Spin polarization dynamics in the non-boost invariant background

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Zimanyi School 2021

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Related papers: 1901.09655 & 2112.01856

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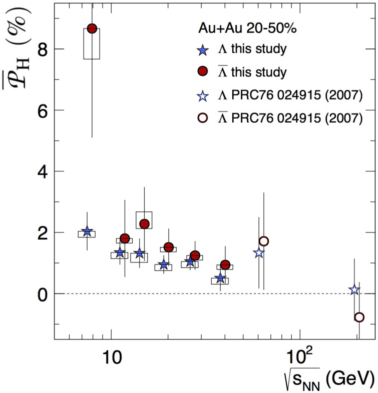
Recall about the Spin Polarization Measurement

More details in Rajeev Singh' Talk

In 2017, a *Nature* reference,



Nature 548, 62 (2017)

- Have been first **positive** measurements of global spin polarization of Λ hyperons particles.
- **Agreement** between experiment and models based on local thermodynamic equilibrium of spin degrees of freedom.

  0711.1253, 1304.4427, 1303.3431, 1501.04468, 1610.02506, 1610.04717, 1605.04024, 1703.03770 and others
- However, still a lot of work to be done concerning the **Longitudinal Spin Polarization.**

Study the Quark Gluon Plasma (QGP) with our approach

1. Include **spin degrees of freedom** into the ideal standard hydrodynamics to form **spin hydrodynamics** formalism.
2. Using the different **law conservations** to generate the perfect fluid evolution in time.
3. Using the **conservation of spin** to generate the spin evolution using the hydrodynamic background.
4. Determination of the **Pauli-Lubański** (PL) vector on the freeze-out hypersurface.
5. Calculation of **momentum dependent** and **momentum integrated** spin polarization of particles in their rest frame which can be directly compared with the experimental findings.

Conservation Laws

More details in Rajeev Singh's Talk

Perfect Fluid Background

- Conservation of **Net Baryon Density**

$$\partial_\alpha N^\alpha(x) = 0\,,$$

- Conservation of **Energy and linear Momentum**

$$\partial_\alpha T^{\alpha\beta}(x) = 0,$$

$\Rightarrow$ 5 Unknowns + 5 Equations = 1 Answer!

Evolution of T, μ, U_x, U_y and ϑ

$U^\mu = (U_0 \cosh[\Phi], U_x, U_y, U_0 \sinh[\Phi])$ where $\Phi = \vartheta + \eta$ is the fluid rapidity and ϑ describing the deviations of the flow from the boost-invariant form

$$U_0 = \sqrt{1 + U_x^2 + U_y^2}$$

Conservation Laws

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Evolution of T, μ, U_x, U_y and ϑ

Spin Evolution

- Conservation of **Spin**

$$\partial_\alpha S^{\alpha,\beta\gamma}(x) = 0\,,$$

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$$U_0 = \sqrt{1 + U_x^2 + U_y^2}$$

Conservation Laws

$\omega^{\beta\gamma}$, ω^{δ} and κ^{ϵ} contain C_κ and C_ω components

More details in Rajeev Singh's Talk

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Evolution of T, μ, U_x, U_y and ϑ

Spin Evolution

- Conservation of **Spin**

$$\partial_\alpha S^{\alpha,\beta\gamma}(x) = 0\,,$$

$$d_\alpha S^{\alpha,\beta\gamma} \equiv$$

$$d_\alpha\Big[\mathcal{A}_1 U^\alpha \omega^{\beta\gamma} + \mathcal{A}_2 U^\alpha U^{[\beta}\kappa^{\gamma]} + \mathcal{A}_3\big(U^{[\beta}\omega^{\gamma]\alpha} + g^{\alpha\beta}\kappa^{\gamma]}\big)\Big] = 0$$

$\Rightarrow$ 6 Unknowns + 6 Equations = Other Answer!

