

Helical vortical effects for thermal fermions and polarization asymmetry

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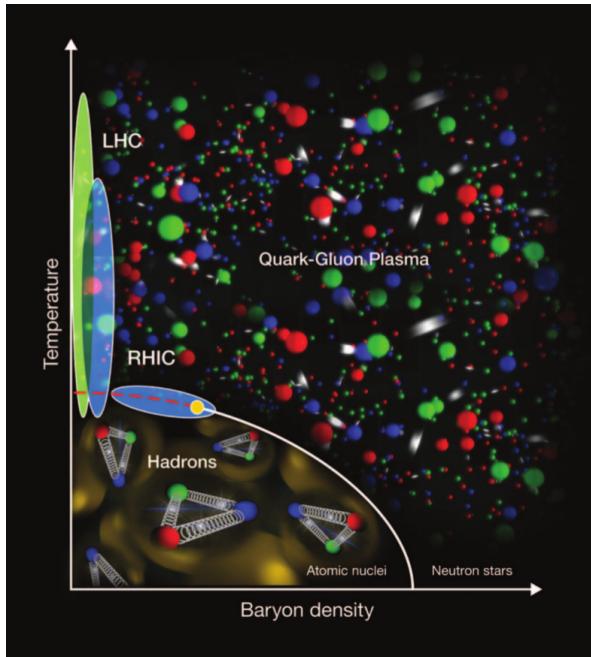


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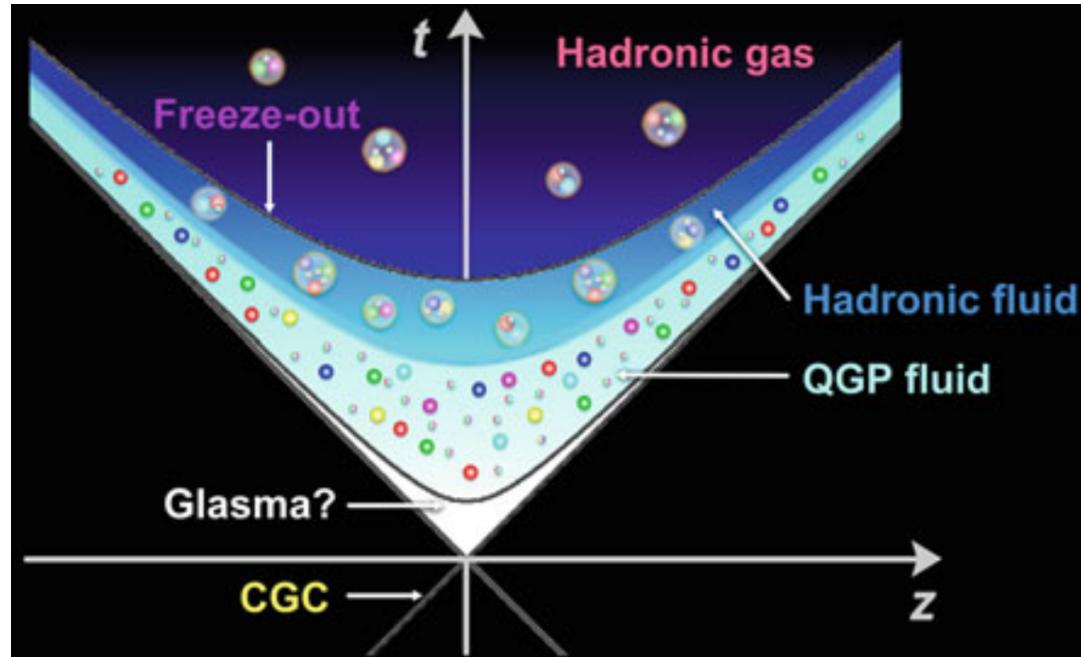


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- 2 Polarisation: Helicity and Chirality
- 3 Helical vortical effects
- 4 Hyperon/anti-hyperon polarisation ratio
- 5 Conclusion

Quark-gluon plasma: hydrodynamic phase



B. V. Jacak, B. Muller, Science
337 (2012) 310.



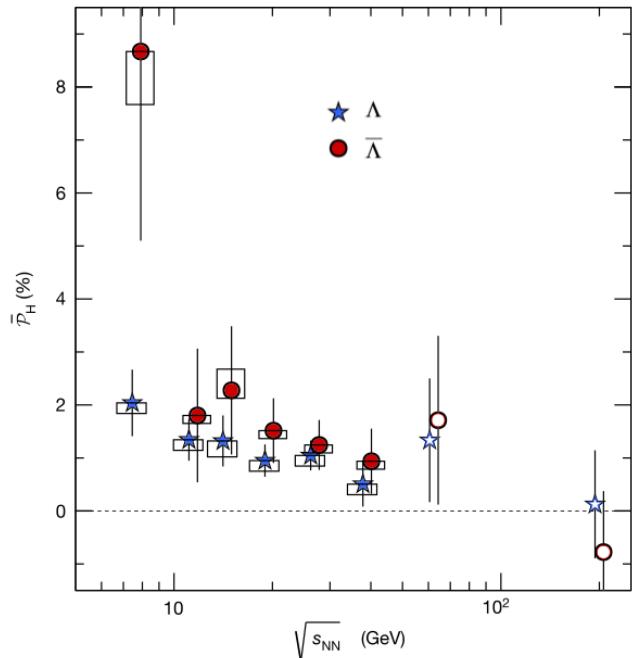
A. Monnai, PhD Thesis (Tokyo, 2014).

- ▶ The QGP produced at RHIC is...
 - the hottest ($k_B T \gtrsim 0.2$ GeV $\Leftrightarrow T \gtrsim 2.3 \times 10^{12}$ K),
 - densest ($p \gtrsim 10$ GeV/fm³ $\simeq 1.6 \times 10^{36}$ Pa)
 - and most vortical ($\omega \simeq 10^{22}$ s⁻¹)...

