

# Thermodynamics and dissipation in quantum fluids

Peter Ván



**wigner** Research Centre for Physics,  
IPNP, Department of Theoretical Physics

Montavid Thermodynamic Research Group

Budapest, 05.12.2021.

# Content

- ① History
- ② Second law restricted gradient expansions
- ③ Korteweg fluids

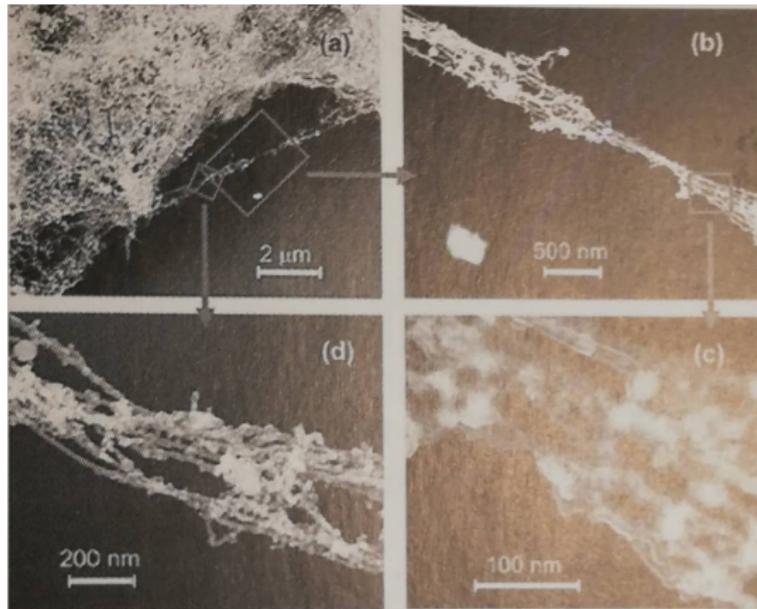
# What is quantum mechanics?

Motivation: Zimányi Winter School 2020, presentation of W. Zajc.  
Thermalization vs. fluidisation. ([Here is a LINK.](#))

## Early history

- Schrödinger, *Quantisierung als Eigenwertproblem I-II.*, AdP (1926), Heft 4, closed 18 March 1926;
- Madelung, *Quantentheorie in hydrodynamischer Form*, ZfP, (1926), submitted 25 October 1926;
- Bohm, *A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables I-II.*, PhysRev (1952);
- de Broglie, Fényes, Nelson, Jánossy, Takabayashi, Bell, Vigier, Holland, etc...
- Jackiw et al. (2004). Every non-Abelian gauge theory can be reformulated as hydrodynamics. QCD, QGP. (See G. Torrieri I guess).
- Cosmology. K. Huang, Khouri, Hossenfelder, etc... Dark matter vs MOND: superfluidity.
- Biró-Ván: *Splitting the source term for the Einstein equation to classical and quantum parts*, FoP (2015). Transformed Klein-Gordon and Einstein equations.

# Quantum fluids exist:



Scanned with CamScanner

Vortice lines in He II, from boundary to boundary. Donelly, 1991.

