

Static quark-antiquark interaction at finite temperature on fine lattices

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Chromo-electric screening length in 2+1 flavor QCD,
P. Petreczky, S. Steinbeißer, JHW, arXiv:2112.00788[hep-lat]

Bottomonium melting from screening correlators at high temperature,
P. Petreczky, S. Sharma, JHW, Phys.Rev.D 104 (2021) 5, 054511
Bottomonia via lattice NRQCD,

R. Larsen, S. Meinel, S. Mukherjee, P. Petreczky, Phys.Rev.D 100 (2019) 7, 074506; Phys.Lett.B 800 (2020) 135119; Phys.Rev.D 102 (2020) 114508

Heavy Quark Potential in QGP: DNN meets LQCD,
S. Shi, K. Zhou, J. Zhao, S. Mukherjee, P. Zhuang, arXiv:2105.07862[hep-ph]

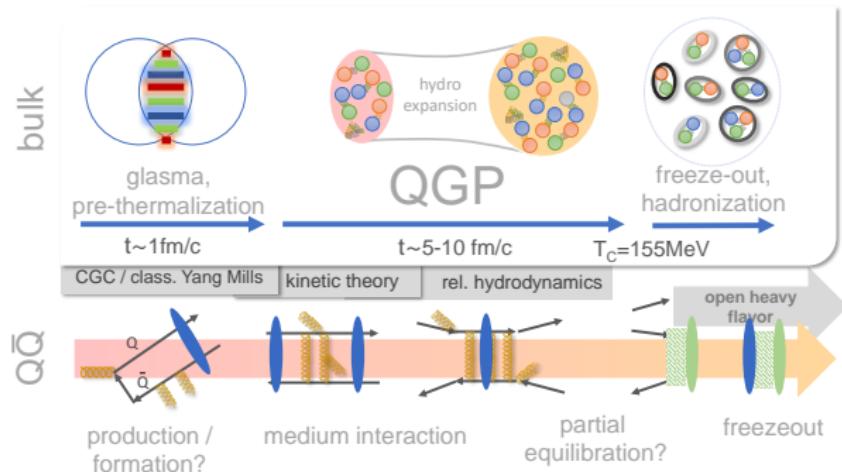
Static quark anti-quark interactions at non-zero temperature from lattice QCD,
D. Bala, O. Kaczmarek, R. Larsen, S. Mukherjee, G. Parkar,
P. Petreczky, A. Rothkopf, JHW, arXiv:2110.11659[hep-lat]

Static Potential At Non-zero Temperatures From Fine Lattices,
+A. Bazavov, D. Hoying, arXiv:2110.00565[hep-lat]

Outline

- ① **Appetizer:** link to heavy-ion phenomenology
 - Motivation
- ② **Main course:** modern perspective on “Heavy quarkonium at $T > 0$ ”
 - Historical perspective on “Heavy quarkonia at $T > 0$ ”
 - In-medium quarkonium at weak coupling
 - Lattice QCD
 - Relativistic bottomonium on the lattice
 - Nonrelativistic bottomonium on the lattice
 - In-medium static quarkonium
- ③ **Dessert:** bringing “Heavy quarkonium at $T > 0$ ” full circle

Why focus on hard probes in heavy-ion collisions?



source: Rothkopf, Phys.Rept. 858 (2020) 1-117

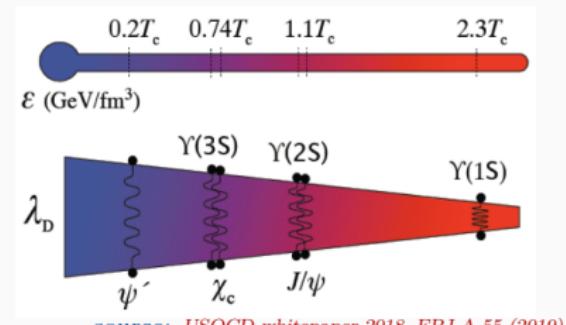
- Hard probes are produced in a few **hard processes** in initial collision, neither created nor destroyed afterwards, but can alter their nature
- Most important probes: *jets*, open heavy flavor & **heavy quarkonia**
- **What happens to quarkonium if we increase the temperature?**

Heavy quarkonia in the hot medium as a local thermometer

- Idea to look at **quarkonia** in the QGP is old and famous

Matsui, Satz, PLB 178 (1986)

- Debye screening length** $1/m_D$ of electric gluons (A_0) limits the radii of hadronic bound states
- Consequence: QGP formation \Leftrightarrow **quarkonia suppression**



- But what is the **correct in-medium potential** for quarkonia?
- Two maximally different scenarios for the in-medium potential predict vastly different **melting points** for all quarkonia...:

① **Weak-binding scenario:** Debye-screened potential goes flat as $V \sim F$

$$T(\Upsilon(1S)) \sim 2 T_{pc}, \quad T(J/\psi) \sim T_{pc}$$

② **Strong-binding scenario:** Remnant of confinement as $V \sim U = F + TS$

$$T(\Upsilon(1S)) \sim 3 T_{pc}, \quad T(J/\psi) \sim 3/2 T_{pc}$$

Screening from Polyakov loop correlators

- Color screening usually studied via Polyakov loop correlator

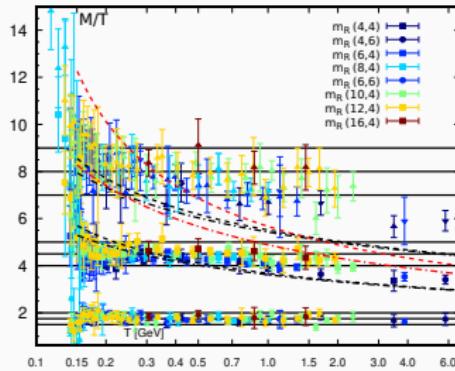
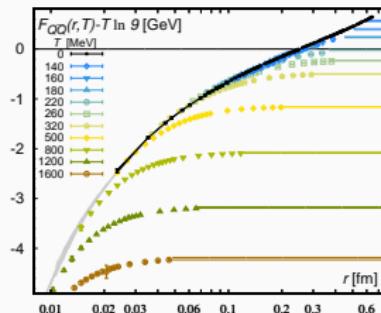
$$C_P(r, T) = \langle P(0)P^\dagger(r) \rangle_T^{\text{ren}} = e^{-F_{Q\bar{Q}}(r, T)/T}$$

- $rT \ll 1$: singlet/octet decomposition

$$C_P(r, T) = \frac{1}{9}e^{-F_S(r, T)/T} + \frac{8}{9}e^{-F_O(r, T)/T}$$

- At $rT \lesssim 0.4$ via $T=0$ potentials and adjoint Polyakov loop: no screening!

$$C_P(r, T) = \frac{1}{9}e^{-V_s(r)/T} + L_A(T) \frac{8}{9}e^{-V_o(r)/T} + \mathcal{O}(\alpha_s^3)$$



- $rm_D \gtrsim 1$: screening regime; decompose

$$C_P(r, T) = C_R(r, T) + C_I(r, T)$$

$$\begin{aligned} C_R(r, T) &= \langle \text{Re } P(0) \text{Re } P(r) \rangle_T^{\text{ren}} \rightarrow \mathcal{C} && \text{even} \\ C_I(r, T) &= \langle \text{Im } P(0) \text{Im } P(r) \rangle_T^{\text{ren}} && \text{odd} \end{aligned}$$

- Asymptotically $C_{R,I}(r, T) \sim e^{-m_{R,I}r}/rT$
- EQCD: $m_R \sim 2m_D$, $m_I \sim 3m_D$ (A_0 exchange)

$$\text{LQCD: } \frac{m_R}{T} \approx 4.5, \quad \frac{m_I}{T} \approx 8, \quad \frac{m_I}{m_R} \approx 1.75$$

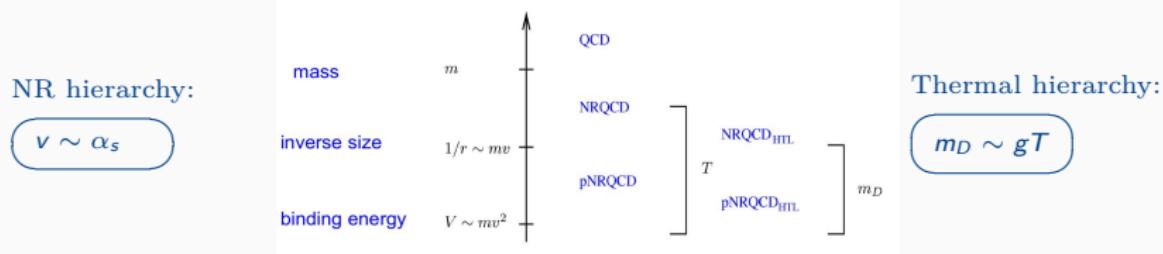
$$\Rightarrow \frac{1}{m_D} \approx \frac{2}{m_R} \approx \frac{3}{m_I} = \{0.38 - 0.44\}/T$$

$$r_{Y(1S)} \approx 0.21 \text{ fm survives until } T \sim 380 \text{ MeV}$$

$$r_{J/\Psi} \approx 0.43 \text{ fm survives until } T \sim 190 \text{ MeV}$$

Screening is not the whole story... (at weak coupling)