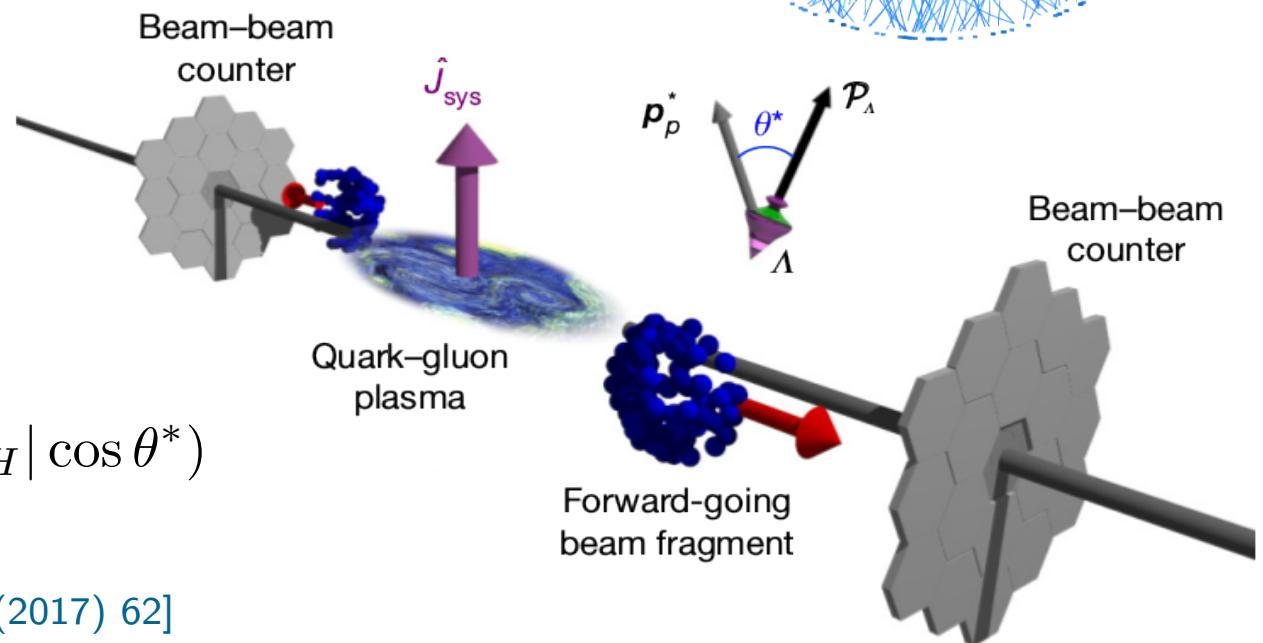
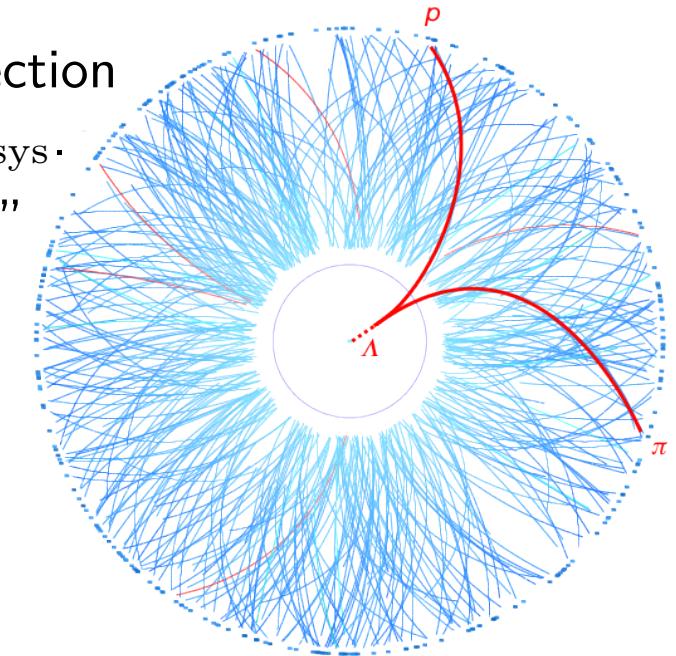
... fluid produced in the laboratory.

[STAR Collaboration, Nature 548 (2017) 62]

QGP: Polarisation of Λ -hyperons



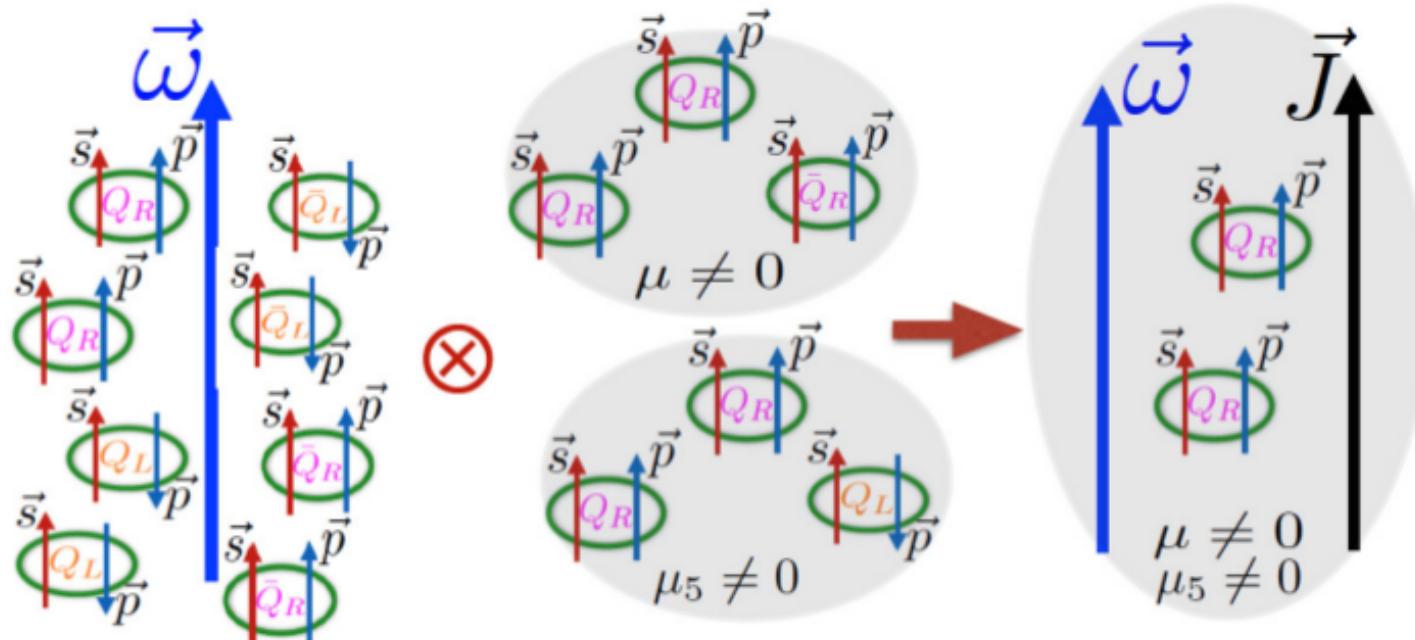
- $\overline{\mathcal{P}}_H \equiv$ average projection of polarization on \hat{J}_{sys} .
- $\Lambda \equiv$ “self-analysing:” proton emitted preferentially along spin.