Evolution of $C_{\kappa X}$, $C_{\kappa Y}$, $C_{\kappa Z}$, $C_{\omega X}$, $C_{\omega Y}$ and $C_{\omega Z}$

$U^\mu = (U_0 \cosh[\Phi], U_x, U_y, U_0 \sinh[\Phi])$ where $\Phi = \vartheta + \eta$ is the fluid rapidity and ϑ describing the deviations of the flow from the boost-invariant form

$$U_0 = \sqrt{1 + U_x^2 + U_y^2}$$

The famous 6 spin components

Relation with Electromagnetic components

$$U^{\mu} = \left(U_0 \cosh[\Phi], U_x, U_y, U_0 \sinh[\Phi]\right)$$

$$X^{\mu} = \left(U_{\perp} \cosh[\Phi], \frac{U_0 \times U_x}{U_{\perp}}, \frac{U_0 \times U_y}{U_{\perp}}, U_{\perp} \sinh[\Phi]\right)$$

$$Y^{\mu} = \left(0, -\frac{U_y}{U_{\perp}}, \frac{U_x}{U_{\perp}}, 0\right) \quad Z^{\mu} = \left(\sinh[\Phi], 0, 0, \cosh[\Phi]\right)$$

$\omega_{\mu\nu}$ is the spin polarization tensor and can be parameterized by the four-vectors κ^{μ} and ω^{μ},

$$\boxed{\omega_{\mu\nu} = \kappa_{\mu} U_{\nu} - \kappa_{\nu} U_{\mu} + \varepsilon_{\mu\nu\alpha\beta} U^{\alpha} \omega^{\beta}},$$

where $\kappa^{\alpha} = C_{\kappa X} X^{\alpha} + C_{\kappa Y} Y^{\alpha} + C_{\kappa Z} Z^{\alpha}, \quad \omega^{\alpha} = C_{\omega X} X^{\alpha} + C_{\omega Y} Y^{\alpha} + C_{\omega Z} Z^{\alpha}.$

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where $\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \quad \omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha.$

Physics of C's components

$$C_{\kappa X} = e^1 \cosh(\vartheta + \eta) - b^2 \sinh(\vartheta + \eta),$$
$$C_{\omega Y} = b^2 \cosh(\vartheta + \eta) - e^1 \sinh(\vartheta + \eta),$$
$$C_{\kappa Y} = e^2 \cosh(\vartheta + \eta) + b^1 \sinh(\vartheta + \eta),$$
$$C_{\omega X} = b^1 \cosh(\vartheta + \eta) + e^2 \sinh(\vartheta + \eta),$$
$$C_{\kappa Z} = e^3, \quad C_{\omega Z} = b^3.$$

Contents

1. Introduction & Recall
2. Non Boost-Invariant Case
   - Initial Conditions
   - Perfect-Fluid and Spin evolution
3. Polarization Calculations
4. Results and Discussions
5. Outlook and Perspectives

System with non-boost invariant flow

- System transversely homogeneous and undergoing a non-trivial dynamics along the beam (z) direction.
- Translational invariance in the transverse plane $\Rightarrow U^\mu$ has vanishing x and y components.

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3D+1 General case

$$U^\mu = (U_0 \cosh[\Phi], U_x, U_y, U_0 \sinh[\Phi])$$
$$X^\mu = \left(U_\perp \cosh[\Phi], \frac{U_0 \times U_x}{U_\perp}, \frac{U_0 \times U_y}{U_\perp}, U_\perp \sinh[\Phi]\right)$$
$$Y^\mu = \left(0, -\frac{U_y}{U_\perp}, \frac{U_x}{U_\perp}, 0\right)$$
$$Z^\mu = (\sinh[\Phi], 0, 0, \cosh[\Phi])$$

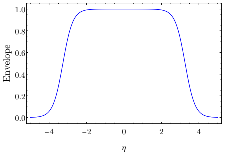
1D+1 Non-Boost Case

$$U^\mu = (\cosh[\Phi], 0, 0, \sinh[\Phi])$$
$$X^\mu = (0, 1, 0, 0)$$
$$Y^\mu = (0, 0, 1, 0)$$
$$Z^\mu = (\sinh[\Phi], 0, 0, \cosh[\Phi])$$

Initial Conditions

Initial Conditions for Background

Θ is the Heaviside Function



- Initialization of **Energy Density** → **Temperature**:

$$\mathcal{E}_0(\eta) = \frac{\mathcal{E}_0^c(T_0)}{2}\left[\Theta(\eta)\Big(\tanh(a-\eta b)+1\Big)+\Theta(-\eta)\Big(\tanh(a+\eta b)+1\Big)\right]$$

- Initialization of **Net-Baryon Density** → **Baryon Chemical Potential**: No Chemical Potential

