Università degli Studi di Milano

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Exact mixed NNLO QCD-EW corrections to the Neutral Current Drell-Yan process

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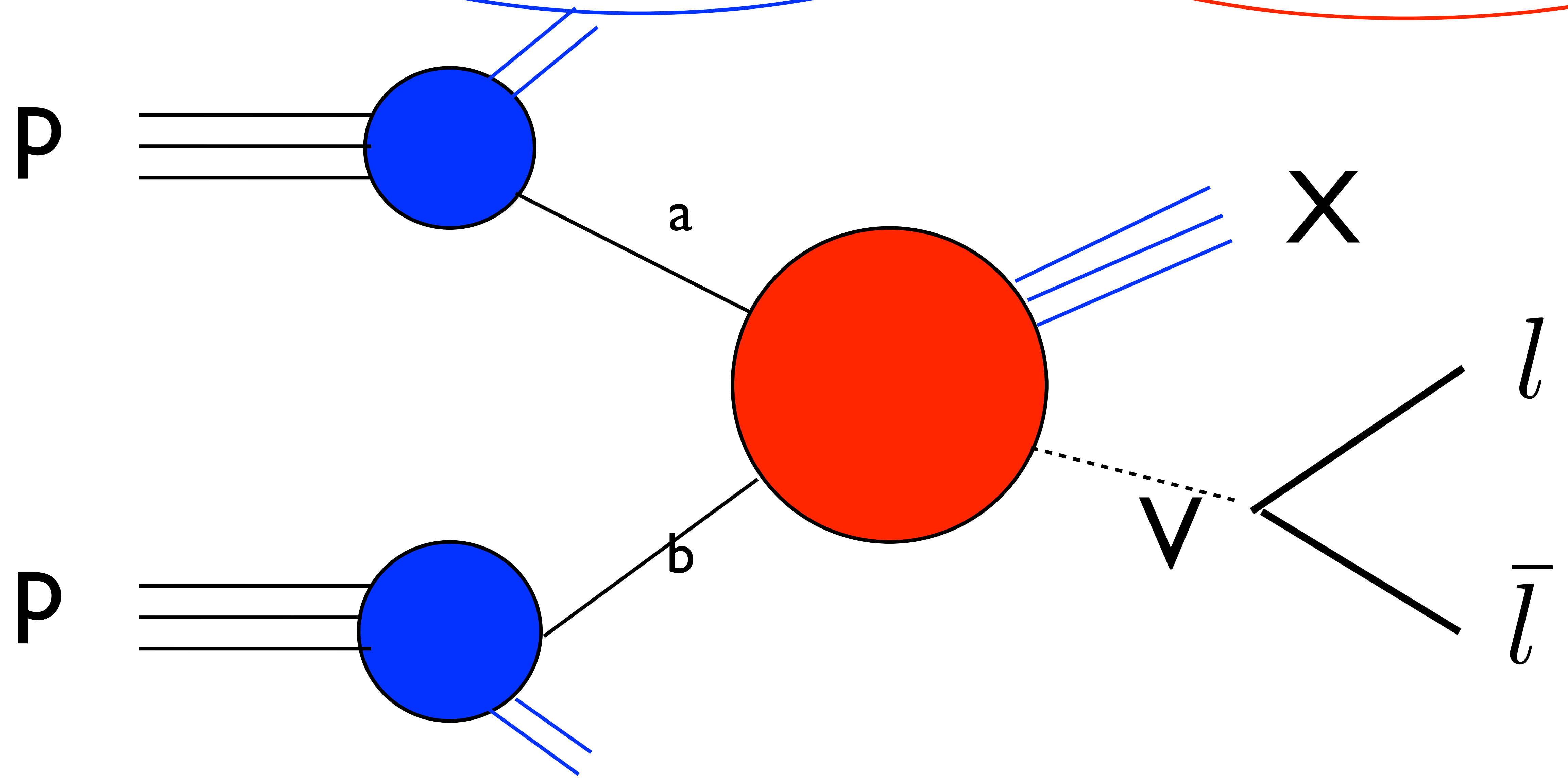
EW WG meeting, CERN/online, December 10th 2021

in collaboration with: R.Bonciani, F.Buccioni, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano,

based on arXiv:2106.11953, arXiv:2007.06518, arXiv:2111.12694

Lepton-pair production at hadron colliders

$$\sigma(P_1, P_2; m_V) = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{h_1,a}(x_1, M_F) f_{h_2,b}(x_2, M_F) \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2, \alpha_s(\mu), M_F)$$

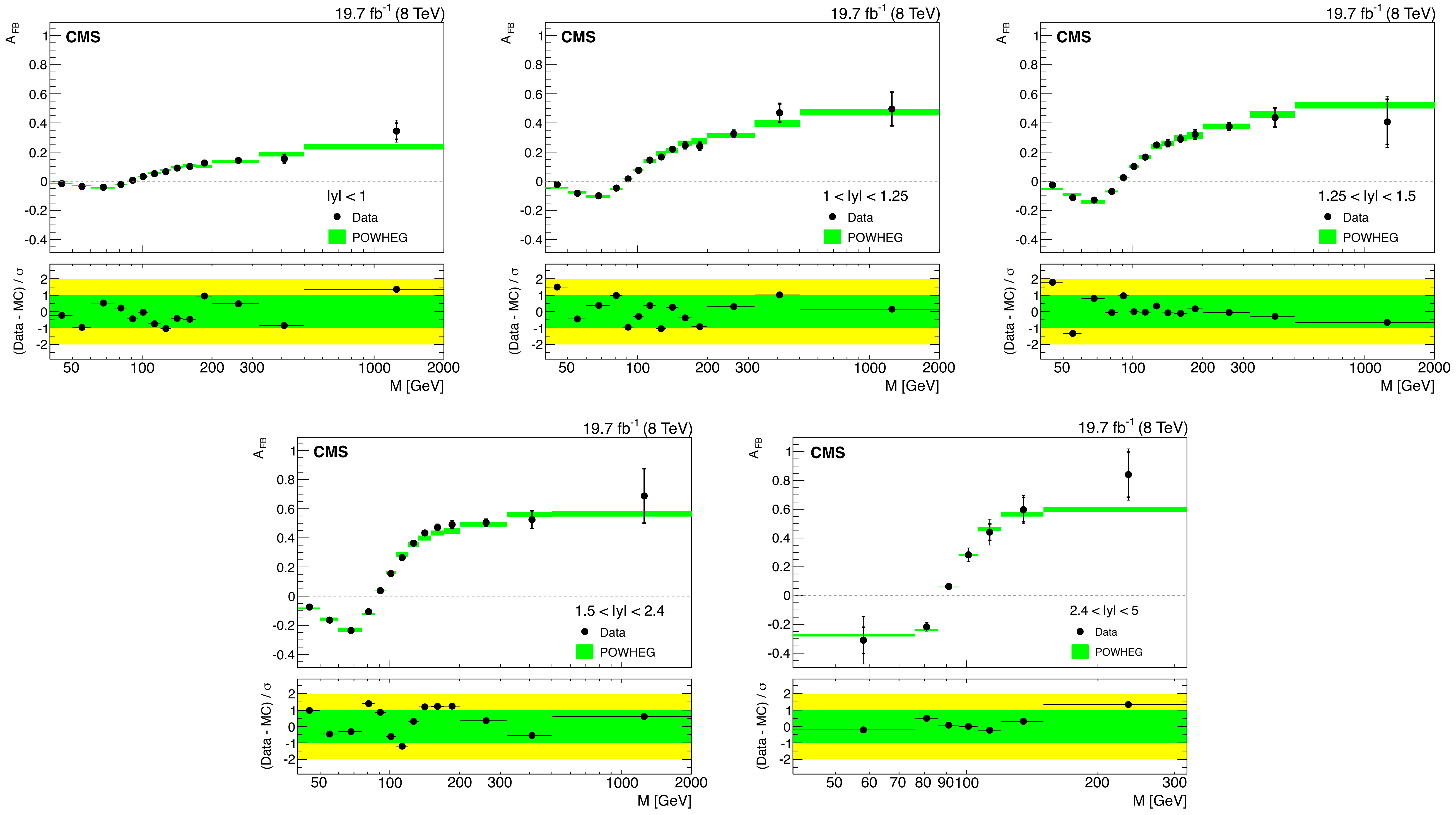


The factorisation theorems guarantee the validity of the above picture up to power correction effects

The interplay of QCD and EW interactions appears both in the partonic cross section and in the proton PDFs

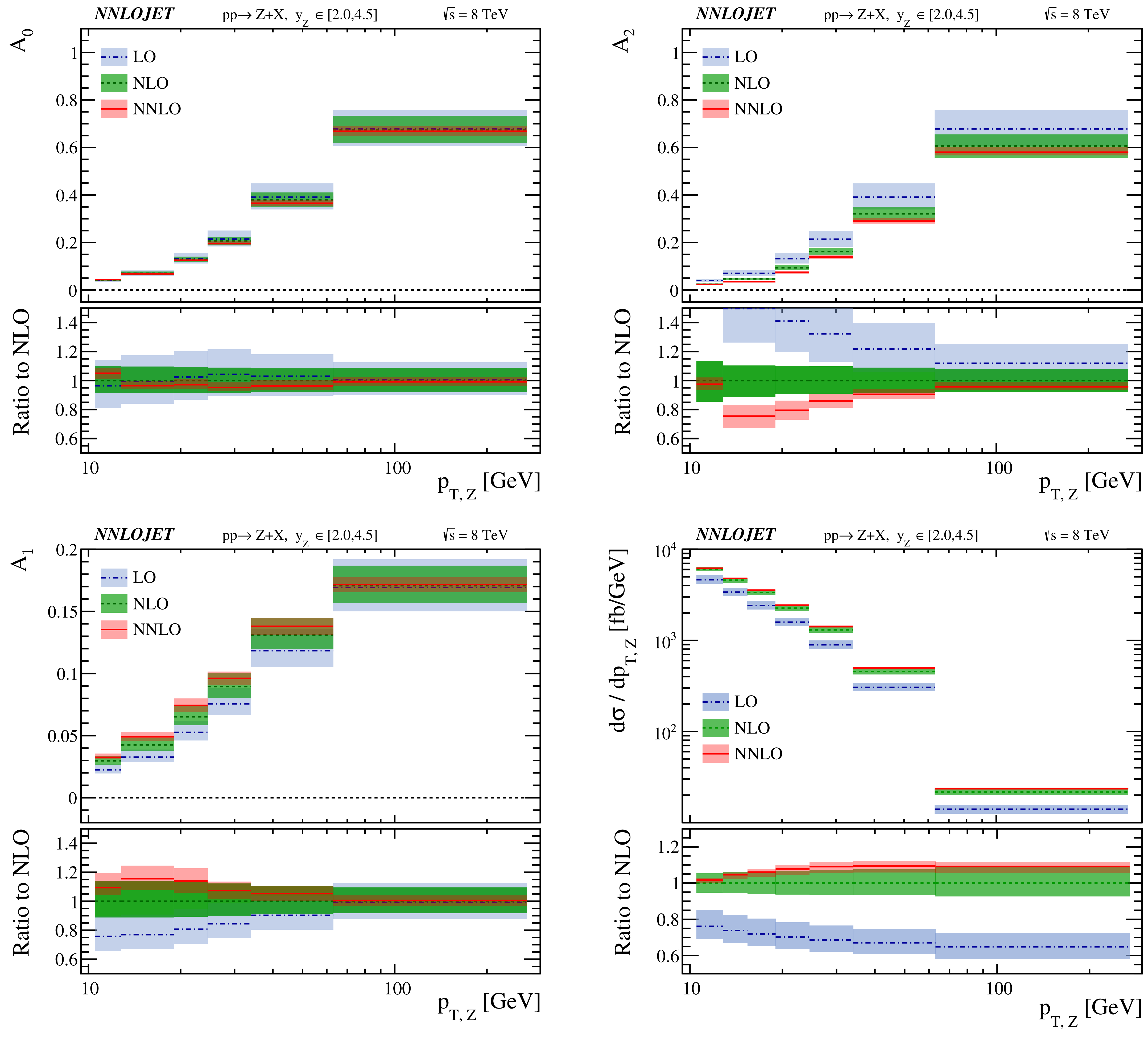
Relevance of NC Drell-Yan measurements: high-precision tests of the Standard Model

CMS, arXiv:1806.00863



Channel	Not constraining PDFs	Constraining PDFs
Muons	0.23125 ± 0.00054	0.23125 ± 0.00032
Electrons	0.23054 ± 0.00064	0.23056 ± 0.00045
Combined	0.23102 ± 0.00057	0.23101 ± 0.00030

Gauld et al., arXiv:1708.00008

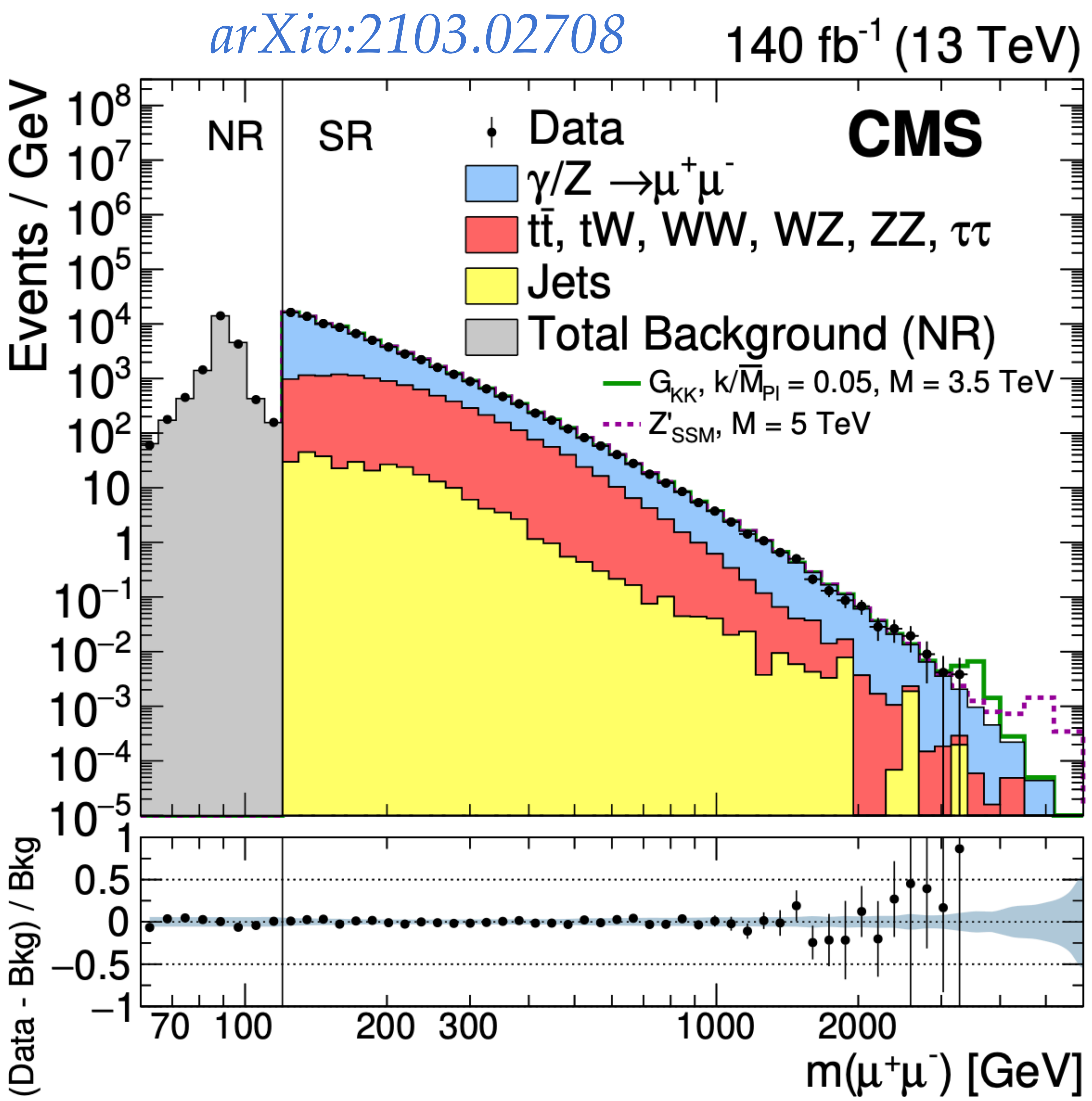


A determination of $\sin^2\theta_{eff}^{lep}$ competitive with the LEP results (0.23152(16)) is becoming possible

QCD corrections are fundamental in the study of the angular coefficients

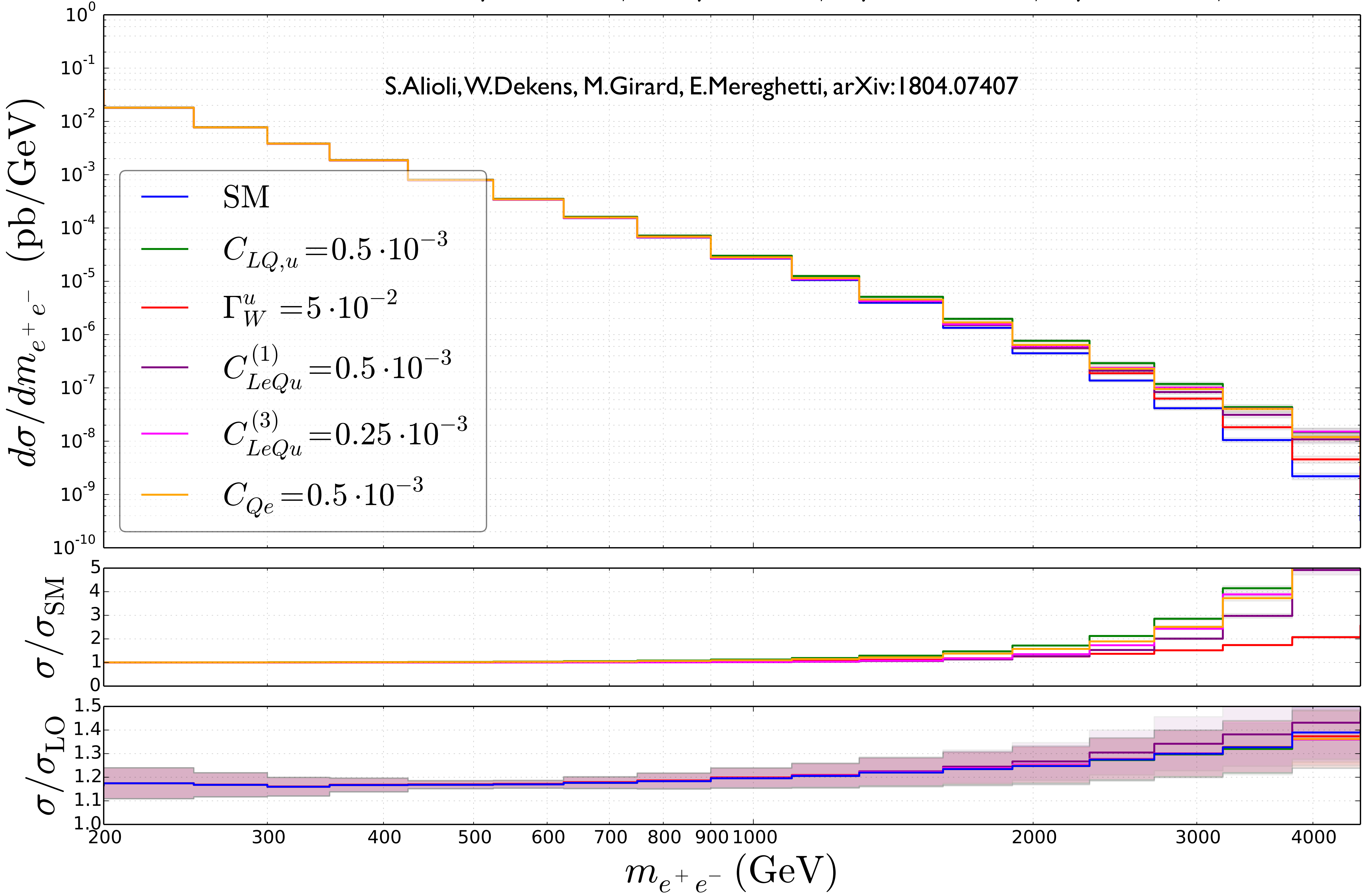
can modulate the shape of $A_{FB}(m_{ll}^2)$ in the Z resonance region

Relevance of NC Drell-Yan measurements: searches for New Physics signals



mass window [GeV]	stat. unc. 140fb^{-1}	stat. unc. 3ab^{-1}
600<$m_{\mu\mu}$<900	1.4%	0.2%
900<$m_{\mu\mu}$<1300	3.2%	0.6%

$$\mathcal{L} = \mathcal{L}_{X^2\varphi^2} + \mathcal{L}_{\psi^2 X\varphi} + \mathcal{L}_{\psi^2\varphi^2 D} + \mathcal{L}_{\psi^2\varphi^3} + \mathcal{L}_{\psi^4}$$



A deviation from the SM prediction can point towards New Physics

Is the SM prediction under control at the O(0.5%) level in the TeV region of the $m_{\ell\ell}$ distribution ?

