A quasiparticle model from a novel Relaxation Time Approximation to the relativistic Boltzmann equation

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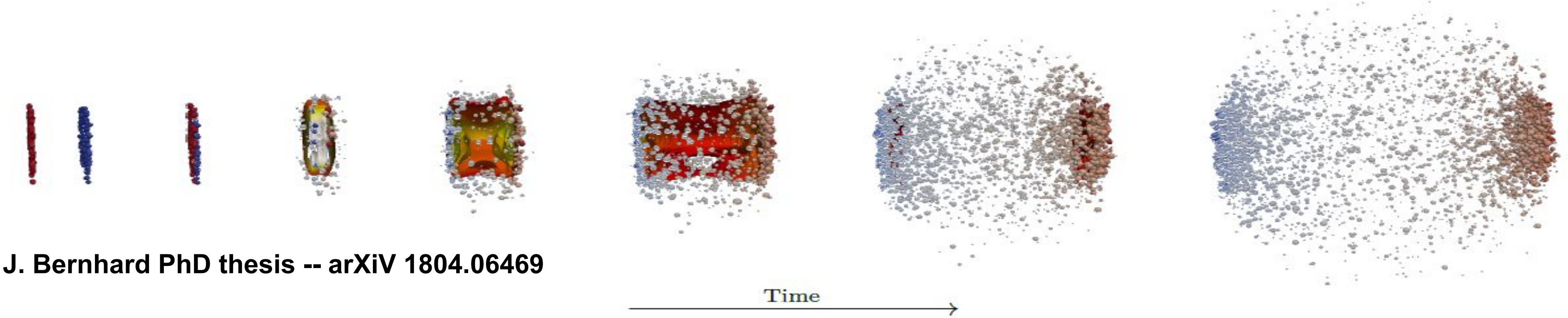
Introduction

- With ultra-relativistic Heavy Ion Collisions, nuclear matter in extreme conditions can be produced and studied;

- Particular interest has been deposited in the computation of transport coefficients.

- Convoluted problem, many approaches: pQCD, Arnold *et al* JHEP11 (2000) 001; Arnold, et al PRD **74**, 085021 (2011)
  LQCD, Nakamura, Sakai PRL. **94**, 072305; Meyer Phys. Rev. Lett. **100**, 162001 *quasi-particle models in Kinetic th*. Jeon (1995) PRD 52 3591-3642; Romantschke *PRD*, *85*(6), 065012, 2012; Bluhm et al PRC 84:025201, 2011
  (...)



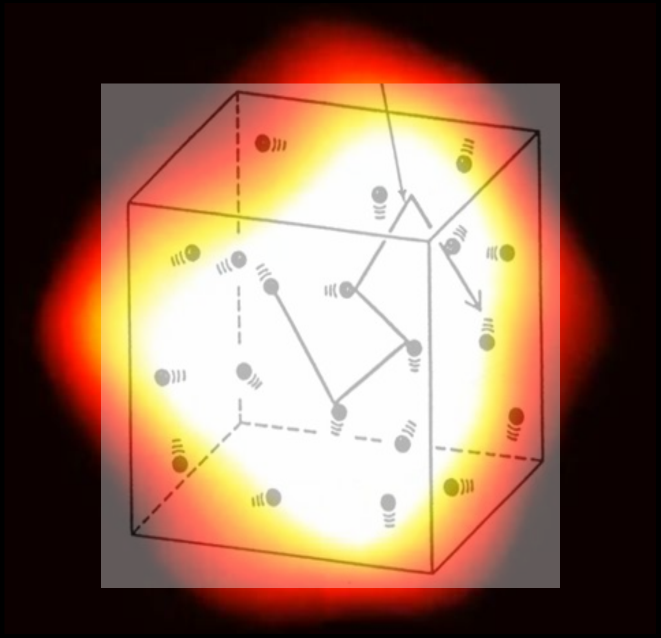
J. Bernhard PhD thesis -- arXiV 1804.06469

Boltzmann equation w/ thermal mass

- The main equation from kinetic theory is the relativistic Boltzmann equation (Most common form)

$$p^\mu \partial_\mu f_p = C[f] = \int dQ\, dQ'\, dP' \tilde{\mathcal{W}}_{pp' \leftrightarrow qq'} (f_p f_{p'} - f_q f_{q'}),$$

Simulation made with JETSCAPE code



- One of the main features of (quasi-)particles in a medium is the acquisition of a T,μ-dependent mass. Implementation: Vlasov-like term

$$p^\mu \partial_\mu f_{\mathbf{p}} + \frac{1}{2} \partial_i M^2(T) \partial^i_{(\mathbf{p})} f_{\mathbf{p}} = C[f_{\mathbf{p}}],$$

Jeon & Yaffe (1996) *PRD*, *53*(10), 5799;
Jeon (1995) PRD 52 3591-3642
Calzetta, E., & Hu, B. L. (1988). *PRD 37*(10), 2878.85,

Boltzmann equation w/ thermal mass

- Redefinition for the EM tensor -- recover consistency with conservation laws

$$T^{\mu\nu} = \int dP\; p^{\mu} p^{\nu} f_p \quad \mapsto \quad T^{\mu\nu} = \int dP\; p^{\mu} p^{\nu} f_p + B(T) g^{\mu\nu}$$

