


$$g \log(2) = \lambda_g \log(2) + \nu_2(2i\pi)$$

# H(x) scaling and the pp and p $\bar{p}$ slope B(s, t)

## A Real Extended Bialas-Bzdak Model Study

based on [T. Csörgő, I. Szanyi, Eur. Phys. J. C \*\*81\*\*, 611 \(2021\)](#) and other recent results

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# Bialas-Bzdak p=(q,d) model

$$\tilde{\sigma}_{in}(\vec{b}) = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} d^2 \vec{s}_q d^2 \vec{s}'_q d^2 \vec{s}_d d^2 \vec{s}'_d D(\vec{s}_q, \vec{s}_d) D(\vec{s}'_q, \vec{s}'_d) \sigma(\vec{s}_q, \vec{s}_d; \vec{s}'_q, \vec{s}'_d; \vec{b})$$

- quark-diquark distribution inside the proton:

$$D(\vec{s}_q, \vec{s}_d) = \frac{1 + \lambda^2}{R_{qd}^2 \pi} e^{-\frac{s_q^2 + s_d^2}{R_{qd}^2}} \delta^2(\vec{s}_q + \lambda \vec{s}_d)$$

$$\begin{aligned} \vec{s}_d &= -\lambda \vec{s}_q \\ \lambda &= \frac{m_q}{m_d} \\ \vec{s}'_d &= -\lambda \vec{s}'_q \end{aligned}$$

A. Bialas, A. Bzdak Acta  
Phys.Polon. B 38, 159-168 (2007)

- interaction probability of the constituents:

$$\sigma(\vec{s}_q, \vec{s}_d; \vec{s}'_q, \vec{s}'_d; \vec{b}) = 1 - \prod_a \prod_b [1 - \sigma_{ab}(\vec{b} + \vec{s}'_a - \vec{s}_b)]$$

$$\sigma_{ab}(\vec{s}) = A_{ab} e^{-|\vec{s}|^2 / S_{ab}^2} \quad S_{ab}^2 = R_a^2 + R_b^2 \quad a, b \in \{q, d\}$$

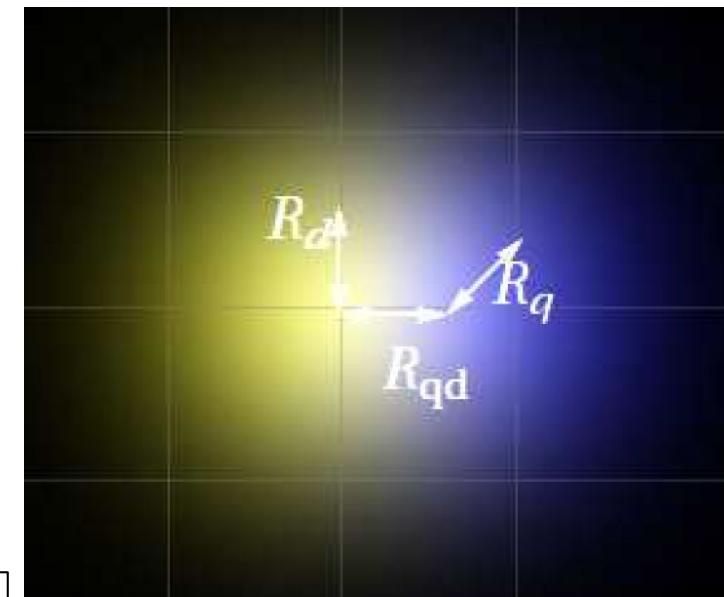
- inelastic cross-sections of quark, diquark scatterings :

$$\sigma_{ab,in} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \sigma_{ab}(\vec{s}) d^2 \vec{s}$$

$$\sigma_{qq,in} : \sigma_{qd,in} : \sigma_{dd,in} = 1 : 2 : 4$$

- free parameters:

$$A_{qq}, \lambda, R_q, R_d, R_{qd}, \quad (A_{qq} = 1 \text{ and } \lambda = 0.5 \text{ can be fixed})$$



Proton-(anti)proton scattering in the quark-diquark model (Glauber style calculation).

# Unitarily Real Extended Bialas-Bzdak (ReBB) model

- elastic scattering amplitude in the impact parameter space:

$$t_{el}(s, \vec{b}) = i [1 - e^{-\Omega(s, \vec{b})}]$$

arXiv:1505.01415

F. Nemes, T. Csörgő, M. Csanád, Int. J. Mod. Phys. A Vol. 30 (2015) 1550076

- the opacity function:

$$\Omega(s, \vec{b}) = Re\Omega(s, \vec{b}) + i Im\Omega(s, \vec{b})$$

$Im\Omega \neq 0$  as the real part of the amplitude is not negligibly small

$$Re\Omega(s, \vec{b}) = -\frac{1}{2} \ln[1 - \tilde{\sigma}_{in}(s, \vec{b})]$$

$$Im\Omega(s, \vec{b}) = -\alpha \tilde{\sigma}_{in}(s, \vec{b})$$



NEW FREE PARAMETER

- elastic scattering amplitude in momentum space:

$$T(s, t) = 2\pi \int_0^\infty t_{el}(s, |\vec{b}|) J_0(|\vec{\Delta}| |\vec{b}|) |\vec{b}| d|\vec{b}|$$

$$|\vec{\Delta}| \equiv \sqrt{-t} \quad \text{as } \sqrt{s} \rightarrow \infty$$

( $t$  is the squared momentum transfer)

# Measurable quantities

- differential cross section:

$$\frac{d\sigma}{dt}(s, t) = \frac{1}{4\pi} |T(s, t)|^2$$

- total, elastic and inelastic cross sections:

$$\sigma_{tot}(s) = 2Im T(s, t = 0)$$

$$\sigma_{el}(s) = \int_{-\infty}^0 \frac{d\sigma(s, t)}{dt} dt$$

$$\sigma_{in}(s) = \sigma_{tot}(s) - \sigma_{el}(s)$$

- ratio  $\rho_0$ :

$$\rho_0(s) = \lim_{t \rightarrow 0} \rho(s, t) \equiv \frac{Re T(s, t \rightarrow 0)}{Im T(s, t \rightarrow 0)}$$

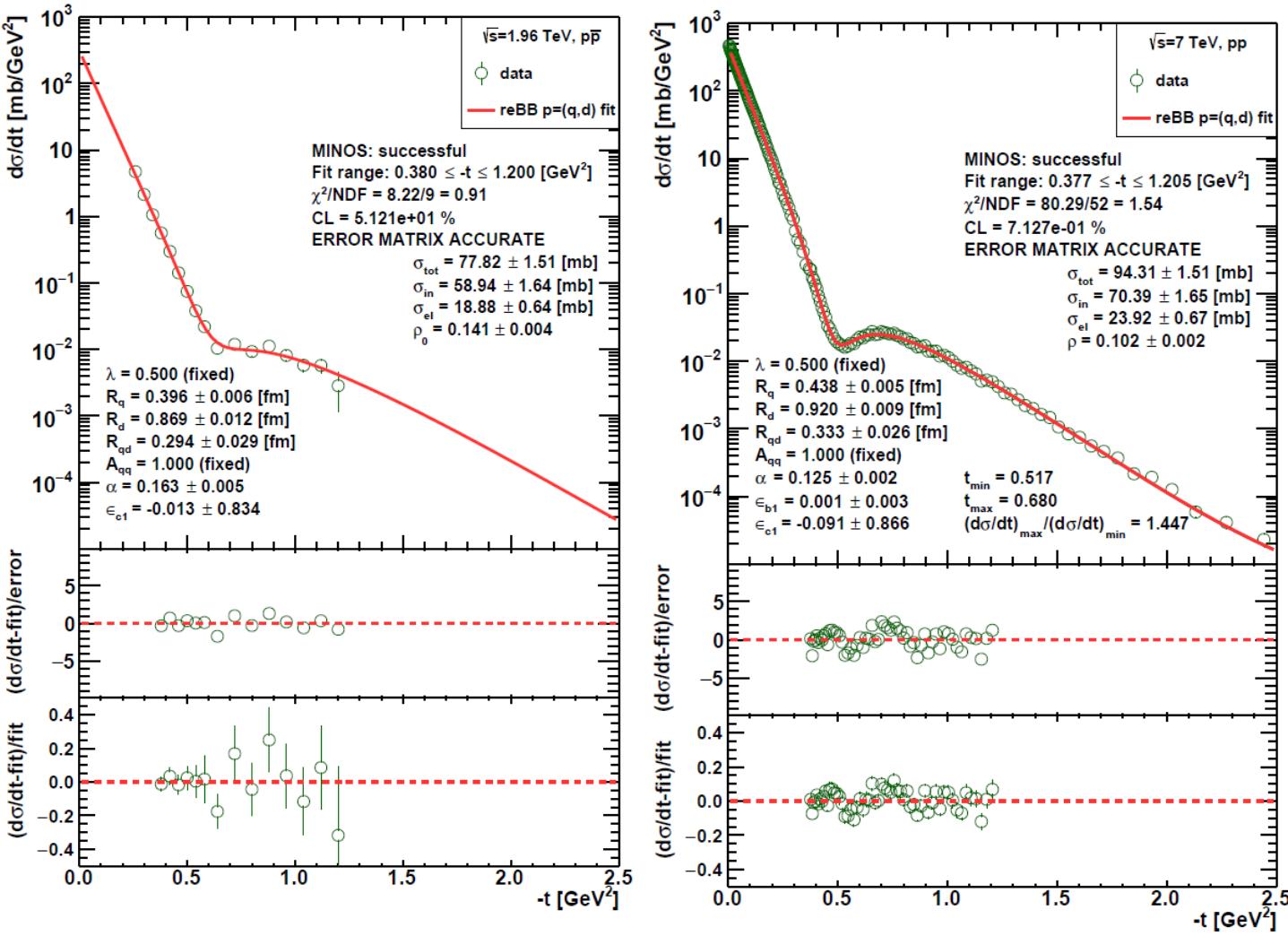
- slope of  $d\sigma/dt$ :

