COVARIANT LINEAR RESPONSE FOR COLLISIONAL PLASMAS

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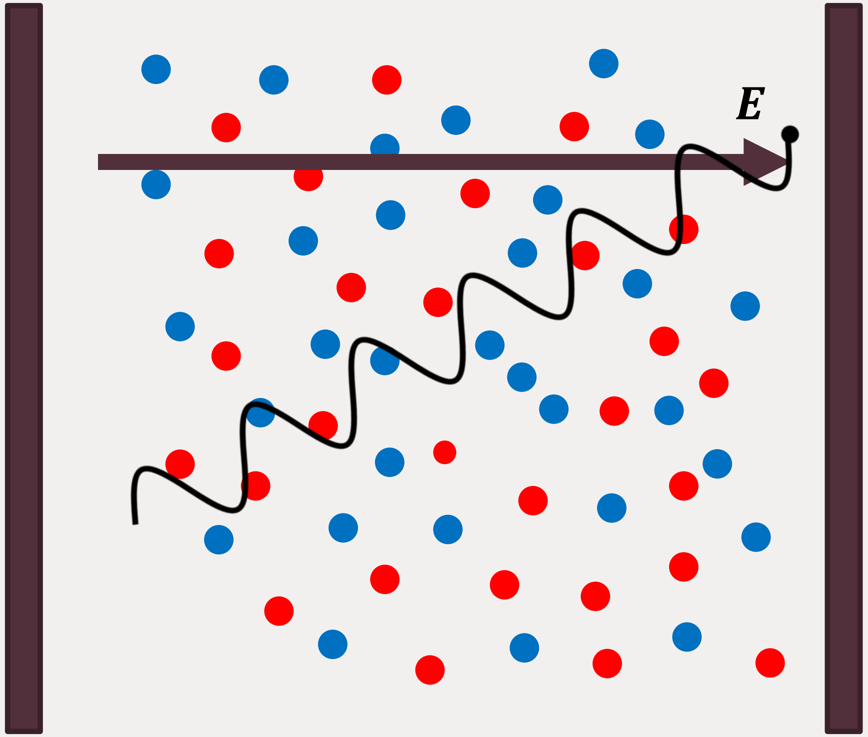
Linear plasma response to time dependent fields



Electron-positron plasma

Boltzmann equation for the distribution function $f(x,p)$

$$p\cdot\partial f(x,p)+\underbrace{\mathrm{q}\mathrm{F}^{\mu\nu}\mathrm{p}_\nu\frac{\partial f(x,p)}{\partial p^\mu}}_{\text{Vlasov force term}}=\underbrace{C[f(x,p)]}_{\text{Collisions}}$$

Linear response:

$$f(x,p)=f_{eq}(p)+\boxed{\delta f(x,p)}\qquad f_{\mathrm{eq}}(p)=\frac{1}{\exp(p\cdot u/T)+1}$$

Induced current from the solution:

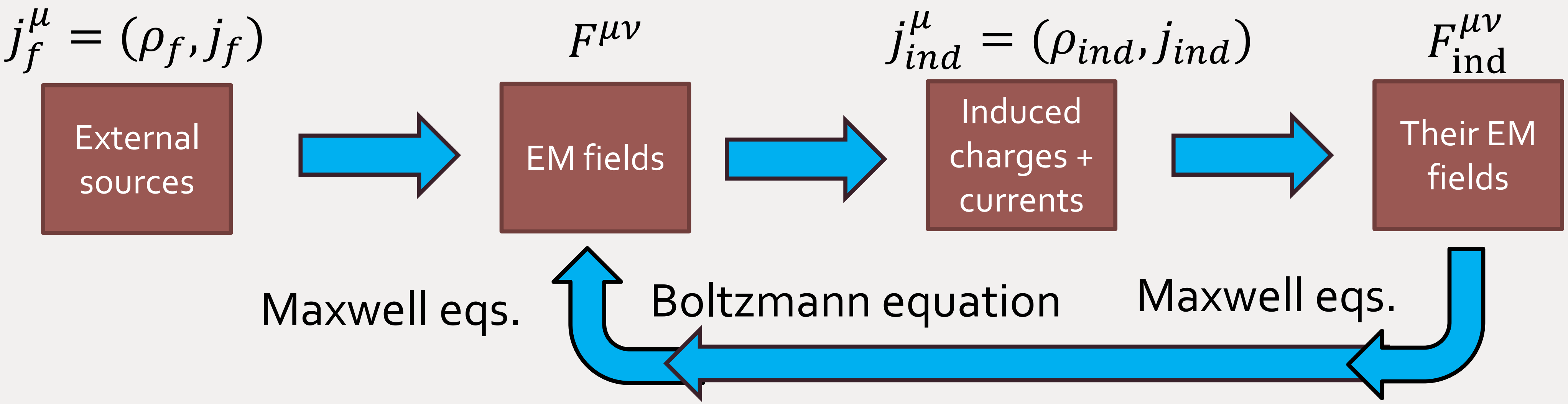
$$j^\mu_{ind}(x)=2q\int(dp)p^\mu f(x,p)\qquad (dp)=\frac{d^4p}{(2\pi)^4}4\pi\delta(p^2-m^2)$$

Two component plasma

$$j^\mu(x)=2q\int(dp)p^\mu[f_+(x,p)-f_-(x,p)]$$

But induced charges also generate fields!

