

Early time anisotropic flow scaling behavior from free-streaming to hydrodynamics

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Motivation & Goal

- Small and dilute systems:



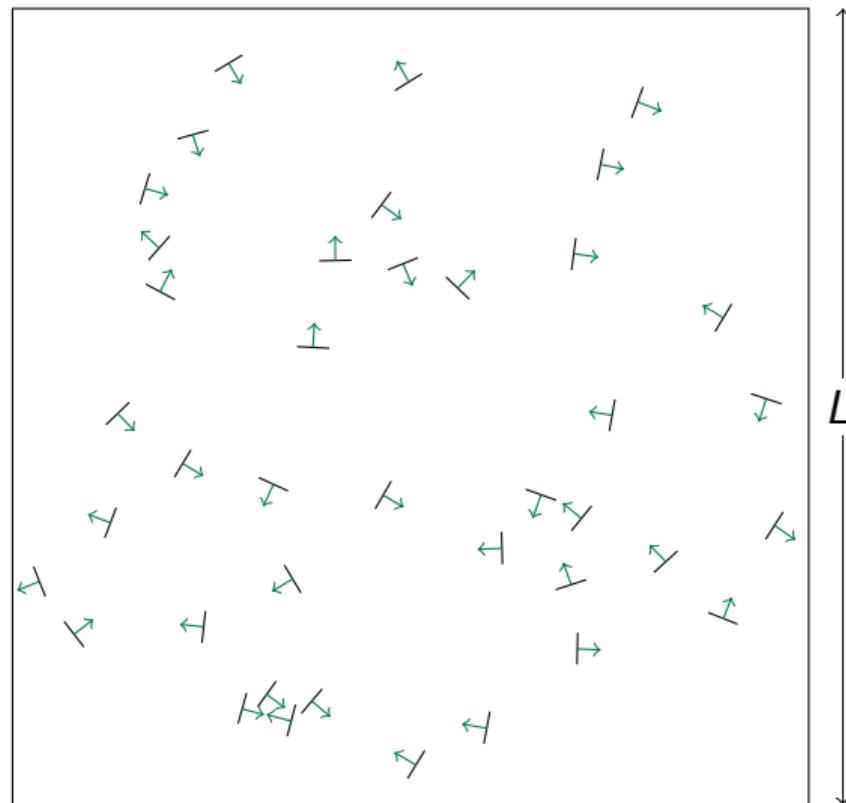
- * Applicability of hydrodynamics is still under debate
- Find the early time scaling behavior of the flow coefficients

$$v_n(t) \propto t^{\gamma_n}$$

along the whole range of Knudsen numbers.

- * Title of the talk "... anisotropic flow ... from free-streaming to hydrodynamics" is strictly speaking not correct → no buildup of flow in free-streaming

- Massless ideal gas of particles in 2D



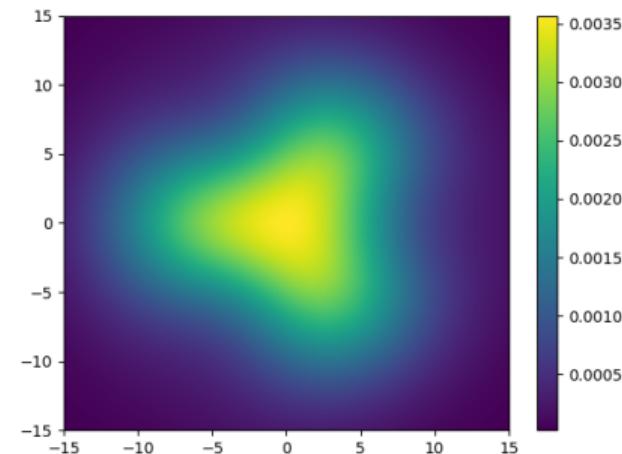
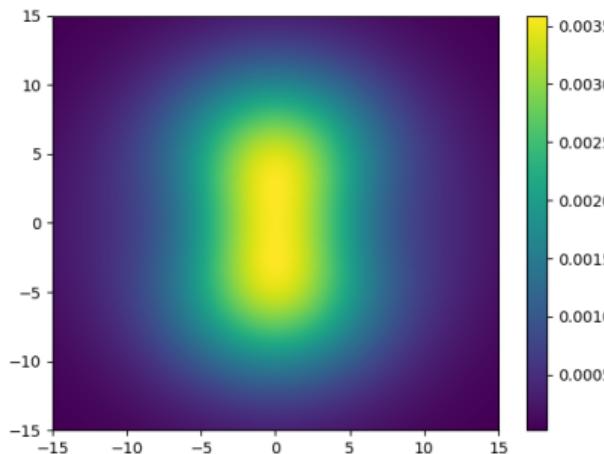
Setup