$$B(s, t) = \frac{d}{dt} \left( \ln \frac{d\sigma}{dt}(s, t) \right)$$

$$B_0(s) = \lim_{t \rightarrow 0} B(s, t)$$

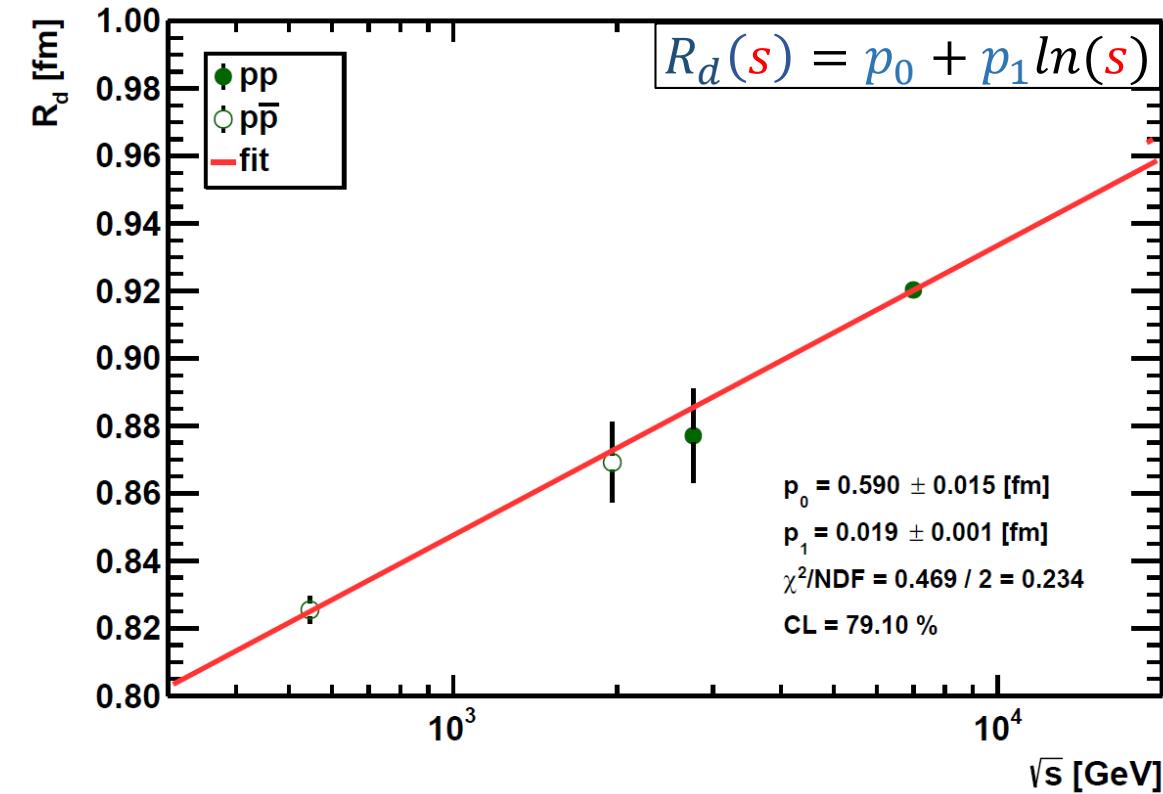
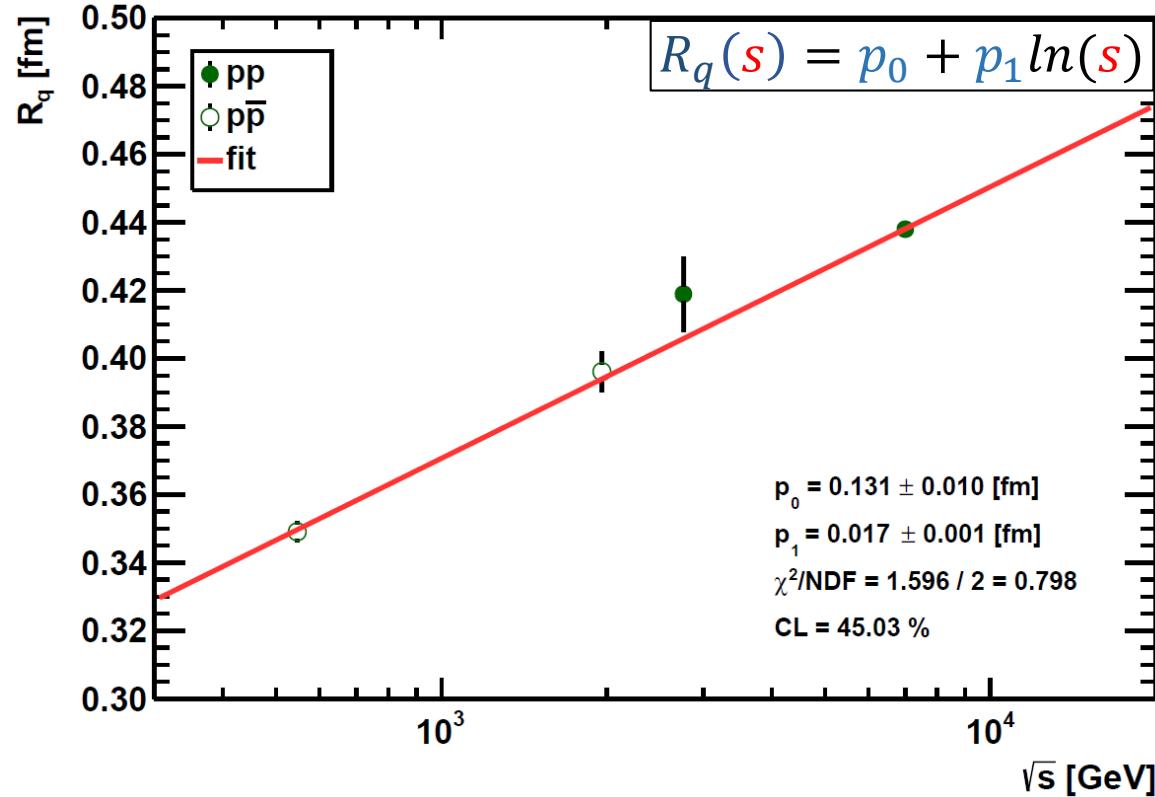
# ReBB model analysis of pp and p $\bar{p}$ data

- fits for pp  $d\sigma/dt$  data at 2.76 TeV and 7 TeV and for p $\bar{p}$   $d\sigma/dt$  data at 0.546 TeV and 1.96 TeV
- use of the  $\chi^2$  definition developed by PHENIX
- determination of the energy dependences of the model parameters
- satisfactory description in the kinematical range:  $0.546 \leq \sqrt{s} \leq 7$  TeV &  $0.37 \leq -t \leq 1.2$  GeV $^2$
- observation of an at least  $7.08\sigma$  Odderon signal



Examples of ReBB model fits for pp and p $\bar{p}$  differential cross section data.

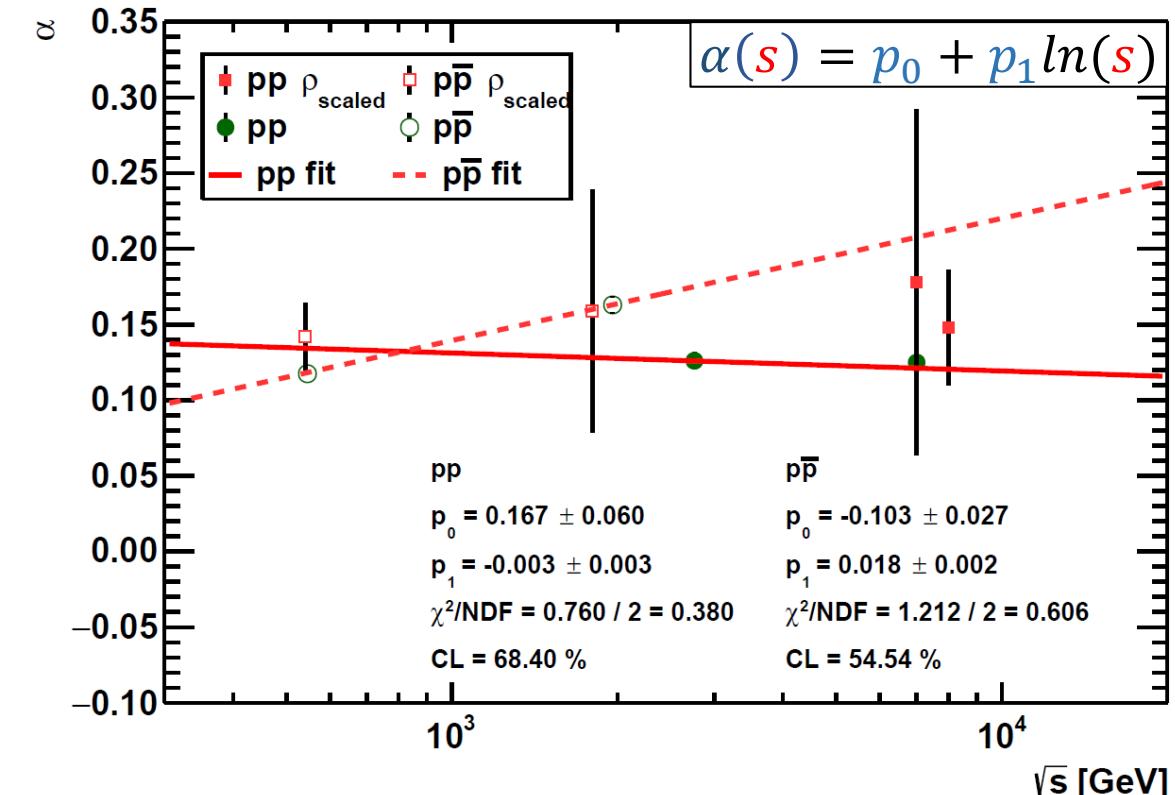
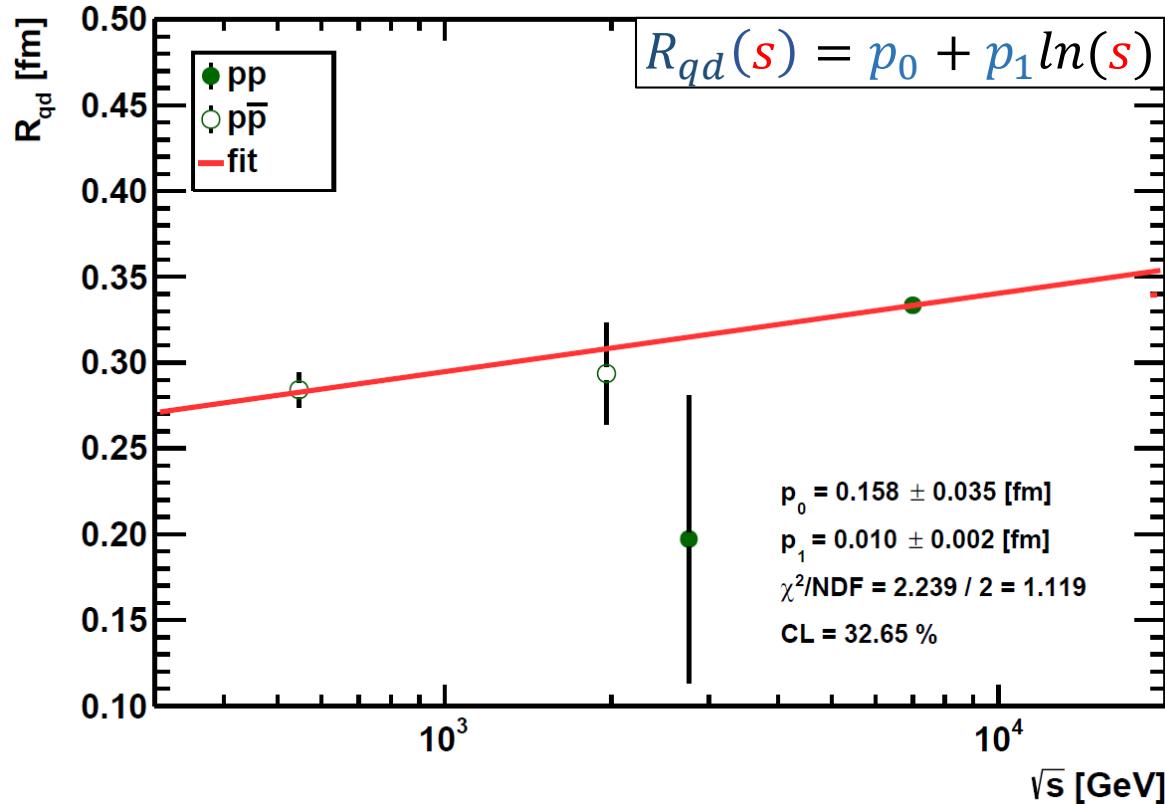
# Energy dependences of the model parameters



The energy dependences of the scale parameters,  $R_q(s)$ ,  $R_d(s)$ , and  $R_{qd}(s)$  are linear logarithmic and the same for  $\text{pp}$  and  $\text{p}\bar{\text{p}}$  processes!

