

Lattice determination of the heavy quark diffusion constant

Viljami Leino

Nora Brambilla,

Julian Mayer-Stedte,

Péter Petreczky,

Antonio Vairo

Technische Universität München, Brookhaven National Laboratory

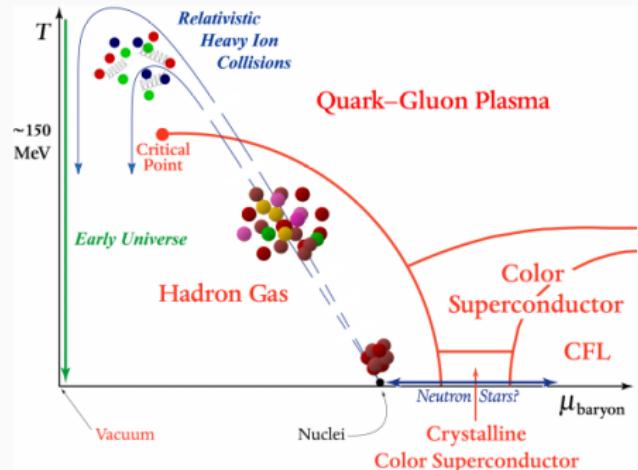


ZIMÁNYI SCHOOL 2021

06.12.2021

Introduction: QCD and QGP

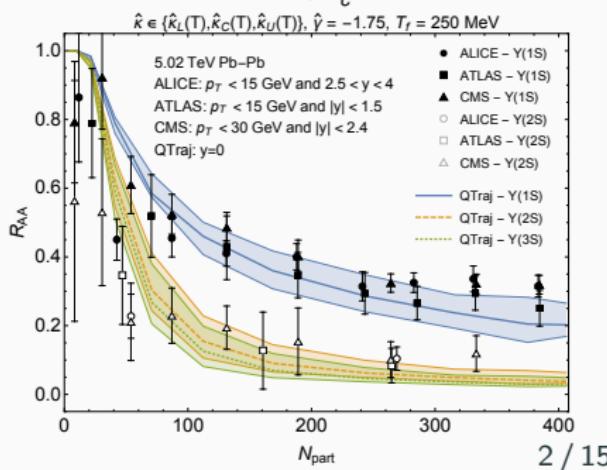
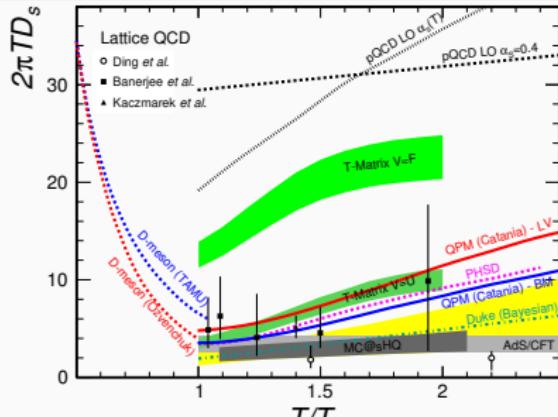
- We aim to understand the strongly coupled Quark Gluon Plasma (QGP)
- QGP generated at particle accelerators such as LHC/RHIC



- The QGP can be described in terms of transport coefficients
- In this talk we focus on the heavy quark momentum diffusion coefficient κ
- κ related to experimental quantities nuclear modification factor R_{AA} and elliptic flow v_2

Motivation for the Diffusion Coefficient

- Nuclear modification factor R_{AA} and the elliptic flow ν_2 described by spatial diffusion coefficient D_x
- Multiple theoretical models predicting wide range of values
- Non-perturbative lattice simulations needed
- κ dominant source of variation in R_{AA}



UP: X. Dong CIPANP (2018)

DOWN: N. Brambilla, M. Escobedo, M. Strickland, A. Vairo,

P. Vander Giend and J. Weber, JHEP 05 (2021) 136

Heavy Quark in medium

- Heavy quark energy doesn't change much in collision with a thermal quark

$$E_k \sim T, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium
→ Brownian motion; Follows Langevin dynamics

$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT} p_i + \xi_i(t), \quad \langle \xi(t) \xi(t') \rangle = \kappa \delta(t - t')$$

