

Search for critical point of strongly interacting matter

(Intermittency analysis by NA61/SHINE at CERN SPS)

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Outline

The goal of this study is to locate the **critical point** of the strongly interacting matter

----- by -----

measuring scaled factorial moments of multiplicity distribution from a selection of

Pb+Pb at 13A GeV/c ($\sqrt{s_{NN}} \approx 5.1$ GeV)

Pb+Pb at 30A GeV/c ($\sqrt{s_{NN}} \approx 7.5$ GeV)

Ar+Sc at 150A GeV/c ($\sqrt{s_{NN}} \approx 17$ GeV)

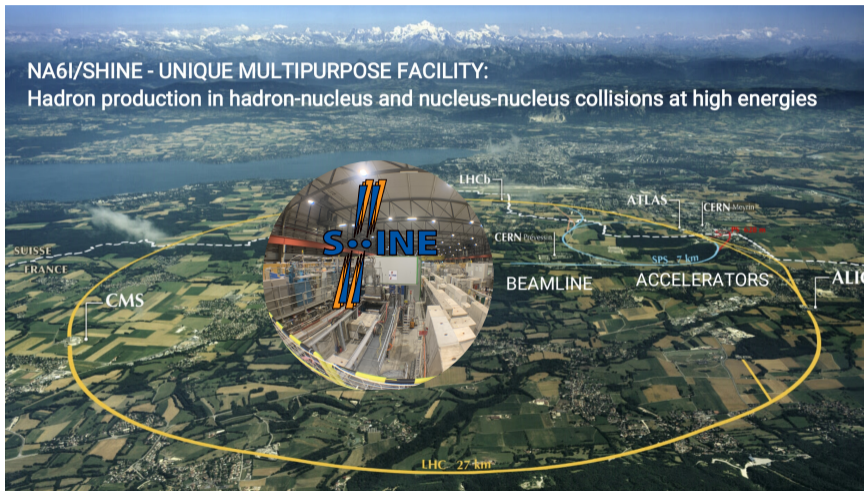
using **statistically independent points** and **cumulative variables**

- Introduction
- QCD critical point search strategies
 - QCD critical point
 - Exploring the phase diagram with heavy-ion collisions
- Experimental measures to search QCD critical point
 - Fluctuations in large momentum bins
 - Multiplicity fluctuations
 - Multiplicity-transverse momentum fluctuations
 - Fluctuation as a function of momentum bin size
 - Proton intermittency analysis
 - h^- intermittency analysis
- Exclusion plot and intermittency analysis result
- Summary

NA61/SHINE at CERN SPS

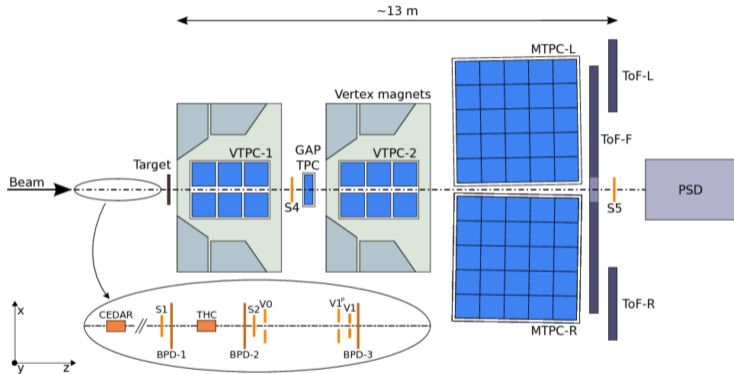
NA61/SHINE - UNIQUE MULTIPURPOSE FACILITY:

Hadron production in hadron-nucleus and nucleus-nucleus collisions at high energies