The energy dependence of the  $\alpha$  parameter,  $\alpha(s)$  is linear logarithmic too, but not the same for  $\text{pp}$  and  $\text{p}\bar{\text{p}}$  processes!

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# Proportionality between $\rho_0(s)$ and $\alpha(s)$

$$t_{el}(s, b) = i \left( 1 - e^{i \alpha \tilde{\sigma}_{in}(s, b)} \sqrt{1 - \tilde{\sigma}_{in}(s, b)} \right)$$

$$\alpha \tilde{\sigma}_{in} \ll 1$$

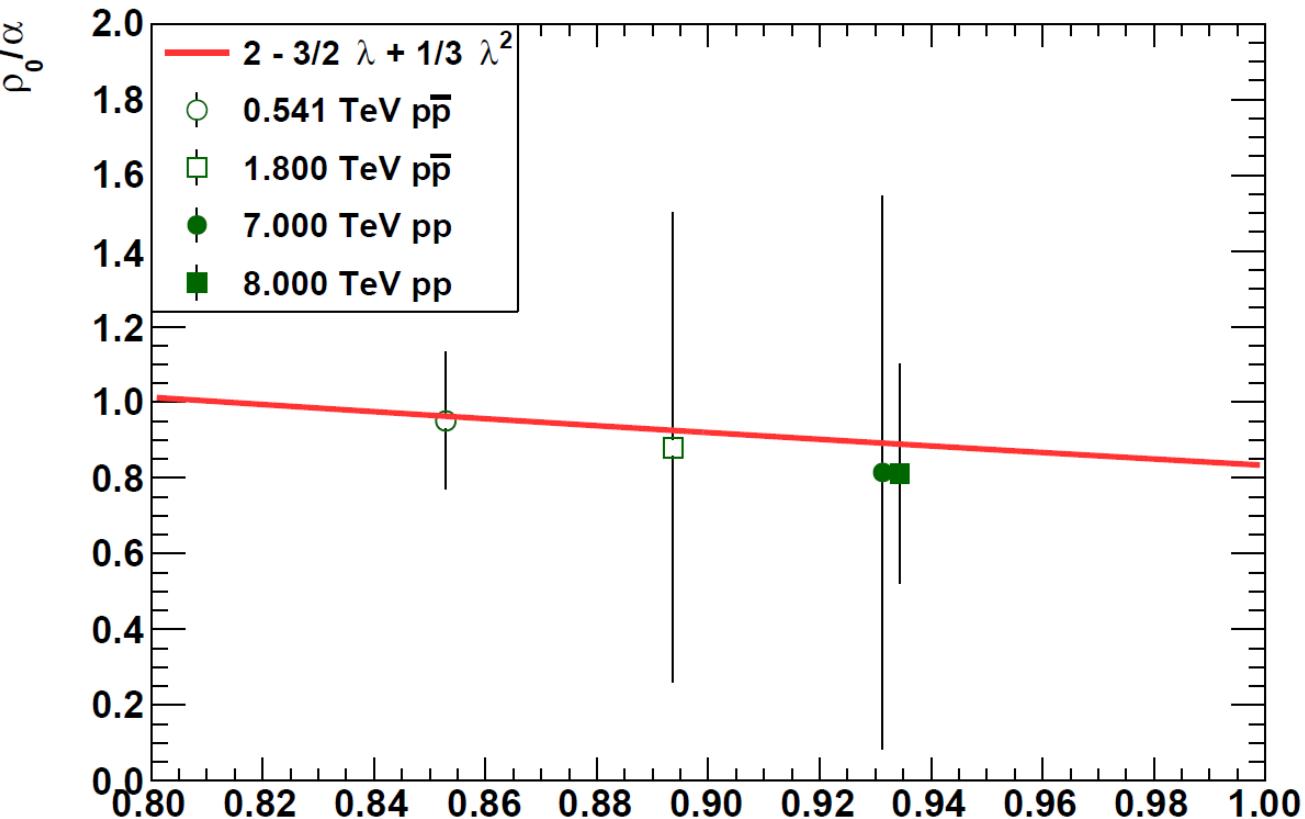
$$\text{Im } t_{el}(s, b) \simeq \lambda(s) \exp \left( -\frac{b^2}{2R^2(s)} \right)$$



$$\rho_0(s) = \alpha(s) \left( 2 - \frac{3}{2} \lambda(s) + \frac{1}{3} \lambda^2(s) \right)$$

$$\lambda(s) = \text{Im } t_{el}(s, b = 0)$$

→ by rescaling one can get additional  $\alpha$  parameter values at energies where  $\rho_0$  is measured (and vice versa)



The dependence of  $\rho_0/\alpha$  on  $\lambda = \text{Im } t_{el}(s, b = 0)$  in the TeV energy range. The data points are generated numerically by using the trends of the ReBB model scale parameters and the experimentally measured  $\rho$ -parameter values.

# $H(x)$ scaling of the ReBB model

- the  $H(x)$  scaling is present in the energy range  $\sqrt{s_1} \leq \sqrt{s} \leq \sqrt{s_2}$  if the

$$H(x, s) = \frac{1}{B_0(s)\sigma_{el}(s)} \left. \frac{d\sigma_{el}}{dt} \right|_{x=-tB_0(s)}$$

scaling function is energy independent in that range i.e.  $H(x, s_1) = H(x, s_2)$ .

- conditions for ReBB model  $H(x)$  scaling to be present:

- energy independence for the  $\alpha$  parameter (or  $\rho_0$ )

$$\alpha(s) = \alpha(s_0) \quad (\text{or} \quad \rho_0(s) = \rho_0(s_0) \quad \text{since } \alpha \sim \rho_0)$$

- the energy dependence of the scale parameters is determined by the same factorizable  $b(s)$  scaling function