Matsui & Satz's idea of the **quarkonium suppression mechanism** was turned inside out by **weak-coupling EFT** results emerging 15 years ago



- For $1/r \sim m_D \ll T$: $\text{Re}[V_s] = F_S + \mathcal{O}(g^4)$ and $\text{Im}[V_s] \sim \mathcal{O}(g^2 T)$

$$V_s(T, r) = -C_F \alpha_s \left\{ \frac{e^{-rm_D}}{r} + m_D + i T \phi(r m_D) \right\}, \quad \phi(x) = 2 \int_0^\infty \frac{dz}{(z^2+1)^2} \left\{ 1 - \frac{\sin(zx)}{zx} \right\}$$

Laine, et al., JHEP 03 (2007)

- For $\Delta V \ll 1/r \ll m_D \ll T$: $\text{Re}[V_s] = V_s + \mathcal{O}(g^4)$ and $\text{Im}[V_s] \sim \mathcal{O}(g^4 r^2 T^3, g^6 T)$

$$V_s(T, r) = \frac{-C_F \alpha_s}{r} + r^2 T^3 \left\{ \mathcal{O}(g^4) + i \mathcal{O}\left(g^4, \frac{g^6}{(rT)^2}\right) \right\}$$

Brambilla, et al., PRD 78 (2008)

But an imaginary part leads to **dissociation** – is **screening** even relevant?

QCD on a lattice

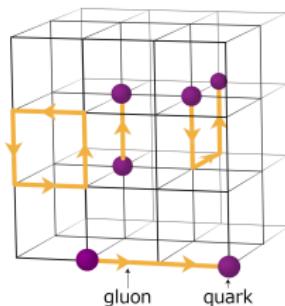
$$S_{QCD}[U, \bar{\psi}, \psi] = a^4 \sum_x \sum_{f=1}^{N_f} \bar{\psi}^f(x) \left(\not{D}[U(x)] + m_f \right) \psi^f(x)$$

$$- a^4 \sum_x \sum_{\mu < \nu} \frac{2}{g_0^2} \text{Re tr} \left\{ 1 - U_{\mu,\nu}(x) + \mathcal{O}(a^2) \right\}$$

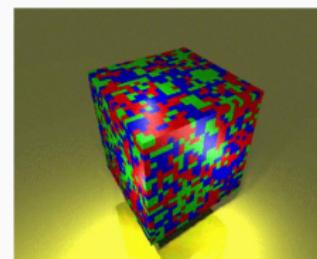
$$D_\mu[U_\mu(x)]\psi^f(x) = \frac{U_\mu(x)\psi^f(x + a\hat{\mu}) - U_\mu^\dagger(x - a\hat{\mu})\psi^f(x - a\hat{\mu})}{2a} + \mathcal{O}(a^2)$$

$$U_\mu(x) = \exp[i a g_0 A_\mu(x)] \quad \text{gauge link}$$

$$U_{\mu,\nu}(x) = U_\mu(x) U_\nu(x + a\hat{\mu}) U_\mu^\dagger(x + a\hat{\nu}) U_\nu^\dagger(x) \quad \text{plaquette}$$



HPC
⇒



Lattice QCD simulations in a box on a computer

Stochastically sample the (Euclidean) QCD path integral

$$\langle \mathcal{O} \rangle_{\text{QCD}} = \frac{1}{Z} \frac{1}{N} \sum_{\{U\}}^N \mathcal{O}[U] \prod_{f=1}^{N_f} \det(\not{D}[U] + m_f) \exp(-S_g[U]) + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$

using MCMC algorithm with importance sampling

QCD on a lattice with spacing a in a box of $N_\sigma^3 \times N_\tau$ points

- scale setting: lattice spacing a is determined a posteriori
control the approach to the continuum limit $a \rightarrow 0$
- time (Euclidean): periodic for gluons, antiperiodic for quarks
- space: periodic for gluons and quarks
always at finite temperature and in finite volume
 $aN_\tau = 1/T$ (volumes only must be large enough)
- quark masses: light quarks at the physical point are expensive
control the quark mass dependence through χ PT
- quark flavors: usually $N_f = 2 + 1$ or $N_f = 2 + 1 + 1$, or $N_f = 0$

Real-time dynamics from Lattice QCD

- Importance sampling requires an imaginary-time formalism
 \Rightarrow **Dissociation** due to real-time dynamics not directly accessible

