



# LLRF Cavity Simulation for SPL

Simulink Model for HP-SPL Extension to  
LINAC4 at CERN from RF Point of View

Acknowledgement:

CEA team, in particular O. Piquet (simulink model)

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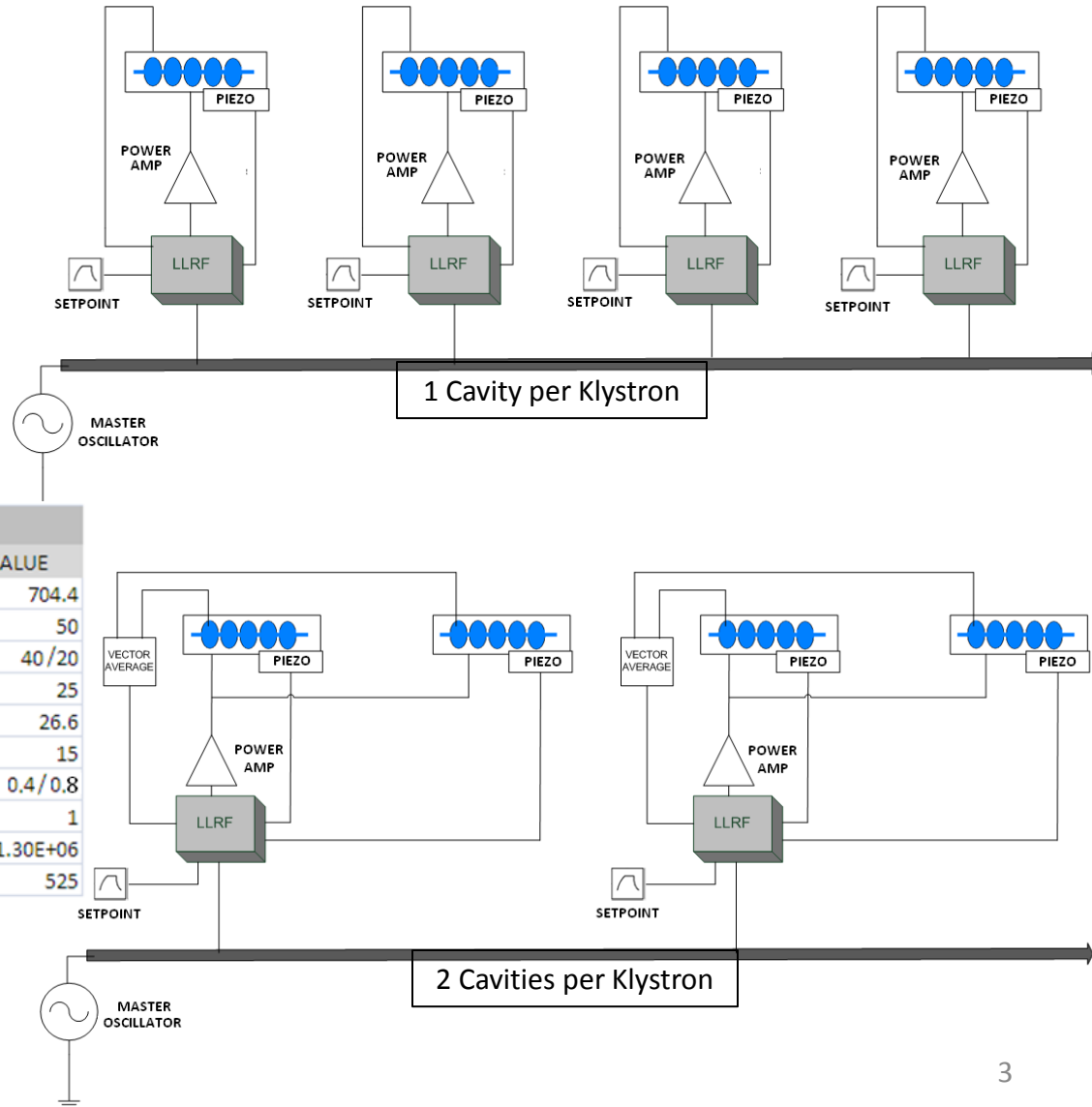


# Presentation Overview

- SPL Characteristics
- Single Cavity Model and Simulation Results
- Dual Cavity Model and Simulation Results
- Error Analysis

# SPL High Current Operation

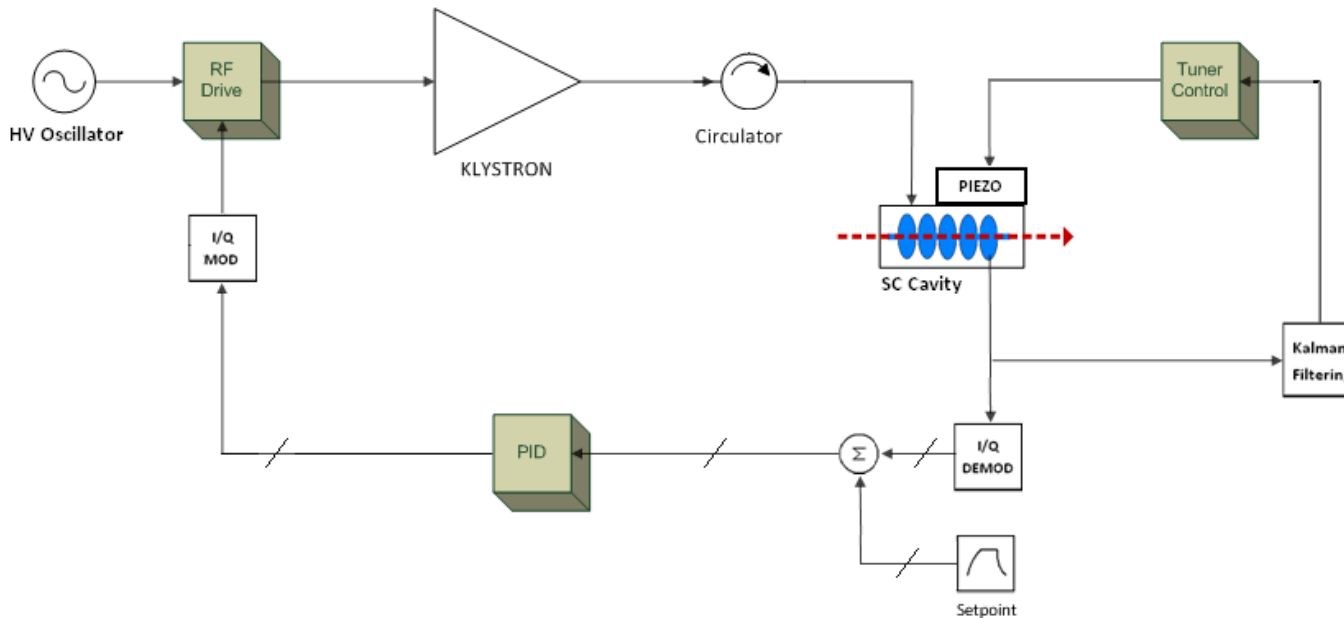
- Possible operation using 1, 2 and 4 cavities fed by a single power amplifier.



## GENERAL PARAMETERS FOR HIGH POWER SPL

PARAMETER	UNIT	VALUE
Resonant Frequency	MHz	704.4
Repetition Rate	Hz	50
Average Pulse Current	mA	40/20
Accelerating Field	MV/m	25
Accelerating Voltage	MV	26.6
Beam Synchronous Angle	Degrees (LINAC)	15
Length of Beampulse	ms	0.4/0.8
Power Delivered to Beam per Cavity	MW	1
Cavity/Generator Coupling Loaded Quality Factor		1.30E+06
Geometry Factor (R/Q)	Ohm (LINAC)	525

# High-Level Diagram of Single Cavity + Control System



$$f_{RF} = 704.4 \text{ MHz}$$

$$I_{b,DC} = 40 \text{ mA}$$

$$\phi_s = 15^\circ \text{ (LINAC)}$$

$$P_b = V_{acc} \times I_{b,DC} \times \cos(\phi_s) = 1.0285 \text{ MW}$$

$$Q_L = 1.3113 \times 10^6$$

$$\frac{R}{Q} = 525 \text{ } \Omega \text{ (LINAC)}$$

$$Q$$

$$\tau_{\text{beam pulse}} = 0.4 \text{ ms}$$

$$\text{rep period} = 20 \text{ ms}$$

$$R_L = 680 \text{ M}\Omega$$

$$I_g = \frac{V_{acc}}{R_L} + I_{b,DC} \cos(\phi_s) = 77.3 \text{ mA}$$

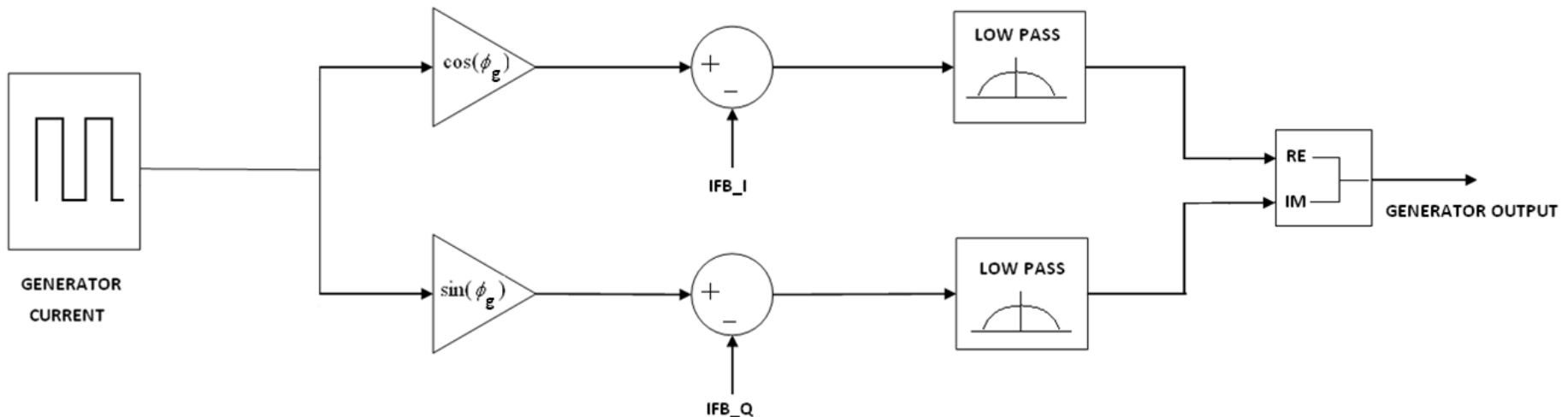
$$\tau_{fill} = \frac{2Q_L}{\omega_{RF}} = 0.5926 \text{ ms}$$

$$\alpha = \frac{I_g}{I_{b,DC} \cos(\phi_s)} = 2$$

$$t_{inj} = \tau_{fill} \ln(\alpha) = 0.4108 \text{ ms}$$

# RF Drive and Generator Model

- Generator current modeled as square pulse for the duration of injection + beam pulse time
- High bandwidth compared to feedback loop and cavity (1 MHz)

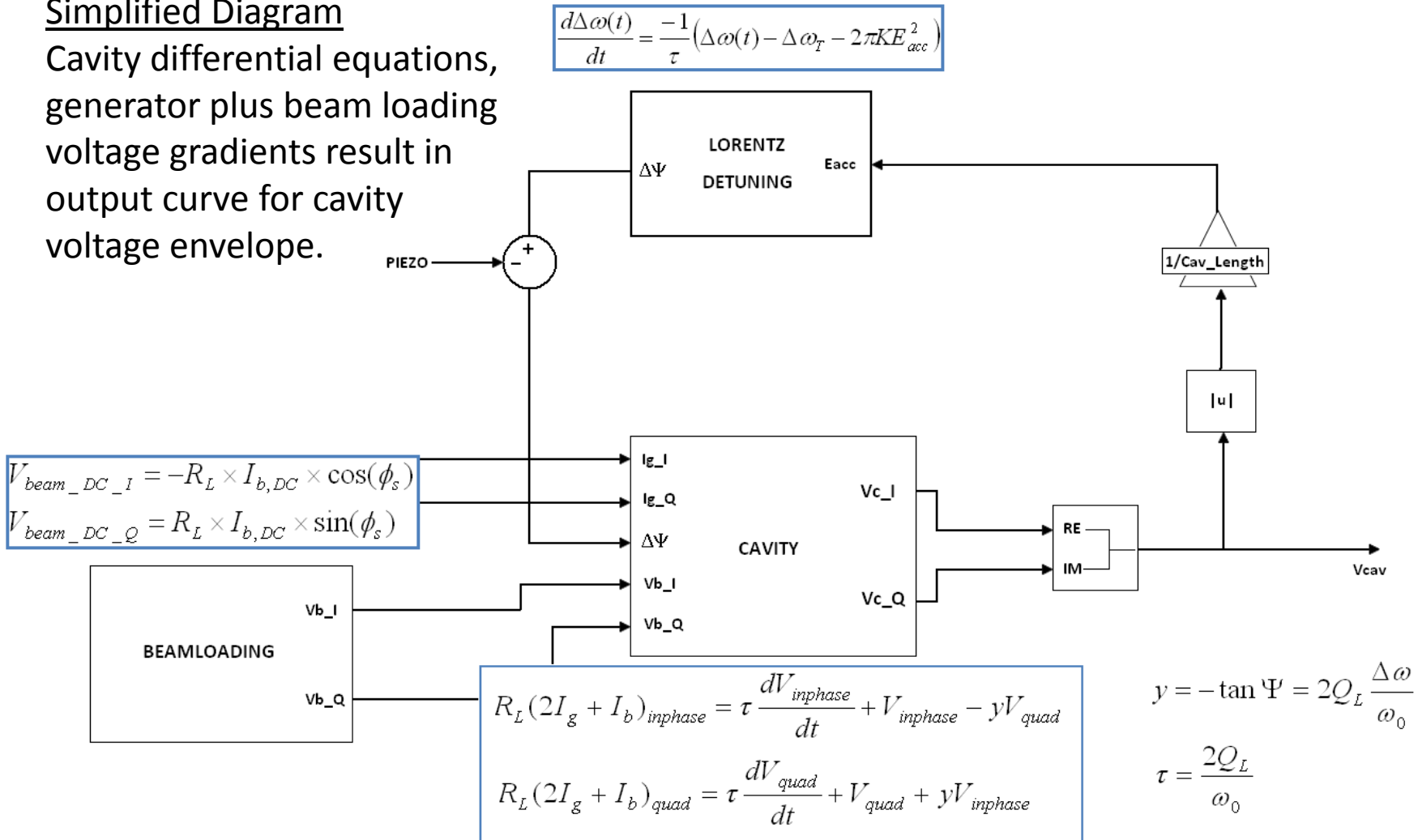


# Cavity Model (cont)

## Simplified Diagram

Cavity differential equations, generator plus beam loading voltage gradients result in output curve for cavity voltage envelope.