- What do we need?
 - * Initial condition / distribution
 - * Time evolution
 - * Method to find out what “early-time” means

Initial distribution function

- Initial distribution in position space:

$$G(\vec{r}) \propto \exp\left(-\frac{r^2}{2R^2}\right) \left[1 + \sum_{n=2}^{\infty} \tilde{\varepsilon}_n \left(\frac{r}{R}\right)^n \cos(n(\theta - \Psi_n)) \exp\left(-\frac{r^2}{R^2}\right) \right]$$



Initial distribution function II

- Initial distribution in momentum space:

$$F(p_{\perp}) \propto e^{-\frac{p_{\perp}}{T(x,y)}}$$

- * $T(x, y)$ from ideal gas $n \propto T^2$
- * rotate momenta randomly in the initial state $\rightarrow v_n(t = 0) = 0$
- * subtract $\langle p_x \rangle$ and $\langle p_y \rangle \rightarrow v_1 \equiv 0$

⇒ Anisotropic position space distribution + isotropic momentum space distribution (up to fluctuations $\sim 1/\sqrt{2N}$) in the initial state

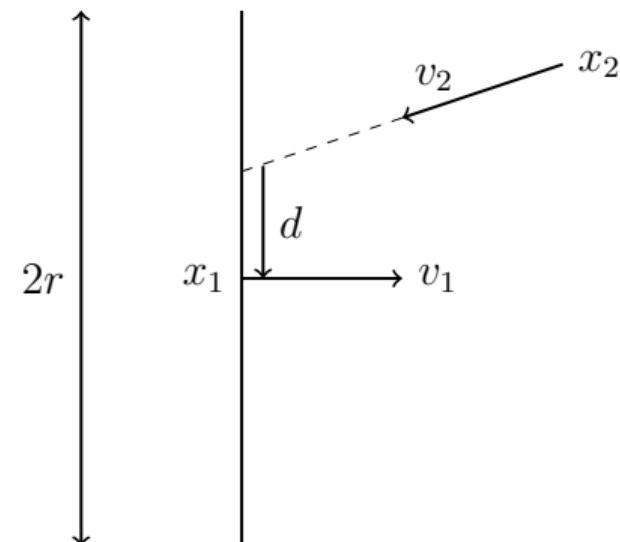
Time evolution → transport algorithm

- Sketch of a collision: particle 1 ($\sigma = 2r$), point-like particle 2
- Impact parameter d :