Perturbative approach (often implicitly assumed in literature on this topic)



Works only if the EM fields of induced charges and currents are much smaller than the fields producing the induced charges and currents in the medium!

Can we do better?

Self consistent solution in Fourier space

$x^\mu = (t, \boldsymbol{x}) \Rightarrow k^\mu = (\omega, \boldsymbol{k})$

Covariant Ohm's law: $\boxed{\tilde{j}^\mu_{\text{ind}} = \Pi^\mu_\nu \tilde{A}^\nu}$

$\tilde{A}^\nu(k) = (\tilde{\phi}, \tilde{\boldsymbol{A}})$ - all the fields including the induced ones

Maxwell equations in medium: $\boxed{k_\mu \tilde{F}^{\mu\nu} = \mu_0(\tilde{J}^\nu_f + \tilde{j}^\nu_{\text{ind}})}$

$\tilde{F}^{\mu\nu}(k) = k^\mu \tilde{A}^\nu(k) - k^\nu \tilde{A}^\mu(k)$

Additionally: Gauge condition $k_\mu \tilde{A}^\mu = 0$ and current conservation $k_\mu \tilde{j}^\mu_{\text{ind}} = 0$

Solution for components $\tilde{\phi}, \tilde{\boldsymbol{A}}$ where $\tilde{\boldsymbol{A}} = \tilde{A}_\parallel \boldsymbol{k} + \tilde{\boldsymbol{A}}_\perp$:

$$\tilde{\phi}(k) = \frac{\tilde{\rho}_f(k)}{(k^2 - \omega^2)(\Pi_L(k)/\omega^2 - 1)} \qquad \tilde{\boldsymbol{A}}_\perp(k) = \frac{\tilde{\boldsymbol{J}}_{\perp,f}(k)}{k^2 - \omega^2 - \Pi_T(k)} \qquad \tilde{A}_\parallel(k) = \frac{\omega}{|\boldsymbol{k}|}\tilde{\phi}(k)$$

Depends only on external charges / currents and polarization tensor properties

Π_T and Π_L - transverse and longitudinal projections of Π^μ_ν (Weldon, PRD 26 (1982) 1394)

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Current-conserving relativistic linear response for collisional plasmas

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Important “sponsor” I should mention:

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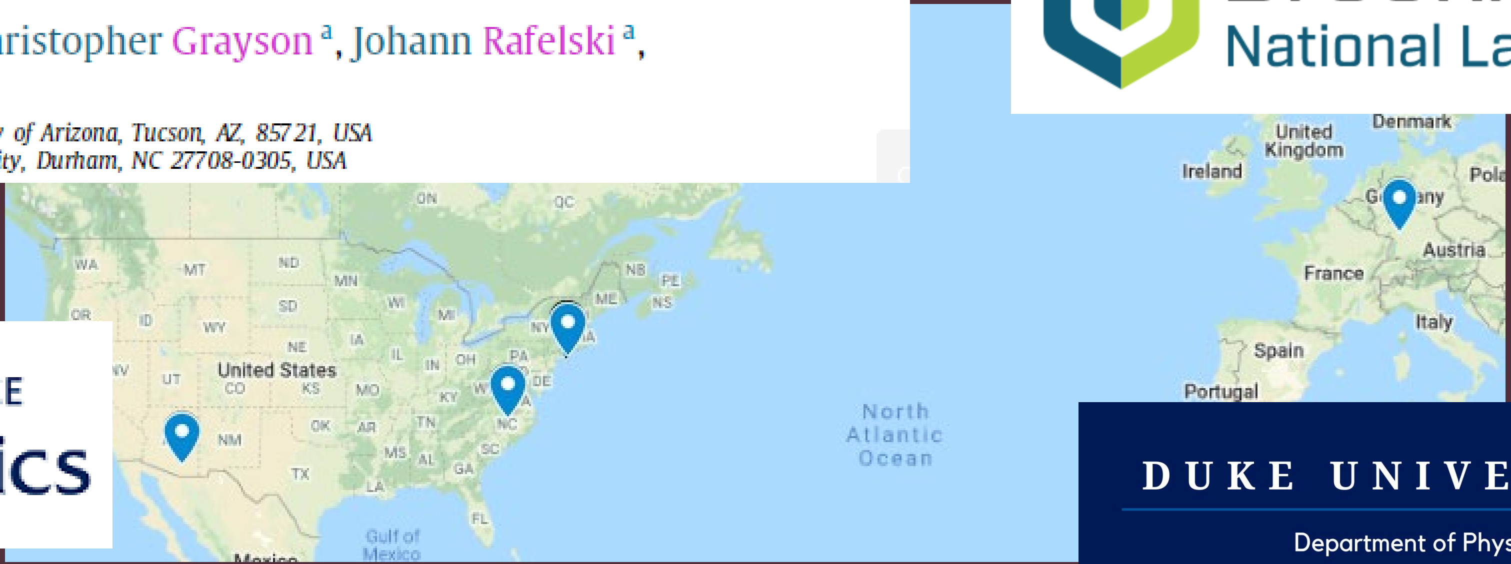

