Spin hydrodynamics: Motivation and Basics

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Related papers: 2112.01856, 2103.02592, 2011.14907, 1901.09655

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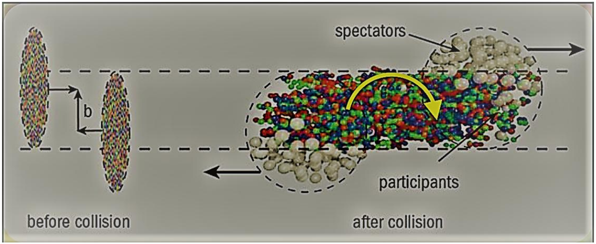
Heavy-ion collisions:

- Non-central relativistic heavy-ion collisions create global rotation of matter, which may induce spin polarization.
- Emerging particles are expected to be globally polarized with their spins on average pointing along the systems angular momentum.

nucl-th/0410079, nucl-th/0410089, arXiv:0708.0035.

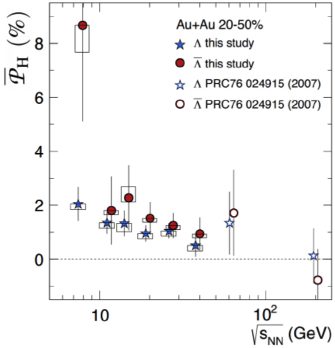


Source: CERN Courier

$$\boldsymbol{J}_{\text{initial}} = \boldsymbol{L}_{\text{initial}} = \boldsymbol{L}_{\text{final}} + \boldsymbol{S}_{\text{final}}$$

Global polarization:

First positive measurements of global spin polarization of Λ hyperons by STAR



thermal approach $\longrightarrow$ $P_{\Lambda} \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_{\Lambda}B}{T}$ $\quad P_{\overline{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_{\Lambda}B}{T}$

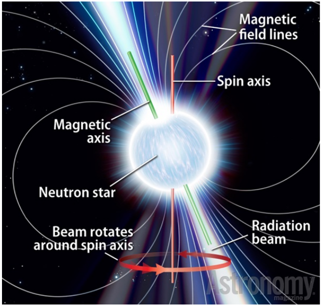
Becattini, F., Karpenko, I., Lisa, M., Upsal, I. , Voloshin, S. , PRC 95, 054902 (2017)

. . . the hottest, least viscous – and now, most vortical – fluid produced in the laboratory . . .

$$\omega = \left(P_{\Lambda} + P_{\overline{\Lambda}}\right) k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22} \mathrm{s}^{-1}$$

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

Even larger than...



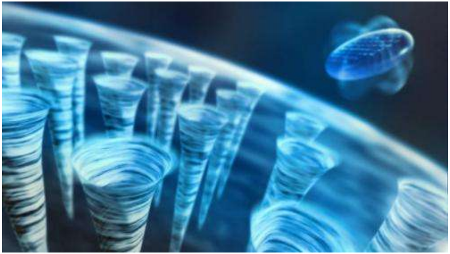


Figure: PSR J1748-2446ad ($716s^{-1}$) & Nanodroplets of superfluid helium ($10^7 s^{-1}$).

10.1126/science.1123430, Science 345, 906–909 (2014)

Global polarization:

- Good agreement between experiment and models based on local thermodynamic equilibrium of spin degrees of freedom.

  0711.1253, 1304.4427, 1303.3431, 1501.04468, 1610.02506, 1610.04717, 1605.04024, 1703.03770, etc...to ∞

- But...

Longitudinal polarization:

- Good agreement between experiment and models based on local thermodynamic equilibrium of spin degrees of freedom.

  0711.1253, 1304.4427, 1303.3431, 1501.04468, 1610.02506, 1610.04717, 1605.04024, 1703.03770, etc...to ∞

- But...