- Initialization of **Rapidity**: Bjorken Flow → $\vartheta = 0$

- **Non-Boost Invariance** case in 1D+1 ⇒ No flow projected on X and Y

Initial Conditions

Initial Conditions for Spin

Recall

$$
\begin{aligned}
C_{\kappa X} &= e^1 \cosh(\vartheta + \eta) - b^2 \sinh(\vartheta + \eta), \\
C_{\omega Y} &= b^2 \cosh(\vartheta + \eta) - e^1 \sinh(\vartheta + \eta), \\
C_{\kappa Y} &= e^2 \cosh(\vartheta + \eta) + b^1 \sinh(\vartheta + \eta), \\
C_{\omega X} &= b^1 \cosh(\vartheta + \eta) + e^2 \sinh(\vartheta + \eta), \\
C_{\kappa Z} &= e^3, \quad C_{\omega Z} = b^3.
\end{aligned}
$$

Initial Conditions

Initial Conditions for Spin

Recall

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C_{\kappa X} &= e^1 \cosh(\vartheta+\eta) - b^2 \sinh(\vartheta+\eta),\\
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C_{\omega X} &= b^1 \cosh(\vartheta+\eta) + e^2 \sinh(\vartheta+\eta),\\
C_{\kappa Z} &= e^3, \quad C_{\omega Z} = b^3.
\end{aligned}
$$

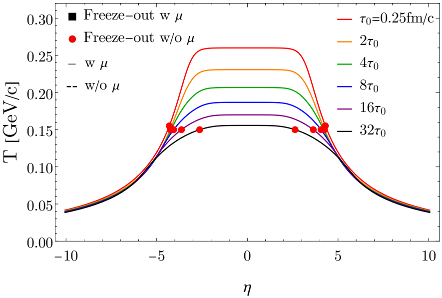
$$
\left.\begin{array}{l}
\text{Initially, there is } \textbf{no Rapidity} \rightarrow \vartheta = 0.\\
\text{Also } e^3 \ \& \ b^3 = 0
\end{array}\right\} C_{\kappa Z} = C_{\omega Z} = C_{\kappa Y} = C_{\omega X} = 0
$$

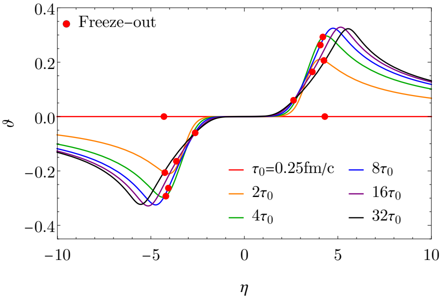
That leads to few initial conditions but we set these ones:

$$\boxed{C_{\kappa X} = 0} \qquad \boxed{C_{\omega Y} = b^2 \operatorname{sech}(\eta)}$$

Perfect-Fluid and Spin evolution

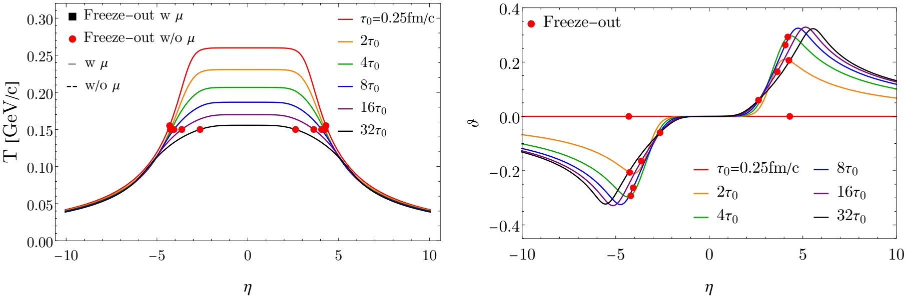
Perfect-Fluid background evolution





Perfect-Fluid and Spin evolution

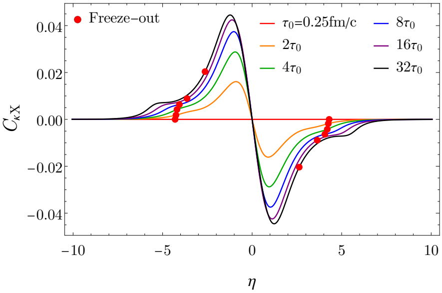
Perfect-Fluid background evolution

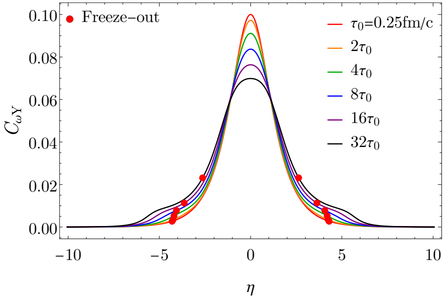


- Envelope of energy density well defined on Temperature
- Flow increasing with rapidity, before to stabilize at forward or backward rapidities.