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Collision term

In general complicated (Groot, Leeuwen, Weert, *Relativistic kinetic theory*, 1980)

$$C[f,f] = \frac{1}{2}\int \frac{d^3p_1}{p_1^0}\frac{d^3p'}{p'^0}\frac{d^3p_1'}{p_1'^0}(f'f_1' - ff_1)W(p',p_1'|p,p_1)\,.$$

Transition rate

Simplest model-relaxation time approximation (RTA) (Anderson, Witting, Physica 74 (1974) 466)

$$C[f(x,p)] = (p^\mu u_\mu)\kappa[f_{\text{eq}}(p) - f(x,p)] \qquad \kappa = 1/\text{relaxation rate}$$

Does not conserve 4-current $\partial_\mu j^\mu_{ind} \neq 0$! Better (BGK) (Bhatnagar, Gross, Krook, Phys. Rev. 94 (1954) 511)

$$C[f(x,p)] = (p^u u_\mu)\kappa\left[f_{\text{eq}}(p)\frac{n(x)}{n_{\text{eq}}} - f(x,p)\right]$$

Designed explicitly to conserve current:

$$\left.\begin{aligned} n(x) &= 2\int (dp)(p\cdot u)f(x,p) \\ n_{eq} &= 2\int (dp)(p\cdot u)f_{\text{eq}}(p)\end{aligned}\right\} \; 2\int (dp)(p\cdot u)C[f(x,p)] = 2\int (dp)(p\cdot u)\left[f_{\text{eq}}(p)\frac{n(x)}{n_{\text{eq}}} - f(x,p)\right] = 0$$

Doesn't conserve energy and momentum!

$$\partial_\mu T^{\mu\nu} = \partial_\mu 2 \int (dp) p^\mu p^\nu f(x,p) \neq 0$$

But can be fixed too by adding more terms as shown very recently (Rocha, Denicol, Noronha, PRL 127 (2021) 042301) For particle-antiparticle plasma $T^{\mu\nu}$ conserved!

Altogether we are solving Boltzmann equation:

$$p \cdot \partial f(x,p) + \mathrm{qF}^{\mu\nu}\mathrm{p}_\nu \frac{\partial f(x,p)}{\partial p^\mu} = (p^u u_\mu)\kappa \left[f_{\mathrm{eq}}(p) \frac{n(x)}{n_{\mathrm{eq}}} - f(x,p) \right]$$

In the plasma rest frame: $p^\mu u_\mu = \mathrm{m}\gamma$

Without collision term solved in Blaizot, Iancu, Phys. Rep. 359 (2002) 355; Satow, PRD 90 (2014) 034018

method of characteristics PDR → ODR along trajectories $m\frac{dx^\mu}{d\tau} = p^\mu$.

Our solution (main result)

We want Fourier transformed current! In the linear order algebraic equation for $\widetilde{\delta f}(k,p)$ which can be solved.

4-current: $$\tilde{j}^{\mu}_{\text{ind}}(k) = 2q\int (dp)p^{\mu}\left[\widetilde{f_+}(k,p) - \widetilde{f_-}(k,p)\right] = 4q\int (dp)p^{\mu}\widetilde{\delta f}(k,p)$$

Ohm's law: $$\tilde{j}^{\mu}_{\text{ind}}(k) = \Pi^{\mu}_{\nu}(k)\tilde{A}^{\nu}(k)$$

Solution: $$\boxed{\Pi^{\mu}_{\nu}(k) = R^{\mu}_{\nu}(k) - \frac{Q^{\mu}(k)H_{\nu}(k)}{1+Q(k)},}$$

Where:

$$Q^{\mu}(k) = -\frac{4qi\kappa}{n_{\text{eq}}}\int (dp)\frac{(p\cdot u)f_{eq}(p)}{p\cdot k + i(p\cdot u)\kappa}p^{\mu} \qquad R^{\mu}_{\nu}(k) = -4q^2\int (dp)f'_{eq}(p)\frac{(u\cdot k)p^{\mu}p_{\nu} - (k\cdot p)p^{\mu}u_{\nu}}{p\cdot k + i(p\cdot u)\kappa}$$

$$Q(k) = -\frac{2i\kappa}{n_{\text{eq}}}\int (dp)\frac{(p\cdot u)^2 f_{\text{eq}}(p)}{p\cdot k + i(p\cdot u)\kappa} \qquad H_{\nu}(k) = -2q\int (dp)(p\cdot u)f'_{eq}(p)\frac{(u\cdot k)p_{\nu} - (k\cdot p)u_{\nu}}{p\cdot k + i(p\cdot u)\kappa}$$