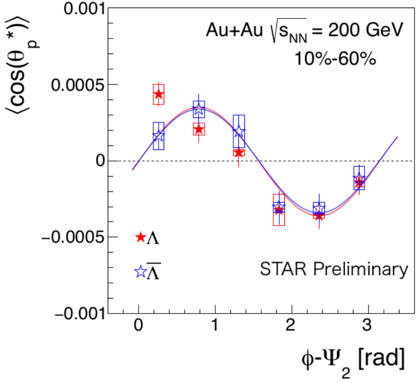


Figure: Longitudinal polarization of Λ-$\bar{\Lambda}$ (1905.11917)

Bigger picture:

- This study will help us to know the formation and characteristics of the QGP, a state of matter believed to exist at sufficiently high energy densities.

- Detecting and understanding the QGP allows us to understand better the universe in the moments after the Big Bang.

Canonical:

$$\mathcal{L}_D(x) = \frac{i\hbar}{2}\,\bar{\psi}(x)\gamma^\mu \overleftrightarrow{\partial}_\mu \psi(x) - m\bar{\psi}(x)\psi(x),$$

Energy-momentum tensor and spin tensor:

$$\hat{T}_C^{\mu\nu} = \frac{i\hbar}{2}\bar{\psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \psi - g^{\mu\nu}\mathcal{L}_D.$$

$$\hat{S}_C^{\lambda,\mu\nu} = -\frac{\hbar}{2}\epsilon^{\lambda\mu\nu\alpha}\bar{\psi}\gamma_\alpha\gamma_5\psi.$$

which are the parts of the total angular momentum tensor as

$$\hat{J}_C^{\lambda,\mu\nu} = x^\mu\,\hat{T}_C^{\lambda\nu} - x^\nu\,\hat{T}_C^{\lambda\mu} + \hat{S}_C^{\lambda,\mu\nu}.$$

Since the total angular momentum is conserved, we have

$$\partial_\lambda \hat{S}_C^{\lambda,\mu\nu} = \hat{T}_C^{[\nu\mu]}.$$

2007.00138

Ambiguity in decomposition of orbital and spin angular momentum:

One can define a new pair of tensors $\hat{T}^{\mu\nu}$ and $\hat{S}^{\lambda,\mu\nu}$ connected to the canonical currents through the so-called pseudo-gauge transformations

$$
\begin{aligned}
\hat{T}^{\mu\nu} &= \hat{T}^{\mu\nu}_{C} + \frac{1}{2}\partial_\lambda(\hat{\Phi}^{\lambda,\mu\nu} + \hat{\Phi}^{\nu,\mu\lambda} + \hat{\Phi}^{\mu,\nu\lambda}) \\
\hat{S}^{\lambda,\mu\nu} &= \hat{S}^{\lambda,\mu\nu}_{C} - \hat{\Phi}^{\lambda,\mu\nu} + \partial_\rho \hat{Z}^{\mu\nu,\lambda\rho}
\end{aligned}
$$

where, $\hat{\Phi}^{\lambda,\mu\nu}$ and $\hat{Z}^{\mu\nu,\lambda\rho}$ are arbitrary differentiable operators called super-potentials satisfying
$\hat{\Phi}^{\lambda,\mu\nu} = -\hat{\Phi}^{\lambda,\nu\mu}$ and $\hat{Z}^{\mu\nu,\lambda\rho} = -\hat{Z}^{\nu\mu,\lambda\rho} = -\hat{Z}^{\mu\nu,\rho\lambda}$

$\rightarrow$ The newly defined tensors preserve the total energy, linear momentum, and angular momentum after integrated over the freeze-out hypersurface.
$\rightarrow$ Conservation laws are unchanged.

$$\partial_\mu \hat{T}^{\mu\nu} = 0, \qquad \partial_\lambda \hat{J}^{\lambda,\mu\nu} = 0,$$

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de Groot, van Leeuwen and van Weert (GLW) Pseudo-gauge:

$$
\begin{aligned}
\hat{\Phi}^{\lambda,\mu\nu} &= \frac{i\hbar^2}{4m}\bar{\psi}(\sigma^{\lambda\mu}\overleftrightarrow{\partial}^{\nu} - \sigma^{\lambda\nu}\overleftrightarrow{\partial}^{\mu})\psi, \\
\hat{Z}^{\mu\nu\lambda\rho} &= 0.
\end{aligned}
$$