Perfect-Fluid and Spin evolution

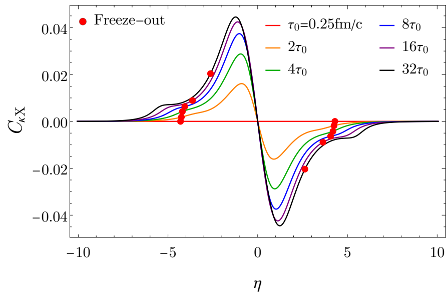
Spin evolution

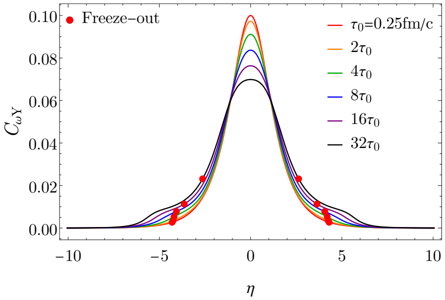




Perfect-Fluid and Spin evolution

Spin evolution





- $C_{\kappa X}$ evolves quickly to vanishing at large eta as $C_{\omega Y}$
- Shape of the sech conserved during time evolutions
- Knee around $\eta \approx 5$ understood by the large mass regime of the factor m/T

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3 Polarization Calculations
   - Hard work and formula
   - Last checks before results

4 Results and Discussions

5 Outlook and Perspectives

Pauli-Lubański (PL) tool

Phase-space density of the PL four-vector Π_μ

$$E_p \frac{d\Delta\Pi_i(x,p)}{d^3p} = -\frac{1}{2}\varepsilon_{i\nu\alpha\beta}\Delta\Sigma_\lambda E_p \frac{dS^{\lambda,\nu\alpha}(x,p)}{d^3p}\frac{p^\beta}{m}.$$

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Phase-space density of the spin tensor

$$E_p \frac{dS^{\lambda,\nu\alpha}}{d^3p} = \frac{\cosh\left(\frac{\mu}{T}\right)}{(2\pi)^3} e^{-\beta\cdot p} p^\lambda \left(\omega^{\nu\alpha} + \frac{2}{m^2} p^{[\nu}\omega^{\alpha]}{}_\delta p^\delta\right).$$

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Integrating over the freeze-out hypersurface give the total value of the momentum density of the PL four-vector

$$E_p \frac{d\Pi^*_i(p)}{d^3p} = -\frac{1}{(2\pi)^3 m}\int \cosh\left(\frac{\mu}{T}\right)\Delta\Sigma_\lambda p^\lambda e^{-\beta\cdot p}\left(\tilde{\omega}_{i\beta}p^\beta\right)^*.$$

* means boost to the **Particle Rest Frame**. $i = (0,x,y,z)$ components

Spin Polarization

1. Mean spin polarization **per particle**:

$$\langle \pi_i \rangle_p = \frac{E_p \dfrac{d\Pi_i^*(p)}{d^3p}}{E_p \dfrac{d\mathcal{N}(p)}{d^3p}} \rightarrow \frac{\text{Total Pauli-Lubański vector}}{\text{Momentum density of all particles}}$$

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2. Momentum **averaged** polarization

$$\langle \pi_i \rangle = \frac{\int d^3p \, \dfrac{d\Pi_i^*(p)}{d^3p}}{\int d^3p \, \dfrac{d\mathcal{N}(p)}{d^3p}}$$

