M_\odot}\right)$$

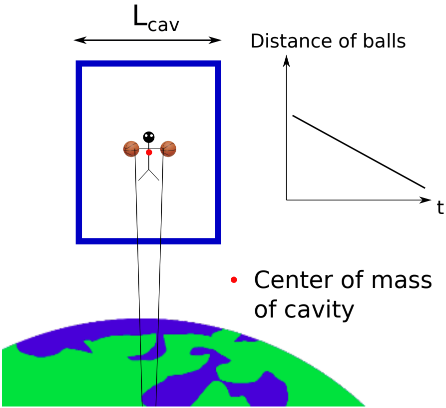
If $10^{-4}\, M_\odot$ PBHs are 1% of DM:

$$D \sim 1\,\text{pc} \quad \Rightarrow \quad \text{best case scenario} \; \Rightarrow h_0 \sim 10^{-23}$$

Proper detector frame

(Fermi Normal coordinates)

Proper detector frame

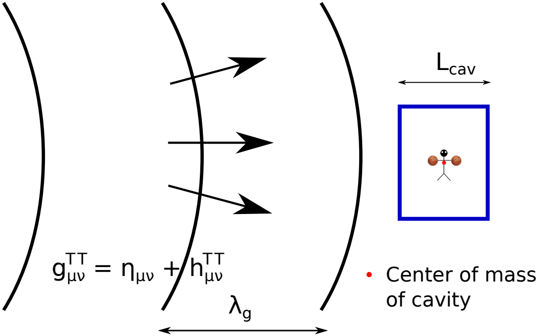


- Cavity freely falling towards Earth
- Coordinate system attached to the center of mass is proper detector frame
- Metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}\left(\frac{L_{\text{cav}}}{K}\right)$$

K is scale on which the gravitational field varies.

Proper detector frame



$$h_{\mu\nu}^{\mathrm{TT}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i\omega_g(t-z)}$$

TT: transverse traceless

Metric in proper detector frame:

$$g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}\left(\frac{L_{\mathrm{cav}}}{\lambda_{\mathrm{g}}}\right)$$

λ_{g} wavelength of GW

Proper detector frame

Resonant excitation of cavity:

$$\lambda_g \simeq L_{\text{cav}}$$

$$g_{00} = \eta_{00} + h_{00} = -1 - 2\sum_{r=0}^{\infty} \frac{r+3}{(r+3)!} R_{0n0n,k_1,\cdots k_r} x^m x^n x^{k_1} \cdots x^{k_r}$$

(Similar eq. for g_{0i} and g_{ij})

[Marzlin 94, Rakhmanov 14]

- Riemann tensor invariant under infinitesimal coordinate transformations. Evaluate therefore in TT-frame and plug in series expansion above.
- GW propagating in z-direction:

$$R_{ijkl} \sim e^{i\omega_g(t-z)}$$

$$h_{00} = -\omega_g^2\, h_{ab}^{\text{TT}}\, x^a\, x^b \left[-\frac{i}{\omega_g z} + \frac{1 - e^{-i\omega_g z}}{(\omega_g z)^2} \right]$$

Proper detector frame

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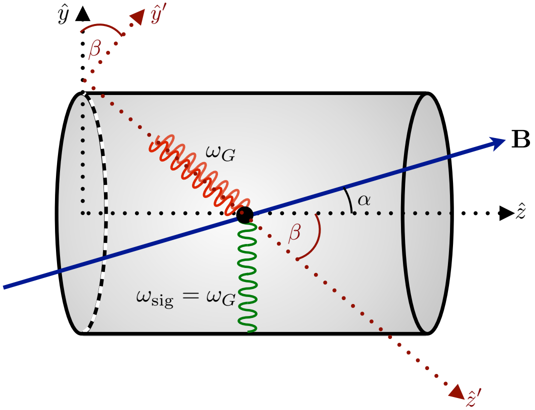
[long wavelength expansion $x^i\omega_g \ll 1$]

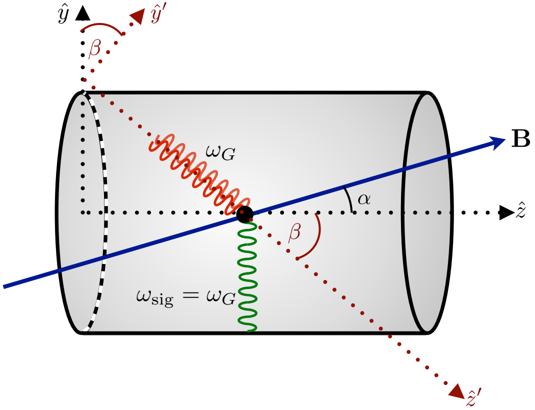
GW detection with cavities



Signals can arise from

Mixing GW with photons

Movement of walls (free charges) in static external *B*-field induce a current



Signals can arise from

Mixing GW with photons

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Maxwell equations on curved spacetime

- After linearization first order eq. are:

$$\begin{aligned} \nabla \cdot \boldsymbol{E} &= \rho_{\text{eff}} + \rho, \\ \nabla \times \boldsymbol{B} - \partial_t \boldsymbol{E} &= \boldsymbol{j}_{\text{eff}} + \boldsymbol{j} \ , \end{aligned}$$

$$j^{\mu}_{\text{eff}} \equiv \partial_{\nu} \left(\frac{1}{2} h \, F^{\mu\nu} + h^{\nu}{}_{\alpha} \, F^{\alpha\mu} - h^{\mu}{}_{\alpha} \, F^{\alpha\nu} \right)$$

- Homogeneous Maxwell eq. are not modified.