- $|\omega| \approx \frac{k_B T}{\hbar} (\overline{\mathcal{P}}_{\Lambda'} + \overline{\mathcal{P}}_{\bar{\Lambda}'})$.
- $\frac{dN_H}{d \cos \theta^*} = \frac{1}{2} (1 + \alpha_H |\mathcal{P}_H| \cos \theta^*)$

[STAR Collaboration, Nature 548 (2017) 62]

Known mechanism: Chiral vortical effect (CVE)



$$J_V = \sigma_V \omega,$$

$$\sigma_V = \frac{\mu_V \mu_A}{\pi^2},$$

$J_A \neq 0$ even when $\mu_A = 0!$

$$J_A = \sigma_A \omega,$$

$$\sigma_A = \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2}.$$

[D. E. Kharzeev *et al.*, Nucl. Phys. **88** (2016) 1]

New mechanism: Helical vortical effect (HVE)

- ▶ Split particles into four groups:

- μ_{\uparrow}^R : particle: $R \Rightarrow \uparrow$
- μ_{\downarrow}^L : particle: $L \Rightarrow \downarrow$
- $\bar{\mu}_{\downarrow}^R$: anti-particle: $R \Rightarrow \downarrow$
- $\bar{\mu}_{\uparrow}^L$: anti-particle: $L \Rightarrow \uparrow$

- ▶ Charge densities:

$$Q_V \equiv (n_{\uparrow}^R + n_{\downarrow}^L) - (\bar{n}_{\downarrow}^R + \bar{n}_{\uparrow}^L),$$

$$Q_A \equiv (n_{\uparrow}^R + \bar{n}_{\downarrow}^R) - (n_{\uparrow}^L + \bar{n}_{\downarrow}^L),$$

$$Q_H \equiv (n_{\uparrow}^R + \bar{n}_{\uparrow}^L) - (n_{\downarrow}^L + \bar{n}_{\downarrow}^R).$$

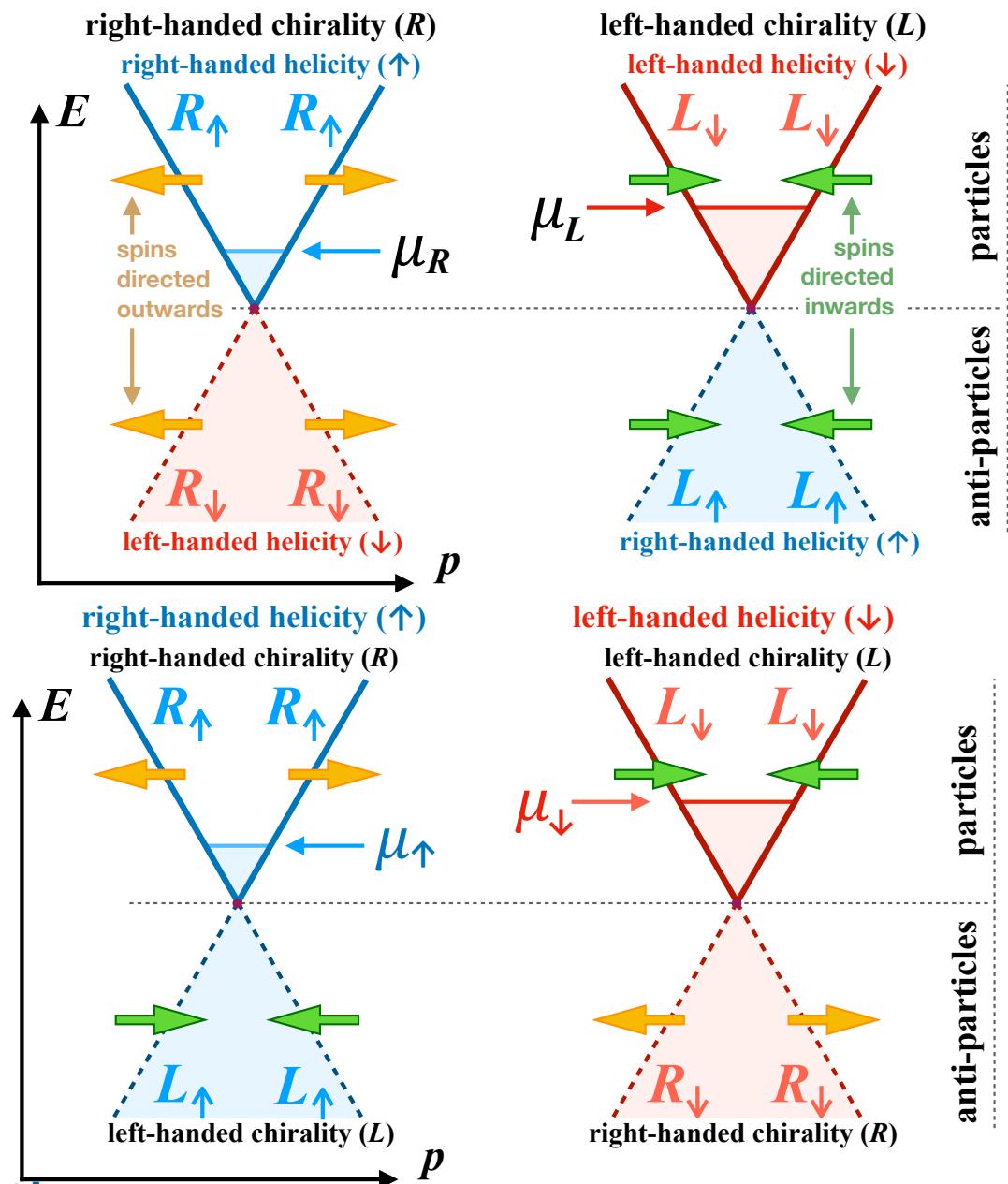
- ▶ Vortical conductivities:

$$\sigma_V \simeq \frac{2\mu_H T}{\pi^2} \ln 2 + \frac{\mu_V \mu_A}{\pi^2},$$

$$\sigma_A \simeq \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2 + \mu_H^2}{2\pi^2},$$

$$\sigma_H \simeq \frac{2\mu_V T}{\pi^2} \ln 2 + \frac{\mu_H \mu_A}{\pi^2}.$$

VEA, M. N. Chernodub, arXiv:1912.11034 [hep-th]



- ▶ For particles ($U_{R/L}$) and anti-particles ($V_{R/L} = i\gamma^2 U_{R/L}^*$):

$$\begin{aligned}\gamma^5 U_R &= + U_R, & \gamma^5 U_L &= - U_L, \\ \gamma^5 V_R &= - V_R, & \gamma^5 V_L &= + V_L.\end{aligned}\tag{1}$$

- ▶ The axial current $J_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$ satisfies (classically)

$$\partial_\mu J_A^\mu = 2im\bar{\psi} \gamma^5 \psi,$$

and is hence conserved when $m = 0$.

- ▶ $Q_A = \int d^3x J_A^0$ can be promoted to a quantum operator:

$$:\hat{Q}_A := \sum_j \chi_j (\hat{b}_j^\dagger \hat{b}_j + \hat{d}_j^\dagger \hat{d}_j), \quad \chi_R = +1, \quad \chi_L = -1, \tag{2}$$

which satisfies $[\hat{Q}_A, \hat{H}] = 0$ when $m = 0$.

Helicity (h)

- ▶ The polarisation of ψ can be characterised using $h = \frac{\mathbf{S} \cdot \mathbf{P}}{p}$, with

$$hU_\lambda = \lambda U_\lambda, \quad hV_\lambda = \lambda V_\lambda, \quad \lambda = \pm \frac{1}{2}. \quad (3)$$

- ▶ The helicity current $J_H^\mu = \bar{\psi}\gamma^\mu h\psi + \bar{h}\psi\gamma^\mu\psi$ is conserved $\forall m$:

$$\partial_\mu J_H^\mu = 0.$$

- ▶ Comparing Eqs. (1) and Eq. (3) shows that for $m = 0$,

$$2\lambda_j = \chi_j. \quad (4)$$

- ▶ $Q_H = \int d^3x J_H^0$ can also be represented as a quantum operator:

$$:\hat{Q}_H := \sum_j 2\lambda_j (\hat{b}_j^\dagger \hat{b}_j - \hat{d}_j^\dagger \hat{d}_j), \quad (5)$$

satisfying $[\hat{Q}_H, \hat{H}] = 0$.

Chirality (γ^5) vs. helicity (h)



- ▶ For $m = 0$, γ^5 and $h = \frac{\mathbf{S} \cdot \mathbf{P}}{p}$ share the eigenmodes U_j and $V_j = i\gamma^2 U_j^*$:

$$\binom{2h}{\gamma^5} U_j = 2\lambda_j U_j, \quad \binom{2h}{-\gamma^5} V_j = 2\lambda_j V_j, \quad (6)$$

- ▶ $J_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$ satisfies $\partial_\mu J_A^\mu = 2im\bar{\psi} \gamma^5 \psi$.
- ▶ $J_h^\mu = \bar{\psi} \gamma^\mu h \psi + \bar{h} \psi \gamma^\mu \psi$ satisfies $\partial_\mu J_H^\mu = 0$ (for all m).
- ▶ Why is chirality good? Chiral vortical / magnetic / separation / etc. effects
- ▶ Why is chirality bad? $m \neq 0$; Axial anomaly ($\partial_\mu J_A^\mu = -\frac{e^2}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$)
- ▶ Why is helicity good? Works at $m \neq 0$; Helical vortical effects
- ▶ Why is helicity bad? interactions; anomaly?; ambiguous when $m \neq 0$

	Q_V	Q_A	Q_H	\mathbf{J}_V	\mathbf{J}_A	\mathbf{J}_H	$\boldsymbol{\omega}$
\mathcal{C}	–	+	–	–	+	–	+
\mathcal{P}	+	–	–	–	+	+	+
\mathcal{T}	+	+	+	–	–	–	–

- ▶ J_V^μ , J_A^μ and J_H^μ form a triad: same \mathcal{T} , different \mathcal{C} and \mathcal{P} .
- ▶ New vortical effects $\mathbf{J}_\ell = \sigma_\ell \boldsymbol{\omega}$ allowed by \mathcal{CPT} symmetries:

$$\begin{aligned}
 (-,-,+)
 & \quad \sigma_V \simeq \frac{2\mu_H T}{\pi^2} \ln 2 + \frac{\mu_V \mu_A}{\pi^2}, \\
 (+,+,+)
 & \quad \sigma_A \simeq \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2 + \mu_H^2}{2\pi^2}, \\
 (-,+,+)
 & \quad \sigma_H \simeq \frac{2\mu_V T}{\pi^2} \ln 2 + \frac{\mu_H \mu_A}{\pi^2}.
 \end{aligned} \tag{7}$$

- ▶ Terms in blue survive when $\mu_A = \mu_H = 0 \Rightarrow$ finite \mathbf{J}_A and \mathbf{J}_H when $\boldsymbol{\omega} \neq 0$.