Freeze-out conditions, last checks

Hypersurface and freeze-out time

Freeze-out conditions, last checks

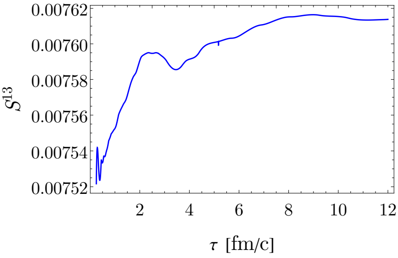
Hypersurface and freeze-out time

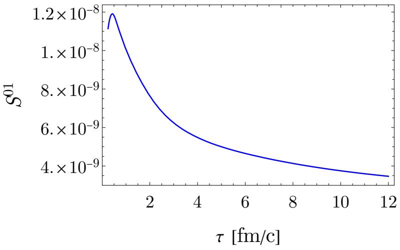
- Concerning the **hypersurface**:

$$S^{\mu\nu}_{\rm FC} = \int \Delta\Sigma_\lambda \, S^{\lambda,\mu\nu}_{\rm GLW} = \int dxdy \, \tau \, d\eta \, U^{\rm B}_\lambda S^{\lambda,\mu\nu}_{\rm GLW}$$

Two important components using our initial conditions: $S^{13}_{\rm FC}$ & $S^{01}_{\rm FC}$

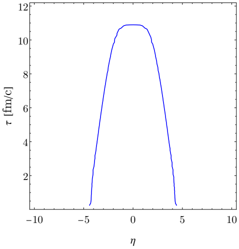
Check if we have $S^{13}_{\rm FC} \neq 0$ & $S^{01}_{\rm FC} = 0$ obtained by symmetry,





Freeze-out conditions, last checks

Hypersurface and freeze-out time



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Two important components using our initial conditions: $S^{13}_{\rm FC}$ & $S^{01}_{\rm FC}$

Check if we have $S^{13}_{\rm FC} \neq 0$ & $S^{01}_{\rm FC} = 0$ obtained by symmetry,

- Concerning the **freeze-out time**:

Temperature of freeze-out set to 150 MeV without μ

Check if we reach the freeze-out temperature at all η points to define the freeze-out time.

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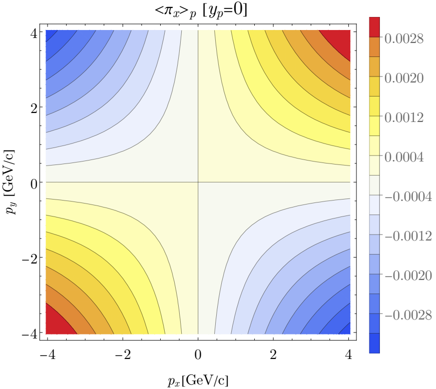
   Momentum dependent Polarization
   Momentum averaged Polarization

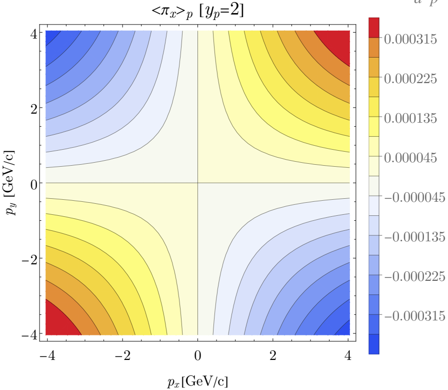
5 Outlook and Perspectives

Momentum dependence of the polarization

Momentum dependence of the polarization

$$\langle \pi_x \rangle_p = \frac{E_p \dfrac{d\Pi_x^*(p)}{d^3p}}{E_p \dfrac{d\mathcal{N}(p)}{d^3p}}$$



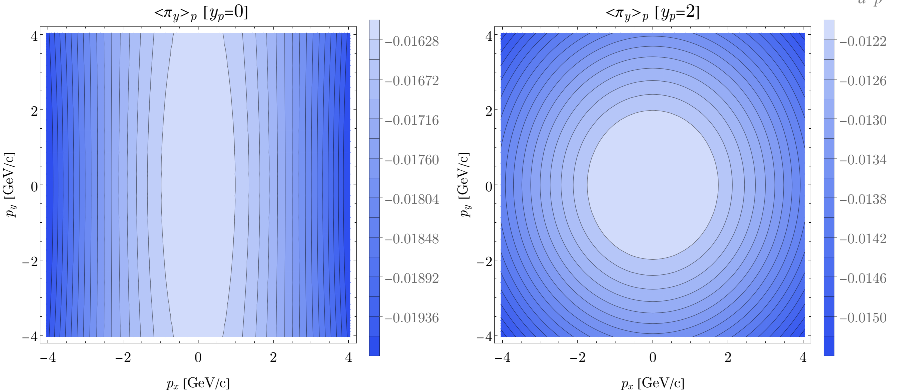


Quadrupole structure in the *x* component in opposite to experiments
Amplitude decreases at larger rapidity
Momentum averaged polarization of the *x* component must be equal to 0.