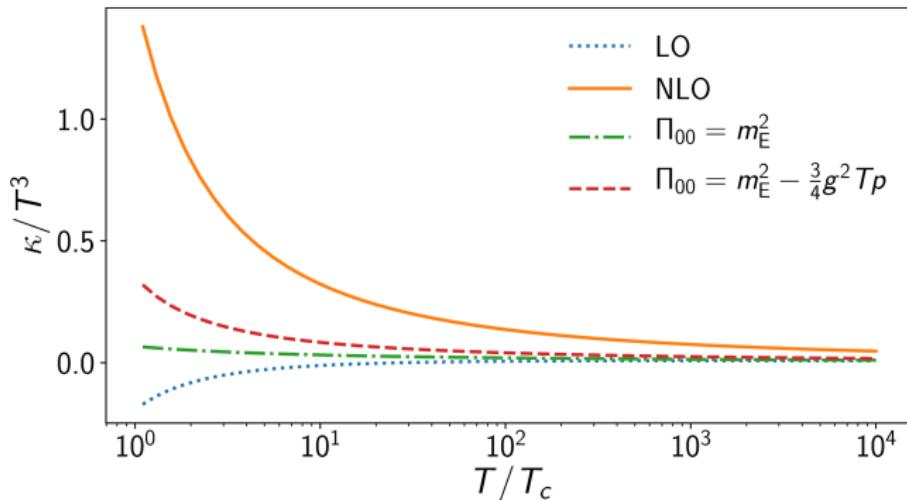
- Heavy quark momentum diffusion coefficient κ related to many interesting phenomena

Such as: Spatial diffusion coefficient $D_s = 2T^2/\kappa$,

Drag coefficient $\eta_D = \kappa/(2MT)$,

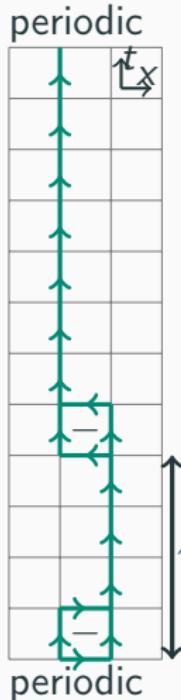
Heavy quark relaxation time $\tau_Q = \eta_D^{-1}$

κ from perturbation theory



- Clearly $m_E \ll T$ is too strict assumption on small T
- Huge perturbative variation
⇒ needs non-perturbative measurements
- Also huge scale dependence through $m_E = g(\mu)T$
- Here we have scale from NLO EQCD $\mu \sim 2\pi T$

Heavy quark diffusion from lattice: Euclidean Correlator



- Traditional approach uses HQ current-current correlators:
Problem: Transport peak at zero

- HQEFT inspired Euclidean correlator is peak free

$$G_E(\tau) = -\frac{1}{3} \sum_{i=1}^3 \frac{\langle \text{Re Tr } [U(\beta, \tau) g E_i(\tau, 0) U(\tau, 0) g E_i(0, 0)] \rangle}{\langle \text{Re Tr } [U(\beta, 0)] \rangle}$$

- Chromoelectric field E needs discretization
- On Lattice E has non-physical self-energy contribution

$$Z_E = 1 + g_0^2 \times 0.137718569 \dots + \mathcal{O}(g_0^4)$$

(Christensen and Laine PLB02 (2016))

- To get momentum diffusion coefficient κ , a spectral function $\rho(\omega)$ needs to be reversed:

$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega, T) K(\omega, \tau T), \quad K(\omega, \tau T) = \frac{\cosh\left(\frac{\omega}{T}(\tau T - \frac{1}{2})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

$$\kappa = \lim_{\omega \rightarrow 0} \frac{2T\rho(\omega)}{\omega}$$

Mass-suppressed effects to HQ diffusion

- Considering full Lorentz force:

$$F(t) = \dot{p} = q(E + v \times B)(t)$$

- $\langle v^2 \rangle \sim \mathcal{O}(\frac{T}{M})$ correction to HQ momentum diffusion

$$\kappa_{\text{tot}} \simeq \kappa_E + \frac{2}{3} \langle v^2 \rangle \kappa_B$$

