Matching in $pp \rightarrow t\bar{t}W/Z/h + \text{jet}$

SMEFT studies

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Introduction

• In this study, we use the framework of EFT to explore the modeling of new physics effects associated with ttX processes:
  
  - With an EFT approach, new physics is written in terms of higher-dimensional operators composed of SM fields and their derivatives
  
  - Benefit of EFT: A model independent approach that allows us to indirectly probe new physics at energy scales larger than what is directly accessible at the LHC

• Goal of the study:
Explore the impact of including extra radiation in the production of LO MC samples for ttX processes within the dimension-six SM EFT framework

In this talk, we'll discuss why the EFT dependence of LO 0+1j calculations can be different from LO, and why that can be interesting
Motivation for including additional jets

- We would like to understand the impact of additional radiation on the EFT interpretation of the ttX inclusive cross section.
- Contributions from diagrams with additional jets are often not just a small correction, especially when the extra jet allows for new initial states.

\[ q' \]
\[ t \]
\[ q \]
\[ b \]

\[ q \]
\[ q' \]
\[ t \]
\[ b \]

- E.g. single top t channel dominates over s, even though t requires an additional parton (in 4f scheme).

- To study the impact of an additional jet, NLO would be the optimal approach.
- But NLO MC can be challenging to produce and to use, for example some specific challenges when generating NLO with MG include:
  - Requires more CPU time to generate
  - A fraction of the events can be negatively weighted
  - Complications for NLO samples involving EFT operators and processes with electroweak vertices.
Complications involving accounting for higher QED order diagrams with NLO MG

- For EFT samples, MG can account for NLO QCD effects, but cannot account for QED loops.

- Tree-level diagrams with QED order $\geq$ two plus the lowest QED order diagrams are not permitted (since they would have the same QED order as a QED loop added to the lowest order diagrams).

- EFT couplings have QED orders assigned to them, so this can affect how the operator contributes to a process at NLO.

For example, this cpt diagram cannot be easily included in an NLO calculation because of the QED order.
Studying additional radiation at tree-level with matching/merging

- In cases where NLO EFT calculations are challenging with MG, it can be useful to study the effects of additional radiation at tree-level using a LO calculation with matching, in this approach:
  - Harder emission is handled with MG (with the extra parton explicitly listed in the final state, i.e. $pp \rightarrow t\bar{t}X + j$)
  - Softer emission is handled by the parton shower
  - A matching procedure is used to remove double counting

- Including an extra jet via matching can allow us to capture the effects of extra radiation, while avoiding some of the challenges of NLO
  - LO MC does not involve negative weights and is faster to generate
  - LO+j calculations can be performed without MG restrictions on the coupling orders, so there are no ambiguities related to dim6 EW contributions
Processes and operators

- This study focuses on $tth$, $ttW$, and $ttZ$ (with the $h/W/Z$ on-shell):

![Diagrams of $tth$, $ttZ$, and $ttW$ processes]

- We use the dim6top model (arxiv 1802.07237), which uses the Warsaw basis, includes operators involving 3rd generation quarks, imposes $U(2)_q \times U(2)_u \times U(2)_d$ flavor symmetry, and sets $\Lambda$ to 1TeV.

- We focus on 9 WC's that can have large impacts on $ttX$.

<table>
<thead>
<tr>
<th>Operator</th>
<th>Definition</th>
<th>Wilson Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_{t\varphi}$</td>
<td>$\bar{q}t\bar{\varphi}(\varphi^\dagger\varphi)$</td>
<td>$c_{t\varphi}$</td>
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<tr>
<td>$O_{\varphi q}^1$</td>
<td>$(\varphi^\dagger i\slashed{D}_\mu\varphi)(\bar{q}\gamma^\mu q)$</td>
<td>$c_{\varphi q}^- + c_{\varphi q}^3$</td>
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<td>$O_{\varphi q}^3$</td>
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<td>$c_{\varphi q}^3$</td>
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<td>$O_{t\varphi t}$</td>
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<td>$O_{\varphi t b}$</td>
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<td>$O_{t W}$</td>
<td>$(\bar{q}\sigma^{\mu\nu}\tau^I t)\bar{\phi}W_{\mu\nu}^I$</td>
<td>$c_{t W}$</td>
</tr>
<tr>
<td>$O_{b W}$</td>
<td>$(\bar{q}\sigma^{\mu\nu}\tau^I b)\varphi W_{\mu\nu}^I$</td>
<td>$c_{b W}$</td>
</tr>
<tr>
<td>$O_{t B}$</td>
<td>$(\bar{q}\sigma^{\mu\nu} t)\bar{\phi}B_{\mu\nu}$</td>
<td>$(c_W c_{t W} - c_{t Z})/s_W$</td>
</tr>
<tr>
<td>$O_{t G}$</td>
<td>$(\bar{q}\sigma^{\mu\nu} T^A t)\bar{\phi}G_{\mu\nu}^A$</td>
<td>$c_{t G}$</td>
</tr>
</tbody>
</table>
Modeling the EFT dependence