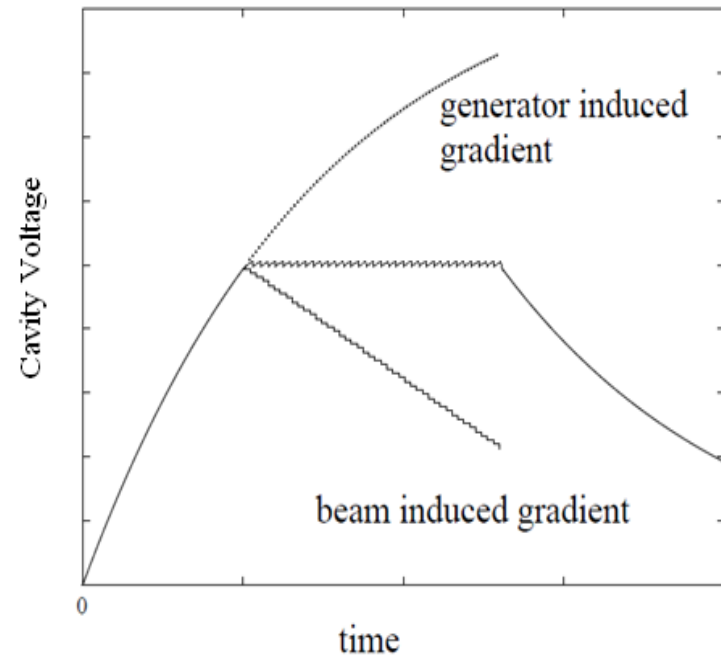
$$\frac{d\Delta\omega(t)}{dt} = \frac{-1}{\tau} (\Delta\omega(t) - \Delta\omega_T - 2\pi KE_{acc}^2)$$



# Beam Loading

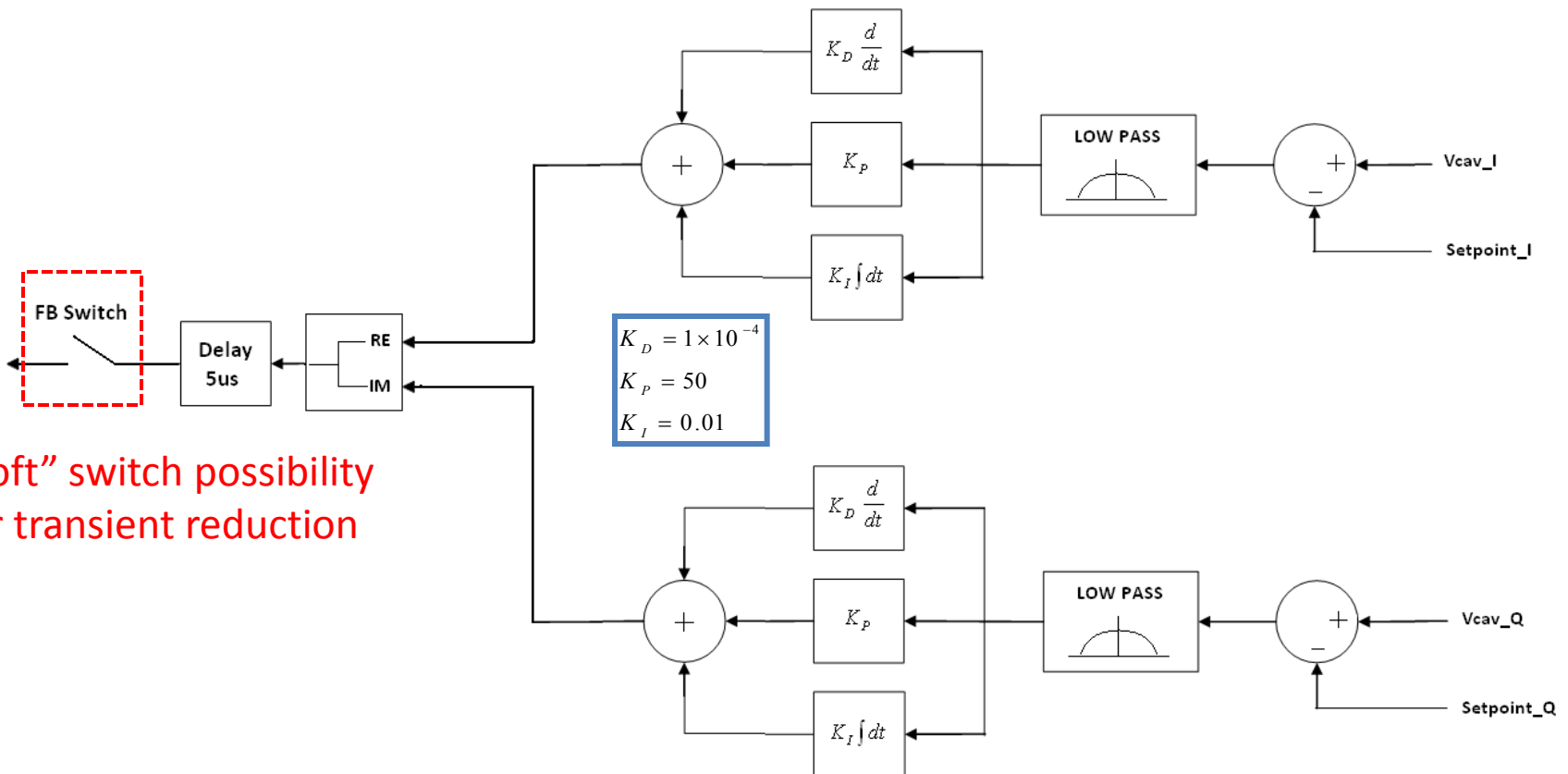
- Infinitely narrow bunches induce instant voltage drops in cavity
- Voltage drop is equal to generator induced voltage increase between bunches creating flattop operation
- Envelope of RF signal in I/Q

$$V_{cav\_bunch} = \omega_{RF} \times \frac{R}{Q} (\text{circuit}) \times q_b$$



# RF Feedback

- PID controller
- Limit bandwidth in feedback loop to 100 kHz
- (Klystron bandwidth is 1 MHz)



“soft” switch possibility  
for transient reduction





# Graphical User Interface

**SPLGUI**

**Start Simulation**

1-Cav  Feedback  Feed-Forward

**Operating Parameters**

Generator Frequency (Hz)	704.4e6	Synchronous Angle (Deg) LINAC	15	R/Q LINAC	525	Loaded (Specify if Fixed)	Input Value
Beam Current (A)	40e-3	Accelerating Field (V/m)	25e6	Tpulse (s)	1.2e-3	Lorentz Coefficient K (Hz/(MV/m)*2)	0
Per Shot Variation (%lb)	5						

Time Elapsed: 108.861

**Cavity Voltage**

Axis Control: Xlims [X1 X2] [0 0.02] Autoscale Ylims [Y1 Y2] AutoZoom

**Power Phasor Diagram**

Axis Control: Xlims [X1 X2] Ylims [Y1 Y2]  Pbeam  PRef  PVcav  Pfor Phasor Diagram

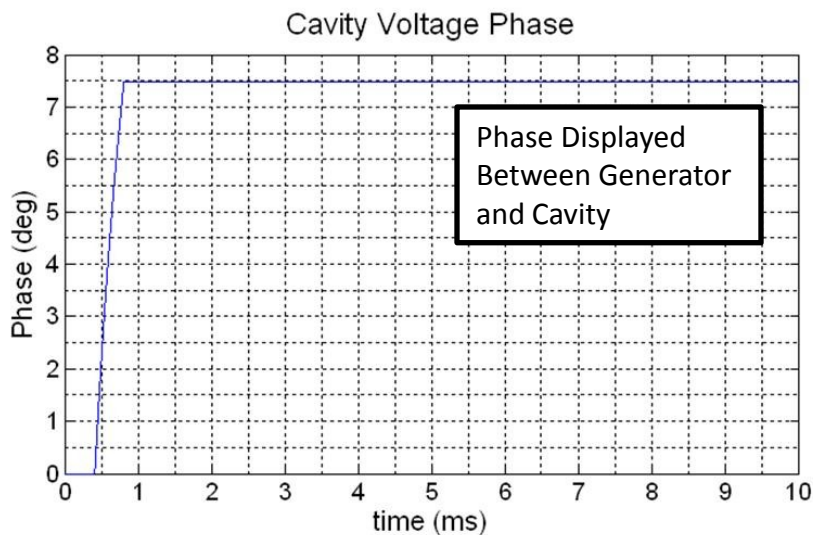
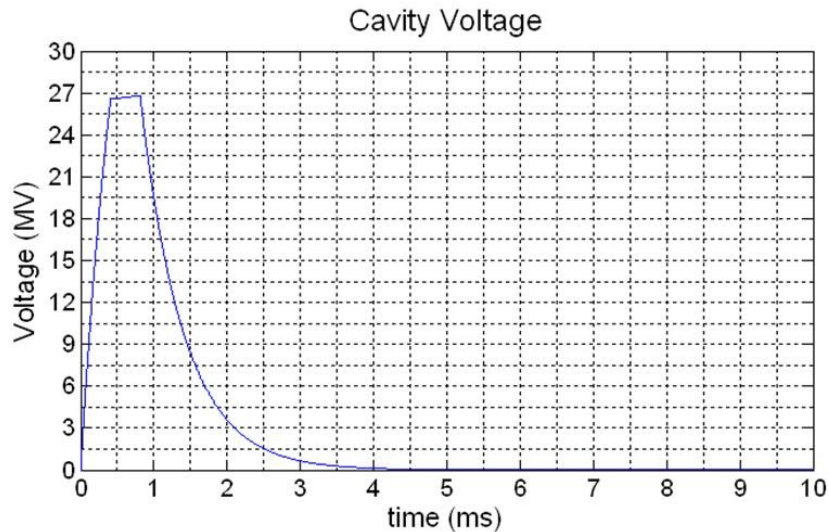
- Cavity Voltage
- Cavity Voltage Phase
- Lorentz Detuning
- Forward Power
- Reflected Power
- Feedback Power
- Phasor Diagram
- Beam Current
- Cavity Voltage



# Results

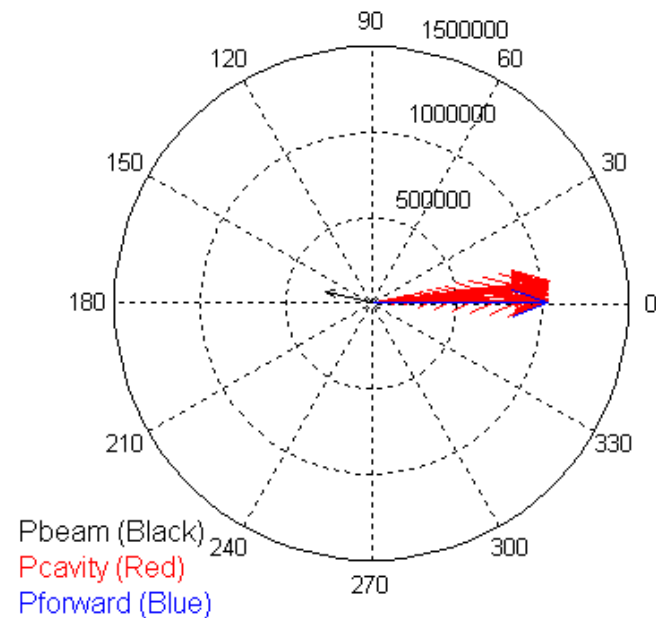
- Cavity Voltage Amplitude and Phase
- Forward and Reflected Power
- Additional Power for Feedback Transients and Control
- Effect of Lorentz Detuning on Feedback Power
- Effect of Source Current Fluctuations
- Mismatched Low-Power Case
- Effects of Beam Relativistic Beta Factor on Cavity Voltage During Beamloading

