Neutron star dissipative hydrodynamics

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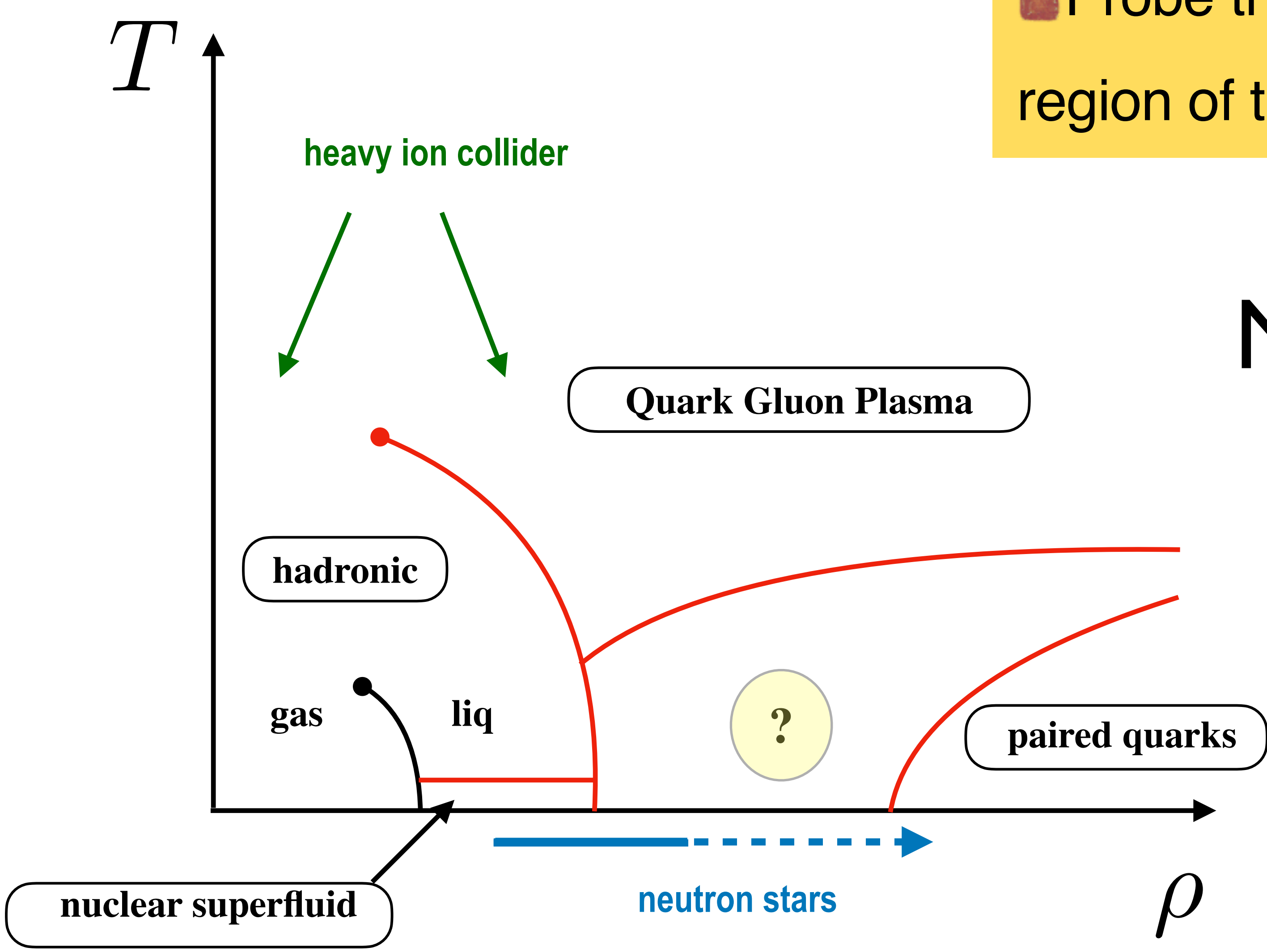
Neutron Stars - theoretical importance

What is the Equation of State of dense matter?

