Linear transverse flow responses at small and large opacities in conformal kinetic theory

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CRC-TR 211
Strong-interaction matter under extreme conditions

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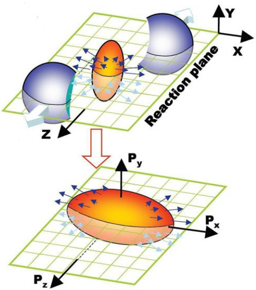
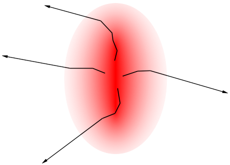
Aims of our Work

- describe spacetime evolution of QCD fireball created in a hadronic collision
- examine how spatial anisotropies in the initial state (ϵ_n) dynamically create momentum anisotropies in the final state (v_n) in small vs. large systems
- small densities, large gradients: hydro not necessarily applicable; alternative: microscopic description in terms of kinetic theory
- employ simplified description in conformal kinetic theory to understand parametric dependences and differences of flow response in small and large systems



Hiroshi Masui (2008)



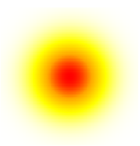
Quantifying Anisotropies

- azimuthal momentum anisotropies are quantified in the "flow harmonics" defined as

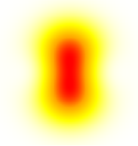
$$\frac{\mathrm{d}N}{\mathrm{d}p_\perp \mathrm{d}\phi_p} = \frac{\mathrm{d}N}{2\pi \mathrm{d}p_\perp}\left(1 + 2\sum_{n=1}^{\infty} v_n(p_\perp)\cos\left[n(\phi_p - \Psi_n)\right]\right) \tag{1}$$

  - will consider $p_\perp$-weighted average of $v_n(p_\perp)$
- models of the fireball dynamics relate the v_n to spatial anisotropies in the initial state
- great correlation with $x_\perp^n$-weighted "eccentricities"; leading order: $v_n \propto \epsilon_n$

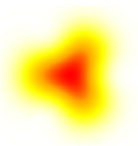
$$\epsilon_n = -\frac{\left\langle x_\perp^n \cos\left[n(\phi_r - \Psi_n)\right]\right\rangle_\epsilon}{\left\langle x_\perp^n \right\rangle_\epsilon} \tag{2}$$



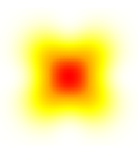
(no ecc.)



$(n = 2)$



$(n = 3)$



$(n = 4)$

Setup

- microscopic description in terms of averaged on-shell phase-space distribution:

$$f(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y) = \frac{(2\pi)^3}{\nu_{\text{eff}}} \frac{\mathrm{d}N}{\mathrm{d}^3x\,\mathrm{d}^3p}(\tau, \mathbf{x}_\perp, \eta, \mathbf{p}_\perp, y) \tag{3}$$

- time evolution: Boltzmann equation in relaxation time approximation

$$p^\mu \partial_\mu f = C_{RTA}[f] = \frac{p_\mu u^\mu}{\tau_R}(f_{eq} - f)\ , \quad \tau_R = 5\frac{\eta}{s}T^{-1} \tag{4}$$

  - time evolution depends on opacity $\hat{\gamma}$ $\quad C_{RTA}[f] \sim \hat{\gamma} = \left(5\frac{\eta}{s}\right)^{-1} \left(\frac{30}{\nu_{\text{eff}}\pi^2}\frac{1}{\pi}\frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}R\right)^{1/4}$

    Kurkela, Wiedemann, Wu EPJC 79 (2019) 965

  - restriction to energy weighted degrees of freedom yields closed set of equations

$$\mathcal{F}(\tau, \mathbf{x}_\perp; \phi_p, v_z) \propto \int_0^\infty \mathrm{d}p^\tau (p^\tau)^3 f(\tau, \mathbf{x}_\perp; p^\tau, \phi_p, v_z) \tag{5}$$

- specify initial energy density to be isotropic Gaussian with anisotropic perturbation

$$\epsilon(\tau_0, \mathbf{x}_\perp) = \frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta}\frac{1}{\pi R^2 \tau_0}\exp\left(-\frac{x_\perp^2}{R^2}\right)\left\{1 + \delta_n \exp\left(-\frac{x_\perp^2}{2R^2}\right)\left(\frac{x_\perp}{R}\right)^n \cos(n\phi_x)\right\} \tag{6}$$