Background field

P. Romantschke *PRD*, *85*(6), 065012, 2012 .
Bluhm et al PRC 84:025201, 2011

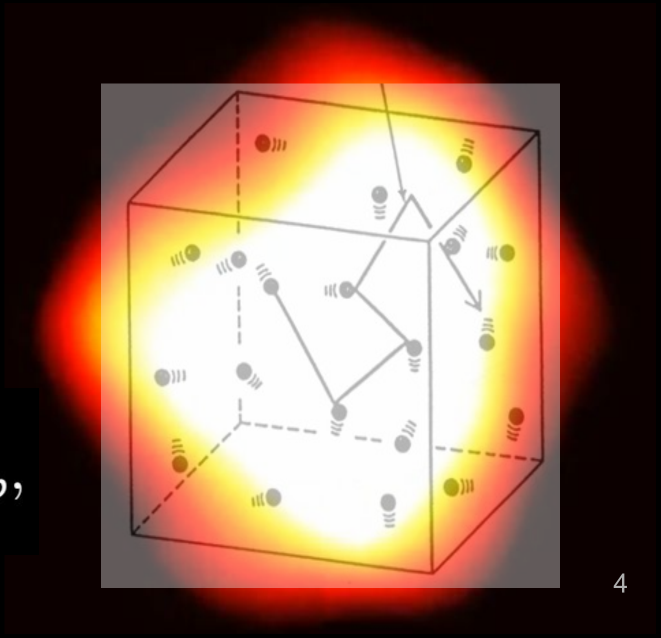
$$P \neq nT$$

Non-ideal gas!

No redefinition required for current tensors N^{μ}, when present

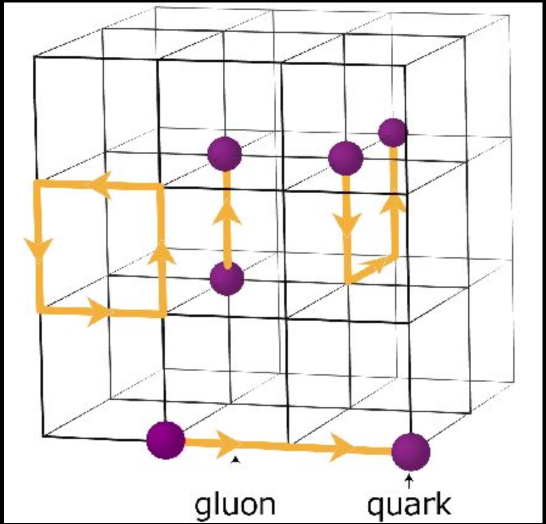
- Relation between thermal mass and background field

$$\partial_{\mu} T^{\mu\nu} = 0 \Rightarrow \frac{\partial B(T)}{\partial T} = -\frac{1}{2}\left(\frac{\partial M^2(T)}{\partial T}\right) \int dP\; f_{0p},$$



Boltzmann equation w/ thermal mass

- Our interest is modelling a QCD plasma. Extraction of the th. mass from LQCD EoS (μ_B≡0) Borsanyi et al, JHEP 11, 077 (2010), arXiv:1007.2580 [hep-lat];



https://www.researchgate.net/figure/Lattice-QCD-discretization-illustration-Quark-fields-lie-on-the-grid-points-and-gluon_fig1_346904487