- Probe the low-temperature high-density region of the QCD phase diagram!



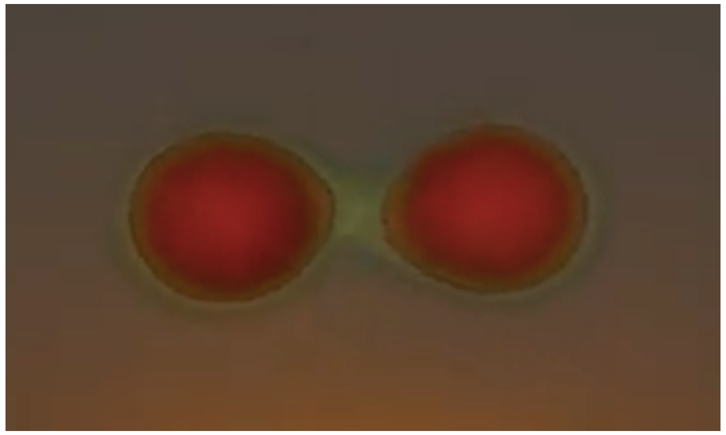
Neutron Stars are cold

$$T \approx 1 \ \ \mathrm{keV}$$

$$E_{\mathbf{Fermi}} \approx 50 \ \ \mathrm{MeV}$$

(see Haskell & Sedrakian 2018 for a review)