Linearized Analytical and Numerical Desciption

- typical values of $\hat{\gamma}$
  - pp: $\hat{\gamma} \approx 0.88 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{0.4\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{5\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{40}\right)^{-1/4}$
  - PbPb: $\hat{\gamma} \approx 9.2 \left(\frac{\eta/s}{0.16}\right)^{-1} \left(\frac{R}{6\,\mathrm{fm}}\right)^{1/4} \left(\frac{\mathrm{d}E_\perp^{(0)}/\mathrm{d}\eta}{4000\,\mathrm{GeV}}\right)^{1/4} \left(\frac{\nu_{\mathrm{eff}}}{40}\right)^{-1/4}$

$\Rightarrow$ treat problem both analytically (for small $\hat{\gamma}$) and numerically

- linearized analytical treatment
  - "opacity expansion" in number of scatterings

$$\text{0th order}: \ p^\mu \partial_\mu f^{(0)} = 0 \ , \qquad \text{1st order}: \ p^\mu \partial_\mu f^{(1)} = C[f^{(0)}] \ , \qquad \text{etc...}$$

Heiselberg, Levy PRC 59 (1999) 2716
Borghini, Gombeaud EPJC 71 (2011) 1612

Romatschke EPJC 78 (2018) 636
Kurkela, Wiedemann, Wu PLB 783 (2018) 274
Borghini, Feld, Kersting EPJC 78 (2018) 832
Kurkela, Mazeliauskas, Törnkvist arXiv:2104.08179

  - to simplify coupling of isotropic and anisotropic part, we also linearize in δ_n

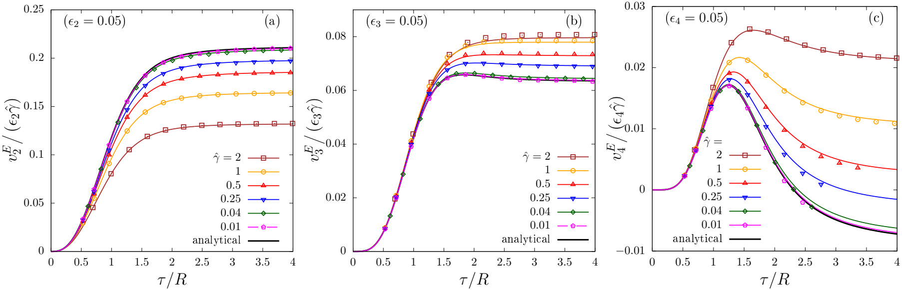
- numerical treatment:
  fully nonlinear in $\hat{\gamma}$, ϵ_n, examine opacity dependence up to "hydro"-limit
  - using two different codes that produce similar results

Kamata, Martinez, Plaschke, Ochsenfeld, Schlichting PRD 102 (2020) 056003

Ambrus, Blaga PRC 98 (2018) 035201

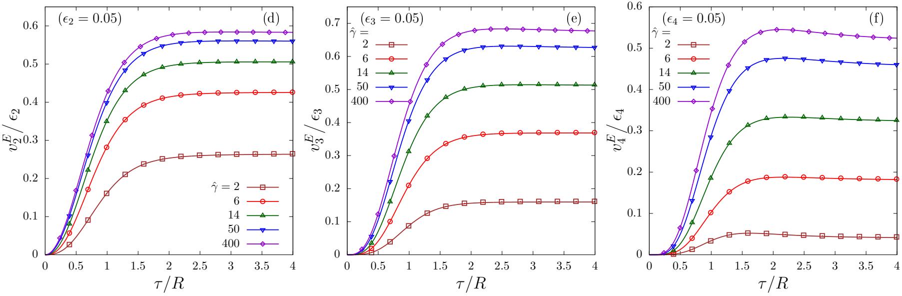
Numerical v_n-Results at small $\hat{\gamma}$ and Comparison

linear behaviour in ϵ_n, $\hat{\gamma}$ $\Rightarrow$ flow harmonics normalized as $v_n/(\epsilon_n\hat{\gamma})$



- initial buildup: $0.5 \lesssim \tau/R \lesssim 1.5$,
  $v_2/\epsilon_2 > v_3/\epsilon_3 > v_4/\epsilon_4$,
  v_4 has strong negative late time trend
- small $\hat{\gamma}$: agreement with linearized result
- larger opacities: $v_2/\hat{\gamma}$ decreases, $v_3/\hat{\gamma}$, $v_4/\hat{\gamma}$ increase