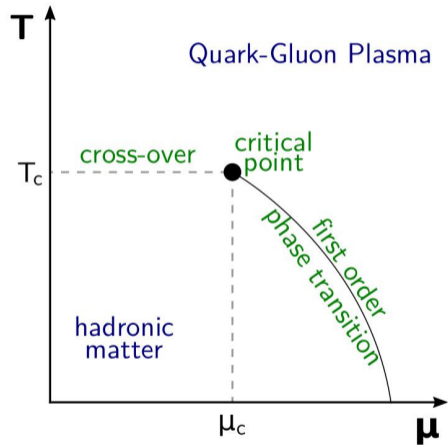
NA61/SHINE (SPS Heavy Ion and Neutrino Experiment) is a particle physics fixed-target experiment at CERN SPS

NA61/SHINE detector



- VTPC-1, VTPC-2 are placed in the magnetic field
- TPC system: [track reconstruction](#) and [particle identification](#) based on specific energy loss
- Projectile Spectator Detector (PSD): hadronic calorimeter, measures projectile spectators energy

Critical point of strongly interacting matter



- Critical Point (CP): a hypothetical end point of first order phase transition line (QGP-HM) that has properties of second order phase transition
- Second order phase transition \rightarrow scale invariance \rightarrow power-law form of correlation function
- These expectations are for fluctuations and correlations in the configuration space
- They are expected to be *projected* to the momentum space via quantum effect and/or collective flow
- Predictions on the CP existence, its location and what and how should fluctuate are model-dependent

Asakawa, Yazaki NPA 504 (1989) 668

Barducci, Casalbuoni, De Curtis, Gatto, Pettini, PLB 231 (1989) 463

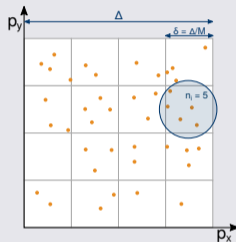
Scaled factorial moments of order r

Second order phase transition \rightarrow scale invariance \rightarrow power-law form of correlation function \rightarrow enhanced multiplicity fluctuations that can be revealed by scaled factorial moments of order r:

$$F_r(\delta) = \frac{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i(n_i - 1) \dots (n_i - r + 1) \right\rangle}{\left\langle \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right\rangle^r}$$

δ : size of each of the M sub-division intervals of Δ

n_i : number of particles in i-th bin



When the system is a simple fractal and $F_r(\delta)$ follows a power law dependences:

$$F_r(\delta) = F_r(\Delta) \cdot \left(\frac{\Delta}{\delta}\right)^{D \cdot \phi_r}$$

Additionally, the exponent (intermittency index) ϕ_r obeys the relation:

$$D \cdot \phi_r = (r - 1) \cdot d_r$$

Where the anomalous fractal dimension d_r is independent of r

Wosiek, APPB 19 (1988) 863
 Bialas, Hwa, PLB 253 (1991) 436
 Bialas, Peschanski, NPB 273(1986) 703
 Antoniou, Diakonou, Kapoyannis, Kousouris, PRL 97 (2006) 032002

Intermittency analysis

In NA61/SHINE , intermittency analysis is performed at mid-rapidity and particle fluctuations are studied in transverse momentum plane to locate the **critical point** of the strongly interacting matter measuring scaled factorial moments of multiplicity distribution

modified equivalent formula

$$F_r(M) = \frac{r!(M^2)^{r-1}}{\langle N \rangle^r} \left\langle \sum_{m=1}^{M^2} \binom{n_m}{r} \right\rangle$$

for $r=2$

$$F_2(M) = \frac{2M^2}{\langle N \rangle^2} \langle N_{pp} \rangle$$

M: number of bins in p_x and p_y

N: event multiplicity

n_i : numbers of particles in i th bin

$\langle \dots \rangle$: averaging over events

N_{pp} : total number of pairs in M^2 bins in an event

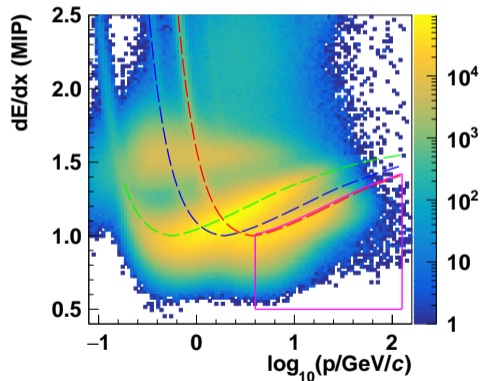
Error propagation:

$$\frac{\sigma_{F_r}}{|F_r|} = \sqrt{\frac{\text{Var}(N_r)}{\langle N_r \rangle^2} + r^2 \frac{\text{Var}(N)}{\langle N \rangle^2} - 2r \frac{\text{Cov}(N_r, N)}{\langle N \rangle \langle N_r \rangle}}$$

Particle identification

Example: proton candidates selection

(Pb+Pb at 30A GeV/c)



- - - - : theoretical BB function for protons
- - - - : theoretical BB function for kaons
- - - - : theoretical BB function for pions

selection of protons is based on dEdx measurements in TPCs

- $0.60 < \log_{10}(p/\text{GeV}/c) < 2.10$
- $0.5 \leq dE_{dx} \leq BB_{proton} + 0.15(BB_{kaon} - BB_{proton})$

around 60% protons are selected with few percent kaon contamination

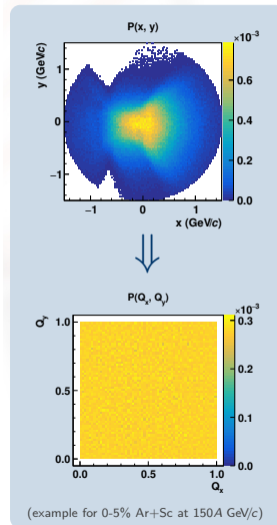
BB: theoretical Bethe-Bloch function

Cumulative variables

Instead of using p_x and p_y , one can use cumulative quantities:

$$Q_x = \int_{x_{min}}^x \rho(x) dx / \int_{x_{min}}^{x_{max}} \rho(x) dx \quad Q_y = \int_{y_{min}}^y P(x, y) dy / P(x)$$

- transform any distribution into uniform distribution (0,1)
- remove the dependence of F_r on the shape of the single-particle distribution
- intermittency index of an ideal power-law correlation function system described in two dimensions in momentum space was proven to remain approximately invariant after the transformation



Preliminary result on intermittency analysis

Pb+Pb at 13A GeV/c($\sqrt{s_{NN}} \approx 5.1$ GeV)

Pb+Pb at 30A GeV/c($\sqrt{s_{NN}} \approx 7.5$ GeV)

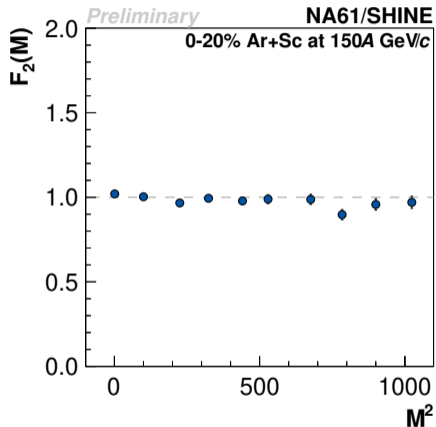
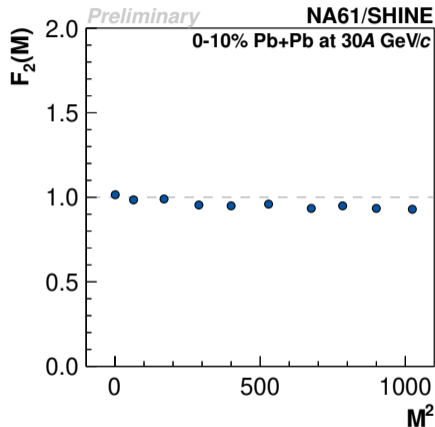
Ar+Sc at 150A GeV/c($\sqrt{s_{NN}} \approx 17$ GeV)

Cumulative variables

Independent sub-sample for each M points

Proton intermittency analysis result

(Experimental result on fluctuations as a function of momentum bin size)



CPOD 2021: <https://indico.cern.ch/event/985460>

ICNFP 2021: <https://indico.cern.ch/event/1025480/>

ISNP ERICE 2021: <http://crunch.iikp.physik.tu-darmstadt.de/erice/2021>

No indication for power-law increase with bin size

Simple power-law model

A simple model that generates momentum of particles for a given number of events with a given multiplicity distribution.