Momentum dependence of the polarization

Momentum dependence of the polarization

$$\langle \pi_y \rangle_p = \frac{E_p \dfrac{d\Pi^*_y(p)}{d^3p}}{E_p \dfrac{d\mathcal{N}(p)}{d^3p}}$$



Only negative values

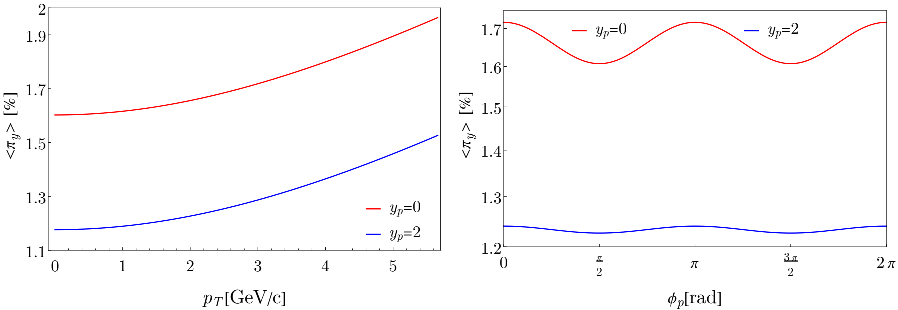
Amplitude decreases at larger rapidity

Momentum averaged Polarization not equal to 0.

Polar coordinates of polarization

Polar coordinates of polarization

$p_T = \sqrt{p_x^2 + p_y^2}$
ϕ_p the azimuthal angle



- Confirmation of decreasing amplitude in function of the rapidity,
- Confirmation of the amplitude for both rapidities,
- Expected Results and behaviors.

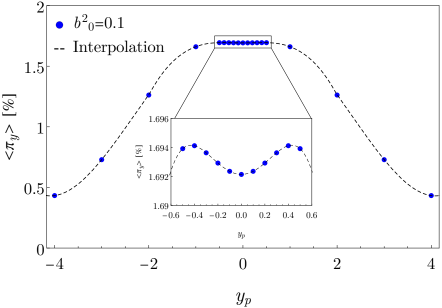
Momentum averaged polarization

Two ways of study

Momentum averaged polarization

Two ways of study

$$\langle \pi_i \rangle = \frac{\int d^3p \, \frac{d\Pi_i^*(p)}{d^3p}}{\int d^3p \, \frac{d\mathcal{N}(p)}{d^3p}}$$



- Large rapidity dependence of the polarization
- Reproduction of the qualitative dependence of STAR

Momentum averaged polarization

Two ways of study

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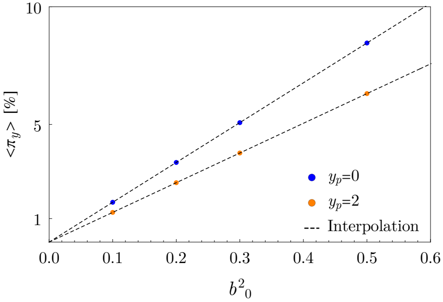

- Large rapidity dependence of the polarization
- Reproduction of the qualitative dependence of STAR
- Linear dependence of the Initial conditions of spin components

Outlook

Outlook

- Defined **Spin hydrodynamic** using the conservation of spin.
- Used the **Non-Boost invariance** case to define the initial condition of **thermodynamic variables** and **spin components**.
- Used the spin polarization tensor to calculate the **spin polarization** using the Pauli-Lubański vector.
- Calculated the **momentum dependence** of the polarization with **quadrupole** structure in the **x-component**.
- **Non-zero** global polarization found in the **y-component**.
- Found a **large rapidity dependence** of the **momentum averaged polarization**.

Perspectives

- Implement the **chemical potential**, already done in the *2112.01856*, at this small values of initial **Temperature**, **no large effects** have been found,
- **Extend to 3D+1** Case using the general expressions derived in *2112.01856*,
- Implement **realistic Equation of State**,
- Implement realistic **Initial Conditions** for Spin and Background like Glauber to do the particlization procedure and see the effect of spin on other observables in same time (spectra, anisotropic flow, etc ...).

