The controversial fate of superluminal perturbations

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Many dissipative equations violate causality

- Navier-Stokes equation
- Heat equation
- Eckart and Landau-Lifschitz first-order theories

…

They all violate causality! For example, take

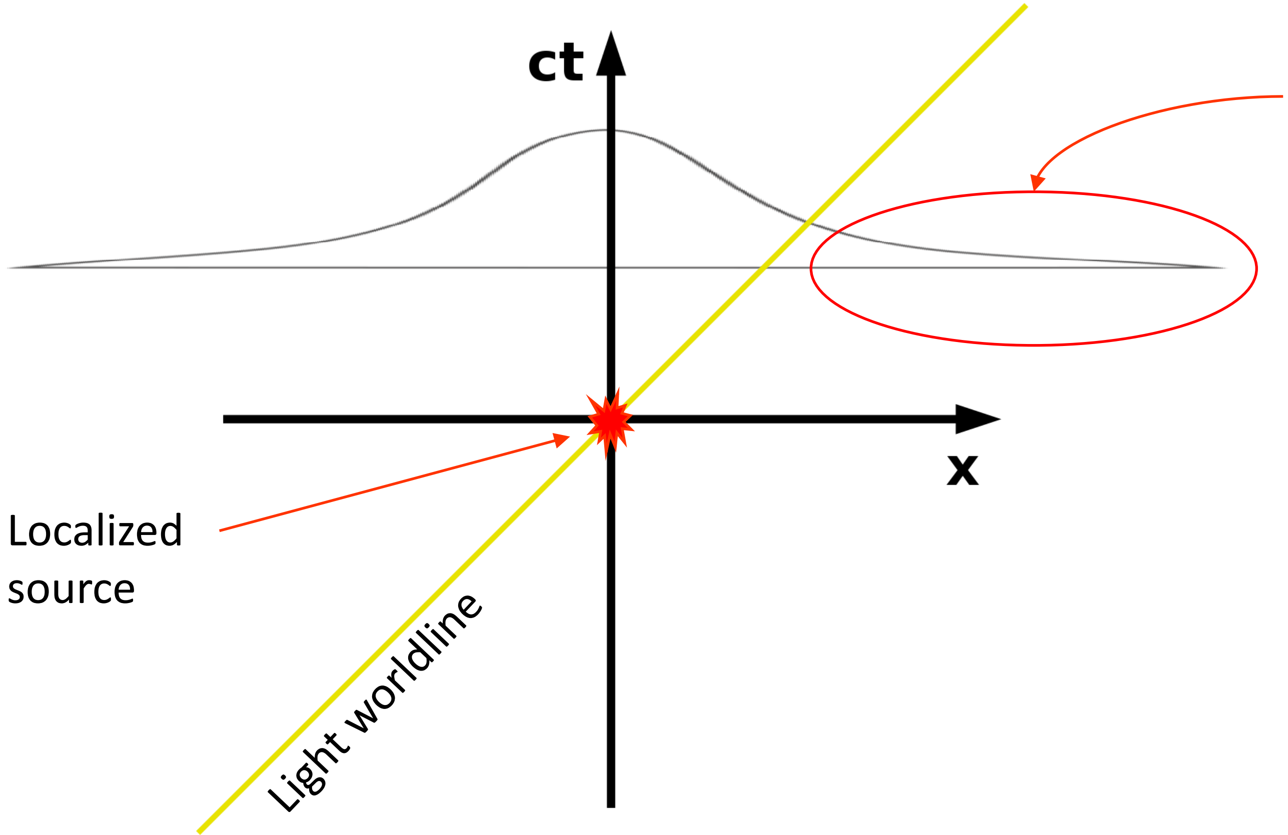
$$\partial_t T = D\partial_x^2 T$$

Heat is acausal!