- Spectral functions** encode the entire dynamics
 - Stable bound states \Rightarrow Delta functions
 - Unstable quasiparticles \Rightarrow regularized peaks, locally Breit-Wigner
 - On top of a UV continuum due to scattering or merged excited states
 - At $T > 0$ potentially a substantial IR tail below the “ground state”
- Same **spectral functions** yield real- or imaginary-time correlators via different, analytically known integral kernels

$$G_T \left(\frac{t}{\tau} \right) = \int d\omega \begin{pmatrix} K^M(T, \omega; t) \\ K^E(T, \omega; \tau) \end{pmatrix} \rho_T(\omega)$$

- \Rightarrow Strategy for lattice QCD:
- Compute imaginary-time correlators on the lattice
 - Reconstruct **spectral functions** by inverting spectral representation
 - Directly read off some state's properties from $\rho_T(\omega)$
- Spectral reconstruction is challenging: at best n_τ resp. $n_\tau/2$ data

At which T are there either bound states or melted $q\bar{q}$ pairs?

- **Euclidean Correlators** are towers of exponential decays $G(\tau) = \sum_i A_i e^{-E_i \cdot \tau}$

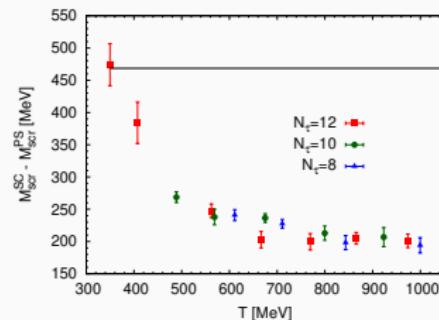
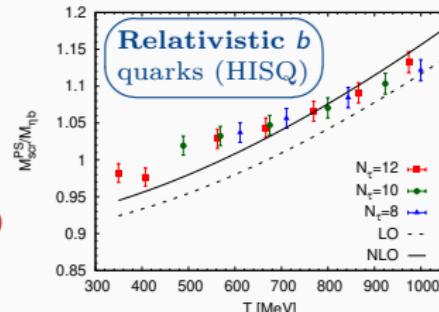
For mesons: same E_0 in temporal or spatial directions

- **Spatial $q\bar{q}$ pair correlators** are a model-independent analysis tool
charm sector \Rightarrow Bazavov, et al., PRD 91 (2015)

$$\begin{aligned} G(z, T) &= \int_0^{1/T} d\tau \int d^2x_\perp \langle J(\tau, x_\perp, z) J^\dagger(0) \rangle \\ &= \int_0^\infty \frac{2d\omega}{\omega} \int_{-\infty}^\infty dp_z e^{ip_z z} \rho(\omega, p_z, T) \end{aligned}$$

with spectral function $\rho(\omega, p_z, T)$

$$\sim \begin{cases} \delta[\omega^2 - p_z^2 - M_0^2] & \text{mesons} \\ \delta[\omega - \sum_{qi} \sqrt{m_{qi}^2 + [\pi T]^2}] & \text{free quarks} \end{cases}$$



source: Petreczky, et al., PRD 104 (2021)

- Survival of η_b & $\Upsilon(1S)$ until $T \approx 400$ MeV; cf. η_c & J/Ψ until $T \approx 200$ MeV
- Survival of χ_{b0} & h_b until $T \approx 350$ MeV; cf. χ_{c0} & χ_{c1} until $T \sim T_{pc}$
- How can we understand the **melting mechanism** at work?

Nonrelativistic bottomonium with extended sources (HotQCD)

- Lattice NRQCD: no continuum limit
⇒ upper limit on T for fixed N_τ
- NRQCD correlator study on $T = 0$ or $N_\tau = 12$ lattices: **extended sources**
Larsen, et al., ...

① Point source vs Gaussian smearing,
new scheme for removing UV part:
thermal width, small mass shift

PRD 100 (2019)

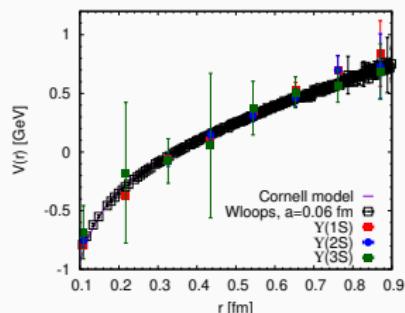
- ② Cornell pot. eigenstates → GEVP
⇒ lattice **NBS amplitudes**
- ③ Recover potential from NBS amp.

$$\left(\frac{-\Delta}{m_b} + V(r) \right) \phi_\alpha = E_\alpha \phi_\alpha$$

PLB 800 (2020) + PRD 102 (2020)

