Electromagnetic Fields in Heavy Ion Collisions: QGP Plasma Response

Manuscript in Preparation: C. Grayson, M. Formanek, B. Müller and J. Rafelski. "Electromagnetic polarization of QGP"*

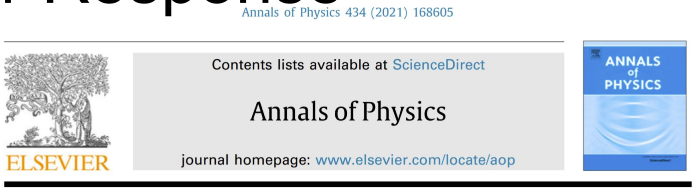
**Duke University*

Slides prepared for 21st Zimanyi School

Current-Conserving Relativistic Linear Response

Applying previous work to QGP

- Calculate induced electromagnetic fields of QGP due to colliding heavy Ion fields.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Current-conserving relativistic linear response for collisional plasmas

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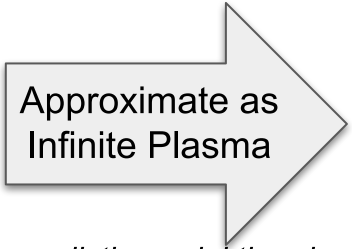
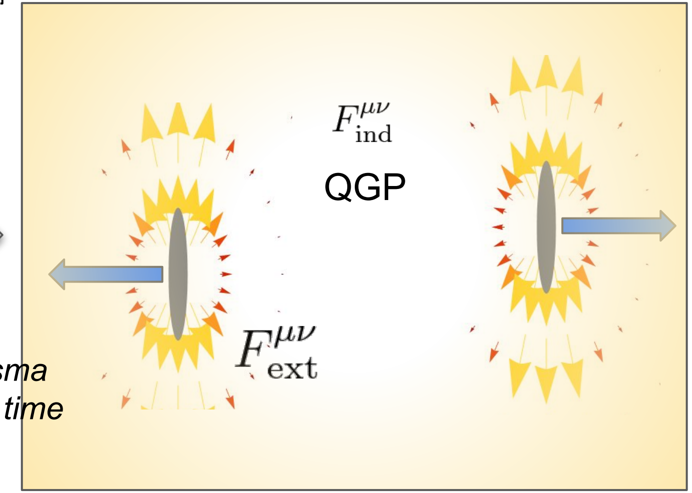
My 1rst paper

Electromagnetic Perturbations $\leftrightarrow$ QCD Plasma





In a more realistic model the plasma would be truncated in space and time

The polarization tensor can also be derived from other methods. For review see,

S. Mrowczynski, B. Schenke and M. Strickland, Phys. Rept. 682 (2017)
J. P. Blaizot and E. Iancu, Phys. Rept. 359, 355-528 (2002)

Ohm’s Law from the Vlasov Boltzmann Equation

Using the Vlasov-Boltzmann equation for each quark flavor one can calculate the induced electromagnetic current in linear response and identify the polarization tensor $\Pi^{\mu}_{\nu}(k)$ M. Formanek, C. Grayson, J. Rafelski and B. Müller, Annals Phys. 434, 168605 (2021)

$$(p \cdot \partial) f(x,p) + qF^{\mu\nu} p_{\nu} \frac{\partial f(x,p)}{\partial p^{\mu}} = (p \cdot u)\kappa \left(f_{\text{eq}}(p) \frac{n(x)}{n_{\text{eq}}} - f(x,p) \right)$$

κ is the damping parameter for QGP and is related to the inverse of the relaxation time 1/$\boldsymbol{\tau}$.

Where the current due to up, down, and strange quarks is,

$$\tilde{j}^{\mu}(k) = 2N_c \int \frac{d^4p}{(2\pi)^4} 4\pi\delta_{+}(p^2 - m^2) p^{\mu} \sum_{u,d,s} q_f (\tilde{f}_f(k,p) - \tilde{f}_{\bar{f}}(k,p))$$

The first order response of a medium to fields can be described in fourier space using a simple relation,

Ohm’s Law

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\tilde{j}^{\mu}_{\text{ind}}(k) = \Pi^{\mu}_{\nu}(k) \tilde{A}^{\nu}(k)$$

k is the 4 vector $k^{\mu} = (\omega, \mathbf{k})$

(R. Starke and G. A. H. Schober, Int. J. Mod. Phys. D 25, 1640010 (2016))

Where $\Pi^{\mu}_{\nu}(k)$ Is the polarization tensor $\tilde{j}^{\mu}_{\text{ind}}(k)$ is the fourier transformed induced current in the medium and $\tilde{A}^{\nu}(k)$ is the fourier transformed perturbing 4-potential of the colliding ions.

The polarization tensor can also be derived from other methods. For review see,
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$$(p\cdot\partial)f(x,p) + qF^{\mu\nu}p_{\nu}\frac{\partial f(x,p)}{\partial p^{\mu}} = (p\cdot u)\underline{\kappa}\left(f_{\mathrm{eq}}(p)\frac{n(x)}{n_{\mathrm{eq}}} - f(x,p)\right)$$

κ is the <u>QCD damping parameter</u> for QGP and is related to the inverse of the relaxation time $1/\tau$.

Where the current due to up, down, and strange quarks is,

$$\tilde{j}^{\mu}(k) = 2N_c\int\frac{d^4p}{(2\pi)^4}4\pi\delta_{+}(p^2-m^2)p^{\mu}\sum_{u,d,s}q_f(\tilde{f}_f(k,p)-\tilde{f}_{\bar{f}}(k,p))$$

Using this current one can calculate the polarization tensor using the method shown in the previous talk,

$$\begin{aligned}\tilde{j}^{\mu}(k) &= 2N_c\int(dp)p^{\mu}\sum_{u,d,s}q_f(\tilde{f}_f(k,p)-\tilde{f}_{\bar{f}}(k,p))\\ &= 4N_c\int(dp)p^{\mu}\sum_{u,d,s}q_f^2\widetilde{\delta f}(k,p)\\ &= 4N_c e^2\int(dp)p^{\mu}\left(\frac{1}{9}+\frac{4}{9}+\frac{1}{9}\right)\widetilde{\delta f}(k,p)\\ &= 8e^2\int(dp)p^{\mu}\widetilde{\delta f}(k,p)\,.\end{aligned}$$

$$\Pi^{\mu}_{\nu}(k) = \left(R^{\mu}_{\nu}(k) - \frac{Q^{\mu}(k)H_{\nu}(k)}{1+Q(k)}\right)$$

See previous talk

The polarization tensor is identical besides a change in the debye mass due to the extra degrees of freedom.

$$m_{D(EM)}^{2} = \sum_{u,d,s}q_f^2T^2\frac{N}{3} = 2\frac{e^2T^2}{3}$$

Electromagnetic Polarization Tensor: Infinite Medium

The polarization tensor for an infinite homogeneous plasma is composed of two independent response functions which can be found by projecting onto ***k*** and considering longitudinal and transverse polarization functions.

$$\Pi^{\mu}_{\nu} = \begin{bmatrix} -\frac{|\mathbf{k}|^2}{\omega^2}\Pi_L & 0 & 0 & \frac{|\mathbf{k}|}{\omega}\Pi_L \\ 0 & \Pi_T & 0 & 0 \\ 0 & 0 & \Pi_T & 0 \\ -\frac{|\mathbf{k}|}{\omega}\Pi_L & 0 & 0 & \Pi_L \end{bmatrix}$$

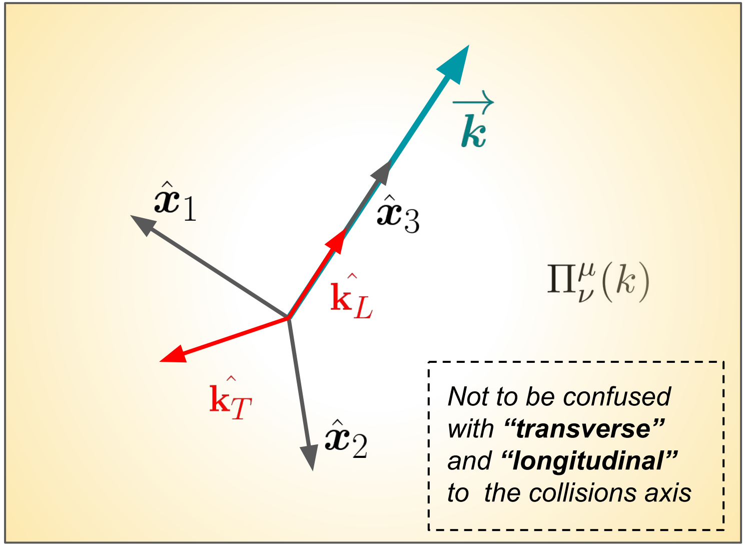
Calculated in the ultrarelativisitic limit

$$\Pi_L = m_D^2 \frac{\omega}{|\mathbf{k}|}\left(1 + \frac{1}{\frac{1}{\frac{i\kappa}{2|\mathbf{k}|}\ln\frac{\omega+i\kappa+|\mathbf{k}|}{\omega+i\kappa-|\mathbf{k}|}} - 1}\right)\left(1 - \frac{\omega+i\kappa}{2|\mathbf{k}|}\ln\frac{\omega+i\kappa+|\mathbf{k}|}{\omega+i\kappa-|\mathbf{k}|}\right)$$

$$\Pi_T = m_D^2 \frac{\omega}{4|\mathbf{k}|}\left(\left(\frac{\omega^2}{|\mathbf{k}|^2} - 1\right)\ln\frac{\omega+i\kappa+|\mathbf{k}|}{\omega+i\kappa-|\mathbf{k}|} - \frac{2(\omega+i\kappa)}{|\mathbf{k}|}\right)$$

