Some thoughts on theory uncertainties (2)

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Including photon exchange and photon form factor estimate:

(neglecting boxes and s-dependence of Z form factors)

$$A_4 = \frac{\sum_q X_q \, 4\left(\frac{v_\ell}{a_\ell}\frac{v_q}{a_q} + \frac{v_{\ell q}(s)}{a_\ell a_q}\right)}{\sum_q X_q \left(1 + \frac{v_\ell^2}{a_\ell^2} + \frac{v_q^2}{a_q^2} + \frac{v_{\ell q}^2(s)}{a_\ell^2 a_q^2}\right)}$$

$$X_q = f_q(x_1) f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1) f_q(x_2)$$

$$v_{\ell q}(s) = v_\ell v_q + \frac{s - M_Z^2 - i M_Z \Gamma_Z}{s} e^2 e_q \, (1 + \overline{\Delta}_q)$$

$$\frac{v_\ell}{a_\ell} = 1 - 4 s_\ell^2, \qquad s_\ell^2 \equiv \sin^2 \theta_{\text{eff}}^\ell$$

$$\frac{v_q}{a_q} = 1 - 4|e_q|(s_\ell^2 + \Delta_q)$$

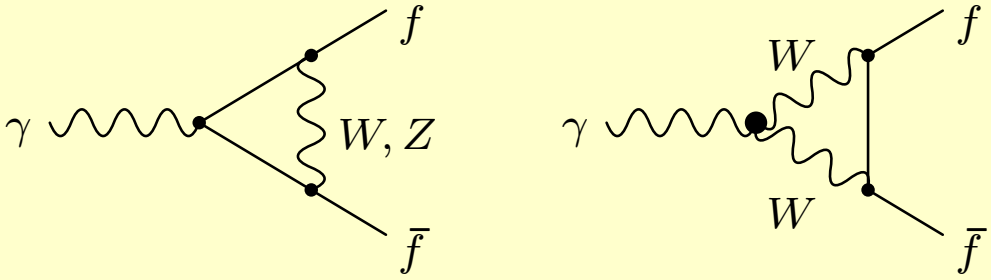
$$\Delta_q = \Delta_{q(1)} + \Delta_{q(2)}$$

$$\Delta_q = \underbrace{\overline{\Delta}_{q(1)}}_{\text{known}} + \underbrace{\overline{\Delta}_{q(2)}}_{\text{unknown}}$$

$\Delta_{q(2)}$ is known (in SM) for leading Z pole term

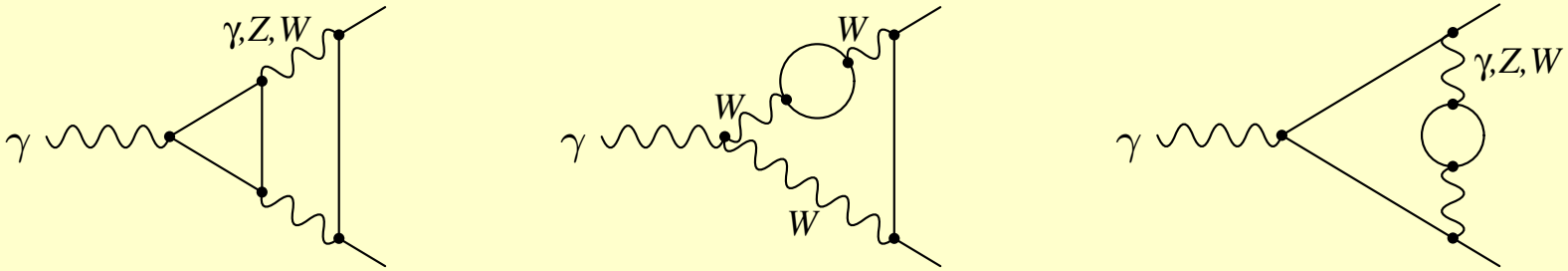
$\overline{\Delta}_{q(2)} = \pm \overline{\Delta}_{q(1)} \times \frac{g^2}{16\pi^2} n_f, \qquad n_f = 6 + 6N_c$ (maybe underestimate?)