- We consider diagrams with a single dim-6 EFT vertex, so the amplitudes will depend linearly on the WCs:

\[ A = A_{SM} + c A_{dim6} \]

The \( A_{dim6} \) includes the \( 1/\Lambda^2 \), where \( \Lambda \) is the scale of the new physics, which we will set to 1 TeV.

- Therefore the cross section depends quadratically on the WCs:

\[ \sigma \sim |A|^2 = |A_{SM}|^2 + c^2 \text{Re}(A_{SM}^* A_{dim6}) + c^2 |A_{dim6}|^2 \]

\[ \uparrow \quad \text{SM contribution} \quad \uparrow \quad \text{Interference between EFT and SM} \quad \uparrow \quad \text{Quadratic EFT contribution} \]
Method of comparison: $\mu = \sigma / \sigma_{SM}$

- The goal is to understand how the additional jet impacts the EFT dependence of the inclusive cross section.
- To focus on these effects, should carefully choose which quantities to compare.
  - For example, we expect to see differences in the overall normalization obtained from the different methods calculations.
  - We don't want known differences like these to distract us when comparing LO $ttX$ to LO $ttX+j$.
  - We will therefore choose to look at the value of the inclusive cross section relative to the SM cross section, and call this ratio $\mu$.

Schematic example of the type of comparisons we'll look at. The xsec for the given calculation is scaled to the SM xsec from that same method of calculation. Note that $\mu$ passes though 1 at $c=0$ by definition, as the overall normalization cancels.
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Using $\mu$ to evaluate the impact of LO+$j$

- We want to evaluate whether there are any cases where LO 0+1j produces different results from the LO 0j calculation
  - If LO 0+1j is always essentially equivalent to the simple LO calculation, then it might not be advantageous to introduce the complexities of the matching procedure
  - On the other hand, if including the additional parton results in significant differences, then it will be important to ensure that these contributions are always included

$$\mu = \frac{\sigma}{\sigma_{SM}}$$

Schematic example of the type of comparisons we'll look at. The xsec for the given calculation is scaled to the SM xsec from that same method of calculation. Note that $\mu$ passes through 1 at c=0 by definition, as the overall normalization cancels.
Using $\mu$ to evaluate the impact of LO+$j$

- We want to evaluate whether there are any cases where LO $0+1j$ produces different results from the LO $0j$ calculation
  - If LO $0+1j$ is always essentially equivalent to the simple LO calculation, then it might not be advantageous to introduce the complexities of the matching procedure
  - On the other hand, if including the additional parton results in significant differences, then it will be important to ensure that these contributions are always included

The limits from arxiv 1901.05965 (SMEFiT)

If $\mu_{t\bar{t}X+j}$ at the 1901.05965 limit is $\geq 10\%$ different from $\mu_{t\bar{t}X}$, we categorize the effect of the additional parton as significant.
An example of a case where the impact of the extra parton is very small, and where the impact is large

- The dependence of $tth$ on $ctG$ is not impacted significantly by the inclusion of the additional parton, as $\mu_{t+iX+j}$ and $\mu_{t+iX}$ agree throughout the range we consider.

- However, the dependence of $tth$ on $ctW$ is impacted fairly significantly by the inclusion of the additional parton (at $ctW = -1.8$, $\mu_{t+iX+j}$ is about 14% larger than $\mu_{t+iX}$).
Evaluating $\mu_{t\bar{t}X+j}/\mu_{t\bar{t}X}$ for all WCs and processes

- Repeating the comparison for all 9 WCs and 3 processes, we find 7 cases in total where $\mu_{t\bar{t}X+j}$ is at least 10% different from $\mu_{t\bar{t}X}$ (at the 1901.05965 limits)
Summary of combinations of processes and WCs with large $\mu_{t\bar{t}X+j} / \mu_{t\bar{t}X}$ ratios

- We have found that there are multiple cases where the LO+j calculation results in a different EFT dependence than the LO calculation.
- Next we'll step through some of the factors that can contribute to the sizes of the $\mu_{t\bar{t}X+j} / \mu_{t\bar{t}X}$ ratios.