Mode decomposition

$$\nabla \times \nabla \times \boldsymbol{E} + \partial_t^2 \boldsymbol{E} = -\partial_t \boldsymbol{j}_{\text{eff}} - \partial_t \boldsymbol{j}$$

Cavity mode decomposition:

$$\boldsymbol{E}(\boldsymbol{x}, t) = \sum_n e_n(t)\, \boldsymbol{E}_n(\boldsymbol{x}) \,,$$

$$\boldsymbol{E}_{\text{sig}}(\boldsymbol{x}, t) = -\frac{\int_{V_{\text{cav}}} d^3\boldsymbol{x}' \; \boldsymbol{E}_n^* \cdot \boldsymbol{j}_{\text{eff}}}{\int_{V_{\text{cav}}} d^3\boldsymbol{x}' \; |\boldsymbol{E}_n|^2} \; \frac{Q}{\omega_g} \; \boldsymbol{E}_n(\boldsymbol{x})\, e^{i\omega_g t}$$

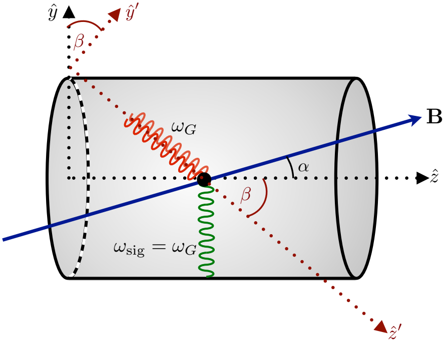
Signal power

$$P_{\rm sig} = \frac{1}{2}\, Q\, \omega_g^3\, V_{\rm cav}^{5/3}\, (\eta_n\, h_0\, B_0)^2$$

$$\eta_n \equiv \frac{\left| \int_{V_{\rm cav}} d^3\boldsymbol{x}\ \boldsymbol{E}_n^* \cdot \hat{\boldsymbol{j}}_{+,\times} \right|}{V_{\rm cav}^{1/2} \left(\int_{V_{\rm cav}} d^3\boldsymbol{x}\ |\boldsymbol{E}_n|^2 \right)^{1/2}}$$

$$\boldsymbol{j}_{\rm eff}(\boldsymbol{x}) \equiv B_0\, \omega_g^2\, V_{\rm cav}^{1/3}\, (h_+\, \hat{\boldsymbol{j}}_+(\boldsymbol{x}) + h_\times\, \hat{\boldsymbol{j}}_\times(\boldsymbol{x}))$$

Sensitivity estimate

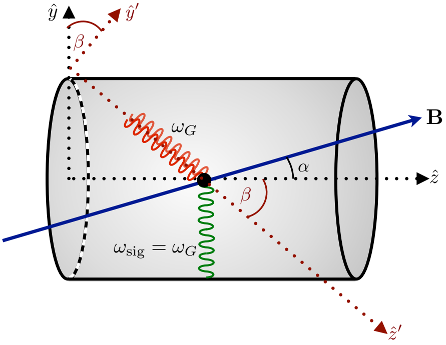


$$\text{SNR} \simeq \frac{P_{\rm sig}}{T_{\rm sys}} \sqrt{\frac{t_{\rm int}}{\Delta\nu}}$$

$$h_0 \gtrsim 3 \times 10^{-22} \times \left(\frac{1\ \text{GHz}}{\omega_g/2\pi}\right)^{3/2} \left(\frac{0.1}{\eta_n}\right) \left(\frac{8\ \text{T}}{B_0}\right) \left(\frac{0.1\ \text{m}^3}{V_{\rm cav}}\right)^{5/6} \times$$
$$\times \left(\frac{10^5}{Q}\right)^{1/2} \left(\frac{T_{\rm sys}}{1\ \text{K}}\right)^{1/2} \left(\frac{\Delta\nu}{10\ \text{kHz}}\right)^{1/4} \left(\frac{1\ \text{min}}{t_{\rm int}}\right)^{1/4}$$

Bandwidth $\Delta\nu = \frac{\omega_g}{2\pi Q}$.