Where m_D is the electromagnetic debye mass of the plasma

$$m_{D(EM)}^2 = \sum_{u,d,s} q_f^2 T^2 \frac{N}{3} = 2\frac{e^2T^2}{3}$$



*Here we have chosen **k** to point along the third component **x_3**.*

For a detailed description of infinite homogeneous plasmas see
D. Melrose, Quantum Plasmadynamics: Unmagnetized Plasmas, Lect. Notes Phys. 735 (Springer, New York, 2008).

Estimating Quark-Quark Collision Rate

To get a perturbative estimate of κ we multiply the parton-quark transport cross section by the parton density, (Mrowczynski, Acta Phys. Polon 1988)

$$\kappa_{(\mathrm{QCD})} = \frac{1}{136\pi^3} 5(9N_f + 16)\zeta(3) g^4 \ln(4\pi/g^2) T$$

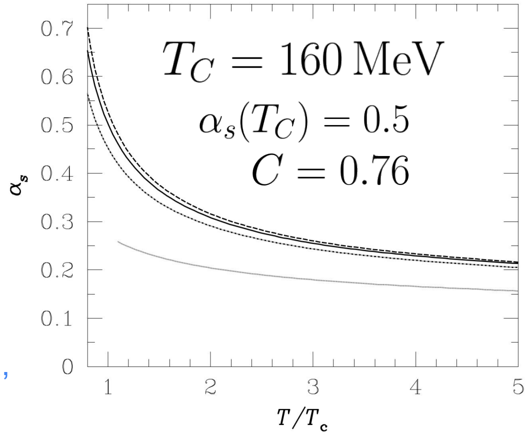
Where we model how the strong coupling varies with temperature using a fit.

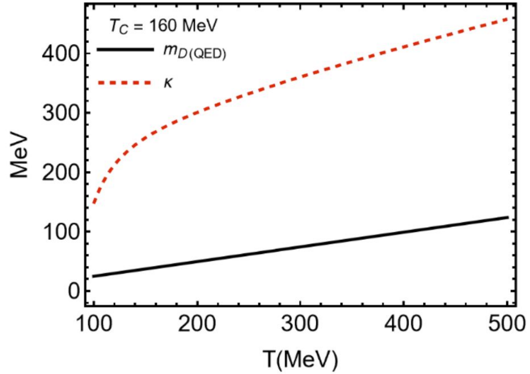
$$\alpha_s(T) \approx \frac{\alpha_s(T_C)}{1 + C\ln(T/T_C)}$$

$$\alpha_s = g^2/4\pi$$

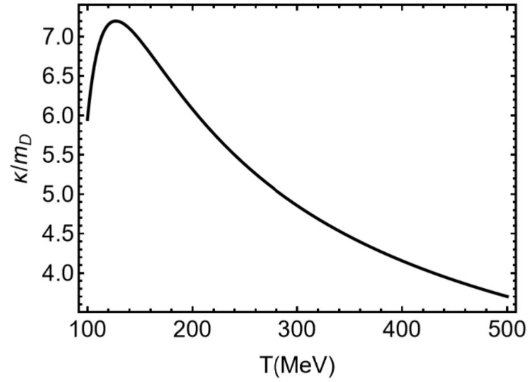
(Hadrons and Quark Gluon plasma, Letessier, J.,Rafelski 2002)

QCD, hadronic structure and high temperature





Plot of the QED debye mass and the QCD dampening rate κ as a function of temperature



Plot of the ratio of the QCD dampening rate κ to the QED debye mass as a function of temperature, At temperature 300 MeV used in the plots below, $\kappa = 4.86\, m_D$

Estimating Quark-Quark Collision Rate

To get a rough estimate of κ we multiply the parton-quark transport cross section by the parton density, (Mrowczynski, Acta Phys. Polon 1988)

$$\kappa_{(\mathrm{QCD})} = \frac{1}{136\pi^3} 5(9N_f + 16)\zeta(3) g^4 \ln(4\pi/g^2) T$$

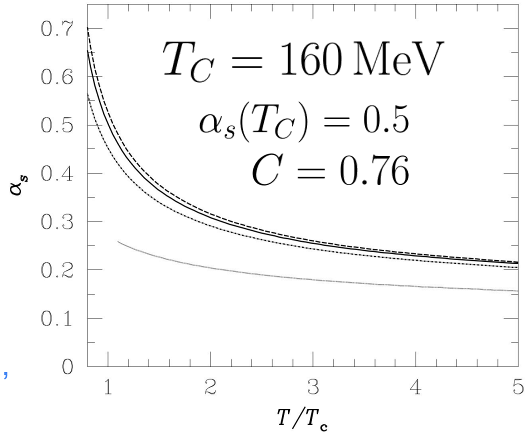
Where we have also included that the coupling is a function of temperature

$$\alpha_s(T) \approx \frac{\alpha_s(T_C)}{1 + C\ln(T/T_C)}$$

$$\alpha_s = g^2/4\pi$$

(Hadrons and Quark Gluon plasma, Letessier, J.,Rafelski 2002)

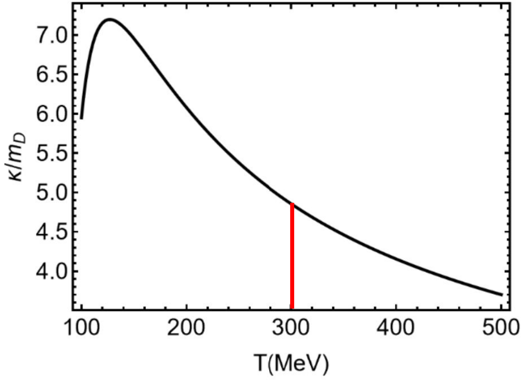
QCD, hadronic structure and high temperature



QCD dampening is much larger than QED debye mass, plasma oscillations are mostly damped.

$$\kappa_{(\mathrm{QCD})} \approx 5\, m_{D(\mathrm{EM})}$$

At T = 300 MeV.



Plot of the ratio of the QCD dampening rate κ to the QED debye mass as a function of temperature, At temperature 300 MeV used in the plots below, $\kappa = 4.86\, m_D$

Free Charge Distribution

For simplicity we prescribe the external fields using a gaussian charge distribution normalized to the radius and charge of the nucleus.