- LQCD computes trace anomaly directly

$$I \equiv \langle T^{\mu}_{\mu} \rangle_{\mathrm{th}} = \varepsilon_0 - 3P_0$$

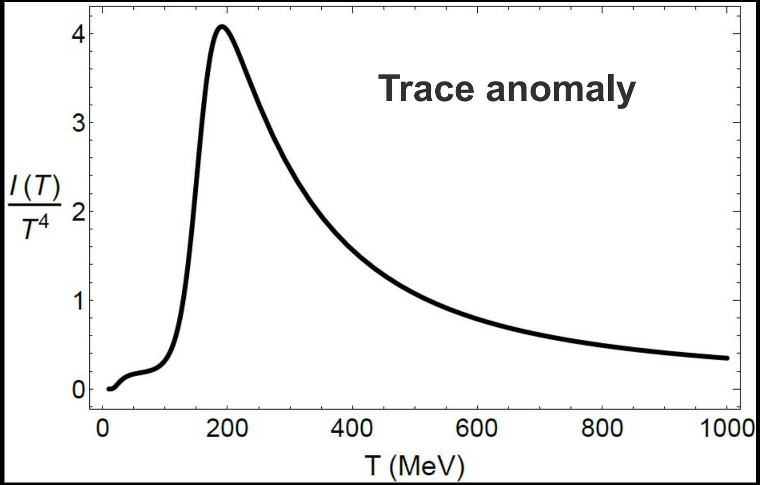
Thermodynamic identities =>

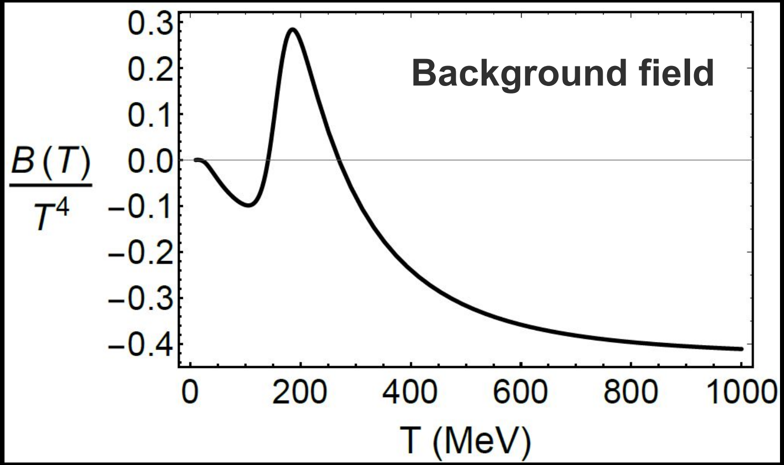
$$\frac{P_0(T)}{T^4} = \int_0^T \frac{dX}{X} \frac{I(X)}{X^4}$$

- Now the thermal mass can be computed

$$Ts_0 = \varepsilon_0 + P_0 \qquad s_0 = \frac{ZM(T)^3}{2\pi^2} K_3\left(\frac{M(T)}{T}\right)$$

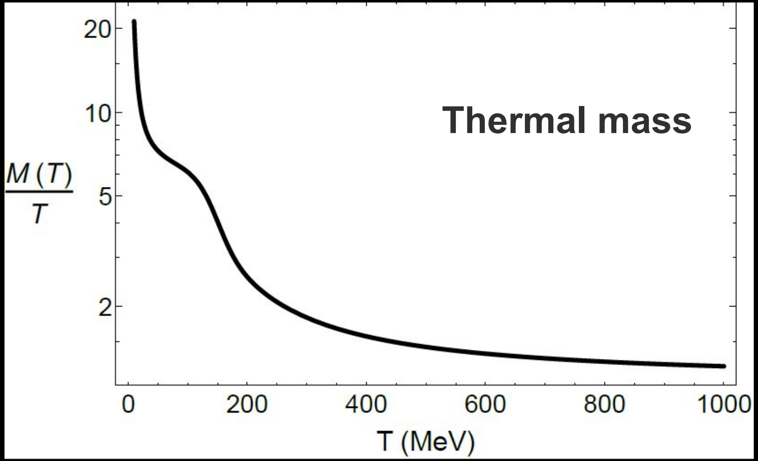
Thermal mass





- Global analytic parametrization ↑

Borsanyi et al, JHEP 11, 077 (2010), arXiv:1007.2580 [hep-lat];



- Large deviation from a conformal theory near the crossover
- Large T ~ conformal

The Relaxation Time Approximation

- The most non-trivial term in the EdB is the collision term

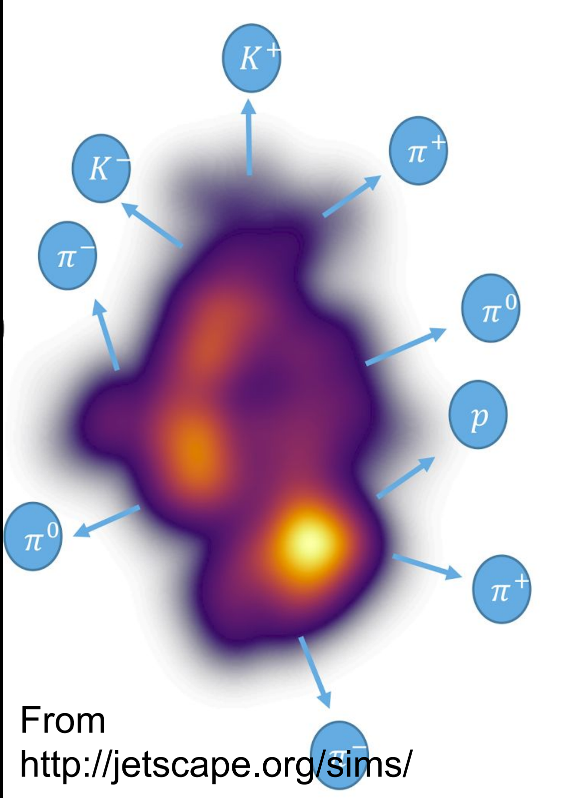
$$C[f] = \int dQ\, dQ'\, dP' \tilde{W}_{pp' \leftrightarrow qq'} (f_p f_{p'} - f_q f_{q'}),$$

- Widely used simplification: Relaxation time approximation (RTA) J. L. Anderson and H. Witting, Physica 74, 466 (1974)

$$C[f] \approx -\frac{u_\mu p^\mu}{\tau_R}(f_p - f_{0p}) \qquad f_{0p} = e^{-\beta_0 u^0_\mu p^\mu + \alpha_0}$$

- Importance in HIC modelling: Conversion from fluid d.o.f's to particles (Cooper-Frye), hydrodynamization of QCD matter Kamata et al PRD **102**, 056003, among others.