Neutron Stars - astrophysical laboratories



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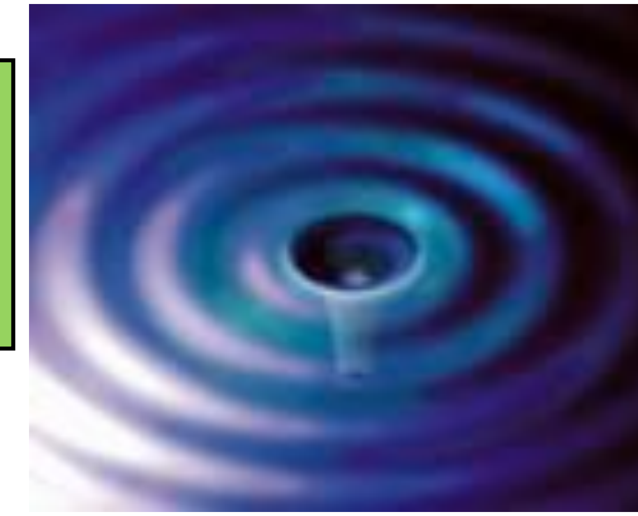
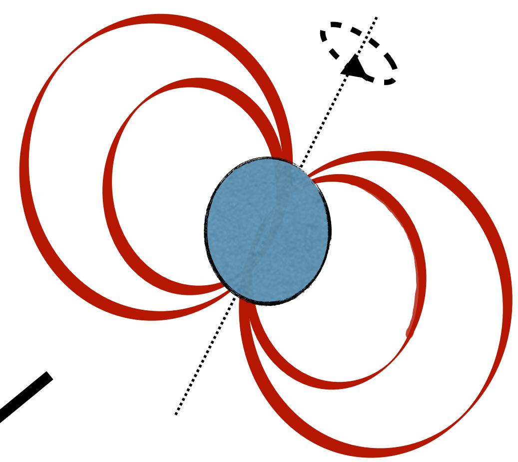
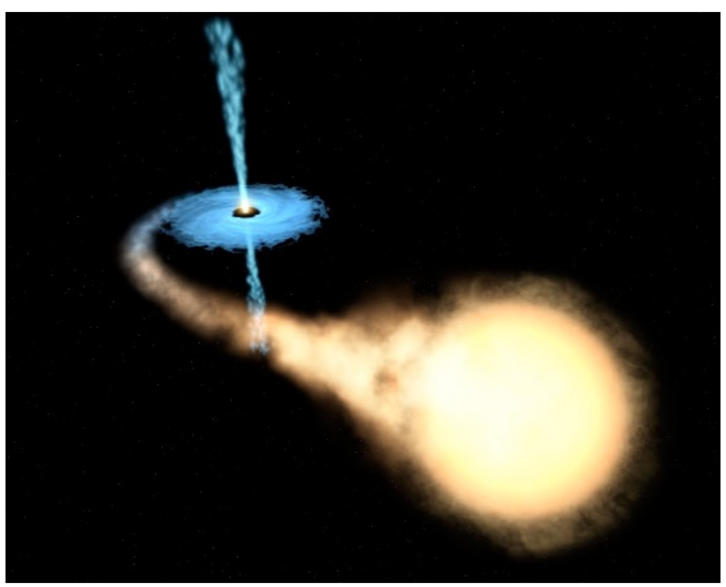
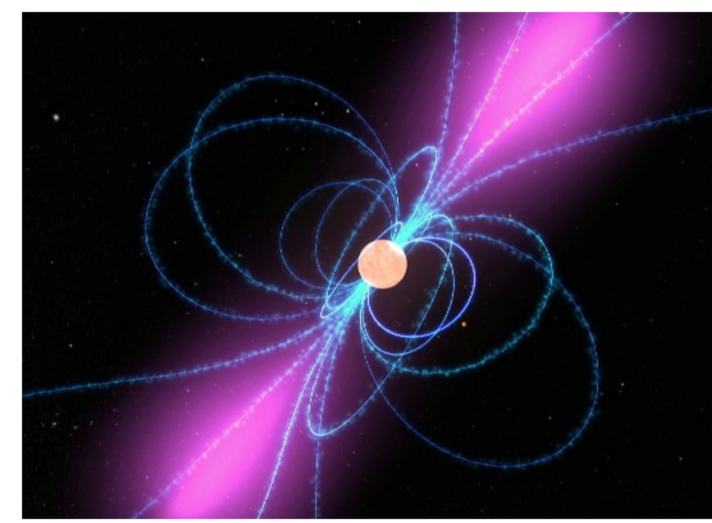
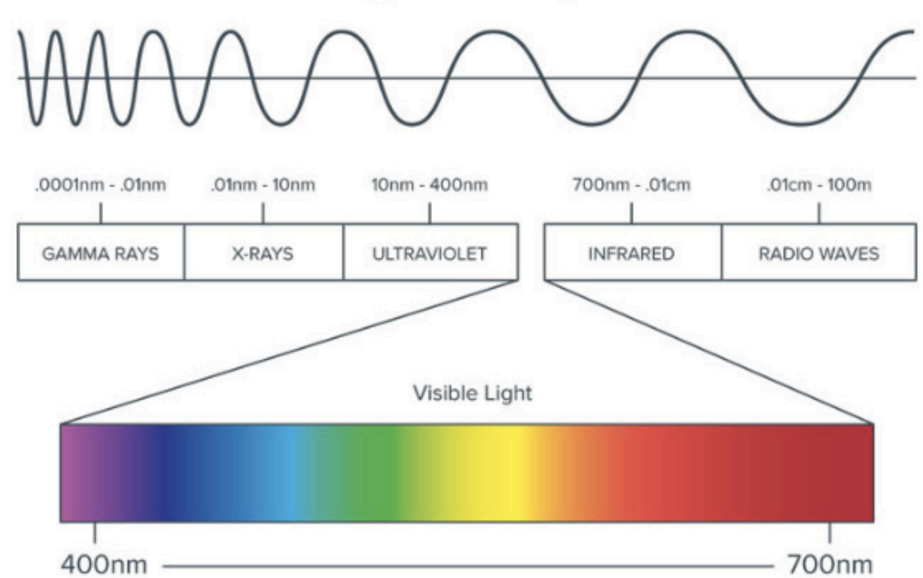
Gravitational waves