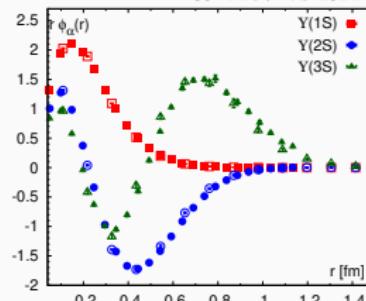
- ④ Small τ : NBS amplitudes at $T = 0$ and $T > 0$ almost **T -independent**
- ⑤ Large τ : $3S$ NBS amp. at $T > 0$ visibly **modified** ⇔ thermal width

$T = 0, a = 0.06 \text{ fm}$: NRQCD vs static $q\bar{q}$



source: *Larsen, et al., PRD 102, (2020)*

$N_\tau = 12, \tau \sim 0.4 \text{ fm}$
 $T = 334 \text{ MeV}$ vs 151 MeV

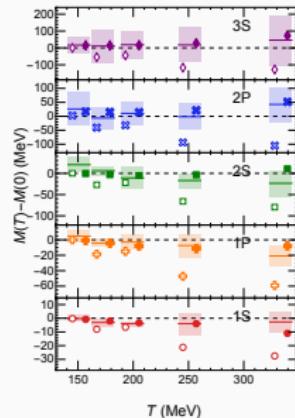


Machine learning the potential from NRQCD amplitudes

Is there room for another interpretation? Let an algorithm figure it out...

Machine learning (DNN) applied to lattice NBS amp.: $T > 0$ potential

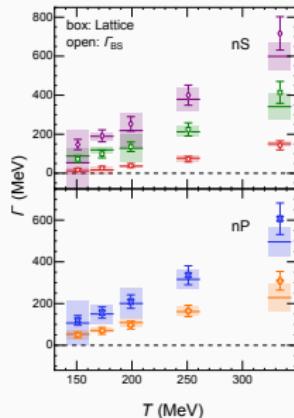
Shi, et al., arXiv:2105.07862



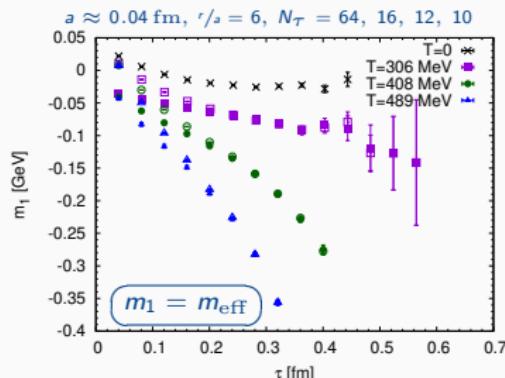
Lattice NBS amplitudes fed into DNN \Rightarrow can reconstruct a nonperturbative potential V^{ML}

Clearly smaller thermal mass shift and larger width than in **Hard Thermal Loop** (HTL) perturbation theory ($\text{Re}(V_s^{\text{HTL}}) \sim F_S$):

$$\begin{aligned} \text{Re}[V^{\text{ML}}] &\sim V_s(T=0) \\ \text{Im}[V^{\text{ML}}] &\gg \text{Im}[V_s^{\text{HTL}}] \end{aligned}$$



Static $q\bar{q}$ pair at $T > 0$ on the lattice



source: Bala, et al., arXiv:2110.11659

- Static $q\bar{q}$ interaction is encoded in (real-time) Wilson loops^a

$$W_{[r, T]}(t) = \left\langle e^{ig \oint_{r \times t} dz^\mu A_\mu} \right\rangle_{\text{QCD}, T}$$

- Stable (ground) state Ω_r exists if

$$\Omega_{[r, T]} \equiv -i \lim_{t \rightarrow \infty} \partial_t W_{[r, T]}(t)$$

^aWe use Wilson line correlators in Coulomb gauge.

- Same spectral functions yield real- or imaginary-time correlators

$$W_{[r, T]} \left(\frac{t}{\tau} \right) = \int d\omega \begin{pmatrix} e^{+i\omega t} \\ e^{-i\omega \tau} \end{pmatrix} \rho_{[r, T]}(\omega)$$

- Motivates generic decomposition

$$\rho_{[r, T]}(\omega) = \rho_{[r, T]}^{\{\Omega; \mathcal{O}(T)\}}(\omega) + \rho_{[r, T]}^{\text{tail}}(\omega) + \rho_{[r, T]}^{\text{UV}}(\omega)$$

- UV continuum $\rho_{[r, T]}^{\text{UV}}(\omega)$ is far above lowest feature $\Omega + \text{effects of } \mathcal{O}(T)$

⇒ Guess $\rho_{[r, T]}^{\text{UV}}(\omega)$ via $\rho_{[r, 0]}^{\text{UV}}(\omega)$ ⇒ subtract

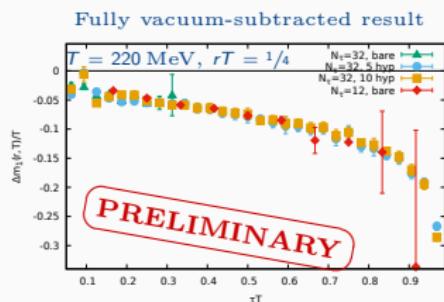
Note: “tail” due to backward propagating UV physics (vacuum excited states) at $\tau \lesssim 1/\tau_c$

Cumulants of spectral functions – what can we expect?