# Superfluidity

## Histocical remarks and more

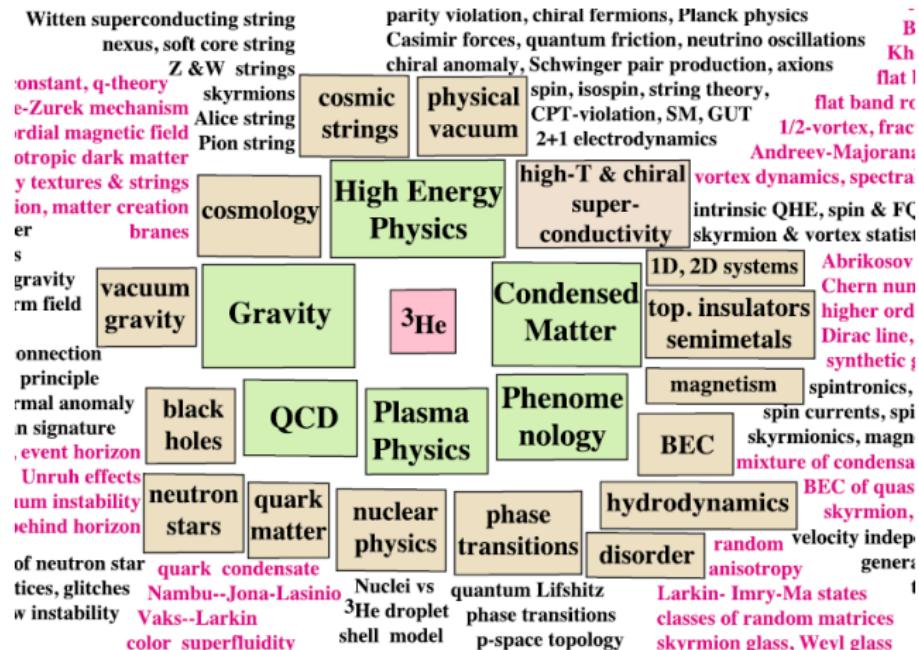
- Landau, ZhETP (1941).
- Bohm and Pines (1952), Gross and Pitaevskii (1961).
- Dense plasmas. Critical survey of Bonitz et al. (2019).
- QGP, T. Kodama et al., stochastic qm., Jackiw et al (2004).
- G.E. Volovik: The Universe in a Helium Droplet (2003).

## Theory notes: there is no quantisation

- There is a Hamiltonian but there is no Lagrangian.
- Hydro is an effective theory and cannot be quantised.
- (Note: Like gravity. Therefore gravity cannot be quantized, q-gravity does not exist.)

# $^3\text{He}$ Universe

A recent view with trimmed boundaries.



From Volovik JLTP (2021). Inflational expansion.

# Connecting hydro to quantum

Gross-Pitaevskii equation:

$$i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m} \Delta \psi + \mu_0 \psi - g |\psi|^2 \psi = 0$$

$\mu_0$  chemical potential,  $g$  interaction parameter ( $\sim$  S-wave scattering length).  
Madelung transformation:

$$\psi = \sqrt{\rho} e^{i\varphi},$$

$\rho$  density (probability or superfluid),  $\varphi$  velocity potential:  $v = \frac{\hbar}{m} \nabla \varphi$ .

$$\frac{i\hbar}{2\rho} \left( \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) \right) \psi - \left( m \frac{\hbar}{m} \frac{\partial \varphi}{\partial t} + m \frac{v^2}{2} - \frac{\hbar^2}{2m} \frac{\Delta \sqrt{\rho}}{\sqrt{\rho}} - \mu_0 + g\rho \right) \psi = 0$$

Continuity and Bernoulli equations of classical rotation free fluids. Its gradient will be:

$$m\dot{v} + \nabla(\mu_0 - g\rho + U_Q) = 0$$

$U_Q(\rho, \nabla\rho, \nabla^2\rho)$  is the Bohm potential. Special momentum balance.

# Interpretations vs. explanations

## Quantum physics

- Canonical quantisation. Do we need a Lagrangian?
- Qhydro as transformation or more?
- Stochastic, path integrals, etc...
- Every theory is quantum. Or not. There are some problems. See the discussion of Weinberg ([Here is a LINK](#)).



Ockham's razor. "*Pluralitas non est ponenda sine neccesitate*"

# Non-equilibrium thermodynamics

# Gradient expansions

Classified by state spaces and constraints

- Heat conduction. Internal energy or temperature.  $\{e, \partial_i e, \partial_{ij} e, \dots\}$ .  
Constraint: balance of internal energy.
- Internal variables (e.g. phase fields).  $\{\varphi, \partial_i \varphi, \partial_{ij} \varphi, \dots\}$ . Tensorial order may be arbitrary.  
Constraint: evolution equation, free or balance  
(Ginzburg-Landau-Alen-Cahn, Cahn-Hilliard).
- Fluid mechanics. Mass, velocity and energy.  $\{\rho, \partial_i \rho, \partial_{ij} \rho, v^i (?), \dots\}$ , + more gradients.  
Constraints: balances of mass, momentum and energy.
- Solid mechanics. Mass, strain and energy  $\{\varepsilon^{ij}, \partial_k \varepsilon^{ij}, \partial_{kl} \varepsilon^{ij}, \dots\}$ , and more gradients.  
Constraints: kinematics, balances of mass, momentum and energy.