Available tools and results

fixed order expansion

$$\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} +$$
$$\alpha_s\,\sigma^{(1,0)} + \alpha\,\sigma^{(0,1)} +$$
$$\alpha_s^2\,\sigma^{(2,0)} + \alpha\,\alpha_s\,\sigma^{(1,1)} + \alpha^2\,\sigma^{(0,2)} +$$
$$\alpha_s^3\,\sigma^{(3,0)} + \ldots$$

Drell-Yan (1970)

Baur, Brein, Hollik, Schappacher, Wackeroth (2001)

Altarelli, Ellis, Martinelli (1979)

Hamberg, Matsuura, van Nerveen, (1991)
Anastasiou, Dixon, Melnikov, Petriello, (2003)
Catani, Cieri, Ferrera, de Florian, Grazzini (2009)

still missing
Sudakov high-energy approximations

C.Duhr, B.Mistlberger, arXiv:2111.10379

R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953

resummation of logarithmically enhanced contributions

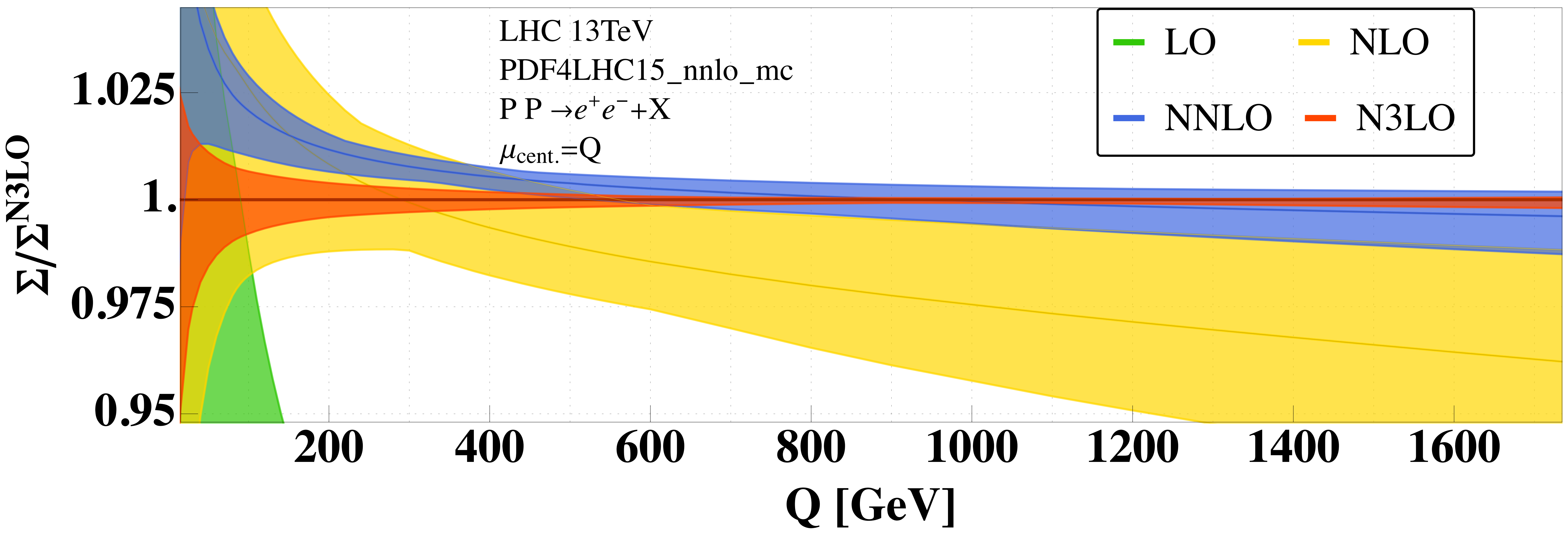
Monte Carlo: different matching algorithms at NLO-QCD +QCD-PS, NNLO-QCD +QCD-PS, NLO-EW +QED-PS
MC@NLO, POWHEG, MiNLO, MiNNLOPS, Geneva, HORACE, POWHEG QCD+EW

Integrators: different resummation formalisms for qt-resummation up to N3LL-QCD
ResBos, DYTurbo, RadIsh, SCETlib

most of the experimental analyses based on the "plug-and-play" convolution of NLO QCD and QED-FSR results
the uncertainties of this combination are not available and are observable dependent
→ it requires a formal and a phenomenological discussions and the development of combined QCD and QED resummation

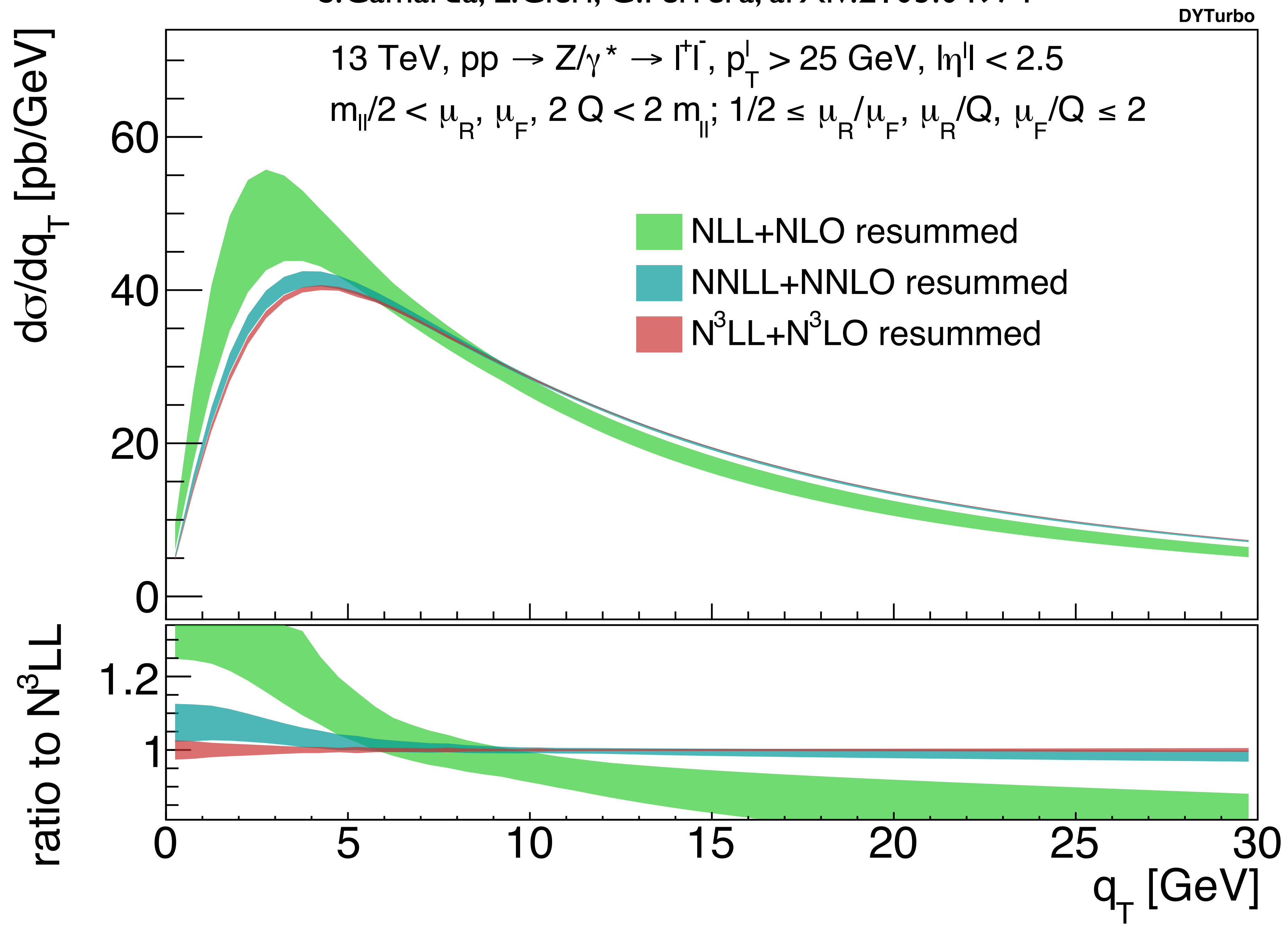
Progress in the QCD calculations and simulations

C.Duhr, B.Mistlberger, arXiv:2111.10379

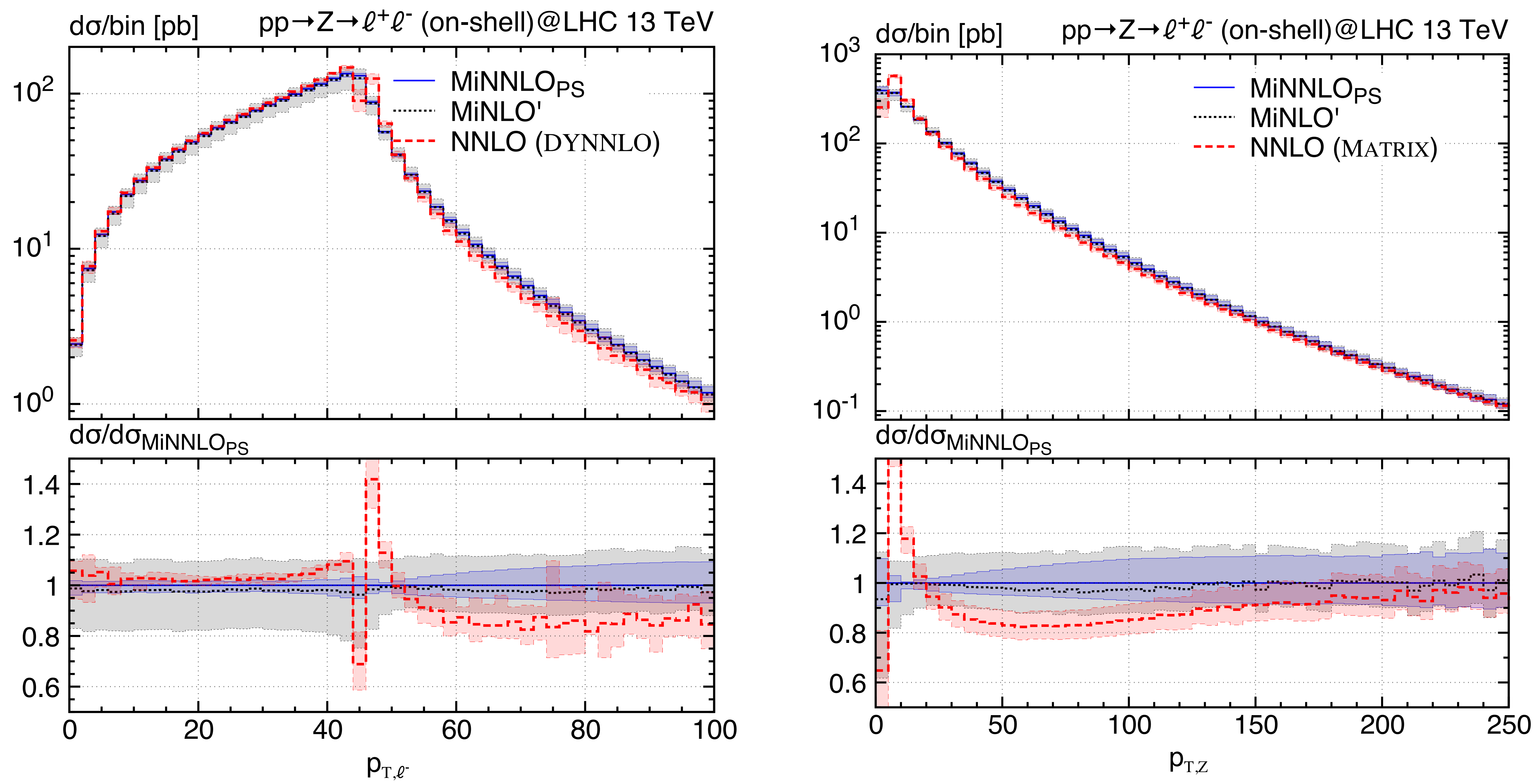


the few per mille precision is becoming reality for various different observables

S.Camarda, L.Cieri, G.Ferrera, arXiv:2103.04974

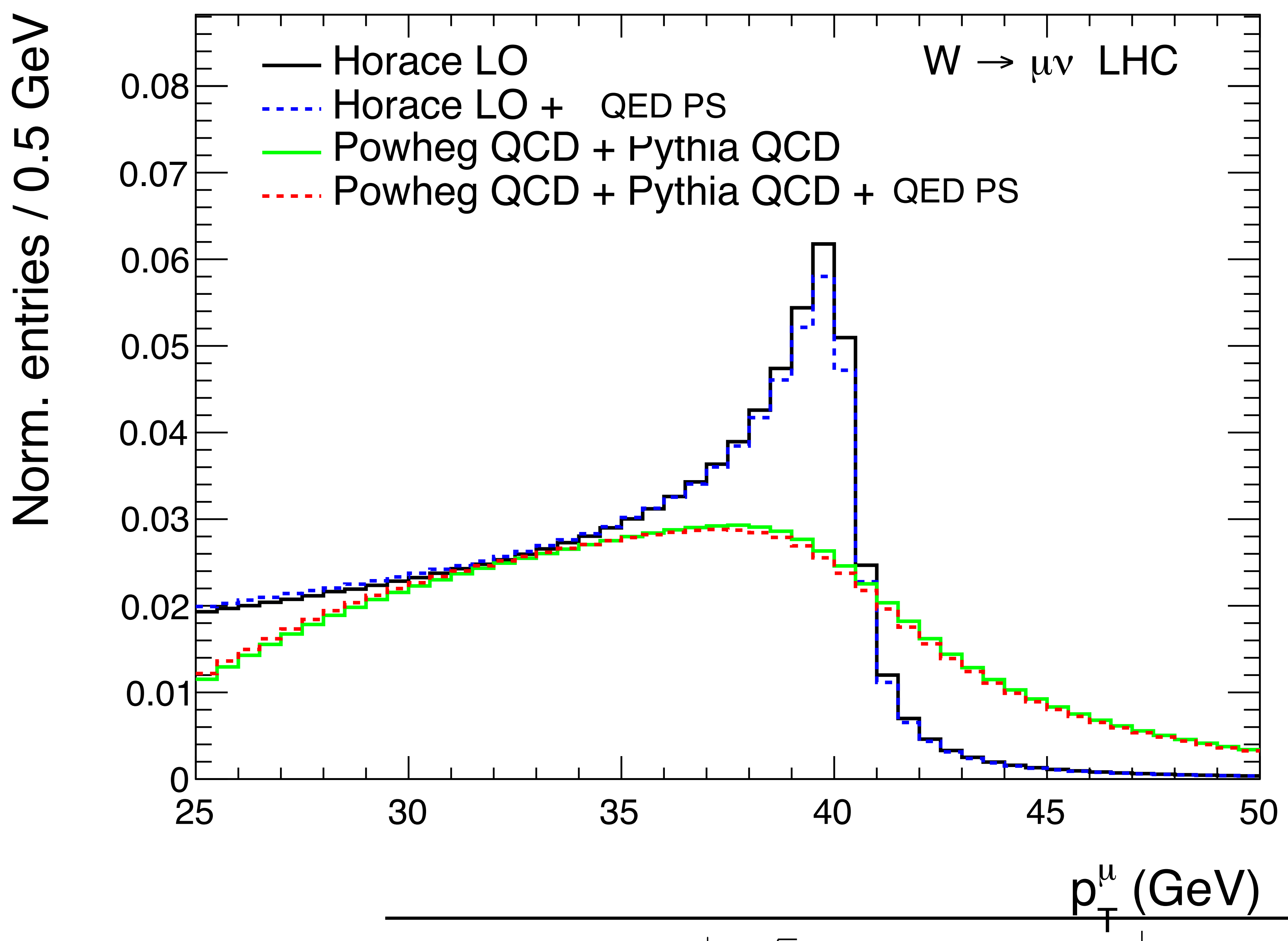


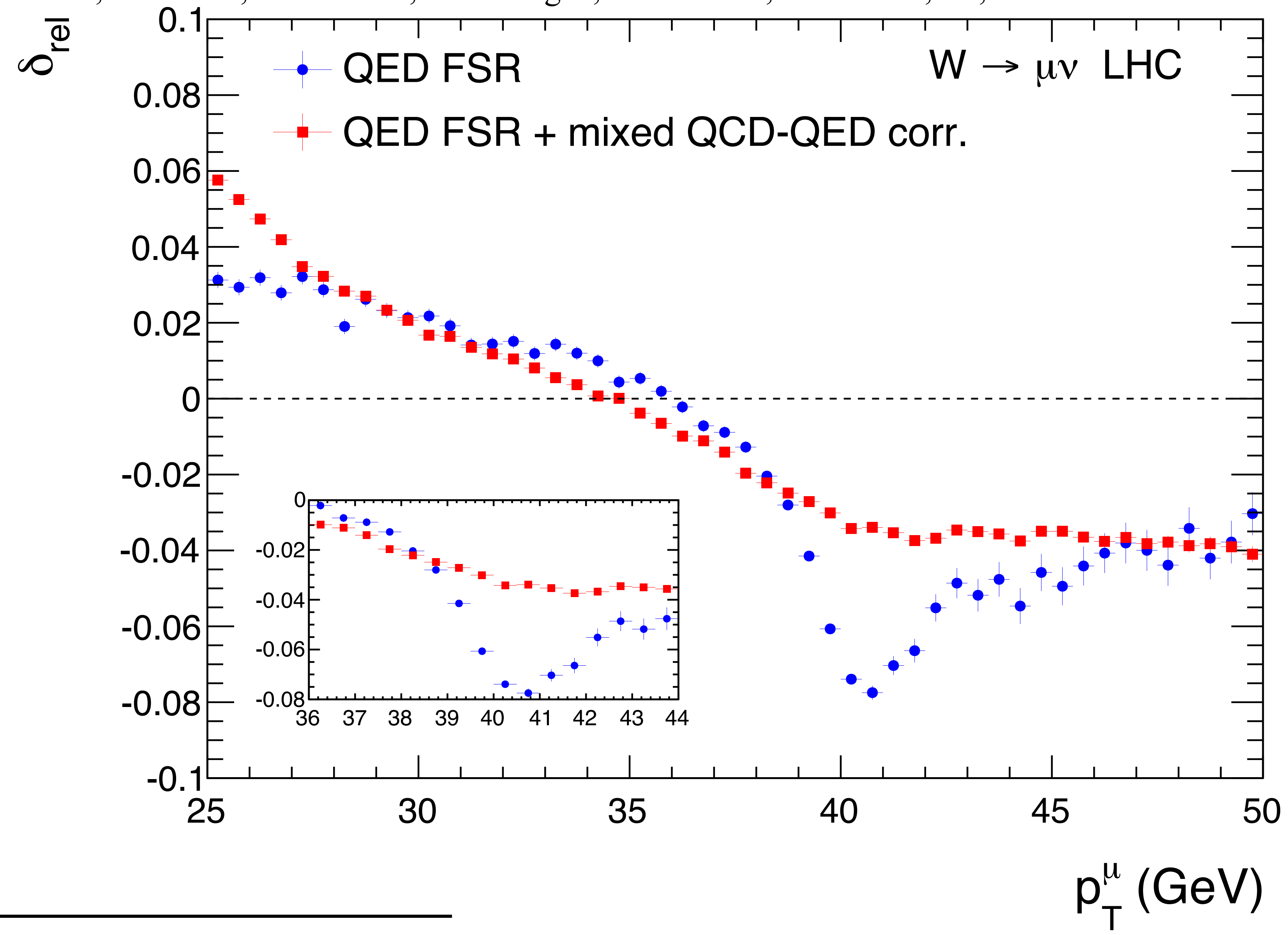
- P.F.Monni, P.Nason, E.Re, M.Wiesemann, G.Zanderighi, arXiv:1908.06987



Combined QCD-EW simulation tools

C.Carloni Calame, M.Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, AV, arXiv:1612.02841





$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: LO Pseudo–data accuracy	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$ M_T	 p_T^ℓ
1 HORACE only FSR-LL at $\mathcal{O}(\alpha)$	-94±1	-104±1	-204±1	-230±2
2 HORACE FSR-LL	-89±1	-97±1	-179±1	-195±1
3 HORACE NLO-EW with QED shower	-90±1	-94±1	-177±1	-190±2
4 HORACE FSR-LL + Pairs	-94±1	-102±1	-182±2	-199±1
5 PHOTOS FSR-LL	-92±1	$\boxed{-100\pm2}$	-182±1	-199±2

$pp \to W^+$, $\sqrt{s} = 14$ TeV Templates accuracy: NLO-QCD+QCD$_{PS}$ Pseudodata accuracy	 QED FSR	M_W shifts (MeV) $W^+ \to \mu^+\nu$ M_T	 p_T^ℓ	 $W^+ \to e^+\nu$(dres) M_T	 p_T^ℓ
1 NLO-QCD+(QCD+QED)$_{PS}$	PYTHIA	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2 NLO-QCD+(QCD+QED)$_{PS}$	PHOTOS	-88.0±0.6	$\boxed{-368\pm2}$	-38.4±0.6	-150±3
3 NLO-(QCD+EW)+(QCD+QED)$_{PS}$two-rad	PYTHIA	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4 NLO-(QCD+EW)+(QCD+QED)$_{PS}$two-rad	PHOTOS	-88.6±0.6	-370±3	-39.2±0.6	-159±2

the impact on MW of the mixed QCD QED-FSR corrections strongly depends on the underlying QCD shape/model

given that the bulk of the corrections is included in the analyses
- what is the associated uncertainty ?
- what happens if we change the underlying QCD model ?

can we constrain the formulation, for the $\alpha\alpha_s$ contribution ?

Progress towards Drell-Yan simulations at NNLO QCD-EW

Strong boost of the activities in the theory community in the last 2 years!