Its Green function is $G(x,t) = \frac{1}{\sqrt{4\pi Dt}} exp\left(-\frac{x^2}{4Dt}\right)$

It describes how an initial condition $\delta(x)$ evolves in time

The tail of the Gaussian is a signal which propagates outside the light-cone

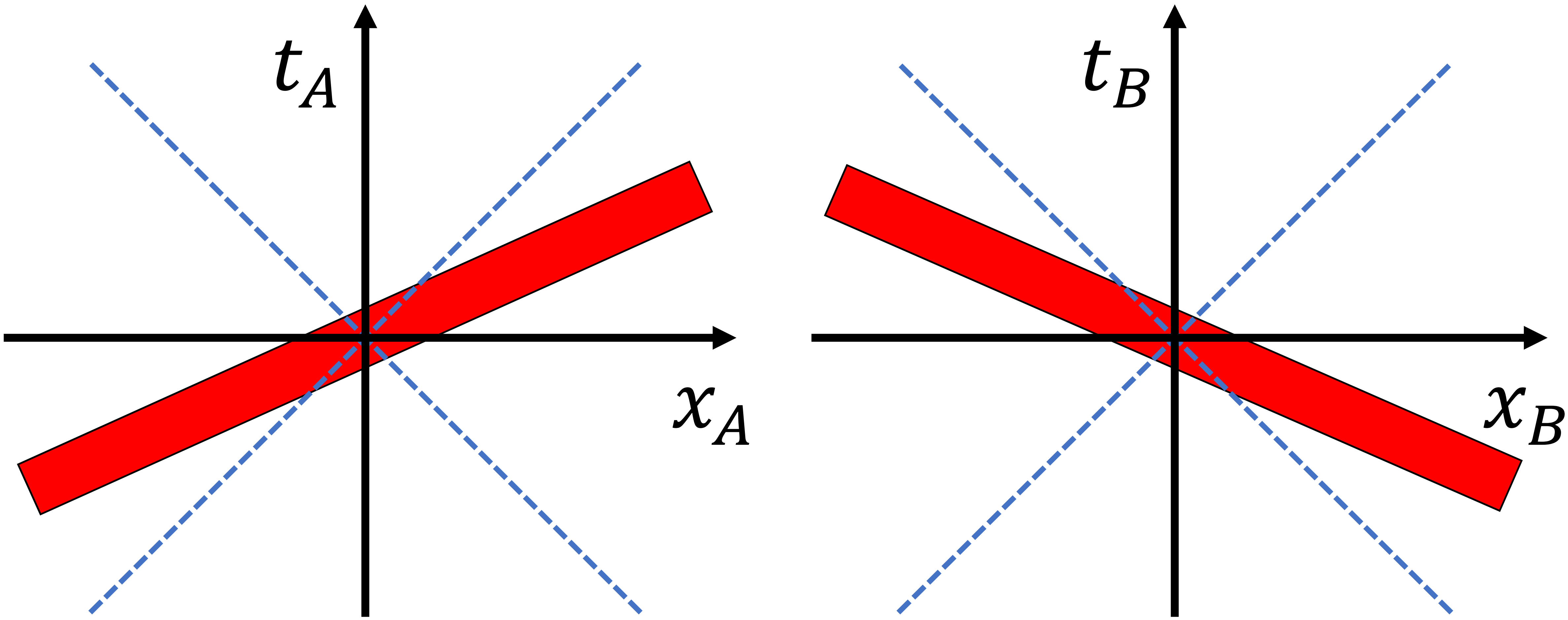


Faster than light communication.