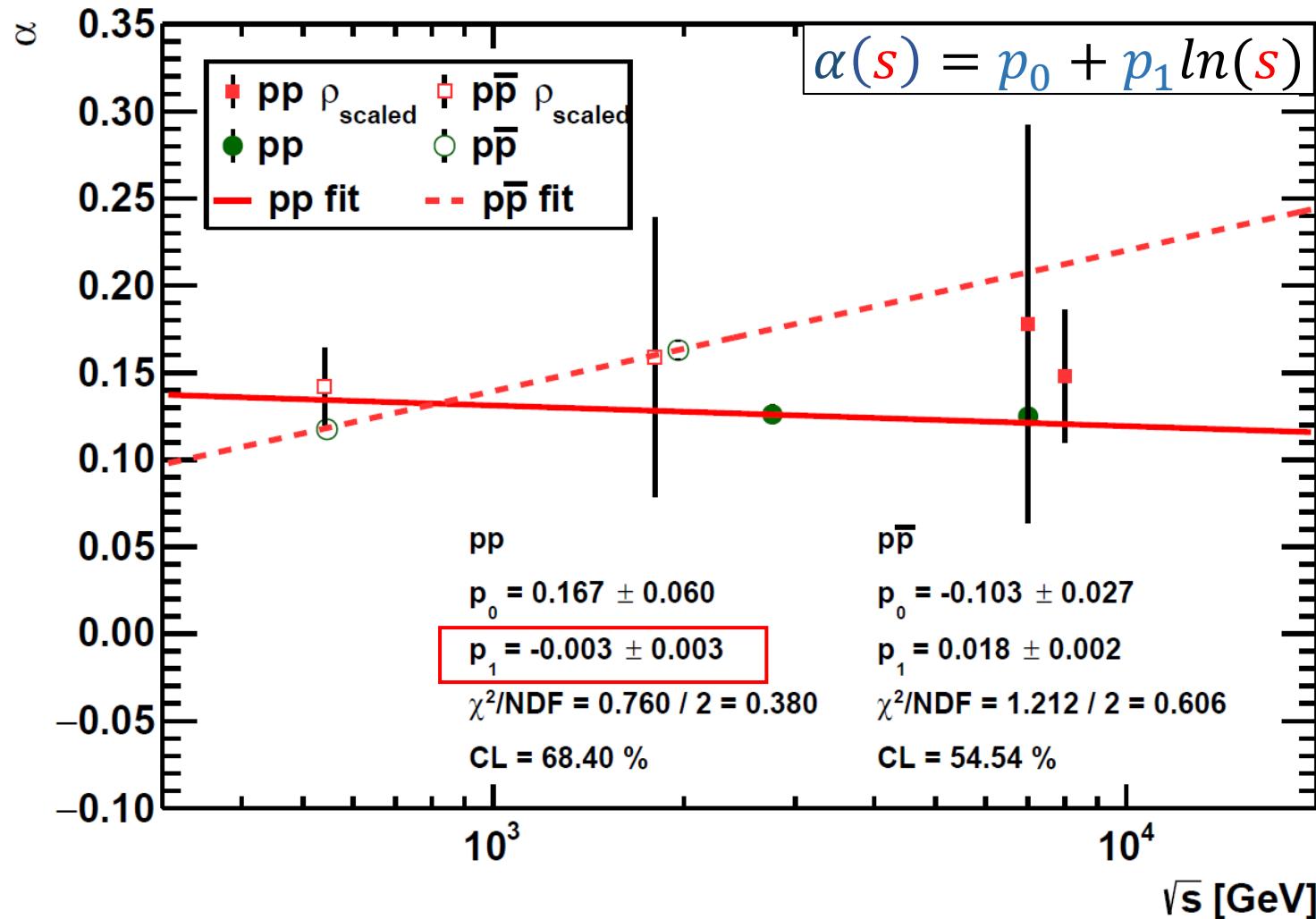
$$R_q(s) = b(s)R_q(s_0)$$

$$R_d(s) = b(s)R_d(s_0)$$

$$R_{qd}(s) = b(s)R_{qd}(s_0)$$

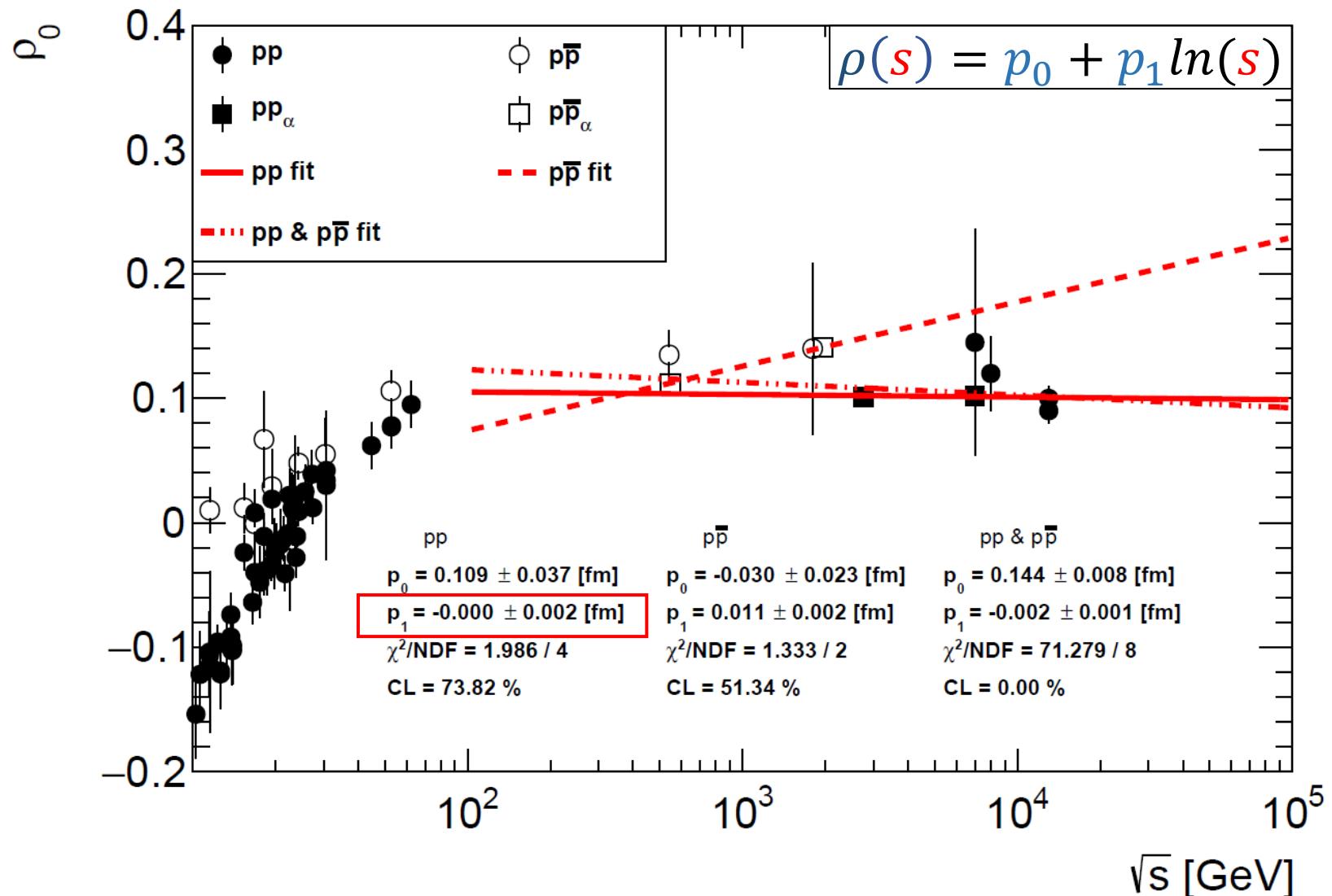
( $\sqrt{s_0}$  is a reference energy to be chosen)

# Energy dependence of $\alpha$



$\alpha(s)$  is constant within errors for pp in the few TeV energy range

# Energy dependence of $\rho_0$



$\rho_0(s)$  is constant within errors for pp in the TeV energy range

# b(s) scaling function

- scaling of the measurables:

$$\frac{d\sigma}{dt}(s, t) = b^2(s) \frac{d\sigma}{dt} \left( s_0, t_0 = \frac{t}{b^2(s)} \right)$$

$$\sigma_{el}(s) = b^2(s) \sigma_{el}(s_0)$$

$$B_0(s) = b^2(s) B_0(s_0)$$

$$\sigma_{tot}(s) = b^2(s) \sigma_{tot}(s_0)$$

- experimental determination of  $b(s)$ :

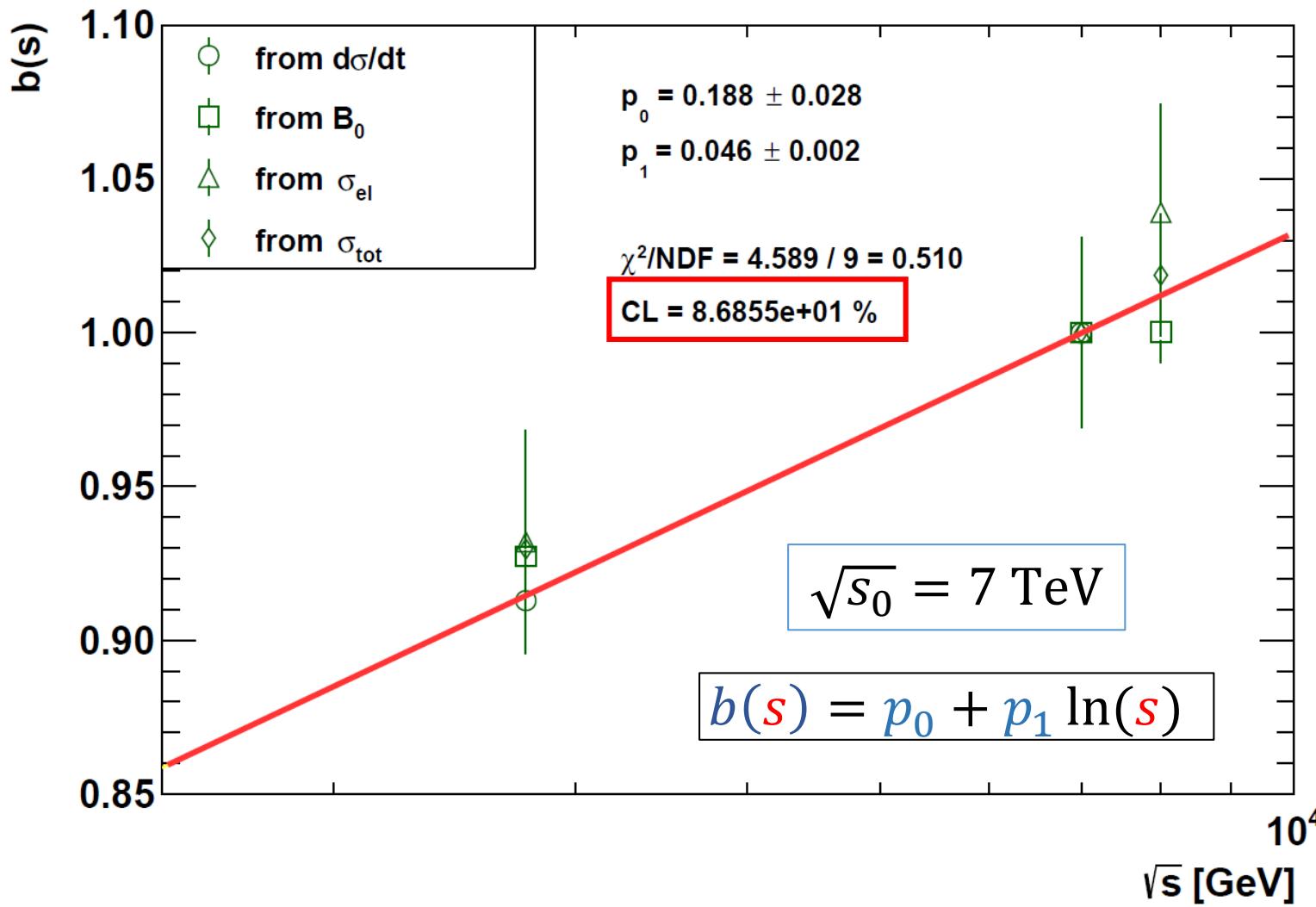
$$b(s) = \sqrt{\frac{d\sigma/dt(s, t)}{d\sigma/dt(s_0, t_0 = t/b^2(s))}}$$

$$b(s) = \sqrt{\frac{B_0(s)}{B_0(s_0)}}$$

$$b(s) = \sqrt{\frac{\sigma_{el}(s)}{\sigma_{el}(s_0)}}$$

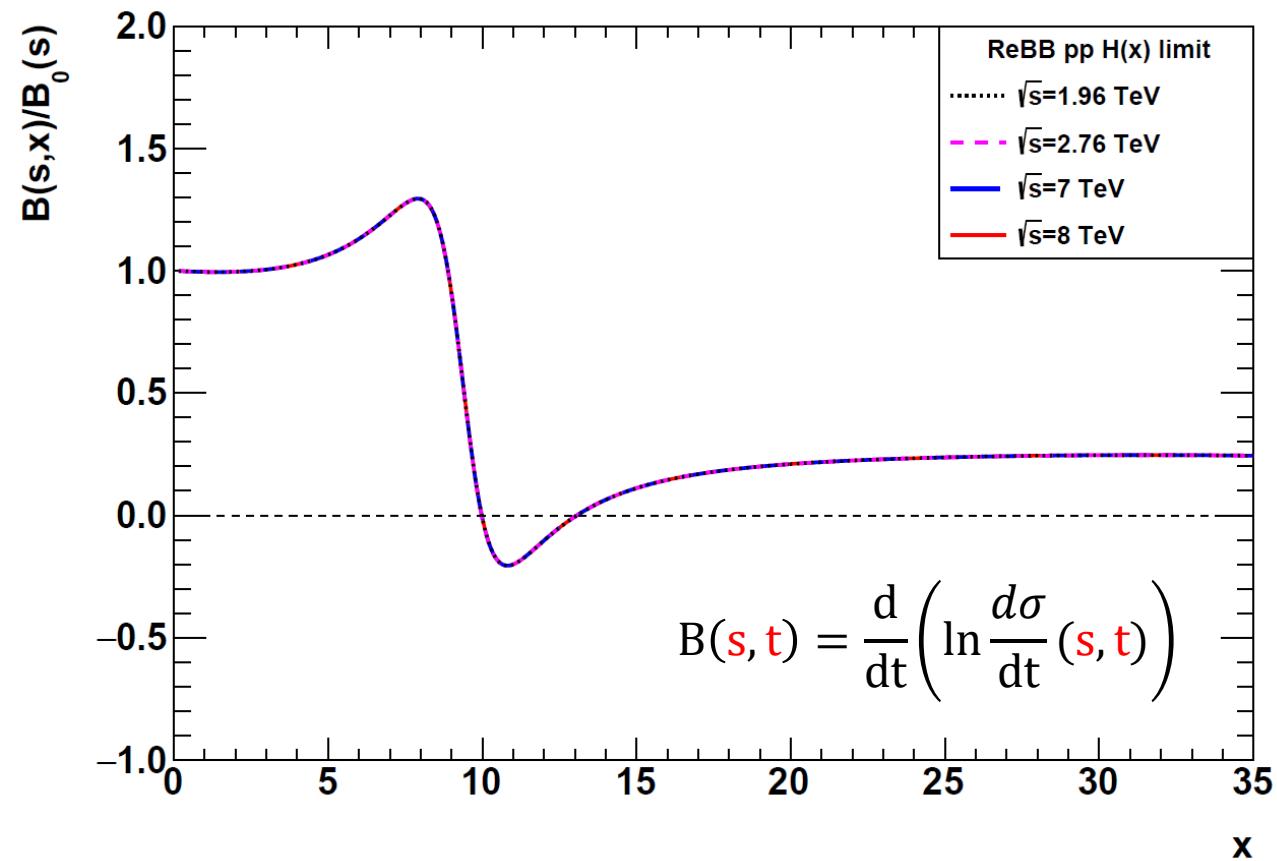
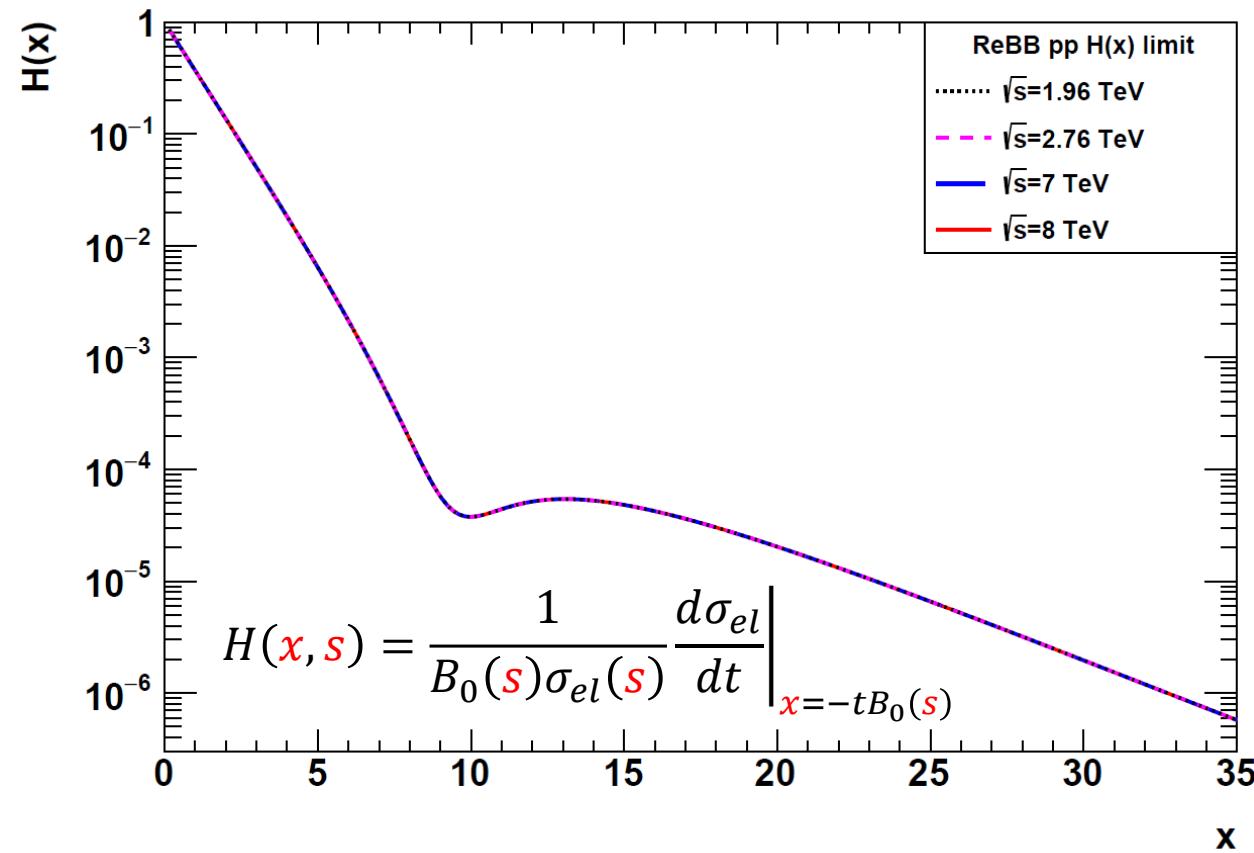
$$b(s) = \sqrt{\frac{\sigma_{tot}(s)}{\sigma_{tot}(s_0)}}$$