It has two main parameters:

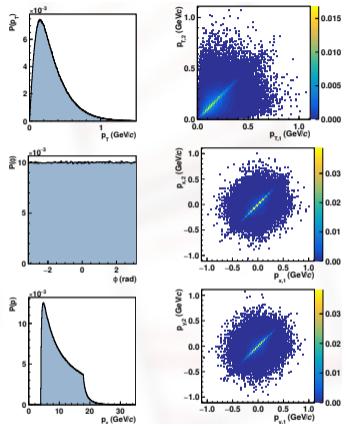
- ratio of correlated to uncorrelated particles
- power-law exponent

Uncorrelated particles (background)

$$\rho_B(p_T) = p_T \cdot e^{-6p_T}$$

Correlated pairs (signal)

$$\rho_S(p_{T,1}, p_{T,2}) = \rho_B(p_{T,1}) \cdot \rho_B(p_{T,2}) \cdot \left[|\Delta p_x|^\phi + \epsilon \right]^{-1} \cdot \left[|\Delta p_y|^\phi + \epsilon \right]^{-1}$$



Example for:

$$\phi = 0.80$$

$$\epsilon = 10^{-5}$$

$$N_B = \text{Poisson}(30)$$

$$N_S = 2$$

Simple power-law model

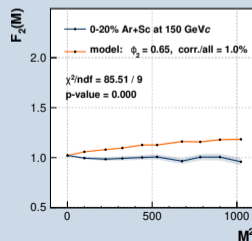
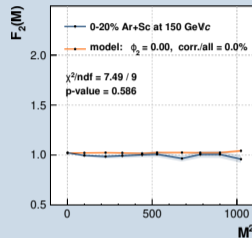
Lots of model data sets are generated:

- correlated-to-all ratio: vary from 0.0 to 4.0% (with 0.2 steps)
- power-law-exponent: vary from 0.0 to 1.0 (with 0.05 steps)

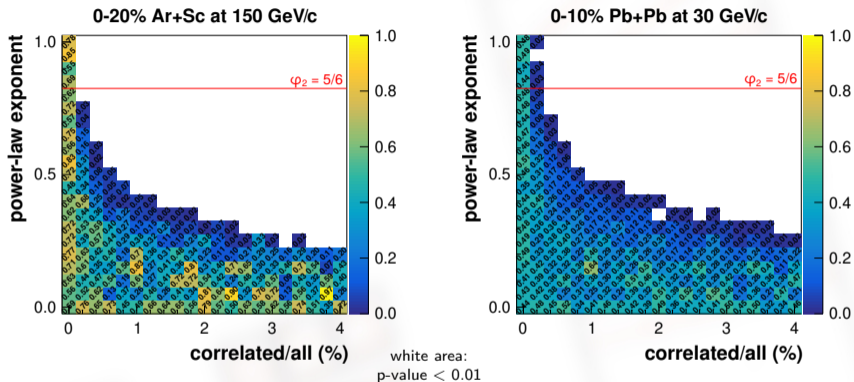
and compared with the experimental data

For the construction of exclusion plots, statistical uncertainties were calculated using model with statistics corresponding to the data.

M = 1 ... 32



Exclusion plot



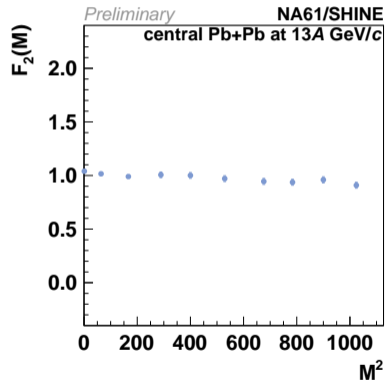
exclusion plots for parameters of simple power-law model

The intermittency index φ_2 for a system freezing out at the QCD critical endpoint is expected to be $\varphi_2 = 5/6$ assuming that the latter belongs to the 3-D Ising universality class.