Sensitivity estimate



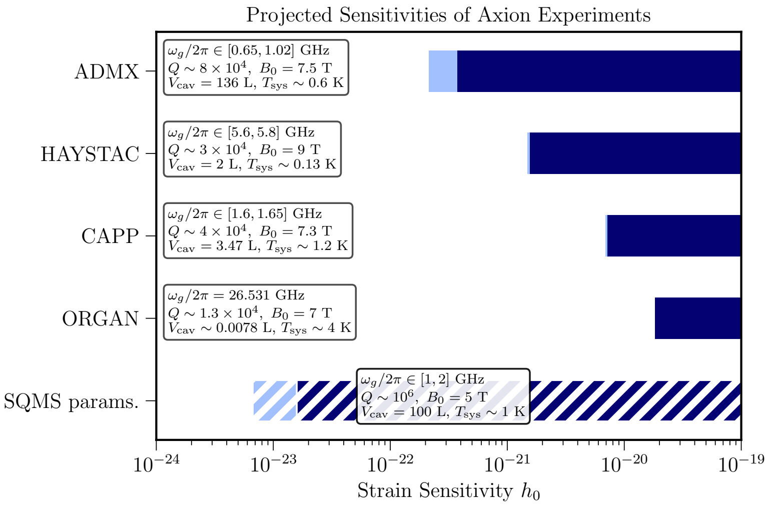
$$\text{SNR} \simeq \frac{P_{\text{sig}}}{T_{\text{sys}}} \sqrt{\frac{t_{\text{int}}}{\Delta\nu}}$$

$$h_0 \gtrsim 1 \times 10^{-23} \times \left(\frac{1\ \text{GHz}}{\omega_g/2\pi}\right)^{3/2} \left(\frac{0.1}{\eta n}\right) \left(\frac{8\ \text{T}}{B_0}\right) \left(\frac{0.1\ \text{m}^3}{V_{\text{cav}}}\right)^{5/6} \left(\frac{10^5}{Q}\right)^{1/2} \left(\frac{T_{\text{sys}}}{1\ \text{K}}\right)^{1/2} \left(\frac{1\ \text{min}}{t_{\text{int}}}\right)^{1/2}$$

Bandwidth $\Delta\nu = \frac{1}{t_{\text{int}}}$

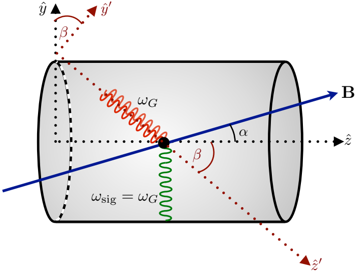
-	Superradiance	Binary mergers
t_{int}	1 min (single scan time)	10^{-4} s (no scan)
best case h_0	10^{-23}	10^{-26}

Sensitivity of existing axion experiments



Existing axion experiments only need to reanalyze their data!

Why the frame matters

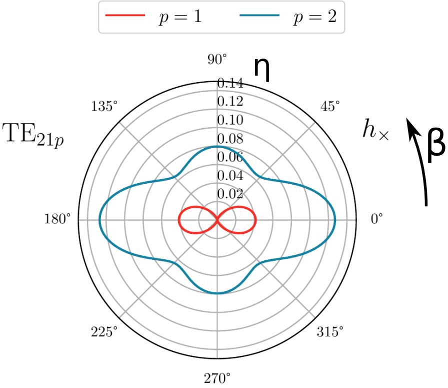


TT frame is not the right frame! Use proper detector frame.

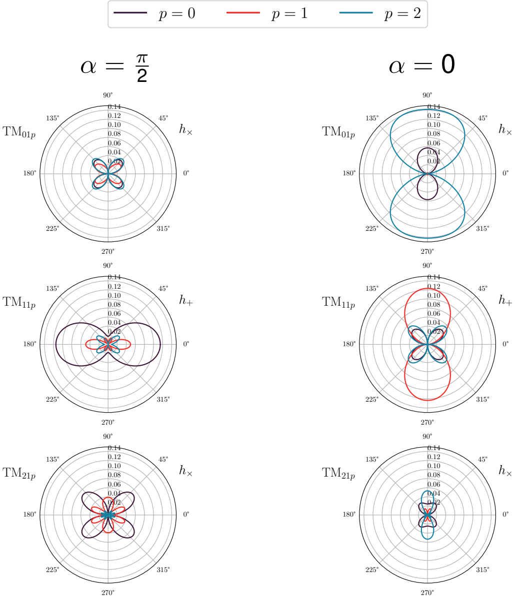
$$\alpha = 0, \beta = 0$$

If we use TT metric $j_{\text{eff}} = 0$. NO SIGNAL!
However this is **WRONG**. Use proper detector metric.

Proper detector frame result ($\alpha = 0$)



Evaluation of coupling coefficient



- Directional sensitivity
- Sensitivity to polarization

Conclusions

- Monochromatic high frequency GWs from superradiance and merging sub solar mass binaries
- Proper detector frame metric resummed to all orders
- Signal calculation in cavities permeated by strong external B-field.
- Existing axion experiments only need to reanalyze data to set limits.

Thank you for your attention