Electromagnetic waves

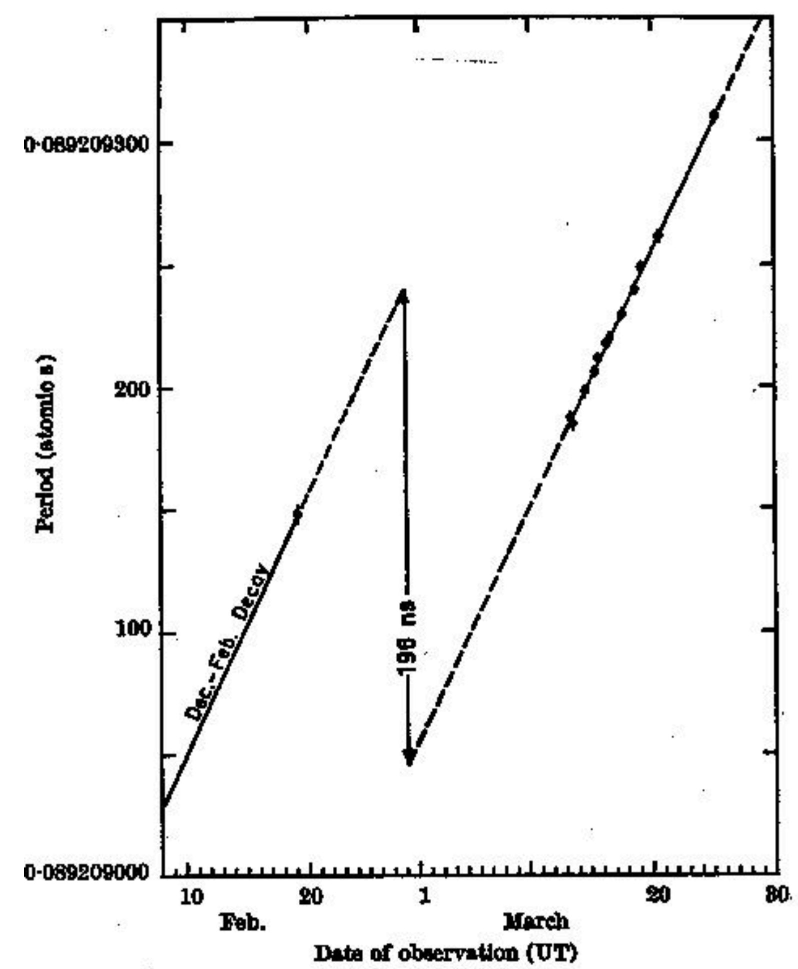
Viscous hydrodynamics: where and how?

NS-NS mergers: matter is hot, shear and bulk viscosity play a role

*(see e.g. Alford et al. 2018, PRL **120**, 041101)*

Mature NSs: Superfluid mutual friction damps oscillations and drives pulsar glitches

(see Haskell & Melatos 2015 for a review)



Multifluid hydrodynamics

Carter's formulation:

$$\partial_t \rho_{\rm x} + \nabla_i (\rho_{\rm x} v^i_{\rm x}) = 0$$

$$(\partial_t + v^j_{\rm x} \nabla_j)(v^{\rm x}_i + \varepsilon_{\rm x} w^{\rm yx}_i) + \nabla_i (\tilde{\mu}_{\rm x} + \Phi) + \varepsilon_{\rm x} w^j_{\rm yx} \nabla_i v^{\rm x}_j = f^{\rm x}_i / \rho_{\rm x} + \nabla_j D^j_i$$

Dissipative terms (bulk viscosity, shear viscosity, etc..)