Properties:

Gauge invariance: $$\left.\begin{aligned} \widetilde{j}'^{\mu}_{\text{ind}} &= \Pi^{\mu}_{\nu}\widetilde{A}'^{\nu} = \Pi^{\mu}_{\nu}(\widetilde{A}^{\nu} - ik^{\nu}\widetilde{\chi}), \\ \widetilde{j}^{\mu}_{\text{ind}} &= \Pi^{\mu}_{\nu}\widetilde{A}^{\nu} \end{aligned}\right\} \Rightarrow \Pi^{\mu}_{\nu}k^{\nu} = 0$$

Current conservation: $$k_\mu \widetilde{j}^{\mu} = 0 = k_\mu \Pi^{\mu}_{\nu}\widetilde{A}^{\nu}\,, \quad \Rightarrow \quad k_\mu \Pi^{\mu}_{\nu} = 0\,,$$

All the integrals can be calculated in ultra-relativistic limit – massless particles. Only two Independent components:

$$\Pi_T(k) = \frac{m_D^2\omega}{4|\boldsymbol{k}|}\left(\frac{\omega'^2}{|\boldsymbol{k}|^2}\Lambda - \Lambda - \frac{2\omega'}{|\boldsymbol{k}|}\right)$$

$$\Pi_L(k) = \frac{m_D^2\omega^2}{|\boldsymbol{k}|^2}\left(1 - \frac{\omega'}{2|\boldsymbol{k}|}\Lambda\right)\frac{2|\boldsymbol{k}|}{2|\boldsymbol{k}| - i\kappa\Lambda}$$

Where:

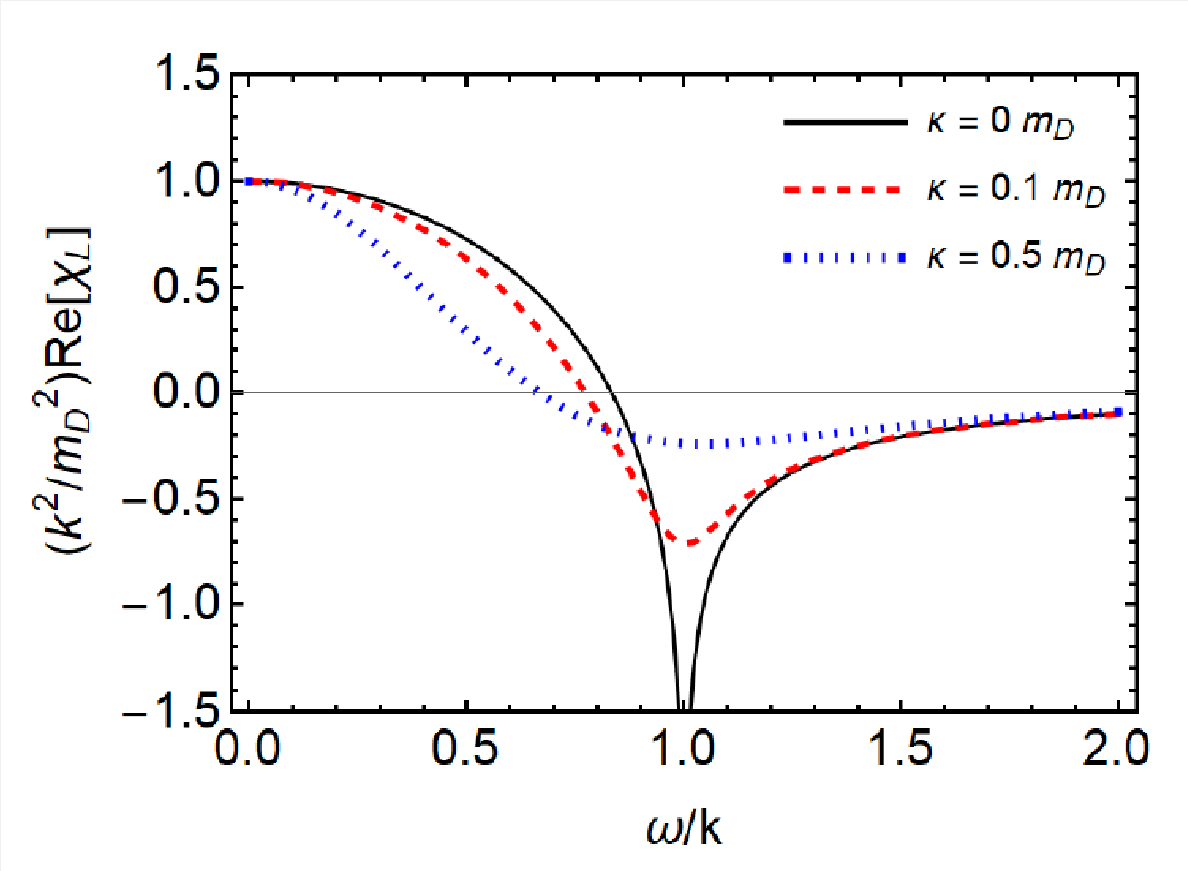
$$\omega' = \omega + i\kappa \qquad \Lambda = \ln\frac{\omega' + |\boldsymbol{k}|}{\omega' - |\boldsymbol{k}|}$$