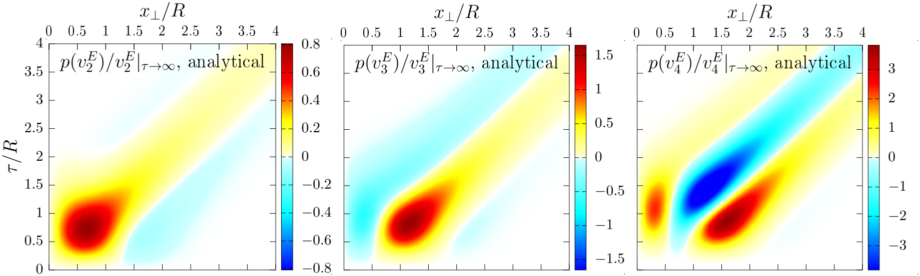
nonlinear in $\hat{\gamma}$ $\Rightarrow$ flow harmonics normalized as v_n/ϵ_n



- all v_ns increase with $\hat{\gamma}$
- simlar scales; curves have no strong distinctive features

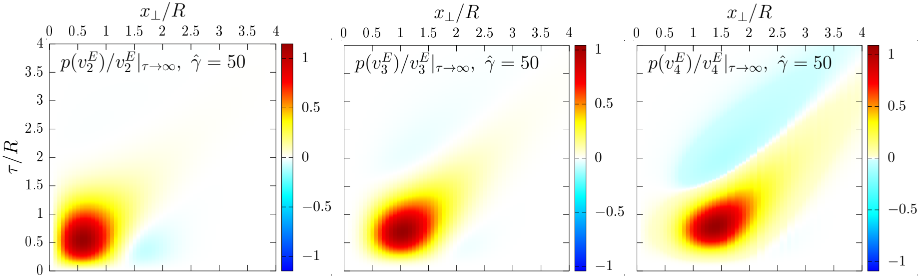
Flow Production Rate at small $\hat{\gamma}$

change in v_n only due to scatterings as rate in τ and density in $x_\perp$: $p(v_n) = \frac{dv_n}{dx_\perp d\tau}\Big|_{\text{coll}}$



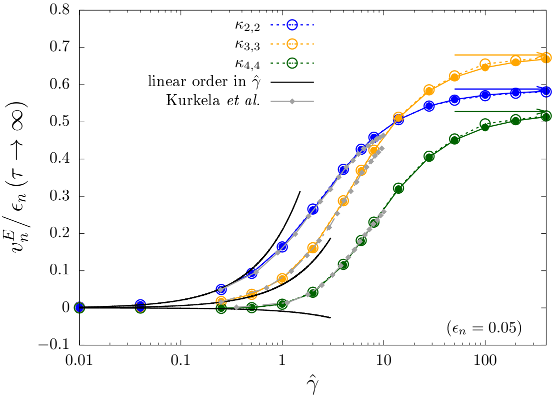
- competing contributions to v_n with different signs from different regions
- this explains ordering of v_n's and negative trend of v_4

change in v_n only due to scatterings as rate in τ and density in $x_\perp$: $p(v_n) = \frac{\mathrm{d}v_n}{\mathrm{d}x_\perp \mathrm{d}\tau|_{\mathrm{coll}}}$



- weight mainly on a single positive contribution, other regions fade away

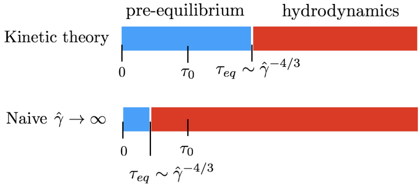
Opacity Dependence



- linear order results have different ranges of validity for different v_n due to peculiarities of small-$\hat{\gamma}$-behaviour
- agreement with previous results in identical setup

  Kurkela, Taghavi, Wiedemann, Wu PLB 811 (2020) 135901
- extension to higher $\hat{\gamma}$, clear signs of saturation

