Hadronic light-by-light scattering and the muon $g-2$

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universität wien

FWF Der Wissenschaftsfonds.

$\int dk\,\Pi$ Doktoratskolleg Particles and Interactions

Introduction

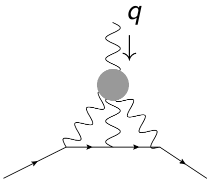
- **definition:**
  $\langle \mu(p_2) | j^\nu(0) | \mu(p_1) \rangle = \bar{u}(p_2) \left[F_E(q^2)\gamma^\nu + F_M(q^2) \frac{i\sigma^{\nu\rho} q_\rho}{2m_\mu} \right] u(p_1)$
  with $q = p_2 - p_1$ and $a_\mu = \frac{1}{2}(g_\mu - 2) = F_M(0)$
- 3 classes of contributions in the **Standard model**:
  - pure QED:
    - more than 99.99 % of the full result
    - **negligible** uncertainty (5 loops) — Aoyama et al. 2012, 2019
  - diagrams involving at least one electroweak gauge or Higgs boson
    - small and sufficiently precise
  - QED diagrams with QCD insertions:
    - $\sim$ 0.006 % of the full result
    - responsible for almost the **complete** uncertainty
    - topic of this talk
- most recent values — BNL 2004, FNL 2021, Aoyama et al. 2020

$$a_\mu^{\text{exp}} = (116\,592\,061 \pm 41) \times 10^{-11}$$
$$a_\mu^{\text{SM}} = (116\,591\,810 \pm 43) \times 10^{-11}$$

$\rightarrow$ 4.2σ discrepancy $\rightarrow$ **new physics?**

Dispersive approach to HLbL

- two important hadronic contributions:
  - hadronic vacuum polarization can be related via dispersion relation to $\sigma(e^+e^- \to \text{Hadrons})$
  - **hadronic light-by-light scattering** (HLbL) focus on this from now on



- Lorentz- and gauge invariance: interaction of four electromagnetic currents described by **scalar functions Π_i**

- analyticity, unitarity and crossing symmetry $\to$ **dispersion relations** for Π_i with input from simpler sub-processes

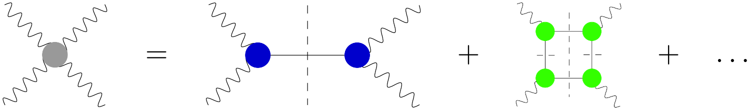
Colangelo et al. 2015, 2017

- a_μ^{HLbL} from 3-dimensional (numerical) integral over linear combinations of Π_i in limit $q \to 0$ ($\bar{\Pi}_i$) with known kernel functions

Intermediate states

- calculation can be organized as sum over **intermediate states**

Colangelo et al. 2015, 2017



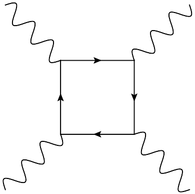
- states with **low mass** and **low multiplicity** most important at low energies
- pseudoscalar poles (π^0, η, η') numerically **dominant**
- dispersion relations allowed for a **model-independent** evaluation of a_μ^{HLbL} with **controlled uncertainties** for the first time
- result $a_\mu^{\text{HLbL}} = (92 \pm 19) \times 10^{-11}$ Aoyama et al. 2020
  consistent with lattice value $(79 \pm 35) \times 10^{-11}$ Blum et al. 2020
- above $\sim 1\,\text{GeV}$ **many** intermediate states become important and a model-independent evaluation of all of them is hardly possible

Short-distance constraints

- at $q = 0$, 3 independent kinematic variables $\to$ choose Euclidean photon virtualities $Q_i^2 = -q_i^2$
- for $Q_3^2, \Lambda_{\text{QCD}}^2 \ll Q_1^2, Q_2^2$ OPE relates HLbL to **VVA correlator**