Debay mass:

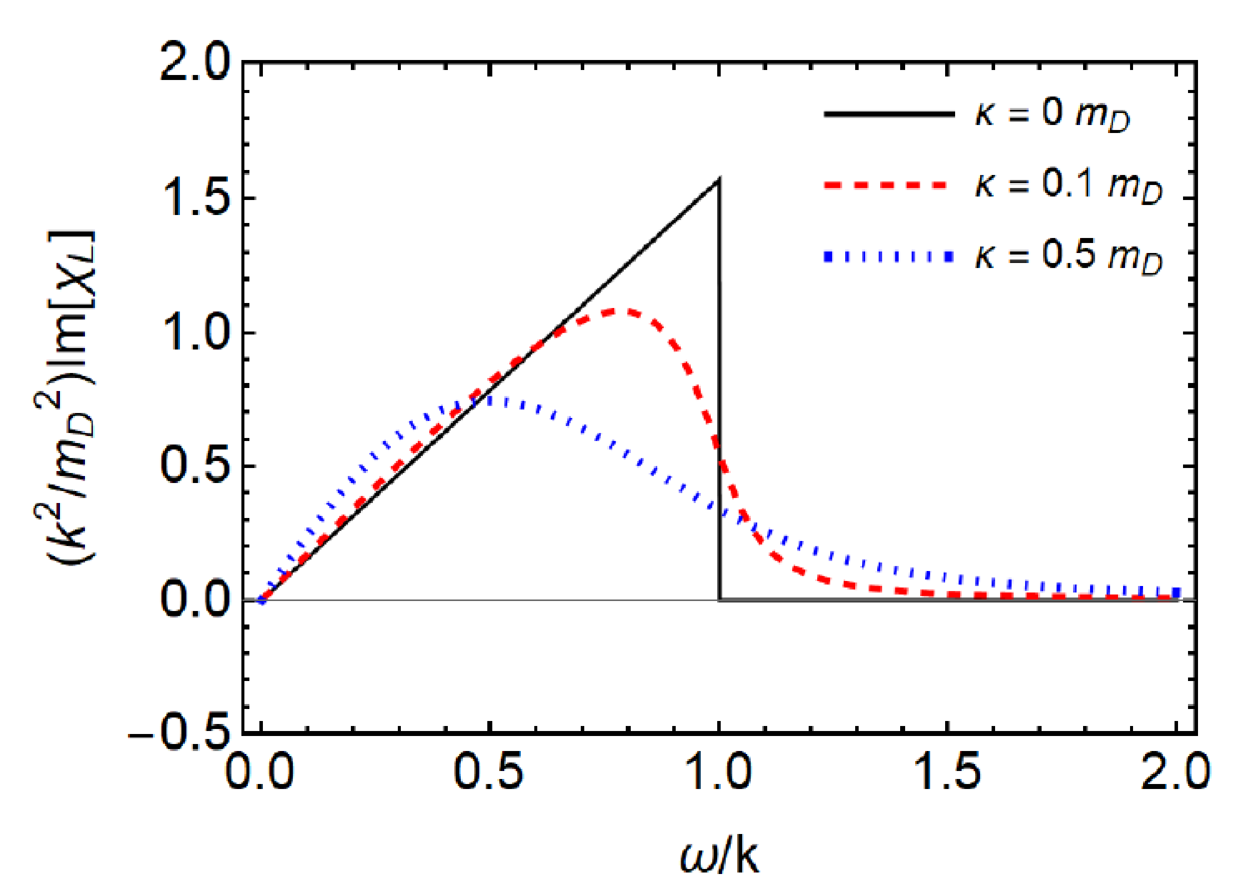
$$m_D^2 = -\frac{2q^2}{\pi^2}\int_0^\infty |\boldsymbol{p}|^2 d|\boldsymbol{p}| f'_{eq}(|\boldsymbol{p}|) = \frac{q^2T^2}{3}$$

Reduces to standard result Weldon, PRD 26 (1982) 1394 in the limit $\kappa \to 0$

Susceptibility $\chi_L = \Pi_L/\omega^2$ $\boldsymbol{D}_L = \varepsilon_0(1+\chi_L)\boldsymbol{E}_\mathrm{L}$