Romatschke, PRL. 120, no.1, 012301 (2018)
Denicol and Noronha, PRL no.15, 152301 (2020)
Heller and Spalinski, PRL 115, no.7, 072501 (2015)



From http://jetscape.org/sims/

Severe limitation of traditional RTA

- RTA is inconsistent with the macroscopic conservation laws;

$$T^{\mu\nu} = \int dP \; p^\mu p^\nu f_p + B(T) g^{\mu\nu} \qquad \partial_\mu T^{\mu\nu} = -\int dP \frac{E_p}{\tau_R} p^\nu \delta f,$$

Traditionally, it is assumed that τ_R =cte and one defines (T, u^μ, α) so that the right-hand side is zero

(Landau matching conditions)

- This happens because an essential property of the collision term was lost:

$$C[\mathcal{Q}_p] = 0.$$

$\mathcal{Q}_p$: **Microscopically Conserved Quantity**

In the present case: p^μ

Our proposal

- To recover the lost properties, we propose schematically

GSR, Denicol, Noronha
PRL 127, 042301 (2021)

$$C[f] \propto -\mathbb{1} + \sum_n |\mathcal{Q}_{n,p}\rangle\langle\mathcal{Q}_{n,p}|,$$

Traditional RTA

Projector in the subspace of conserved quantities in an ***orthogonal basis***

- Our approximation to the rBE in this quasi-particle model is

$$p^\mu \partial_\mu f_{\mathbf{p}} + \frac{1}{2}\partial_i M^2(T)\, \partial^i_{(p)} f_p = -\frac{E_{\mathbf{p}}}{\tau_R} f_{0\mathbf{p}} \left[\phi_{\mathbf{p}} - \frac{\langle \phi_{\mathbf{p}}, E_{\mathbf{p}}\rangle}{\langle E_{\mathbf{p}}, E_{\mathbf{p}}\rangle} - \frac{\langle \phi_{\mathbf{p}}, p^{\langle\mu\rangle}\rangle}{\frac{1}{3}\langle p_{\langle\nu\rangle}, p^{\langle\nu\rangle}\rangle} p_{\langle\mu\rangle} \right]$$

Notation: $p^{\langle\mu\rangle} = \Delta^{\mu\nu} p_\nu \quad \Delta^{\mu\nu} = \eta^{\mu\nu} - u^\mu u^\nu \qquad \langle \psi_p, \phi_p\rangle = \int dP \frac{E_p}{\tau_R} \psi_p \phi_p f_{0p}$

$$\tau_R = t_R \left(\frac{E_p}{T}\right)^\gamma$$

Another application -- MIS EoMs: GSR, Denicol PRD 104, 096016 (2021)

Relativistic generalization of the Bhatnagar-Gross-Krook model -- *C. Cercignani Mathematical methods in kinetic theory (Springer, 1990)*

Non-equilibrium corrections

- We use Chapman-Enskog procedure to derive non-equilibrium corrections

S. Chapman, Phil. Trans. R. Soc. A 216, 279 (1916); 217, 115 (1917);D. Enskog, Dissertation, Uppsala (1917): Arkiv Mat., Ast. och. Fys. 16, 1 (1921).

$$\epsilon\left(p^\mu\partial_\mu f_{\mathbf{p}} + \frac{1}{2}\partial_i M^2(T)\,\partial^i_{(\mathbf{p})} f\right) = -\frac{E_{\mathbf{p}}}{\tau_R} f_{0\mathbf{p}}\left[\phi_{\mathbf{p}} - \frac{\left\langle \phi_{\mathbf{p}}\frac{E^2_{\mathbf{p}}}{\tau_R}\right\rangle_0}{\left\langle \frac{E^3_{\mathbf{p}}}{\tau_R}\right\rangle_0} - \frac{\left\langle \phi_{\mathbf{p}}\frac{E_{\mathbf{p}}}{\tau_R} p^{\langle\mu\rangle}\right\rangle_0}{\frac{1}{3}\left\langle \frac{E_{\mathbf{p}}}{\tau_R}\right\rangle_0} p_{\langle\mu\rangle}\right].$$

$$f_p = \sum_{i=0}^{\infty}\epsilon^i f_p^{(i)}.$$

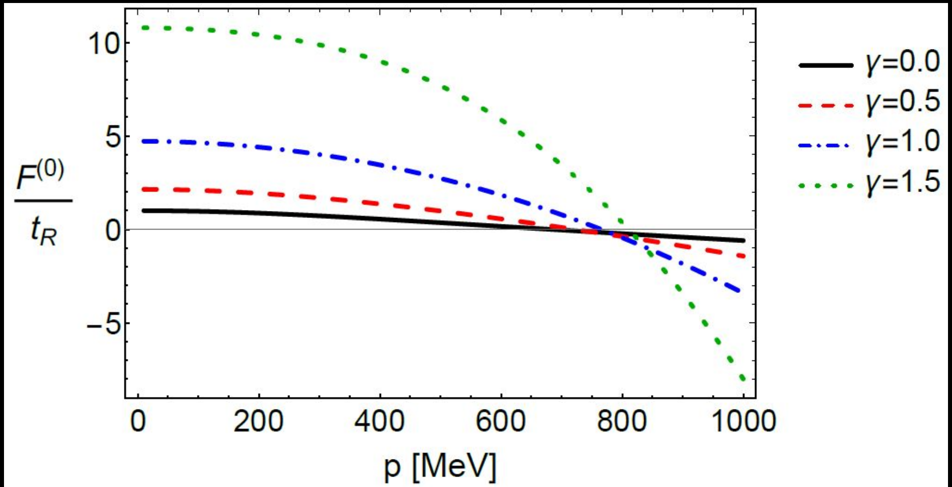
	$\mathcal{O}(\epsilon^0) : f_p^{(0)} = f_{eq,p}$	Ideal hydro
We stop here	$\mathcal{O}(\epsilon^1) : f_p^{(1)}$	Relativistic Navier Stokes

$$\phi^{(1)}_{\mathbf{p}} \equiv \frac{f_p^{(1)} - f_{0p}}{f_{0p}} = F^{(0)}_{\mathbf{p}}\theta + F^{(2)}_{\mathbf{p}} p^{\langle\mu} p^{\nu\rangle}\sigma_{\mu\nu},$$