→ mathematical and theoretical developments and computation of universal building blocks

- 2-loop virtual Master Integrals with internal masses

U. Aglietti, R. Bonciani, arXiv:0304028, arXiv:0401193, R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581, M.Heller, A.von Manteuffel, R.Schabinger arXiv:1907.00491, S.Hasan, U.Schubert, arXiv:2004.14908, M.Long,R,Zhang,W.Ma,Y,Jiang,L.Han,,Z.Li,S.Wang, arXiv:2111.14130

- Altarelli-Parisi splitting functions including QCD-QED effects

D. de Florian, G. Sborlini, G. Rodrigo, arXiv:1512.00612

- renormalization

G.Degrassi, AV, hep-ph/0307122, S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229 S.Dittmaier, arXiv:2101.05154

→ on-shell Z and W production as a first step towards full Drell-Yan

- pole approximation of the NNLO QCD-EW corrections

S.Dittmaier, A.Huss, C.Schwinn, arXiv:1403.3216, 1511.08016

- analytical total cross section including NNLO QCD-QED and NNLO QED corrections

D. de Florian, M.Der, I.Fabre, arXiv:1805.12214

- ptZ distribution including QCD-QED analytical transverse momentum resummation

L. Cieri, G. Ferrera, G. Sborlini, arXiv:1805.11948

- fully differential on-shell Z production including exact NNLO QCD-QED corrections

M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:1909.08428

- total Z production cross section in fully analytical form including exact NNLO QCD-EW corrections

R. Bonciani, F. Buccioni, R.Mondini, AV, arXiv:1611.00645, R. Bonciani, F. Buccioni, N.Rana, I.Triscari, AV, arXiv:1911.06200, R. Bonciani, F. Buccioni, N.Rana, AV, arXiv:2007.06518, arXiv:2111.12694

- fully differential on-shell Z and W production including exact NNLO QCD-EW corrections

F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2005.10221, A. Behring, F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2009.10386, 2103.02671,

→ complete Drell-Yan

- neutrino-pair production including NNLO QCD-QED corrections

L. Cieri, D. de Florian, M.Der, J.Mazzitelli, arXiv:2005.01315

- 2-loop amplitudes

M.Heller, M.von Manteuffel, R.Schabinger, arXiv:2012.05918

- NNLO QCD-EW corrections to charged-current DY including leptonic decay (2-loop contributions in pole approximation).

L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539

The Neutral Current Drell-Yan cross section in the SM: perturbative expansion

$$
\begin{aligned}
\sigma(h_1 h_2 \to \ell\bar{\ell} + X) = \sigma^{(0,0)} &+ \\
&\alpha_s\, \sigma^{(1,0)} + \alpha\, \sigma^{(0,1)} + \\
&\alpha_s^2\, \sigma^{(2,0)} + \alpha\, \alpha_s\, \sigma^{(1,1)} + \alpha^2\, \sigma^{(0,2)} + \\
&\alpha_s^3\, \sigma^{(3,0)} + \ldots
\end{aligned}
$$

$$
\sigma(h_1 h_2 \to l\bar{l} + X) = \sum_{i,j=q\bar{q},g,\gamma} \int dx_1\, dx_2\, f_i^{h_1}(x_1, \mu_F) f_j^{h_2}(x_2, \mu_F)\, \hat{\sigma}(ij \to l\bar{l} + X)
$$

$\sigma^{(1,1)}$ requires the evaluation of the xsecs of the following processes, including photon-induced

$q\bar{q} \to l\bar{l},\ \gamma\gamma \to l\bar{l}$ including virtual corrections of $\mathcal{O}(\alpha_s), \mathcal{O}(\alpha), \mathcal{O}(\alpha\alpha_s)$

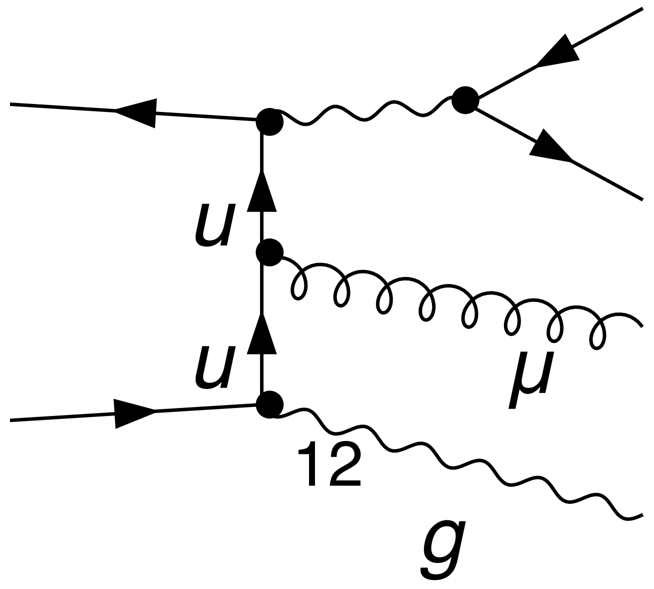
$q\bar{q} \to l\bar{l}g,\ qg \to l\bar{l}q$ including virtual corrections of $\mathcal{O}(\alpha)$

$q\bar{q} \to l\bar{l}\gamma,\ q\gamma \to l\bar{l}q$ including virtual corrections of $\mathcal{O}(\alpha_s)$

$q\bar{q} \to l\bar{l}g\gamma, qg \to l\bar{l}q\gamma, q\gamma \to l\bar{l}qg, g\gamma \to l\bar{l}q\bar{q}$

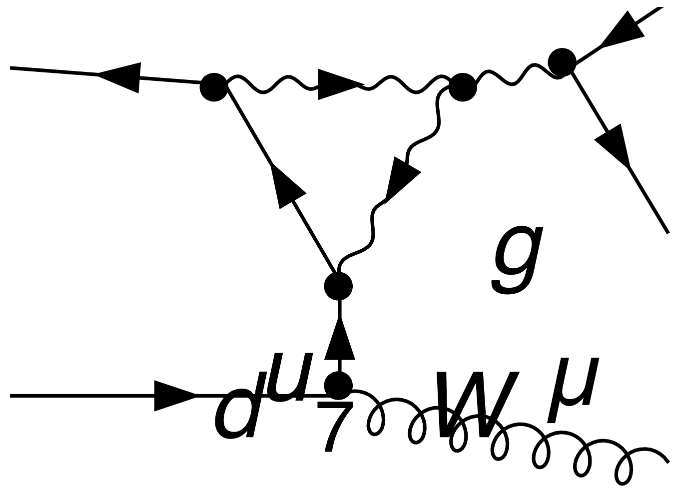
$q\bar{q} \to l\bar{l}q\bar{q},\ q\bar{q} \to l\bar{l}q'\bar{q}',\ qq' \to l\bar{l}qq',\ q\bar{q}' \to l\bar{l}q\bar{q}',\ qq \to l\bar{l}qq$ at tree level

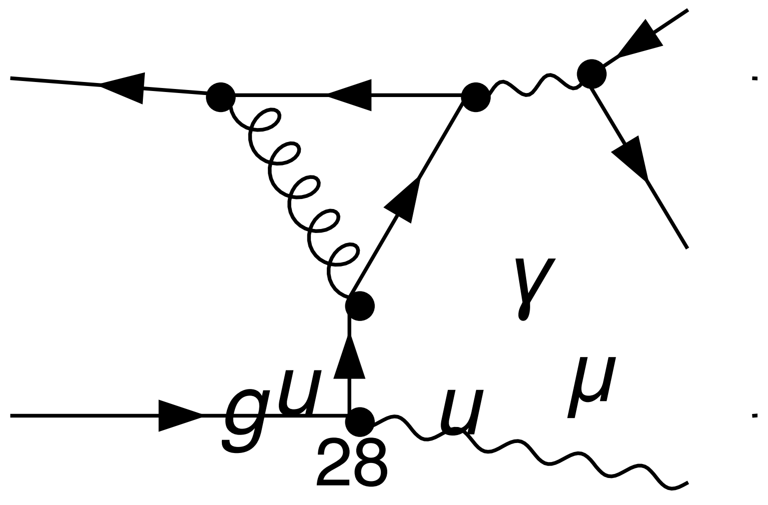
Different kinds of contributions at $\mathcal{O}(\alpha\alpha_s)$ and corresponding problems



double-real contributions

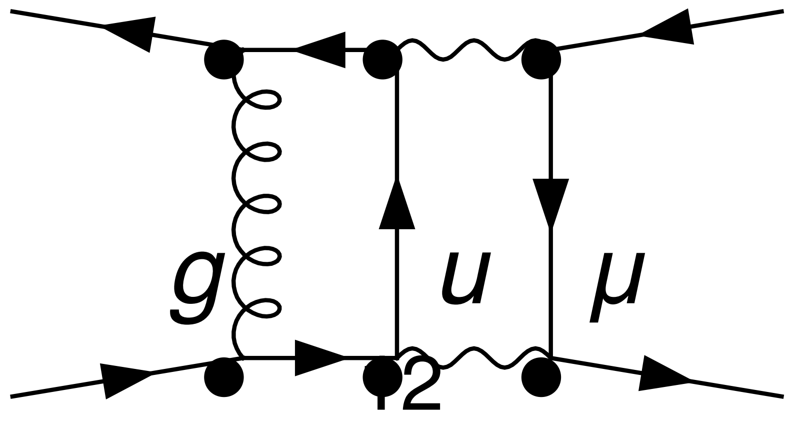
- amplitudes are easily generated with OpenLoops
- IR subtraction
- care about the numerical convergence when aiming at 0.1% precision

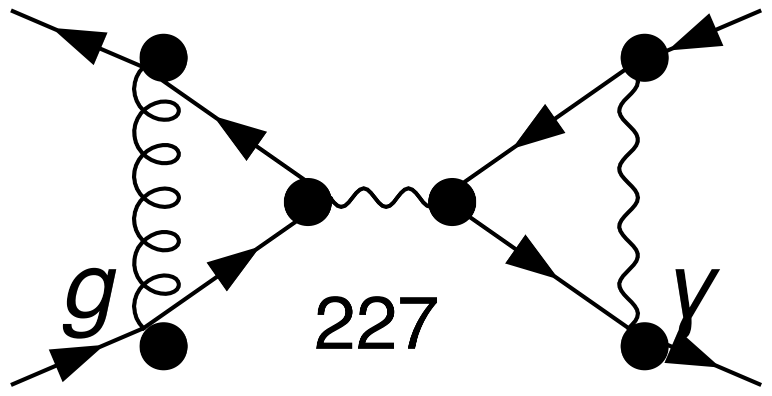




real-virtual contributions

- amplitudes are easily generated with OpenLoops or Recola
- 1-loop UV renormalisation and IR subtraction
- care about the numerical convergence when aiming at 0.1% precision

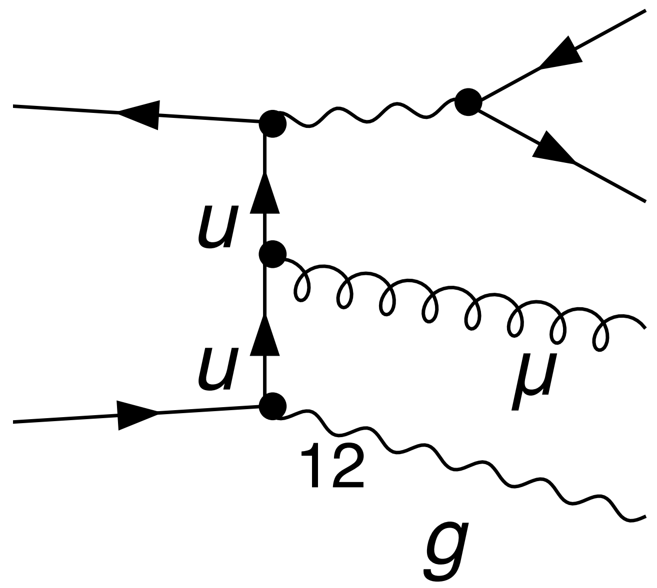




double-virtual contributions

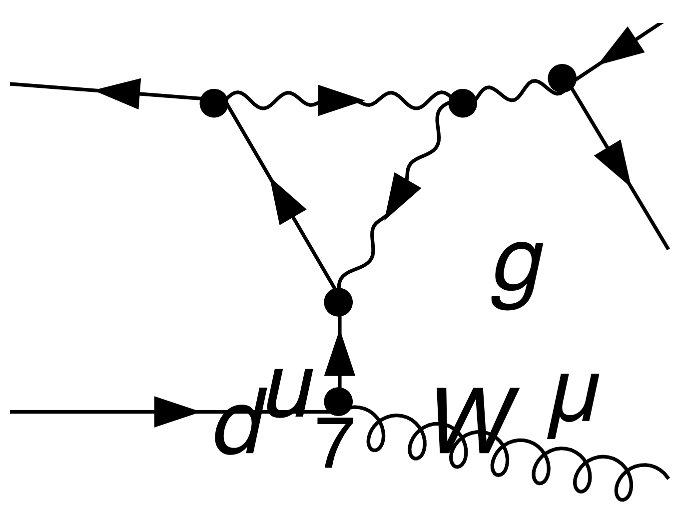
- generation of the amplitudes
- γ_5 treatment
- 2-loop UV renormalization
- solution and evaluation of the Master Integrals
- subtraction of the IR divergences
- numerical evaluation of the squared matrix element

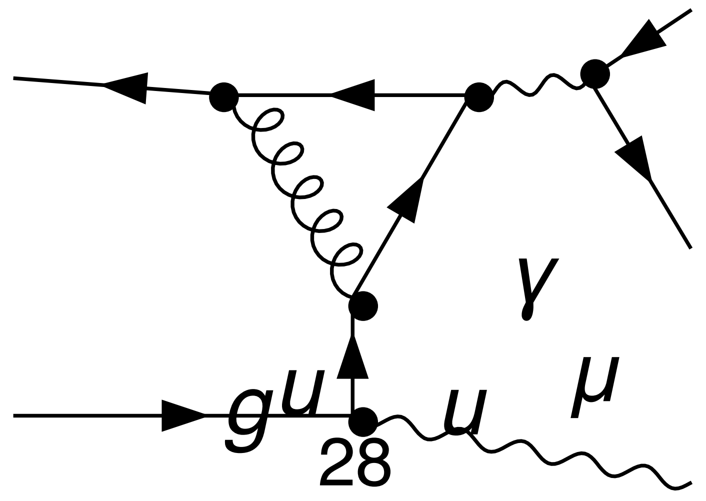
Different kinds of contributions at $\mathcal{O}(\alpha\alpha_s)$ and corresponding problems



double-real contributions

- amplitudes are easily generated with OpenLoops
- IR subtraction
- care about the numerical convergence when aiming at 0.1% precision





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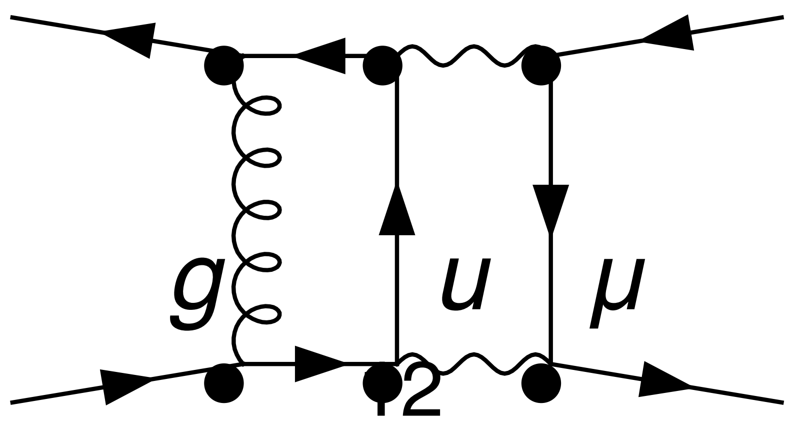

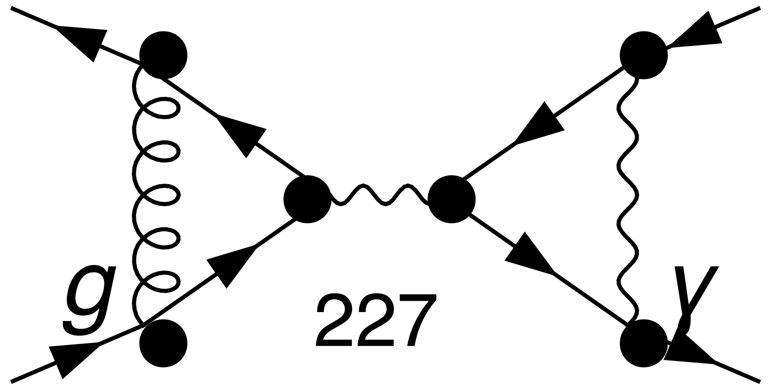

double-virtual contributions

- generation of the amplitudes
- γ_5 treatment
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- subtraction of the IR divergences
- numerical evaluation of the squared matrix element

The double-real and real-virtual corrections already known from studies of the large transverse momentum lepton pair final state

A.Denner, S.Dittmaier, T.Kasprzik, A.Muck, arXiv:1103.0914, A.Denner, S.Dittmaier, M.Hecht, C.Pasold, arXiv:1510.08742 J.Lindert et al., arXiv:1705.04664

Now we can consider the inclusive spectrum, also in the $q_T \to 0$ limit

Computational framework

The complete calculation has been included in the Munich/Matrix framework

- fully automatic generation and bookkeeping of all the double-real and real-virtual contributions based on an interface with OpenLoops and Recola/Collier
- the 2-loop virtual corrections are separately computed and provided in fast-evaluation format

Main compatibility requirement to include the double-virtual corrections:

the q_T-subtraction formalism to handle the IR singularities (Catani, Grazzini, 2007)

General structure of the inclusive cross section and the q_T-subtraction formalism

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

the IR-divergent behaviour of the different processes that contribute to the inclusive cross section and the cancellation of the IR divergences has been handled with the q_T-subtraction formalism

IR structure associated to the QCD-QED part derived from NNLO-QCD results via abelianisation
(de Florian, Rodrigo, Sborlini, 2016, de Florian, Der , Fabre, 2018)

the q_T-subtraction formalism has been extended to the case of final-state emitters (heavy quarks in QCD, leptons in EW)
(Catani, Torre, Grazzini, 2014, Buonocore, Grazzini, Tramontano 2019.)

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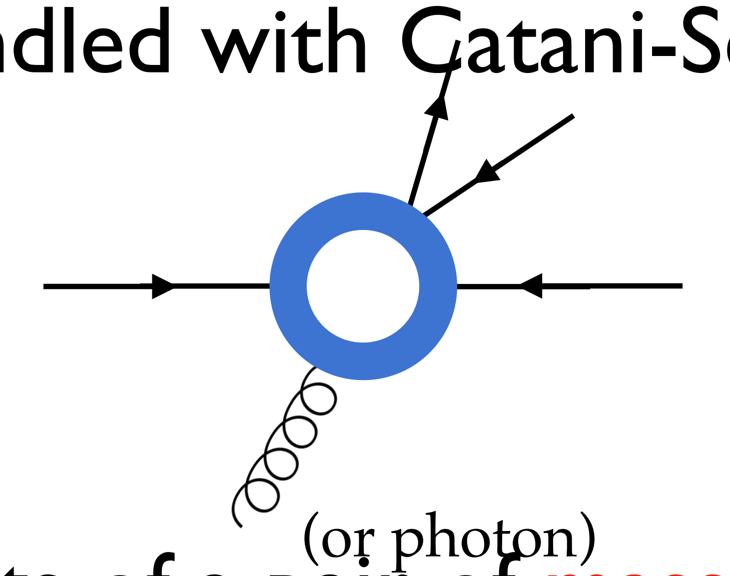
(Catani, Torre, Grazzini, 2014, Buonocore,Grazzini,Tramontano 2019.)