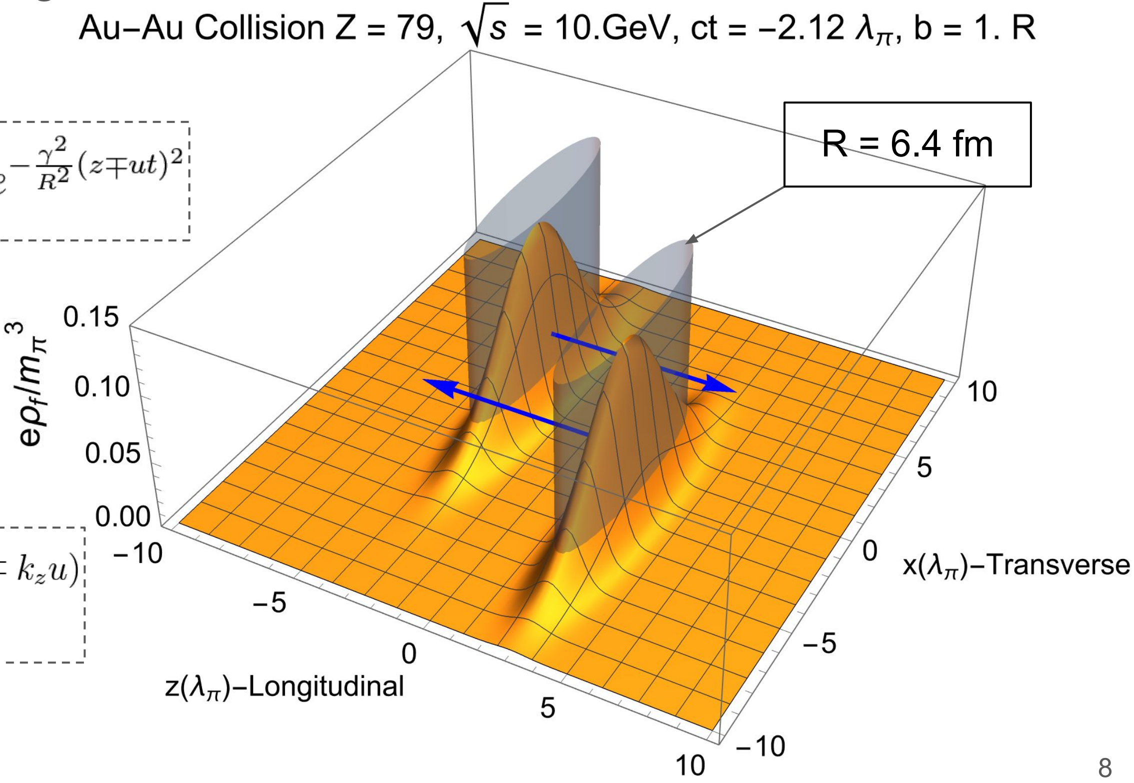
Free Charge in Position Space

$$\rho_{\pm b/2}(t,\boldsymbol{x}) = \frac{Zq\gamma}{\pi^{3/2}R^3} e^{-\frac{1}{R^2}(x\mp b/2)^2} e^{-\frac{1}{R^2}y^2} e^{-\frac{\gamma^2}{R^2}(z\mp ut)^2}$$

Free Charge in Fourier Space

$$\widetilde{\rho}_{\pm b/2}(\omega,\boldsymbol{k}) = 2\pi Zqc\, e^{-(k_x^2+k_y^2+k_z^2/\gamma^2)\frac{R^2}{4}} e^{\mp\frac{ik_x b}{2}} \delta(\omega \mp k_z u)$$

$$\widetilde{\boldsymbol{j}}_{\pm b/2}(\omega,\boldsymbol{k}) = \pm u\hat{\boldsymbol{z}}\widetilde{\rho}_{\pm b/2}(\omega,\boldsymbol{k})$$



Induced Fields and currents

In order to calculate the induced charge and field using the longitudinal and transverse response functions one must project vector perturbations onto ***k***

$$\widetilde{\boldsymbol{A}}_T(\omega,\boldsymbol{k}) = \widetilde{\boldsymbol{A}} - \frac{\boldsymbol{k}\cdot\widetilde{\boldsymbol{A}}}{|\boldsymbol{k}|}\hat{\boldsymbol{k}} \qquad \widetilde{A}_L(\omega,\boldsymbol{k}) = \frac{\boldsymbol{k}\cdot\widetilde{\boldsymbol{A}}}{|\boldsymbol{k}|}$$

Thus are 4-vectors in the different coordinates are,

$$\widetilde{A}^\mu(\omega,\boldsymbol{k}) = (\widetilde{\phi},\ \widetilde{\boldsymbol{A}}) \ \Rightarrow\ (\widetilde{\phi},\ \widetilde{\boldsymbol{A}}_T,\ \widetilde{A}_L)$$

$$\widetilde{j}^\mu(\omega,\boldsymbol{k}) = (\widetilde{\rho},\ \widetilde{\boldsymbol{j}}) \ \Rightarrow\ (\widetilde{\rho},\ \widetilde{\boldsymbol{j}}_T,\ \widetilde{j}_L)$$

Then solving maxwell's equations in projected space one find the self consistent potentials,

$$e\widetilde{\phi} = \alpha\hbar c\frac{\widetilde{\rho}}{(k^2-\omega^2)\left(\frac{\Pi_L}{\omega^2}-1\right)} \qquad \widetilde{A}_L = \frac{\omega}{|\boldsymbol{k}|}\widetilde{\phi}$$

$$ec\widetilde{\boldsymbol{A}}_T = \alpha\hbar c\frac{\widetilde{\boldsymbol{j}}_T}{k^2-\omega^2-\Pi_T}$$

The self consistent potentials can be used to calculate the self consistent fields.

$$\widetilde{\boldsymbol{E}} = -i\boldsymbol{k}\widetilde{\phi} + i\omega\widetilde{\boldsymbol{A}}$$

$$\widetilde{\boldsymbol{B}} = i\boldsymbol{k}\times\widetilde{\boldsymbol{A}} = i\boldsymbol{k}\times\widetilde{\boldsymbol{A}}_T$$

Then we fourier transform the fields back to position space

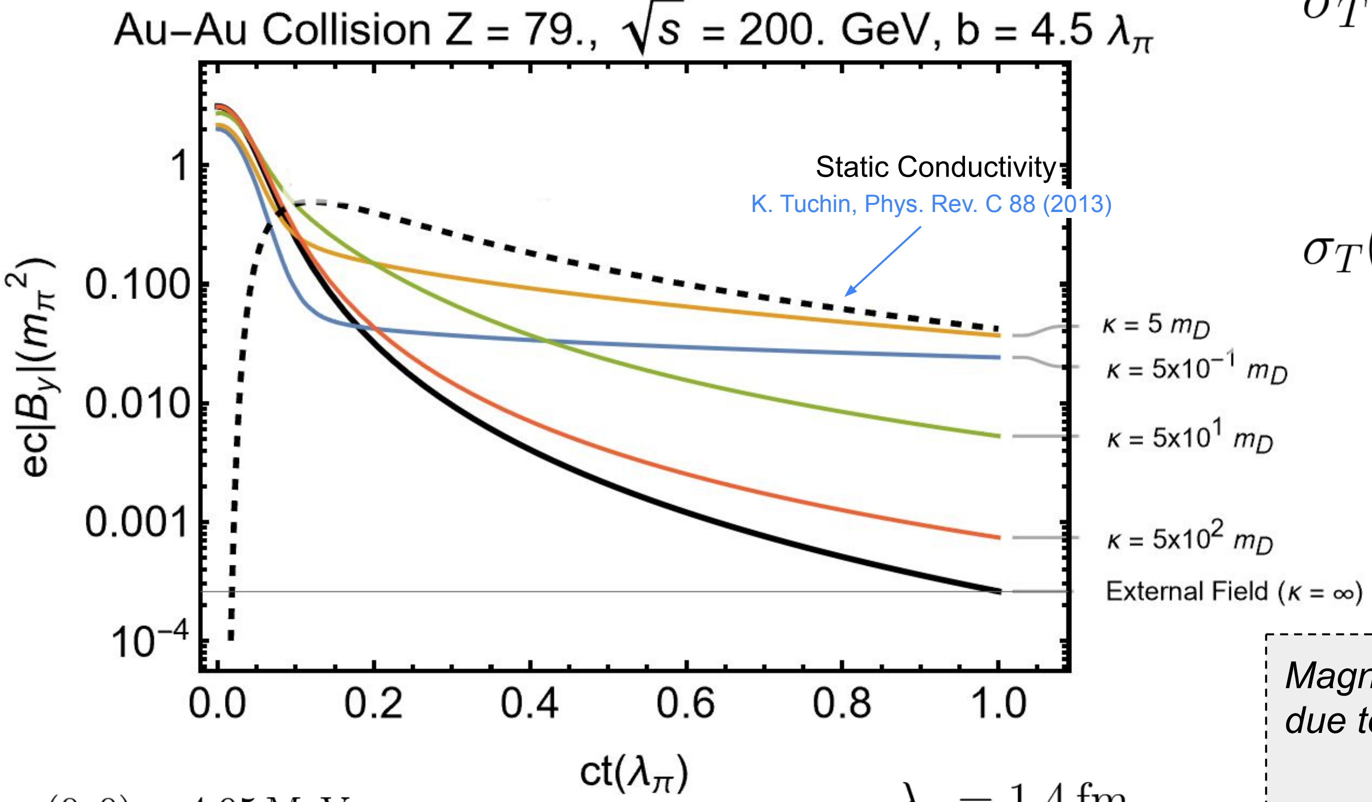
$$B_y(x) = \int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot x}\widetilde{B}_y(k) = \int\frac{d^4k}{(2\pi)^4}e^{-i\omega t+i\boldsymbol{k}\cdot\boldsymbol{x}}\widetilde{B}_y(\omega,\boldsymbol{k})$$

$$B_y(t,0) = \int\frac{d^4k}{(2\pi)^4}e^{-i\omega t}\widetilde{B}_y(\omega,\boldsymbol{k})$$

Magnetic field at the origin

Magnetic Field of Heavy Ion Collisions at the Origin,

Electric field is shown in supplemental slides



$$\sigma_T(\omega, \boldsymbol{k}) = -i\frac{\Pi_T}{\omega}$$

$$\sigma_T(\omega, 0) = \frac{\omega_p^2/\kappa}{1 - i\omega/\kappa}$$

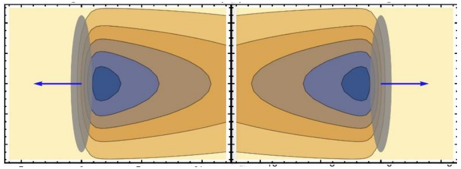
As the QCD dampening κ goes to Infinity the conductivity goes to zero.