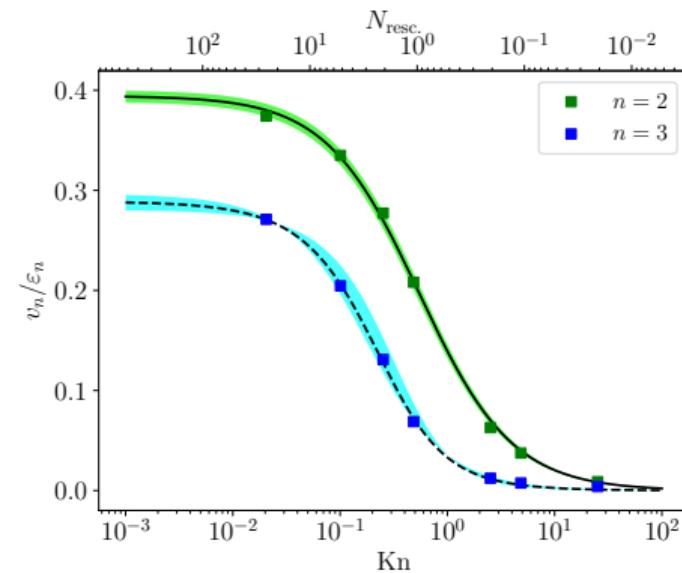
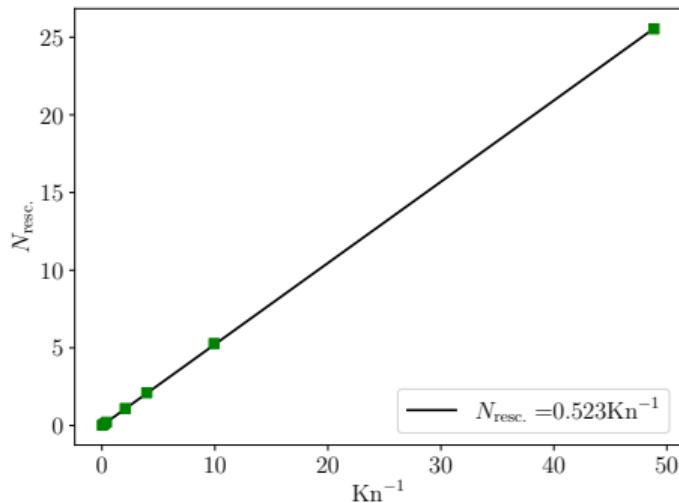
$$d = -\frac{\vec{e}_z \cdot [(\vec{x}_2 - \vec{x}_1) \times (\vec{v}_2 - \vec{v}_1)]}{1 - \vec{v}_1 \cdot \vec{v}_2}$$

- Isotropic scattering angle in center-of-momentum frame:

$$\theta^* = \pi \left(1 - \frac{d}{r} \right)$$



Characterizing the flow regime



$$\text{Kn} \equiv \frac{\lambda_{\text{mfp}}}{R} = \frac{1}{n\sigma R} \propto N_{\text{resc.}}^{-1}$$

$$v_2 = \frac{v_2^{\text{hydro}}}{1 + \frac{\text{Kn}}{\text{Kn}_0}}$$

$$v_3 = \frac{v_3^{\text{hydro}}(1 + B_3\text{Kn})}{1 + (A_3 + B_3)\text{Kn} + C_3\text{Kn}^2}$$

Known limits: Hydro vs. kinetic theory

- Hydro: $v_n \propto t^n$

[Phys.Rev.C 66 (2002) 014907, Nucl.Phys.A 715 (2003) 653-656, nucl-th/0305084, Phys.Rev.C 83 (2011) 064904] (simulations)

[Phys.Rev.C 79 (2009) 044915] (general arguments)

- Kinetic theory in large Kn limit: $v_n \propto t^{n+1}$

[Phys.Rev.C 77 (2008) 054904 (numerical simulation), Eur.Phys.J.C 71 (2011) 1612 (analytical at $\mathcal{O}(\text{Kn}^{-1})$)]

- General analytical behavior:

$$v_n(t) \stackrel{\text{early } t}{\sim} \sum_{j \geq 1} (\text{Kn}^{-1})^j \sum_{k \geq 0} a_{n,k}^{(j)} t^{n+2k+j}$$

[arXiv:2109.15218]

- * systems with elastic $2 \rightarrow 2$ scatterings, inelastic scatterings
- * valid in 2D/3D, classical/quantum, massless/massive particles