Example contributions to $\Delta_{q(1)}$, $\overline{\Delta}_{q(1)}$:



Note: 1-loop boxes and s-dependence of Z vertex form factors also contribute at same order (1-loop without Z pole)

Example contributions to $\Delta_{q(1)}$, $\overline{\Delta}_{q(2)}$:



Z-pole 2-loop flavor dependence:

Assume: no EW 2-loop corrections included in analysis (i.e. they are theory unc.)

Schemes:

- α' : Use $\alpha, M_{\mathsf{W}}, M_{\mathsf{Z}}$ as inputs, perturb. exp. in α
- α : Use $\alpha, G_\mu, M_{\mathsf{Z}}$ as inputs, perturb. exp. in α
- G_μ : Use $G_\mu, M_{\mathsf{W}}, M_{\mathsf{Z}}$ as inputs, perturb. exp. in G_μ

Scheme:	α'	α	G_μ
$\Delta_{u(\alpha^2)}$ $[10^{-5}]$	-1.74	-1.82	-1.45
$\Delta_{d(\alpha^2)}$ $[10^{-5}]$	-1.49	-1.67	-0.88

Inputs: $M_{\mathsf{Z}} = 91.1876$ GeV, $M_{\mathsf{W}} = 80.385$ GeV, $M_{\mathsf{H}} = 125.7$ GeV

$m_{\mathsf{t}} = 173.5$ GeV, $\Delta\alpha = 0.059$, $\alpha_{\mathsf{s}} = 0.1184$, $G_\mu = 1.16638 \times 10^{-5}$ GeV^{-2}

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including non-factorizable EW$\times$QCD corrections:			
$\Delta_{u(\alpha^2+\alpha\alpha_{\mathsf{s}})}\,[10^{-5}]$	$+1.47$	$+1.38$	$+1.74$
$\Delta_{d(\alpha^2+\alpha\alpha_{\mathsf{s}})}\,[10^{-5}]$	$+2.34$	$+2.15$	$+2.95$

Czarnecki, Kühn '96

Harlander, Seidensticker, Steinhauser '97

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Including photon exchange and photon form factor estimate:

Impact of EW 2-loop contributions (without EW$\times$QCD):

$\delta A_4/A_4$: [10^{-4}]

$m_{\ell\ell}$ [GeV]	Scheme:	α'	α	G_μ
60		0.37	0.35	15.50
70		0.52	0.60	8.99
80		1.53	1.61	37.37
M_Z-2		17.54	10.27	208.5
M_Z-1		2.14	1.97	27.6
M_Z		0.58	0.59	0.57
M_Z+2		0.45	0.46	10.61
M_Z+1		0.55	0.55	16.15
100		0.84	0.83	24.85
110		0.80	0.81	21.71
130		0.53	0.56	12.34
150		0.34	0.38	6.04

- dominated by photon form factor unc. $\overline{\Delta}_q$
- artifically large corrections for G_μ scheme

  [same for (G_μ, s_ℓ, M_Z) scheme?]

Comments and discussion points

- Dependence of form factors on $s = m_{\ell\ell}$ and box contributions not taken into account so far
- IFI box and other QED requires separate uncertainty estimate
- Large corrections for G_μ scheme from photon exchange contribution:
  $\alpha = \frac{\sqrt{2} G_\mu s_{\mathsf{w}}^2 M_{\mathsf{W}}^2}{\pi}(1 - \Delta r)$
  $\rightarrow$ Anything different being done in generators?
- Plan for $\mathcal{O}(\alpha\alpha_{\mathsf{s}})$:
  include in analysis, or use available results for error estimate?
  [should be added in quadr. to $\mathcal{O}(\alpha^2)$ estimate]