- ▶ Fermions in equilibrium can be described using

$$f_{\sigma,\lambda} = f^{(\text{eq})}(\beta \cdot k - \alpha_{\sigma,\lambda}), \quad \beta^\mu = \frac{u^\mu}{T}, \quad \alpha_{\sigma,\lambda} = \frac{\mathbf{q}_{\sigma,\lambda} \cdot \boldsymbol{\mu}}{T}, \quad (8)$$

where $\mathbf{q}_{V/A/H} = (\sigma, 2\lambda, 2\lambda\sigma)$ and $\boldsymbol{\mu} = (\mu_V, \mu_A, \mu_H)$.

- ▶ Global equilibrium is achieved when

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0, \quad \nabla_\mu \alpha_{\sigma,\lambda} = 0. \quad (9)$$

- ▶ One possible solution of the *Killing eq.* corresponds to rigid rotation:

$$\beta = \beta_0(\partial_t + \Omega\partial_\varphi), \quad (10)$$

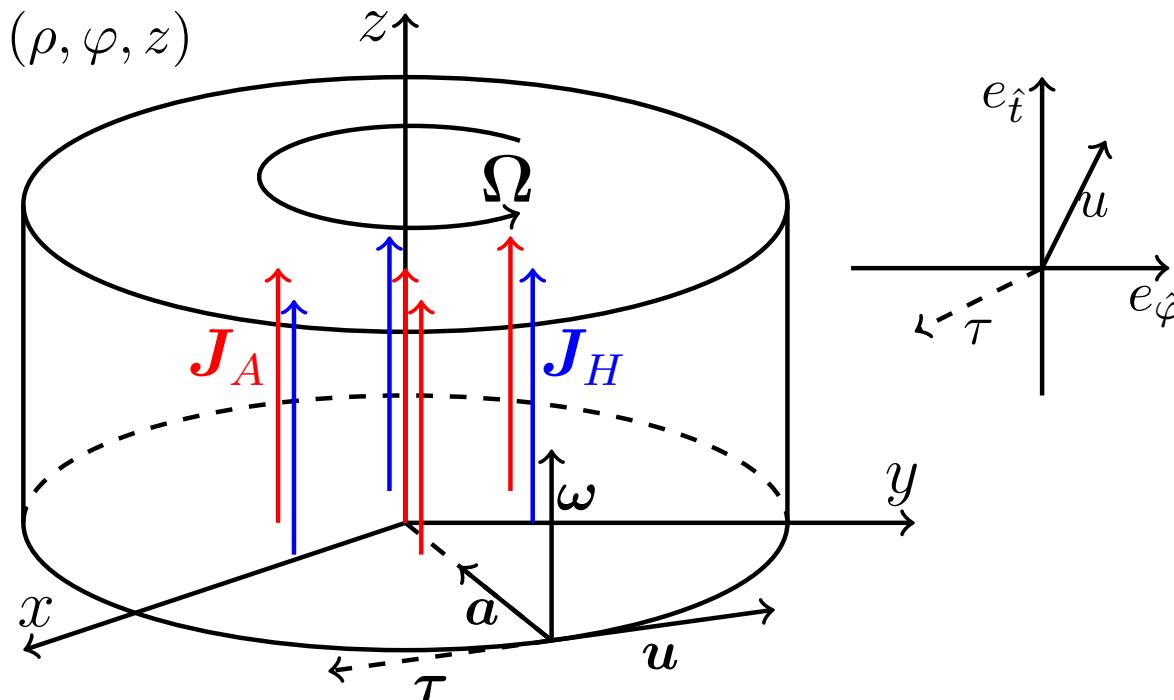
giving rise to

$$u = \Gamma(\partial_t + \Omega\partial_\varphi), \quad \begin{pmatrix} T \\ \mu \end{pmatrix} = \Gamma \begin{pmatrix} T_0 \\ \mu_0 \end{pmatrix}, \quad \Gamma = (1 - \rho^2\Omega^2)^{-1/2}. \quad (11)$$

- ▶ In equilibrium, $T^{\mu\nu} = (E + P)u^\mu u^\nu - Pg^{\mu\nu}$ and $J_\ell^\mu = Q_\ell u^\mu$, where

$$P = -\frac{T^4}{\pi^2} \sum_{\sigma,\lambda} \text{Li}_4(-e^{\mathbf{q}_{\sigma,\lambda} \cdot \boldsymbol{\mu}/T}), \quad Q_\ell = \frac{\partial P}{\partial \mu_\ell}. \quad (12)$$

Kinematic frame for rigid rotation



The *kinematic tetrad* is given by:

[Becattini, Grossi, PRD 2015]

Velocity : $u = \Gamma(e_{\hat{t}} + \rho\Omega e_{\hat{\varphi}}), \quad \Gamma = (1 - \rho^2\Omega^2)^{-1/2},$

Acceleration : $a = \nabla_u u = -\rho\Omega^2\Gamma^2 e_{\hat{\rho}},$

Vorticity : $\omega = \frac{1}{2}\varepsilon^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\sigma}}e_{\hat{\alpha}}u_{\hat{\beta}}(\nabla_{\hat{\gamma}}u_{\hat{\sigma}}) = \Gamma^2\Omega e_{\hat{z}},$