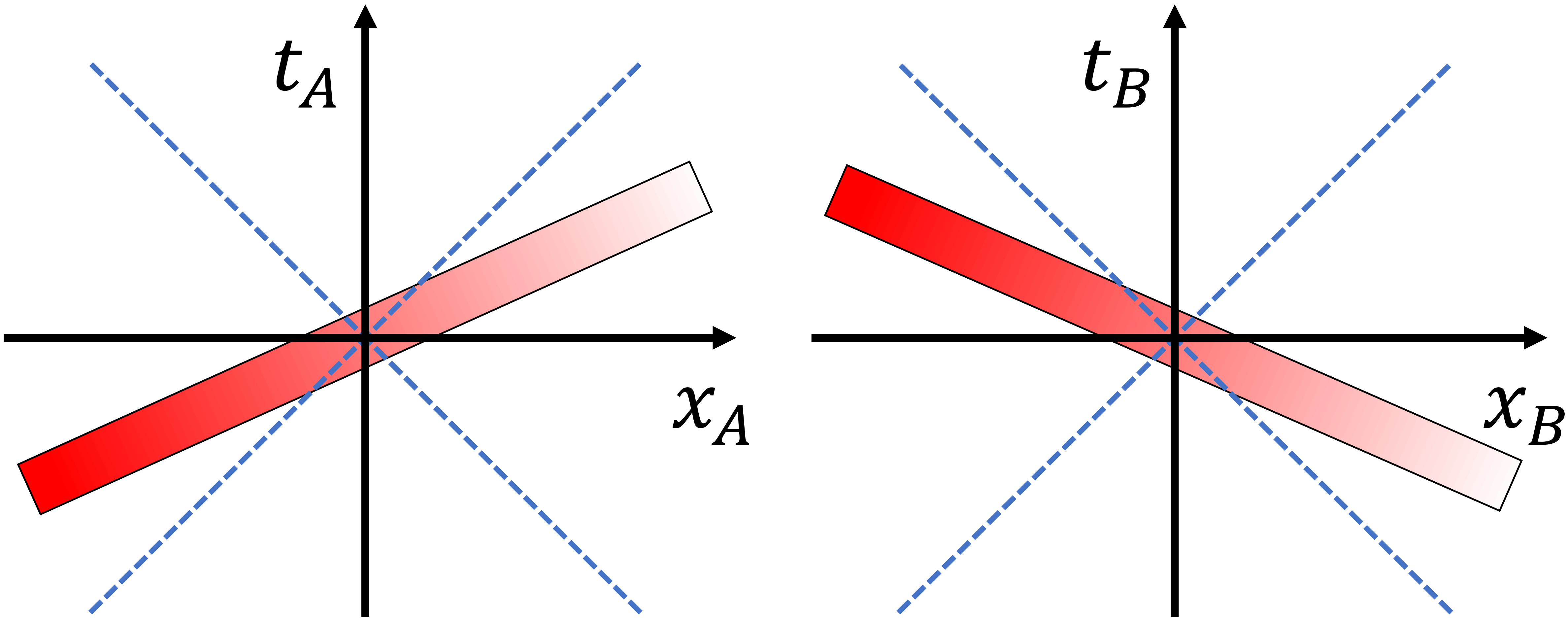
Causality broken!

Is this a problem?