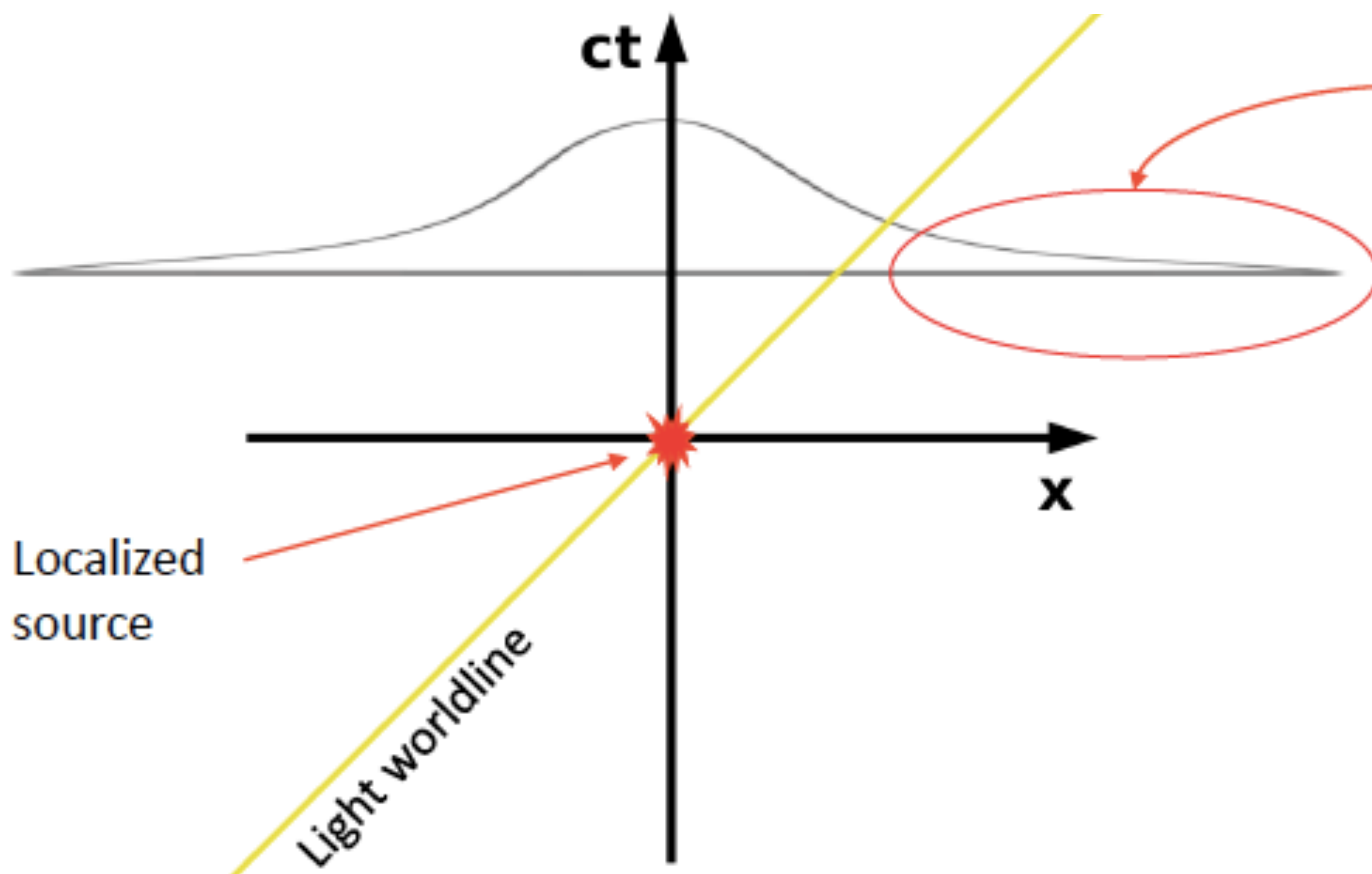
$$f^{\rm x}_i = 2\rho_{\rm n} \mathcal{B}' \epsilon_{ijk} \Omega^j w^k_{\rm xy} + 2\rho_{\rm n} \mathcal{B} \epsilon_{ijk} \hat{\Omega}^j \epsilon^{klm} \Omega_l w^{\rm xy}_m$$

Mutual Friction

Newtonian heat transport

Lets start with heat transport:

$$\frac{\partial T}{\partial t'} = D\frac{\partial^2 T}{\partial x'^2}$$



Tail outside the light cone!