# Cavity Voltage Magnitude and Phase in the Absence of Lorentz Detuning (Open Loop)



Reactive Beamloading  
Results in V<sub>cavc</sub> Deviation

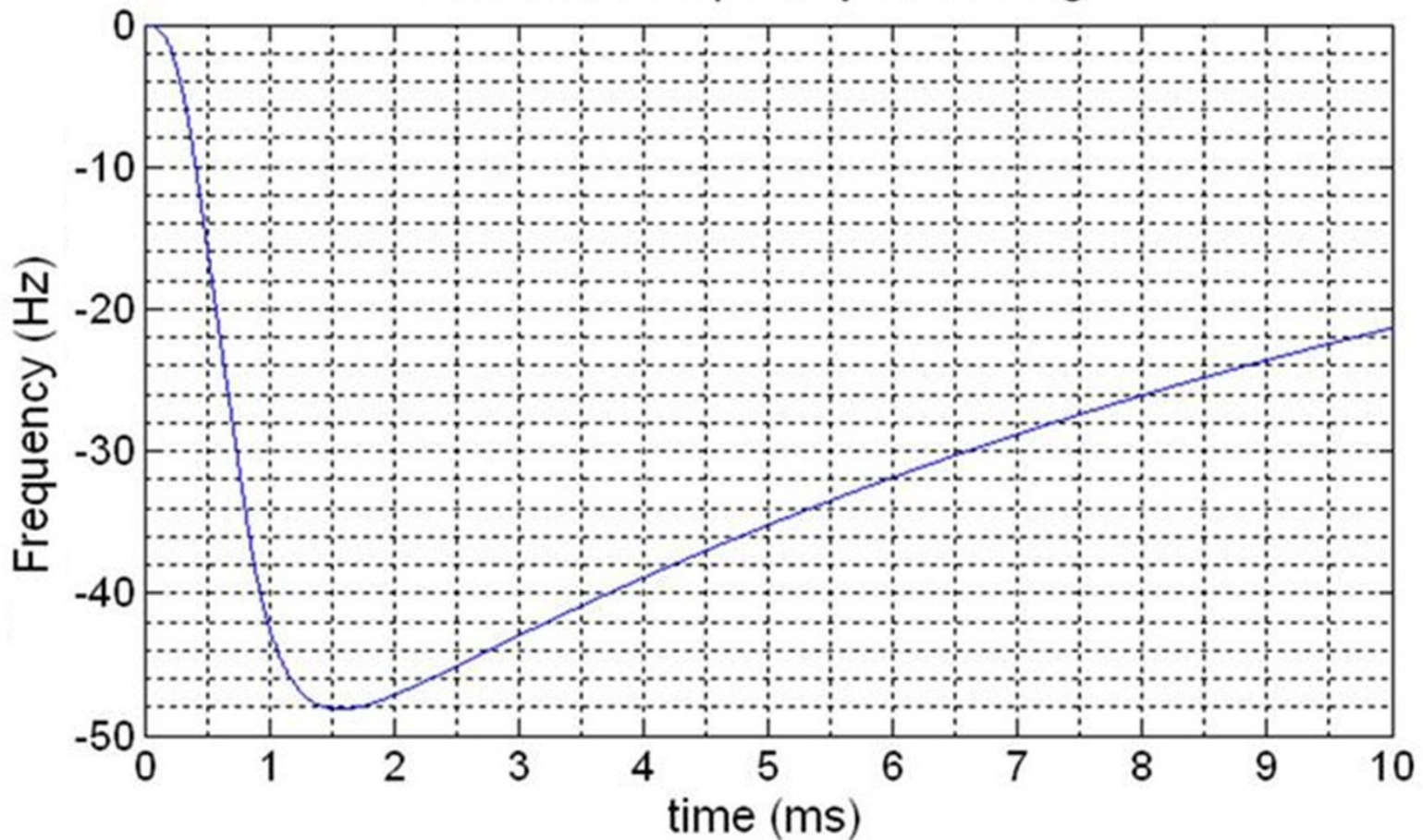
1st Turn Power Phasor Diagram



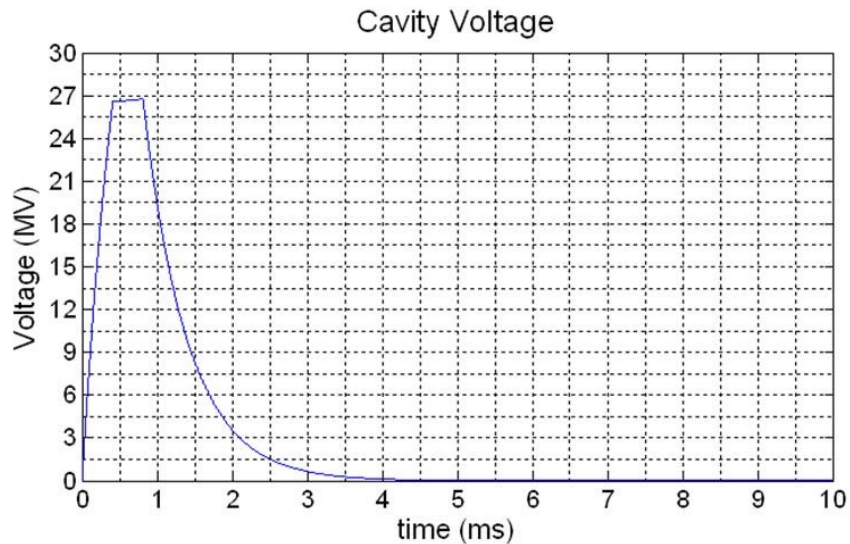
# Effect of Lorentz Detuning on Cavity Voltage and Phase (Lorentz Frequency Shift)

$$\frac{d\Delta\omega(t)}{dt} = \frac{-1}{\tau} (\Delta\omega(t) - \Delta\omega_r - 2\pi KE_{acc}^2)$$

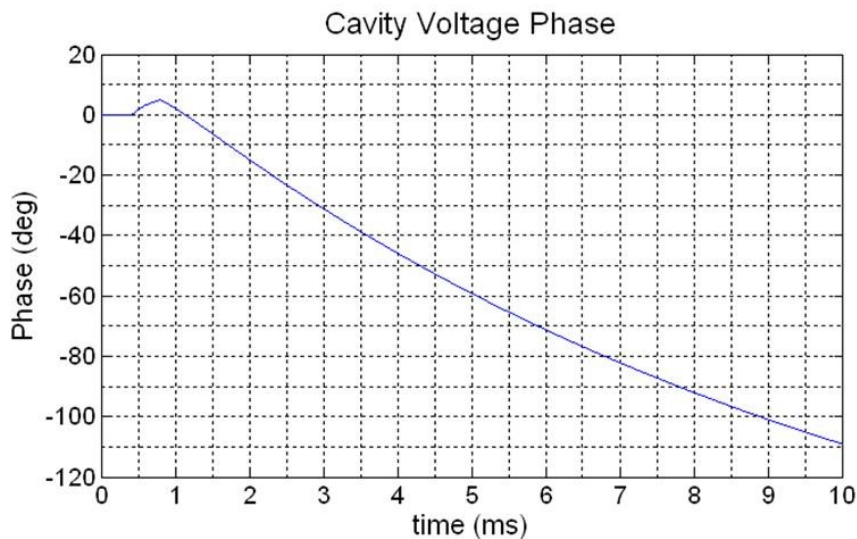
Lorentz Frequency Detuning



# Effect of Lorentz Detuning on Cavity Voltage and Phase (Open Loop)



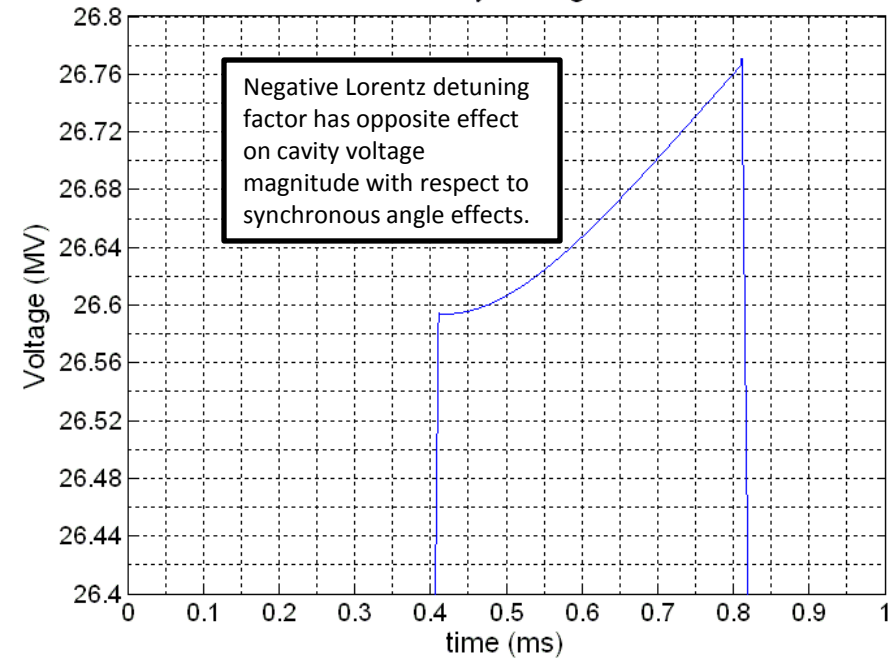
Lorentz effects oppose those of the synchronous angle



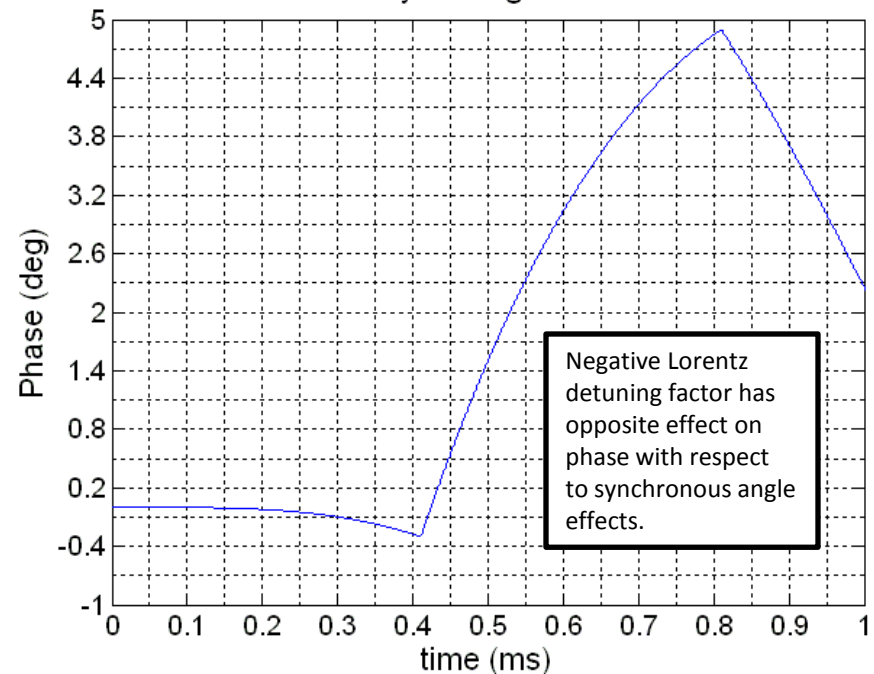
Approximately linear phase shift for undriven cavity during field decay

# Effect of Lorentz Detuning on Cavity Voltage and Phase (Open Loop Close-Up)

Cavity Voltage



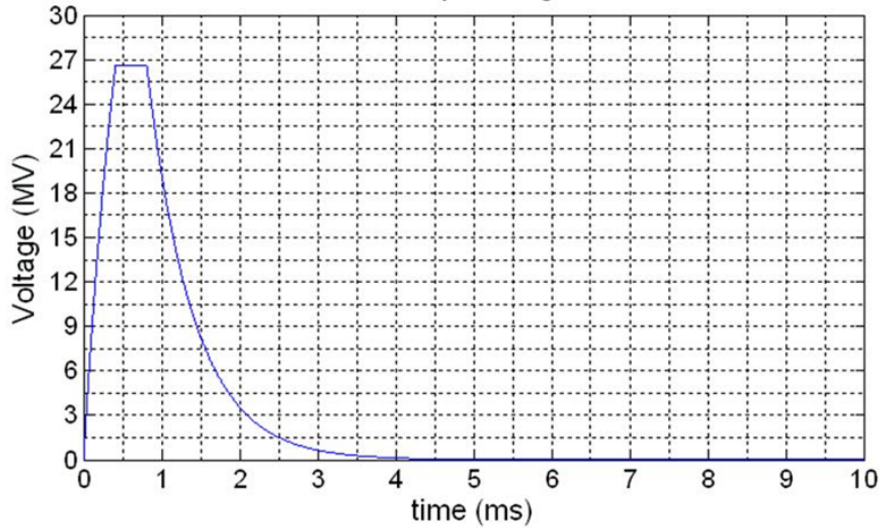
Cavity Voltage Phase



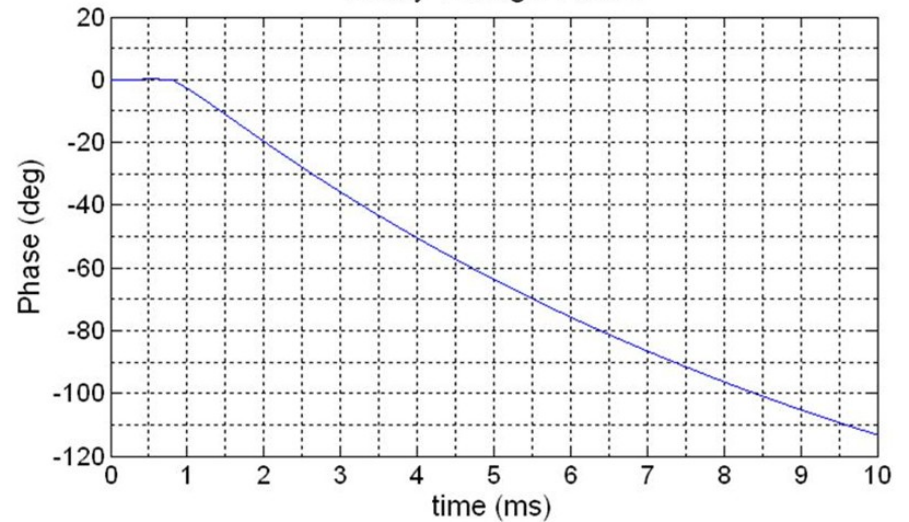


# Cavity Voltage and Phase With Lorentz Detuning (Closed Loop Performance of Fast Feedback)

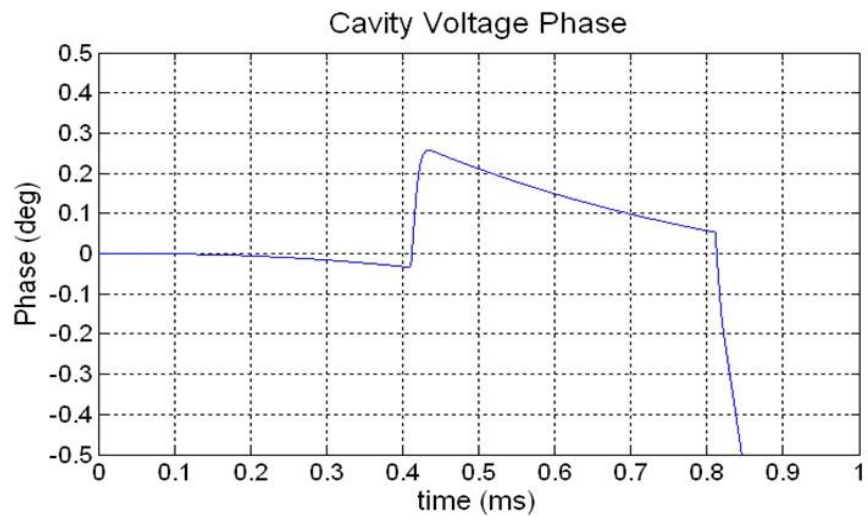
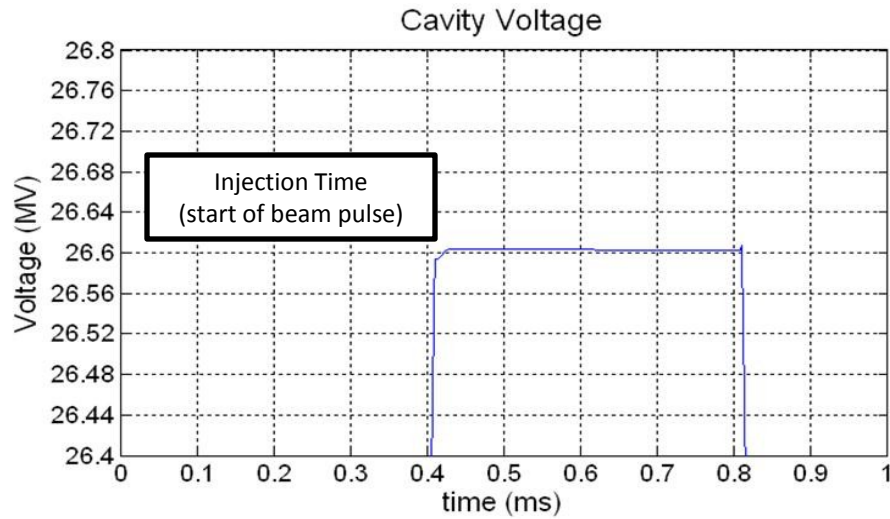
Cavity Voltage