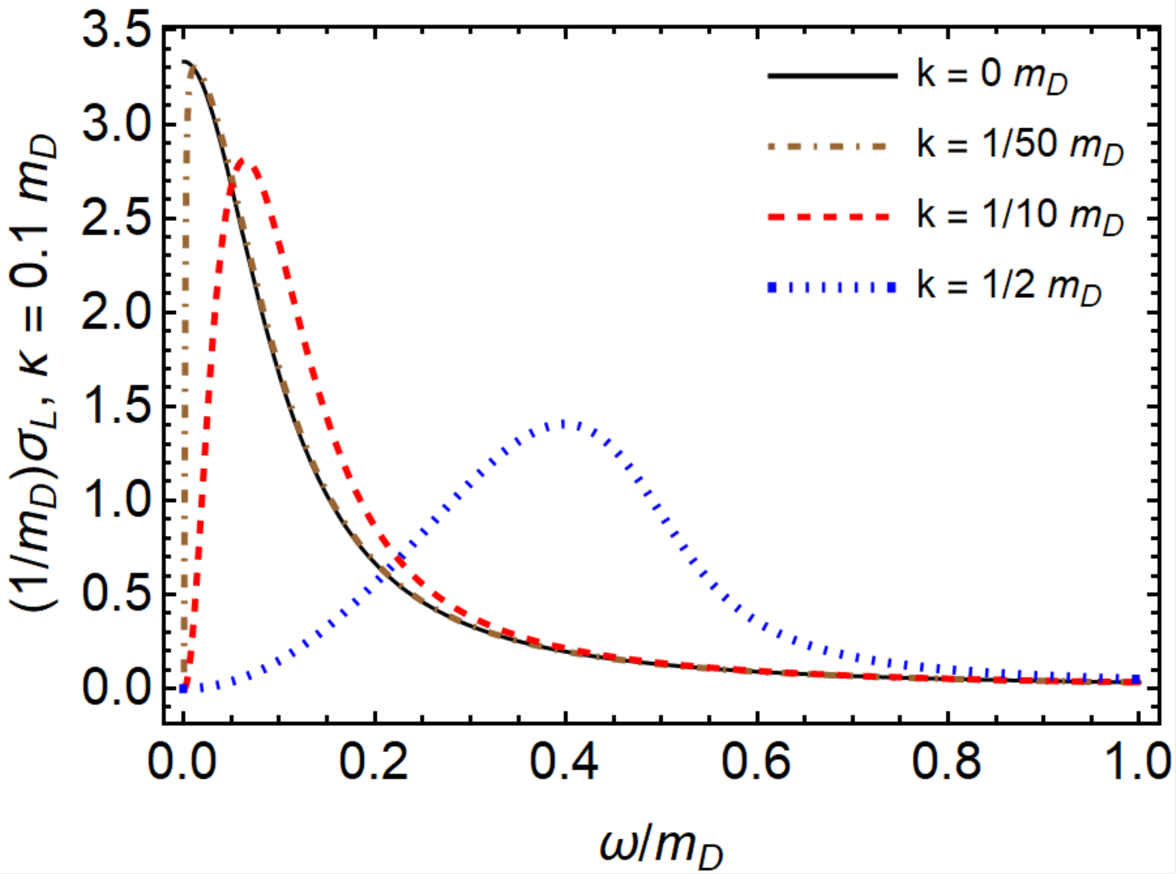
Real part



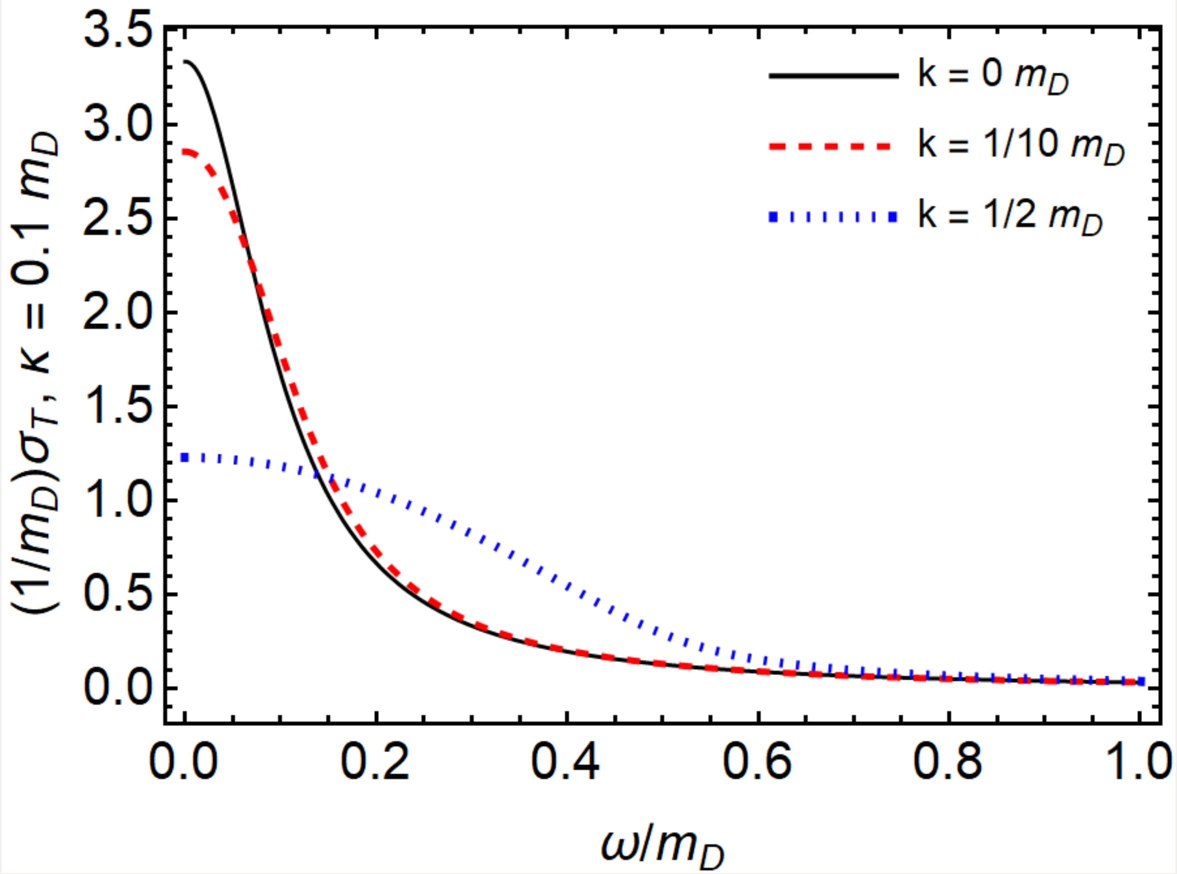
Imaginary part

For $\kappa = 0$ we reproduce Weldon, PRD 26 (1982) 1394

Conductivity $\sigma_{T/L} = -i\omega\Pi_{T/L}$



Real part of longitudinal conductivity



Real part of transversal conductivity