- Access cumulants of $\rho_{[r,T]}(\omega)e^{-\omega\tau}$ via τ (log) derivatives of $W_{[r,T]}(\tau)$

$$m_1^{[r,T]}(\tau) = -\partial_\tau \ln W_{[r,T]}(\tau) \quad [\equiv m_{\text{eff}}^{[r,T]}(\tau)],$$

$$m_n^{[r,T]}(\tau) = -\partial_\tau m_{n-1}^{[r,T]}(\tau), \quad n > 1$$
- For $N_\tau \leq 16$ obtain up to $m_3^{[r,T]}(\tau)$: supports ≤ 5 parameters for $\rho_{[r,T]}(\omega)$
- Higher cumulants at small τ need at least $N_\tau > 16$: bad signal-to-noise

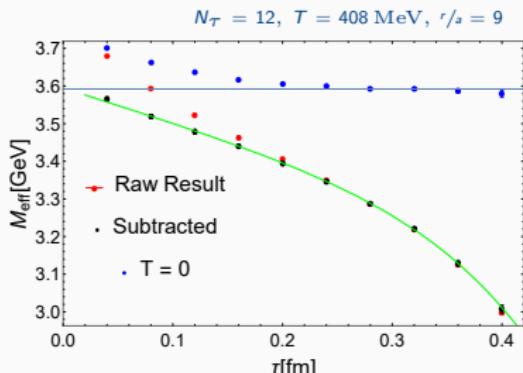


see: Hoying, et al., arXiv:2110.00565 [hep-lat]

Feasibility study with $N_\tau = 32$: $m_n^{[r,T]}, n > 2$?

- Fine lattices: $a^{-1} \approx 7 \text{ GeV}$ $m_\pi \approx 0.3 \text{ GeV}$
- UV filtering (HYP) for noise reduction
→ distortions cancel in vacuum subtraction
- Definitely still work in progress

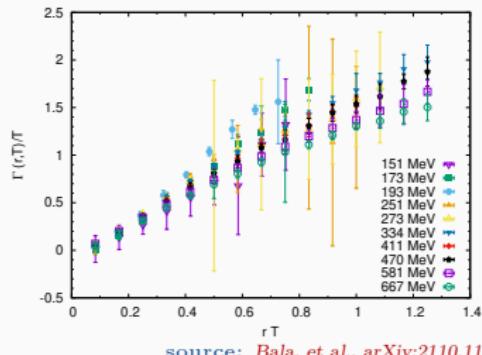
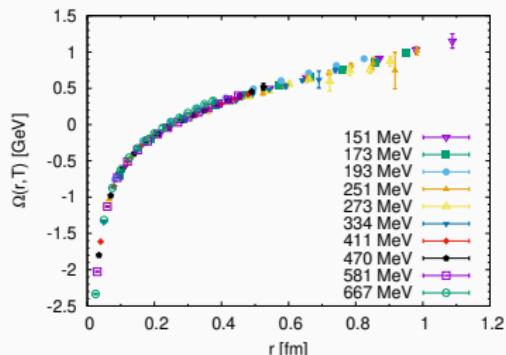
Lowest spectral feature from fits using Gaussian approximation



- Quasiparticles are represented as Breit-Wigner in $\rho_{[r, T]}(\omega)$
- Ansatz: approximate BW of $\rho_r^{\{\Omega; \mathcal{O}(T)\}}(\omega)$ locally as Gaussian, include delta function for $\rho_r^{\text{tail}}(\omega)$

$$W_{[r, T]}(\tau) = A_{[r, T]}^{\{\Omega; \mathcal{O}(T)\}} e^{-\Omega_{[r, T]}\tau + (\Gamma_{[r, T]}^G)^2 \tau^2/2} + A_{[r, T]}^{\text{tail}} e^{-\omega_{[r, T]}^{\text{tail}}\tau}, \quad \omega_{[r, T]}^{\text{tail}} \ll \Omega_{[r, T]}$$