Magnetic field persists due to induced currents.

$\sigma_T(0,0) = 4.95\,\mathrm{MeV}$

$\lambda_\pi = 1.4\,\mathrm{fm}$

Induced Charge - (Single Nuclei)



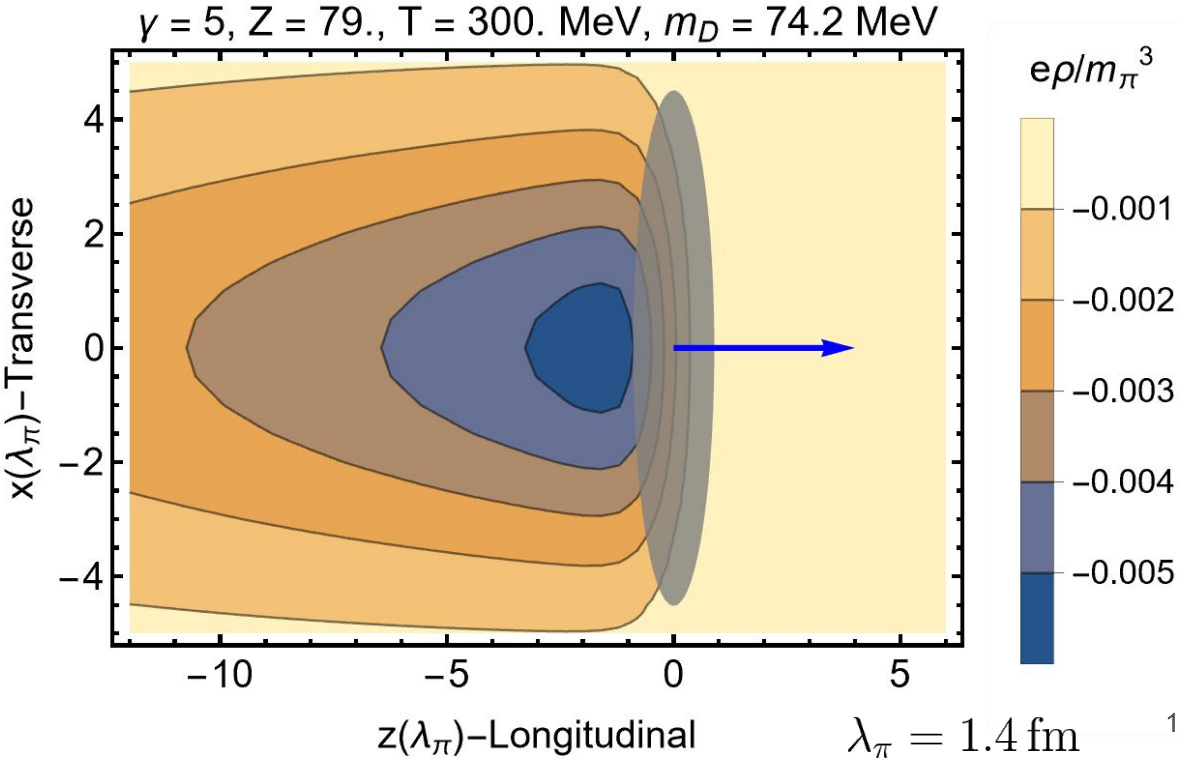
Plot below shows a single ion traveling through the plasma. A trailing negatively charged wake is shown trailing behind the positive Ion.

$$\widetilde{j}^{\mu}_{\text{ind}}(k) = \Pi^{\mu}_{\nu}(k)\widetilde{A}^{\nu}(k)$$

$$\widetilde{\rho}_{\text{ind}} = \Pi_L \widetilde{\phi}\left(1 - \frac{|\boldsymbol{k}|^2}{\omega^2}\right)$$

$$\widetilde{\boldsymbol{j}}^{T}_{\text{ind}} = \Pi_T \widetilde{\boldsymbol{A}}_T \quad \widetilde{j}^{L}_{\text{ind}} = \frac{\omega}{|\boldsymbol{k}|}\widetilde{\rho}_{\text{ind}}$$

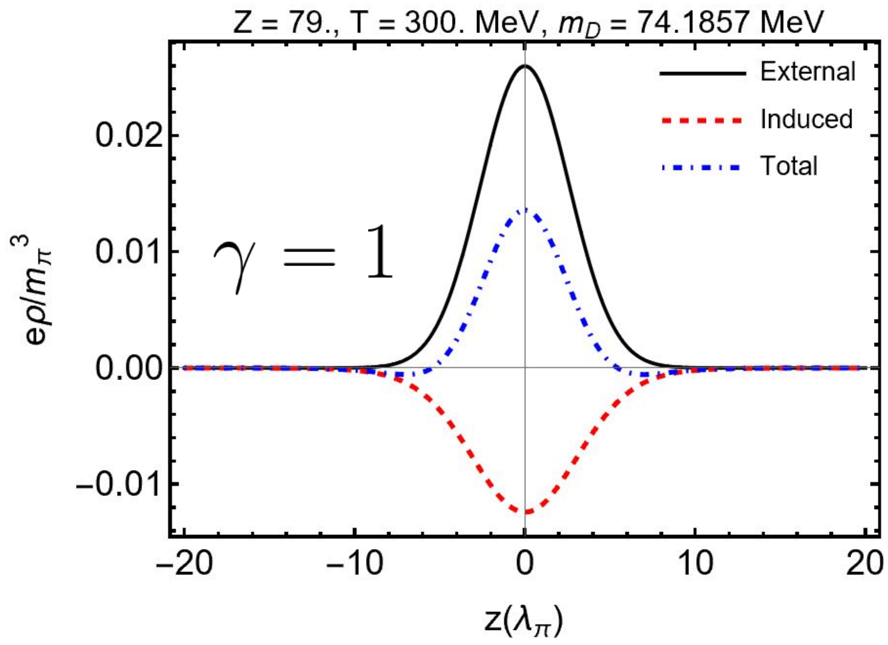
$$\kappa_{(\text{QCD})} \approx 5\, m_{D(\text{EM})}$$

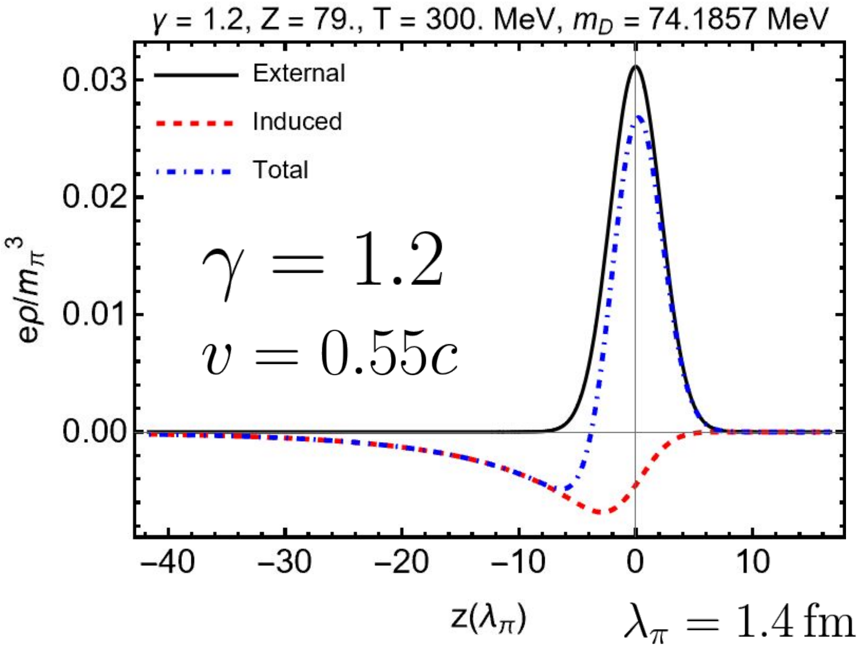


$\lambda_\pi = 1.4\,\text{fm}$

Induced Charge

As γ increases the external charge density is contracted (grows large in magnitude but smaller in space).The functional dependence of the tail changes little so appears to be small and more smeared out at high gamma.