- κ_B related to correlation of chromo magnetic fields:

$$G_B(\tau) = \sum_{i=1}^3 \frac{\langle \text{Re Tr } [U(1/T, \tau) B_i(\tau, 0) U(\tau, 0) B_i(0, 0)] \rangle}{3 \langle \text{Re Tr } U(1/T, 0) \rangle}$$

$$G_B(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_B(\omega, T) K(\omega, \tau T), \quad \kappa_B = \lim_{\omega \rightarrow 0} \frac{2T\rho_B(\omega)}{\omega}$$

- Same tree level expansion as G_E , NLO has divergence:

$$\rho_B = \frac{g^2 C_f \omega^3}{6\pi} \left[1 - \frac{g^2 C_A}{(4\pi)^2} \frac{2}{\varepsilon} + (\text{finite}) \right] + \mathcal{O}(g^6)$$

Multilevel procedure

- Perform multilevel simulations
- Renormalize with the LO perturbative result Z_E
- Perform tree-level improvement by matching LO lattice and continuum perturbation theories
- Normalize the data with perturbative LO result

$$G_E^{\text{norm}} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

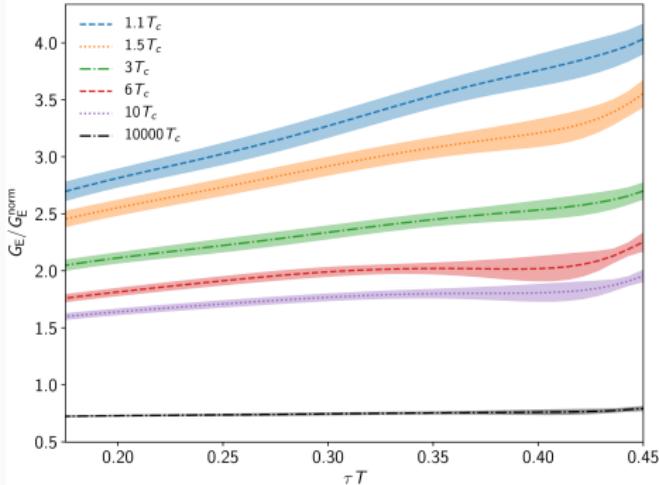
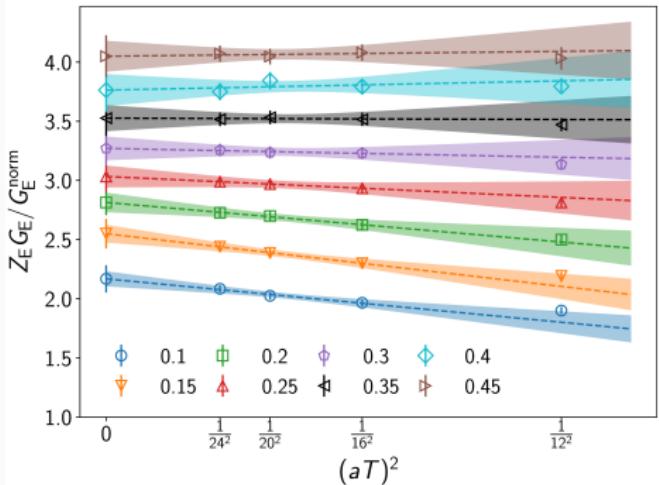
- Model the spectral function by connecting known IR and UV behavior:

$$\rho_{\text{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

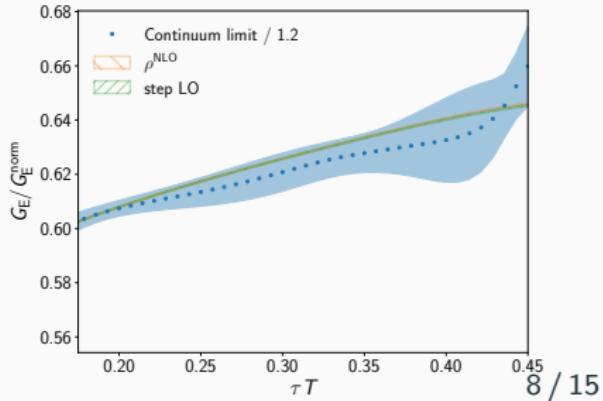
$$\rho_{\text{QCD,naive}}(\omega) = \frac{g^2 C_F \omega^3}{6\pi} \left\{ 1 + \frac{g^2}{(4\pi)^2} \left[N_c \left(\frac{11}{3} \ln \frac{\mu^2}{4\omega^2} + \frac{149}{9} - \frac{8\pi^2}{3} \right) \right] \right\}$$