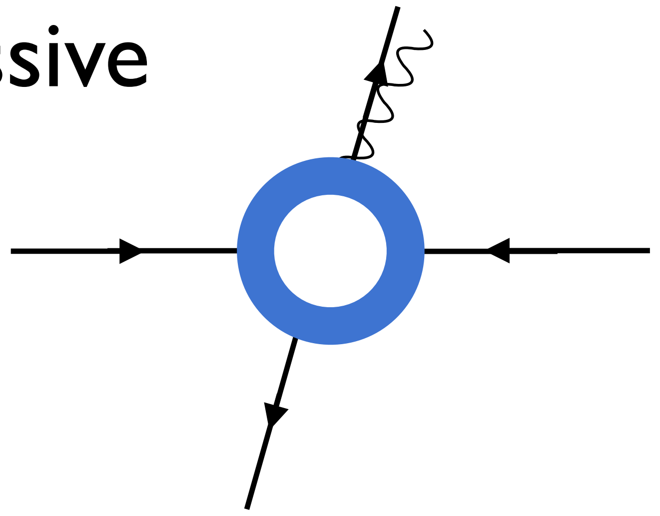
the gauge-boson phase space is split into $q_T = 0$ and $q_T > 0$ regions

$$r_{cut} = q_T^{cut}/Q$$

for ISR, if $q_T > 0$ the emitted parton is always resolved
and the process under study receives only NLO corrections
which can be handled with Catani-Seymour dipoles



in the FSR case, with $q_T > 0$,
the emitted parton is always resolved
only if the emitter is massive



the final state consists of a pair of massive leptons (treated as bare) to regulate the collinear (mass) singularities

The q_T-subtraction and the residual cut-off dependency

$$\boxed{d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathcal{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}}$$

The double-real and real-virtual contributions contribute to $d\sigma_R^{(1,1)}$
are demanding in terms of book-keeping of the many contributions
are generated with a fully automatised procedure for NLO corrections

Since the phase-space integration is numerical, the analytical complexity is that of a NLO calculation

When $q_T/Q > r_{cut}$ the double-real and the real-virtual contributions, subtracted with CS dipoles, are finite

$d\sigma_{CT}^{(1,1)}$ is obtained by expanding to fixed order the q_T resummation formula

The q_T-subtraction and the residual cut-off dependency

$$d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}$$

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$d\sigma_{CT}^{(1,1)}$ is obtained by expanding to fixed order the q_T resummation formula

Logarithmic sensitivity on r_{cut} in the double unresolved limit $\int d\sigma_R^{(1,1)} \sim \sum_{i=1}^{4} c_i \ln^i r_{cut} + c_0 + \mathcal{O}(r_{cut}^m)$

The counter term removes the IR sensitivity to the cutoff variable $\int \left(d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right) \sim c_0 + \mathcal{O}(r_{cut}^m)$

→ we need small values of the cutoff

→ explicit numerical tests to quantify the bias induced by the cutoff choice (cfr. Buonocore, Kallweit, Rottoli, Wiesemann, 2111.13661
Camarda, Cieri, Ferrera, arXiv:2111.14509)

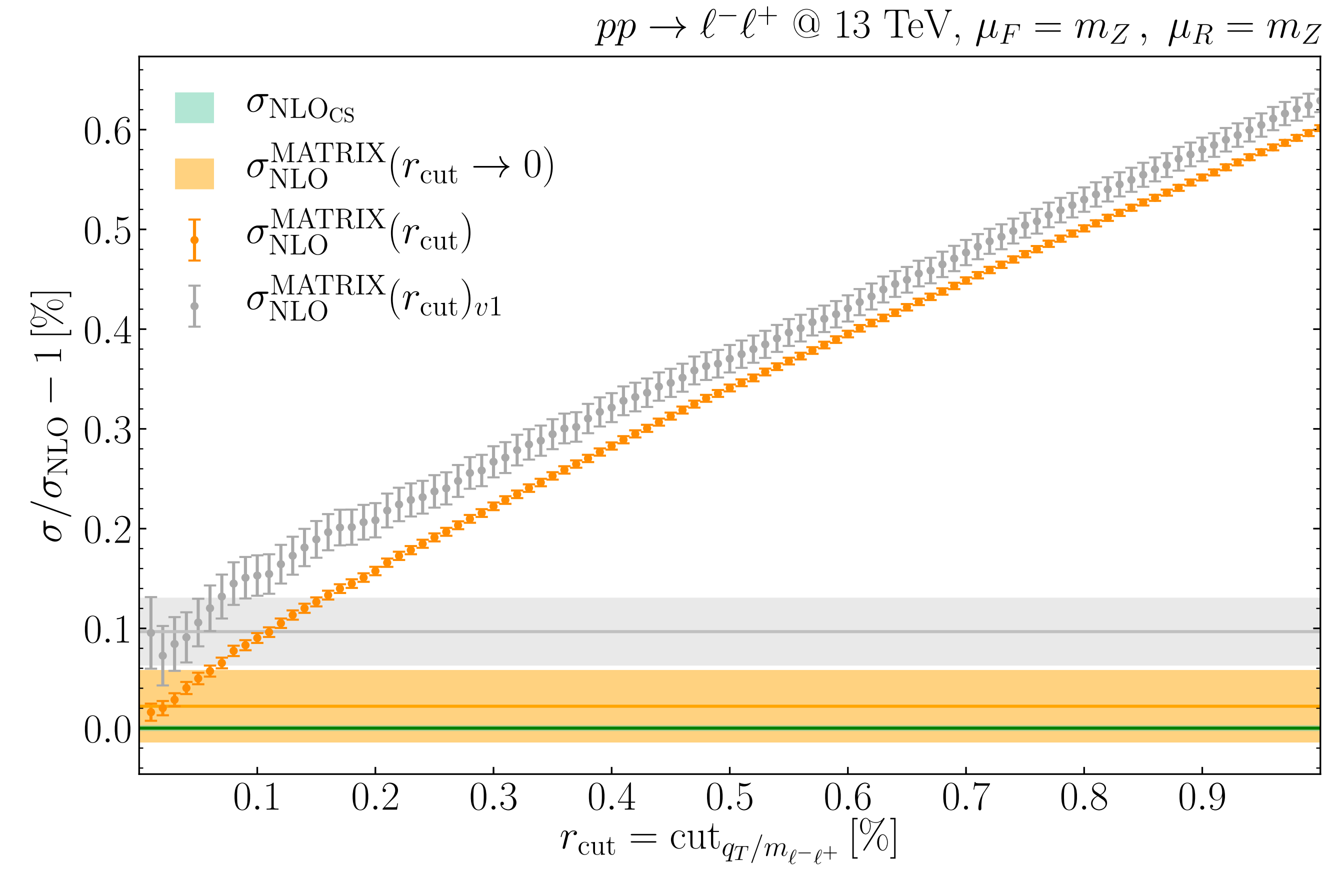
The q_T-subtraction and the residual cut-off dependency in different acceptance setups

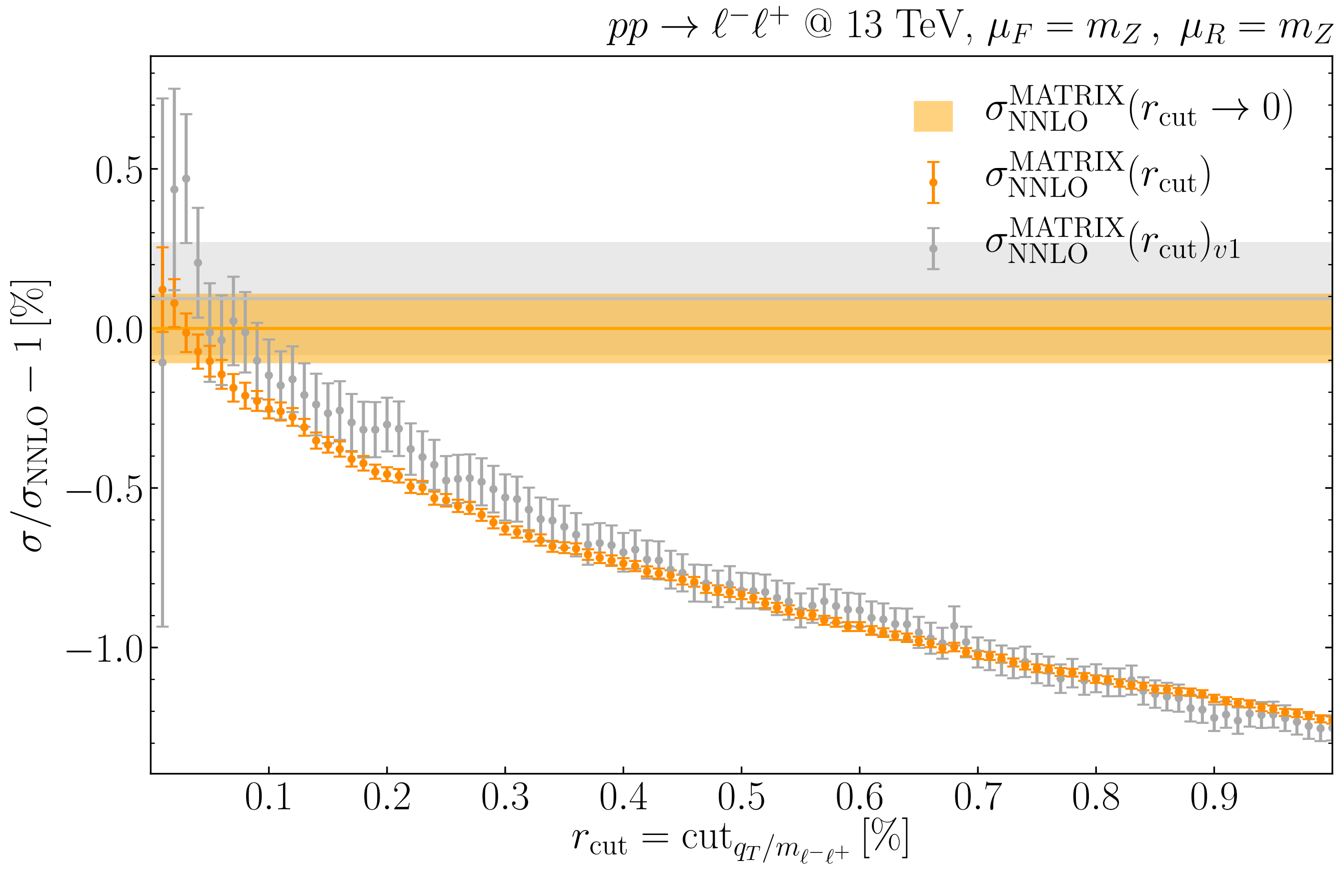
courtesy of S.Kallweit

(cfr. Buonocore, Kallweit, Rottoli, Wiesemann, 2111.13661)