<table>
<thead>
<tr>
<th>Process</th>
<th>WC</th>
<th>Limits from [8]</th>
<th>$\mu_{t\bar{t}X+j} / \mu_{t\bar{t}X}$ at limits</th>
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</thead>
<tbody>
<tr>
<td>$t\bar{t}W$</td>
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<td>1.09, 1.19</td>
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<td>[-13, 18]</td>
<td>1.13, 1.36</td>
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<td>$c_{t\phi}$</td>
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<td>$c_{tZ}$</td>
<td>[-2.1, 4.0]</td>
<td>1.06, 1.14</td>
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</tbody>
</table>
Exploring factors affecting the $\mu_{t\bar{t}X+j}/\mu_{t\bar{t}X}$ ratios

Some factors that may contribute to the $\mu_{t\bar{t}X+j}/\mu_{t\bar{t}X}$ ratios include:

- **Initial states**: The extra parton can allow for new qg initial states, which can lead to a large enhancement, especially when there were no gg initiated diagrams available at 0j.

- **Topology** of diagrams: If all of the 0j diagrams for a given WC involve off-shell s-channel propagators, might expect the additional parton to have a large impact if it opens up new t-channel diagrams.

- **Energy scaling** of the vertices

- **Effects** that impact interference with the SM

- **Limits** where we evaluate the $\mu_{t\bar{t}X+j}/\mu_{t\bar{t}X}$ ratio

Because of the competing influences of these varied factors, it is challenging to predict a priori which WCs and processes will be strongly impacted.
An example: The effect of ctW on tth

- Based on the LO calculation, ctW has a small impact on tth, but when an additional parton is included, the effect is significantly larger.

- At LO without an extra parton ($\mu_{t\bar{t}h}$):
  - All ctW diagrams have qq initial states
  - All ctW diagrams are s-channel, except for bb initiated t-channel diagrams

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$\mu = \sigma/\sigma_{SM}$

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$\mu_{t\bar{t}h}$

---

$\mu_{t\bar{t}h}^{LO}$

---

$\mu_{t\bar{t}h}^{LO}$

---

$\mu_{t\bar{t}h}^{LO}$
When an additional parton is included in the calculation ($\mu_{t\bar{t}h+j}$):

- New gq initiated EFT diagrams become available (including t-channel)
- Since there were no gq initiated EFT diagrams available at 0j, these new diagrams might be expected to have a significant impact
- While the SM contribution also gains new gq initiated diagrams, there were already gg diagrams available at 0j, meaning that the relative impact of the new gq diagrams would not be expected to be as large

An example: The effect of ctW on tth
Summary

• In the effort to obtain more precise predictions that comprehensively model EFT effects for ttX processes, a full NLO treatment (QCD and QED) would be ideal, but can be challenging to produce.

• When NLO is unwieldily or unavailable, LO+j can be a useful approach:
  - LO+j calculations do not require restrictions on the QED order to be imposed.
  - Including an additional parton can significantly impact the inclusive cross section's EFT dependence.
  - The dominant factor accounting for this effect are the new quark-gluon initial states that are able to contribute when an additional parton is included.

• These reasons thus suggest that matched LO calculations are potentially preferable to strictly LO calculations without extra partons because contributions from diagrams with an extra parton can provide important modifications to the dependence of the inclusive tth/W/Z cross section on the WCs.
Backup
Matching procedure summary

- Generating ttX samples with an extra parton requires a matching procedure to avoid double counting between ME and PS
- We use $k_T$-jet version of MLM matching, an event rejection based approach that matches partons generated by MG to jets clustered by Pythia:
  - Final-state partons produced by MG clustered according to $k_T$ algorithm, $k_T$ required to be above a cutoff "$xqcut""
  - After showering and before hadronization, Pythia clusters the final-state objects using the $k_T$ algorithm with a cutoff scale "$qCut"
  - With $qCut$ as maximal $k_T$ distance between jets and partons, the clustered jets are matched to the ME partons
  - Event is saved if all jets are successfully matched to partons
    - Event is otherwise discarded except in highest jet multiplicity sample, where extra jets (with $k_T < $ softest ME parton) are permitted since no danger of double counting
  - To avoid missing a region of phase space, choose $xqcut < qCut$
- We used $xqcut=10$, $qCut=19$ in this study
A potential concern regarding EFT and matching

- In the matching/merging procedure, the phase space of the additional radiation is divided into harder emission (handled by the ME) and softer emission (handled by PS).
- Any overlap must be removed to avoid double counting.
- Since EFT effects are included in the ME but not the PS, this could potentially cause an inconsistency.