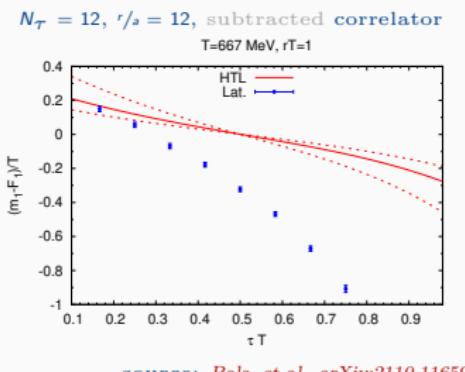
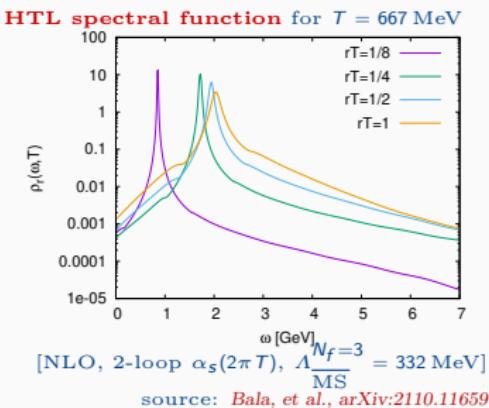
$N_\tau = 12, \Omega(r, T) \equiv \Omega_{[r, T]}, \Gamma(r, T) \equiv \sqrt{2 \ln 2} \Gamma_{[r, T]}^G, \text{ subtracted correlators}$



source: Bala, et al., arXiv:2110.11659

- Almost no τ dependence in $\Omega_{[r, T]}$ (naive correspondence: $\text{Re } V_s(r, T)$)
- Naively expected scaling of $\Gamma(r, T)/T \approx \Gamma(rT)/T$ down to $T \approx T_{pc}$

Comparison: lattice QCD vs HTL



- **HTL** is an attractive proposition: motivated & regularized BW
- **HTL** result is antisymmetric around the midpoint $\tau = 1/2T$:

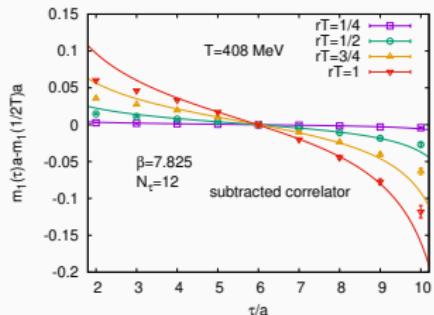
$$\log W_{[r, T]}(\tau) = -\operatorname{Re} V_s(r, T) \times \tau + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left\{ e^{-\omega\tau} + e^{-\omega(1/T-\tau)} \right\} \times \{1 + n_B(\omega)\} \sigma_{[r, T]}(\omega)$$

- Leading singularity of $\sigma_{[r, T]}(\omega)$ (transv. gluon spec. fun.) fixes $\operatorname{Im} V_s(r, T)$

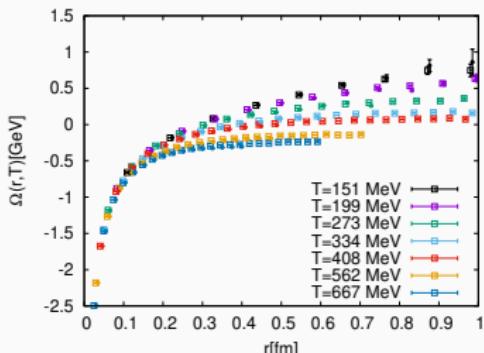
- **HTL** should work at $r \sim 1/m_D$
- Subtleties due to renormalons and regulators: consider $(m_1 - F_S)/T$
Reminder: $\operatorname{Re} [V_s] = F_S + \mathcal{O}(g^4)$ in **HTL**
- No large UV component in **HTL**, compare UV-subtracted result
- **m₁** at midpoint lower than **HTL**, and **m₂** is much more negative

Lowest spectral feature from fits using HTL-motivated Ansatz

$N_\tau = 12, T = 408 \text{ MeV}, r/a = 9$



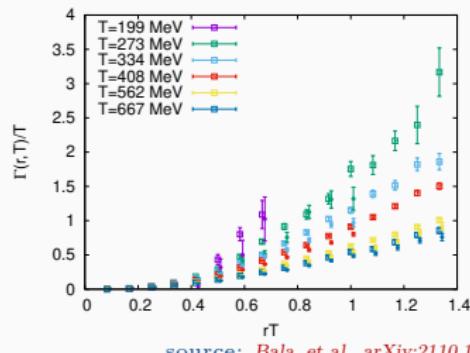
$N_\tau = 12, \Omega(r, T) \equiv \Omega_{[r, T]}^{BD}, \Gamma^{BD}(r, T) \equiv \Gamma_{[r, T]}^{BD}$, (un-)subtracted correlators