Symmetric cuts

- $p_{\mathrm{T},\ell^\pm} > 25$ GeV

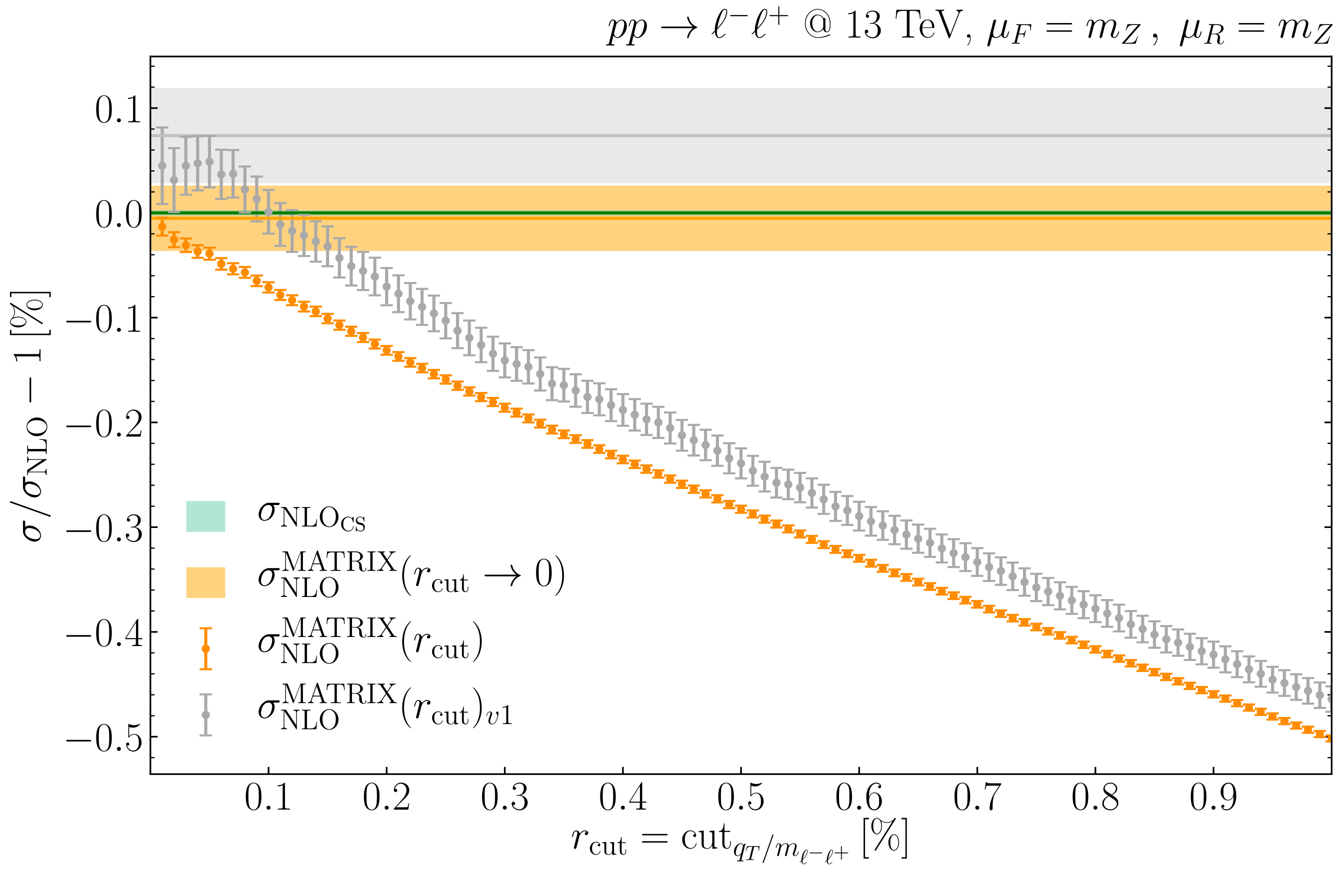


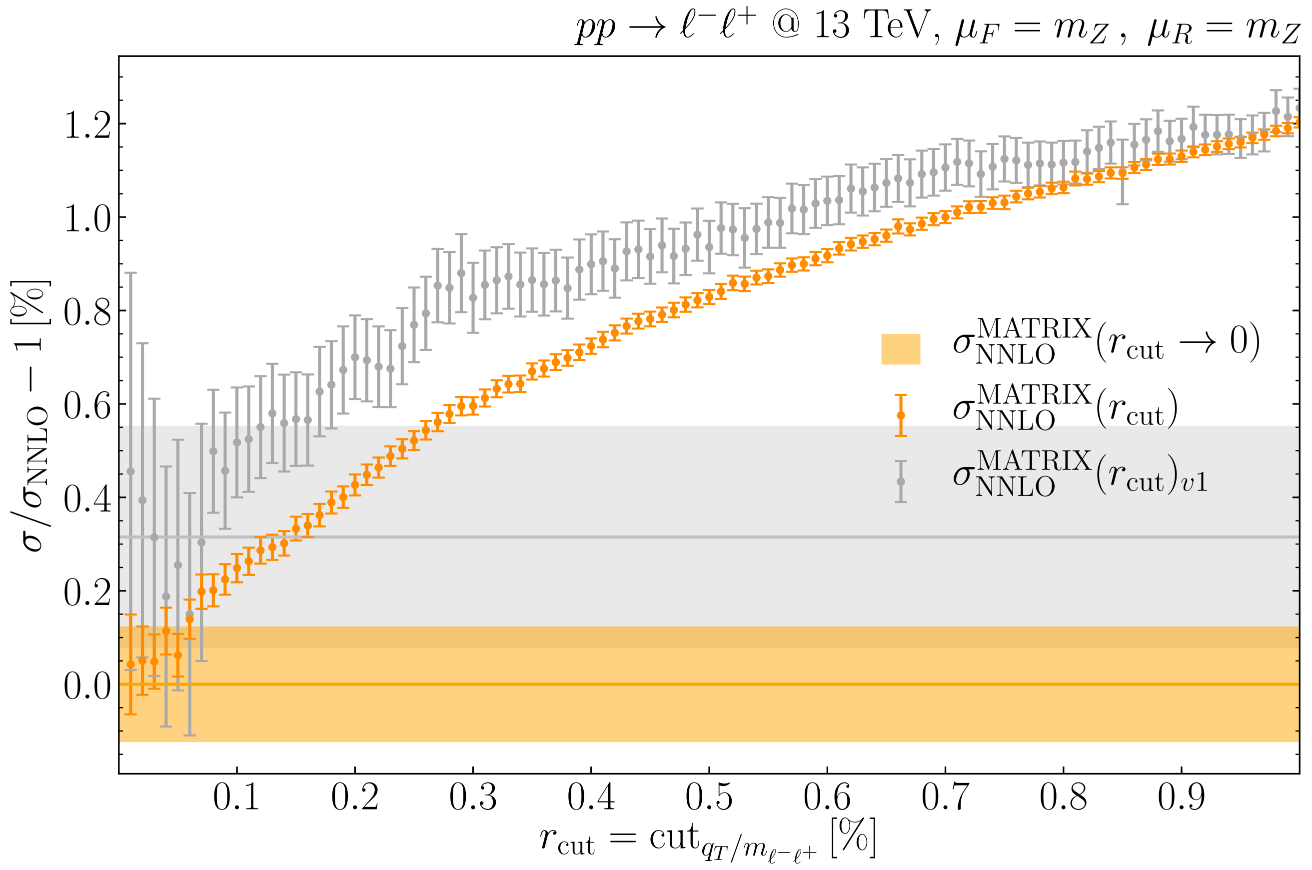


➥ large power corrections in r_{cut}

Asymmetric cuts on ℓ_1 and ℓ_2

- $p_{\mathrm{T},\ell_1} > 25$ GeV $p_{\mathrm{T},\ell_2} > 20$ GeV

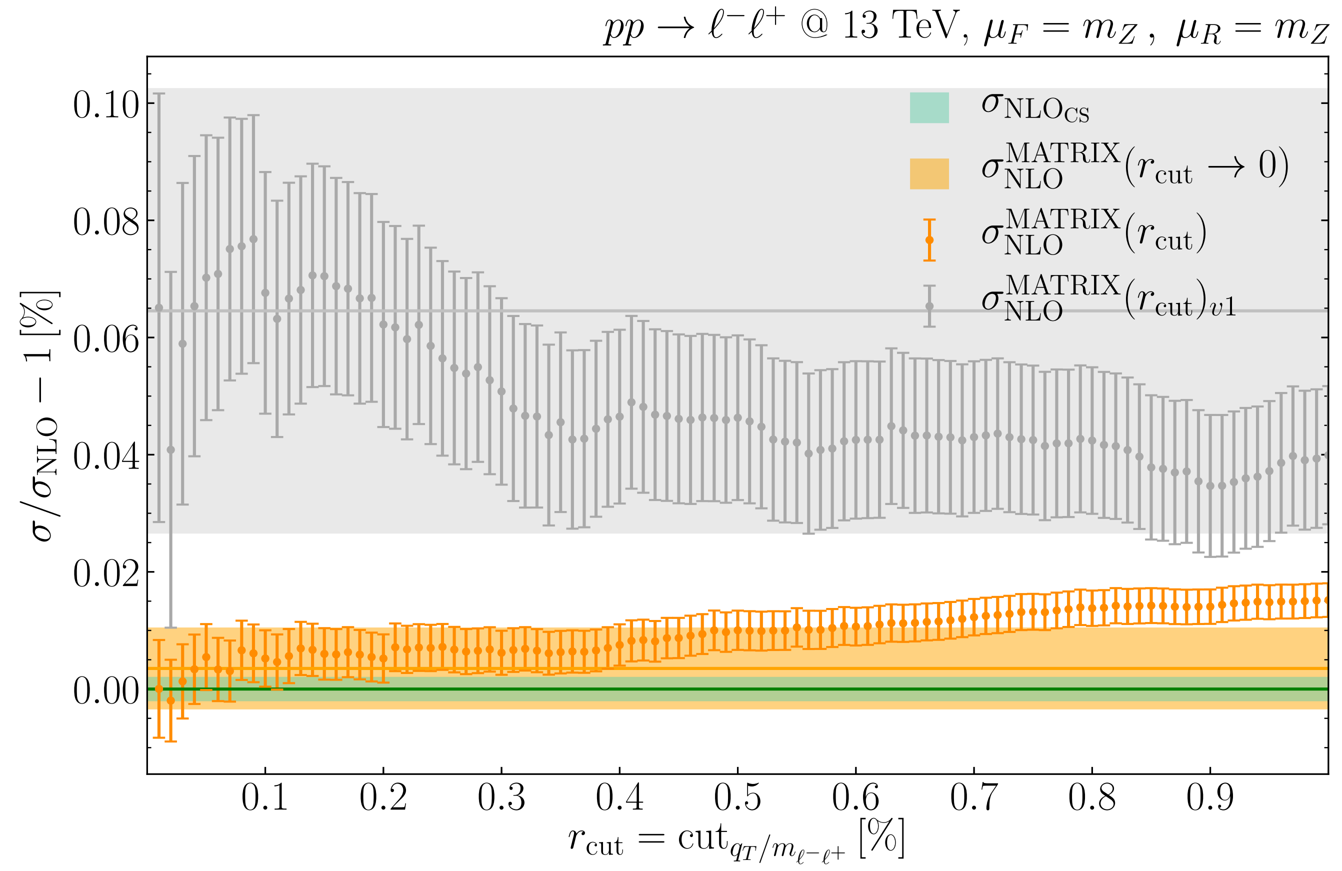


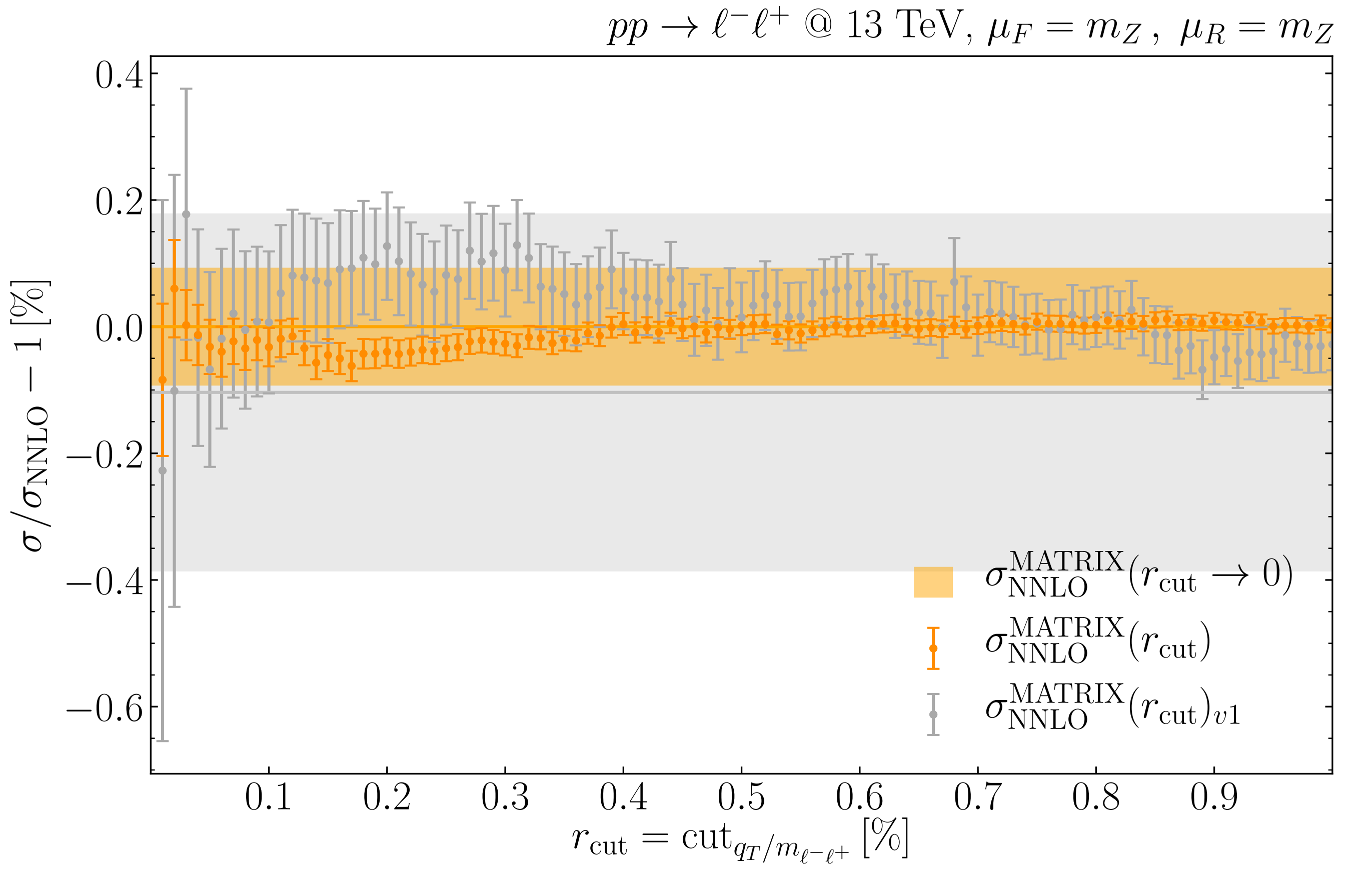


➥ large power corrections in r_{cut}

Asymmetric cuts on ℓ^+ and ℓ^-

- $p_{\mathrm{T},\ell^+} > 25$ GeV $p_{\mathrm{T},\ell^-} > 20$ GeV





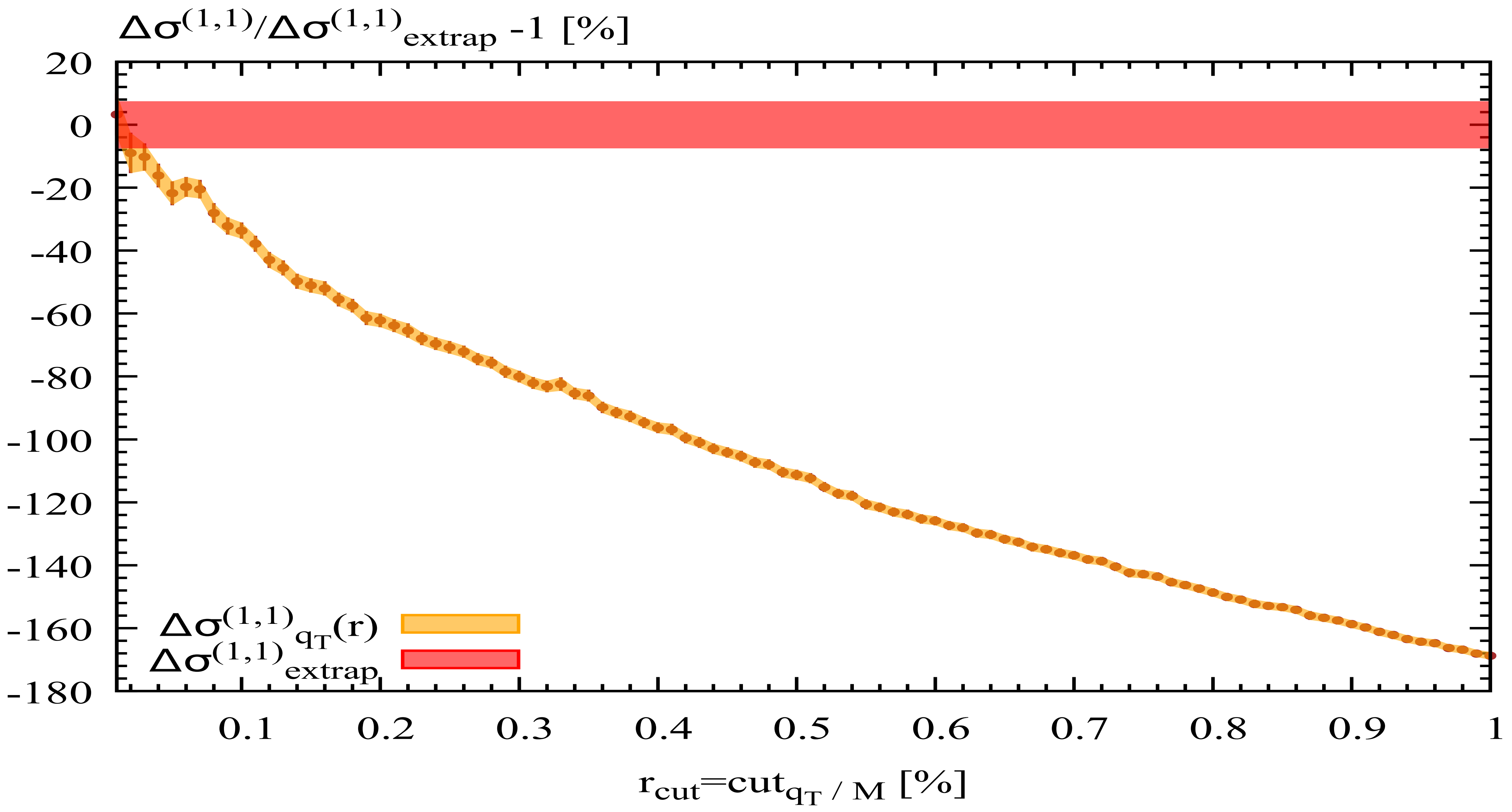
➥ no significant dependence on r_{cut}

Dependence on r_{cut} of the NNLO QCD-EW corrections to NC DY

courtesy of S.Kallweit

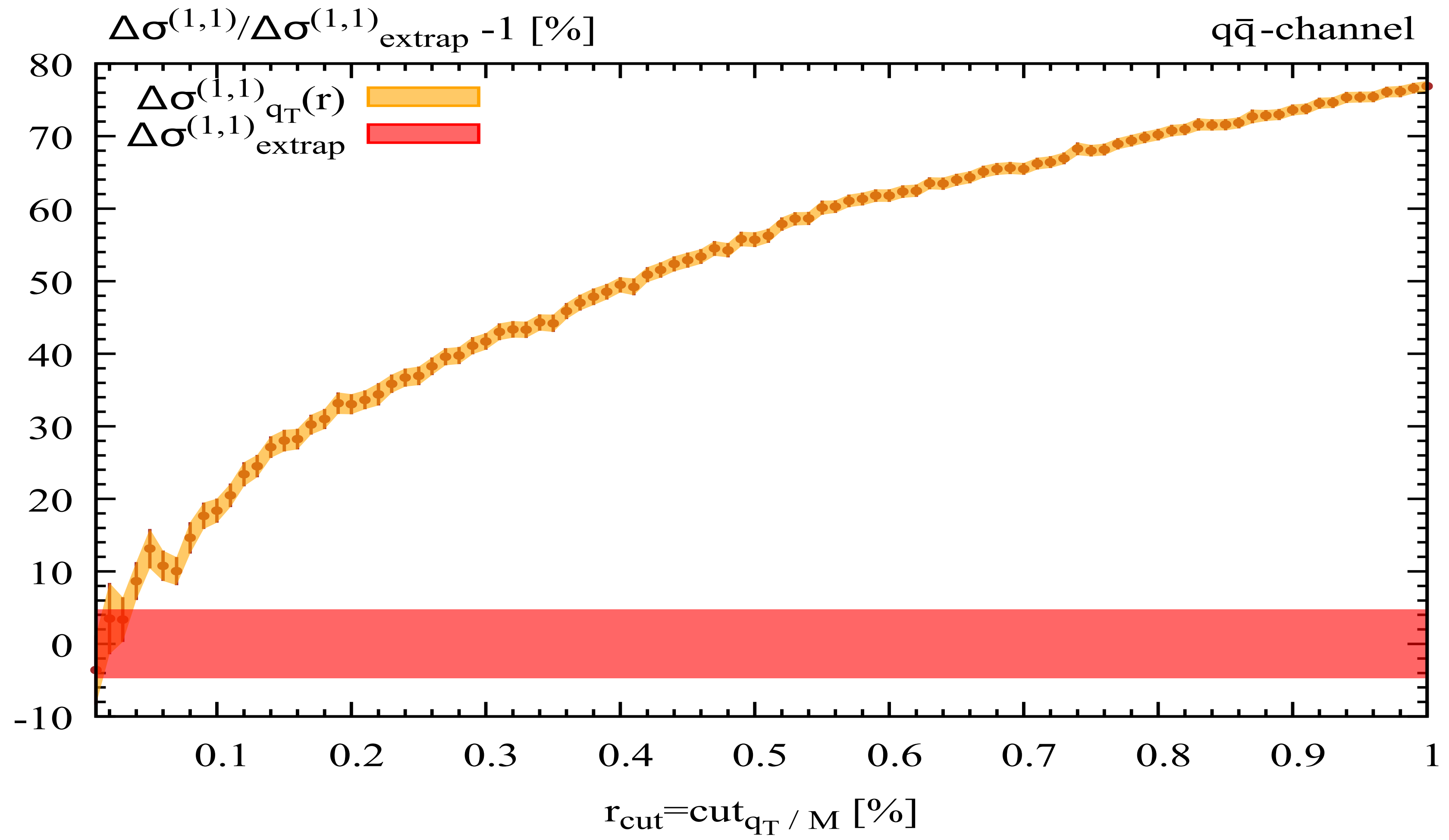
Symmetric-cut scenario

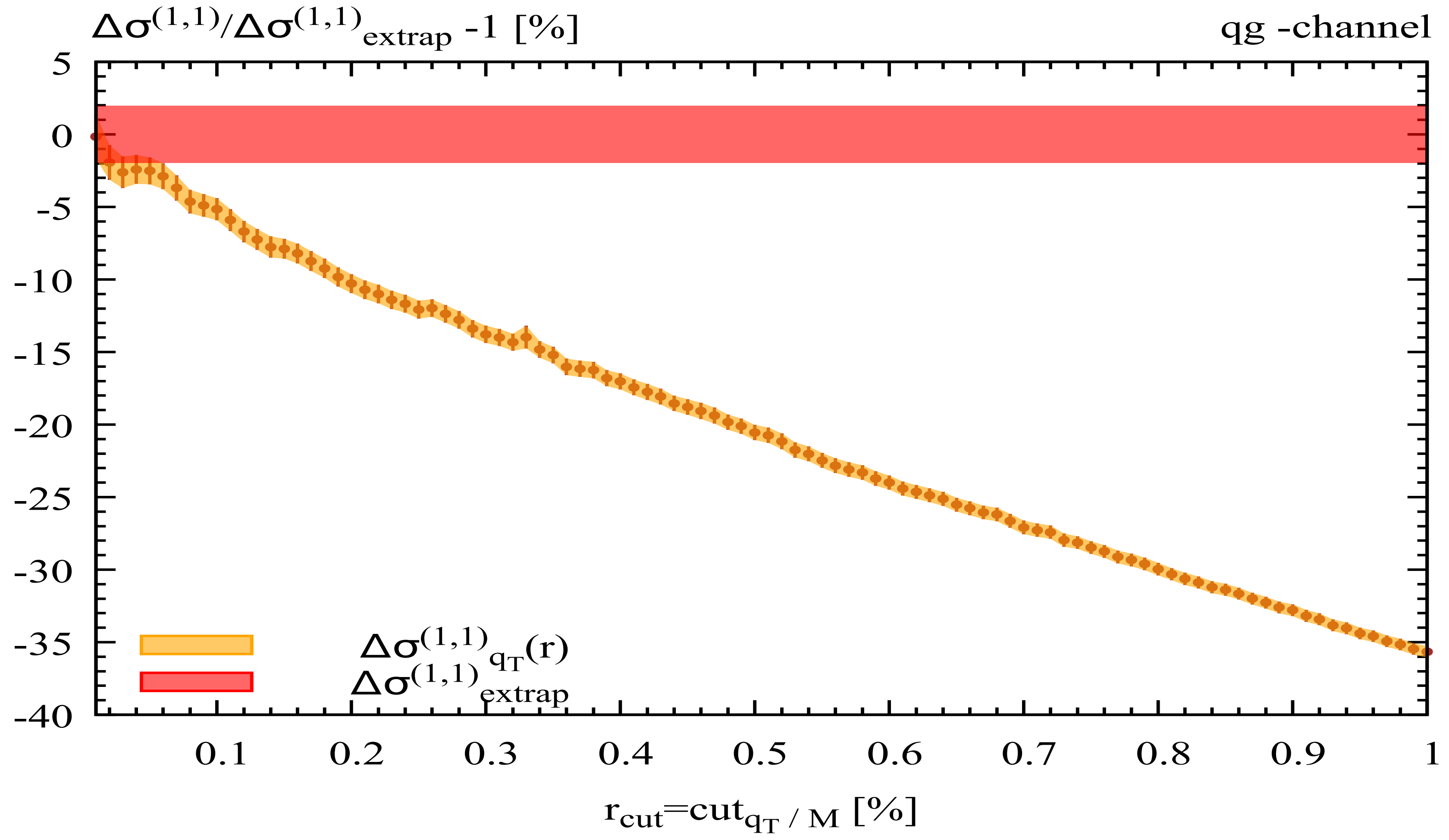
$p_{T,\ell^\pm} > 25$ GeV $y_{\ell^\pm} < 2.5$ $m_{\ell\ell} > 50$ GeV



- **large power corrections in r_{cut} for mixed corrections**
  - ➥ explained by overall small size of corrections, and in parts also by cancellation between partonic channels
- **by far less dramatic dependence at level of cross sections**
  - ➥ better than permille precision at inclusive level

Splitting into partonic channels





The hard-virtual coefficient

$$\boxed{d\sigma = \sum_{m,n=0}^{\infty} d\sigma^{(m,n)} \qquad d\sigma^{(1,1)} = \mathscr{H}^{(1,1)} \otimes d\sigma_{LO} + \left[d\sigma_R^{(1,1)} - d\sigma_{CT}^{(1,1)} \right]_{q_T/Q > r_{cut}}}$$

$$\mathscr{H}^{(1,1)} = H^{(1,1)}\, C_1\, C_2$$

The process independent collinear functions C_1, C_2 are known up to N3LO

The process dependent hard function H is defined upon subtraction of the universal IR contributions

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The process independent collinear functions C_1, C_2 are known up to N3LO

The process dependent hard function H is defined
upon subtraction of the universal IR contributions

$$2\mathrm{Re}\langle \mathscr{M}^{(0,0)} | \mathscr{M}^{(1,1)} \rangle = \sum_{k=-4}^{0} \varepsilon^k f_i(s,t,m)$$

after UV renormalisation the poles are only of IR origin

$$|\mathscr{M}_{fin}\rangle \equiv (1 - I)|\mathscr{M}\rangle \qquad H \propto \langle \mathscr{M}_0 | \mathscr{M}_{fin} \rangle$$

$$H^{(1,0)} = \frac{2\mathrm{Re}\langle \mathscr{M}_{fin}^{(0,0)} | \mathscr{M}_{fin}^{(1,0)} \rangle}{|\mathscr{M}^{(0,0)}|^2}, \qquad H^{(0,1)} = \frac{2\mathrm{Re}\langle \mathscr{M}_{fin}^{(0,0)} | \mathscr{M}_{fin}^{(0,1)} \rangle}{|\mathscr{M}^{(0,0)}|^2}, \qquad H^{(1,1)} = \frac{2\mathrm{Re}\langle \mathscr{M}_{fin}^{(0,0)} | \mathscr{M}_{fin}^{(1,1)} \rangle}{|\mathscr{M}^{(0,0)}|^2}$$

NLO-QCD NLO-EW NNLO QCD-EW

Subtraction of the IR divergences from the 2-loop amplitude

we identify QCD-QED (poles up to $1/\varepsilon^4$) and QCD-weak (poles up to $1/\varepsilon^2$ with cumbersome coefficients) diagrams

$$|\mathcal{M}^{(1,0),fin}\rangle = |\mathcal{M}^{(1,0)}\rangle - \mathcal{I}^{(1,0)}|\mathcal{M}^{(0)}\rangle\,,$$

standard NLO-QCD subtraction

$$|\mathcal{M}^{(0,1),fin}\rangle = |\mathcal{M}^{(0,1)}\rangle - \mathcal{I}^{(0,1)}|\mathcal{M}^{(0)}\rangle\,.$$

NLO-EW subtraction, with massive leptons

$$|\mathcal{M}^{(1,1),fin}\rangle = |\mathcal{M}^{(1,1)}\rangle - \mathcal{I}^{(1,1)}|\mathcal{M}^{(0)}\rangle - \tilde{\mathcal{I}}^{(0,1)}|\mathcal{M}^{(1,0),fin}\rangle - \tilde{\mathcal{I}}^{(1,0)}|\mathcal{M}^{(0,1),fin}\rangle\,.$$

$$\mathcal{I}^{(1,0)} = \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{s}{\mu^2}\right)^{-\epsilon} C_F\left(-\frac{2}{\epsilon^2}-\frac{1}{\epsilon}(3+2i\pi)+\zeta_2\right)\,,$$

$$\mathcal{I}^{(0,1)} = \left(\frac{\alpha}{4\pi}\right)\left(\frac{s}{\mu^2}\right)^{-\epsilon}\left[Q_u^2\left(-\frac{2}{\epsilon^2}-\frac{1}{\epsilon}(3+2i\pi)+\zeta_2\right)+\frac{4}{\epsilon}\Gamma_l^{(0,1)}\right],$$

$$\Gamma_l^{(0,1)} = Q_u Q_l \log\left(\frac{2p_1.p_3}{2p_2.p_3}\right)+\frac{Q_l^2}{2}\left(-1-\frac{1+x_l^2}{1-x_l^2}\log(x_l)\right).$$

$$\mathcal{I}^{(1,1)} = \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha}{4\pi}\right)\left(\frac{s}{\mu^2}\right)^{-2\epsilon} C_F Q_u^2\left(\frac{4}{\epsilon^4}+\frac{1}{\epsilon^3}(12+8i\pi)+\frac{1}{\epsilon^2}(9-28\zeta_2+12i\pi)\right.$$
$$\left.+\frac{1}{\epsilon}\left(-\frac{3}{2}+6\zeta_2-24\zeta_3-4i\pi\zeta_2\right)\right).$$

$2\mathrm{Re}\langle\mathscr{M}^{(0,0)}|\mathscr{M}^{(1,1),fin}\rangle$ is free of any singularity

the analytical check of the cancellation of the IR poles in the QCD-weak sector is one very demanding test of the calculation

The double virtual amplitude: regularisation of the IR divergences

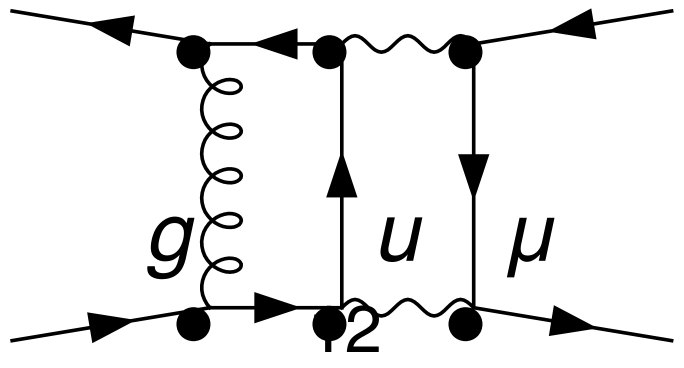
The evaluation of the amplitudes is done in $n = 4 - 2\varepsilon$ dimensions

In the q_T-subtraction formalism, the final state leptons are massive, yielding mass singular logarithms
→ also the 2-loop virtual corrections should be evaluated with massive leptons

We start with a fully massive final state 2-loop amplitude
We retain only collinear singular terms ($\sim \log(m_l^2/M_Z^2)$) and discard those suppressed by a power of m_l^2/M_Z^2