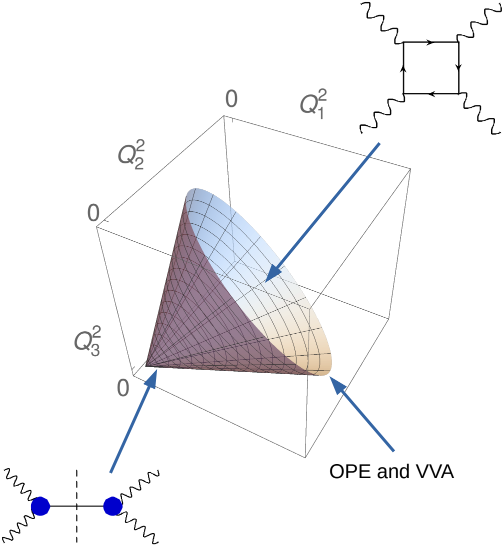
  Melnikov & Vainshtein 2004
- for massless quarks: **VVA correlator** given by axial anomaly (up to gluon anomaly)
- for $Q_1^2 \sim Q_2^2 \sim Q_3^2 \gg \Lambda_{\text{QCD}}^2$ HLbL given by OPE with massless **pQCD quark loop** as leading term

  Bijnens et al. 2019, 2021
- perturbative and non-perturbative (OPE) corrections small (for not too small Q_i^2)



Interpolation

JL & Procura, EPJ C **80** (2020) 1108



Strategy:

- construct functions **interpolating** between constrained regions
- calculate effect on a_μ via master formula
- **conservatively** estimate uncertainties

Results

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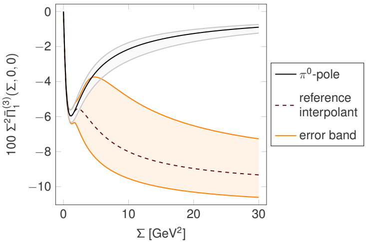
- effect of longitudinal short-distance constraints on HLbL is $(9.1 \pm 5.0) \times 10^{-11}$

- **compatible** with recent results from Regge model and holographic QCD

  Colangelo et al. 2020, Cappiello et al. 2019, Leutgeb & Rebhan 2019, 2021, Anton's talk

- But: far smaller than often-used model by Melnikov and Vainshtein (2004), discrepancy well understood



- uncertainties **well below** near-term experimental goal of $\pm 16 \times 10^{-11}$

- uncertainties almost exclusively due to **1–2 GeV** region

- improved knowledge of **resonances** in that mass-range needed to considerably improve

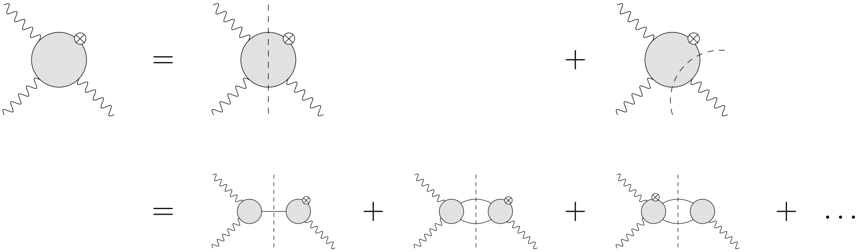
Novel dispersive formalism

JL, M. Procura and P. Stoffer, in preparation

- current formalism plagued by **singularities** for intermediate states with angular momentum ≥ 2
- singularities **cancel in** (infinite) **sum** over intermediate states due to sum rules $\rightarrow$ truncation leads to regularization ambiguities
- small effect at low energies, but important around **1–2 GeV** and for transition to perturbative regime
- idea: **dispersion relations** in the limit $q \rightarrow 0$, i.e. for the $\bar{\Pi}_i$
- free of such singularities **by construction**
- leads to reshuffling of contributions compared to current formalism

Unitarity

- prize to pay: implementation of **unitarity** more difficult



- some sub-processes known, others have to be reconstructed from dispersion relations and unitarity again
- additional complication: full amplitude finite at $q = 0$, but some sub-processes not $\rightarrow$ cancellation necessary **before** limit $q \rightarrow 0$

Conclusions

- SM prediction for muon anomalous magnetic moment limited by **hadronic** uncertainties
- **connection** between dispersive representation and asymptotic constraints in HLbL among most important open questions (conceptually and numerically)
- presented method how to **model-independently** combine QCD constraints on HLbL
- introduced **novel** dispersive formalism
- will allow to calculate also contributions due to **higher-spin** states

Thank you for your attention!