Fourth vector : $\tau = \varepsilon^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\sigma}}e_{\hat{\alpha}}u_{\hat{\beta}}a_{\hat{\gamma}}\omega_{\hat{\sigma}} = -\rho\Omega^3\Gamma^5(\rho\Omega e_{\hat{t}} + e_{\hat{\varphi}}).$

Quantum rigidly-rotating thermal states



- ▶ Computing $J_{V/A/H}^\mu = \langle \hat{J}_{V/A/H}^\mu \rangle$ using $[Z = \text{Tr}(\hat{\varrho})]$

$$\langle \hat{A} \rangle = Z^{-1} \text{Tr}(\hat{\varrho} \hat{A}), \quad \hat{\varrho} = \exp \left[-\frac{1}{T_0} \left(\hat{H} - \Omega \hat{M}^z - \sum_\ell \mu_{\ell;0} \hat{Q}_\ell \right) \right],$$

the charge currents can be seen to deviate from the perfect fluid form,

$$J_\ell^\mu = Q_\ell u^\mu + \sigma_\ell^\omega \omega^\mu + \sigma_\ell^\tau \tau^\mu, \quad (13)$$

where at leading order w.r.t. Ω we have:

$$\begin{aligned} Q_\ell &= \frac{\partial P}{\partial \mu_\ell} = -\frac{T^3}{\pi^2} \sum_{\sigma,\lambda} q_{\sigma,\lambda}^\ell \text{Li}_3 \left[-\exp \left(\frac{\mathbf{q}_{\sigma,\lambda} \cdot \boldsymbol{\mu}}{T} \right) \right], \\ \sigma_\ell^\omega &= \frac{1}{2} \frac{\partial^2 P}{\partial \mu_A \partial \mu_\ell} = -\frac{T^2}{2\pi^2} \sum_{\sigma,\lambda} q_{\sigma,\lambda}^A q_{\sigma,\lambda}^\ell \text{Li}_2 \left[-\exp \left(\frac{\mathbf{q}_{\sigma,\lambda} \cdot \boldsymbol{\mu}}{T} \right) \right], \\ \sigma_\ell^\tau &= \frac{1}{12} \frac{\partial^3 P}{\partial^2 \mu_A \partial \mu_\ell} = \frac{T}{12\pi^2} \sum_{\sigma,\lambda} q_{\sigma,\lambda}^\ell \ln \left[1 + \exp \left(\frac{\mathbf{q}_{\sigma,\lambda} \cdot \boldsymbol{\mu}}{T} \right) \right]. \end{aligned} \quad (14)$$

where the circular term is suppressed since $\tau = -\Omega^3 \Gamma^5 (\rho^2 \Omega \partial_t + \partial_\varphi)$.

Axial/helical vortical effects: Constitutive relations



- ▶ For high temperatures, $\sigma_\ell^\tau = \mu_\ell / 6\pi^2$ and

$$\begin{aligned}\sigma_V^\omega &= \frac{2\mu_H T}{\pi^2} \ln 2 + \frac{\mu_A \mu_V}{\pi^2} + O(T^{-1}), \\ \sigma_A^\omega &= \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2 + \mu_H^2}{2\pi^2} + O(T^{-1}), \\ \sigma_H^\omega &= \frac{2\mu_V T}{\pi^2} \ln 2 + \frac{\mu_A \mu_H}{\pi^2} + O(T^{-1}).\end{aligned}\tag{15}$$

- ▶ At finite T and μ_V , ω generates both J_A and J_H !

Particle/anti-particle polarisation from $J_A \pm J_H$

- Considering now a system with $\Omega_{\text{sys}} = n_{\text{sys}} |\Omega_{\text{sys}}|$ and $J_\ell \equiv \mathbf{J}_\ell \cdot \mathbf{n}$, we can identify:

$$J_V = J_\uparrow + J_\downarrow - \bar{J}_\uparrow - \bar{J}_\downarrow,$$

$$J_A = J_\uparrow + \bar{J}_\uparrow - J_\downarrow - \bar{J}_\downarrow,$$

$$J_H = J_\uparrow + \bar{J}_\downarrow - J_\downarrow - \bar{J}_\uparrow,$$

where $(\uparrow, \downarrow) \equiv$ (right-, left-)handed helicity, while $\bar{}$ \equiv anti-particles.

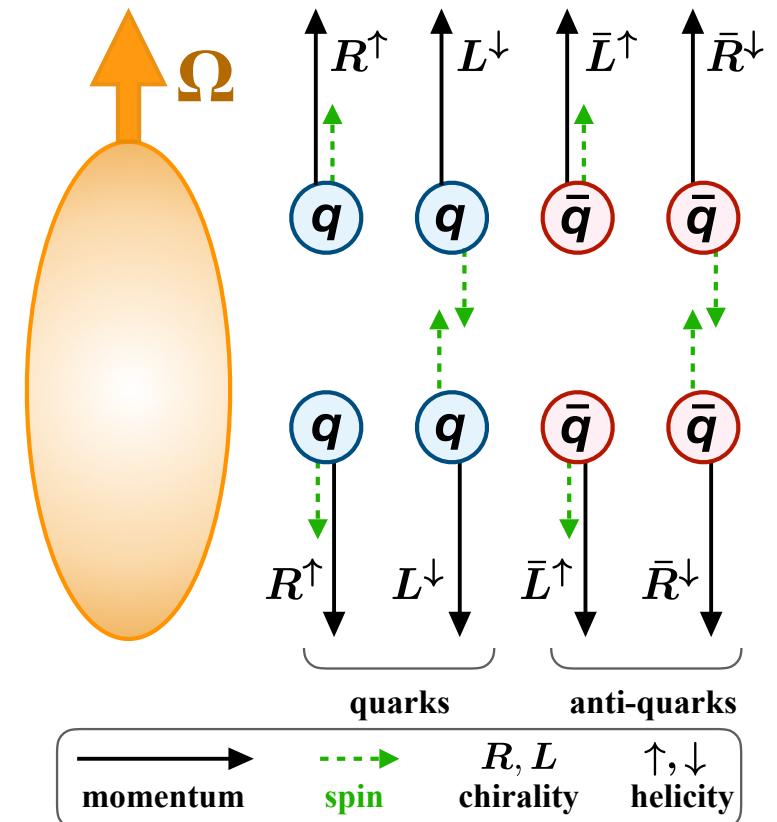
- The net helicity current of particles and anti-particles can be obtained as