# Energy dependence of the $b(s)$ scaling function



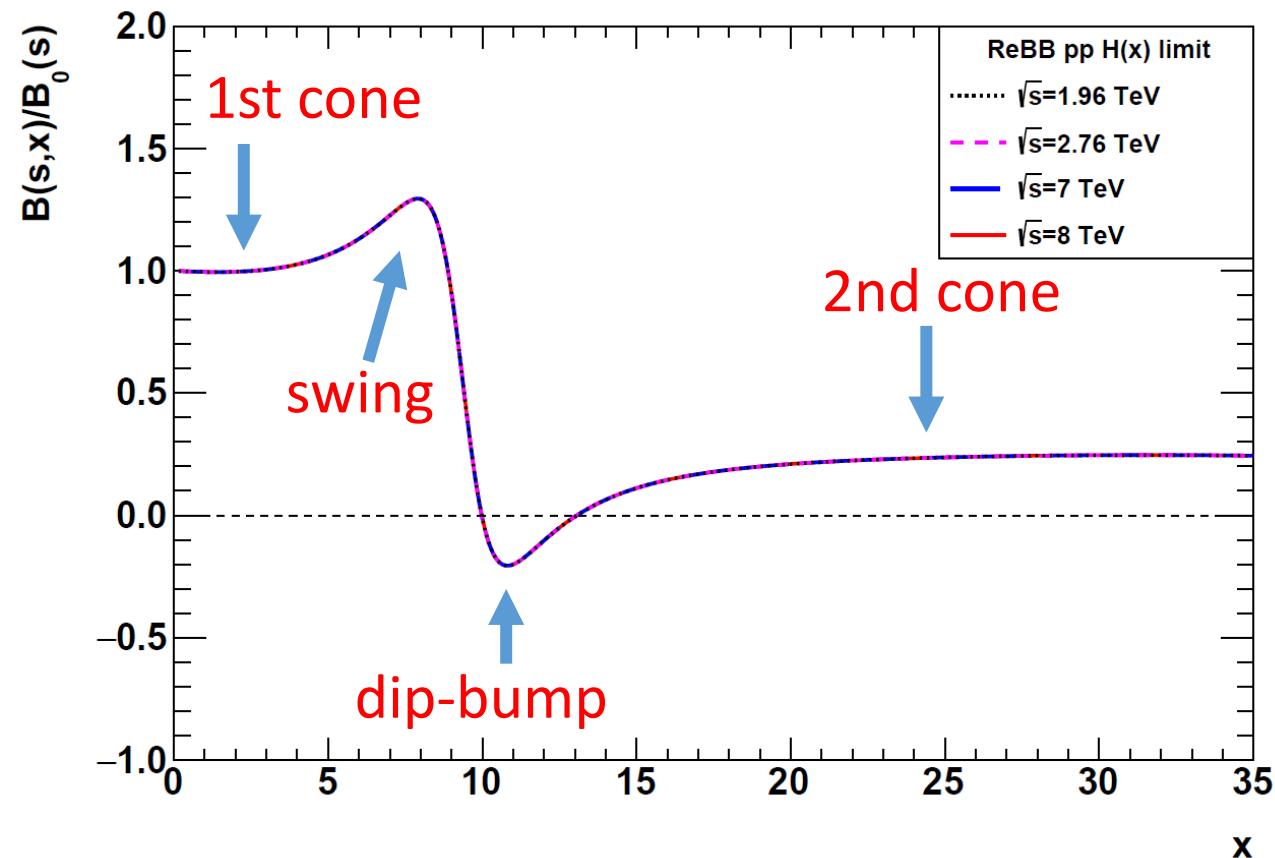
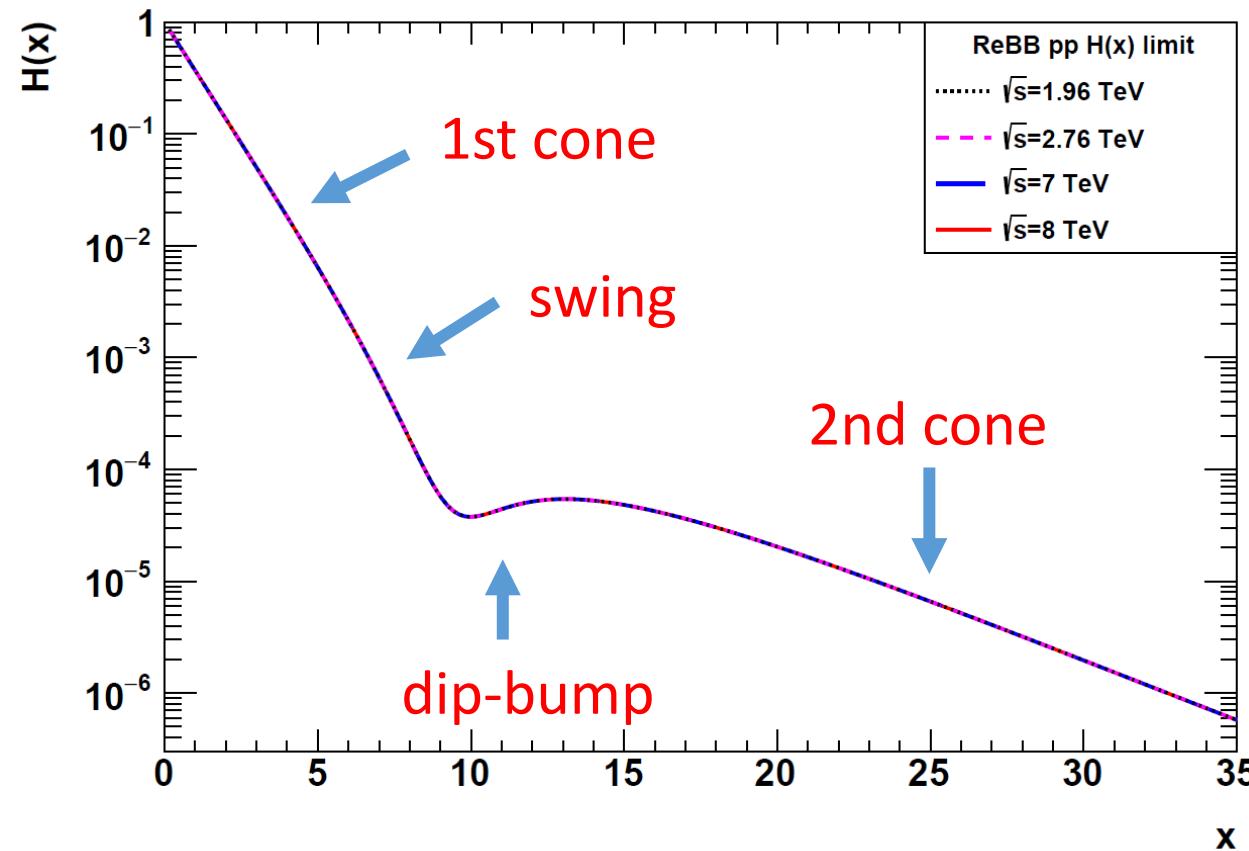
measurables scale with the same  $b(s)$  within errors in the TeV energy range

# $H(x)$ & $B(s, x)/B_0(s)$ in the $H(x)$ -ReBB model



$H(x)$  and the ratio  $\frac{B(s,x)}{B_0(s)}$  is  $x$  dependent but energy independent if  $H(x)$  scaling holds