Heat transport: problem in GR

$$\frac{\partial T}{\partial t'} = D \frac{\partial^2 T}{\partial x'^2}$$

What happens if we apply a boost:

$$t^{'} = \gamma(t - vx)$$
$$x^{'} = \gamma(x - vt)$$

$$\frac{\partial T}{\partial t'} - \nu \frac{\partial T}{\partial x'} = D\gamma \left(\frac{\partial^2 T}{\partial x'^2} - 2\nu \frac{\partial^2 T}{\partial t' \partial x'} \partial t^{'} + \nu^2 \frac{\partial^2 T}{\partial t'^2} \right)$$

Homogeneous solution is unstable! Only in Relativity, no Newtonian analogue

$$\frac{\partial T}{\partial t'} = D\gamma\nu^2 \frac{\partial^2 T}{\partial t'^2}$$

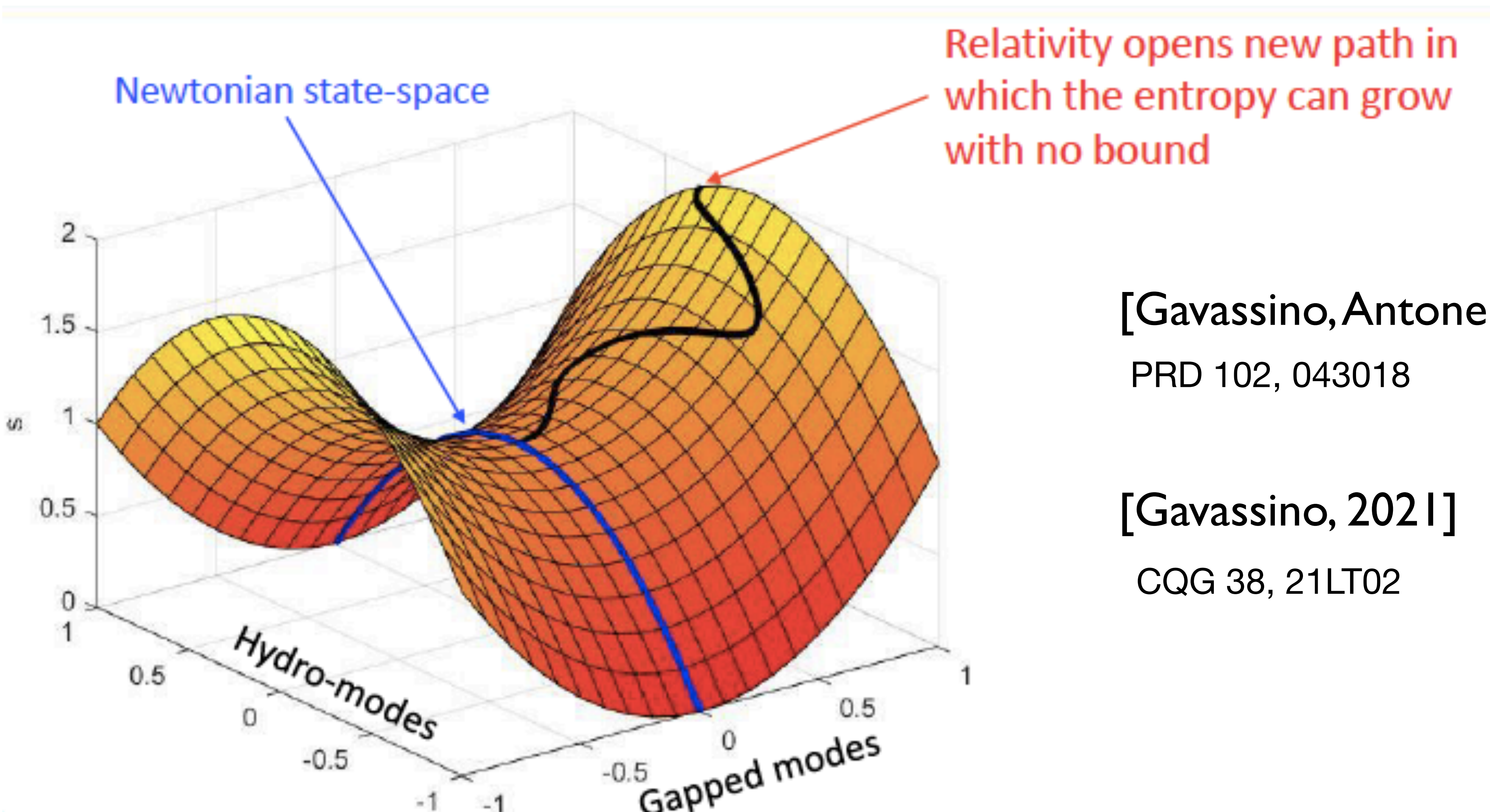
Exponentially growing solutions

Viscous hydrodynamics: problem in GR

Navier Stokes suffers from a similar instability

$$\rho \frac{DV}{Dt} = -\nabla P + \rho g = \mu \nabla^2 V$$

What is going wrong?



[Gavassino, Antonelli & BH, 2020]
PRD 102, 043018

[Gavassino, 2021]
CQG 38, 21LT02

How to solve things in GR

Causality is key (Gavassino will explain)

$$\tau_0 \dot{q} + q = -K \frac{\partial T}{\partial x}$$

Maxwell Cattaneo for heat

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} - \tau_0 \frac{\partial^2 T}{\partial t^2}$$

Similar approach to Navier Stokes (most used Israel Stewart)

Link between coefficients and microphysics unclear though….

Bulk viscosity