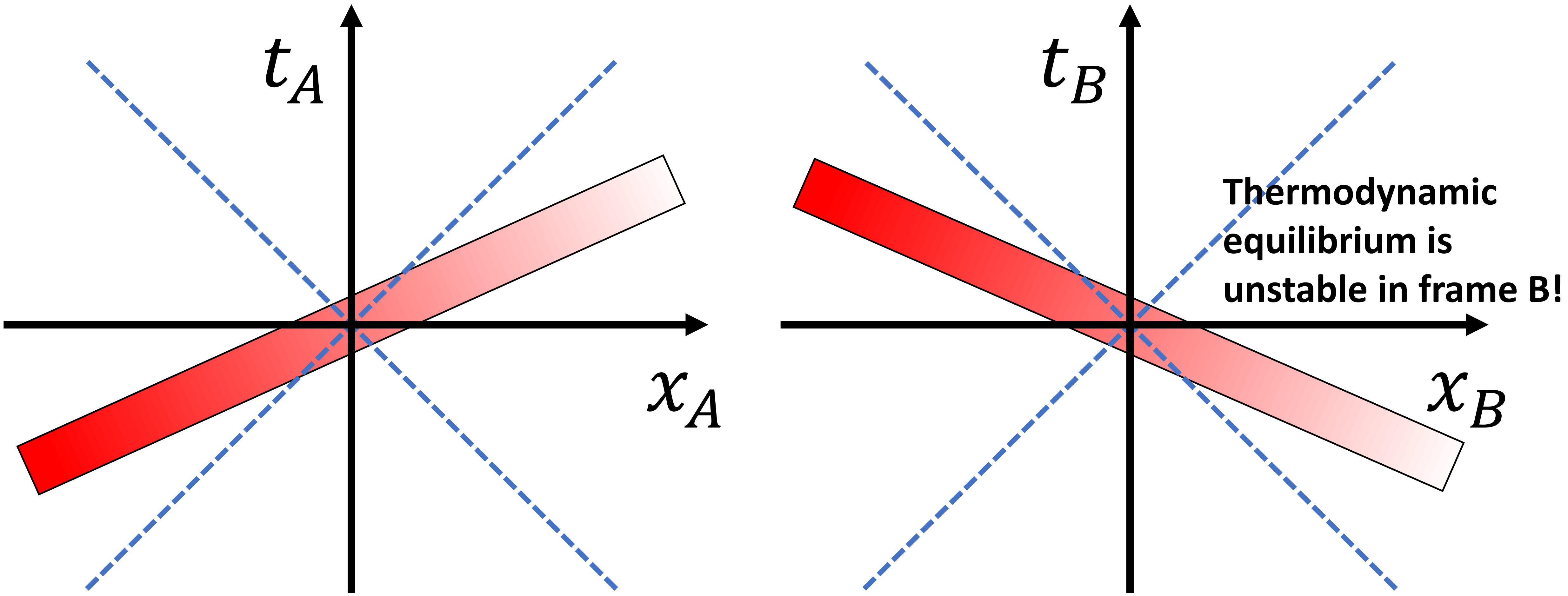
A simple argument



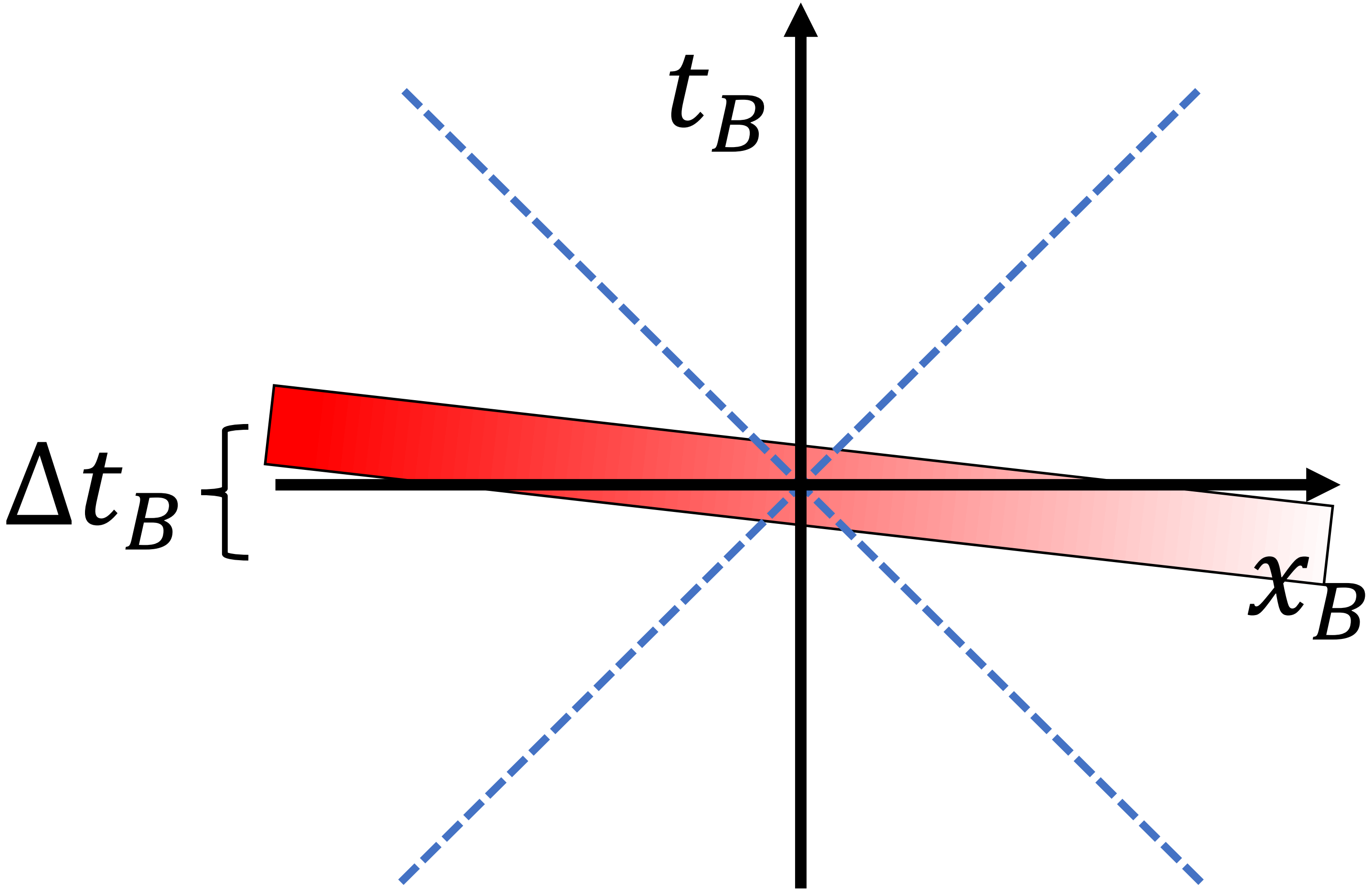
A simple argument



A simple argument



The instability is violent!