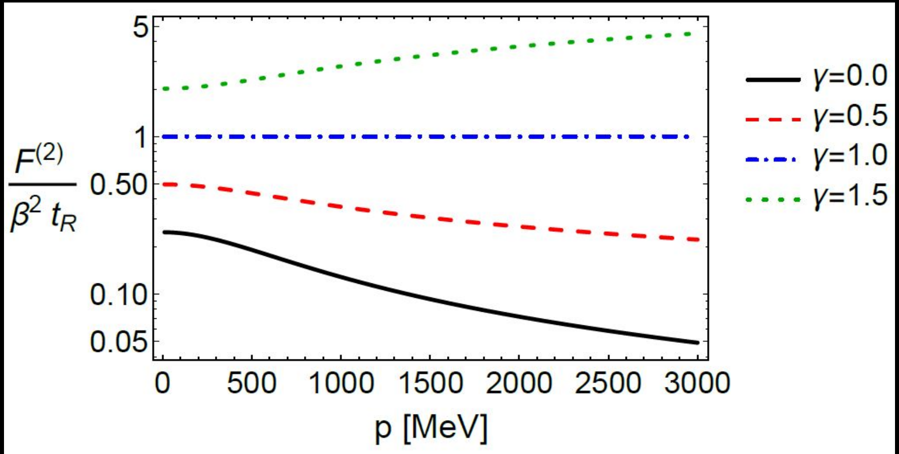
Chapman-Enskog solution to BE

$$\tau_R = t_R \left(\frac{E_p}{T}\right)^{\gamma},$$

$$\phi_{\mathbf{p}}^{(1)} \equiv \frac{f_p^{(1)} - f_{0p}}{f_{0p}} = F_{\mathbf{p}}^{(0)}\theta + F_{\mathbf{p}}^{(2)} p^{\langle\mu} p^{\nu\rangle}\sigma_{\mu\nu},$$



Scalar component



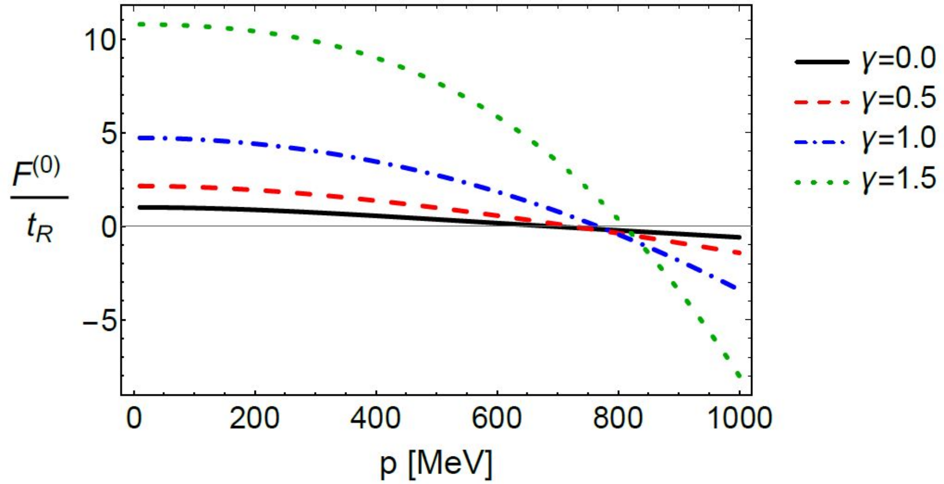
Tensor component

T = 150MeV

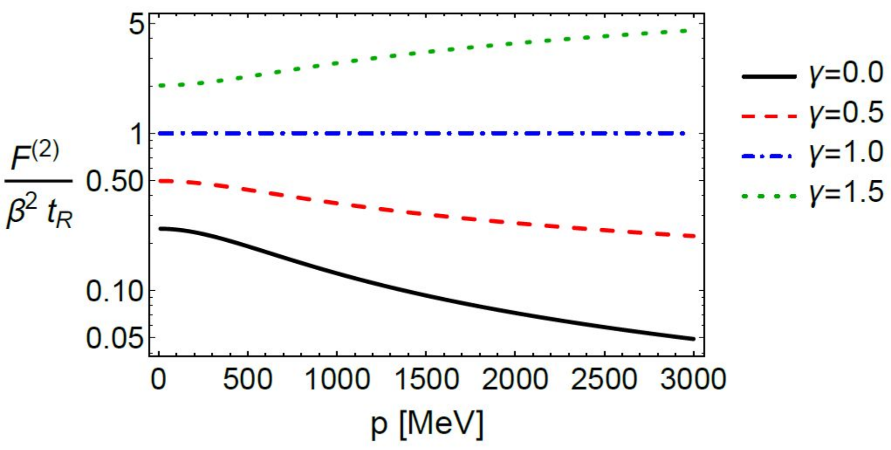
Chapman-Enskog solution to BE

$$\tau_R = t_R \left(\frac{E_p}{T} \right)^{\gamma},$$

$$\phi_{\mathbf{p}}^{(1)} \equiv \frac{f_p^{(1)} - f_{0p}}{f_{0p}} = F_{\mathbf{p}}^{(0)} \theta + F_{\mathbf{p}}^{(2)} p^{\langle \mu} p^{\nu \rangle} \sigma_{\mu\nu},$$