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Thank you for your attention!

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Thank you for your attention!

Questions?

$d_\alpha S^{\alpha,\beta\gamma} \equiv d_\alpha\Big[\mathcal{A}_1 U^\alpha \omega^{\beta\gamma} + \mathcal{A}_2 U^\alpha U^{[\beta}\kappa^{\gamma]} + \mathcal{A}_3\left(U^{[\beta}\omega^{\gamma]\alpha} + g^{\alpha\beta}\kappa^{\gamma]}\right)\Big] = 0$ where
$\mathcal{A}_1 = \cosh(\mu/T)\left(n_{(0)} - \mathcal{B}_{(0)}\right), \quad \mathcal{A}_2 = \cosh(\mu/T)\left(\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)}\right),$
$\mathcal{A}_3 = \cosh(\mu/T)\,\mathcal{B}_{(0)}$
with, $\mathcal{B}_{(0)} = -\frac{2}{(m/T)^2}(\mathcal{E}_{()} + \mathcal{P}_{()})/\mathcal{T}$ and $\mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2\mathcal{N}_{()}$.
Here, $\omega^{\beta\gamma}$ is known as spin polarization tensor.

Spin polarization tensor:

from Rajeev Singh

$\omega_{\mu\nu}$ is an anti-symmetric tensor of rank 2 and can be parameterized by the four-vectors κ^μ and ω^μ,

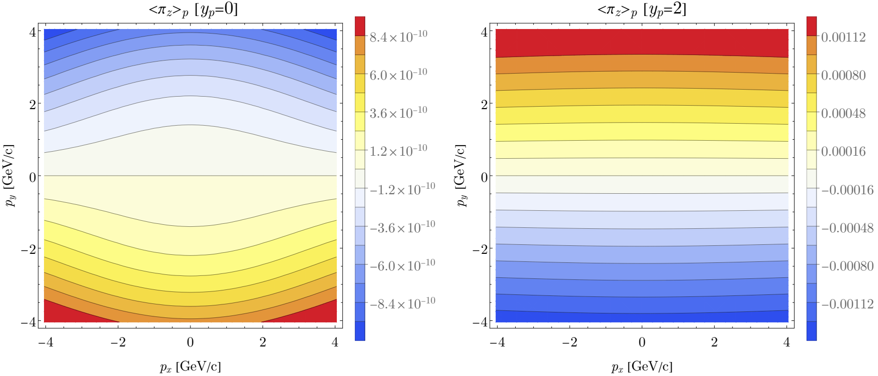
$$\boxed{\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \varepsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta},$$

where $\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \quad \omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha$. U, X, Y and Z form a 4-vector basis satisfying the following normalization conditions: $U \cdot U = 1, \qquad X \cdot X = Y \cdot Y = Z \cdot Z = -1.$

$$\omega_{\alpha\beta} = \begin{bmatrix} 0 & e^1 & e^2 & e^3 \\ -e^1 & 0 & -b^3 & b^2 \\ -e^2 & b^3 & 0 & -b^1 \\ -e^3 & -b^2 & b^1 & 0 \end{bmatrix}$$

Momentum dependence of the polarization

$$\langle \pi_x \rangle_p = \frac{E_p \dfrac{d\Pi^*_x(p)}{d^3p}}{E_p \dfrac{d\mathcal{N}(p)}{d^3p}}$$



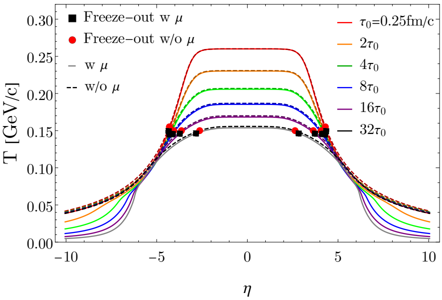
Quadrupole structure in the z component in opposite to experiments

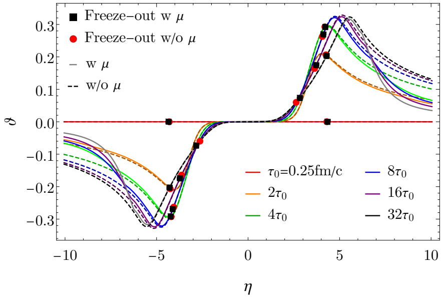
Amplitude decreases at larger rapidity

Momentum averaged polarization of the x component must be equal to 0.