It is the growth **time** (not the growth rate!) that changes sign smoothly when we go from the stable to the unstable reference frames.

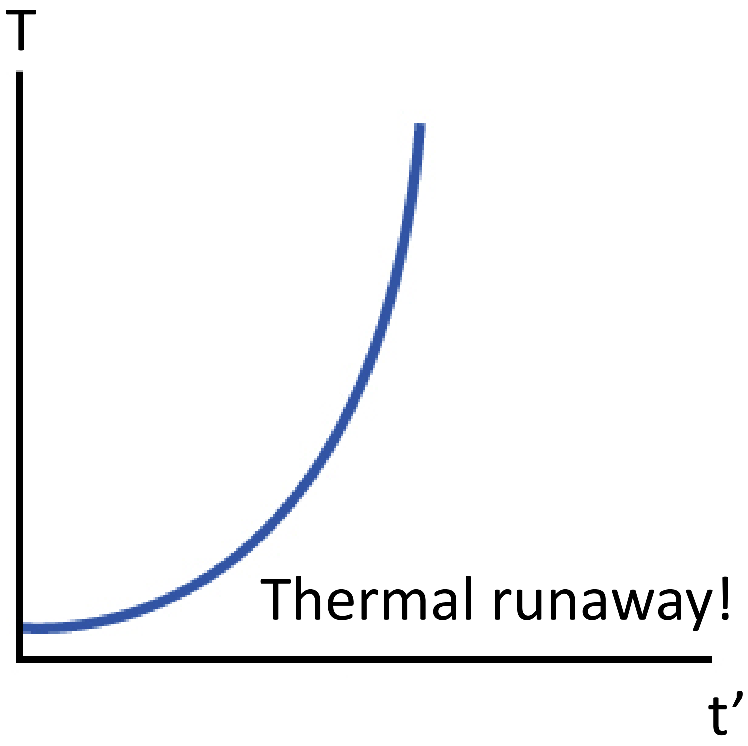
In some frame, the instability is infinitely fast!

Example: boosted heat equation

$$\frac{\partial T}{\partial t'} - v\frac{\partial T}{\partial x'} = D\gamma\left(\frac{\partial^2 T}{\partial x'^2} - 2v\frac{\partial^2 T}{\partial x'\partial t'} + v^2\frac{\partial^2 T}{\partial t'^2}\right)$$

Homogeneous limit:

$$\frac{\partial T}{\partial t'} = D\gamma v^2\frac{\partial^2 T}{\partial t'^2}$$



$$T = T_0 + \frac{\dot{T}_0}{\Gamma_+}\left(e^{\Gamma_+ t'} - 1\right) \qquad \Gamma_+ = \frac{1}{D\gamma v^2} > 0$$