Cavity Voltage Phase

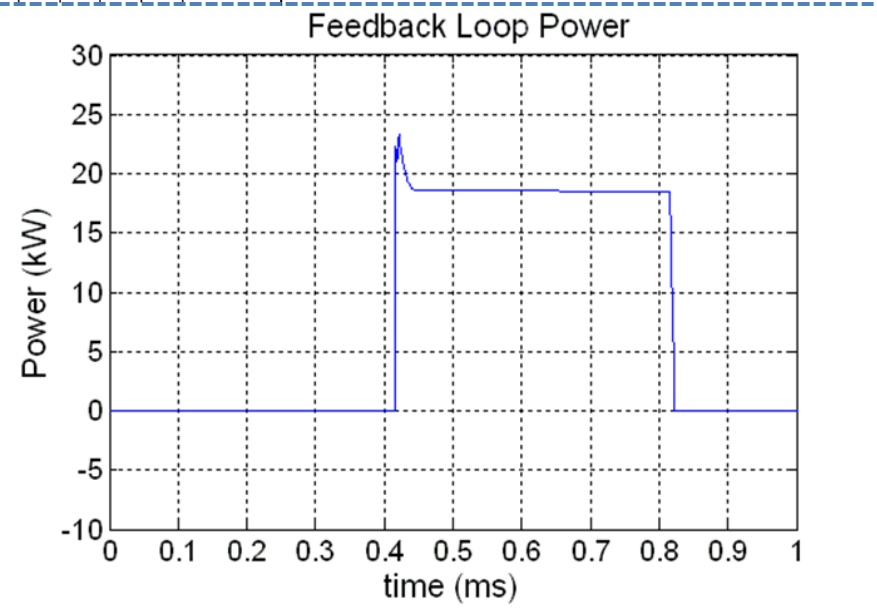
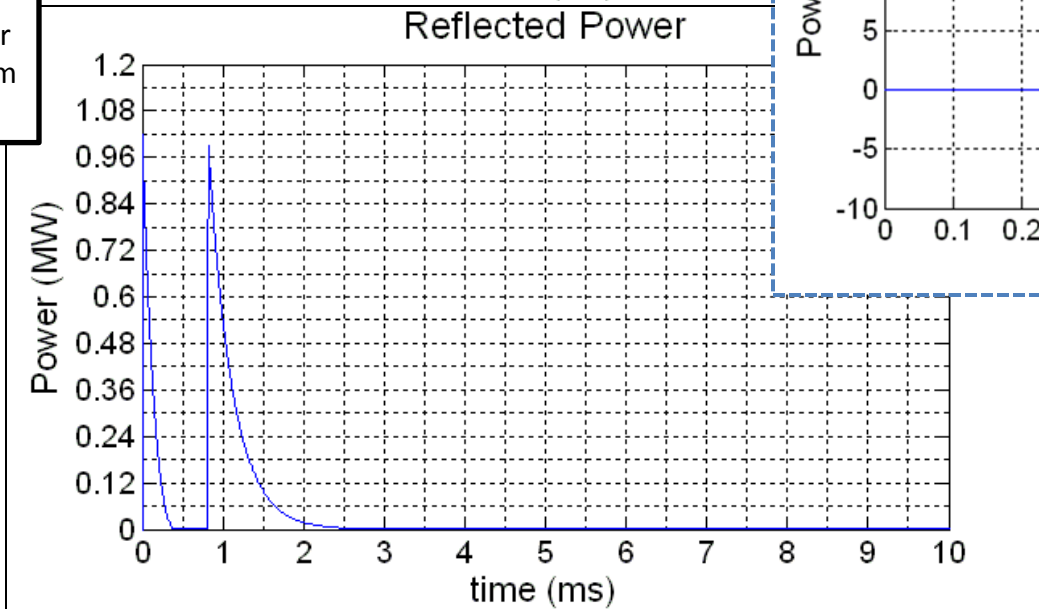
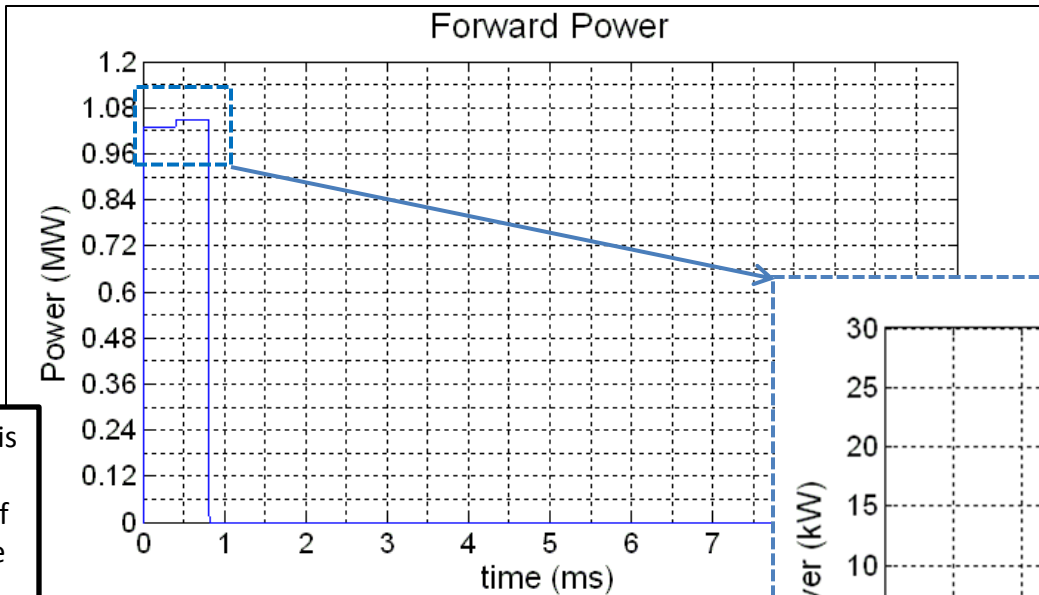


# Cavity Voltage and Phase Close-up



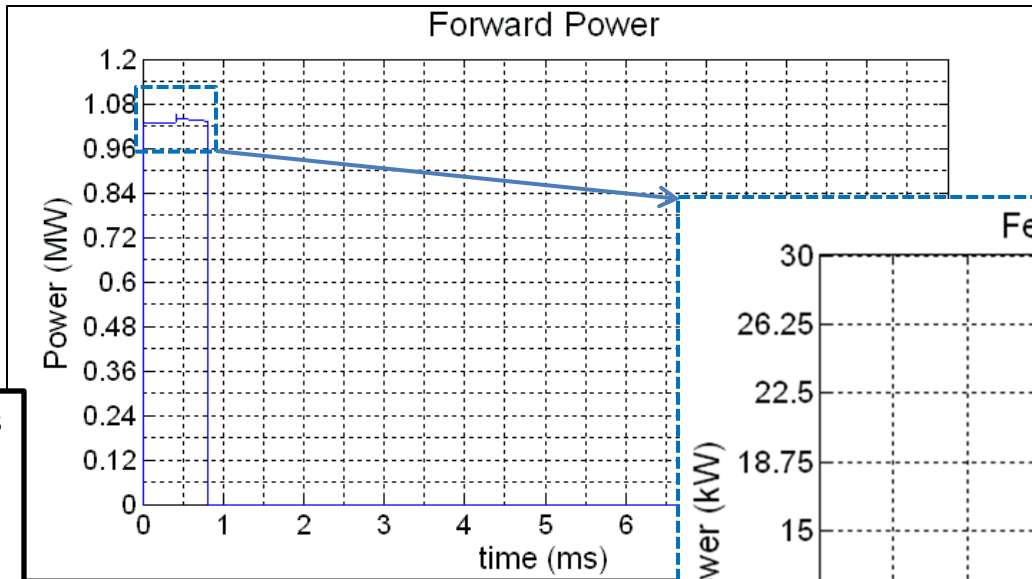


# Forward and Reflected Power without Lorentz Detuning

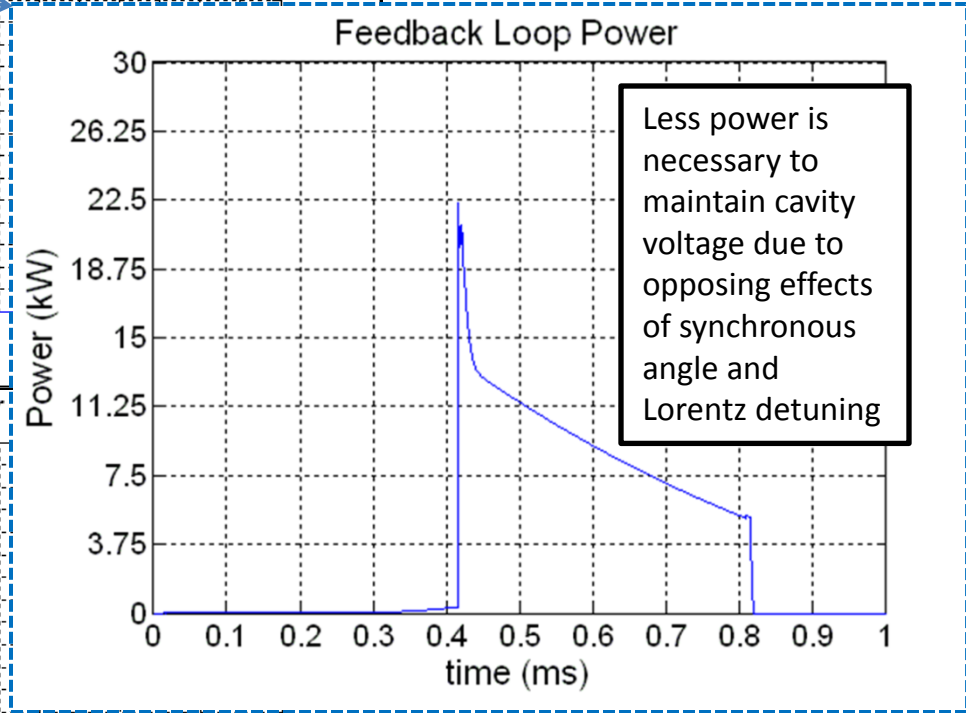
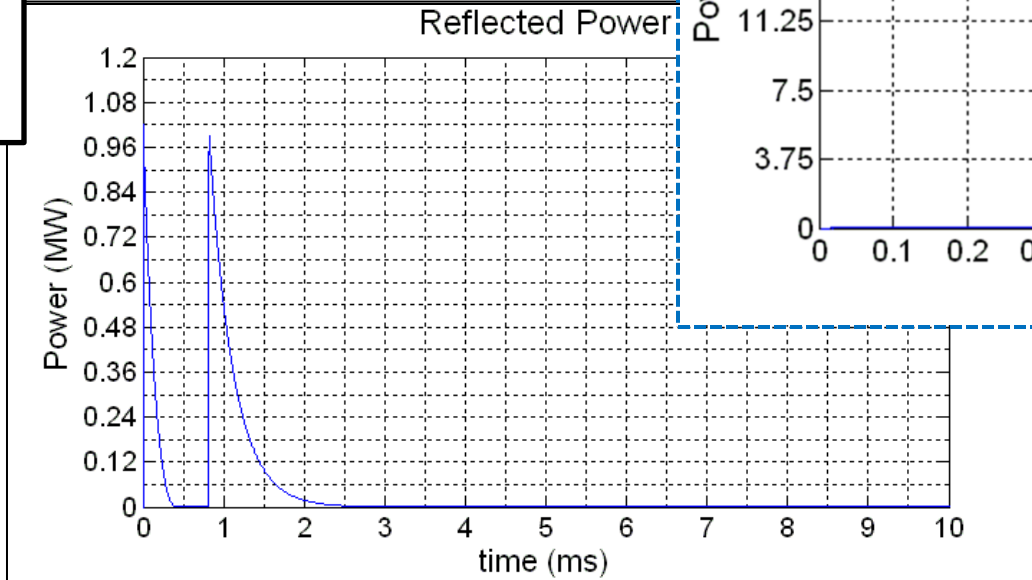


Feedback loop is closed (ON) 10 us after start of generator pulse and opened (OFF) 10 us after end of the beam pulse.