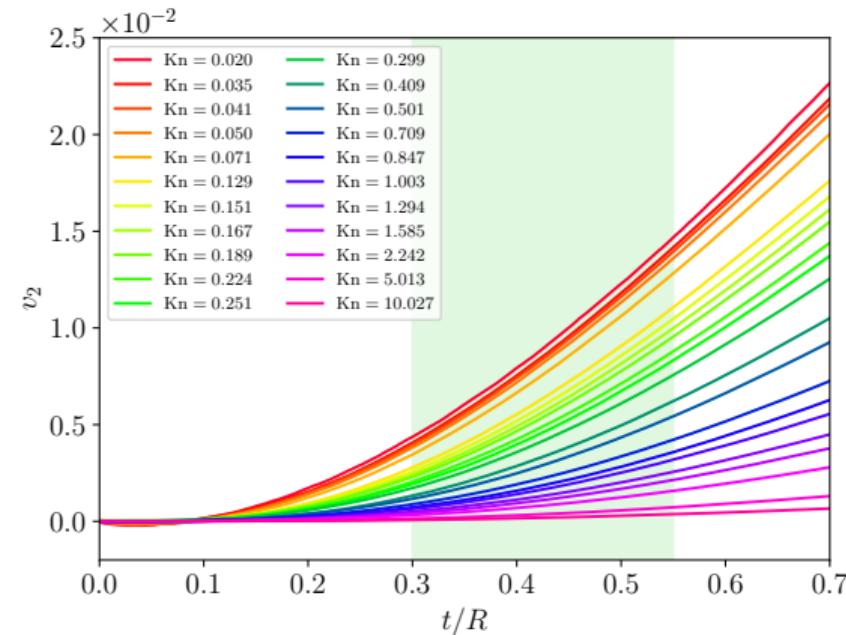
What is small t ?

- It is not obvious where the first order of the Taylor expansion breaks down!
- Solution: We use different fits up to a different t_{\max}
- Perform weighted average over the fits:

$$\langle \gamma_n \rangle = \frac{\sum_i \gamma_{n,i} / \sigma_{\gamma_{n,i}}^2}{\sum_i 1 / \sigma_{\gamma_{n,i}}^2}$$

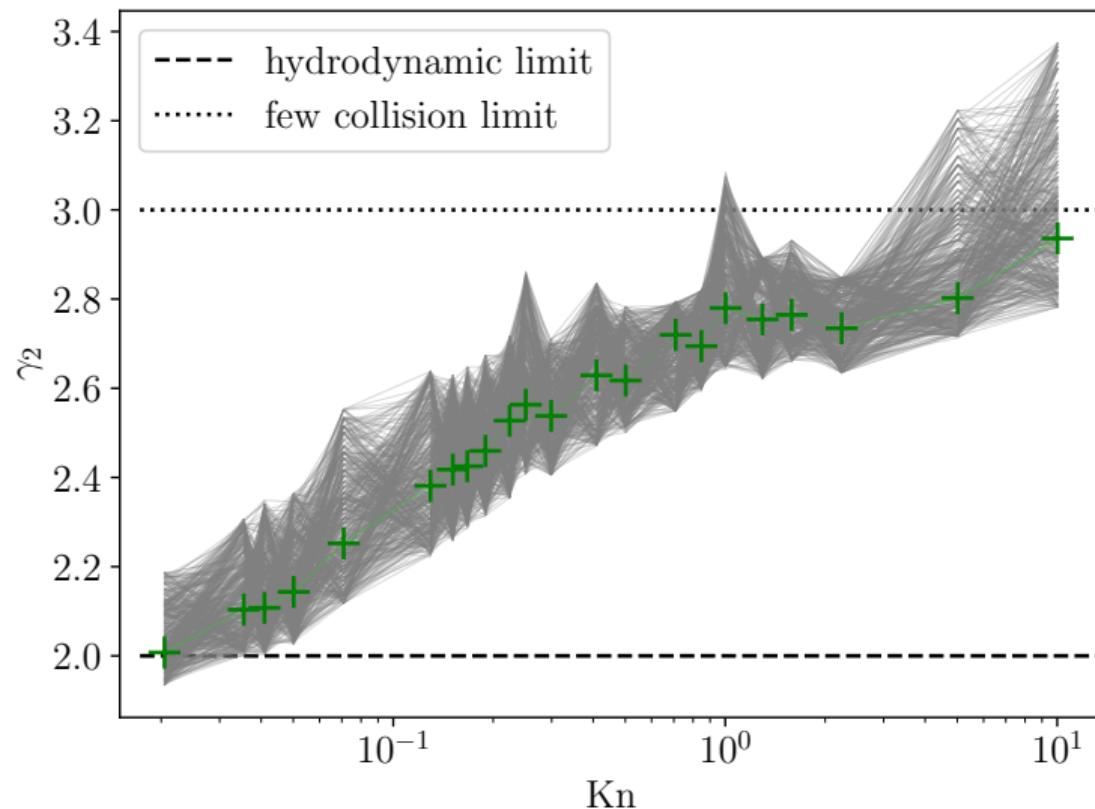
$$\sigma_{\gamma_n}^2 = \frac{\sum_i 1}{\sum_i 1 / \sigma_{\gamma_{n,i}}^2}$$