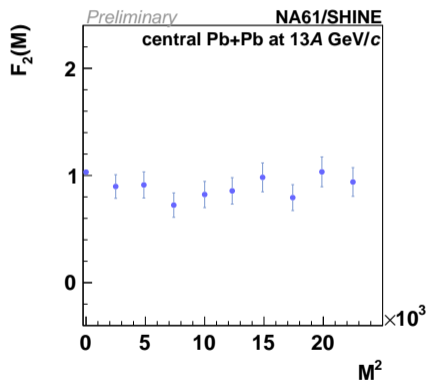
New proton intermittency analysis result

(using Pb+Pb at 13A GeV/c)

small M range (1,8,...32):



full M range (1,50,...,150):

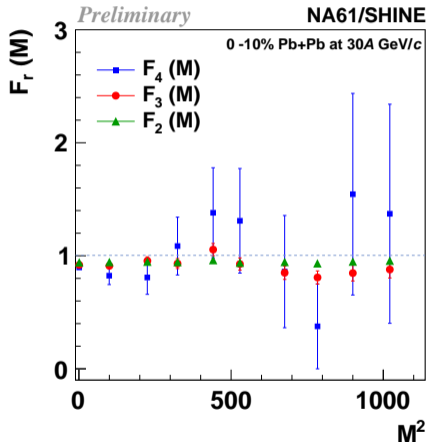


SSFMs are calculated using **cumulative variable** and **independent sub-sample** are used for each M points

No indication for power-law increase with bin size

Negatively charged hadron intermittency analysis

(higher order moments in Pb+Pb at 30A GeV/c data)



modified equivalent formula

Factorial moment in general for any order r is given by;

$$F_r(M) = \frac{r!(M^2)^{r-1}}{\langle N \rangle^r} \left\langle \sum_{m=1}^{M^2} \binom{n_m}{r} \right\rangle$$

M : number of bins in p_x and p_y

N : event multiplicity

n_m : numbers of particles in i th bin

$\langle \dots \rangle$: averaging over events

$N_2(M)$: number of pairs of particles in M bins

$N_3(M)$: number of triplets of particles in M bins

$N_4(M)$: number of quadruplets of particles in M bins

$$F_2(M) = \frac{2M^2}{\langle N \rangle^2} \langle N_2(M) \rangle$$

$$F_3(M) = \frac{6M^4}{\langle N \rangle^3} \langle N_3(M) \rangle$$

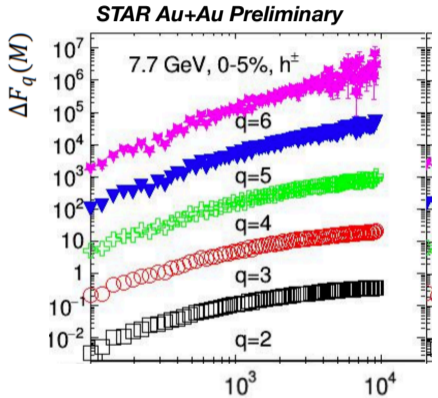
$$F_4(M) = \frac{24M^6}{\langle N \rangle^4} \langle N_4(M) \rangle$$

relationship between intermittency indices:

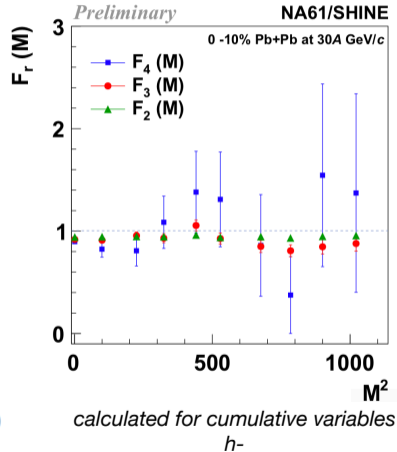
$$\phi_2 = \frac{\phi_3}{2} = \frac{\phi_4}{3}$$