$\lambda_\pi = 1.4\,\mathrm{fm}$

$$m_{D\,(EM)}^{\;2} = \sum_{u,d,s} q_f^2 T^2 \frac{N}{3} = 2\frac{e^2T^2}{3}$$

$$\kappa_{(\mathrm{QCD})} \approx 5\, m_{D(\mathrm{EM})}$$

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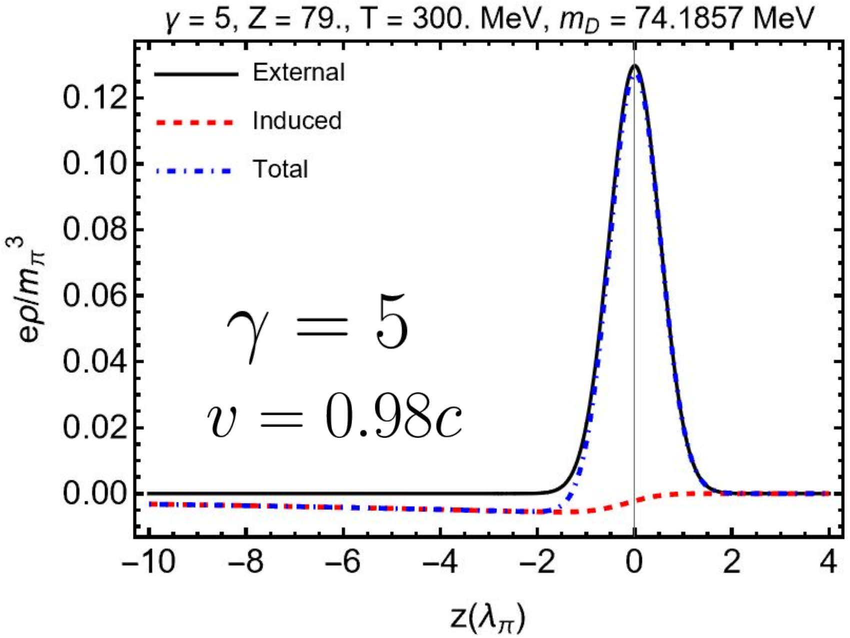
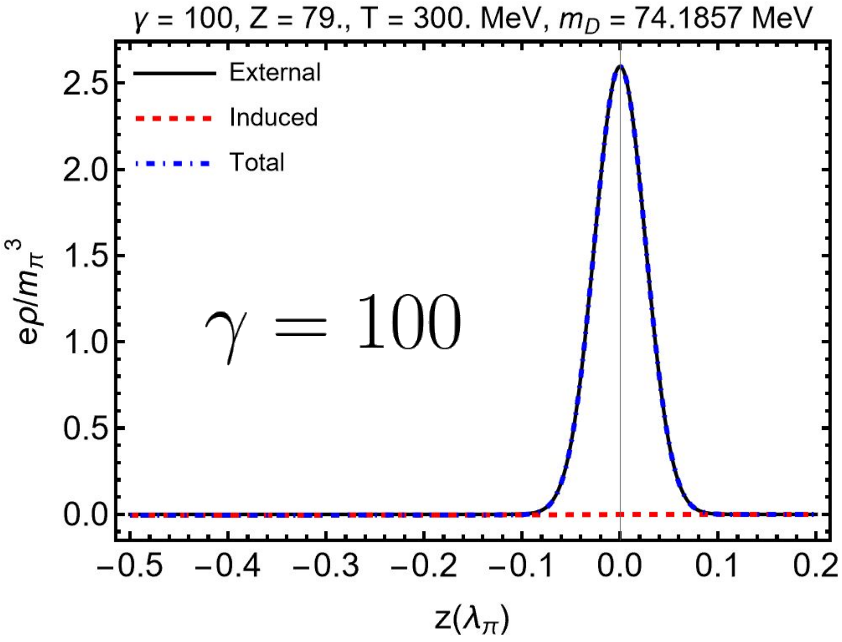




$$m_{D\,(EM)}^{\;2} = \sum_{u,d,s} q_f^2 T^2 \frac{N}{3} = 2\frac{e^2 T^2}{3}$$

$$\kappa_{(\mathrm{QCD})} \approx 5\, m_{D(\mathrm{EM})}$$

Outlook: Just at the Beginning

- **QGP Switch-On and Evolution:** In order to accurately model in space and time QGP we would like to create a realistic spacetime picture of the plasma.
  - This can be done by adding a source to the neutral plasma in the boltzmann equation

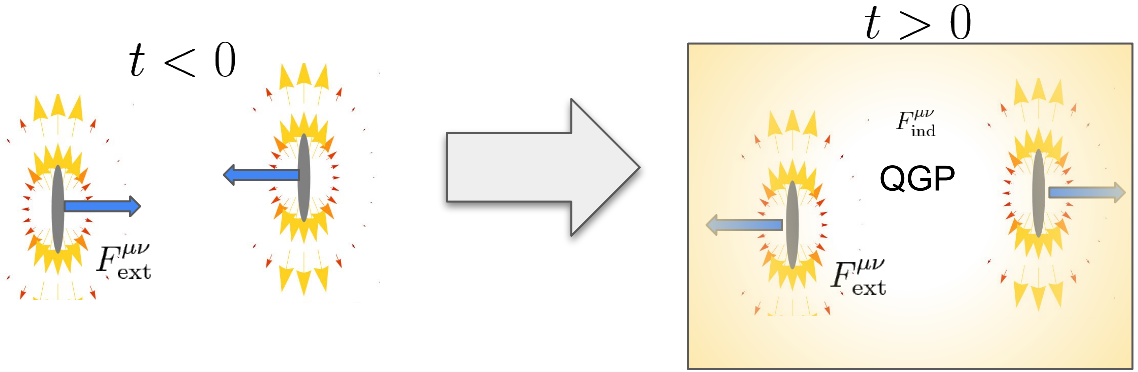
$$(p\cdot\partial)f(x,p)+qF^{\mu\nu}p_\nu\frac{\partial f(x,p)}{\partial p^\mu}=(p\cdot u)\kappa\left[f_{\rm eq}(p)\frac{n(x)}{n_{\rm eq}}-f(x,p)\right]+\boxed{(p\cdot u)f_{\rm eq}(p)\Delta(u_\mu(x^\mu-x_0^\mu))}$$

The leads to a convolution integral in Ohm's Law of the form,

$$\widetilde{j}^{\mu}_{\rm ind}(\omega,\boldsymbol{k})=\int\frac{d\omega'}{2\pi}\Pi^\mu_\nu(\omega',\omega,\boldsymbol{k})\widetilde{A}^\nu(\omega',\boldsymbol{k})\tilde{\Theta}(\omega-\omega')$$