$$i \in \{1, \dots, 500\}$$



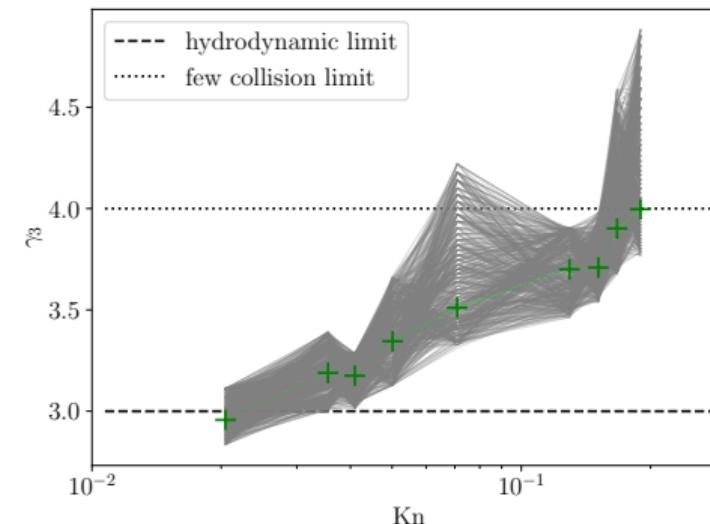
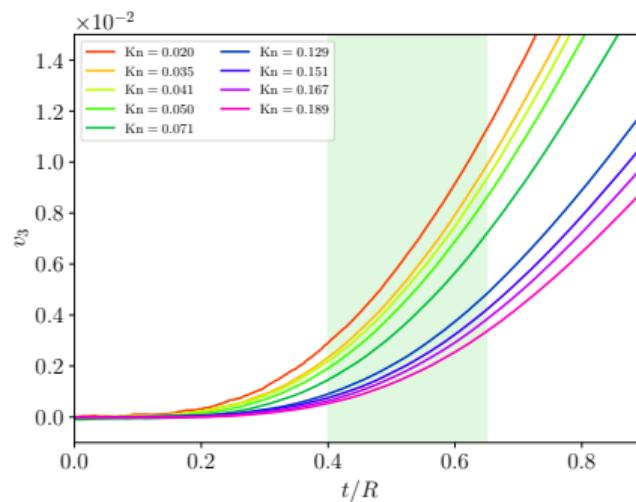
v_2 scaling as a function of Kn

- $v_2 \propto t^{\gamma_2}$



v_3 scaling as a function of Kn

- $v_3 \propto t^{\gamma_3}$



PRELIMINARY (more statistics needed)