![Diagram](chart)

- We use MG for the ME calculation, so EFT effects are included.
- We use Pythia for the PS calculation, so EFT effects are not included.

- For the subset of operators this paper focuses on, only $\hat{O}_{tG}$ involves gluons, so any potential issues should be limited to this operator.
The potential issue for $\mathcal{O}_{tG}$ is mitigated

- The contributions from $\mathcal{O}_{tG}$ in the soft and collinear region turn out to be small, so the phase space overlap with the SM contribution to the PS is also small, meaning this operator should not be problematic.

- As additional validation, the differential jet rate (DJR) can be used to check if the matching procedure is smoothly filling the overlapping phase space without any discontinuities.

- For the $k_T$ algorithm, the DJR histogram represents the distribution of $k_T$ values for which an $n$ jet event transitions to an $n+1$ jet event.

This smooth transition indicates MG and pythia are working together to smoothly fill the overlapping phase space (this plot is for $tth$, but $ttW$ and $ttZ$ are similarly smooth).
Validation of matching procedure with EFT

- The differential jet rate (DJR) can be used to determining whether the matching procedure is able to smoothly fill the overlapping phase space without any discontinuities.
- For the $k_T$ algorithm, the DJR histogram represents the distribution of $k_T$ values for which an n jet event transitions to an n+1 jet event.
- The smooth transitions between the n and n+1 curves for tth, ttW, and ttZ indicate that the chosen matching scales have allowed MG and Pythia to work together to smoothly populate the overlapping region of phase space.

The x axis shows the log of the scale where an n jet event transitions into an n+1 jet event. Here all 9 WCs are set to non-SM values.
Systematic uncertainties

- We explored the size of systematic uncertainties associated with the matched samples. As we did in the rest of the study, we focus on $\mu = \sigma/\sigma_{SM}$. This will allow us to focus on uncertainties that change the quadratic shape of the inclusive cross section's dependence on the Wilson coefficients without becoming distracted by effects impacting only the overall normalization, even in the SM. The following uncertainties are included:
  - We varied the nominal matching scale (i.e. $q_{cut} = 19$ GeV) between 15 GeV and 25 GeV.
  - We varied the renormalization ($\mu_R$) and factorization scale ($\mu_F$) scales of the hard process. Each scale was varied independently up or down by a factor of two and an envelope was constructed from the various combinations of up/down variations of the two scales.
  - We varied the initial and final state radiation (ISR and FSR, respectively) scales in the parton shower up and down by a factor of $\sqrt{2}$.
  - We use the PDF set NNPDF3.1, so the uncertainties are computed with eigenvector PDF members using the LHAPDF tools as described in 1412.7420.
- The effects of the systematics are combined in quadrature
Systematic uncertainties

- As seen in JHEP 06 (2021) 151 table 4, the size of the systematic uncertainties for the ttX+j processes (relative to the SM) are less than 2%. This is much smaller than effect of including an extra parton that we find for some combinations of WCs and processes. An example is shown in JHEP 06 (2021) 151 Figure 12.

Figure 12. Systematic uncertainties on the inclusive tt\bar{h} and tt\bar{h}+j cross sections for $c_{\varphi t}$ and the inclusive ttW and ttW+j cross section for $c_{\varphi Q}^3$. Both ttX and ttX+j curves are normalized to the SM as discussed in the text, and both curves include the uncertainty band, but the uncertainties on the ttX curve are too small to be seen on this plot. We note that the size of the uncertainties are much smaller than the size of the effect of including an extra parton. As described in the text, the range for each Wilson coefficient is taken from the marginalized limits presented in ref. [9].
Comparison of CPU time for LO vs NLO calculations

- Though the overhead time (indicated by the y-intercept of the fit) for the LO samples is somewhat larger than the NLO overhead, the slopes of the NLO fits are larger than the LO slopes by a factor of about two to three, depending on the process.

- The same random seed is used for each running of each gridpack. These tests were performed using a single core from an AMD Opteron 6276 2.3 GHz processor.

**Figure 13.** Plots showing how the CPU time scales with the number of events generated for the NLO and LO models. For $\bar{t}\bar{t}h$ (left), the slope of the fit to the NLO points is about 3 times larger than the slope of the line fit to the LO points. For $\bar{t}\bar{t}W$ (center), the slope of the fit the NLO points is about 2 times larger than the slope of the line fit to the LO points. For $\bar{t}\bar{t}Z$ (right), the slope of the line fit to the NLO points is about 3 times larger than the slope of the line fit to the LO points. For consistency with the NLO samples, the LO samples were generated with a QED=1 constraint.
## Summary of tth diagram QED and QCD orders by WC

<table>
<thead>
<tr>
<th>WC</th>
<th>0p diagrams</th>
<th>+1p diagrams</th>
<th>Notes</th>
<th>Affects ttH based on our LO MC</th>
<th>Affects ttH according to 1901.05965 Tab 3.5</th>
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## Summary of ttW diagram QED and QCD orders by WC

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<th>+1p diagrams</th>
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## Summary of ttZ diagram QED and QCD orders by WC

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<th>Notes</th>
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<td>sort of (but not really at 0j)</td>
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<tr>
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<tr>
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<td>QED=3, QCD=1</td>
<td>sort of (but not really at 0j)</td>
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Summary of how initial states contribute to the size of the $ttX \rightarrow ttX+j$ effects

- The inclusion of an additional parton allows for new diagrams with $gq$ initial states, which can affect the predicted cross section.
  - Note that for some of the WCs, the $q$ in the $gq$ must be a $b$, these would not be expected to have a large impact, so in the following discussion "$gq$" implies that the $q$ is not a $b$.

- However, both the EFT and SM contribution will gain new $gq$ diagrams, so the expected impact depends on the initial states available to each.

- For each WC, we can categorize the effects as follows (starting with the case where we might expect to see the largest impact when including an additional parton):
  - Case 1: At 0j, already gg initiated SM diagrams, but only qq initiated EFT diagrams.
  - Case 2: At 0j, only qq diagrams for both SM and EFT.
  - Case 3: At 0j, already gg initiated SM diagrams and gg initiated EFT diagrams.
Summary of $ttX\rightarrow ttX+j$ effects

- WCs with $\mu_{tt\bar{h}+j}/\mu_{tt\bar{h}}$ larger than 1.1 at either the high or low limit from arxiv 1901.05965: cpt, ctW, ctZ
  - All three of these WCs fall under Case 1
  - cpQM also falls under Case 1, but has tighter limits from 1901.05965, but shows a similar impact when a comparable x axis range is considered

- WCs with $\mu_{ttZ+j}/\mu_{ttZ}$ larger than 1.1 at either the high or low limit from arxiv 1901.05965: ctZ
  - ctZ falls under Case 3
  - Other WCs falling under Case 1 or Case 3: ctG, ctW, cpt, cpQM, ctp
  - For ctG and ctW: When comparable limits to ctZ are considered, these also see a significant impact
  - For cpt, cpQM, ctp: These scale less strongly with energy than ctZ

- WCs with $\mu_{ttW+j}/\mu_{ttW}$ larger than 1.1 at either the high or low limit from arxiv 1901.05965: cpQ3, cpt, ctp
  - All WCs fall under Case 2
  - All WCs impacted to varying degrees, except for cptb and cbW, which do not affect $ttW$ at either 0j or 1j (since all diagrams involve an EFT t-b-W vertex connected to a SM t-b-W vertex via the b, which requires a b with the opposite chirality required by the EFT vertex, so the contribution from these diagrams is small in the 4f scheme, and zero in the 5f scheme)
Comparisons to some NLO computations

• While not the primary goal of this study, we can compare a set of LO+j calculations to NLO calculations (produced with SMEFTatNLO)
  - This can provide another example of how our method of comparison is insensitive to overall normalization while providing a clear comparison between the cross section's dependence on a given WC

• Because of the challenges associated with including higher-order QED effects in MG NLO calculations, we'll focus on WCs that enter at the lowest QED order
  - For tth and ttZ, it has already been shown that the inclusive cross section's dependence on these operators calculated at LO and NLO are in reasonable agreement, i.e. the K factors are close to 1 (1607.05330 and 1601.08193, respectively)
  - For the WCs we'll be comparing, the inclusion of the additional parton did not have a significant impact on the EFT dependence
  - We'll therefore expect to also see agreement between the EFT dependence predicted by the LO+j calculation and the NLO calculation
Summary of NLO comparisons

- The EFT dependence of our LO+j calculations (blue dashed line) are consistent with the SMEFTatNLO comparison points, as expected.