- Fit via HTL-motivated Ansatz
Bala, Datta, PRD 101 (2020)

$$W_{[r, T]}(\tau) = A_{[r, T]}^{BD} e^{-\Omega_{[r, T]}^{BD} \tau - i \frac{\Gamma_{[r, T]}^{BD}}{\pi} \log \sin(\pi \tau)}$$

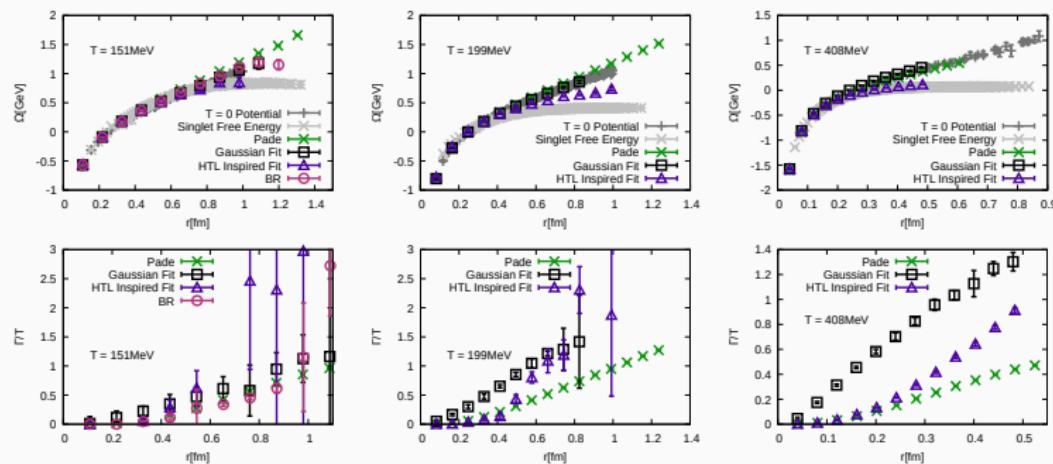
- Note: similar result via Gaussian around midpoint $\tau = 1/2T$



source: Bala, et al., arXiv:2110.11659

- Significant τ dependence in $\Omega_{[r, T]}$ (naive correspondence: $\text{Re } V_s(r, T)$)
- Weaker than naive scaling of $\Gamma(r, T)/T \approx \Gamma(rT)/T$

Comparison: lowest spectral feature from four different methods



source: Bala, et al., arXiv:2110.11659

- Applied two further, independent methods (Padé rational approximation, Bayesian reconstruction) not discussed in detail
- $T \approx 150$ MeV conclusive: $\Omega_{[r, T]} \approx F_S(r, T) \approx V_s(r)$ for $r \lesssim 0.8$ fm
- $T \lesssim 250$ MeV: all three methods yield $\Omega_{[r, T]} \gg F_S(r, T)$
- $T \approx 400$ MeV inconclusive: $\Omega_{[r, T]}^{BD} \approx F_S(r, T)$ vs $\Omega_{[r, T]}^G \approx \Omega_{[r, T]}^P \approx V_s(r)$
- All methods find for all T nontrivial $\Gamma_{[r, T]}$ that increases with r or T



Heavy quarkonium at finite temperature

- Spatial correlation functions using **relativistic heavy quarks**
 - Model-independent study of quarkonia melting/survival in LQCD
 - η_b or $\Upsilon(1S)$ until $T \approx 400$ MeV; χ_{b0} or h_b until $T \approx 350$ MeV
 - η_c or J/ψ until $T \approx 200$ MeV; χ_{c0} or χ_{c1} until $T \approx 150$ MeV
- Polyakov correlators (melting in static picture → screening length)
 - η_b or $\Upsilon(1S)$ until $T \approx 380$ MeV
 - η_c or J/ψ until $T \approx 190$ MeV
- Nonrelativistic bottomonia in lattice NRQCD
 - Extended sources or BS wave functions boost resolving power of LQCD
 - Indicate **finite thermal widths**, but no significant thermal mass shifts
- Static quarkonia ($q\bar{q}$ pair)
 - Lowest spectral feature $\{\Omega; \mathcal{O}(T)\} + \text{tail} + \text{UV continuum}$
 - Model-independent cumulant analysis → robust evidence for significant **thermal width being important for quarkonia melting**
 - Consistent with minor (Gaussian fit, Padé) or major (HTL-inspired fit) medium effects in real part → $N_\tau \leq 16$ has insufficient resolution

- So far, no conclusive statement about weak vs strong binding.
- Lattice + EFT are in good shape to deliver more robust and more realistic results needed for HIC phenomenology in the near future.

Thank you for your attention!