Scalar component



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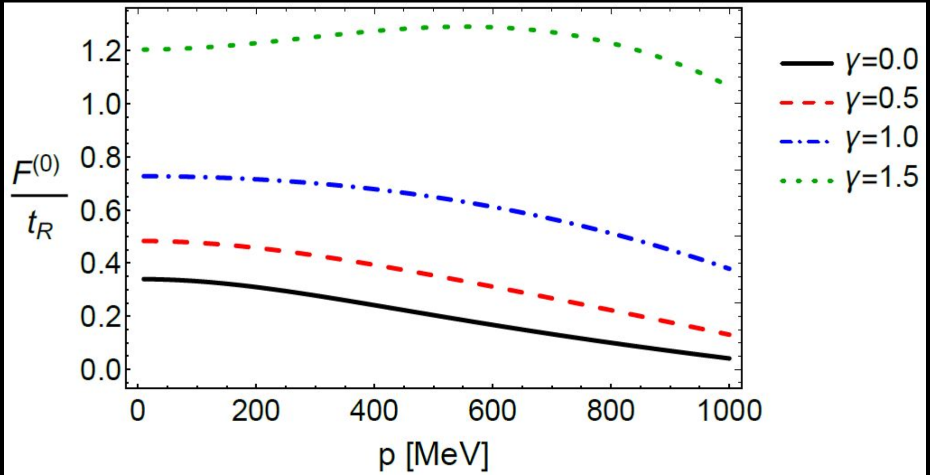
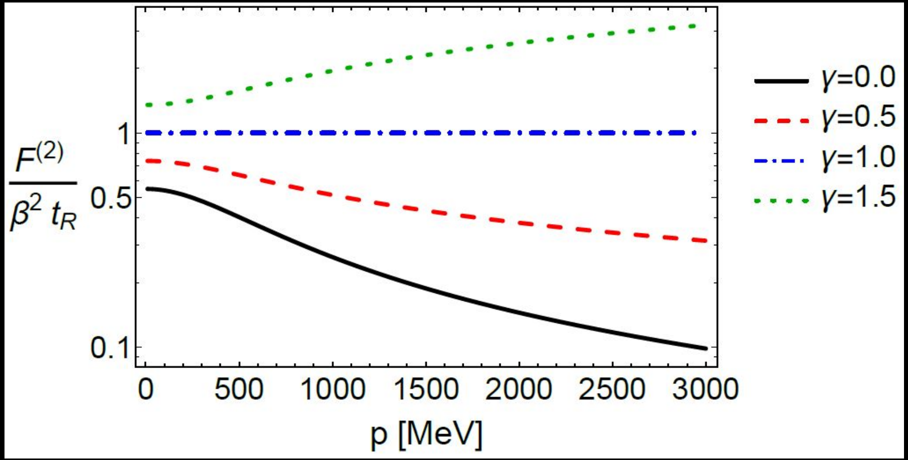




Scalar component

Tensor component

T = 300MeV

Transport coefficients

$$\tau_R = t_R \left(\frac{E_p}{T}\right)^{\gamma},$$

- Irreducible decomposition of the EMT

$$T^{\mu\nu} = T_E^{\mu\nu} + \tilde{T}^{\mu\nu} = \varepsilon_0 u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

(Landau matching)

- We use the results for $\phi_{\mathbf{p}}^{(1)}$ to derive Navier-Stokes const. relations (Landau matching)

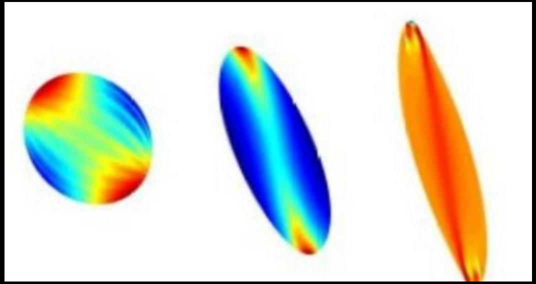
C. Cercignani Mathematical methods in kinetic theory (Springer, 1990)

$$\Pi \equiv -\zeta \partial_\mu u^\mu$$

$$\pi^{\mu\nu} \equiv 2\eta\sigma^{\mu\nu}$$

The diffusion coefficient is zero because we consider that $\mu\equiv 0$

Bulk viscosity



Shear viscosity

Transport coefficients

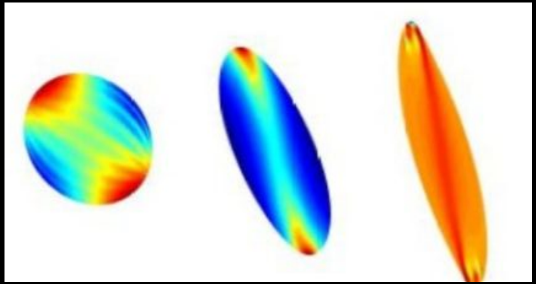
$$\tau_R = t_R \left(\frac{E_p}{T} \right)^{\gamma},$$