# Forward and Reflected Power with Lorentz Force detuning

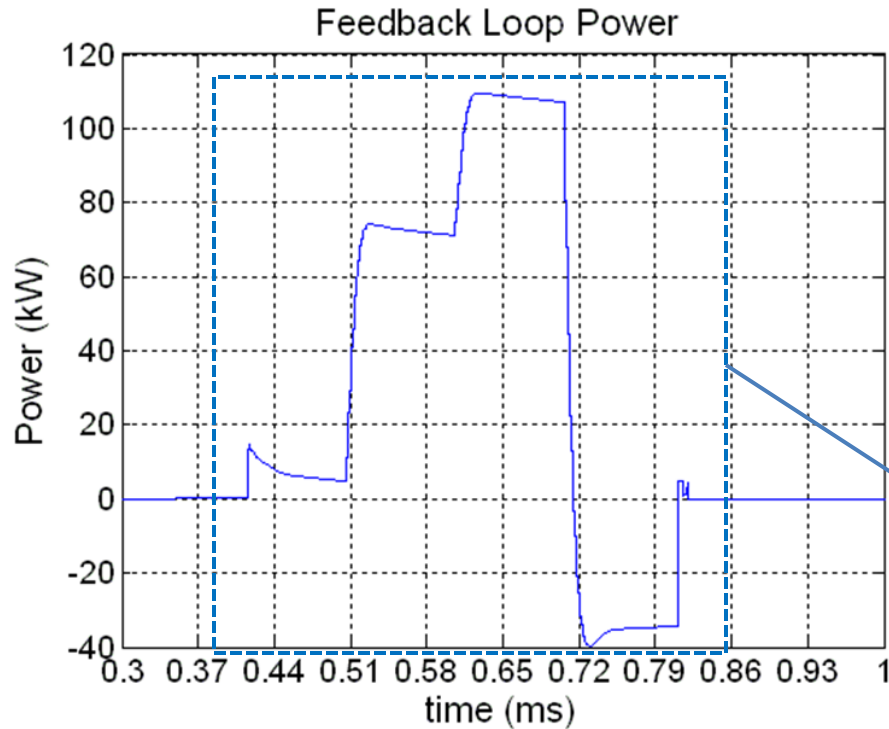


Feedback loop is closed 10 us after start of generator pulse and opened 10 us after the end of the beam pulse.

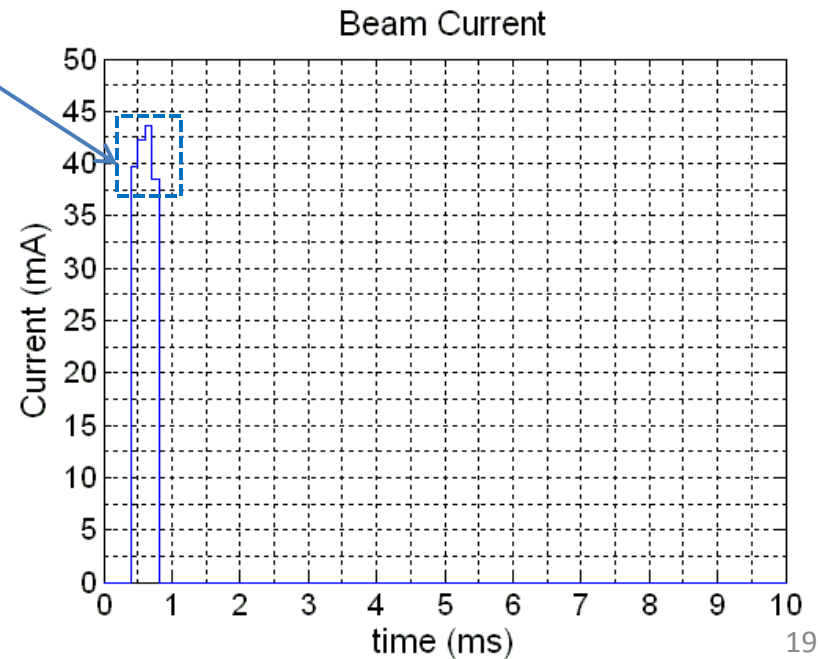


Less power is necessary to maintain cavity voltage due to opposing effects of synchronous angle and Lorentz detuning

# Effects of Source Beam Current Variation

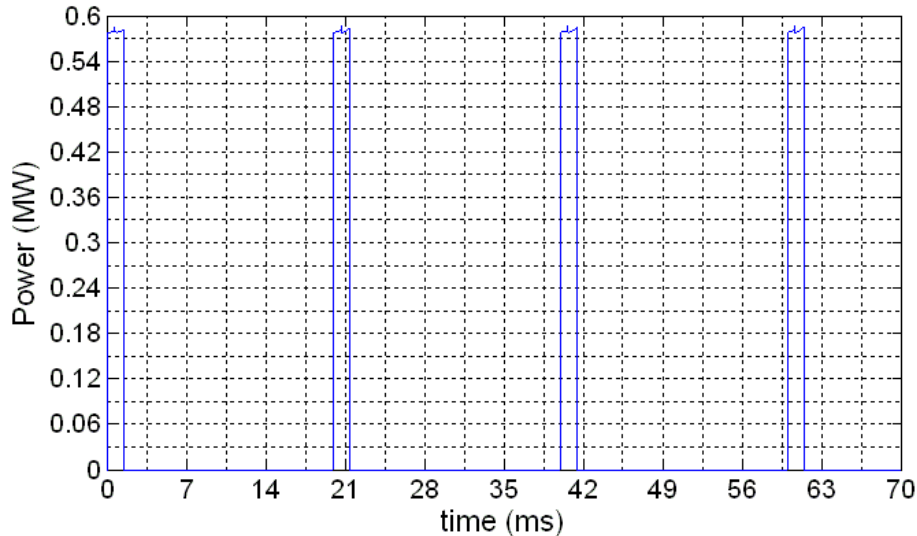


5% variation in  $I_b$  for 40mA case requires approximately 60kW of additional power.

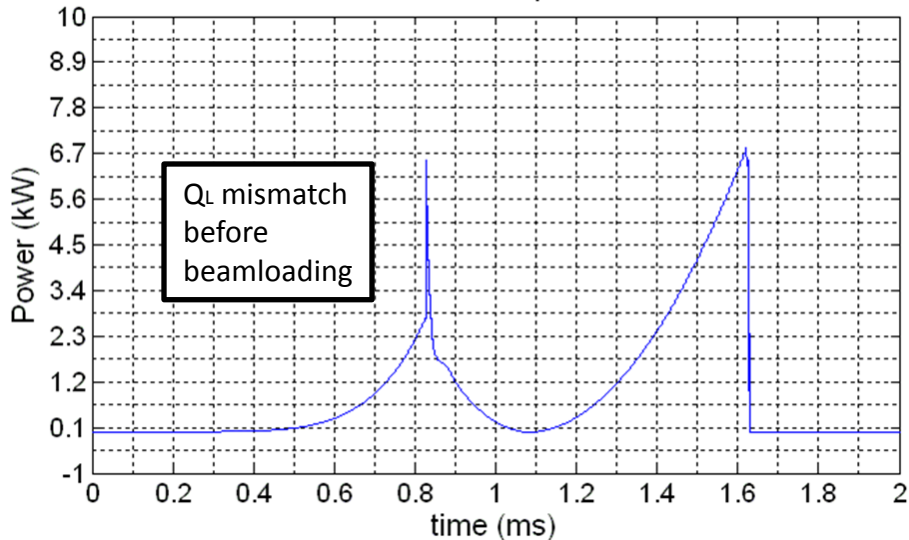


# SPL Low Current Operation (Power Analysis)

Forward Power



Feedback Loop Power



$$I_{b,DC} \cong 20 \text{ mA}$$

$$P_b = V_{acc} \times I_{b,DC} \times \cos(\phi_s) \cong 514 \text{ kW}$$

$$Q_{L, \text{fixed}} = \frac{V_{acc}}{\frac{R}{Q} \times I_{b,40 \text{ mA}} \times \cos(\phi_s)} \cong 1.3113 \times 10^6$$

$$I_g = \frac{V_{acc}}{R_L} + I_{b,DC} \cos(\phi_s) = 58 \text{ mA}$$

$$\alpha = \frac{I_g}{I_{b,DC} \cos(\phi_s)} = 3$$

$$\tau_{fill} = \frac{2Q_L}{\omega_{RF}} = 0.5926 \text{ ms}$$

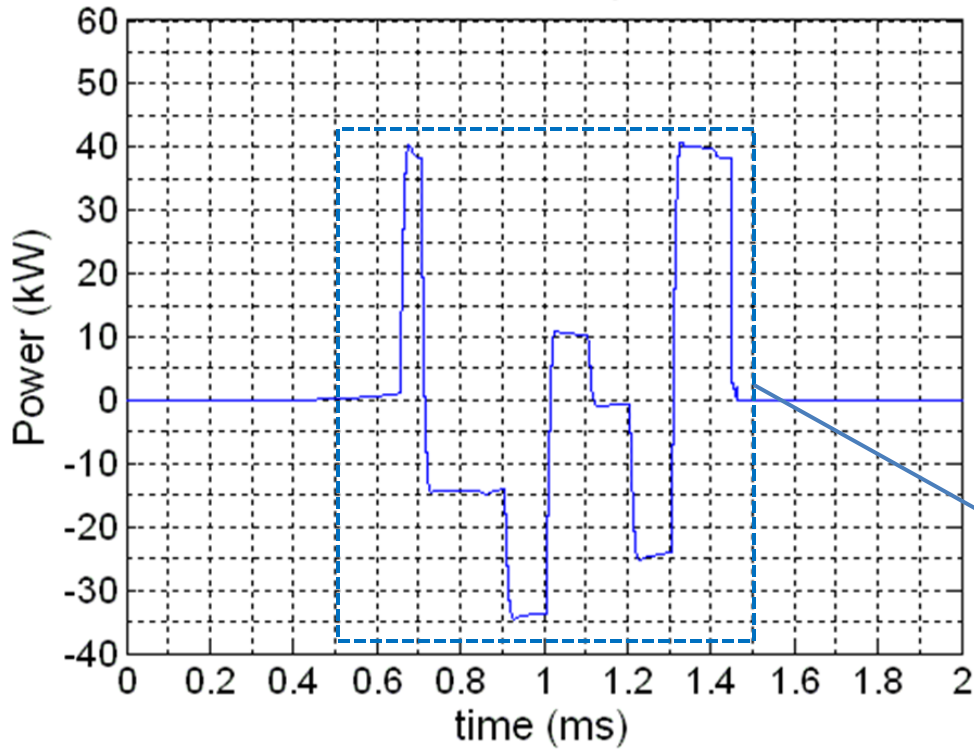
$$t_{inj} = \tau_{fill} \ln(\alpha) = 0.6510 \text{ ms}$$

$$P_{fwd} = \frac{1}{4} R_L |I_g|^2 = 578 \text{ kW}$$

$$t_{pulse} = 0.8 \text{ ms}$$

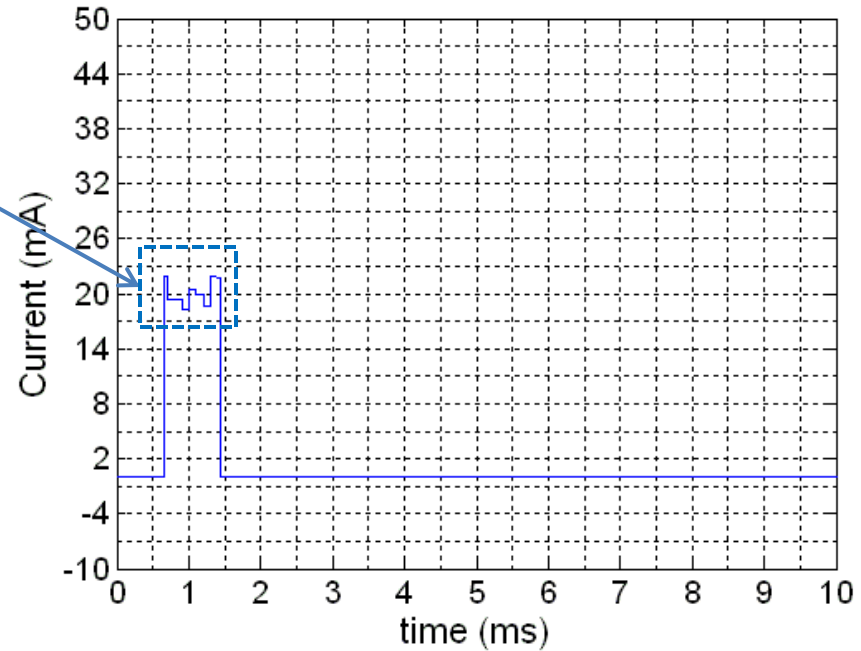
# Effects of Source Beam Current Variation

Feedback Loop Power



5% (1mA) results  
in approx. 20kW  
FB power  
increase

Beam Current





# Transit Time Factor Variation with Relativistic Beta (SPL $\beta=1$ cavities)

- The shunt impedance relates the voltage in the cavity gap to the power dissipated in the cavity walls.

$$R_{sh} = \frac{V^2}{P_d}$$

- The corrected shunt impedance (effective shunt impedance) relates the accelerating voltage in the cavity to the power dissipated. This quantity describes the voltage that a particle travelling at a certain speed will “see” when traversing the cavity.

$$V_{cav} = \int_{gap} E(t=0, z) dz$$
$$V_{acc}(\beta) = \int_{gap} E(t=0, z) \cos\left(\frac{2\pi}{\beta\lambda_{RF}} z\right) dz$$

- The correction applied is known as the “Transit Time Factor”. For even symmetric field distributions:

$$T(\beta) = \frac{\int_{gap} E(t=0, z) \cos\left(\frac{2\pi}{\beta\lambda_{RF}} z\right) dz}{\int_{gap} E(t=0, z) dz}$$



# Transit Time Factor Variation with Relativistic Beta (SPL $\beta=1$ cavities)

- Until now, the cavity dynamics have been modeled from the point of view of a beam travelling at the speed of light ( $\beta = 1$ ).
- We now investigate how the cavity voltage is affected during beam loading with a “slower” beam.