Alternative definition of anisotropy: $\varepsilon_{2,p}$

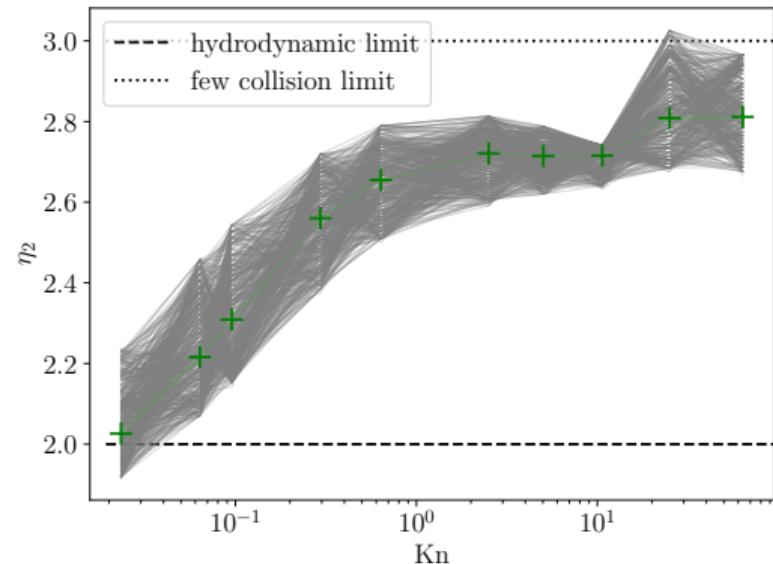
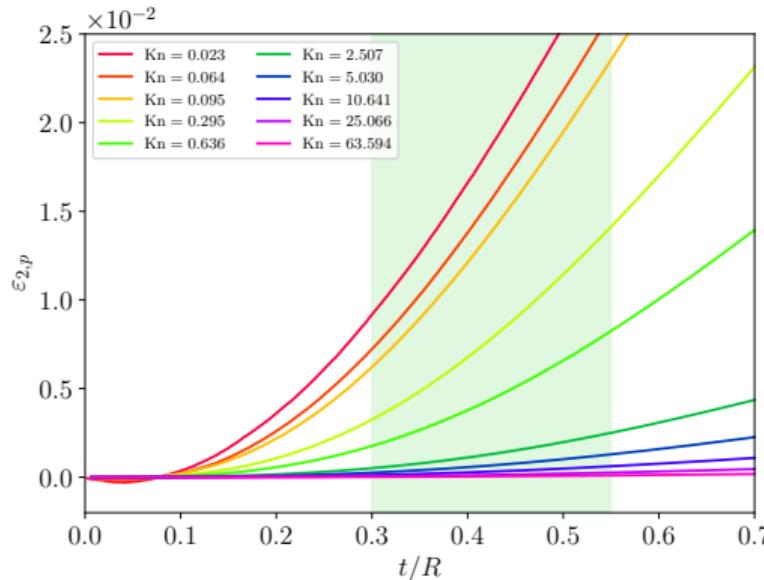
- Scaling behavior at early times is more general for anisotropies in momentum space.

$$\varepsilon_{2,p} \equiv \frac{\langle T^{xx} - T^{yy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$$

- Analytical solution in kinetic theory: $\varepsilon_{2,p}(t) \propto (\text{Kn})^{-1} t^3 + \mathcal{O}(t^4)$
- Problem: No intuitive generalization to higher harmonics.

$\varepsilon_{2,p}$ scaling as a function of Kn

- $\varepsilon_{2,p} \propto t^{\eta_2}$



PRELIMINARY (more statistics needed)

Alternative definition of anisotropy: $\alpha_{2,p}$

- Definition: [Phys.Rev.C 83 (2011) 064904]

$$\alpha_{2,p} \equiv \frac{\langle T^{0x} u_x - T^{0y} u_y \rangle}{\langle T^{00} u_0 \rangle}$$

- Implementation in the transport code:

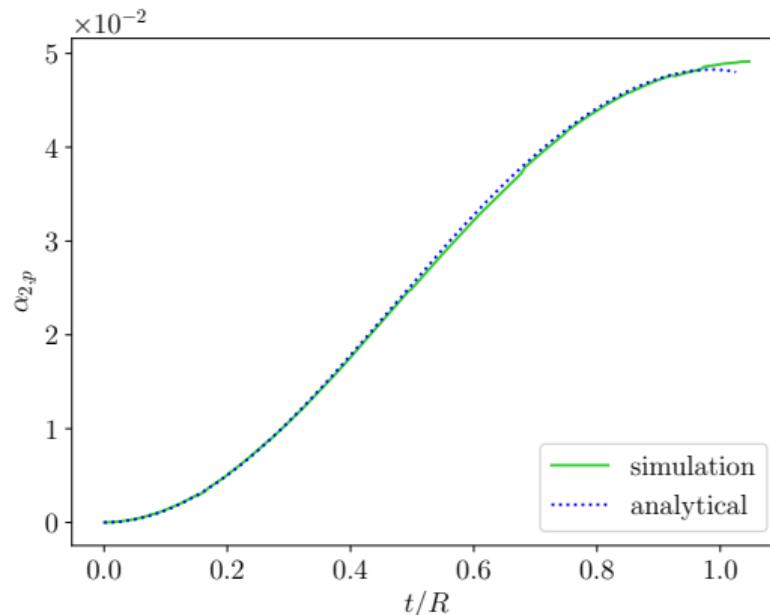
$$\langle T^{0x} u_x - T^{0y} u_y \rangle \approx \frac{1}{N_{\text{cells}}} \sum_{i=1}^{N_{\text{cells}}} \frac{P_{x,i}\bar{v}_{x,i} - P_{y,i}\bar{v}_{y,i}}{\sqrt{1 - \bar{v}_{x,i}^2 - \bar{v}_{y,i}^2}}$$

$$\langle T^{00} u_0 \rangle \approx \frac{1}{N_{\text{cells}}} \sum_{i=1}^{N_{\text{cells}}} \frac{E_{\text{tot},i}}{\sqrt{1 - \bar{v}_{x,i}^2 - \bar{v}_{y,i}^2}},$$

- * $P_{x/y}$: $\sum p_{x/y}$ in cell, $\bar{v}_{x/y}$: mean velocity per particle in cell
- * $N_{\text{cells}} = 40$, results are independent of N_{cells} → tested with 20-120

Free-streaming case

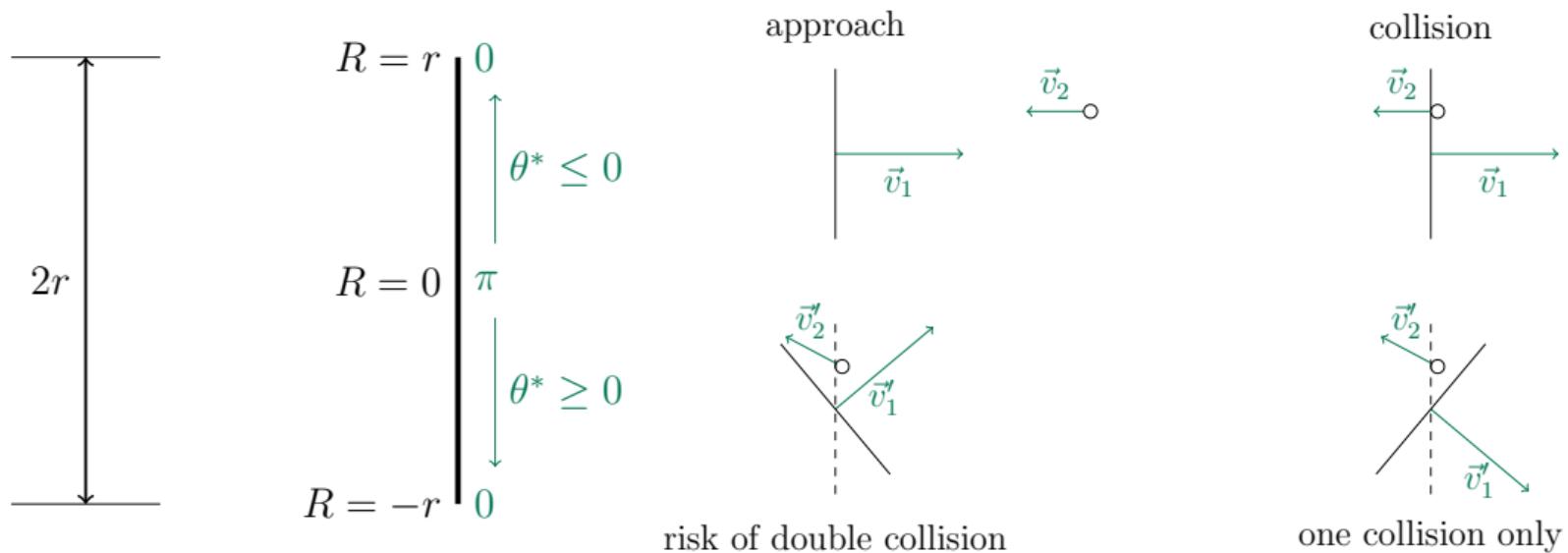
- Analytical result in free-streaming system: $\alpha_{2,p} \propto t^2 + \mathcal{O}(t^3)$
- $\alpha_{2,p}$ does not behave well in the free-streaming case!
 - * Expectation: vanishing anisotropy in free-streaming