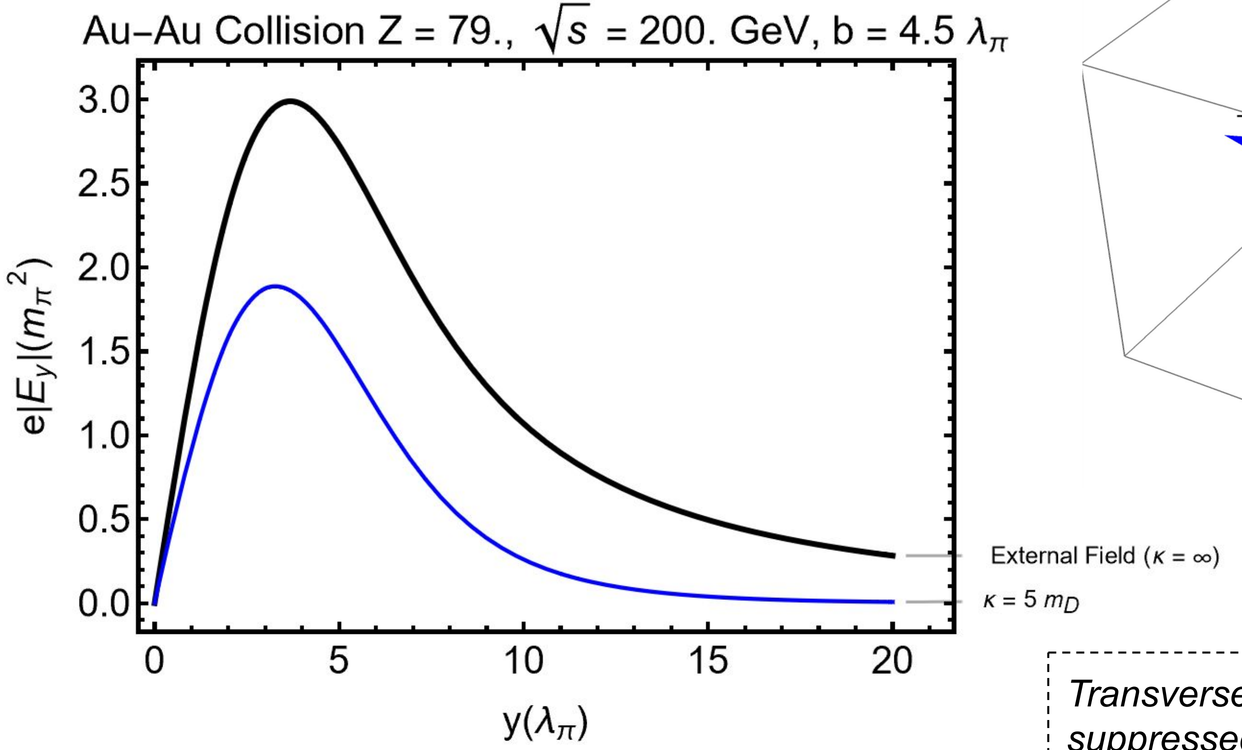
where,

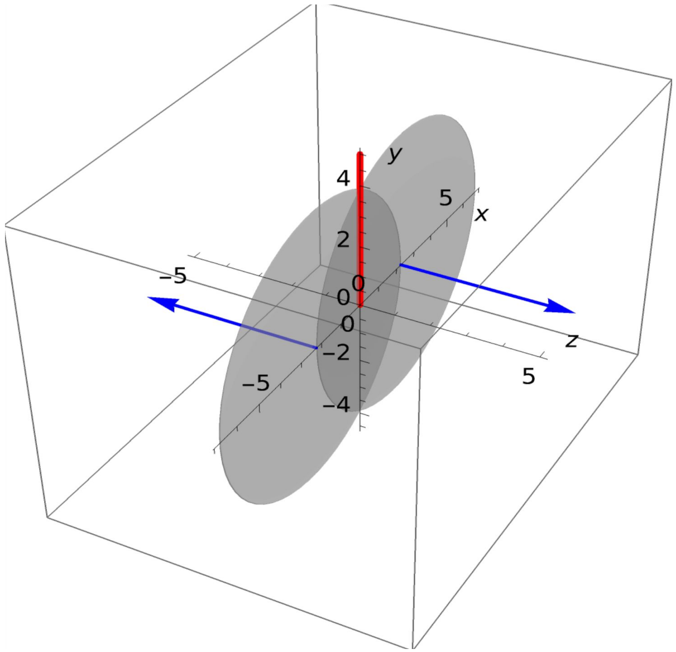
$$\Theta(t)=\int_{-\infty}^{t}dt'\Delta(t')$$



Supplemental Slides

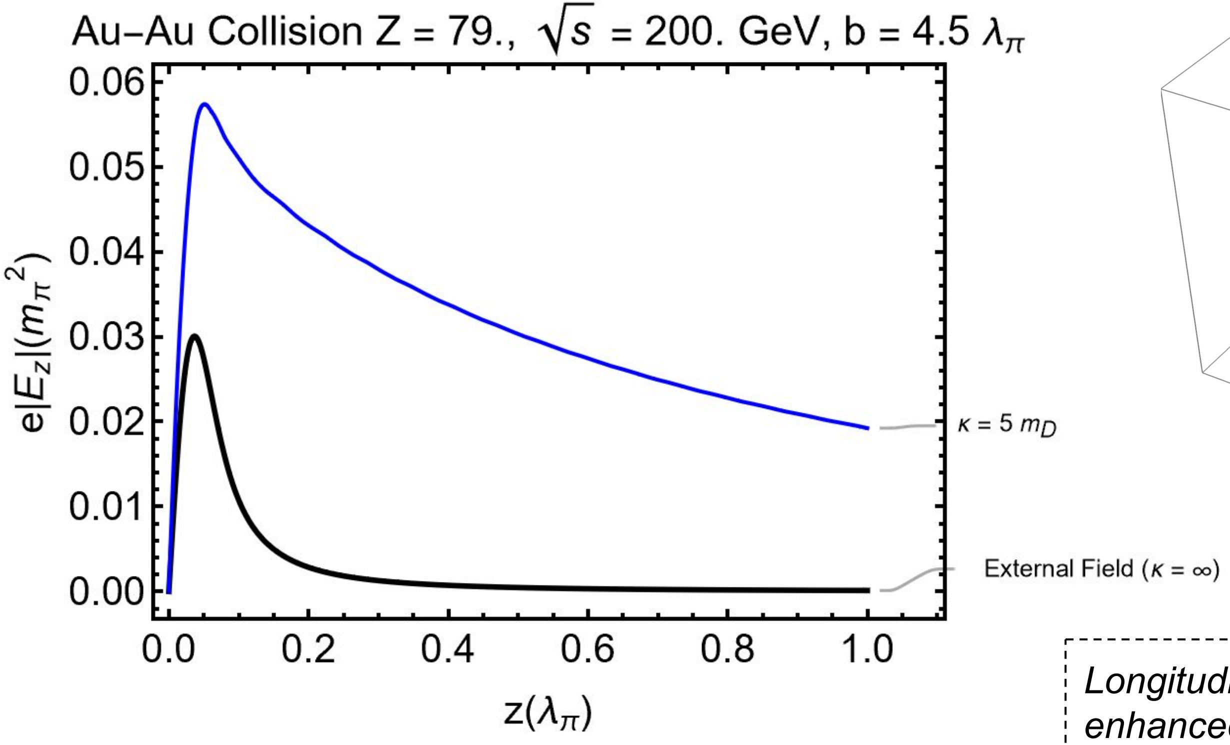
Electric Field

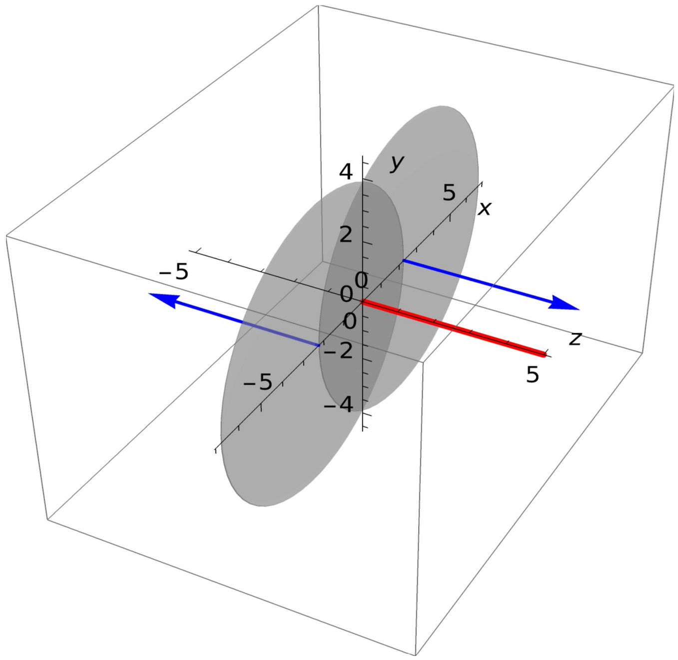




Transverse electric field is suppressed.

Electric Field



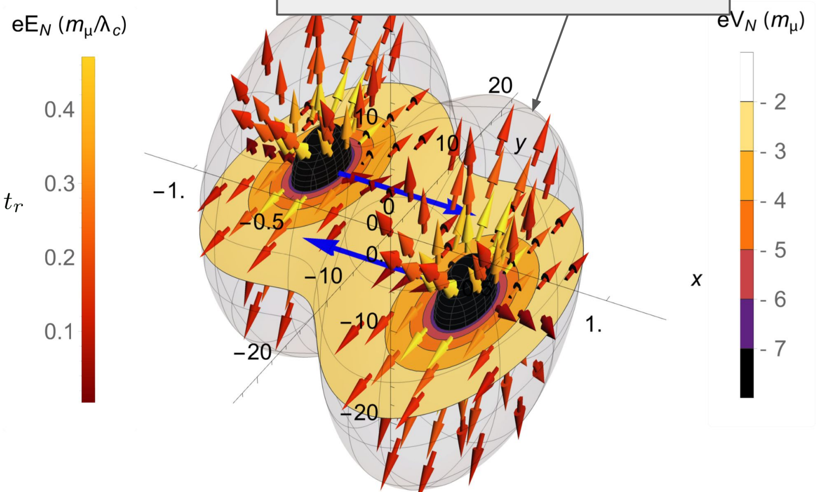


Longitudinal Electric field is enhanced

Fields of Colliding Ions

$$
e\mathbf{E}(\mathbf{r},t) = Z\alpha\hbar c \left(\underbrace{\frac{(\mathbf{n}-\beta)}{\gamma^2(1-\mathbf{n}\cdot\beta)^3|\mathbf{r}-\mathbf{r}_s|^2}}_{\text{Velocity Field}} + \underbrace{\frac{\mathbf{n}\times((\mathbf{n}-\beta)\times\dot{\beta})}{c(1-\mathbf{n}\cdot\beta)^3|\mathbf{r}-\mathbf{r}_s|}}_{\text{Radiation Field}} \right)_{t_r}
$$