Chemical potential effect:

Perfect Fluid Background

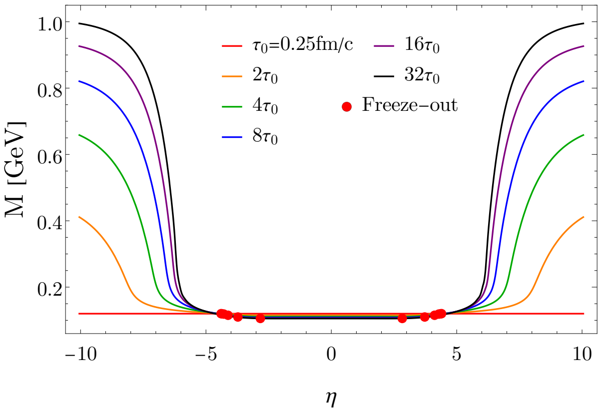




- effect of chemical potential more important at the edges in η

Chemical potential effect:

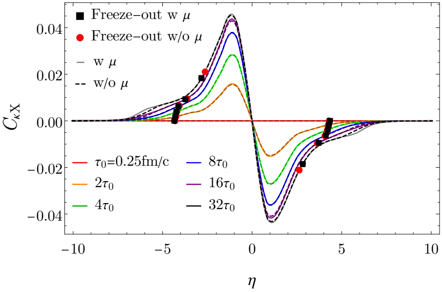
Chemical Potential

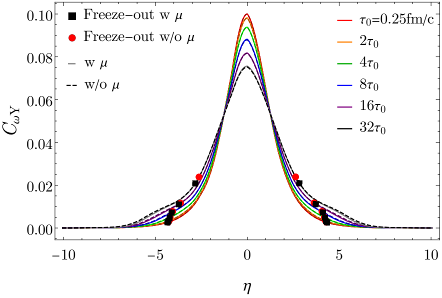


- Chemical potential more important at the edges in η

Chemical potential effect:

Spin components

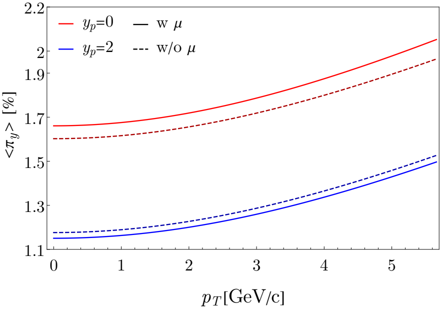


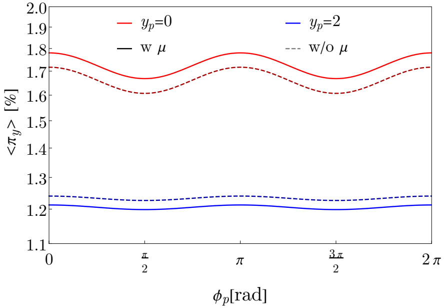


- effect of chemical potential more important at the edges in η ...
- but the effect is tiny everywhere

Chemical potential effect:

Polarization using polar coordinates

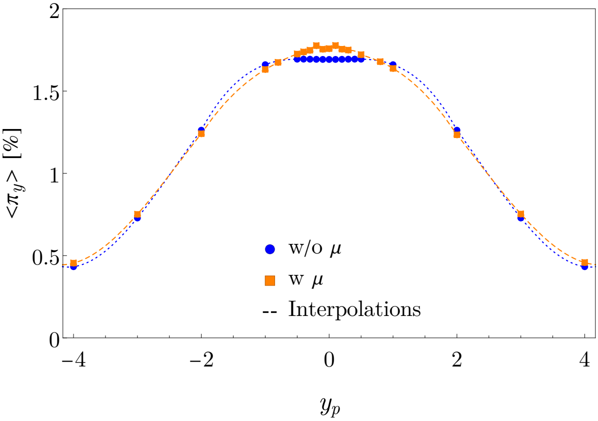




- Momentum averaged polarization more importent at the center in rapidity and smaller at $y_p = 2$.

Chemical potential effect:

Momentum averaged polarization



- Momentum averaged polarization more importent at the center in rapidity and smaller at the edges.