Conclusion

- Mapped out the early time scaling of v_2 and $\varepsilon_{2,p}$ as a function of Kn for the first time
- Different classes of models (kinetic theory & hydrodynamics) lead to different early time behavior → no universality, (attractor?)
- Relevant for HIC?
 - * Experiment: only final state v_n and indirectly ε_n
 - * Study particles which decouple from the bulk at early times (photons or dilepton pairs in an appropriate invariant mass interval) [Phys. Lett. B 821, 136626 (2021)]
- $\alpha_{2,p}$ is not well behaved in the free-streaming limit

Scattering



Free-streaming: $\alpha_{2,p}$

- Alternative argumentation: use small τ_{fs} expansion of [Phys.Rev.C 103 (2021) 5, 054909]

$$T^{\mu\nu}(x, y) = \frac{1}{2\pi\tau_{\text{fs}}} \int_0^{2\pi} d\phi \hat{p}^\mu \hat{p}^\nu T(x - v_{\text{fs}}\tau_{\text{fs}} \cos \phi, y - v_{\text{fs}}\tau_{\text{fs}} \sin \phi)$$

$$\hat{p}^\mu \hat{p}^\nu = \begin{pmatrix} 1 & v_{\text{fs}} \cos \phi & v_{\text{fs}} \sin \phi \\ v_{\text{fs}} \cos \phi & v_{\text{fs}}^2 \cos^2 \phi & v_{\text{fs}}^2 \cos \phi \sin \phi \\ v_{\text{fs}} \sin \phi & v_{\text{fs}}^2 \cos \phi \sin \phi & v_{\text{fs}}^2 \sin \phi \end{pmatrix}$$

$$\Rightarrow T^{\mu\nu} = \begin{pmatrix} \frac{T}{\tau_{\text{fs}}} & -\frac{\tau v_{\text{fs}}^2}{2} \partial_x \log T & -\frac{\tau v_{\text{fs}}^2}{2} \partial_y \log T \\ -\frac{\tau v_{\text{fs}}^2}{2} \partial_x \log T & \frac{\tau v_{\text{fs}}^2}{2\tau_{\text{fs}}} & 0 \\ -\frac{\tau v_{\text{fs}}^2}{2} \partial_y \log T & 0 & \frac{\tau v_{\text{fs}}^2}{2\tau_{\text{fs}}} \end{pmatrix}$$

$$u^\mu = \left(1, -\frac{v_{\text{fs}}^2 \tau_{\text{fs}}}{2 + v_{\text{fs}}^2} \partial_x \log T, -\frac{v_{\text{fs}}^2 \tau_{\text{fs}}}{2 + v_{\text{fs}}^2} \partial_y \log T \right) + \mathcal{O}(\tau_{\text{fs}}^2) \Rightarrow \alpha_{2,p} \equiv \frac{\langle T^{0x} u_x - T^{0y} u_y \rangle}{\langle T^{00} u_0 \rangle} \propto \tau_{\text{fs}}^2$$