- Vary κ parameter to find models that match lattice measurements

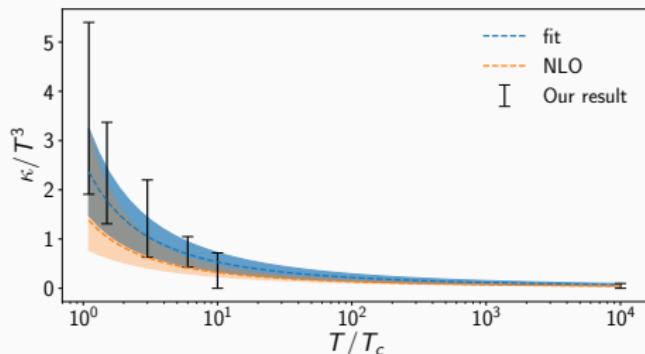
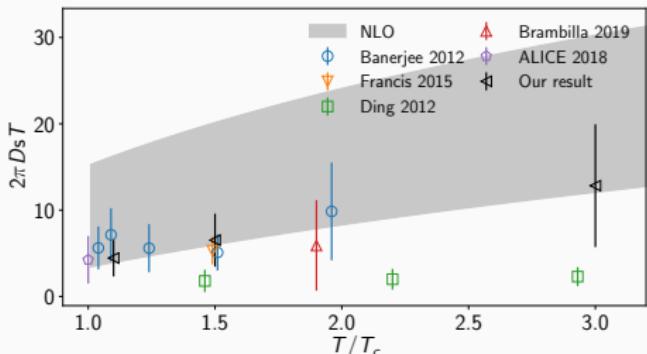
Continuum limit



- Data needs additional normalization, do this at $\tau T = 0.19$
- Great agreement to perturbation theory at very high temperatures



Multilevel results



- Recent multilevel results

Brambilla *et.al.* PRD102 (2020)

- Quenched multilevel simulations, very wide temperature range $1.1 T_c - 10^4 T_c$
- Can fit temperature dependence:

$$\frac{\kappa^{\text{NLO}}}{T^3} = \frac{g^4 C_F N_c}{18\pi} \left[\ln \frac{2T}{m_E} + \xi + C \frac{m_E}{T} \right].$$

- Other lattice studies

Meyer NJP13 (2011),

Ding *et.al.* JPG38 (2011),

Banerjee *et.al.* PRD85 (2012),

Francis *et.al.* PRD92 (2015)

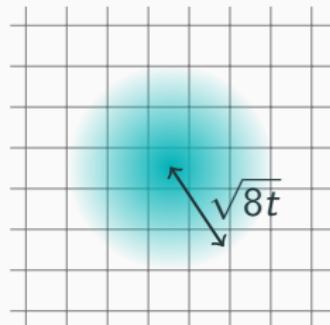
Altenkort *et.al.* PRD103 (2021)

New approach: Gradient flow

$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu},$$

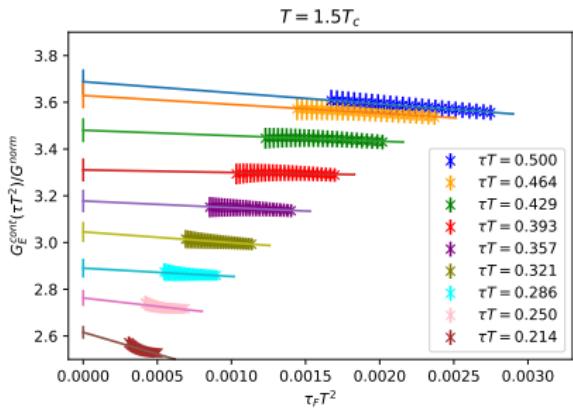
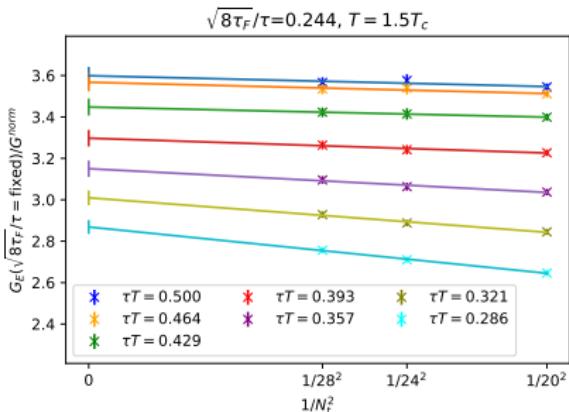
$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}].$$