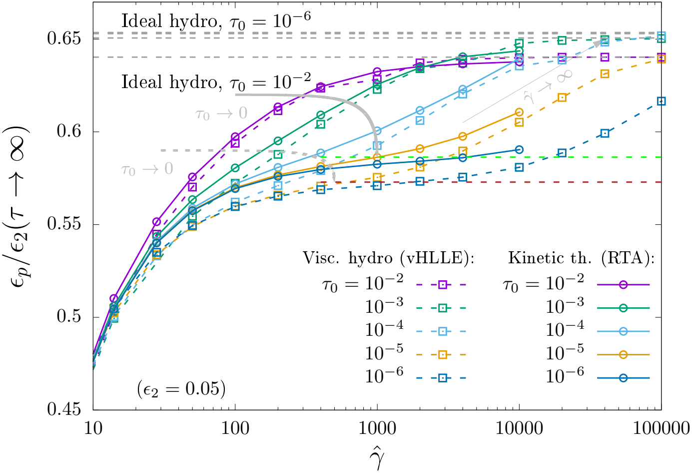
- expectation: in large opacity regime, approximately described by viscous (eventually ideal) hydrodynamics
- hydrodynamics describes pre-equilibrium stage differently than kinetic theory: free-streaming until $\tau_{\text{eq}}/R \sim \hat{\gamma}^{-4/3}$, naively taking $\hat{\gamma} \to \infty$ at fixed τ_0 will cut this out



- investigate this by comparing to relativistic viscous hydrodynamics: vHLLE Karpenko, Huovinen, Bleicher Comput. Phys. Commun. 185, 3016 (2014)
- $\hat{\gamma}$ ill-defined in $\tau_0 \to 0$ initialize at several finite τ_0
- instead of v_2, compare ϵ_p:

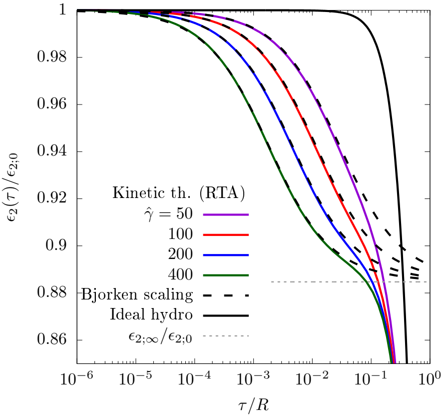
$$\epsilon_p = \frac{\int_{\mathbf{x}_\perp} T^{11} - T^{22} + 2iT^{12}}{\int_{\mathbf{x}_\perp} T^{11} + T^{22}} \tag{7}$$

Discrepancies in Comparison to Hydro



- discrepancy from cutting out pre-equilibrium period
  - need non-equilibrium description of early time dynamics even at large $\hat{\gamma}$
- small τ_0: curves plateau at physical large-opacity asymptote in the limit $\tau_0 \to 0$
- fixed τ_0: for $\tau_{eq} \lesssim \tau_0$, responses reach the (unphysical) ideal hydro limit $\hat{\gamma} \to \infty$

Eccentricity Evolution in pre-equilibrium Period



- due to inhomogeneous cooling, eccentricity decays in pre-equilibrium phase
- normalization with ϵ_2 at onset of transverse expansion brings kinetic theory results into agreement with ideal hydro

- able to describe flow responses in full opacity range from linearized to saturated regime
  - features in small opacity results explained via production rate
- different description of pre-equilibrium stage introduces discrepancy with hydrodynamic results: limits $\tau_0 \to 0$ and $\hat{\gamma} \to \infty$ do not commute
  - pre-equilibrium phase decreases eccentricity

Outlook: comparison of wider range of different descriptions of the pre-equilibrium period for more realistic initial conditions

Backup

coordinates:

$$\tau = \sqrt{t^2 - z^2} \qquad \eta = \mathrm{artanh}(z/t) \qquad y = \mathrm{artanh}(p_z/E)$$

Boltzmann equation:

$$[\underbrace{p_T \cosh(y-\eta)}_{p^\tau} \partial_\tau + p_\perp^i \partial_i + \underbrace{\frac{p_T}{\tau} \sinh(y-\eta)}_{p^\eta} \partial_\eta] f = C[f]$$

initial condition:

$$f^{(0)}(\tau_0, \mathbf{x}_\perp, \mathbf{p}_\perp, y-\eta) = \frac{(2\pi)^3}{\nu_{eff}} \frac{\delta(y-\eta)}{\tau_0 p_\perp} F\left(\frac{Q_s(\mathbf{x}_\perp)}{p_\perp}\right)$$

position dependent momentum scale $Q_s(\mathbf{x}_\perp)$ chosen such that

$$\epsilon(\tau_0, \mathbf{x}_\perp) = \frac{\mathrm{d}E_\perp^{(0)}}{\mathrm{d}\eta} \frac{1}{\pi R^2 \tau_0} \exp\left(-\frac{{\mathbf{x}_\perp}^2}{R^2}\right) \left\{1 + \delta_n \left(\frac{x_\perp}{R}\right)^n \exp\left(-\frac{x_\perp^2}{2R^2}\right) \cos[n(\phi_x - \psi_n)]\right\}$$