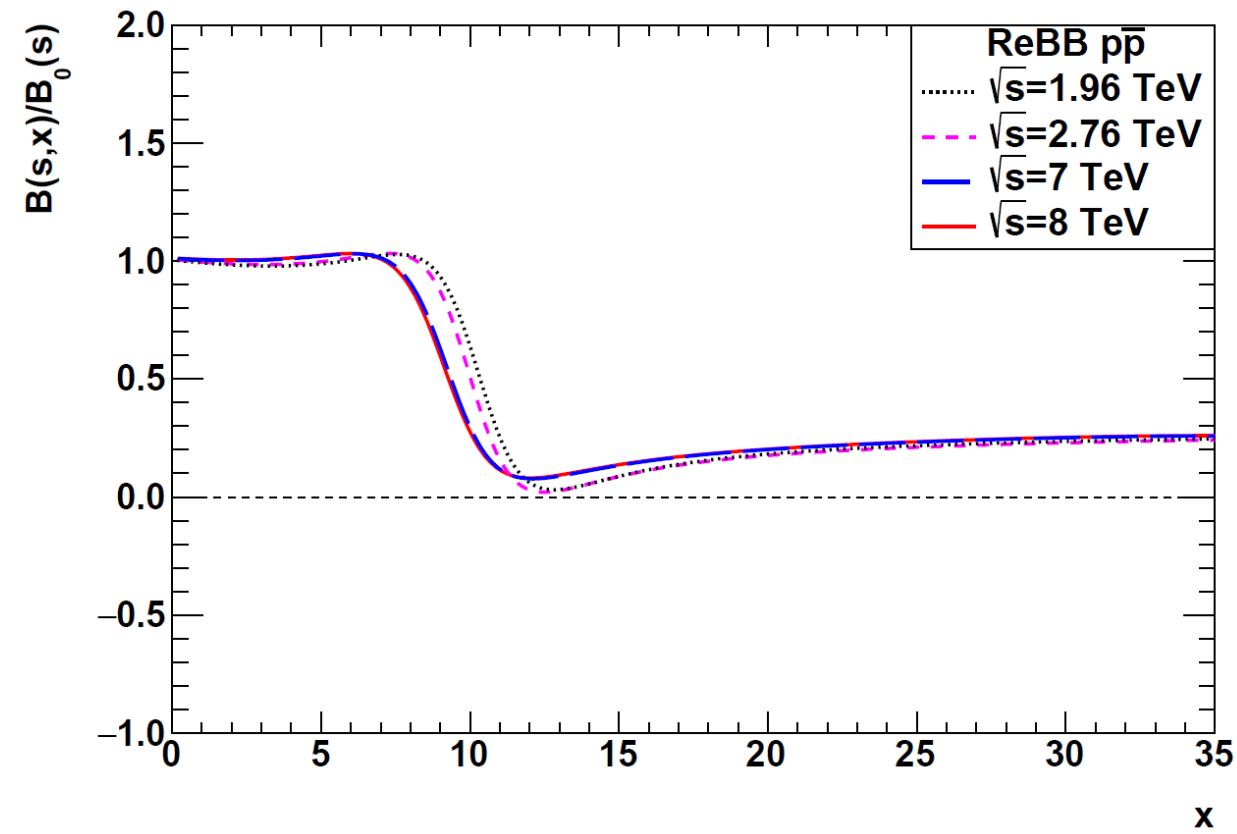
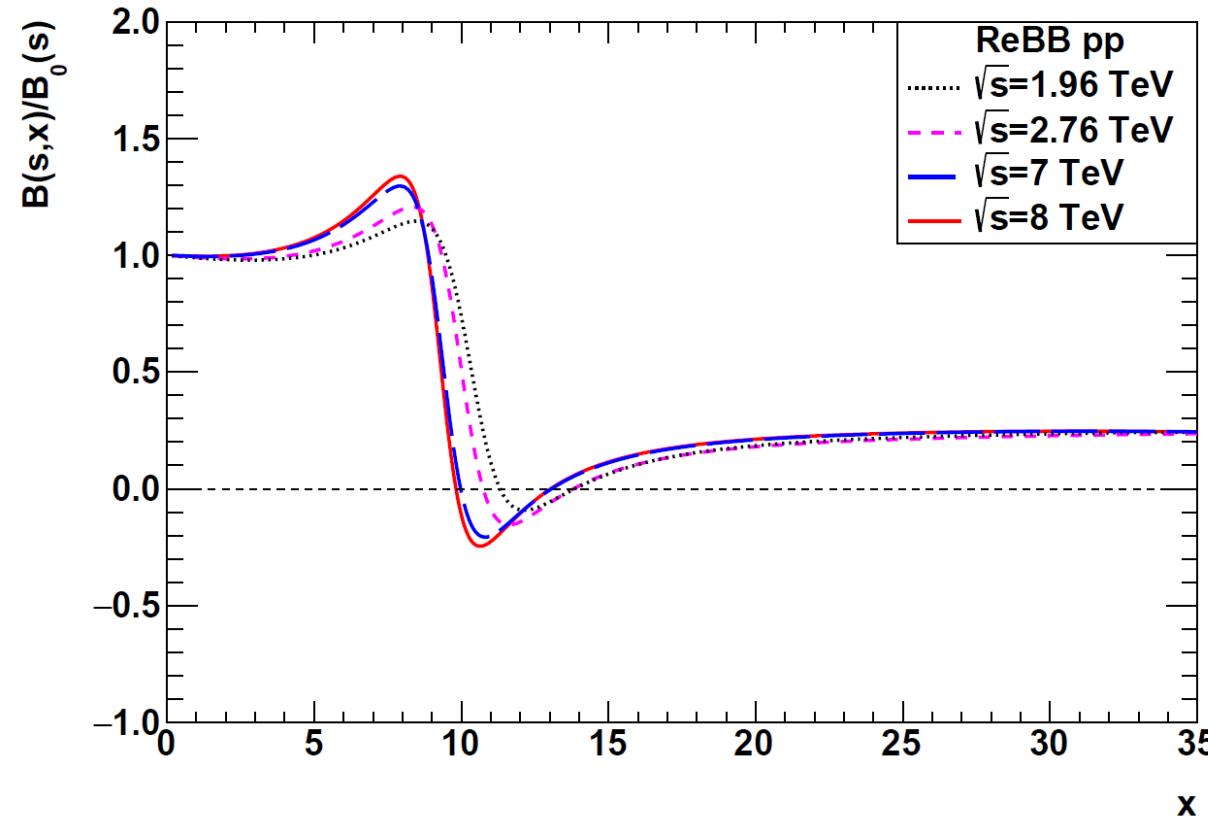
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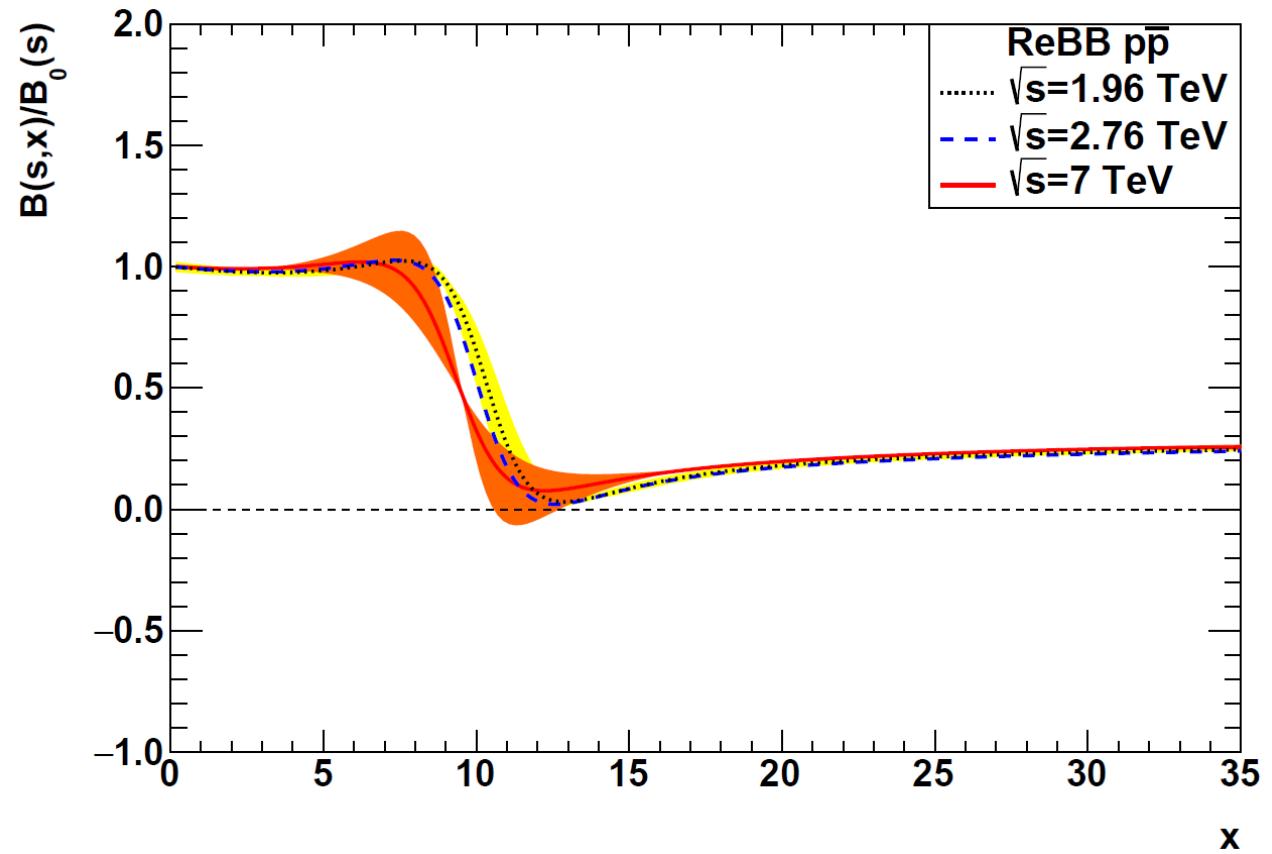
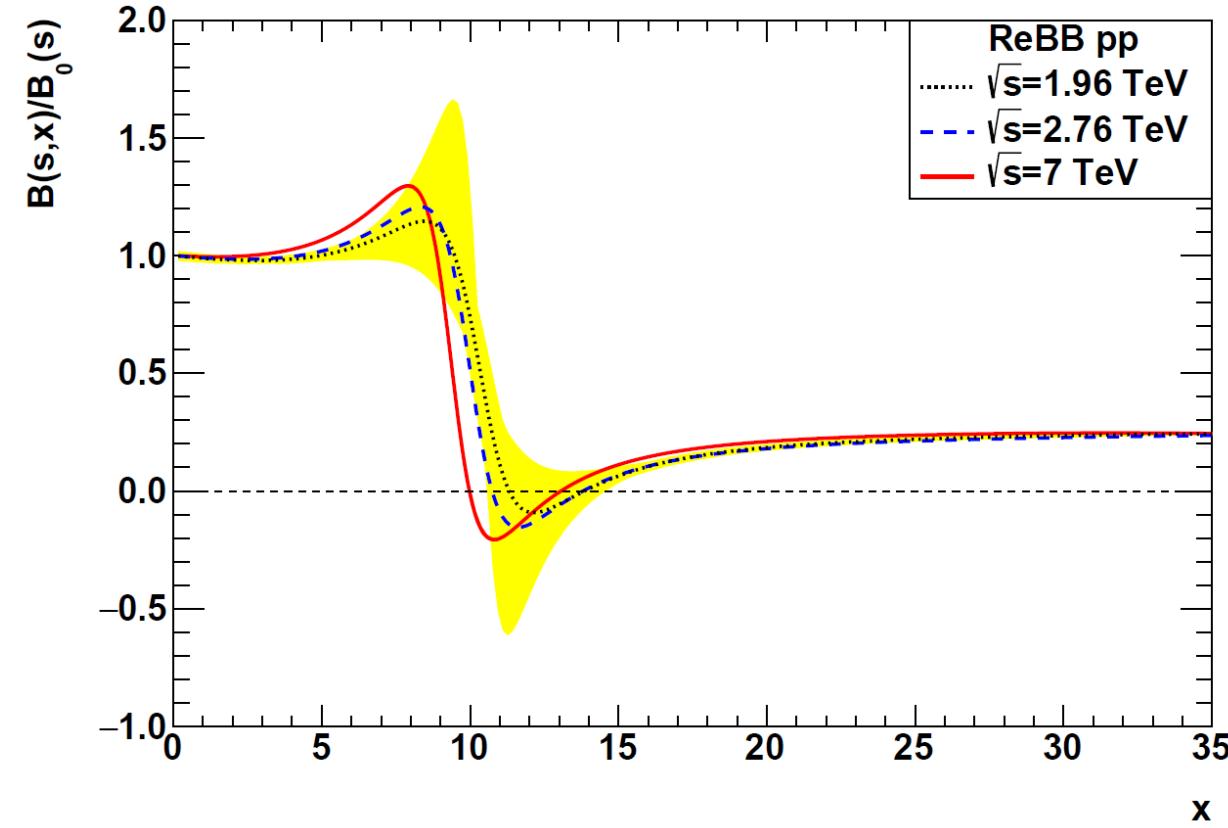
# $B(s, x)/B_0(s)$ in the ReBB model

similar qualitative results from model-independent Lévy imaging: T. Csörgő, R. Pasechnik, A. Ster, Eur. Phys. J. C 79, 62 (2019)