Thus, we obtain

$$
\begin{aligned}
\hat{T}^{\mu\nu}_{GLW} &= -\frac{\hbar^2}{4m}\bar{\psi}\overleftrightarrow{\partial}^{\mu}\overleftrightarrow{\partial}^{\nu}\psi, \\
\hat{S}^{\lambda,\mu\nu}_{GLW} &= \frac{i\hbar^2}{4m}\left(\bar{\psi}\sigma^{\mu\nu}\overleftrightarrow{\partial}^{\lambda}\psi - \partial_{\rho}\epsilon^{\mu\nu\lambda\rho}\bar{\psi}\gamma^5\psi\right),
\end{aligned}
$$

2007.00138, 1705.00587, 1712.07676, 1806.02616, 1811.04409, S. R. De Groot *et. al.*, Relativistic Kinetic Theory: Principles and Applications (1980).

Our approach:

- Include spin degrees of freedom into the ideal standard hydrodynamics to form spin hydrodynamics formalism.
- $J^{\mu,\alpha\beta}(x) = x^{\alpha} T^{\mu\beta}(x) - x^{\beta} T^{\mu\alpha}(x) + S^{\mu,\alpha\beta}(x)$
- And, conservation of total angular momentum, $\partial_\lambda J^{\lambda,\mu\nu}(x) = 0$ gives $\partial_\lambda S^{\lambda,\mu\nu}(x) = T^{\nu\mu}(x) - T^{\mu\nu}(x)$
- For symmetric energy-momentum tensor, $T^{\nu\mu}_{\rm GLW}(x) = T^{\mu\nu}_{\rm GLW}(x)$, we have $\partial_\lambda S^{\lambda,\mu\nu}_{\rm GLW}(x) = 0$
- Hence conservation of the angular momentum implies the conservation of its spin part in the de Groot-van Leeuwen-van Weert (GLW) formulation.

1705.00587, 1712.07676, 1806.02616, 1811.04409, S. R. De Groot *et. al.*, Relativistic Kinetic Theory: Principles and Applications (1980).

Steps of spin hydrodynamic framework:

- Solving the standard perfect-fluid hydrodynamic equations without spin.
- Determination of the spin evolution in the hydrodynamic background.
- Determination of the Pauli-Lubański (PL) vector on the freeze-out hypersurface.
- Then we calculate momentum dependent and momentum integrated spin polarization of particles in their rest frame which can be directly compared with the experimental findings.

  1901.09655

Conservation laws:

- $d_\alpha N^\alpha \equiv d_\alpha(\mathcal{N} U^\alpha) = 0 \rightarrow$ **Conservation of net baryon number**
  where $\mathcal{N} = 4\ \sinh(\mu/T)\ \mathcal{N}_{(0)}$.
- $d_\alpha T^{\alpha\beta} \equiv d_\alpha \left[(\mathcal{E}+\mathcal{P})\, U^\alpha U^\beta - \mathcal{P} g^{\alpha\beta}\right] = 0 \rightarrow$ **Conservation of EMT**
  where $\mathcal{E} = 4\cosh(\mu/T)\mathcal{E}_{(0)}$ and $\mathcal{P} = 4\cosh(\mu/T)\mathcal{P}_{(0)}$.

  These laws provide closed system of 5 eqns. for 5 funcs: μ, T, and three independent components of U^μ which need to be solved to get the perfect-fluid background evolution.
- $d_\alpha S^{\alpha,\beta\gamma} \equiv d_\alpha \left[\mathcal{A}_1 U^\alpha \omega^{\beta\gamma} + \mathcal{A}_2 U^\alpha U^{[\beta}\kappa^{\gamma]} + \mathcal{A}_3 (U^{[\beta}\omega^{\gamma]\alpha} + g^{\alpha[\beta}\kappa^{\gamma]})\right] = 0$
  $\downarrow$
  Conservation of spin

  where $\mathcal{A}_1 = \cosh(\mu/T)\left(n_{(0)} - \mathcal{B}_{(0)}\right)$, $\mathcal{A}_2 = \cosh(\mu/T)\left(\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)}\right)$,
  $\mathcal{A}_3 = \cosh(\mu/T)\,\mathcal{B}_{(0)}$
  with, $\mathcal{B}_{(0)} = -\frac{2}{(m/T)^2}(\mathcal{E}_{(0)} + \mathcal{P}_{(0)})/T$ and $\mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2\mathcal{N}_{(0)}$.
  Here, $\omega^{\beta\gamma}$ is known as spin polarization tensor.