Among the 2-loop boxes

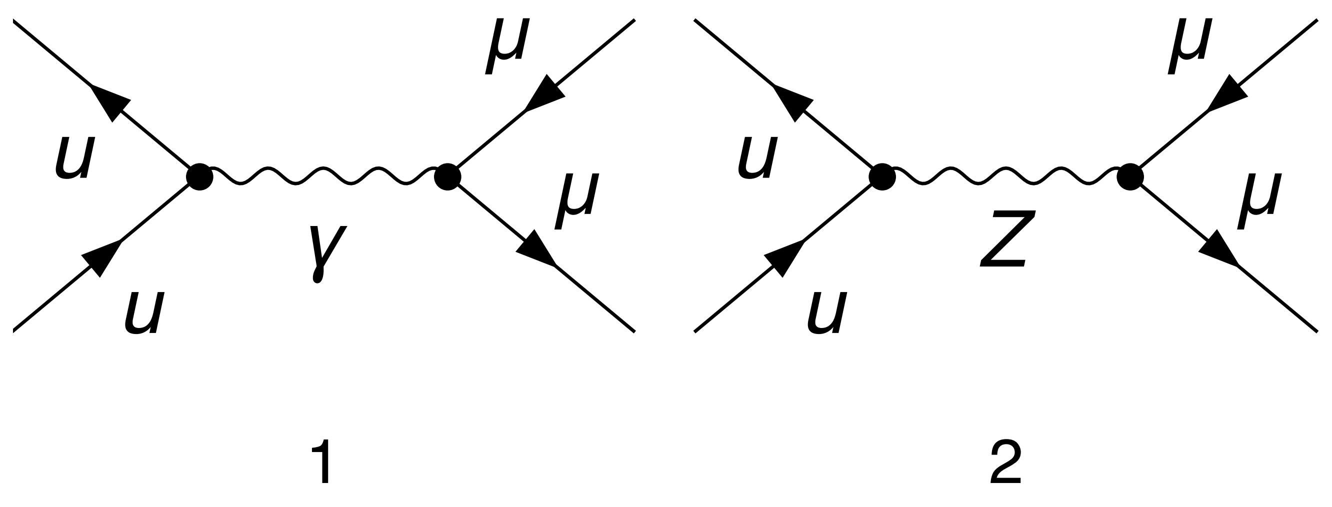
WW and ZZ boxes do not develop collinear singularities
→ evaluated with Master Integrals with massless external lines



$\gamma\gamma$ and γZ boxes individually develop collinear singularities, but in the sum they exactly cancel
→ explicit check in the $\gamma\gamma$ case, based on the massive MIs known from $t\bar{t}$ production
in the γZ check that the residual singularity is the soft divergence

The double virtual amplitude: generation of the amplitude

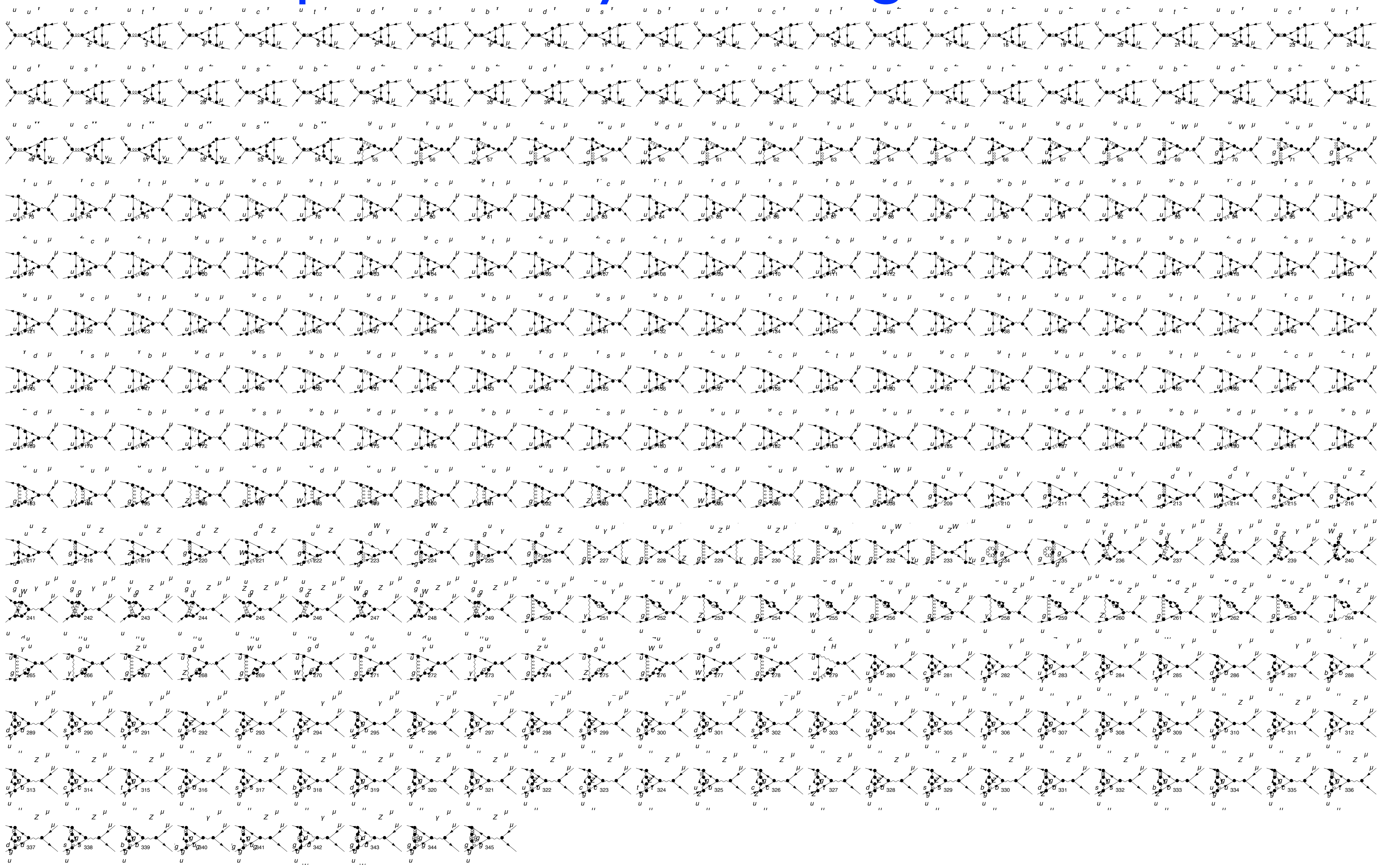
$\mathscr{M}^{(0,0)}(q\bar{q} \to l\bar{l}) =$



$\mathscr{M}^{(1,1)}(q\bar{q} \to l\bar{l}) =$ O(1000) self-energies + O(300) vertex corrections +O(130) box corrections + 1loop x 1loop
(before discarding all those vanishing for colour conservation, e.g. no fermonic triangles)

we compute $2\mathrm{Re}\left(\mathscr{M}^{(1,1),fin}\,(\mathscr{M}^{(0,0)})^\dagger\right)$ which then enters in the hard coefficient function

2-loop virtual Feynman diagrams: vertex and box corrections



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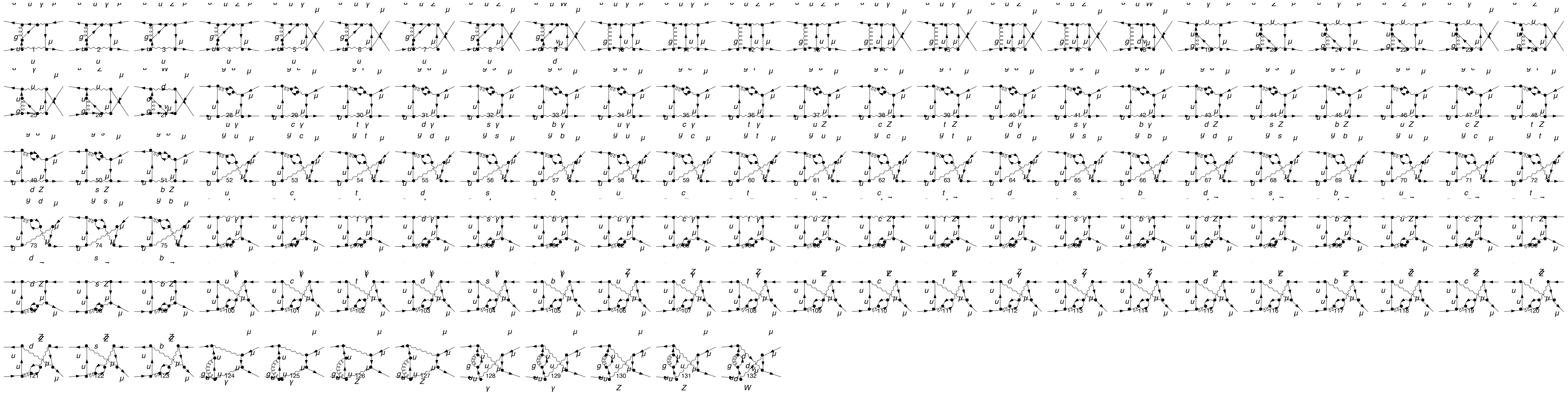

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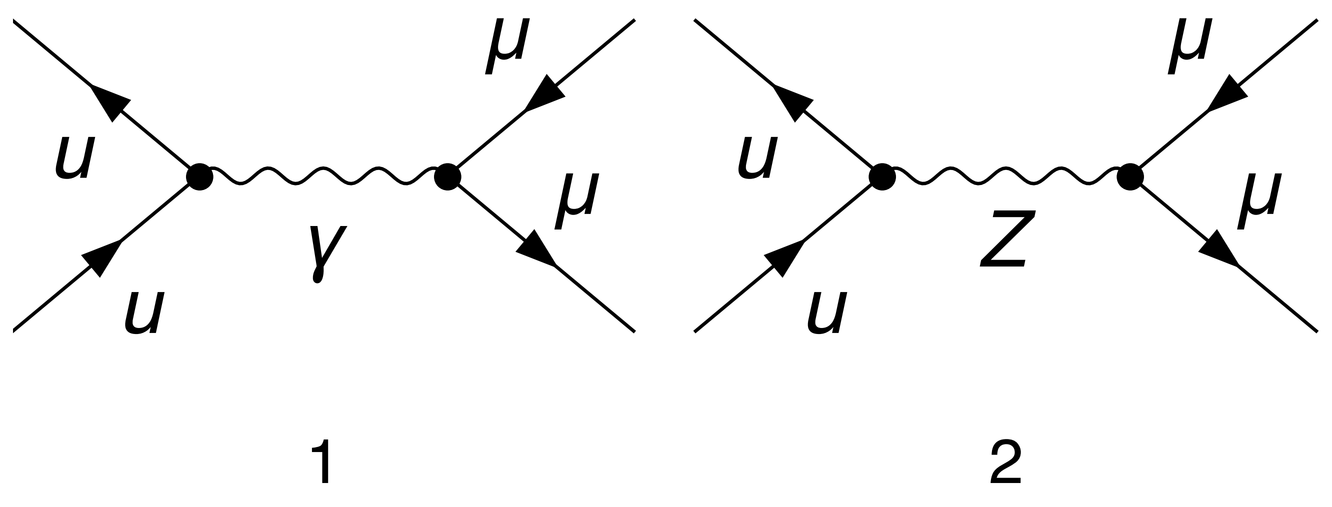

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we compute $2\mathrm{R}e\left(\mathscr{M}^{(1,1),fin}(\mathscr{M}^{(0,0)})^\dagger\right)$ which then enters in the hard coefficient function

Two independent calculations based on QGraf and FeynArts in the EW Background Field Gauge

The BFG choice guarantees the validity of EW Ward identities for the initial state vertex → additional technical checks

e.g. UV finiteness when combining 2-loop vertex and quark WF in the full EW SM

→ the discussion of the UV renormalisation is automatically confined to the gauge-boson propagators sector

The 1-loop check of the gauge-parameter independence identifies those subsets of diagrams yielding the cancellation.

The 2-loop calculation is organised splitting the total amplitude in the combination of different subsets, according to their EW charges (# of Ws, Zs, γs)

The double virtual amplitude: UV renormalization

G.Degrassi, AV, hep-ph/0307122, S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229 S.Dittmaier, arXiv:2101.05154

Complex mass scheme

$$\mu_{W0}^2 = \mu_W^2 + \delta\mu_W^2, \quad \mu_{Z0}^2 = \mu_Z^2 + \delta\mu_Z^2, \quad e_0 = e + \delta e$$

$$\frac{\delta s^2}{s^2} = \frac{c^2}{s^2}\left(\frac{\delta\mu_Z^2}{\mu_Z^2} - \frac{\delta\mu_W^2}{\mu_W^2}\right)$$

the mass counterterms are defined at the complex pole of the propagator

the weak mixing angle is complex valued $c^2 \equiv \mu_W^2/\mu_Z^2$

BFG EW Ward identity → cancellation of the UV divergences combining vertex and fermion WF corrections

The bare couplings of Z and photon to fermions in the (G_μ, μ_W, μ_Z) input scheme are given by

$$\frac{g_0}{c_0} = \sqrt{4\sqrt{2}G_\mu\mu_Z^2}\left[1 - \frac{1}{2}\Delta r + \frac{1}{2}\left(2\frac{\delta e}{e} + \frac{s^2 - c^2}{c^2}\frac{\delta s^2}{s^2}\right)\right] \equiv \sqrt{4\sqrt{2}G_\mu\mu_Z^2}\left(1 + \delta g_Z^{G_\mu}\right)$$

$$g_0 s_0 = \sqrt{4\sqrt{2}G_\mu\mu_W^2 s^2}\left[1 + \tfrac{1}{2}\left(-\Delta r + 2\tfrac{\delta e}{e}\right)\right] \equiv e_{ren}^{G_\mu}\left(1 + \delta g_A^{G_\mu}\right)$$

Gauge boson renormalised propagators

$$\Sigma_{R,T}^{AA}(q^2) = \Sigma_T^{AA}(q^2) + 2\,q^2\,\delta g_A$$

$$\Sigma_{R,T}^{ZZ}(q^2) = \Sigma_T^{ZZ}(q^2) - \delta\mu_Z^2 + 2\,(q^2 - \mu_Z^2)\,\delta g_Z$$

$$\Sigma_{R,T}^{AZ}(q^2) = \Sigma_T^{AZ}(q^2) - q^2\,\frac{\delta s^2}{sc}$$

$$\Sigma_{R,T}^{ZA}(q^2) = \Sigma_T^{ZA}(q^2) - q^2\,\frac{\delta s^2}{sc},$$

After the UV renormalisation, the singular structure is entirely due to IR soft and/or collinear singularities

The double virtual amplitude: γ_5 treatment

The absence of a consistent definition of γ_5 in $n = 4 - 2\varepsilon$ dimensions yields a practical problem

The trace of Dirac matrices and γ_5 is a polynomial in ε
The UV or IR divergences of Feynman integrals appear as poles $1/\varepsilon$

$$\mathrm{Tr}(\gamma_\alpha \ldots \gamma_\mu \gamma_5) \times \int d^n k \frac{1}{[k^2 - m_0^2][(k+q_1)^2 - m_1^2][(k+q_2)^2 - m_2^2]} \sim (a_0 + a_1 \varepsilon + \ldots) \times \left(\frac{c_{-2}}{\varepsilon^2} + \frac{c_{-1}}{\varepsilon} + c_0 + \ldots \right)$$

If a_1 is evaluated in a non-consistent way,
then poles might not cancel and the finite part of the xsec might have a spurious contribution

- 't Hooft-Veltman treat γ_5 (anti)commuting in (4) $n-4$ dimensions preserving the cyclicity of the traces
  (one counterterm is needed)
- Kreimer treats γ_5 anticommuting in n dimensions, abandoning the cyclicity of the traces (→ need of a starting point)

- Heller, von Manteuffel, Schabinger verified that the IR-subtracted squared matrix element are identical in the two approaches

- we adopted the naive anticommuting prescription (Kreimer); we use $\gamma_5 = \frac{i}{4!} \epsilon_{\mu\nu\rho\sigma} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma$ to compute traces with one γ_5

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  - we computed the 2-loop amplitude and, independently, the IR subtraction term; both depend on the prescription chosen
  - the cancellation of the lowest order poles is checked (and non trivial)
  - absence of fermionic triangles because of colour conservation

The double virtual amplitude: reduction to Master Integrals

$$2\mathrm{Re}\left(\mathcal{M}^{(1,1)}(\mathcal{M}^{(0,0)})^\dagger\right) \;=\; \sum_{i=1}^{N_{MI}} c_i(s,t,m;\varepsilon)\;\mathcal{T}_i(s,t,m;\varepsilon)$$

The coefficients c_i are rational functions of the invariants, masses and of ε
The size of the total expression can rapidly "explode"
→ careful work to identify the patterns of recurring subexpressions keeping the total size in the O(1-10 MB) range

MIs relevant for the QCD-QED corrections, with massive final state

Bonciani, Ferroglia,Gehrmann, Maitre, Studerus., arXiv:0806.2301, 0906.3671

MIs with 1or 2 internal mass relevant for the EW form factor

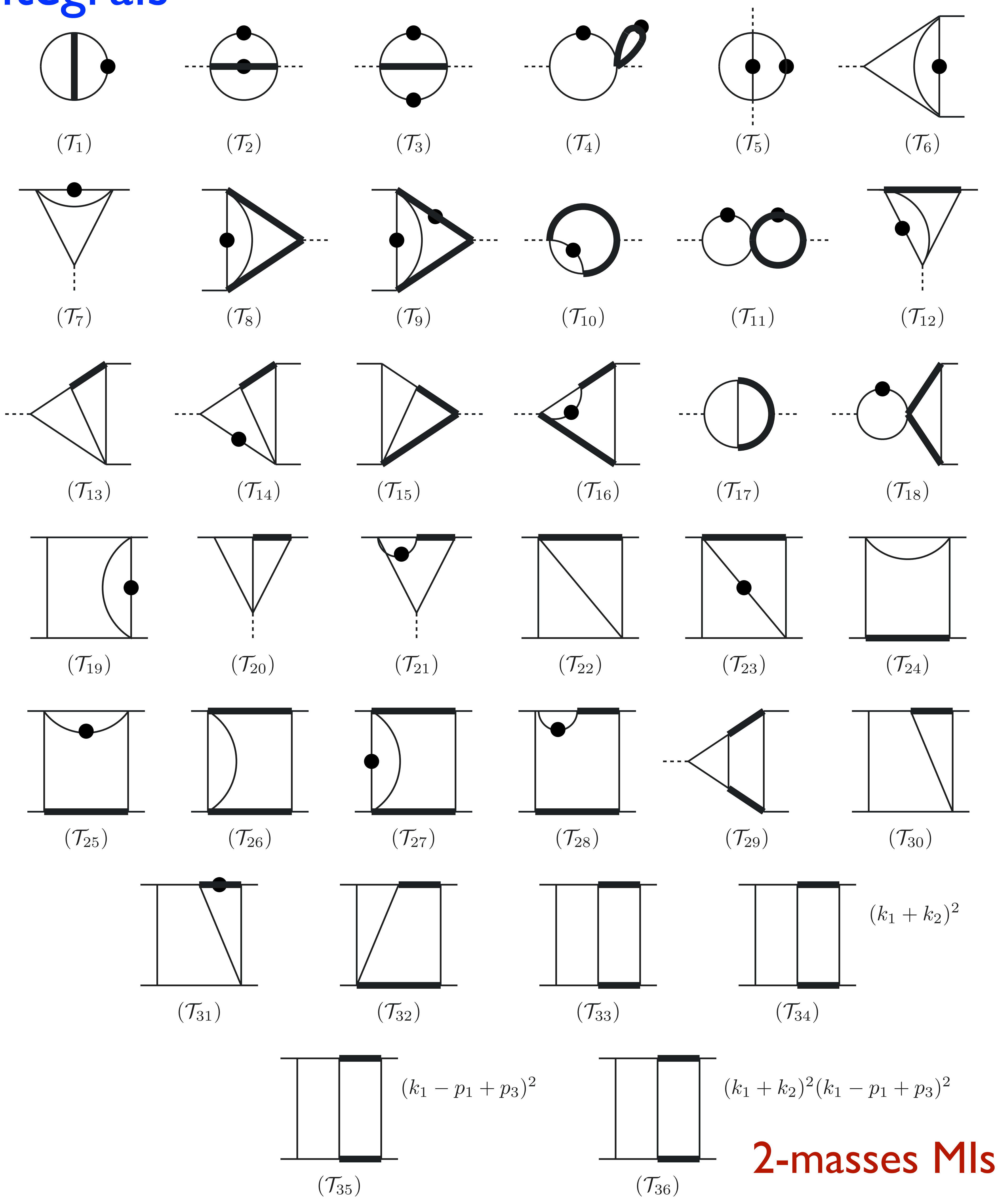
Aglietti, Bonciani, hep-ph/0304028, hep-ph/0401193

31 MIs with 1 mass and 36 MIs with 2 masses including boxes, relevant for the QCD-weak corrections to the full Drell-Yan

Bonciani, Di Vita, Mastrolia, Schubert., arXiv:1604.08581

An alternative representation of the latter has been discussed in

Heller, von Manteuffel, Schabinger, arXiv:1907.00491



The double virtual amplitude: solution and evaluation of the Master Integrals

The system of first-order linear differential equations satisfied by the 36 QCD-weak Master Integrals chosen by Bonciani et al. can be written in dlog form

$$d\mathbf{I} = \epsilon\, d\mathbb{A}\, \mathbf{I}\,, \qquad d\mathbb{A} = \sum_{i=1}^{n} \mathbb{M}_i \; d\log \eta_i\,.$$

The letters η_i provide the complete information about the singular structure of the amplitude

Master Integrals 1-31

When the letters have a rational (linear) expression, it is possible to integrate the system in terms of GPLs

The appearance, for kinematical reasons, of four square roots among the letters is handled with a change of variables that makes all the new letters linear, leading to a GPL solution in the new variables

Master Integrals 32-36

The appearance of another distinct square root among the letters, makes it impossibile to linearise the weights
→ the equations are formally solved with a Chen-Goncharov iterated representation

- the poles of these MIs contain Chen-Goncharov functions, but they cancel in the physical amplitude
- in the finite part, the Chen-Goncharov functions remain → problems to evaluate the amplitude in the physical region

Boundary Conditions

The BCs have been evaluated outside the physical phase space and are expressed in exact form

The double virtual amplitude: solution and evaluation of the Master Integrals

- Alternative semi-analytical approach: solution of the system by series expansions of the unknown functions

  S.Pozzorini, E.Remiddi, hep-ph/0505041, F.Moriello, arXiv:1907.13234

  The differential equations are replaced by a set of algebraic eqs for the unknown coefficients of the series.