$B_{0,\mu} = A_\mu$ ← the original gauge field



- Evolve gauge along fictitious time t
- Drives B_μ towards minima of S_{YM}
- Diffuses the initial gauge field with radius $\sqrt{8t}$
- We use Lüscher-Weisz action for S_{YM}
- Automatically renormalizes gauge invariant observables
- Zero flowtime limit $\mathcal{O}(x, t) \xrightarrow{t \rightarrow 0} \sum_j c_j(t) \mathcal{O}_j^R(x)$

Continuum and Zero flow time limits



- Measure $G_E(\tau_F, \tau)$, τ_F : flowtime, τ : E -field separation
- Take continuum limit
- Take zero flowtime limit.

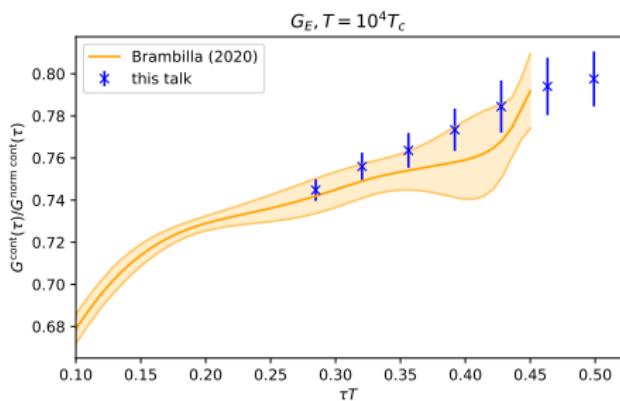
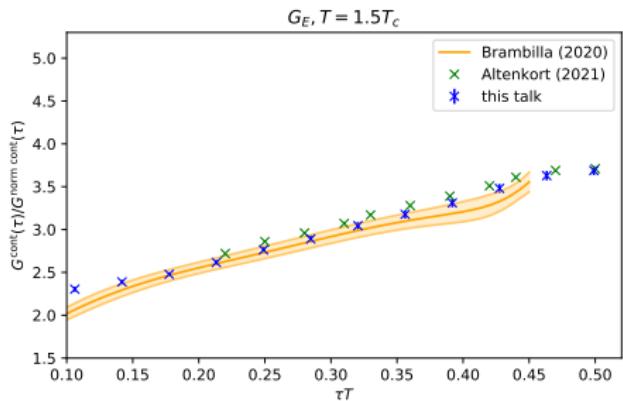
Must be taken before solving $\rho(\omega)$: ([Altenkort et.al. PRD103 \(2021\)](#))

Limit flow regime:

$$a \lesssim \sqrt{8\tau_F} \lesssim \frac{\tau - a}{3}$$

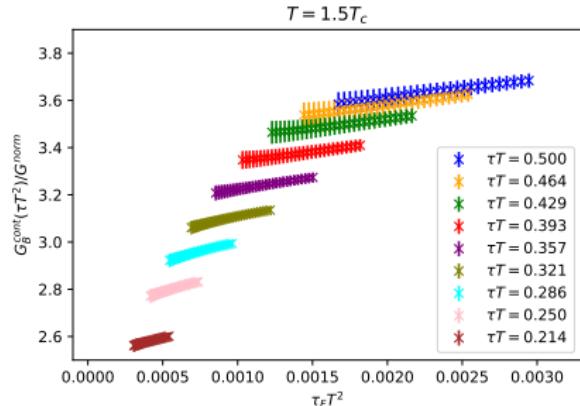
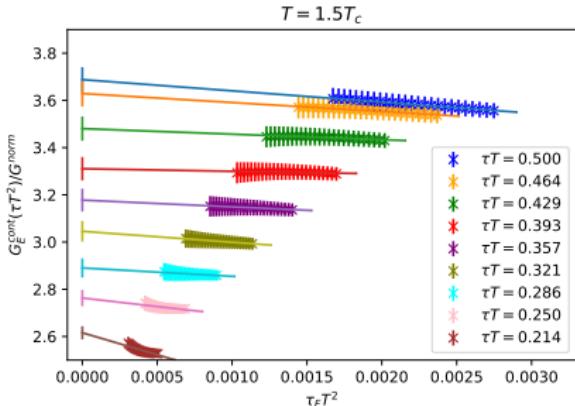
- Find κ through $\rho(\omega)$. (In this talk we focus on Euclidean correlators)