1811.04409

Spin polarization tensor:

$\omega_{\mu\nu}$ is an anti-symmetric tensor of rank 2 and can be parameterized by the four-vectors κ^μ and ω^μ,

$$\boxed{\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta},$$

where $\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \quad \omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha$.
U, X, Y and Z form a 4-vector basis satisfying the following normalization conditions: $U \cdot U = 1, \qquad X \cdot X = Y \cdot Y = Z \cdot Z = -1$.

$$\omega_{\alpha\beta} = \begin{bmatrix} 0 & e^1 & e^2 & e^3 \\ -e^1 & 0 & -b^3 & b^2 \\ -e^2 & b^3 & 0 & -b^1 \\ -e^3 & -b^2 & b^1 & 0 \end{bmatrix}$$

$\boldsymbol{e}$ and $\boldsymbol{b}$ are LAB frame spin components.

Spin polarization of emitted particles:

$$\langle\pi_\mu\rangle^* = \frac{E_p \frac{d\overset{*}{\Pi}_\mu(p)}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}} \quad \frac{\to Total\ \ Pauli\ \ Lubanski\ \ vector}{\to Momentum\ \ density\ \ of\ \ all\ \ particles} \sim \ \boldsymbol{p}\ dependent$$

$$\langle\pi_\mu\rangle = \frac{\int dP \langle\pi_\mu\rangle_p E_p \frac{d\mathcal{N}(p)}{d^3p}}{\int dP E_p \frac{d\mathcal{N}(p)}{d^3p}} = \frac{\int d^3p \ \frac{d\overset{*}{\Pi}_\mu(p)}{d^3p}}{\int d^3p \ \frac{d\mathcal{N}(p)}{d^3p}} \sim \ \boldsymbol{p}\ integrated$$

where, $$E_p \frac{d\overset{*}{\Pi}_\mu(p)}{d^3p} = \frac{1}{(2\pi)^3 m} \int \cosh\left(\frac{\mu}{T}\right) \Delta\Sigma_\lambda p^\lambda \, e^{-\beta\cdot p} \left(\tilde{\omega}_{\beta\mu} \ p^\beta\right)^*$$

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4}{(2\pi)^3} \int \cosh\left(\frac{\mu}{T}\right) \Delta\Sigma_\lambda p^\lambda \, e^{-\beta\cdot p}$$

* meaning quantities calculated in particle rest frame.

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Boost-invariant and transversely homogeneous flow

Initialization of spin components:

$\omega_{\mu\nu}$ is an anti-symmetric tensor of rank 2 and can be parameterized by the four-vectors κ^μ and ω^μ,

$$\boxed{\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta},$$

where $\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \quad \omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha$.
U, X, Y and Z form a 4-vector basis satisfying the following normalization conditions: $U \cdot U = 1, \qquad X \cdot X \ = \ Y \cdot Y \ = \ Z \cdot Z \ = \ -1.$

Perfect-fluid background and spin components evolution:

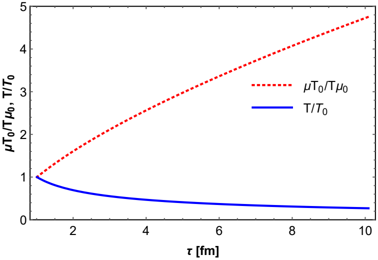
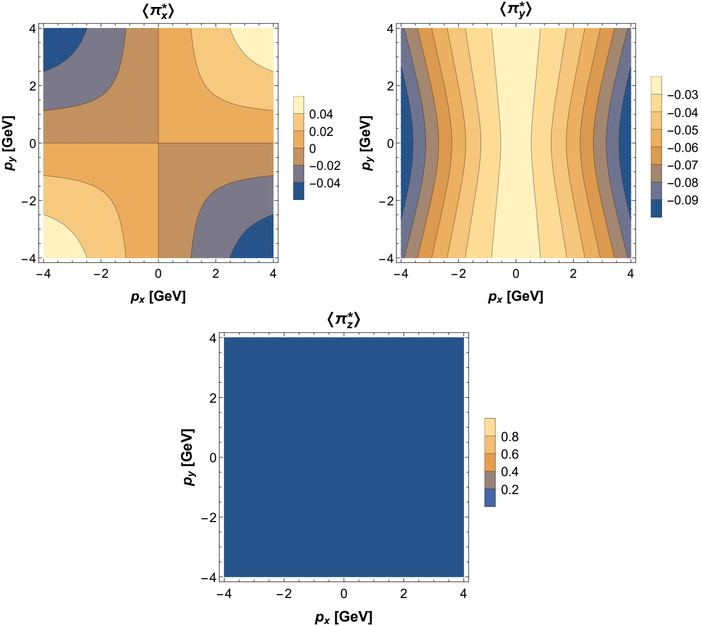


Figure: Evolution of T divided by its initial value T_0 and the ratio of baryon chemical potential μ and temperature T (left). Evolution of the spin coefficients $\boldsymbol{C_\kappa}$ and $\boldsymbol{C_\omega}$ (right).

For the spin dynamics in 1p1, see Dr. Gabriel Sophys talk.

Momentum dependence of polarization:

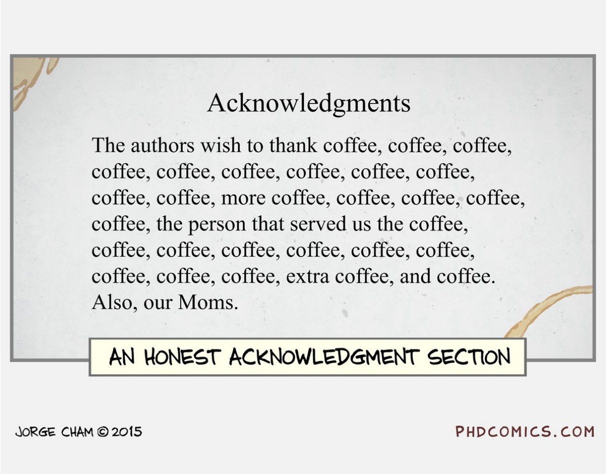


Summary:

- Discussed relativistic hydrodynamics with spin based on the GLW forms of energy-momentum and spin tensors.
- Showed how our formalism can be compared with the experiments.
- Obtained dynamics of spin polarization in the boost-invariant background.
- Incorporation of spin in full 3+1D hydro model required to address the problem of longitudinal polarization (which will be out pretty soon, stay tuned).

For the spin dynamics in 1p1, see Dr. Gabriel Sophys talk.

Thank you for your time!



Various choices of Pseudo-gauge:

Belinfante and Rosenfeld:

$$\hat{\Phi}^{\lambda,\mu\nu} = \hat{S}_C^{\lambda,\mu\nu}, \qquad \hat{Z}^{\mu\nu,\lambda\rho} = 0,$$

with

$$\hat{T}_B^{\mu\nu} = \frac{i\hbar}{4}\bar{\psi}(\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^\nu \overleftrightarrow{\partial}^\mu)\psi - g^{\mu\nu}\mathcal{L}_D,$$
$$\hat{S}_B^{\lambda,\mu\nu} = 0.$$

Hilgevoord-Wouthuysen:

$$\hat{\Phi}^{\lambda,\mu\nu} = \hat{M}^{[\mu\nu]\lambda} - g^{\lambda[\mu}\hat{M}_\rho{}^{\nu]\rho},$$
$$\hat{Z}^{\mu\nu\lambda\rho} = -\frac{\hbar}{8m}\bar{\psi}(\sigma^{\mu\nu}\sigma^{\lambda\rho} + \sigma^{\lambda\rho}\sigma^{\mu\nu})\psi,$$

where

$$\hat{M}^{\lambda\mu\nu} \equiv \frac{i\hbar^2}{4m}\bar{\psi}\sigma^{\mu\nu}\overleftrightarrow{\partial}^\lambda\psi.$$

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