  Once we have the solution as a series centered at the BCs, how do we evaluate the solution at a different point ?

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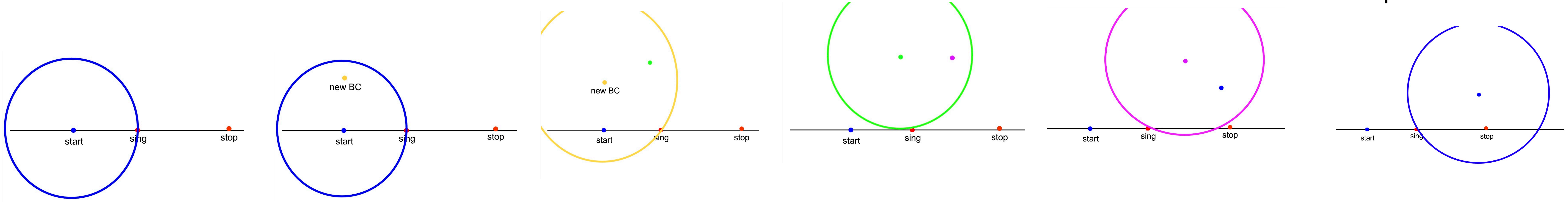

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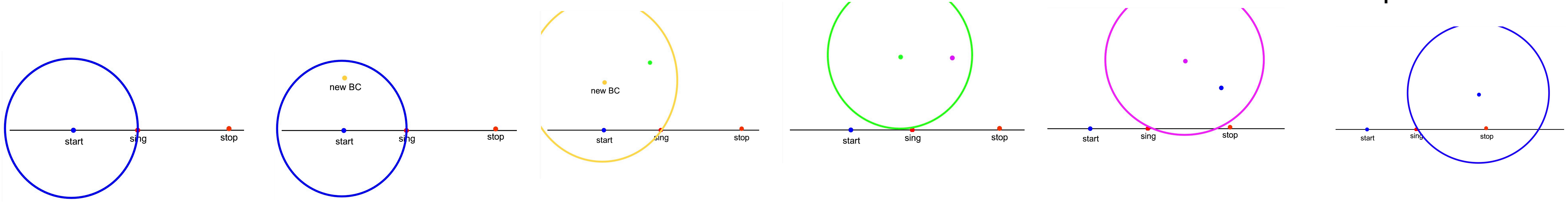

The package DiffExp by M.Hidding, arXiv:2006.05510 implements this idea, for real valued masses.

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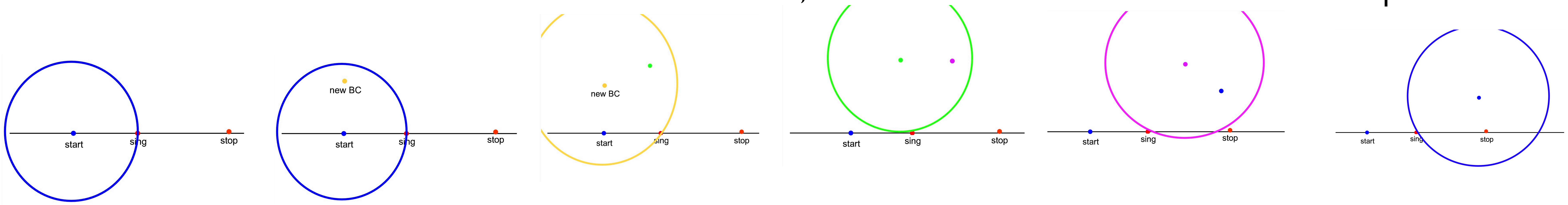

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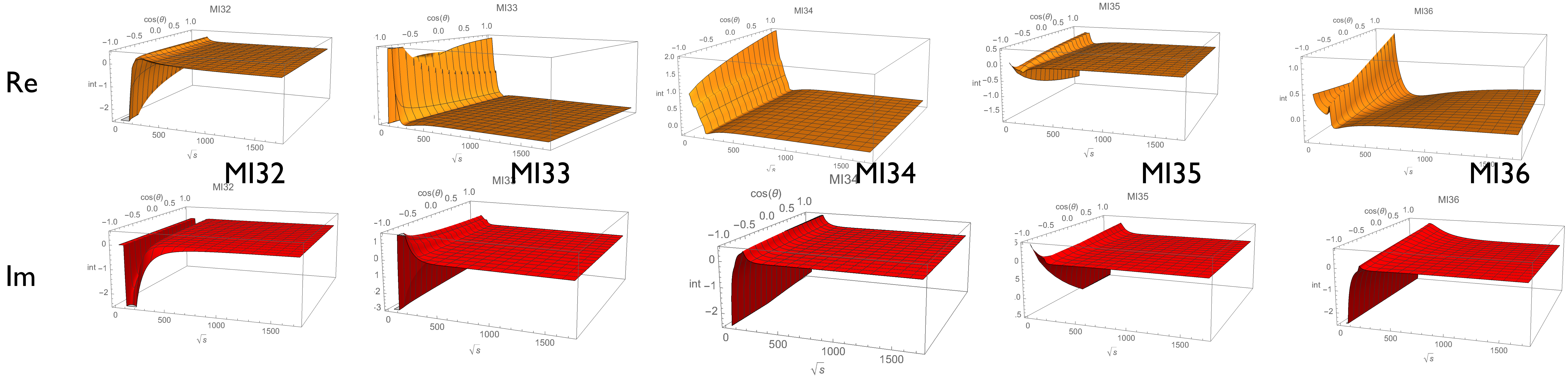


The solution can be computed with an arbitrary number of significant digits, but not in closed form → semi-analytical

Numerical evaluation of the hard coefficient function

The interference term $2\text{Re}\langle \mathcal{M}^{(1,1),fin} | (\mathcal{M}^{(0,0)} \rangle$ contributes to the hard function $H^{(1,1)}$

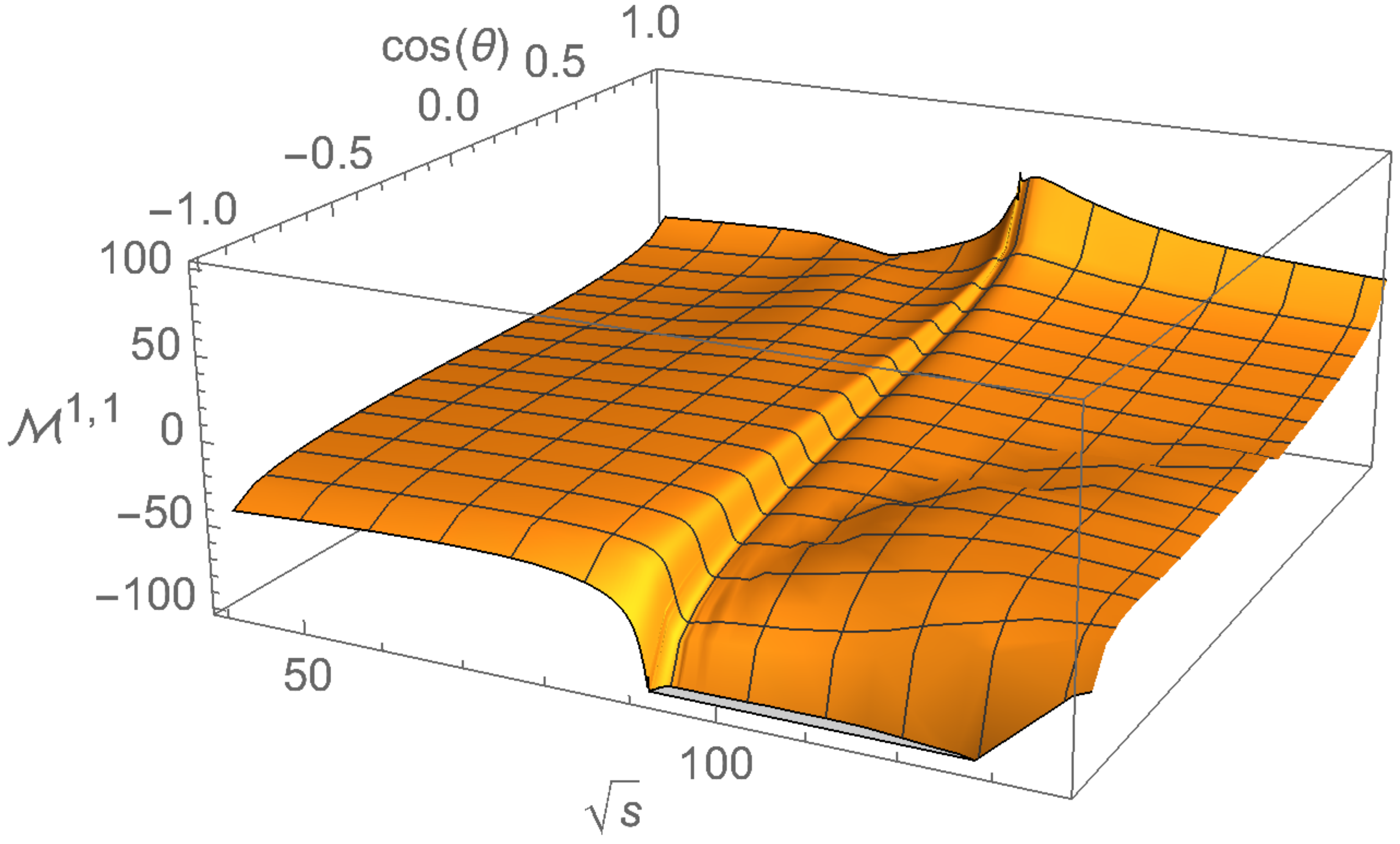
After the subtraction of all the universal IR divergences, it is a finite correction

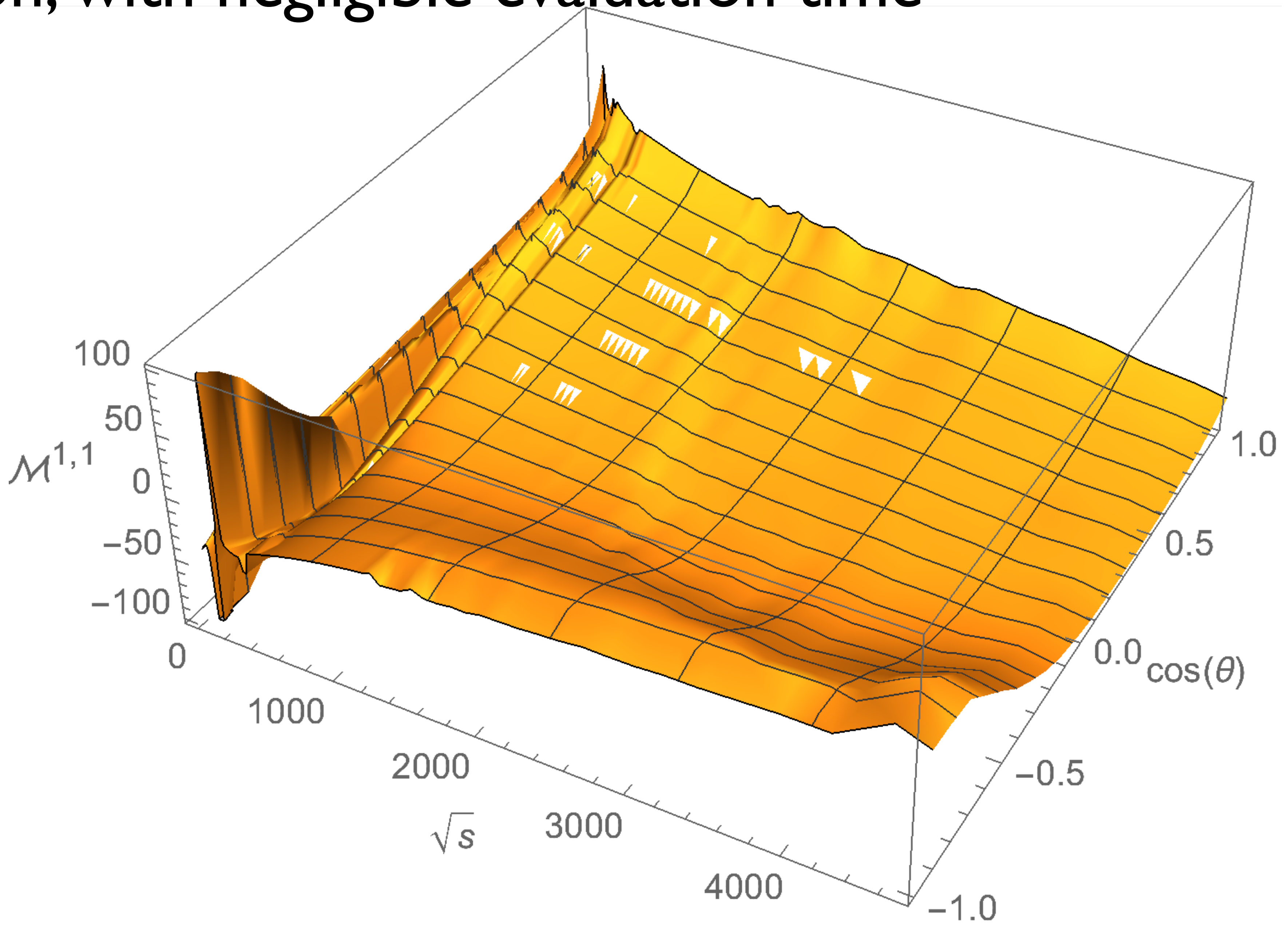
Several checks of the MIs performed with $\mathtt{Fiesta}$ and $\mathtt{PySecDec}$

A numerical grid has been prepared for all the 36 MIs, with $\mathtt{GiNaC}$ and $\mathtt{DiffExp}$,
covering the whole $2 \to 2$ phase space in (s,t),
in O(3 h) on one 32-cores machine

→ a numerical grid for $2\text{Re}\langle \mathcal{M}^{(1,1),fin} | (\mathcal{M}^{(0,0)} \rangle$ has been prepared
arbitrary values can be obtained with excellent accuracy via interpolation, with negligible evaluation time

in units $\frac{\alpha}{2\pi} \frac{\alpha_s}{2\pi} \sigma_0$





Total cross section in the fiducial region

$G_\mu = 1.1663787 \times 10^{-5}\,\mathrm{GeV}^{-2},\ M_W = 80.358\ \mathrm{GeV}, \Gamma_W = 2.084\ \mathrm{GeV}, M_Z = 91.1535\ \mathrm{GeV}, \Gamma_Z = 2.4943\ \mathrm{GeV}$

$M_H = 125.09\ \mathrm{GeV},\ m_t = 173.07\ \mathrm{GeV}$ NNPDF31_nnlo_as_0118_luxqed

$p_T^{\mu^\pm} > 25\,\mathrm{GeV}, \quad |\eta^{\mu^\pm}| < 2.5, \quad m_{\mu^+\mu^-} > 50\,\mathrm{GeV}$

σ [pb]	σ_{LO}	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	809.56(1)	191.85(1)	−33.76(1)	49.9(7)	−4.8(3)
qg	—	−158.08(2)	—	−74.8(5)	8.6(1)
$q(g)\gamma$	—	—	−0.839(2)	—	0.084(3)
$q(\bar{q})q'$	—	—	—	6.3(1)	0.19(0)
gg	—	—	—	18.1(2)	—
$\gamma\gamma$	1.42(0)	—	−0.0117(4)	—	—
tot	810.98(1)	33.77(2)	−34.61(1)	−0.5(9)	4.0(3)
$\sigma^{(m,n)}/\sigma_{\mathrm{LO}}$		$+4.2\,\%$	$-4.3\,\%$	$\sim 0\,\%$	$+0.5\,\%$

Accidental cancellation of NLO-QCD and NLO-EW, small contribution from NNLO-QCD

→ the NNLO QCD-EW is comparable (or larger) in size than the combination of the previous orders

Differential distributions: exact vs approximated predictions

The exact $\mathcal{O}(\alpha\alpha_s)$ corrections allow to test the validity of different recipes based on NLO-QCD and NLO-EW results

factorised Ansatz

$$\frac{d\sigma_{fact}}{dX} = \frac{d\sigma^{(0,0)}}{dX}\left[1+\frac{d\sigma^{(1,0)}}{dX}\left(\frac{d\sigma^{(0,0)}}{dX}\right)^{-1}\right]\times\left[1+\frac{d\sigma^{(0,1)}}{dX}\left(\frac{d\sigma^{(0,0)}}{dX}\right)^{-1}\right]$$

$$\simeq \frac{d\sigma^{(0,0)}}{dX}+\frac{d\sigma^{(1,0)}}{dX}+\frac{d\sigma^{(0,1)}}{dX}+\frac{d\sigma^{(0,1)}}{dX}\frac{d\sigma^{(1,0)}}{dX}\left(\frac{d\sigma^{(0,0)}}{dX}\right)^{-1}$$

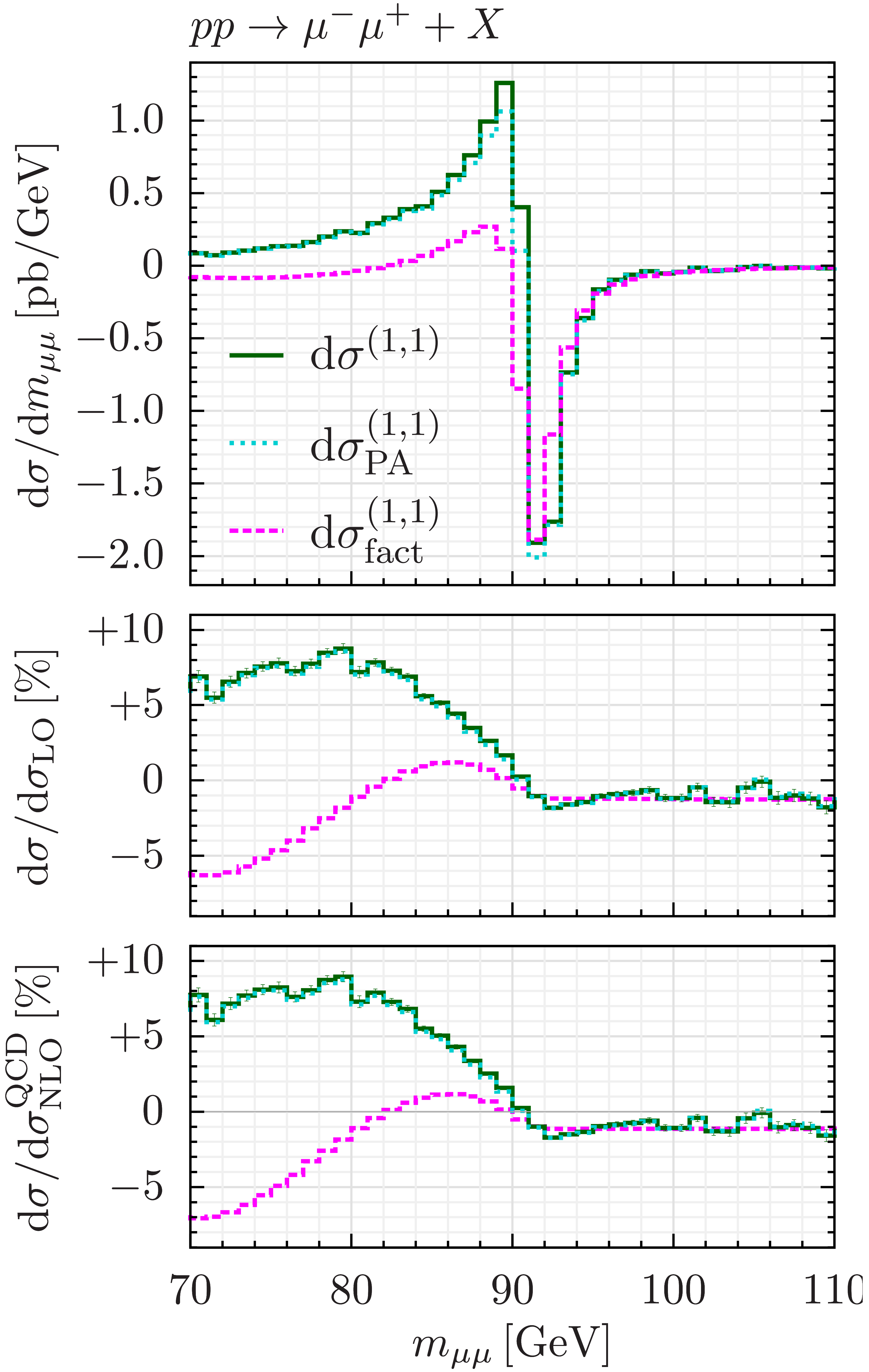
- the last term is absent in a purely additive formulation
- Factorisation is expected to work when
  both QCD and EW corrections factorise w.r.t. the gauge boson production
- the giant K-factors (qg and qγ processes) should not be applied to photon-induced channels

pole approximation

in the hard coefficient, the 2-loop virtual is approximated by $H_{PA}^{(1,1)} = \dfrac{2\mathrm{Re}(\mathcal{M}^{(1,1)}\mathcal{M}^{(0,0)*})_{PA}}{|\mathcal{M}_{PA}^{(0,0)}|^2}$

the 2-loop virtual corrections are evaluated in pole approximation
- on-shell Z boson form factor
- the resonant contributions of the γZ box diagrams cancel

The lepton-pair invariant mass distribution

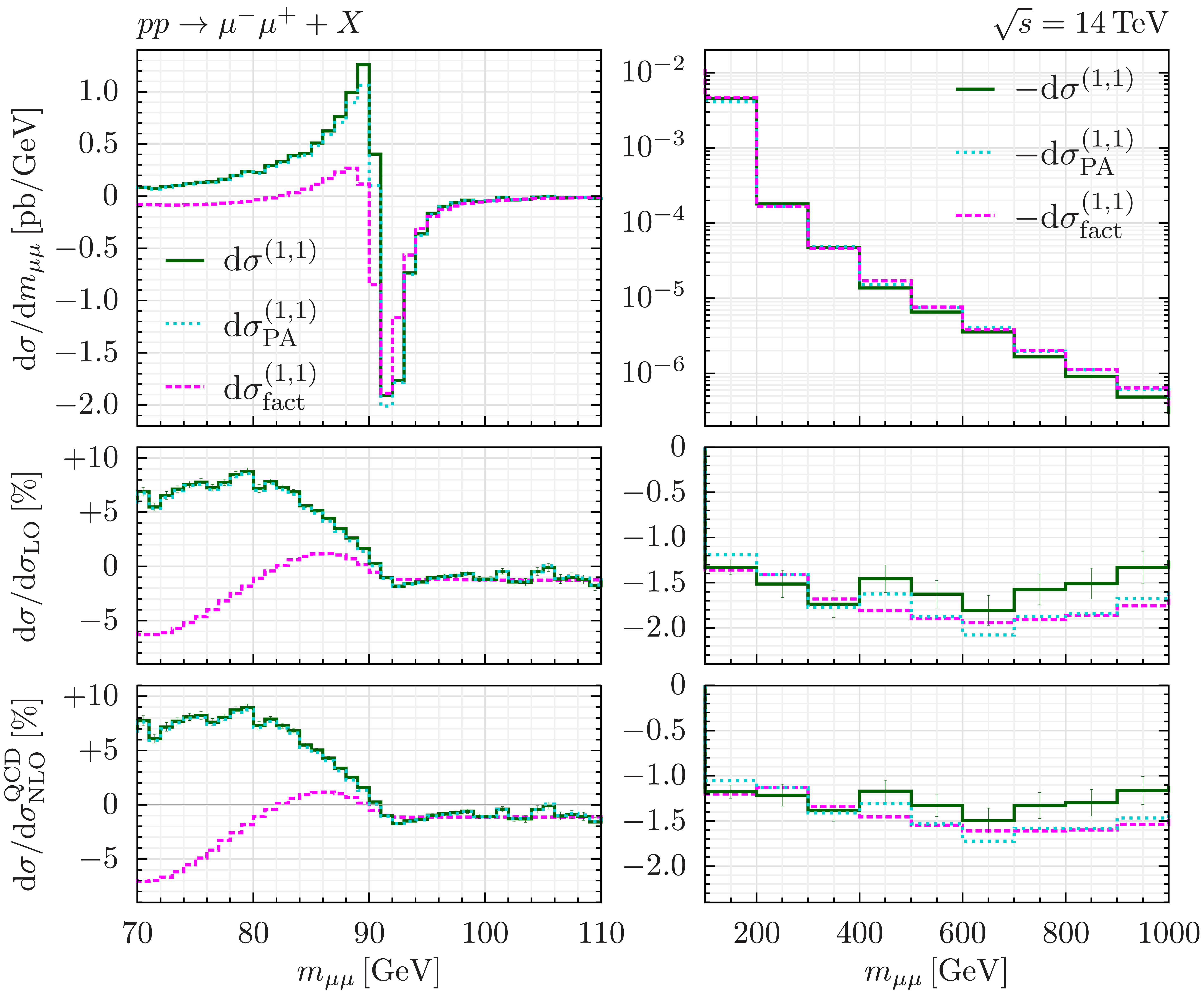


below the Z resonance,
QCD and QED effects do not factorise

- a proper treatment of kinematics is needed (cfr. e.g. POWHEG QCD+EW)
- the full calculation has a non-trivial correction

the pole approximation provides an excellent description of the full calculation confirming S.Dittmaier, A.Huss, C.Schwinn, arXiv:1403.3216, 1511.08016

The lepton-pair invariant mass distribution: QCD-EW corrections

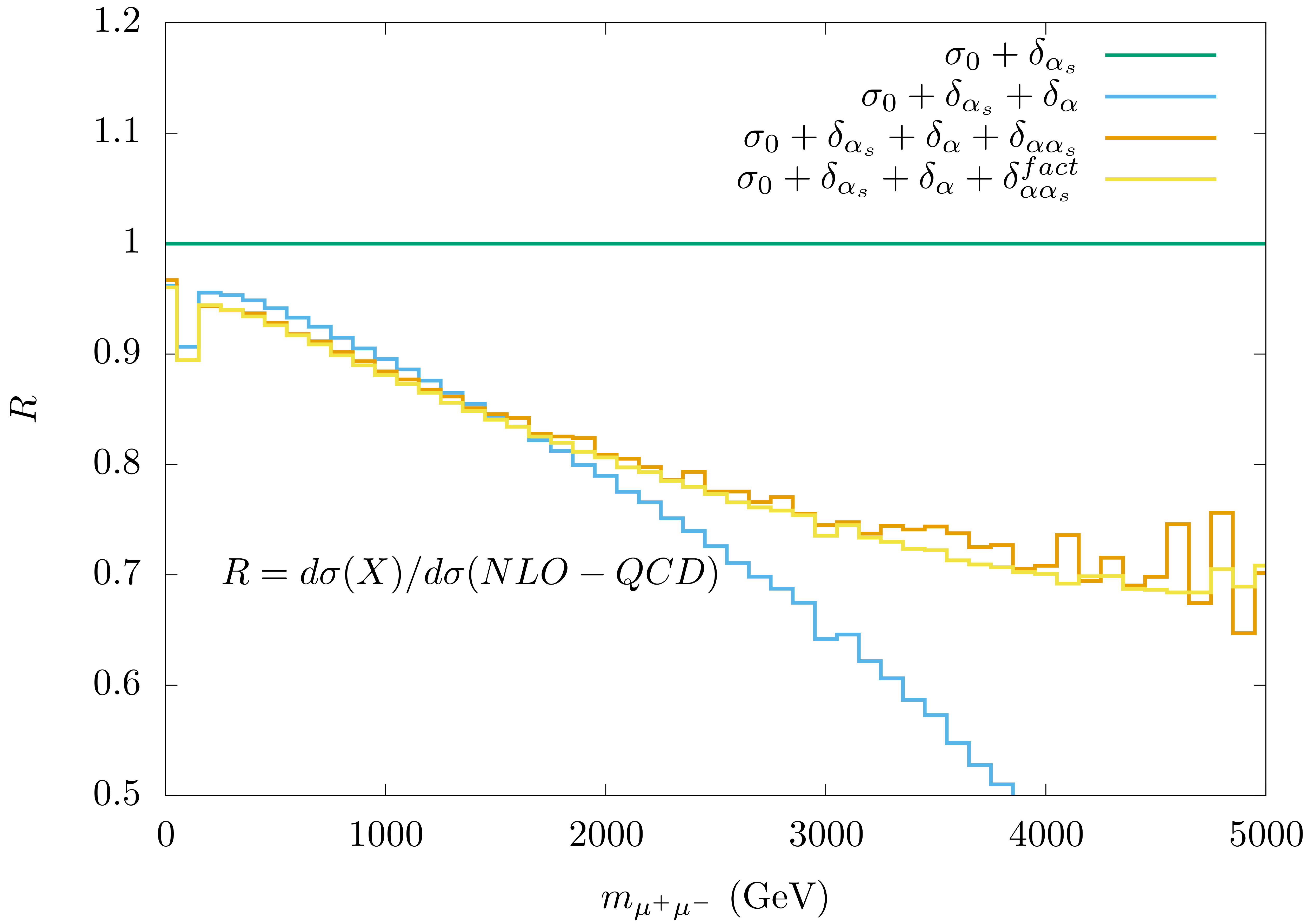


in the large invariant mass tail
the QCD-EW corrections are negative O(-1.5%)

these contributions are absent in an additive combination of NNLO-QCD + NLO-EW (implemented e.g. in `FEWZ`)

the exact calculation deviates from the pole and factorised approximations at the O(0.4%) level

The lepton-pair invariant mass distribution: QCD-EW corrections



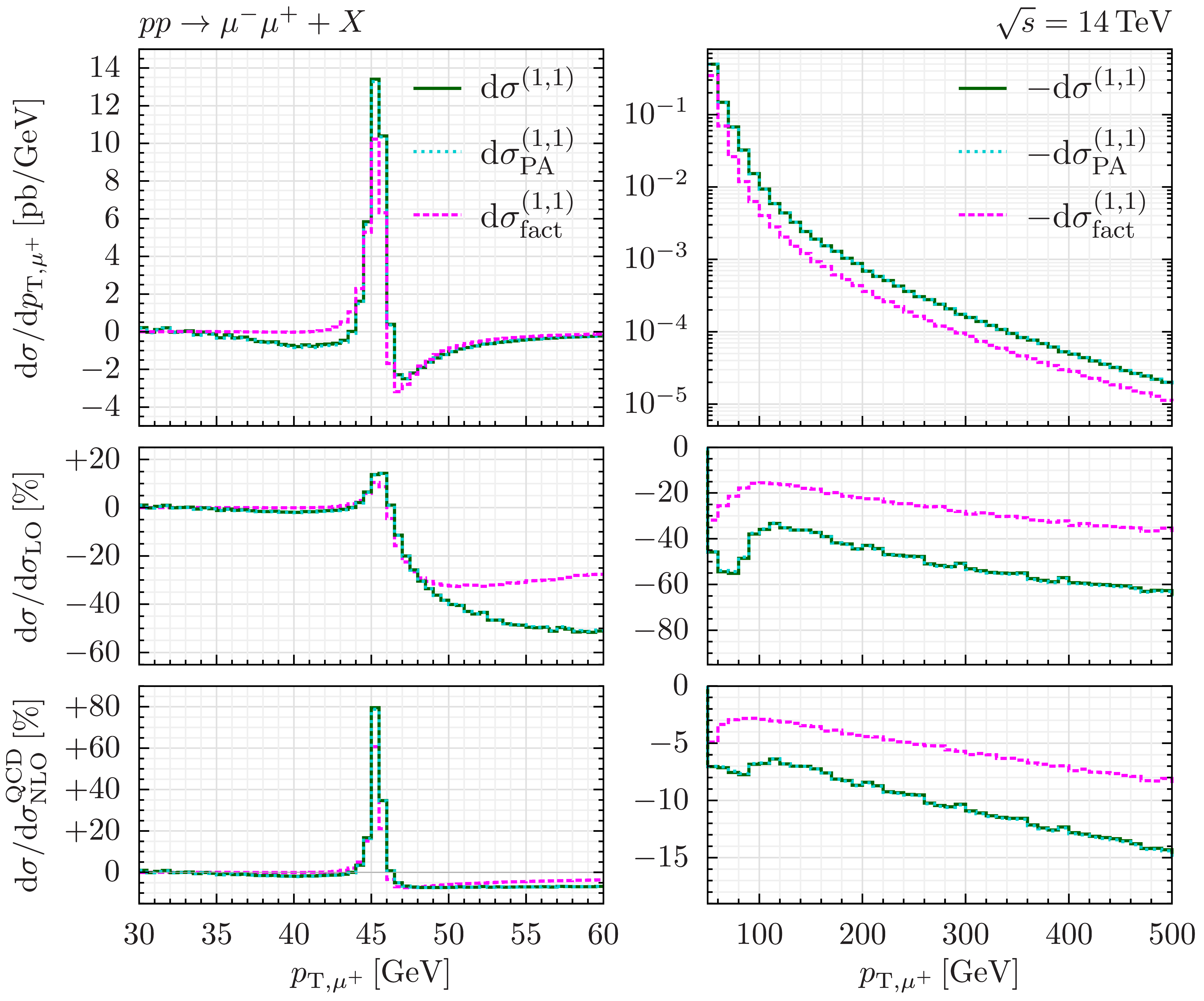
in the very high invariant mass range
QCD-EW effects are large and positive

at 3 TeV they are O(+10%), still comparable
with the statistical uncertainty at the end of HL-LHC

the factorised approximation
catches the bulk of the QCD-EW correction
but
a residual O(1%) non-factorisable effect emerges
(but more statistics is needed)

1% is large in units $\alpha\alpha_s$

The lepton transverse momentum distribution: QCD-EW corrections



in the jacobian peak region the xsec is very much affected by the fixed-order nature of the results

in the large transverse momentum tail
the dominant topology is Z+jet
which receives negative and large EW corrections

the exact calculation deviates from the factorised approximation (like MiNLO for Z+j rescaled with DY EW corrections)
because
the EW correction $d\sigma^{(0,1)}$ applies correctly to the $p_T^Z = 0$ bin
but
misses the large Sudakov logs which develop at $p_T^Z \gg 0$

the pole approximation on the 2-loop virtual corrections
affects only the $q\bar{q}$ process with $p_T^Z = 0$
which is negligible at large p_T^μ

Estimate of the residual uncertainties

The impact of the NNLO QCD-EW corrections is twofold: more accurate predictions (additional higher orders)
reduced uncertainties (scale, inputs, matching)

Ongoing phenomenological studies for full NC DY

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A representative example from the results for the on-shell Z production total cross section R.Bonciani, F.Buccioni, N.Rana, AV, arXiv:2007.06518
→ dependence on the EW input-scheme choice

comparison of (G_μ, M_W, M_Z) and $(\alpha(0), M_W, M_Z)$ (very conservative choice that maximises the spread of the results)

order	G_μ	$\alpha(0)$	$\delta(G_\mu-\alpha(0))$ (%)
NNLO-QCD	55787	53884	3.53
NNLO-QCD+NLO-EW	55501	55015	0.88
NNLO-QCD+NLO-EW+ NNLO QCD-EW	55469	55340	0.23

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the LO + NLO-EW result would suffer of only 0.55% spread;
the NLO-QCD and NNLO-QCD corrections are only LO-EW and reintroduce a dependence (→0.88%)
which is reduced by the NNLO QCD-EW (→0.23%)

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The availability of N3LO-QCD and NNLO QCD-EW results can bring the study of EW gauge bosons in the per mille arena !!!
Is the full NNLO-EW calculation negligible at this level ?

Conclusions

The evaluation of the NNLO QCD-EW corrections is not yet a "pressing-just-one-button" game
but
the main obstacles to compute the 2-loop virtual corrections have been understood and solved for NC DY

- amplitude manipulation
- IR structure
- evaluation of Master Integrals with 2 internal masses

The complete set of corrections has been combined in the Matrix framework to compute phenomenological predictions

The systematic automation of the progresses in the 2-loop virtual section is ongoing and will
allow the study of NNLO QCD-EW corrections to other scattering processes
be the starting point for the evaluation of NNLO-EW corrections

The phenomenological impact of mixed NNLO QCD-EW corrections is not negligible
in the precision physics program at the LHC

A precise SM prediction is the mandatory starting point for any SMEFT study or search in a UV-complete model

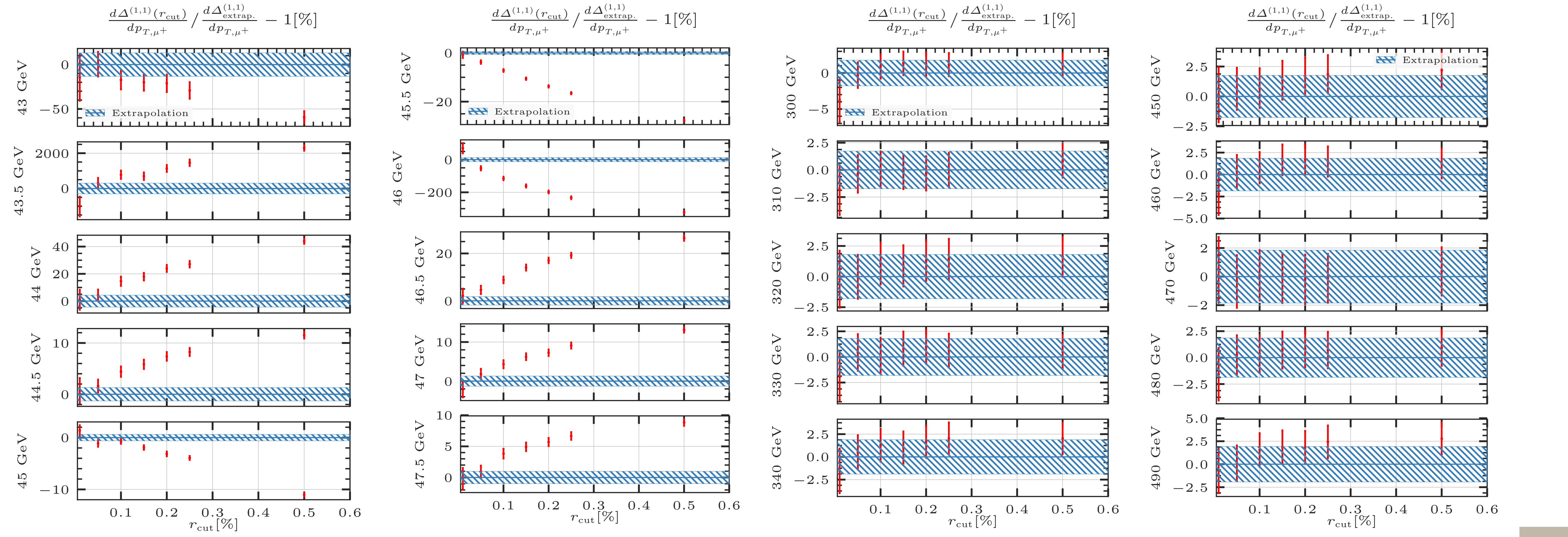
Few per mille precision of the theoretical predictions at large masses/momenta is
assuming a perfect proton PDF → a severe constraint on New Physics models
assuming the SM validity → a strong constraint in the proton PDF fit

Back-up

Differential sensitivity to r_{cut}

Binwise $r_{\rm cut}$ dependence of the mixed NNLO QCD–EW corrections for NC Drell–Yan

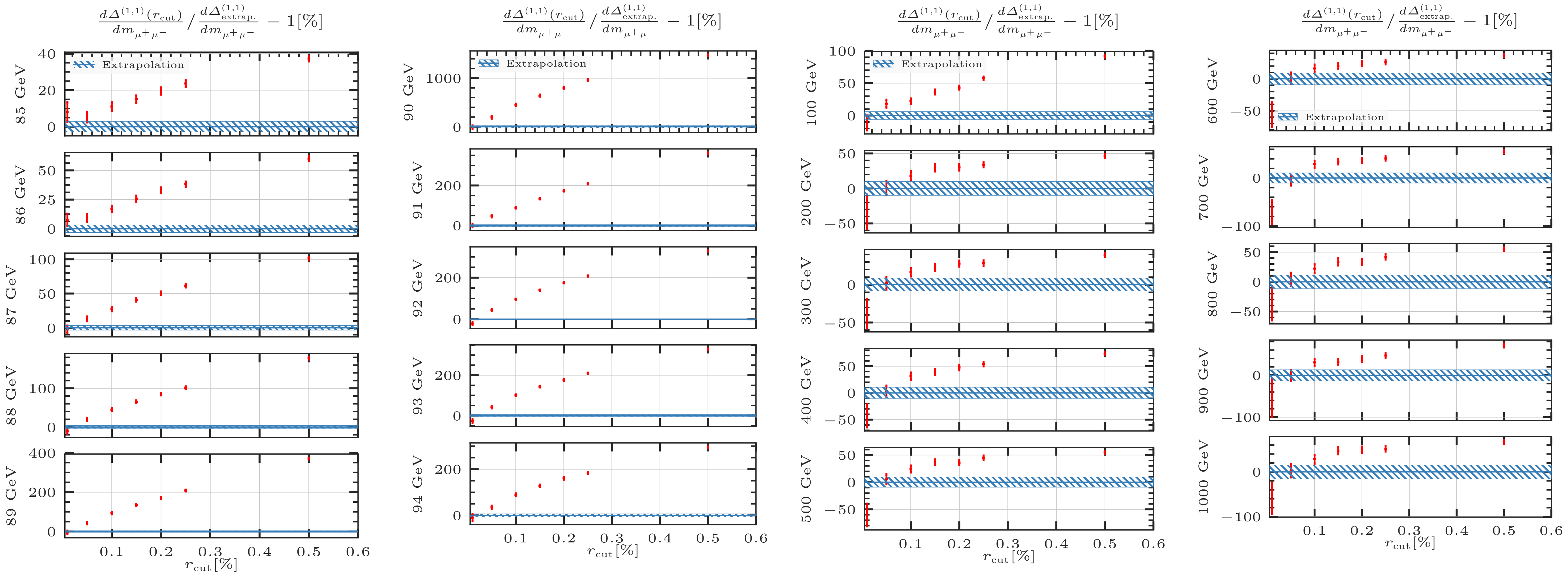
Differential distribution in p_{T,μ^+}: peak (left panels) and tail (right panels) regions



➥ large $r_{\rm cut}$ dependence in particular around the peak of the distribution, and typically precision of $\lesssim 3\%$ on the relative mixed QCD–EW corrections (artificially large where corrections are basically zero)

ependence of the mixed NNLO QCD–EW corrections for NC Drell–Yan

Differential distribution in $m_{\mu^+\mu^-}$: peak (left panels) and tail (right panels) regions



➥ quite large $r_{\rm cut}$ dependence throughout, and lower numerical precision of $\lesssim 10\%$ on the relative mixed QCD–EW corrections (but still permille-level precision at the level of cross sections)