Surface indicates where potential reaches $2m_\mu$



Blue arrows indicate direction of motion

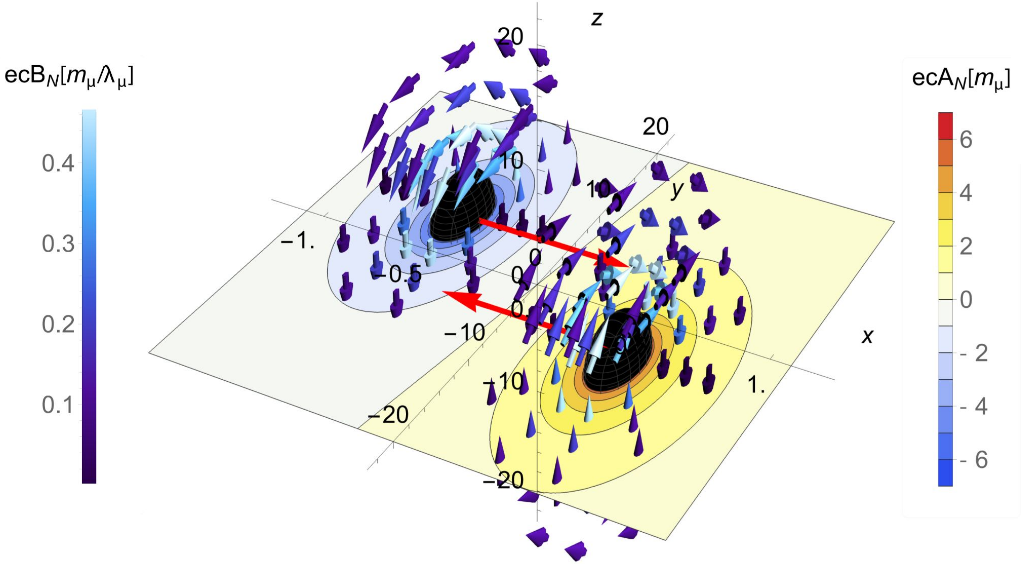
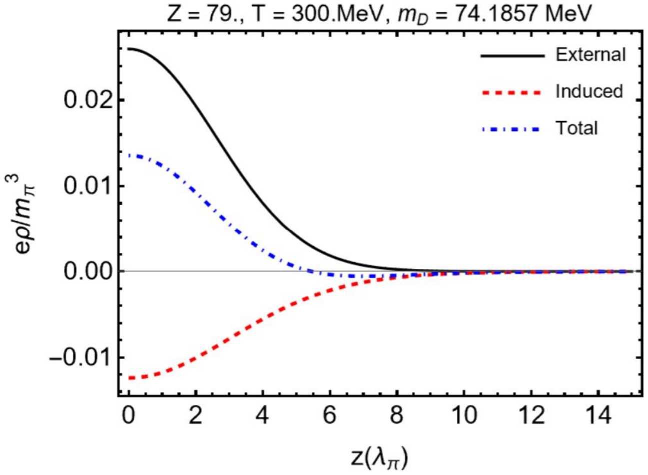
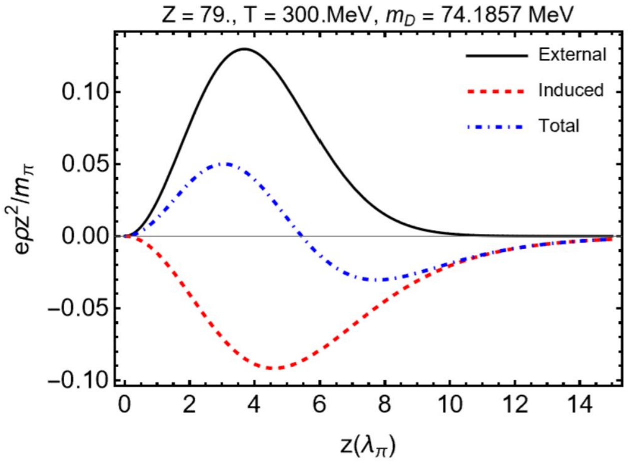


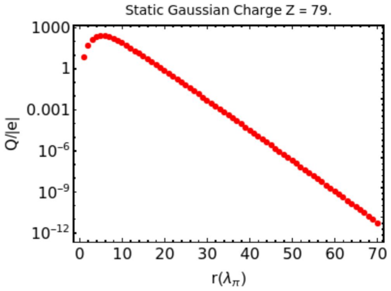
Figure 19: Here we see the fields and gauge potential of two colliding Pb ions at $\gamma = 37$ (about 74 GeV per nucleon pair in the CM frame) and impact parameter $b = 3R$ where the ions are separated by $\Delta x = 2\beta ct = 1\lambda_\mu$, where λ_μ is the muon Compton wavelength, approximately 1.87 fm. At this collision energy the potential of the ions surpasses $2m_\mu$ over a distance larger than the muon compton wavelength λ_μ The lorentz contracted nuclei are indicated by black surfaces traveling in the x direction (collision axis). (a) the potential is plotted in units of m_μ in the xy-plane (collision plane) and the electric field vectors are shown in units $\frac{m_\mu}{\lambda_\mu}$ where in these units the Schwinger field $eE_s = 1\frac{m_\mu}{\lambda_\mu}$. The $2m_\mu$ barrier, at which the Dirac equation predicts boundstates to dive in to the negative continuum, [110] is indicated by a gray surface.

Charge Conservation in the Infinite Plasma

In the case of an infinite plasma in both space and time the induced charge balancing the induced screening charges has moves to infinity.







Plasma Frequency

We can use the dispersion relation to solve for the natural oscillations of the effective particles within the plasma i.e. plasmons, This is done by solving the dispersion relation in the limit k→0,

$$\frac{1}{(k\cdot\tilde{u})}\left((k\cdot\tilde{u})^2+\mu_0\Pi_L(k)\right)\left(k^2+\mu_0\Pi_T(k)\right)^2=0$$

for both modes one finds,

$$\omega=-i\frac{\kappa}{2}\pm\sqrt{\omega_p^2-\frac{\kappa^2}{4}} \qquad \text{Where} \qquad \omega_p^2=\frac{m_D^2}{3}$$

For an oscillatory electric field, these modes are damped by the relaxation parameter

$$\mathbf{E}=E_0e^{-i\omega t} \longrightarrow \mathbf{E}=E_0e^{-\frac{\kappa}{2}t\pm it\sqrt{\omega_p^2-\frac{\kappa^2}{4}}}$$

Susceptibility

$$\Pi^i_j = \begin{bmatrix} \Pi^x_x & 0 & 0 \\ 0 & \Pi^x_x & 0 \\ 0 & 0 & -\frac{\omega^2}{|\mathbf{k}|^2}\Pi^0_0 \end{bmatrix}$$

$$K^i_j(\omega, \mathbf{k}) = 1 + \frac{\Pi^i_j(\omega, \mathbf{k})}{\omega^2 \varepsilon_0} = 1 + \chi^i_j(\omega, \mathbf{k})$$

(Rukhadze and Silin, 1961)

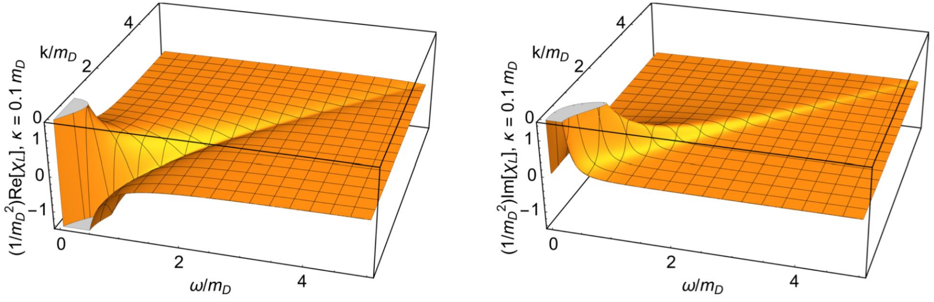


FIG. 3. Longitudinal susceptibility $\chi_L(\omega, k)$ in units of m_D^2 for $\kappa = 0.1\, m_D$. Left panel: real part of $\chi_L(\omega, k)$; right panel: imaginary part of $\chi_L(\omega, k)$.