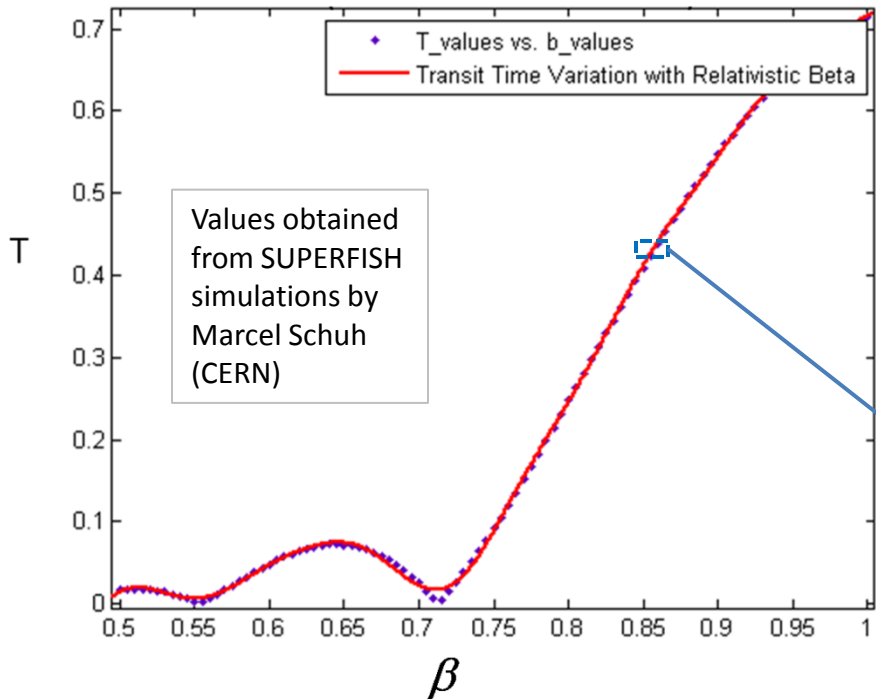
$$R_{sh,eff} = R_{sh} T_{\beta < 1}^2 = R_{sh, \beta = 1} \left( \frac{T_{\beta < 1}}{T_{\beta = 1}} \right)^2$$

$$V(t)_{gen} = R_{sh, \beta = 1} I_g \left( 1 - e^{-\frac{t}{\tau}} \right) \quad t_{ON} < t < t_{OFF}$$

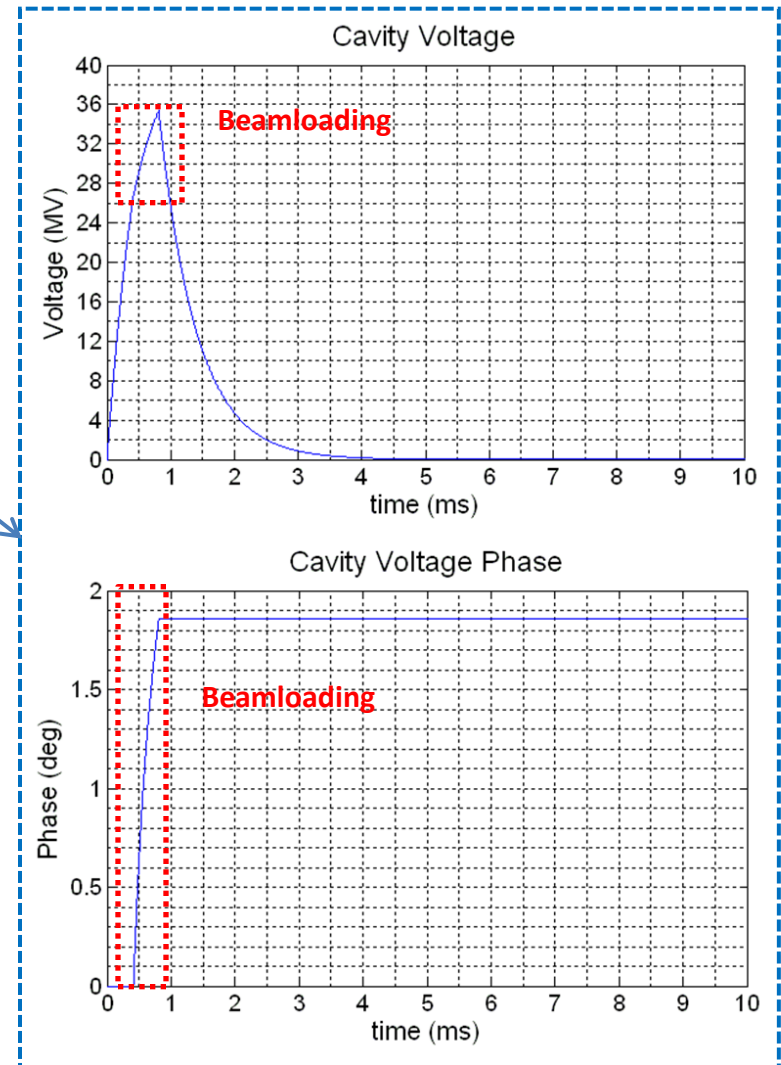
$$V(t)_{beam} = -R_{sh,eff} I_{b,DC} \left( 1 - e^{-\frac{-(t-t_{inj})}{\tau}} \right) \quad t_{inj} < t < t_{inj+pulse}$$

$$V_{cav, \beta = 1}(t) = V(t)_{gen} + V(t)_{beam} = R_{sh, \beta = 1} I_g \left( 1 - e^{-\frac{t}{\tau}} \right) - R_{sh,eff} I_{b,DC} \left( 1 - e^{-\frac{-(t-t_{inj})}{\tau}} \right) \quad t_{inj} < t < t_{inj+pulse}$$

# Transit Time Factor Variation with Relativistic Beta (SPL $\beta=1$ cavities, open loop simulation)

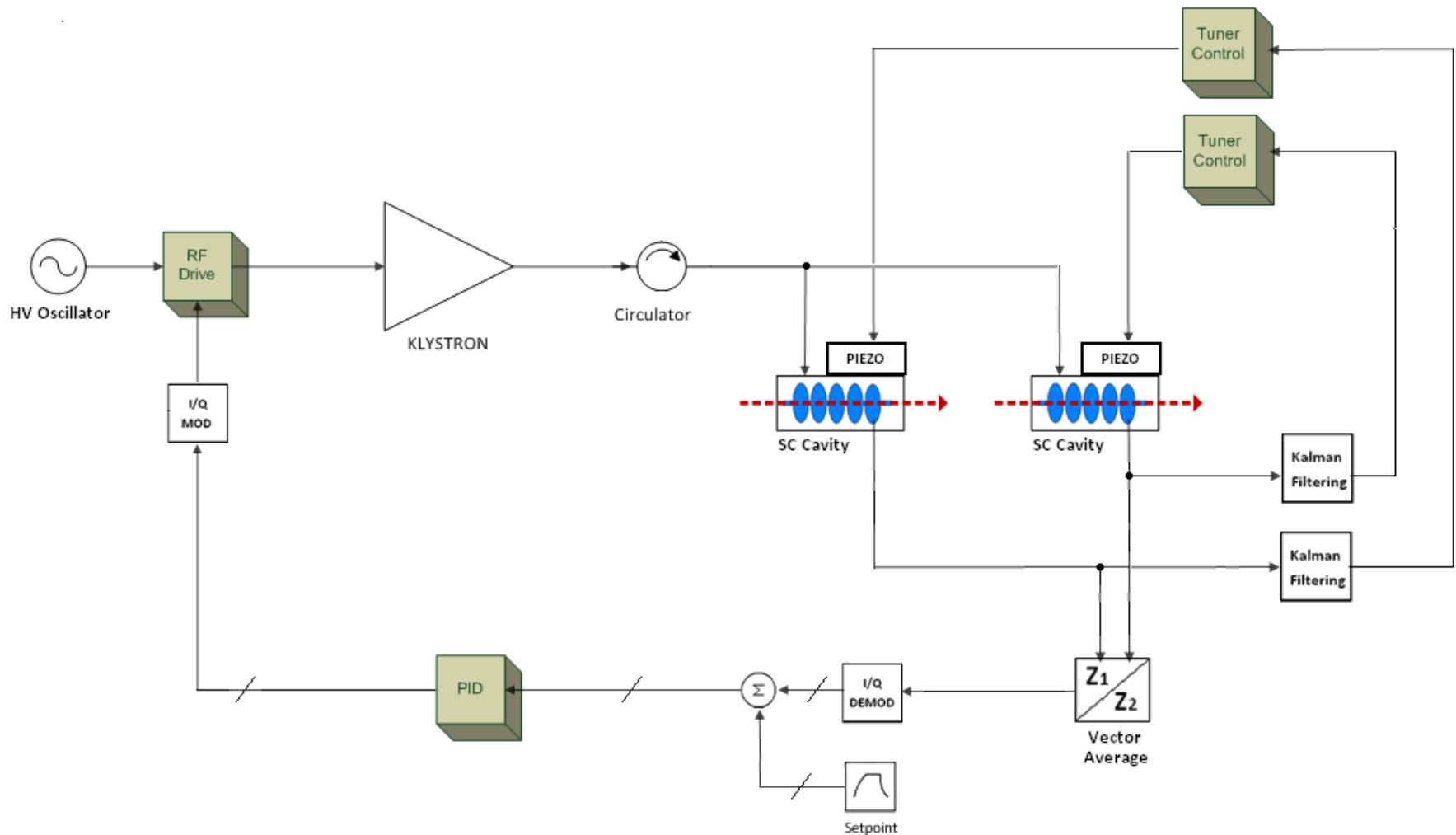


- Weaker beamloading will result in a higher flattop equilibrium and less phase detuning of the cavity for the same generator power.
- Beta value taken from beam energy at beginning of SPL  $\beta=1$  section.





# High Level Diagram for Dual Cavity + Control System





# 2-Cavity GUI

**SPLGUI**

**Start Simulation**

2-Cav  Feedback  Feed-Forward

**Operating Parameters**

Generator Frequency (Hz)	704.4e6	Synchronous Angle (Deg) LINAC	15	R/Q LINAC	525	Qloaded (Specify if Fixed)	1.312e6
Beam Current (A)	40e-3	Accelerating Field (V/m)	25e6	Tpulse (s)	1.2e-3	Ql 2 (Specify if Fixed)	1.31e6
Per Shot Variation (%lb)	0					Lorentz Coefficient K (Hz/(MV/m) <sup>2</sup> )	-1
							-0.8

K 2

Time Elapsed: 319.352

**3rd Turn Phase**

Phase (deg) vs time (s)

Axis Control: Xlims [X1 X2] [0 0.002] Autoscale Ylims [Y1 Y2] [-0.5 1.5] AutoZoom

**Lorentz Frequency Detuning (1)**

Frequency Deviation (Hz) vs time (s)

Axis Control: Xlims [X1 X2] Autoscale Ylims [Y1 Y2] AutoZoom

**Lorentz Frequency Detuning (2)**

Frequency Deviation (Hz) vs time (s)

Axis Control: Xlims [X1 X2] Autoscale Ylims [Y1 Y2] AutoZoom

Lorentz Detuning

- Cavity Voltage Vsum
- Cavity Voltage (1)
- Cavity Voltage (2)
- Cavity Voltage Phase Vsum
- Cavity Voltage Phase (1)
- Cavity Voltage Phase (2)
- Kalman Filter Outputs
- Lorentz Detuning
- Cavity Voltage Phase (1)



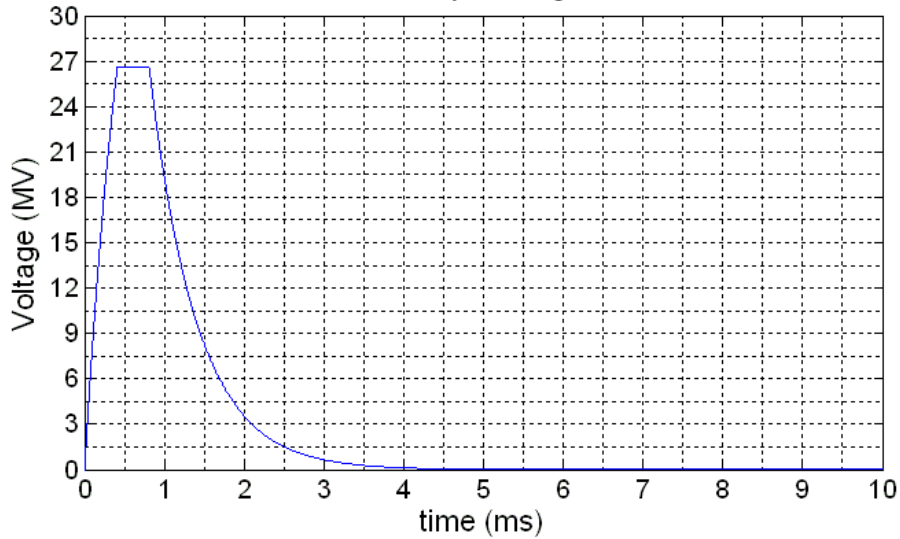
# Results

- Cavity Phase Variation Without Feed-Forward
- Effects of Adaptive Feed-Forward
- Effects of Loaded Quality Factor Variation



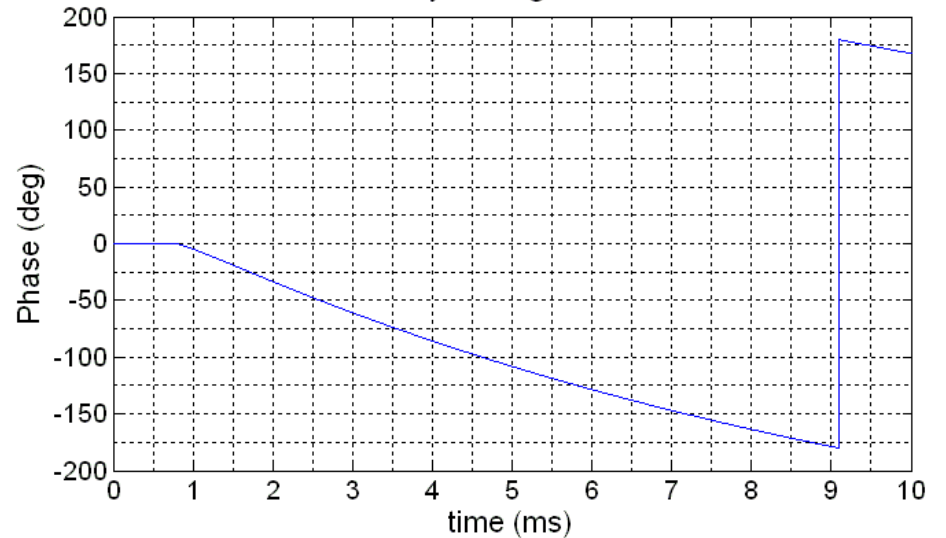
# Vcav Magnitude and Phase for Dual Cavity Case ( $K=-1$ and $-0.5$ )

Cavity Voltage



Voltage magnitude and  
phase of vector average

Cavity Voltage Phase

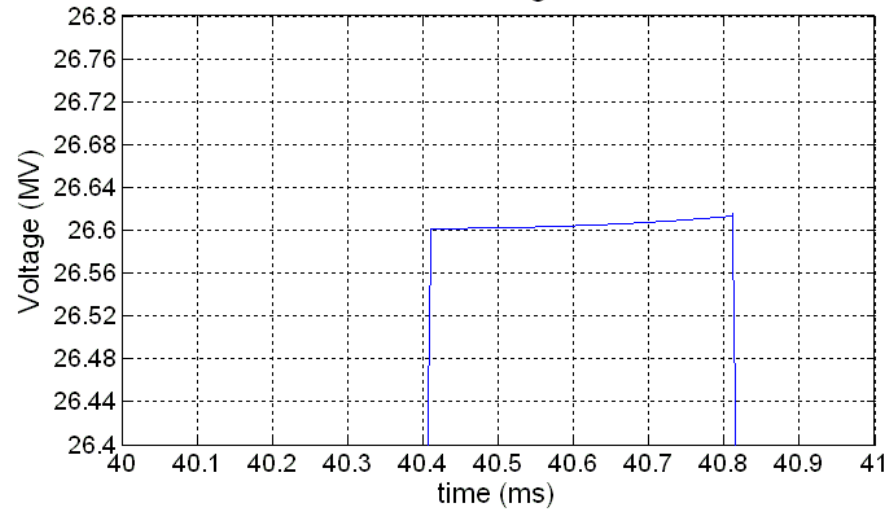




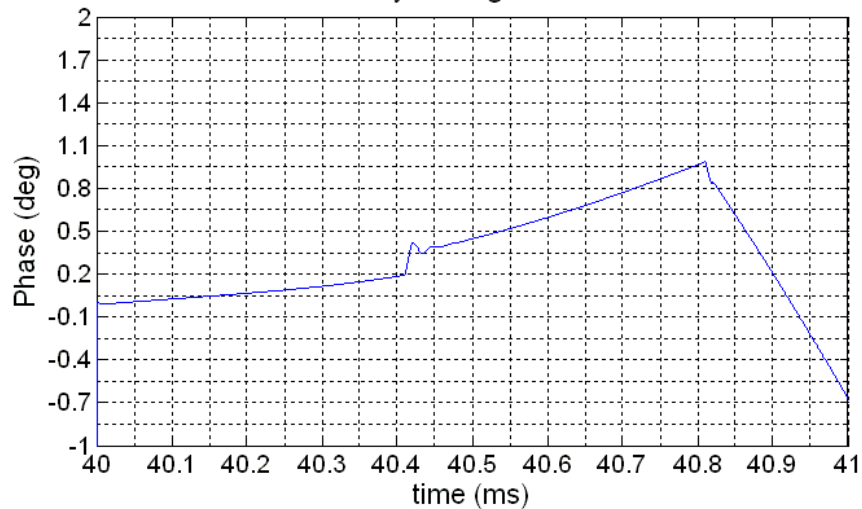
# Vcav Magnitude and Phase for Dual Cavity Case (Without Feed-Forward)

$$K = -0.5 \text{ Hz} \left( \frac{MV}{m} \right)^2$$

3rd Turn Magnitude

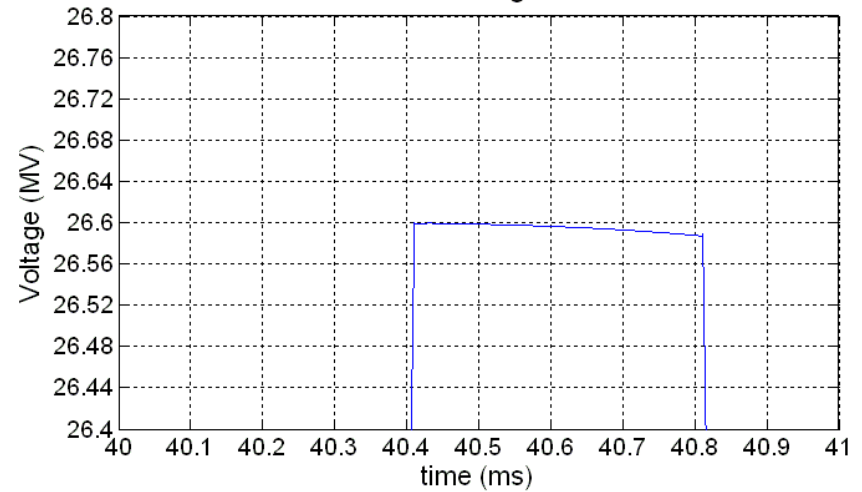


Cavity Voltage Phase

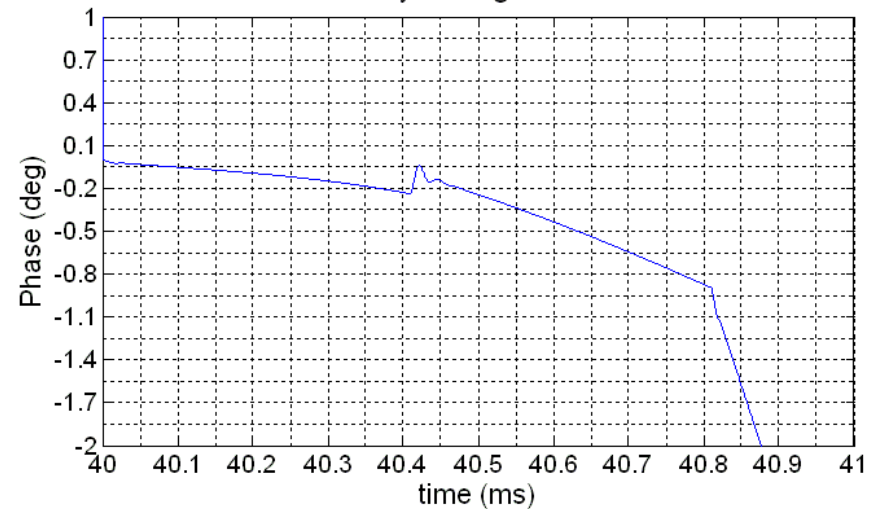


$$K = -1 \text{ Hz} \left( \frac{MV}{m} \right)^2$$

3rd Turn Magnitude

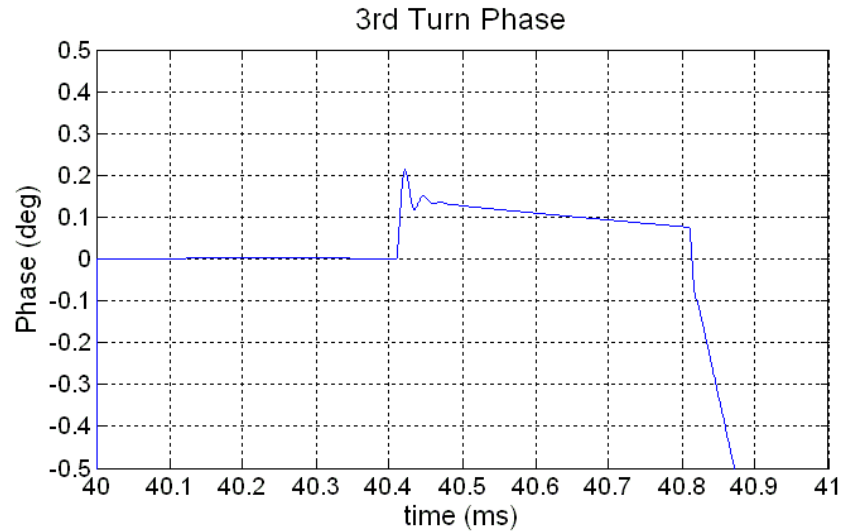


Cavity Voltage Phase

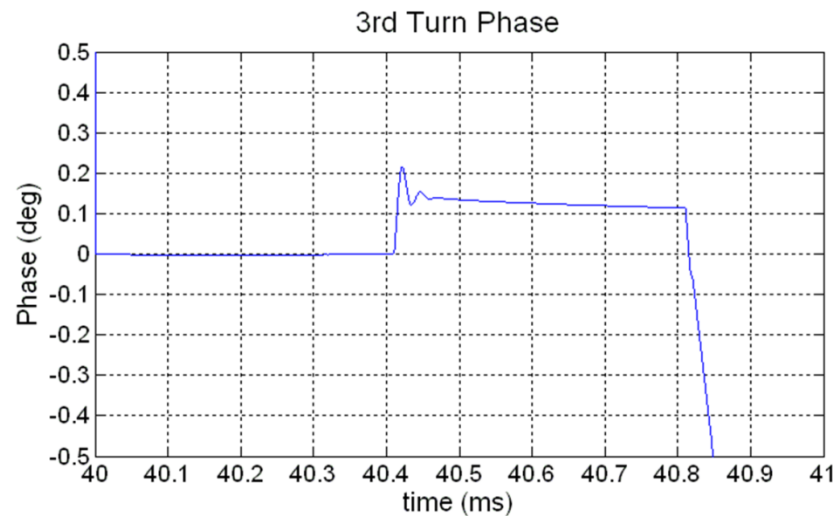


# Vcav Magnitude and Phase for Dual Cavity Case (With Feed-Forward)

Cavity 1 ( $K=-1 \text{ Hz} \left(\frac{MV}{m}\right)^2$ )

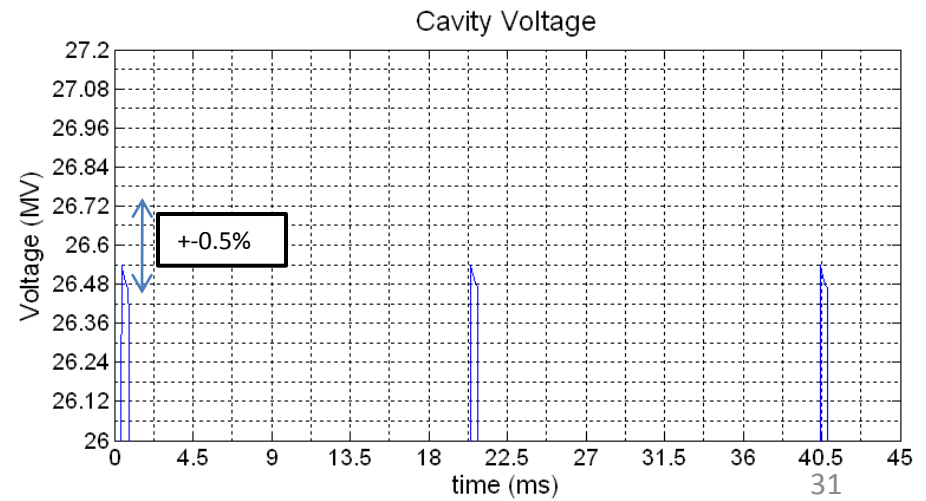
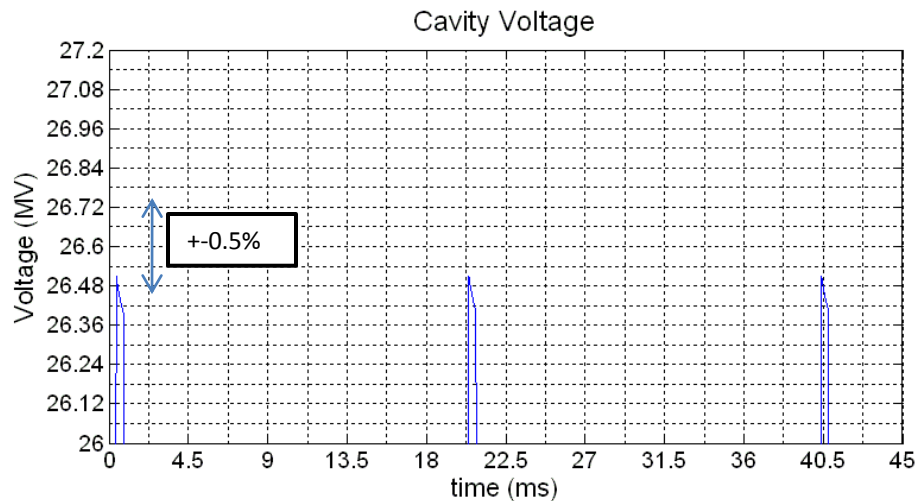
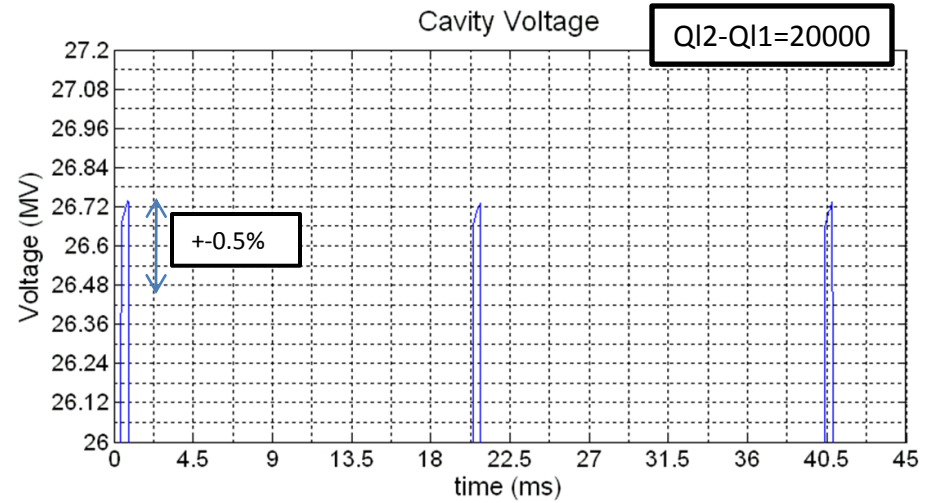
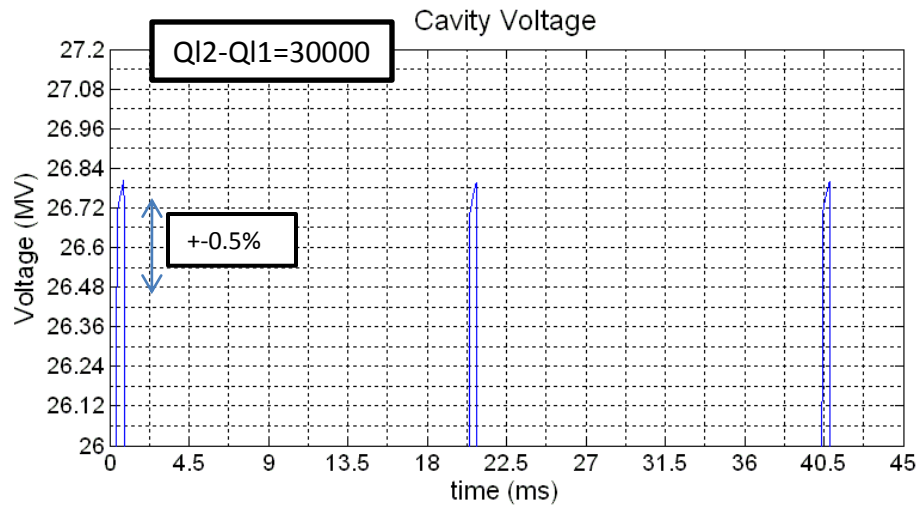


Cavity 2 ( $K=-0.5 \text{ Hz} \left(\frac{MV}{m}\right)^2$ )





# Loaded Quality Factor Fluctuation Effects on Cavity Voltage Magnitude





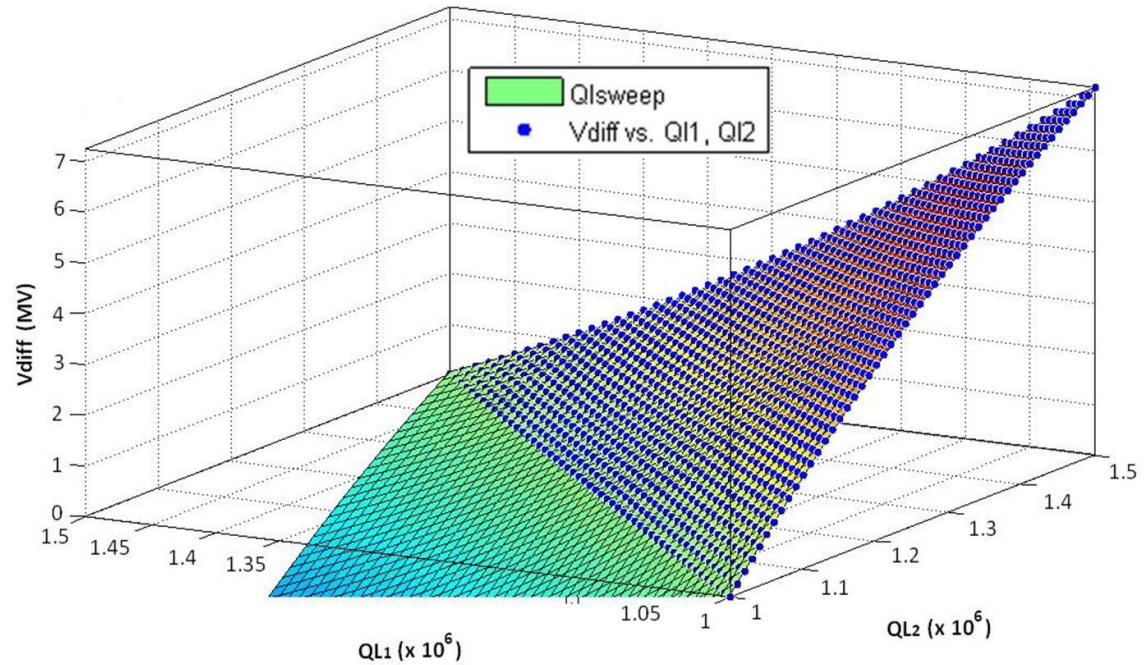
# Error Analysis

- Vector average is maintained within specifications with RF feedback loop, but individual cavities deviate depending on their parameters.
- Characterize deviation of cavity voltage with variations in loaded quality factor and Lorentz detuning coefficients
- Curves fitted for difference in cavity voltage magnitude and phase between 2 cavities controlled by a single RF feedback loop, with a setpoint at nominal accelerating voltage magnitude and phase.
- With this information, the overall effects of the cavity voltage deviation due to Lorentz detuning and loaded quality factor mismatches can be investigated with a model for the whole length of the SPL (investigated at CERN by Piero Antonio Posocco).



# Effects of Varying Loaded Quality Factor on Cavity Voltage Magnitude

$p_{00}=1.725e+006$   
 $p_{10}=34.88$   
 $p_{01}=-37.66$   
 $p_{20}=-8.311e-006$   
 $p_{11}=1.527e-007$   
 $p_{02}=9.27e-006$



$$V_{Diff} = f(Q_{L1}, Q_{L2}) = f(x, y) = V_{c2} - V_{c1}$$

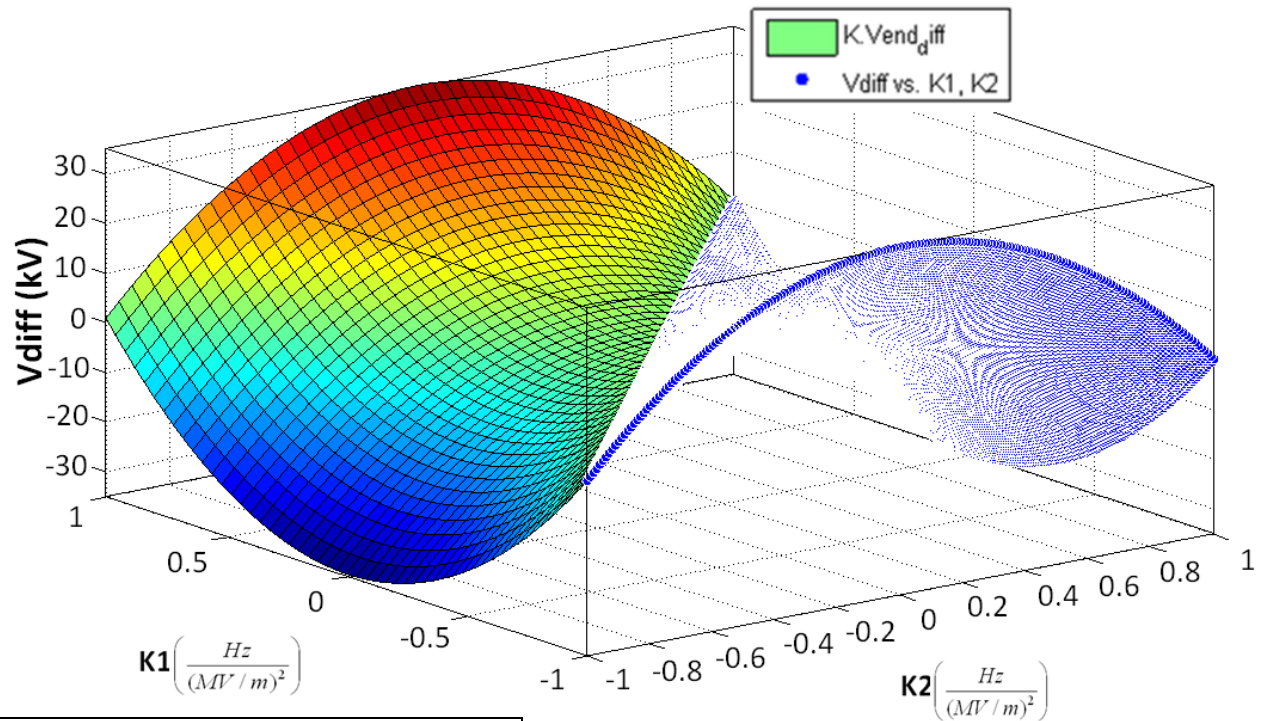
$$V_{c1} = V_{acc} + \frac{V_{Diff}}{2}$$

$$V_{c2} = V_{acc} - \frac{V_{Diff}}{2}$$

$$V_{Diff}(x, y) = p_{00} + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2$$

# Effects of Varying Lorentz Detuning Coefficient on Cavity Voltage Magnitude

$p_{00} = 25.8$   
 $p_{10} = -2.05e+014$   
 $p_{01} = 2.014e+014$   
 $p_{20} = -3.496e+028$   
 $p_{11} = -1.565e+026$   
 $p_{02} = 3.505e+028$



$$V_{\text{Diff}} = V_{c2} - V_{c1} = f(K_1, K_2) = f(x, y)$$

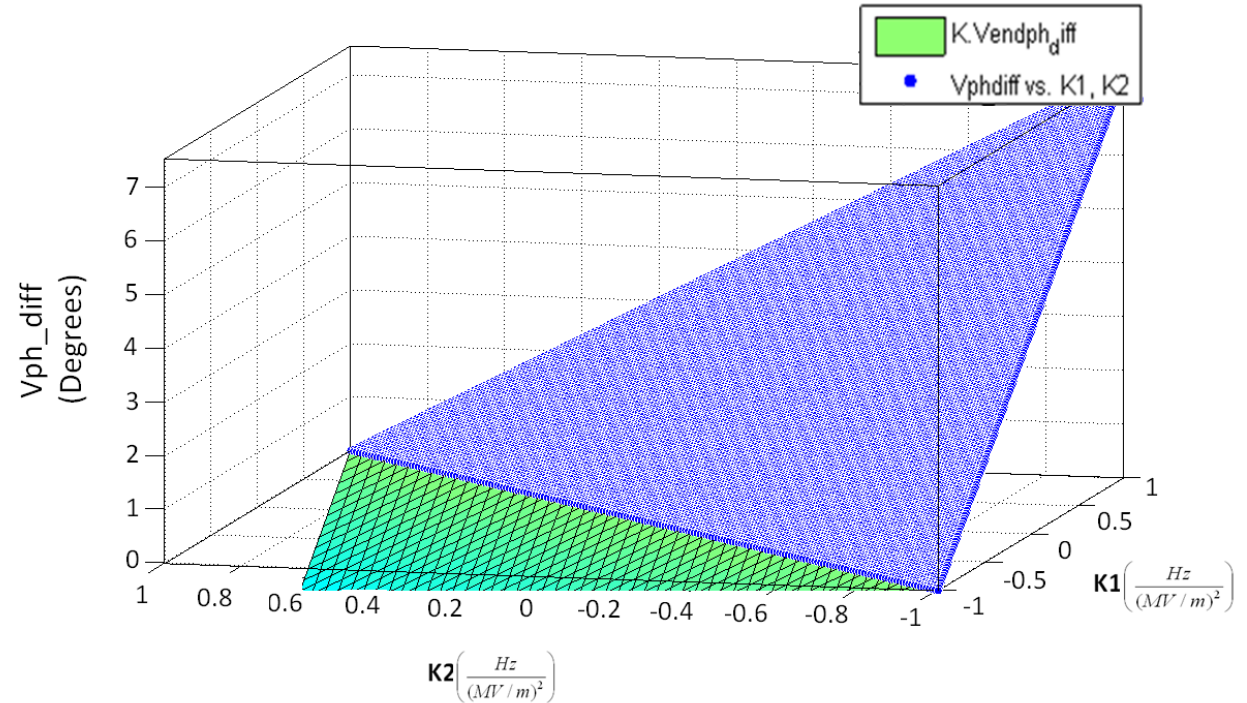
$$V_{c1} = V_{\text{acc}} + \frac{V_{\text{Diff}}}{2}$$

$$V_{c2} = V_{\text{acc}} - \frac{V_{\text{Diff}}}{2}$$

$$V_{\text{Diff}}(x, y) = p_{00} + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2$$

# Effects of Varying Lorentz Detuning Coefficient on Cavity Voltage Phase

$p_{00} = -0.0004408$   
 $p_{10} = 3.768e+012$   
 $p_{01} = -3.768e+012$



$$V_{Diff} = V_{c2} - V_{c1} = f(K_1, K_2) = f(x, y)$$

$$V_{c1} = V_{acc} + \frac{V_{Diff}}{2}$$

$$V_{c2} = V_{acc} - \frac{V_{Diff}}{2}$$

$$V_{Diff}(x, y) = p_{00} + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2$$



## In Summary...

- In order to cater for the needs of project specifications, a high flexibility simulation model was developed.
- Flexible graphical user interface allows for efficient handling of simulation data.
- 1, 2 and 4 cavities can be observed from RF point of view for a wide set of parameters.
- Can estimate practical issues that can arise during development of the real LLRF system in terms of power, stability of accelerating field and technology necessary for operation.



## Next Step

- Investigate different possible optimisations to transit time factor effects in terms of forward power, loaded quality factor and injection time along the LINAC.
- Characterize power amplifier and other components from real measurements in terms of their transfer functions.
- Characterize the behavior of the piezo-electronic tuner within the control loop.
- Develop a full digital/analogue control system using hardware and test in a cold cavity.