Discontinuity at $k = 0$ comes from infinite extent of plasma (Baranger, 1989)

Dispersion relations - $\boldsymbol{\omega(k)}$

$$\underbrace{\left[(k\cdot u)^2+\mu_0\Pi_L(k)\right)}_{\text{Longitudinal modes}}\underbrace{\left[k^2+\mu_0\Pi_T(k)\right]}_{\text{Transverse modes}}=0$$

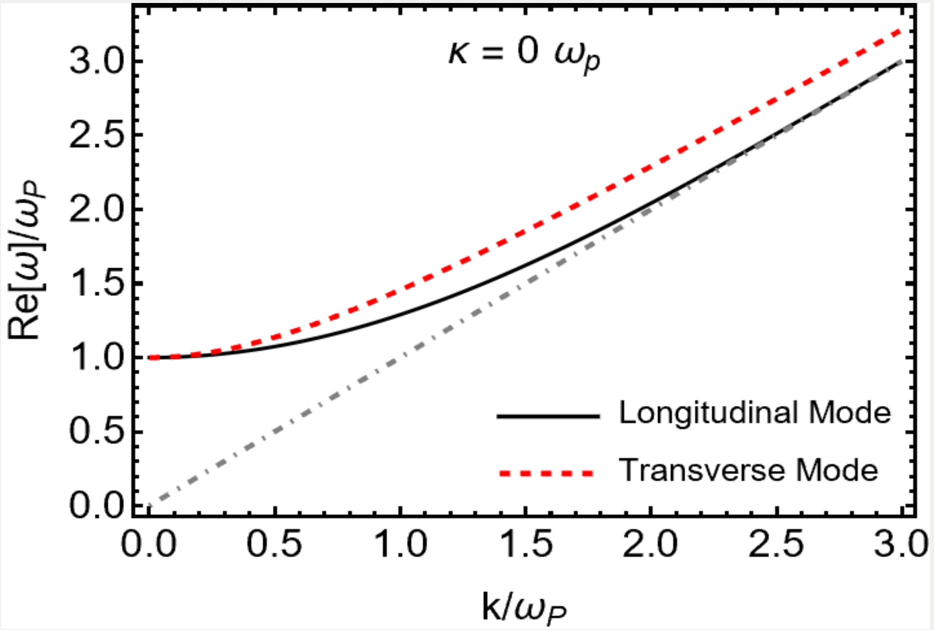
For $|\boldsymbol{k}|\to 0$ we have for both L,T:

$$\omega_\pm=-\frac{i\kappa}{2}\pm\sqrt{\omega_p^2-\frac{\kappa^2}{4}}$$

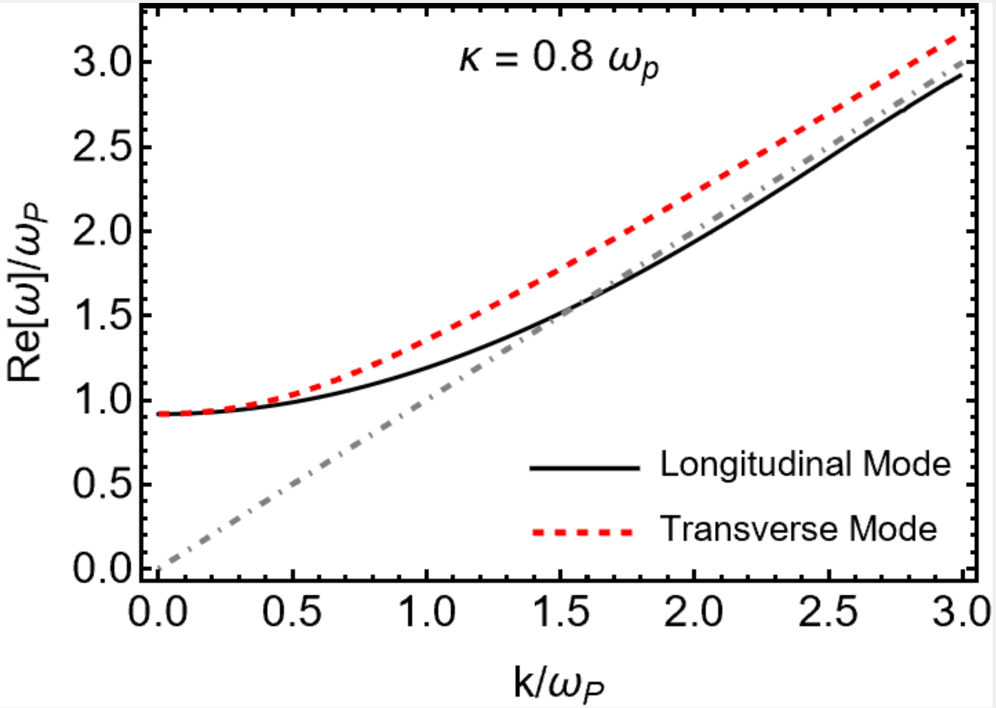
Modified plasma frequency $\omega_p=\frac{1}{3}m_D^2$

- $\kappa \ll \omega_p$ weekly damped case
- $\kappa > 2\omega_p$ overdamped case

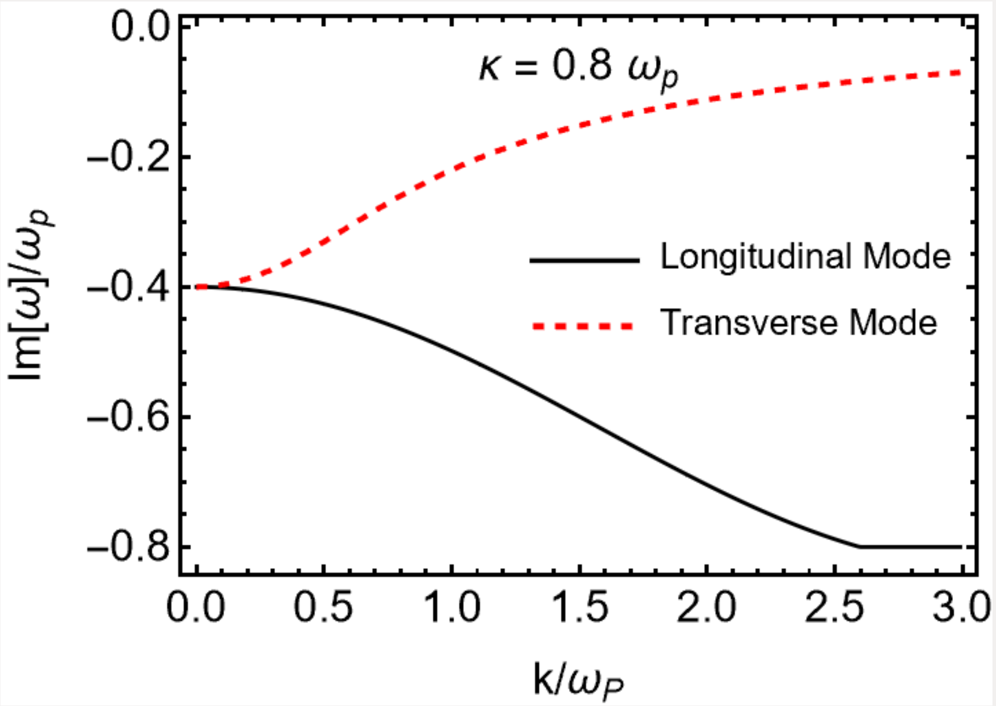
$\kappa=0, |\boldsymbol{k}|\neq 0$



$|\boldsymbol{k}| \neq 0, \kappa \neq 0$



Real part



Imaginary part

See also results of Carrington, Fugleberg, Pickering, Thoma, Can. J. Phys. 82 (2004) 671; B. Schenke, M. Strickland, C. Greiner, M.H. Thoma, Phys. Rev. D 73 (2006) 125004

Summary:

- Manifestly Lorentz covariant formulation of the plasma perturbation
- Including a collision term which preserves 4-current and energy momentum tensor
- Polarization tensor manifestly gauge invariant
- The results reduce to Weldon in the limit $\kappa \to 0$

Outlook:

- Generalization for multicomponent plasma
- application to QGP in heavy ion collisions (following talk by Chris)
- Finite plasma in space and time

Thank you for your attention!

Thank you for your hospitality in Budapest in Spring 2019! Hopefully visit will be possible again soon!

Stefan Evans Martin Formanek

Lake Balaton, June 2019