- We use Chapman-Enskog expansion to derive Navier-Stokes const. relations (Landau matching) *C. Cercignani Mathematical methods in kinetic theory (Springer, 1990)*

$$\Pi \equiv -\zeta \partial_\mu u^\mu \qquad \pi^{\mu\nu} \equiv 2\eta\sigma^{\mu\nu}$$

The diffusion coefficient is zero because we consider that $\mu \equiv 0$

Bulk viscosity

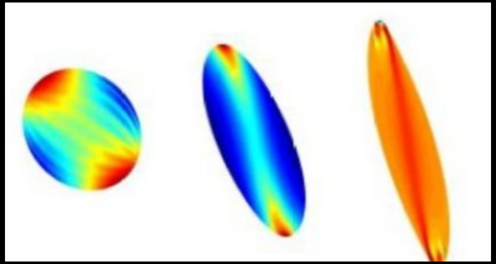


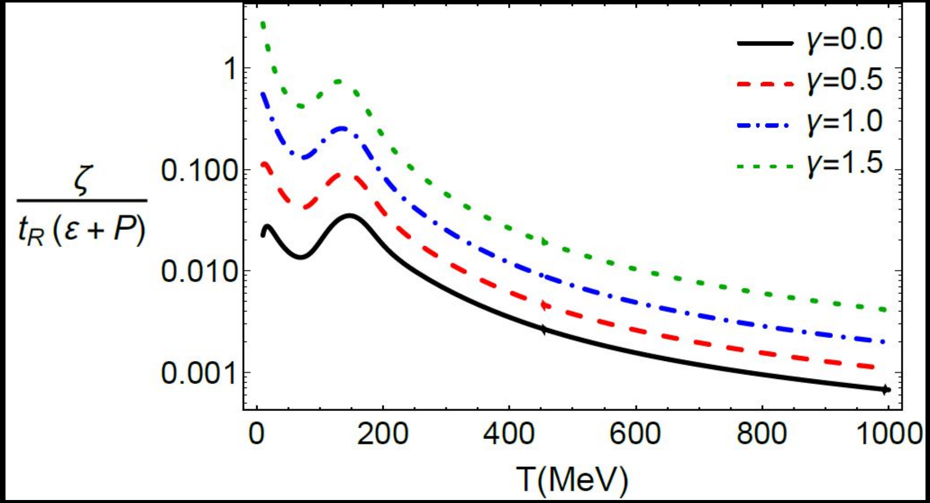
Shear viscosity

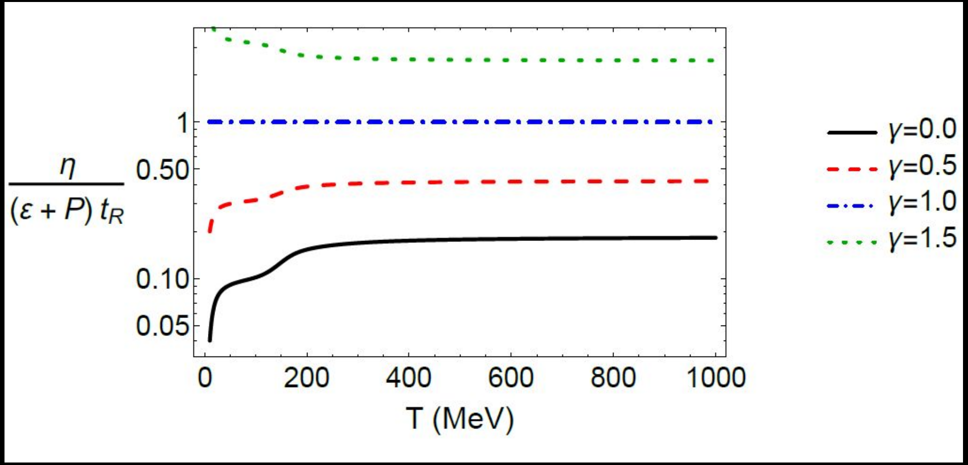
- Our contribution is to compute transport coefficients for energy-dependent relaxation times and alternative matching conditions (Previous contributions: Bluhm et al PRC 84:025201, 2011; P. Romantschke PRD, 85(6), 065012, 2012; Alqahtani et al PRC 92, 054910 (2015);Chakraborty and Kapusta PRC 83, 014906, 2011 etc)

Transport coefficients

$$\tau_R = t_R \left(\frac{E_p}{T}\right)^{\gamma},$$







Bulk viscosity

Shear viscosity

Conclusions

- To consistently use the RTA with energy-dep. τ_R or matching conditions other than Landau, one *must* modify it;
- In this work we used our ansatz to derive transport coefficients in a quasi-particle model;
- We also explored alternative matching conditions. Both affect the transport coefficients. It can be shown that $\partial_\mu S^\mu \geq 0$

THANK YOU FOR THE ATTENTION!

https://dailynewshungary.com/19-photos-that-will-make-you-regret-you-did-not-visit-budapest-in-the-winter/