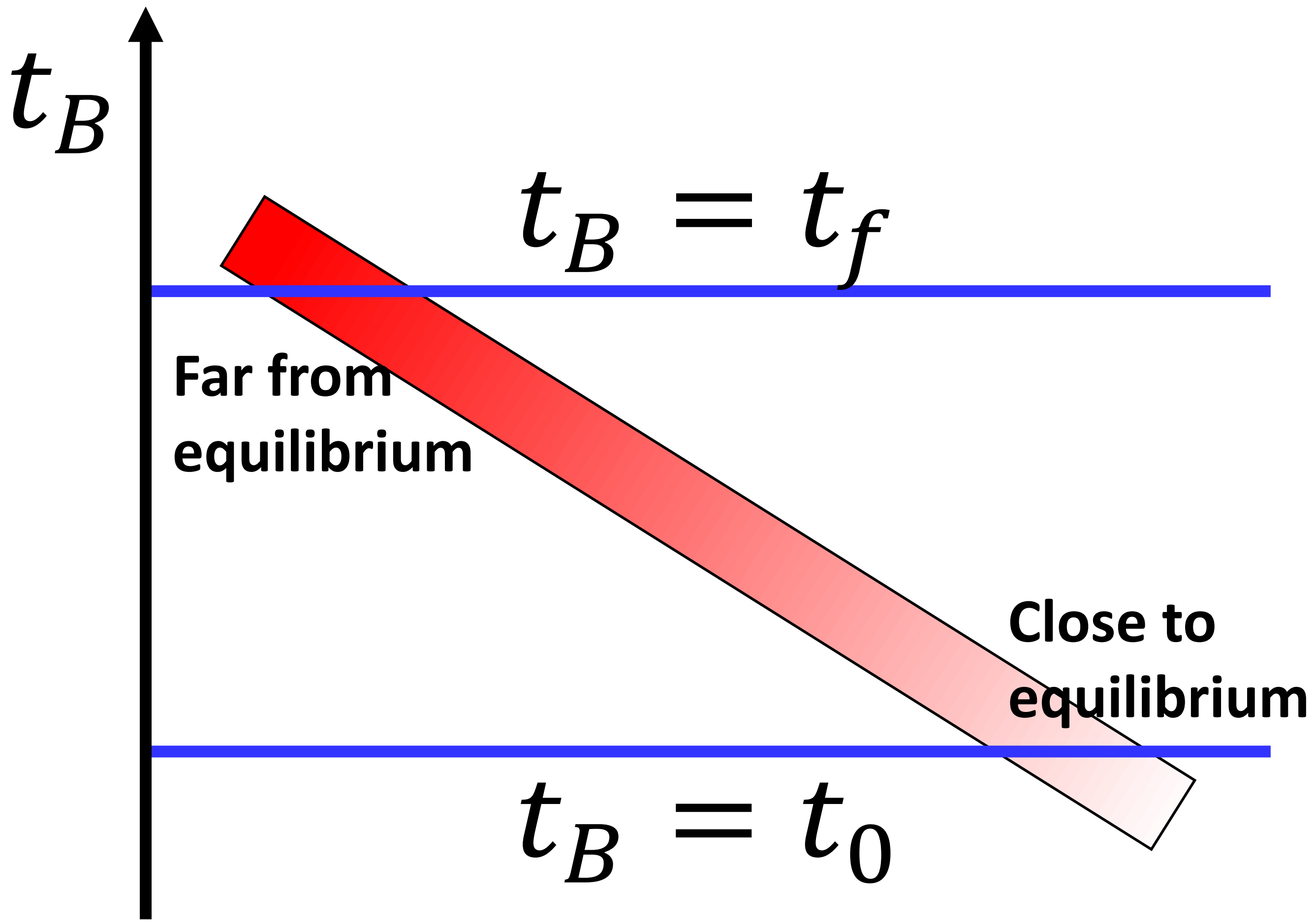
The rate diverges when we approach $v = 0$… as expected.

More examples

- “Generic instabilities of first-order theories”;
- Acausal telegraph equation is unstable;
- Acausal Israel-Stewart theory is unstable;
- Acausal divergence-type theories are unstable;
- Acausal BDNK theory is unstable;

…

Acausality destroys the entropy principle



$\nabla_a s^a \geq 0$ is a Lorentz-invariant statement.

$$S_B(t_f) - S_B(t_0) = \int \nabla_a s^a d\Omega \geq 0$$

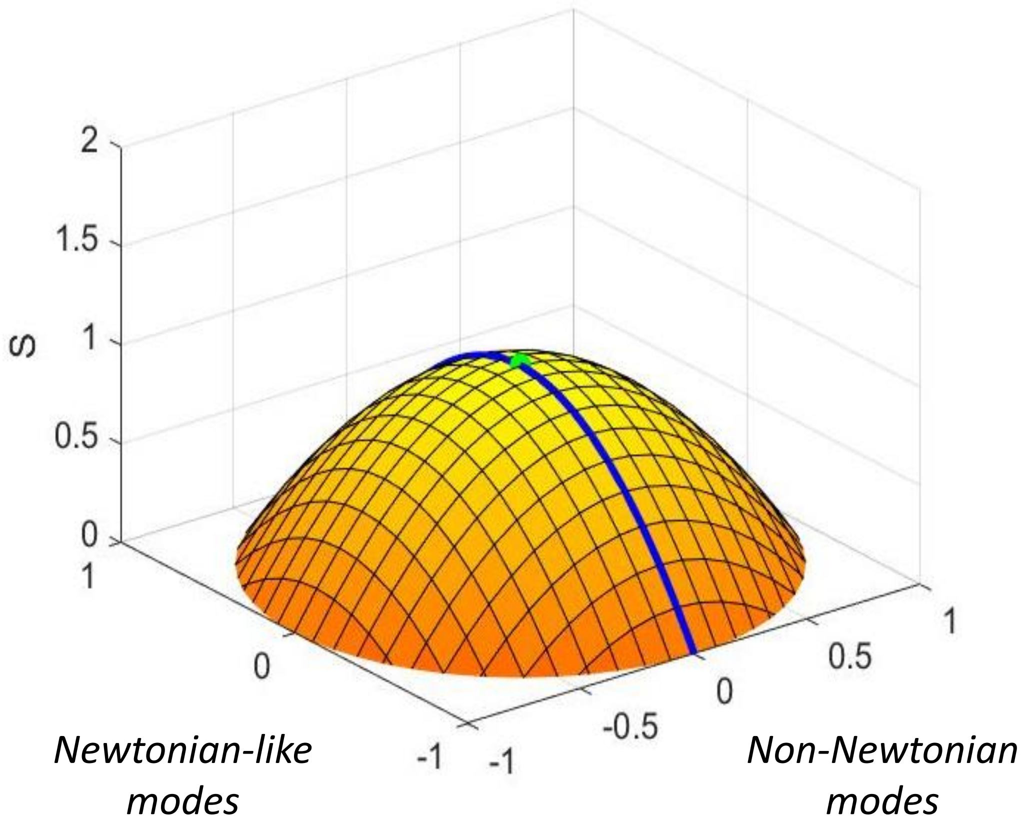
Therefore:

$$\frac{dS_B}{dt_B} \geq 0$$

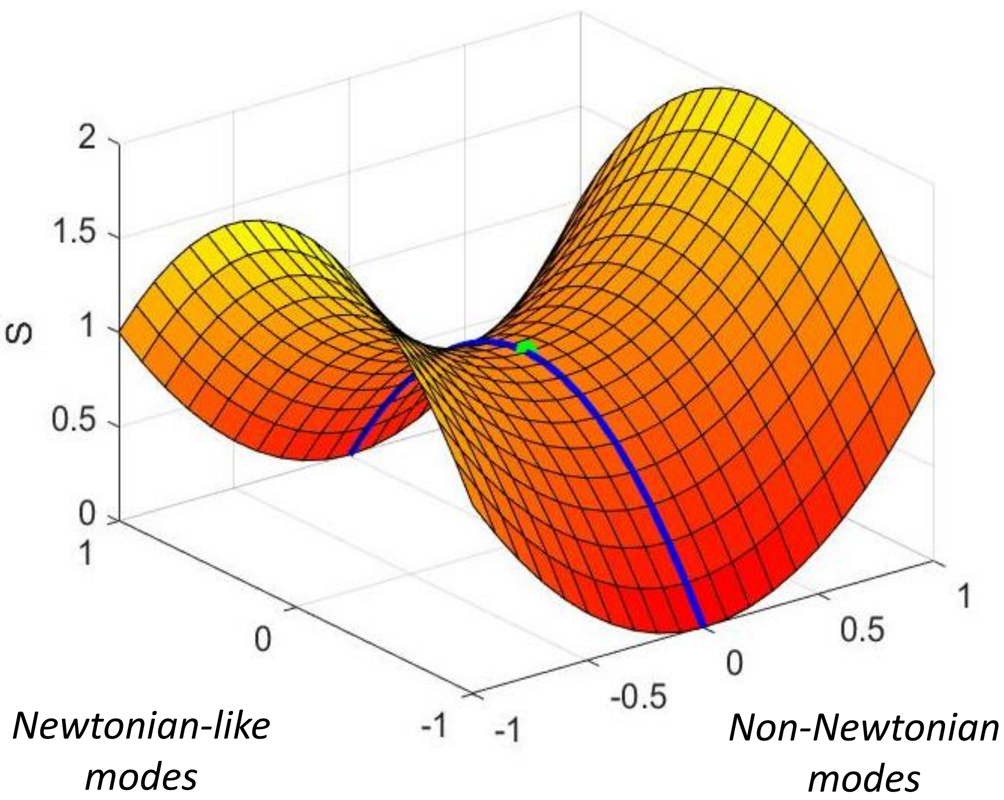
The equilibrium is not the maximum entropy state in B's frame!