GF results for G_E



- After continuum and zero flowtime limits, we replicate the previous studies:
 - Brambilla *et.al.* PRD102 (2020)** Previous multilevel
 - Altenkort *et.al.* PRD103 (2021)** Previous Gradient flow

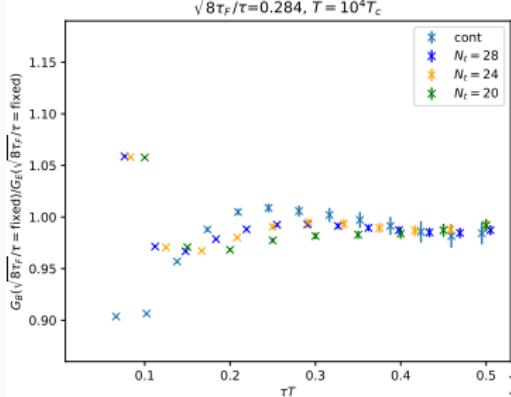
Flowtime dependence of G_E and G_B



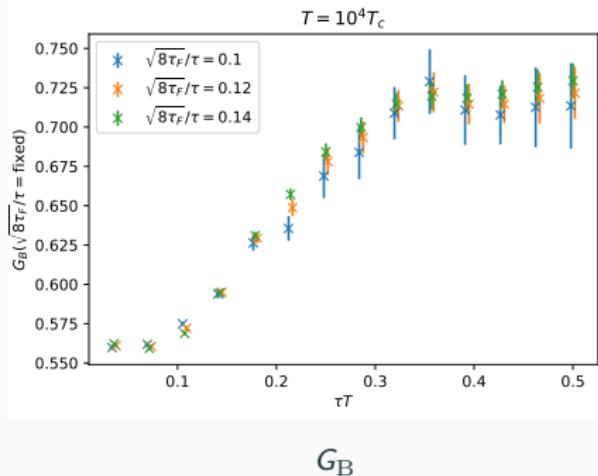
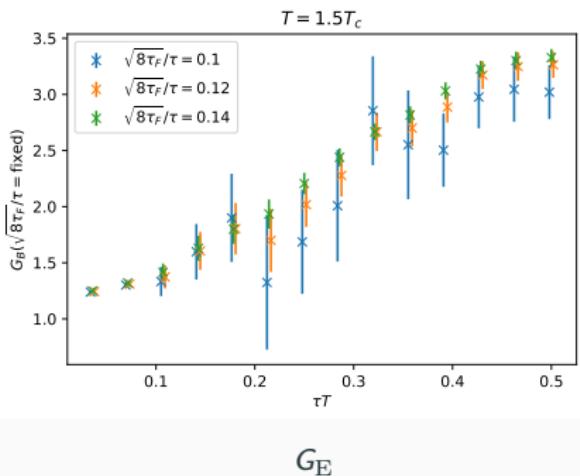
G_E

G_B

- We observe different small flow time scaling between G_E and G_B
- Possible indication of divergence or $\log(\tau_F)$ contributions



Current results for G_B



Conclusions and Future prospects

- Heavy quark momentum diffusion coefficient κ
 - Measured in wide range of temperatures
 - Fit temperature dependence
 - Agreement to perturbation theory at high T
 - Agreement to previous results at small T
- $1/M$ corrections:
 - Initial lattice result promising
 - Logs still need to be understood
- Future prospects:
 - Measure γ from our data
 - Go un-quenched
 - Check adjoint representation versions of these operators

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Thank you for your attention!