EXTRA SLIDES

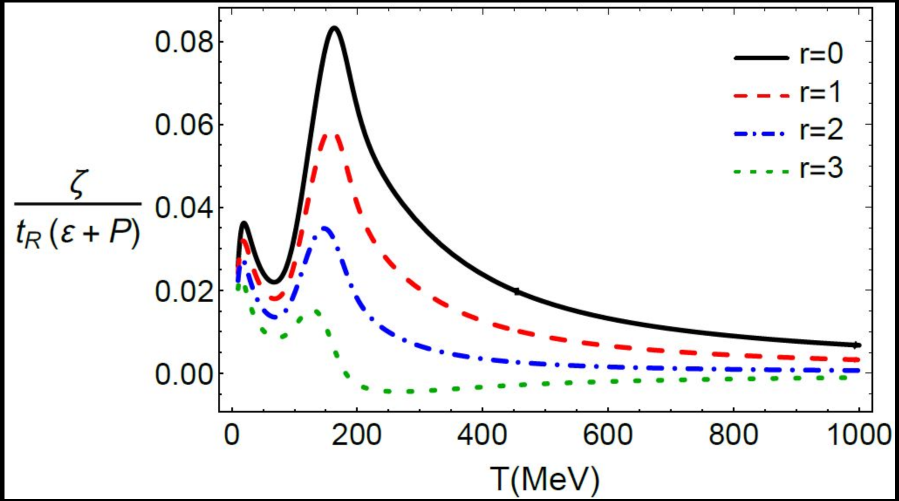
Transport coefficients

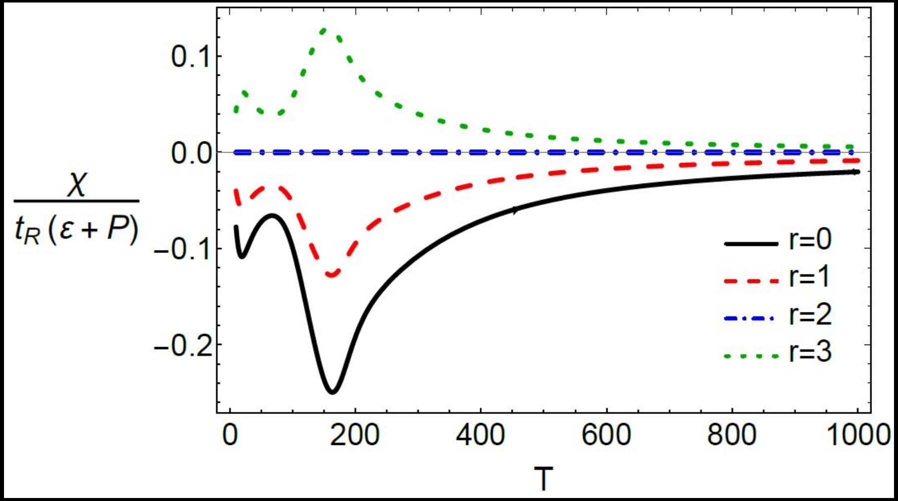
$$\tau_R = t_R \left(\frac{E_p}{T}\right)^{\gamma},$$

- Inspired in some recent works*, we have also explored the effect of alternative matching conditions $\delta\varepsilon = \rho_2 \equiv 0 \mapsto \rho_r \equiv 0$

*Bemfica et al PRD, 98(10):104064, (2018); PRD 100(10):104020, (2019); arxiv 2009.11388 [gr-qc]

$$\Pi \equiv -\zeta \partial_\mu u^\mu \qquad \delta\varepsilon = \chi \partial_\mu u^\mu$$





Bulk viscosity **γ =0** New transport coefficient

Expressions for the coefficients

$$\tau_R = t_R \left(\frac{E_p}{T}\right)^{\gamma},$$

$$\zeta = -\frac{1}{3}\left\langle \left(\Delta^{\mu\nu} p_\mu p_\nu\right) A_{\mathbf{p}} \frac{\tau_R}{E_{\mathbf{p}}} \right\rangle_0 - \left\langle \tau_R E_{\mathbf{p}}^{r-1} A_{\mathbf{p}} \right\rangle \frac{I_{3,1}}{I_{r+1,0}}$$

$$\chi = -\left\langle A_{\mathbf{p}} \tau_R E_{\mathbf{p}} \right\rangle + \left\langle \tau_R E_{\mathbf{p}}^{r-1} A_{\mathbf{p}} \right\rangle \frac{I_{3,0}}{I_{r+1,0}},$$

$$\eta = \frac{\beta}{15}\left\langle \left(\Delta^{\mu\nu} p_\mu p_\nu\right)^2 \frac{\tau_R}{E_{\mathbf{p}}} \right\rangle,$$

$$A_{\mathbf{p}} = -\beta c_s^2 E_p^2 - \frac{\beta}{3}\Delta^{\lambda\sigma} p_\lambda p_\sigma - \beta^2 M \frac{\partial M}{\partial \beta} c_s^2$$

$$I_{nq} = \frac{1}{(2q+1)!!}\int dP \left(-\Delta^{\lambda\sigma} p_\lambda p_\sigma\right)^q E_{\mathbf{p}}^{n-2q} f_{0\mathbf{p}}. \qquad \langle\cdots\rangle_0 = \int dP(\cdots) f_{0p}.$$