Entropy in the frame B

How it should be



How it is in acausal theories



Entropy principle

Equilibrium Gibbs-like state: $\hat{\rho}_{eq} \propto \exp(\alpha_I^\star \hat{Q}^I)$ [$\hat{Q}^I$: conserved charges]

It maximizes $\Phi = S + \alpha_I^\star Q^I$ ["Entropy principle with multipliers"]

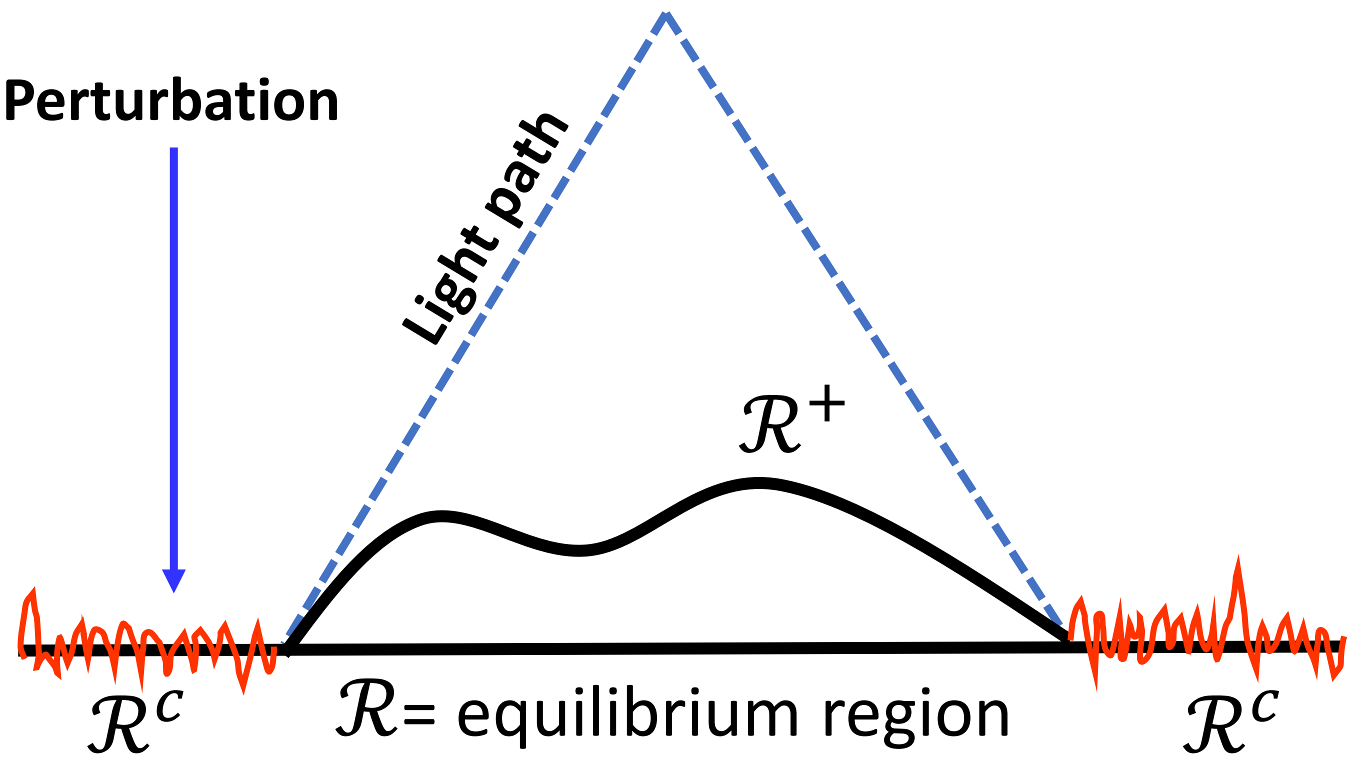
Let's write $-\delta\Phi$ as the flux of a current:

$$E := -\delta\Phi = \int_\Sigma E^a d\Sigma_a \geq 0 \qquad\qquad E^a = -\delta(s^a + \alpha_I^\star J^{Ia})$$

Properties of E^a:

1. Time-like future-directed
2. Vanishes only at equilibrium
3. Non-positive divergence: $\nabla_a E^a \leq 0$

A quick proof of linear causality



Gauss theorem:

$$E[\mathcal{R}^+] - E[\mathcal{R}] = \int \nabla_a E^a \leq 0$$

But $E[\mathcal{R}] = 0$, then

$$E[\mathcal{R}^+] = \int_{\mathcal{R}^+} E^a d\Sigma_a \leq 0$$

However, $E^a d\Sigma_a \geq 0$;
Therefore, $E^a = 0$ on $\mathcal{R}^+$.
Linear causality!

The End

In summary: The “arrow of time” cannot exist without the light-cone.

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Thank you for your attention!