$$J_\uparrow - J_\downarrow = \frac{J_A + J_H}{2}, \quad \bar{J}_\uparrow - \bar{J}_\downarrow = \frac{J_A - J_H}{2}. \quad (16)$$

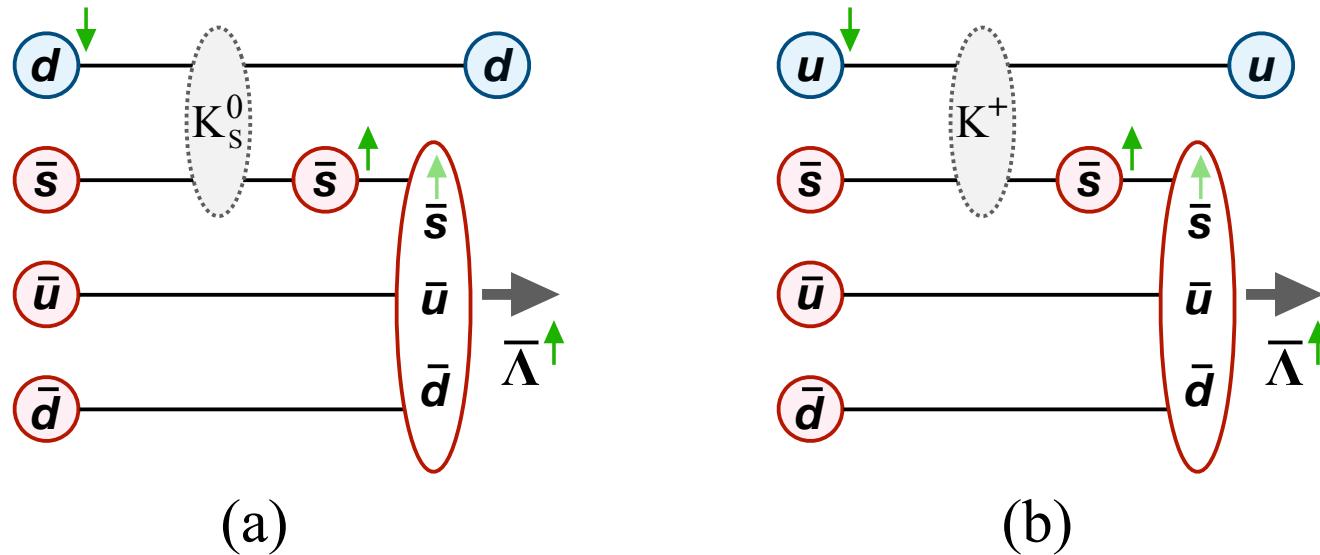
- The polarisation of (light flavour) quarks / anti-quarks can be related to the above via

$$\mathcal{P}_q = \kappa_{qj} (J_\uparrow - J_\downarrow), \quad \mathcal{P}_{\bar{q}} = \kappa_{\bar{q}\bar{j}} (\bar{J}_\uparrow - \bar{J}_\downarrow), \quad (17)$$

where $\kappa_{qj} = \kappa_{\bar{q}\bar{j}}$ are (C-even) kinematical factors.



(Anti-)hyperon polarisation from q/\bar{q}



- ▶ The discussion above applies to $q = (u, d)$. [strange-neutrality requires $\mu_s = 0$]
- ▶ \mathcal{P}_Λ comes predominantly from \mathcal{P}_s . [QCDSF Collaboration, PLB 545 (2002) 112.]
- ▶ \mathcal{P}_q can be transferred to $\mathcal{P}_{\bar{s}}$ via intermediate K_S^0 , K^+ states:

$$\mathcal{P}_s = \kappa_{s\bar{q}} \mathcal{P}_{\bar{q}}, \quad \mathcal{P}_{\bar{s}} = \kappa_{\bar{s}q} \mathcal{P}_q, \quad \kappa_{s\bar{q}} = \kappa_{\bar{s}q}. \quad (18)$$

- ▶ The intermediate Kaons donate their polarised \bar{s} quarks to the antihyperon:

$$\mathcal{P}_\Lambda = \kappa_{\Lambda s} \mathcal{P}_s, \quad \mathcal{P}_{\bar{\Lambda}} = \kappa_{\bar{\Lambda}\bar{s}} \mathcal{P}_{\bar{s}}, \quad \kappa_{\Lambda s} = \kappa_{\bar{\Lambda}\bar{s}}. \quad (19)$$

Freezeout calculation

$a, \text{ GeV}$	$b, \text{ GeV}$	$c, \text{ GeV}$	$d, \text{ GeV}$	$f, \text{ GeV}^{-1}$
0.166(2)	0.139(16)	0.053(21)	1.308(28)	0.273(8)