Solutions in opacity expansion

<u>zeroth order</u> $p^\mu \partial_\mu f^{(0)} = 0$:

$$f^{(0)}(\tau, \mathbf{x}_\perp, \mathbf{p}_\perp, y-\eta) = f^{(0)}\left(\tau_0, \mathbf{x}_\perp - \vec{v}_\perp t(\tau, \tau_0, y-\eta), \mathbf{p}_\perp, \operatorname{arsinh}\left(\frac{\tau}{\tau_0}\sinh(y-\eta)\right)\right)$$

<u>first order</u> $p^\mu \partial_\mu f^{(1)} = C[f^{(0)}]$:

$$f^{(1)}(\tau, \mathbf{x}_\perp, \mathbf{p}_\perp, y-\eta) = \int_{\tau_0}^{\tau} \mathrm{d}\tau' \left(\frac{C[f^{(0)}]}{p^\tau}\right)\left(\tau', \mathbf{x}_\perp{}', \mathbf{p}_\perp, y-\eta'\right)$$

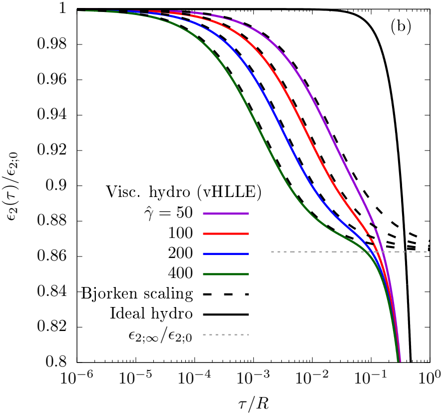
<u>collision kernel:</u> find local rest frame and temperature using Landau matching to compute $C_{RTA}[f^{(0)}] = \frac{p_\mu u^\mu T}{5\,\eta/s}(f_{eq} - f)$ where $f_{eq} = \frac{1}{\exp(p_\mu u^\mu/T)-1}$

$$T^{\mu\nu} = \nu_{eff}\tau \int \frac{\mathrm{d}^3 p}{(2\pi)^3 p^\tau} p^\mu p^\nu f^{(0)} \qquad \epsilon u^\mu = u_\nu T^{\nu\mu} \qquad \epsilon = \frac{\nu_{eff}\pi^2}{30} T^4$$

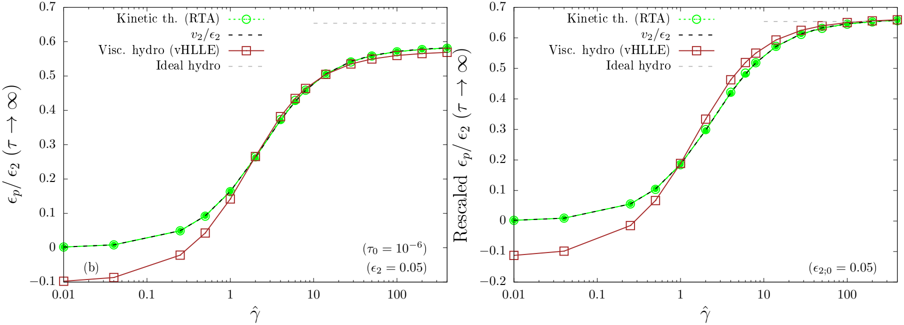
<u>free-streamed $\delta\epsilon$-cosine:</u>

$$|\mathbf{x}_\perp - \mathbf{v}_\perp \Delta\tau|^n \cos(n\phi_{\mathbf{x}_\perp - \mathbf{v}_\perp \tau}) = \sum_{j=0}^{n} \binom{n}{j} x_T^{n-j} (-\tau)^j \cos[n\phi_{\mathbf{x}_\perp} + j(\phi_{\mathbf{x}_\perp} - \phi_{\mathbf{v}_\perp})]$$

Eccentricity Evolution in pre-equilibrium Period: Hydro



Rescaled results



- Rescaling could be done with ghat dependent factors.