General requirements: second law and objectivity.

# Thermodynamic origin of evolution equations

Scalar field evolution:  $\dot{\varphi} = f(\varphi, \partial_i \varphi)$

$$\begin{aligned}\dot{S}(\varphi, \partial_i \varphi) - \lambda(\dot{\varphi} - f(\varphi, \partial_i \varphi)) &= (\partial_\varphi S - \lambda)\dot{\varphi} + \partial_{\partial_i \varphi} S \partial_i \dot{\varphi} + \lambda f \geq 0 \\ \partial_\varphi S - \lambda &= 0, \quad \partial_{\partial_i \varphi} S = 0, \\ \boxed{0 \leq f \partial_\varphi S} \quad \rightarrow \quad f &= l \partial_\varphi S, \quad (l \geq 0)\end{aligned}$$

Extended approach:  $\dot{\varphi} = f(\varphi, \partial_i \varphi, \partial_{ij} \varphi)$

- Higher order state space:  $(\varphi, \partial_i \varphi, \partial_{ij} \varphi)$ ;
- Constitutive entropy flux;
- Gradient constraints:  $\partial_i \dot{\varphi} = \partial_i f$

$$\begin{aligned}\dot{S} + \partial_i J^i - \lambda(\dot{\varphi} - f) - \Lambda^i(\partial_i \dot{\varphi} - \partial_i f) &\geq 0 \\ \partial_\varphi S = \lambda, \quad \partial_{\partial_i \varphi} S = \Lambda^i, \quad \partial_{\partial_\varphi} S = 0\end{aligned}$$

$$J^i = -\partial_{\partial_i \varphi} S f + \tilde{J}^i(\varphi, \partial_i \varphi) \quad \boxed{0 \leq f(\partial_\varphi S - \partial_i(\partial_{\partial_i \varphi} S))} = f \frac{\delta S}{\delta \varphi} \quad 11/33$$

# Korteweg fluids

Ván-Fülöp (Proc. Roy. Soc., 2004)  
Ván-Kovács (Phil. Trans. Roy. Soc. A, 2020)

# Korteweg fluids I.

Capillarity.

Van der Waals: gradient energy expansion.

Korteweg (1905): gradient pressure expansion.

Balances of mass, momentum and internal energy:

$$\begin{aligned}\dot{\rho} + \rho \nabla \cdot \mathbf{v} &= 0, \\ \rho \dot{\mathbf{v}} + \nabla \cdot \mathbf{P} &= 0, \\ (\rho \dot{e} + \nabla \cdot \mathbf{q}) &= -\mathbf{P} : \nabla \mathbf{v}.\end{aligned}$$

$$\mathbf{P} = (p - \alpha \Delta \rho - \beta (\nabla \rho)^2) \mathbf{I} - \delta \nabla \rho \circ \nabla \rho - \gamma \nabla^2 \rho$$

$\alpha, \beta, \gamma, \delta$  are density dependent material parameters.

Violently unstable. Second law? Eckart fluids (1948)!

## Korteweg fluids II.

Extended state space:  $(e, \rho, \nabla\rho, \nabla^2\rho)$

$s(e, \rho, \nabla\rho)$ . Gibbs reláció:

$$de = Tds + \frac{p}{\rho^2} d\rho + \mathbf{A} \cdot d\nabla\rho.$$

Balances of mass, momentum and internal energy:

$$\dot{\rho} + \rho \nabla \cdot \mathbf{v} = 0, \quad (\nabla(\dot{\rho} + \rho \nabla \cdot \mathbf{v})) = 0,$$

$$\rho \dot{\mathbf{v}} + \nabla \cdot \mathbf{P} = 0,$$

$$\rho \dot{e} + \nabla \cdot \mathbf{q} = -\mathbf{P} : \nabla \mathbf{v}.$$

Entropy balance:

$$\boxed{\rho \dot{s} + \nabla \cdot \mathbf{J} = \Sigma \geq 0}$$

# Korteweg fluids III.

$$\begin{aligned} \rho \dot{s} + \nabla \cdot \mathbf{J} &= \mathbf{q} \cdot \nabla \left( \frac{1}{T} \right) - \\ &- \left[ \mathbf{P} - p\mathbf{I} - \frac{\rho^2}{2} \left( \nabla \cdot \frac{\partial s}{\partial \nabla \rho} \mathbf{I} + \nabla \frac{\partial s}{\partial \nabla \rho} \right) \right] : \frac{\nabla \mathbf{v}}{T} \geq 0 \end{aligned}$$

- Mathematical methods beyond Eckart (Liu or Coleman-Noll procedures).
- The pressure of an ideal, non-dissipative Korteweg fluid:

$$\boxed{\mathbf{P} = p(e, \rho)\mathbf{I} + \frac{\rho^2}{2} \left( \nabla \cdot \frac{\partial s}{\partial \nabla \rho} \mathbf{I} + \nabla \frac{\partial s}{\partial \nabla \rho} \right)}$$

# Korteweg fluids – quantum mechanics

$$\mathbf{P}_K = \frac{\rho^2}{2} \left( \nabla \cdot \frac{\partial s}{\partial \nabla \rho} \mathbf{I} + \nabla \frac{\partial s}{\partial \nabla \rho} \right)$$

- 'Holographic' property:

$$\boxed{\nabla \cdot \mathbf{P}_K = \rho \nabla \phi}, \quad \text{where} \quad \phi = \nabla \cdot \frac{\partial(\rho s)}{\partial \nabla \rho} - \frac{\partial \rho s}{\partial \rho} = -\delta_\rho(\rho s)$$

- Momentum balance for both continuum and point mass:

$$\rho \dot{\mathbf{v}} + \nabla \cdot \mathbf{P}_K = \rho (\dot{\mathbf{v}} + \nabla \phi) = 0 \quad \rightarrow \quad \dot{\mathbf{v}} = -\nabla \phi$$

- Mass scale independent quadratic free energy  $\rightarrow$  general Gross-Pitaevskii equation

$$s(e, \rho, \nabla \rho) = s_Q \left( e - \frac{\hbar^2}{2m} \frac{(\nabla \rho)^2}{4\rho^2} \right) \rightarrow \boxed{m \dot{\mathbf{v}} = -\nabla \left( \frac{\hbar^2}{2} \frac{\Delta \sqrt{\rho}}{\sqrt{\rho}} \right)}$$

Bohm potential  $\rightarrow$  inverse Madelung transformation  $\rightarrow$  nonlinear Schrödinger equation

# Notes and contemplation

## Hidden problems in quantum mechanics

Galilei relativity and Schrödinger equation :

- Partial time derivative is not objective. Local and comoving.
- Wave function is not scalar. Its phase transforms like energy.
- Hydrodynamics is Galilean objective , see Fülöp-Katz quant-ph/9806067.

## The role of thermodynamics (here)

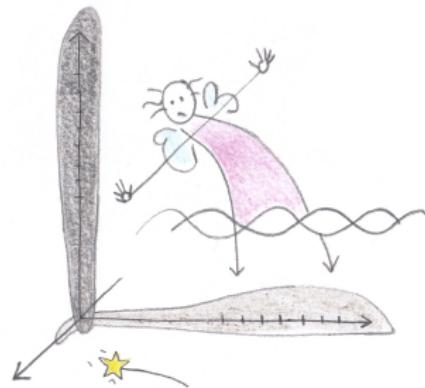
- Thermodynamics  $\neq$  statistical physics.
- Universality. Numerous consequences. Gravity.
- Euler-Lagrange equations without variational principles. A uniform derivation of ideal and dissipative evolution equations.
- Holographic property, quantisation method, dissipation...

# Thank you for the attention!

I.



II.



N. Janke