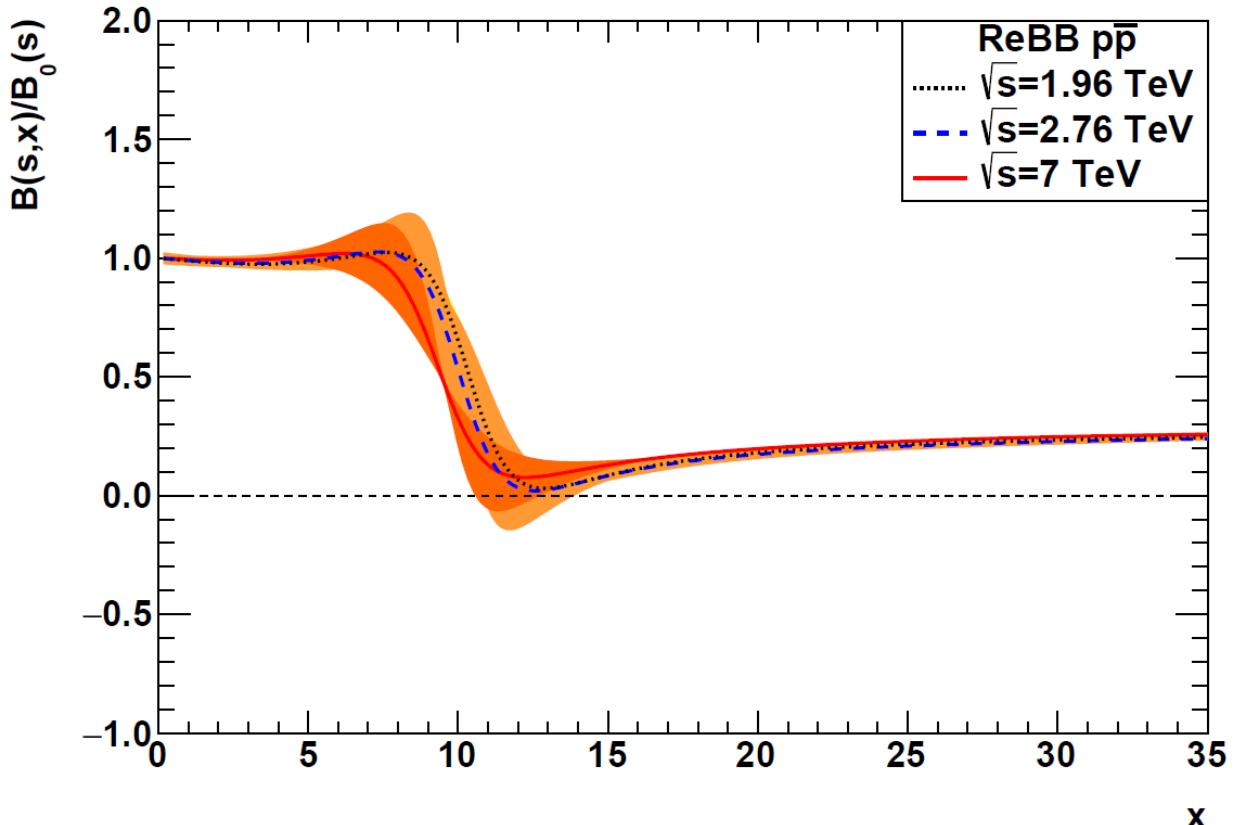
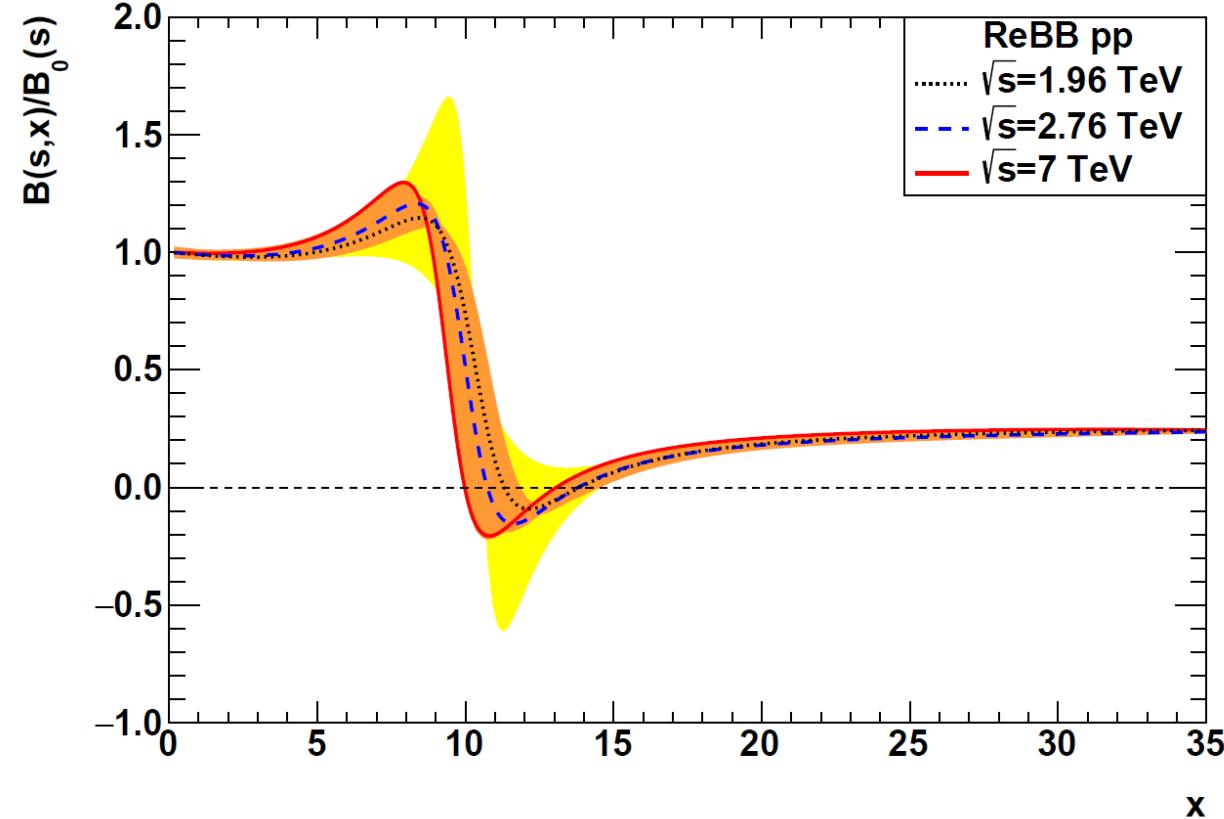
in the original ReBB model the ratio  $\frac{B(s,x)}{B_0(s)}$  is not completely energy independent,  
i.e. small scaling violations are present

# Error estimation for $B(s, x)/B_0(s)$ in the ReBB model



very small scaling violations are present between 7 and 1.96 TeV in the ReBB model even if the estimated systematic errors are considered

# Error estimation for $B(s, x)/B_0(s)$ in the ReBB model



no scaling violations are present between 7 & 2.76 TeV and between 2.76 and 1.96 in the ReBB if the estimated systematic errors are considered

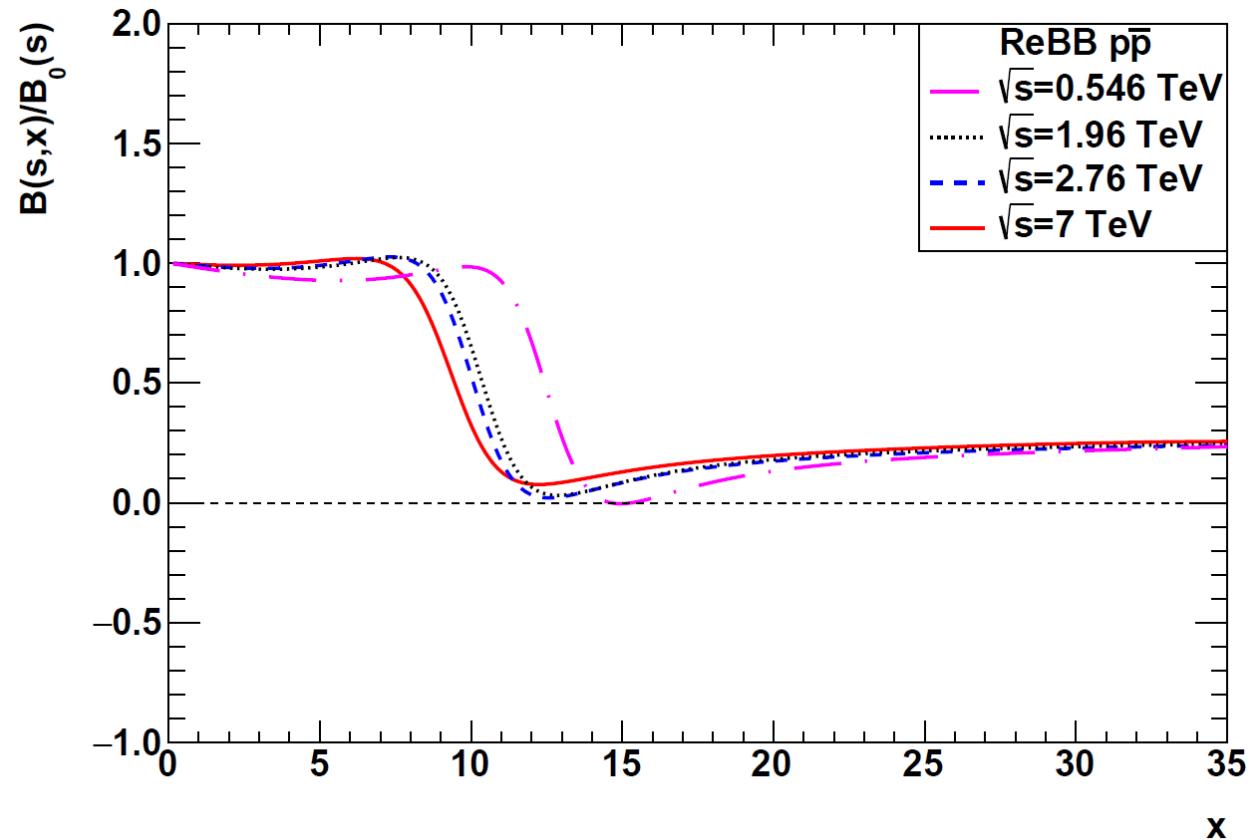
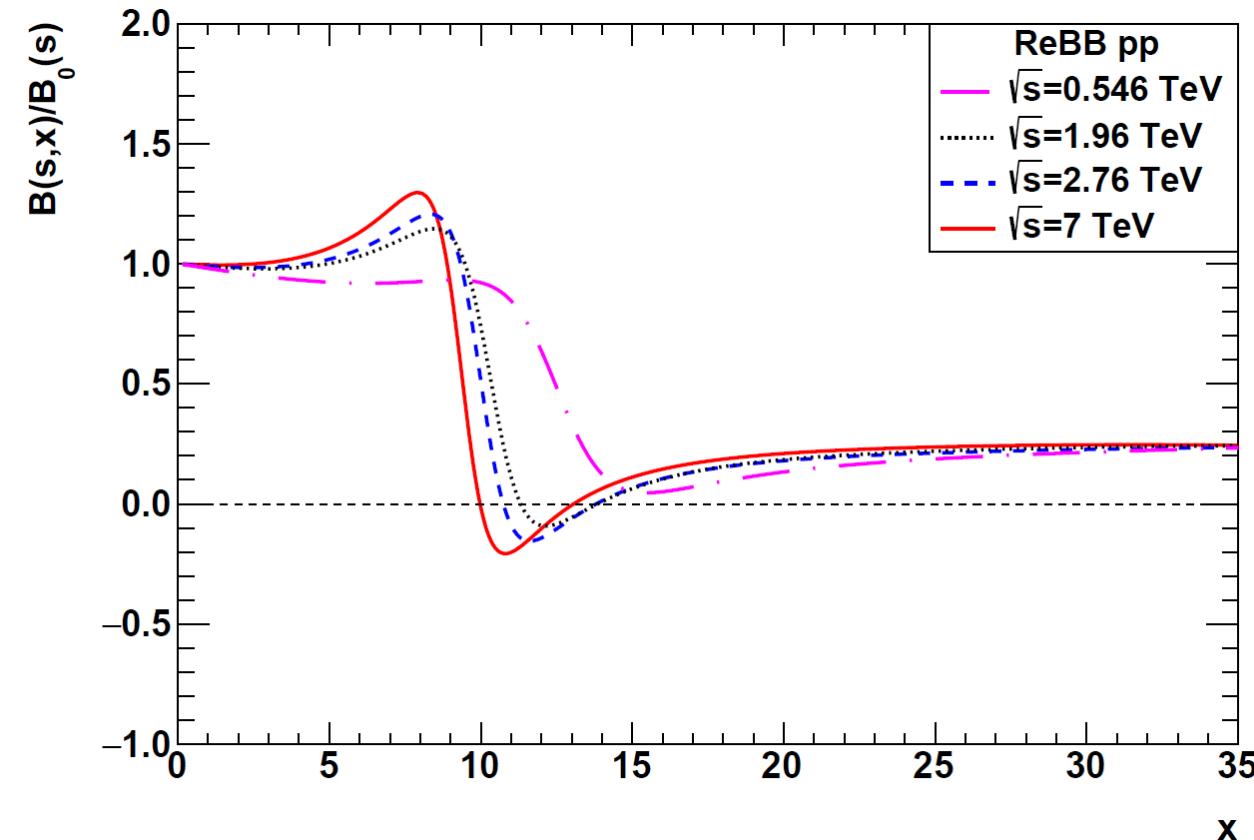
# Summary