Consider a system of particles (or `fluids') and increase the volume v

[Gavassino, Antonelli & BH, 2020]

CQG 38, 075001,

$$\mathcal{U} = \mathcal{U}(s, n, \boxed{n_A})$$

`Slow' degrees of freedom

$$d\mathcal{U} = \Theta ds + \mu dn - \sum_{A=1}^{l-1} \mathbb{A}^A dn_A$$

generalised affinities

$$T^{\mu\nu} = (\mathcal{U} + \Psi) u^\mu u^\nu + \Psi g^{\mu\nu}$$

$$\Psi = P + \Pi$$

$$\Pi = -\mathbb{A}^A \frac{\partial x_A^{eq}}{\partial v}$$

Bulk viscosity in neutron stars: protons and neutrons

reaction rates

$$\boxed{\tau_M}\dot{\mathbb{A}} + \mathbb{A} = \kappa \nabla_\nu u^\nu$$

telegraph kind equation

relaxation timescales

$$\Pi = -\zeta \nabla_\nu u^\nu + \tau_M \dot{\mathbb{A}} \frac{\partial x_p^{eq}}{\partial v}$$

parabolic limit $\tau_M \longrightarrow 0$ $\Pi = -\zeta \nabla_\nu u^\nu$

otherwise causal (Bemfica et al. 2019)

$$\Pi = -\zeta(\nabla_\nu u^\nu + \chi \dot{\Pi})$$

Bulk viscosity in neutron stars

several `fluids' are needed in general

$$\tau^A_B \dot{\mathbb{A}}^B + \mathbb{A}^A = -\kappa^A \nabla_\nu u^\nu$$

(cfr. Israel Stewart where one has only Π as a variable)

numerical implementations work in progress...

Conclusions

- NS are an awesome fundamental physics lab
- EM and GW observations allow quantitative constraints
- Essential to model dissipation in GR
- Stability is closely linked to causality and the 2nd law