- ▶ Applying the vortical effects for $\mathcal{P}_{q/\bar{q}}$, we get

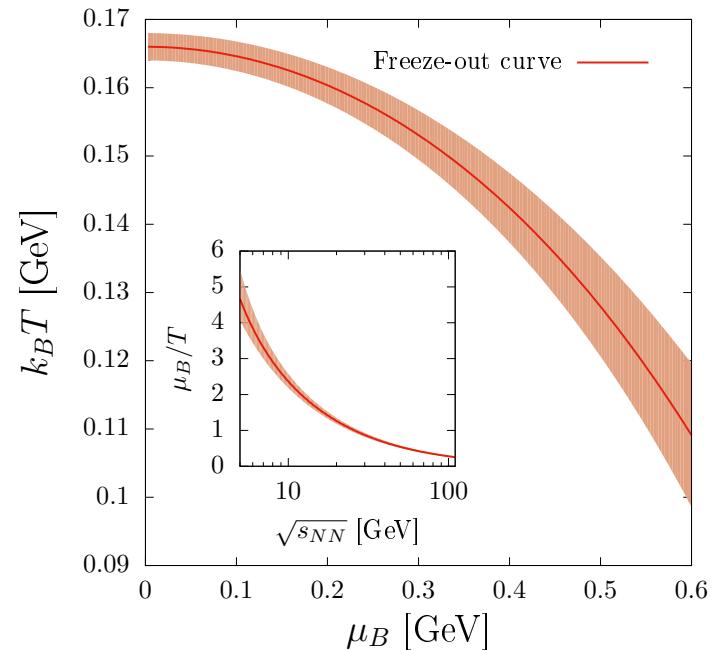
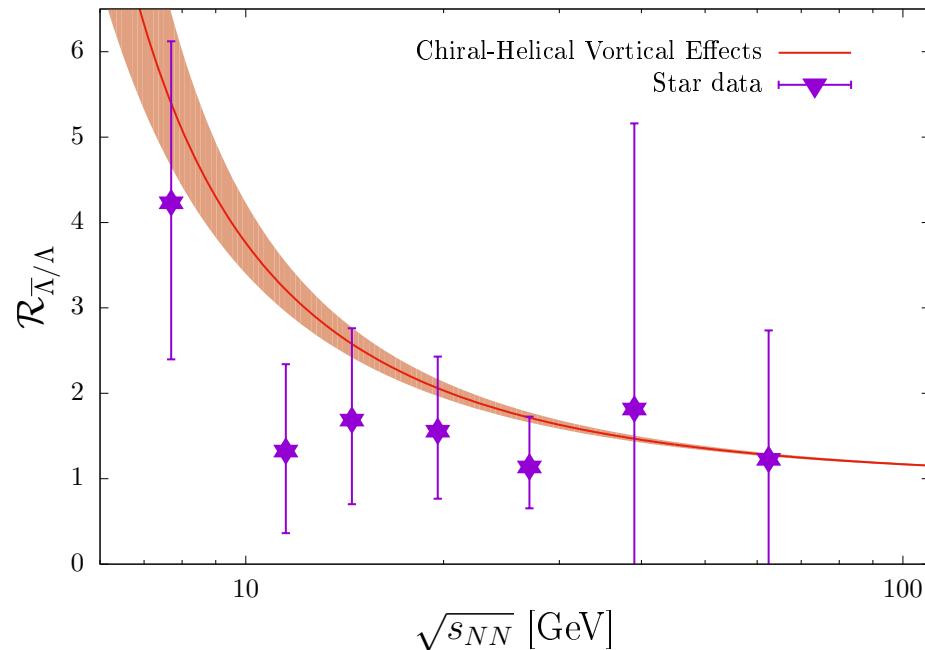
$$\mathcal{P}_\Lambda = \frac{1}{2} \kappa_{\Lambda s} \kappa_{s\bar{q}} \kappa_{\bar{q}\bar{j}} (\sigma_A^\omega - \sigma_H^\omega) \omega, \quad \mathcal{P}_{\bar{\Lambda}} = \frac{1}{2} \kappa_{\bar{\Lambda}\bar{s}} \kappa_{\bar{s}q} \kappa_{qj} (\sigma_A^\omega + \sigma_H^\omega) \omega. \quad (20)$$

- ▶ At freezeout, [Cleymans, Oeschler, Redlich, Wheaton, PRC **73** (2006) 034905]

$$T \equiv T(\mu_B) = a - b\mu_B^2 - c\mu_B^4, \quad \mu_B(\sqrt{s}) = \frac{d}{1 + f\sqrt{s}}. \quad (21)$$

- ▶ The total polarisation can be obtained by integrating \mathcal{P} over the FO hypersurface:

$$\mathcal{P}_{q/\bar{q}} = \frac{1}{2} \kappa_{qj} (\sigma_A^\omega \pm \sigma_H^\omega) \int d\Sigma_\mu \omega^\mu. \quad (22)$$



- ▶ The anti-hyperon / hyperon polarisation ratio becomes simply

$$\mathcal{R}_{\bar{\Lambda}/\Lambda} = \frac{\mathcal{P}_{\bar{\Lambda}}}{\mathcal{P}_{\Lambda}} = \frac{\mathcal{P}_q}{\mathcal{P}_{\bar{q}}} = \frac{\sigma_A^\omega + \sigma_H^\omega}{\sigma_A^\omega - \sigma_H^\omega} = 1 + \frac{8 \ln 2}{\pi^2} \frac{\mu_B}{T} + O(\mu_B^2/T^2). \quad (23)$$

- ▶ The (V, A, H) triad uncovers the *helical vortical effects* (HVE).
- ▶ \mathbf{J}_A generated at finite T and/or finite μ_V , even when $\mu_A = \mu_H = 0$.
- ▶ \mathbf{J}_H generated at finite T and μ_V , even when $\mu_A = \mu_H = 0$.
- ▶ Polarisation of light quarks /antiquarks can be expressed via $J_A \pm J_H$.
- ▶ Assuming $\mathcal{P}_{q/\bar{q}} \rightarrow \mathcal{P}_{\bar{s}/s} \rightarrow \mathcal{P}_{\bar{\Lambda}/\Lambda}$, it is easy to derive $\mathcal{R}_{\bar{\Lambda}/\Lambda} \simeq 1 + \frac{8 \ln 2}{\pi^2} \frac{\mu_B}{T}$.

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THANK YOU FOR YOUR ATTENTION!