- the ReBB model gives satisfactory description for  $p\bar{p}$  and  $p\bar{p}$  data in the kinematical ranges  $0.546 \leq \sqrt{s} \leq 7 \text{ TeV}$  and  $0.37 \leq -t \leq 1.2 \text{ GeV}^2$
- the ReBB model manifests  $H(x)$  scaling if the parameters  $R_q$ ,  $R_d$ , and  $R_{qd}$  have the same factorizable energy dependence and the parameter  $\alpha$  is energy independent
- the conditions for  $H(x)$  scaling is satisfied for  $p\bar{p}$  scattering in the few TeV energy range (and presumably also for  $p\bar{p}$  but new measurements would be needed to test)
- $H(x)$  and the ratio  $B(s, x)/B_0(s)$  are energy independent if  $H(x)$  scaling is present
- the original ReBB model shows small  $H(x)$  scaling violations in the few TeV energy range which almost vanish within the estimated systematic errors

Thank you for your attention!

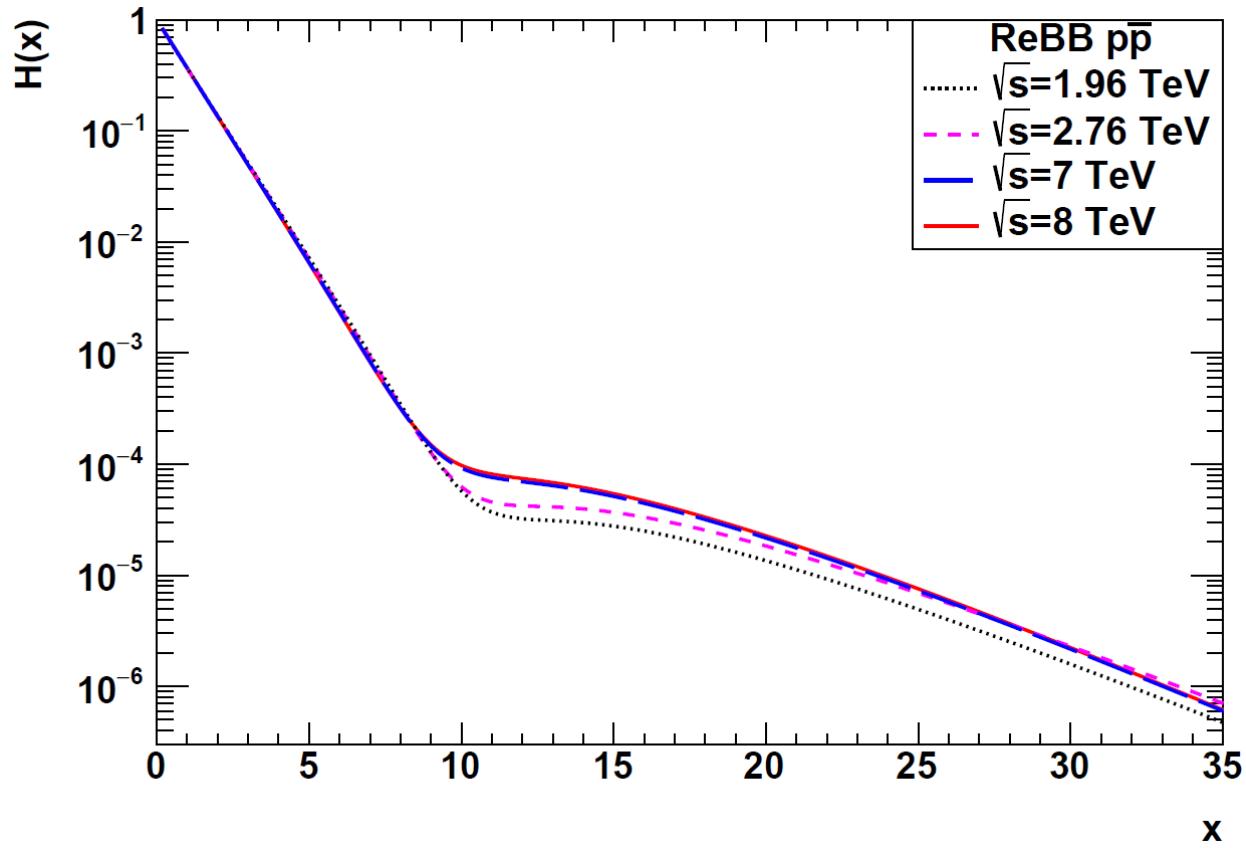
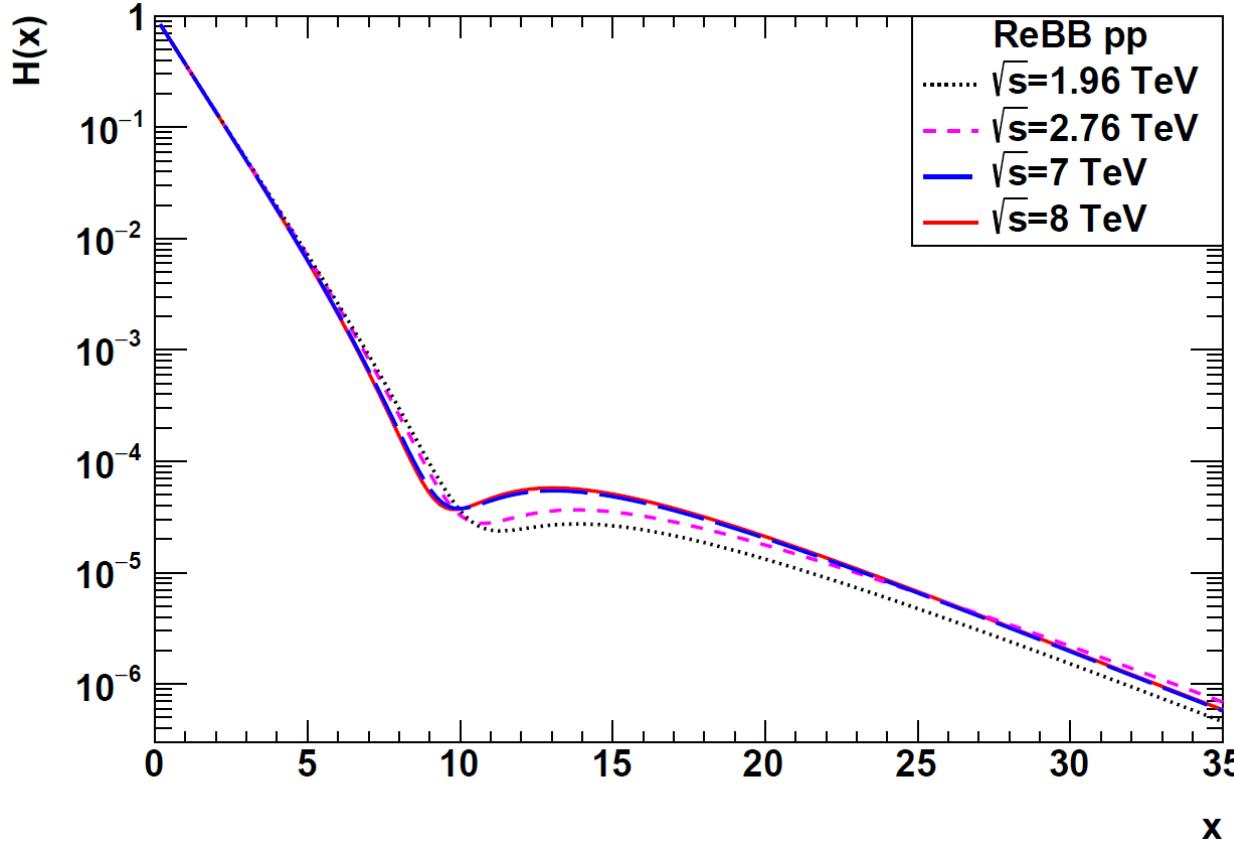
# Backup slides

# $B(s, x)/B_0(s)$ in the ReBB model 0.546 TeV included



scaling violations in ReBB model become more significant at  $\sqrt{s} < 1 \text{ TeV}$

# $H(x)$ in the ReBB model



in the original ReBB model the ratio  $H(x)$  is not completely energy independent,  
i.e. small scaling violations are present (error estimation is required)