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$$

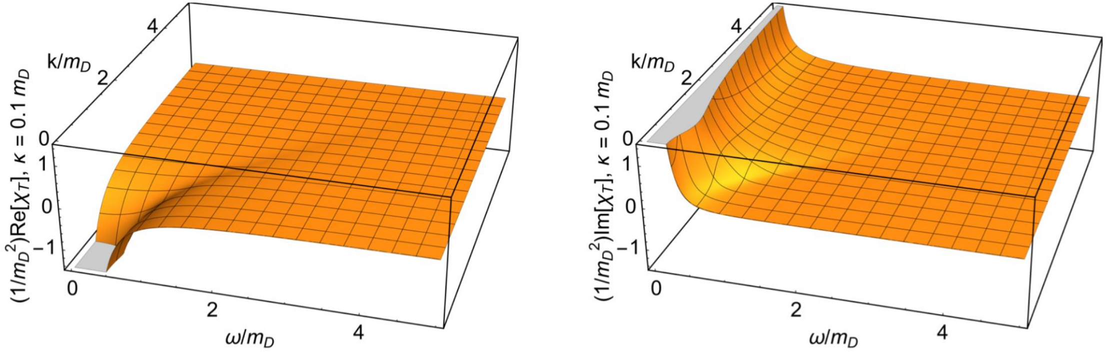


FIG. 4. Transverse susceptibility $\chi_T(\omega, k)$ in units of m_D^2 for $\kappa = 0.1\, m_D$. Left panel: real part of $\chi_T(\omega, k)$; right panel: imaginary part of $\chi_T(\omega, k)$.

Conductivity

$$\sigma^i_j(\omega, \mathbf{k}) = -i\frac{\Pi^i_j(\omega, \mathbf{k})}{\omega}$$

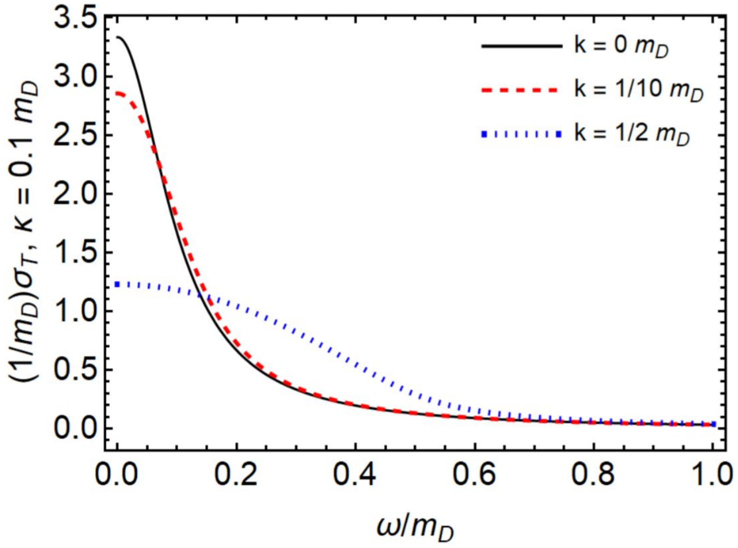


FIG. 8. Real part of σ_T for different values of k.

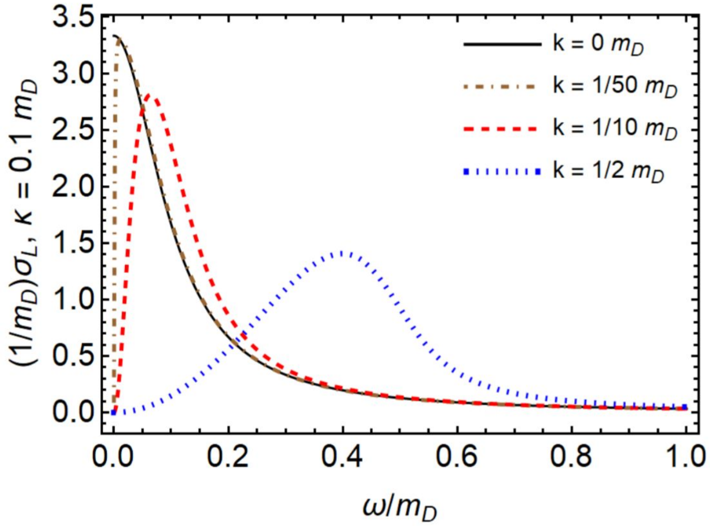


FIG. 7. Real part of σ_L for different values of k.

Discontinuity at k = 0 comes from infinite extent of plasma (Baranger 1989)

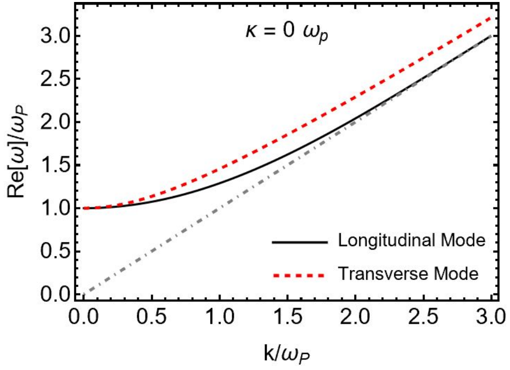
Modes - Dispersion Relation

$$((k \cdot u)^2 + \mu_0 \Pi_L(\omega, k))(k^2 + \mu_0 \Pi_T(\omega, k))^2 = 0$$

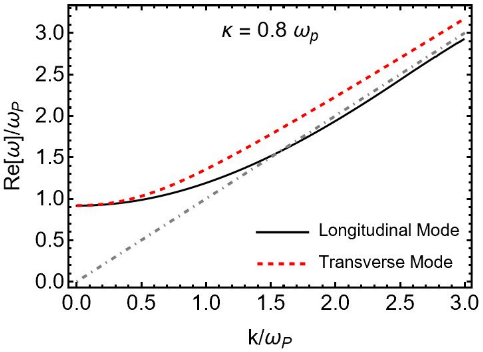
Negative imaginary part means modes are damped

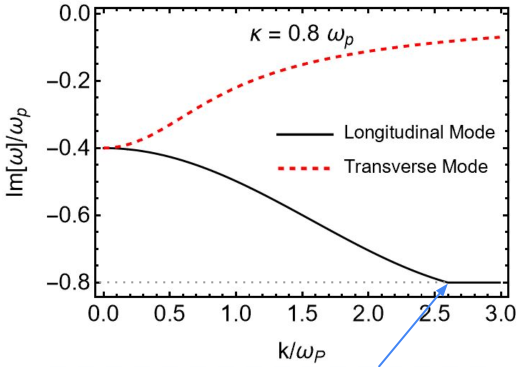
<u>Longitudinal Modes</u>: Electric Charge Oscillations → Langmuir waves (density fluctuations), Debye screening (charge screening)

<u>Transverse Modes:</u> Current Oscillations → Electromagnetic waves in vacuum



Finte κ





Longitudinal solution runs into the branch cut at -*iκ*

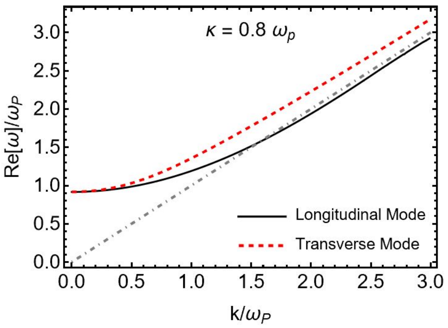
Modes - Dispersion Relation

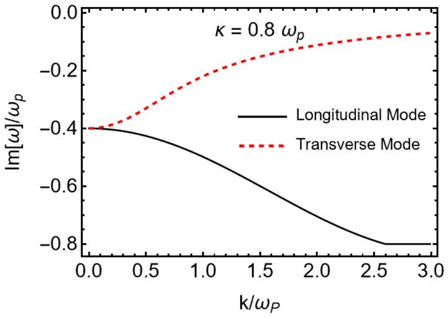
$$((k \cdot u)^2 + \mu_0 \Pi_L(\omega, k))(k^2 + \mu_0 \Pi_T(\omega, k))^2 = 0$$

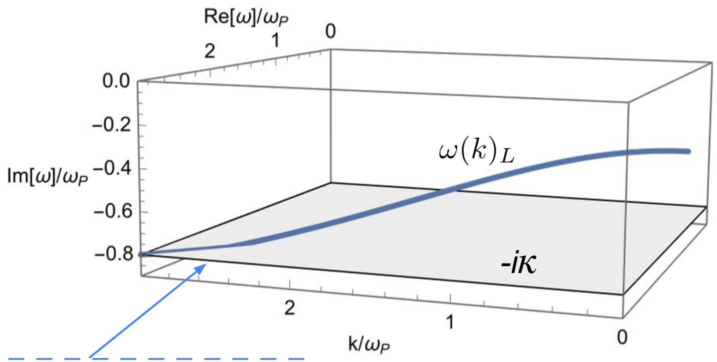
$$\ln \frac{\omega + i\kappa + |\mathbf{k}|}{\omega + i\kappa - |\mathbf{k}|}$$

<u>Longitudinal Modes</u>: Electric Charge Oscillations → Langmuir waves (density fluctuations), Debye screening (charge screening)

<u>Transverse Modes:</u> Current Oscillations → Electromagnetic waves in vacuum







Longitudinal solution runs into the branch cut at *-iκ*

(Schenke et al. 2006)