No indication for power-law increase with bin size

STAR vs NA61/SHINE preliminary results on $\Delta F_q(M)$



$$\Delta F_q(M) = F_q^{data}(M) - F_q^{mix}(M)$$



****This seems to be in tension with corresponding results by the STAR Collaboration****

Summary

- Results on the dependence of **scaled factorial moments** of multiplicity distribution on cumulative momentum bin size for:
 - protons in Pb+Pb at $30A \text{ GeV}/c (\sqrt{s_{NN}} \approx 7.5 \text{ GeV})$
 - protons in Pb+Pb at $13A \text{ GeV}/c (\sqrt{s_{NN}} \approx 5.1 \text{ GeV})$
 - protons in Ar+Sc at $150A \text{ GeV}/c (\sqrt{s_{NN}} \approx 17 \text{ GeV})$
 - negatively charged hadrons in Pb+Pb at $30A \text{ GeV}/c (\sqrt{s_{NN}} \approx 7.5 \text{ GeV})$show **no indication of a power-law increase**
- **Exclusion plots** for parameters of a simple model (ratio of correlated to background particles and power-law exponent) are shown
- We are continuing proton and negatively charged hadron intermittency analysis for Pb+Pb at $13A \text{ GeV}/c$ and Ar+Sc at $13A - 75A \text{ GeV}/c$ data

Acknowledgement

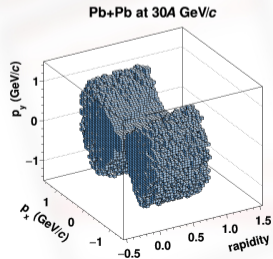
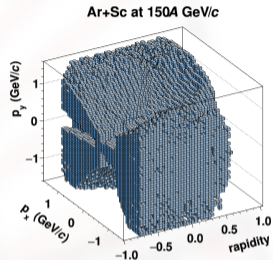
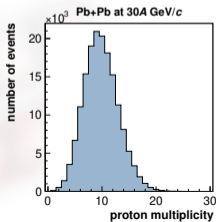
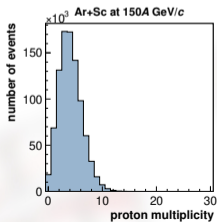
My work is supported by the National Science Centre, Poland under grant no.
2018/30/A/ST2/0026 & NA61/SHINE SPS CERN Collaboration



Back-ups!

Analysis acceptance

$|p_x| < 1.5 \text{ GeV}/c$
 $|p_y| < 1.5 \text{ GeV}/c$
Ar+Sc: $-0.75 < \text{rapidity} < 0.75$
Pb+Pb: $0.00 < \text{rapidity} < 0.75$



mTTD cut

- Time Projection Chambers (TPCs) do not allow to reconstruct tracks too close to each other
- To reject those close tracks momentum based two track distance cut introduced for Pb+Pb at 30A GeV/c and Ar+Sc at 150A GeV/c data

Momentum based two-tracks distance cut

Coordinates of momentum will be used:

$$s_x = \frac{p_x}{p_{xz}} = \cos(\Psi)$$

$$s_y = \frac{p_y}{p_{xz}} = \sin(\lambda)$$

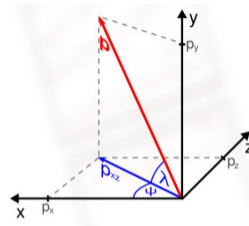
$$\rho = \frac{1}{p_{xz}}$$

For pairs:

$$\Delta s_x = s_{x2} - s_{x1}$$

$$\Delta s_y = s_{y2} - s_{y1}$$

$$\Delta \rho = \rho_2 - \rho_1$$



Effect of mTTD cut:

