# Quark-hadron continuity for neutron stars

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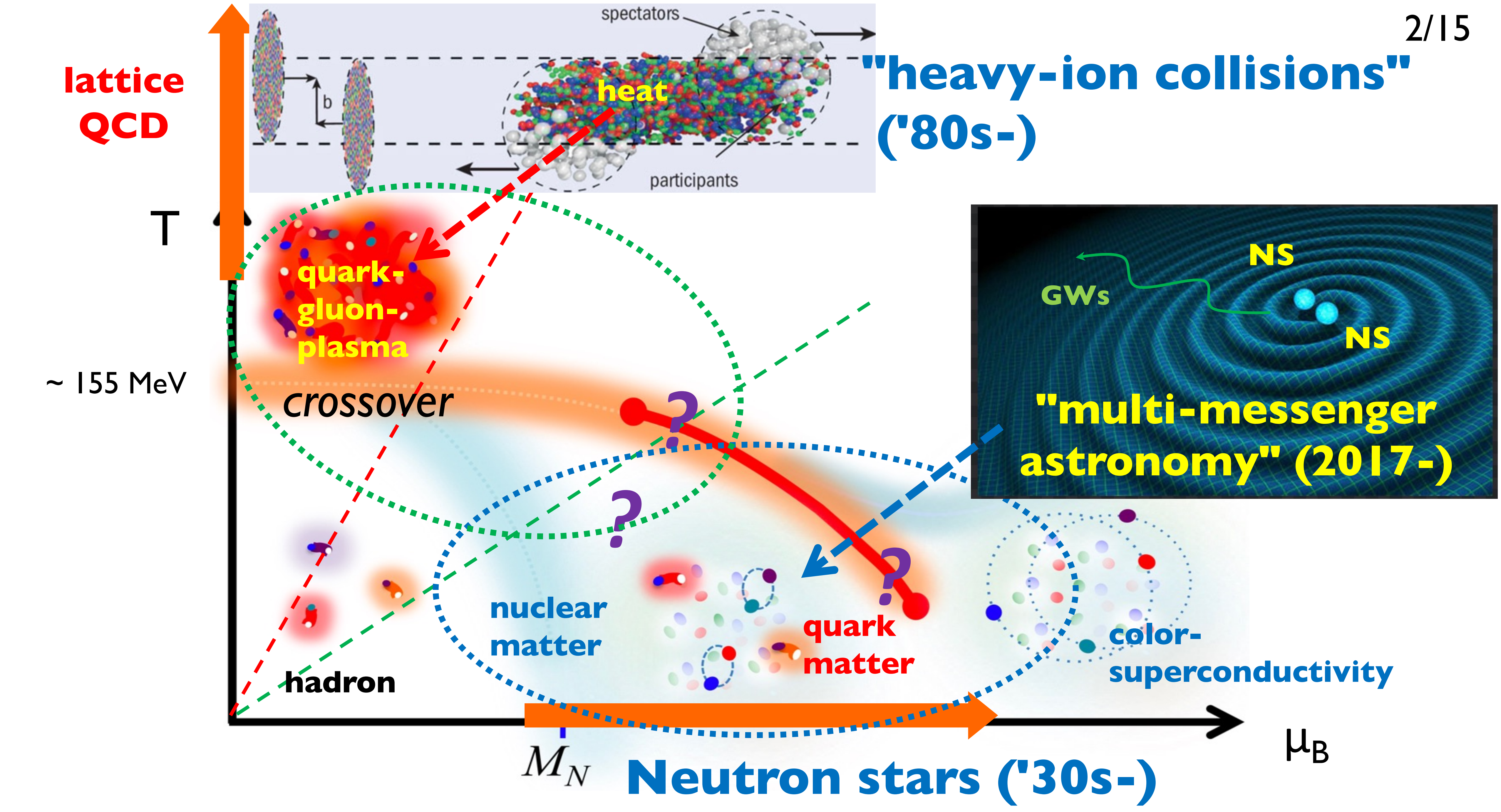
Refs)
- Baym-Hatsuda-TK-Powell-Song-Takatsuka (2018): review
- TK (2021): mini-review, AAPPS Bull. 31 (2021) 1, 11
- TK (2021): cs2 peak, PRD104 (2021); TK-Suenaga: 2110.02100
- TK-Baym-Hatsuda (2021): QHC21 EOS, 2111.11919



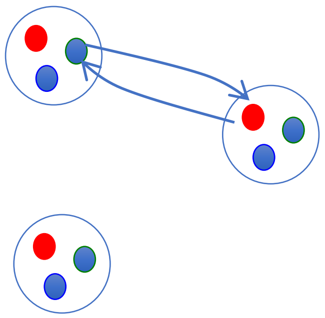
lattice QCD
spectators
b
heat
participants
"heavy-ion collisions" ('80s-)
T
quark-gluon-plasma
~ 155 MeV
crossover
?
?
?
NS
GWs
NS
"multi-messenger astronomy" (2017-)
nuclear matter
quark matter
color-superconductivity
hadron
$M_N$
$\mu_B$
Neutron stars ('30s-)

# State of matter: overview

($n_0$ = 0.16 fm$^{-3}$ )

[Masuda+ '12; TK+ '14]

- few meson exchange
- nucleons only

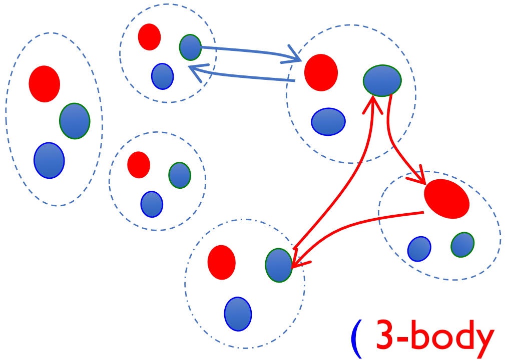


ab-initio nuclear cal.

laboratory experiments

steady progress

- many-quark exchange
- structural change,...
- hyperons, Δ, ...

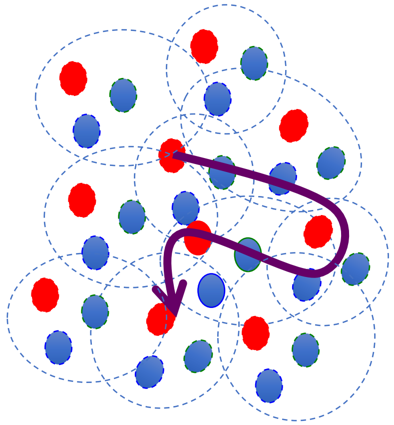


( 3-body )

**most difficult**

**(d.o.f ??)**

- Baryons overlap
- Quark Fermi sea



**strongly correlated**

(d.o.f : quasi-particles??)

not explored well

(pQCD)

[Freedman-McLerran, Kurkela+, Fujimoto+...]

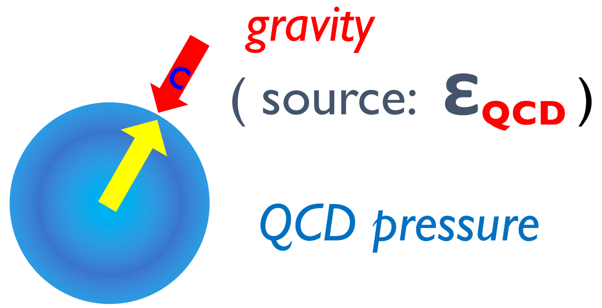
~ 1.4 $M_\odot$ ~ 2 $M_\odot$

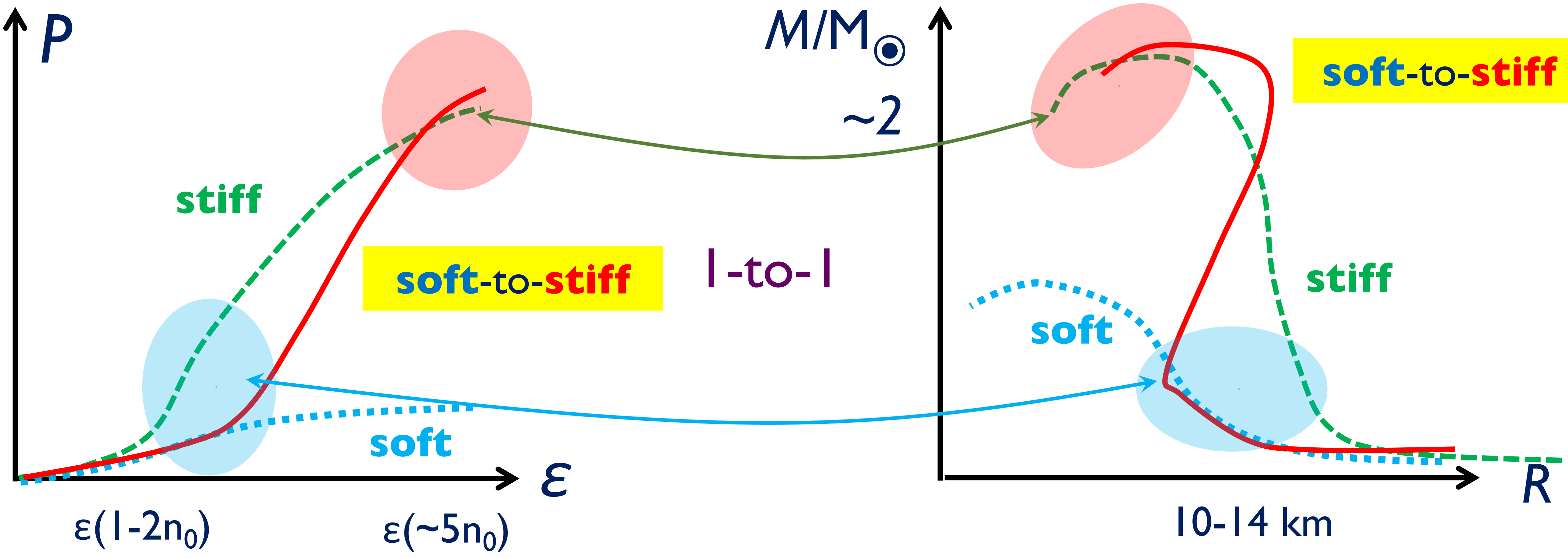
$n_B$

~ $2n_0$ **Hints from NS** ~ $5n_0$ ~ $40n_0$

# EoS & Neutron Star M-R relation

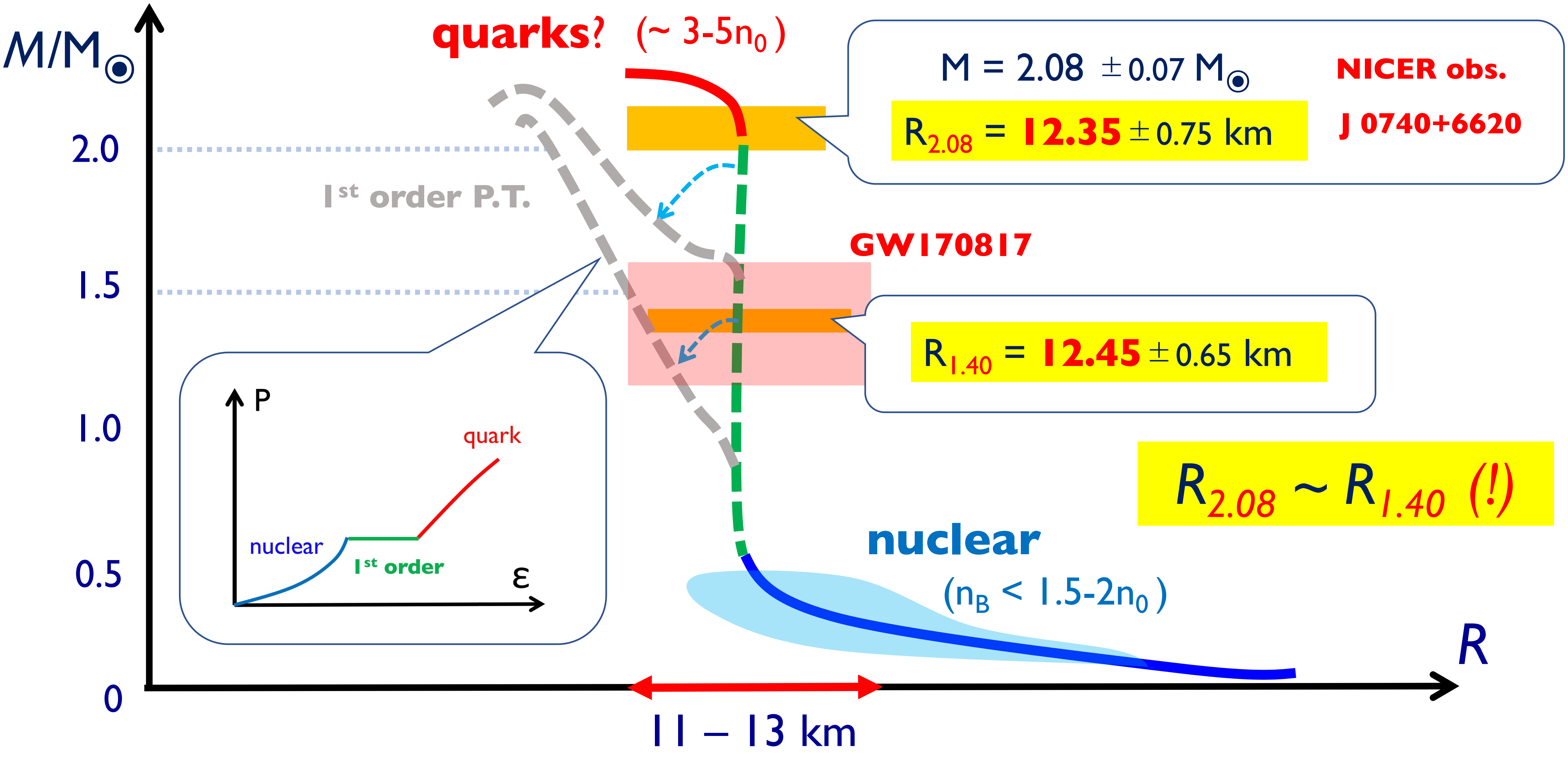
Einstein eq. : $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ QCD (+EW) EoS



# Observations: summary (2010-)

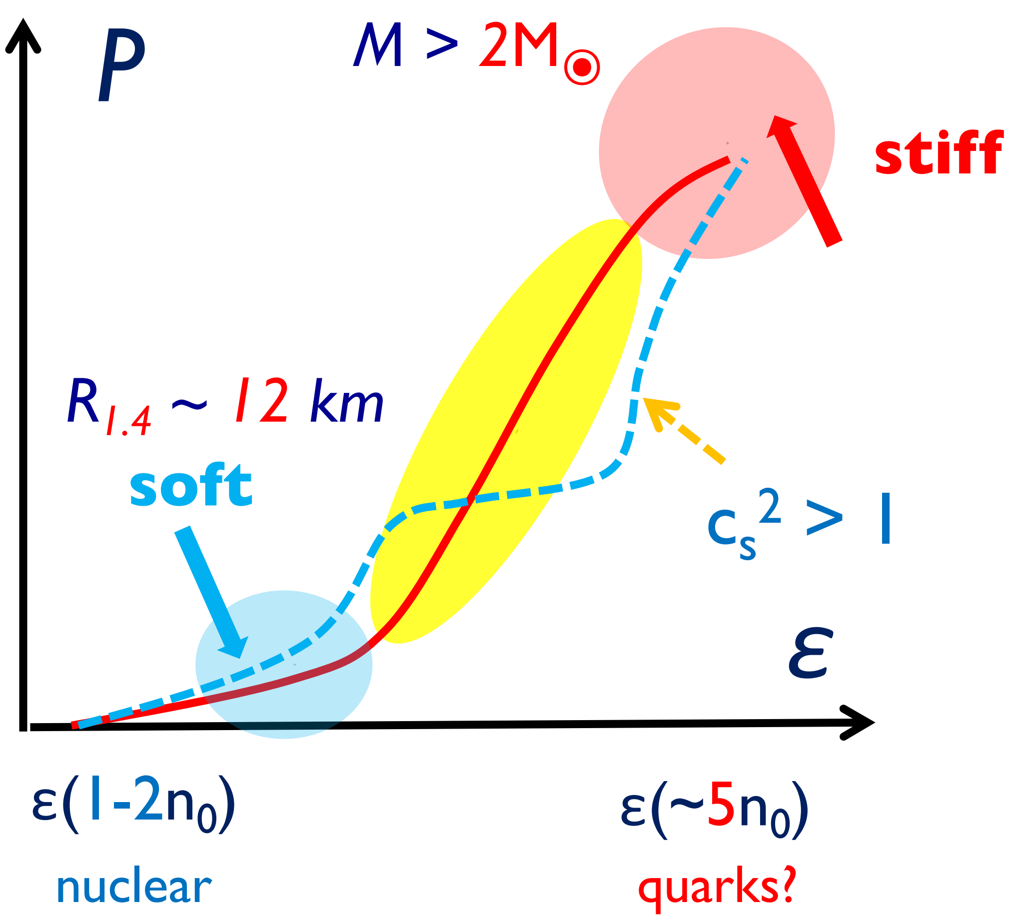
[Miller+ '21]

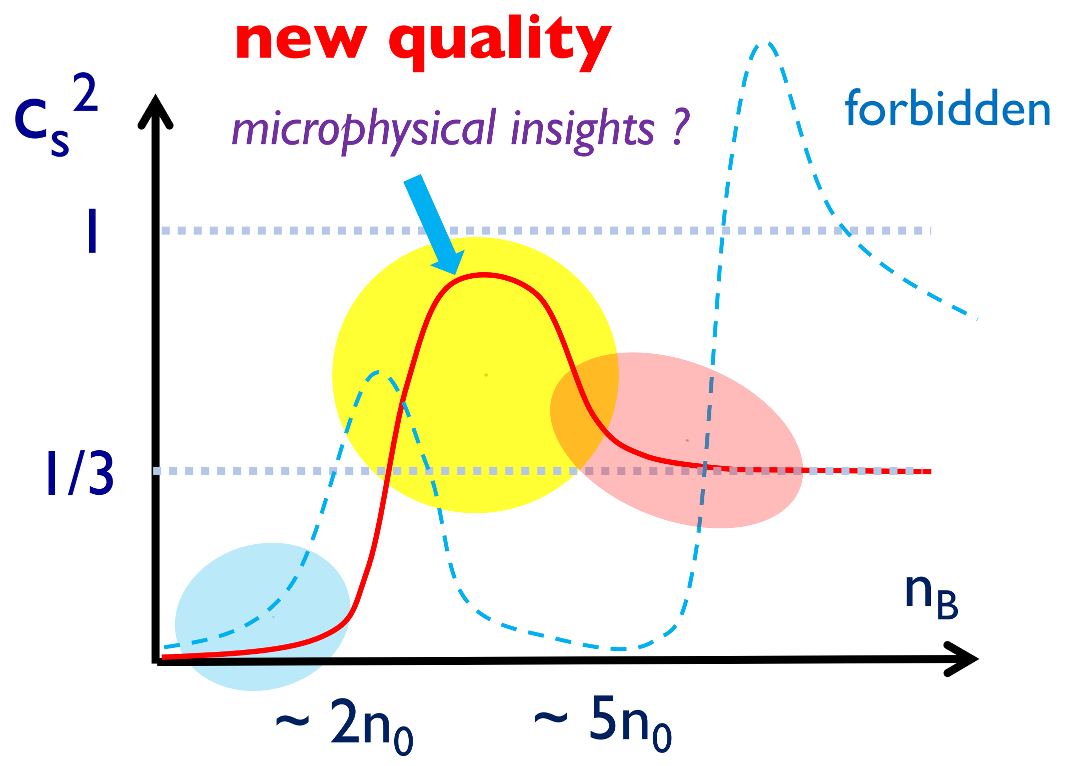


# *Soft* *to* *stiff* *is challenging:* **a new quality**

sound velocity: $c_s^2 = dP/d\varepsilon < 1$ (*causality*)

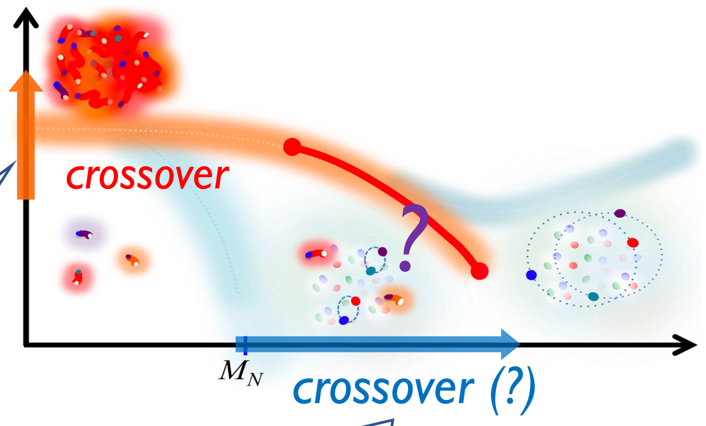
*nuclear* & *quark* physics constrain each other

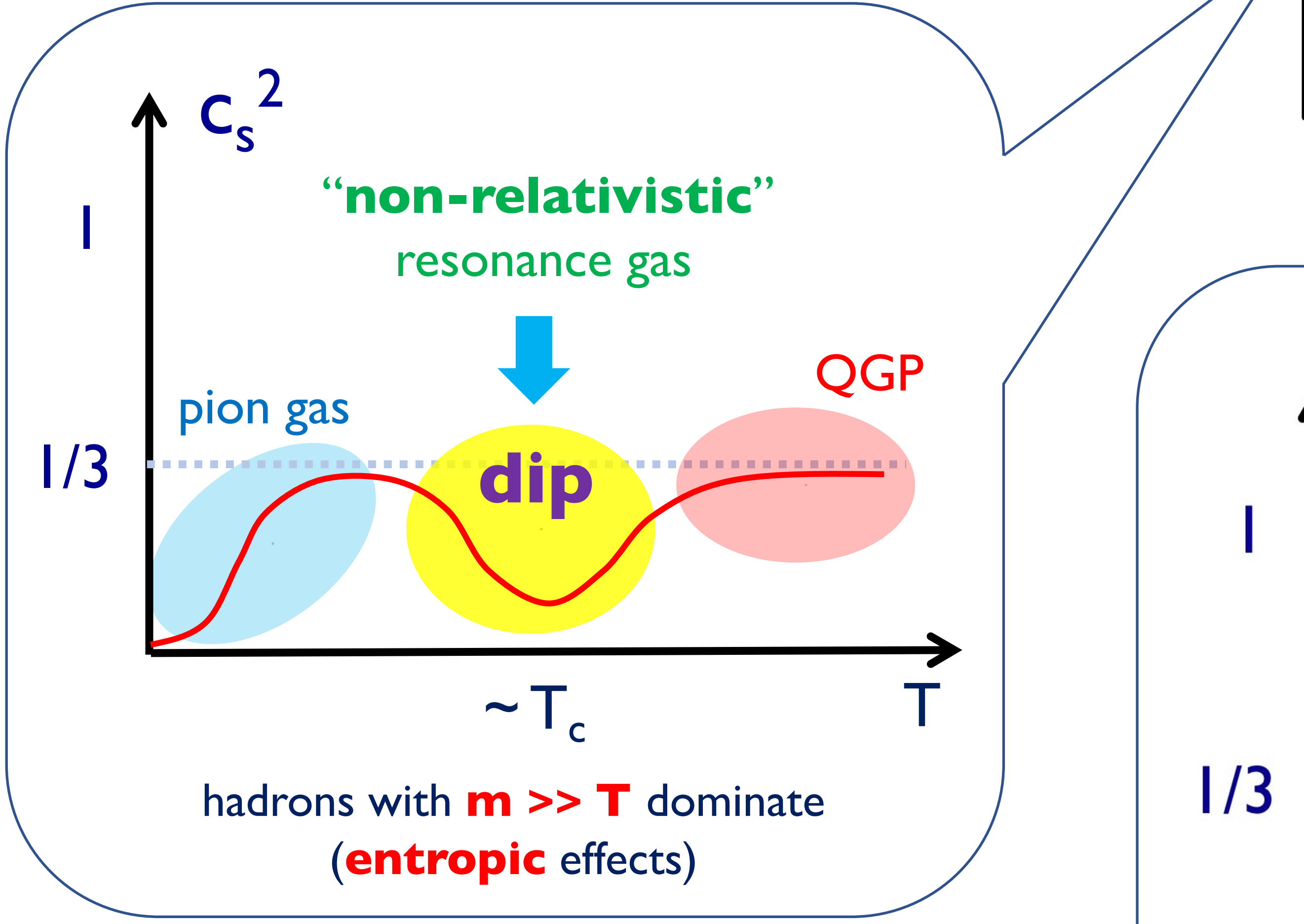


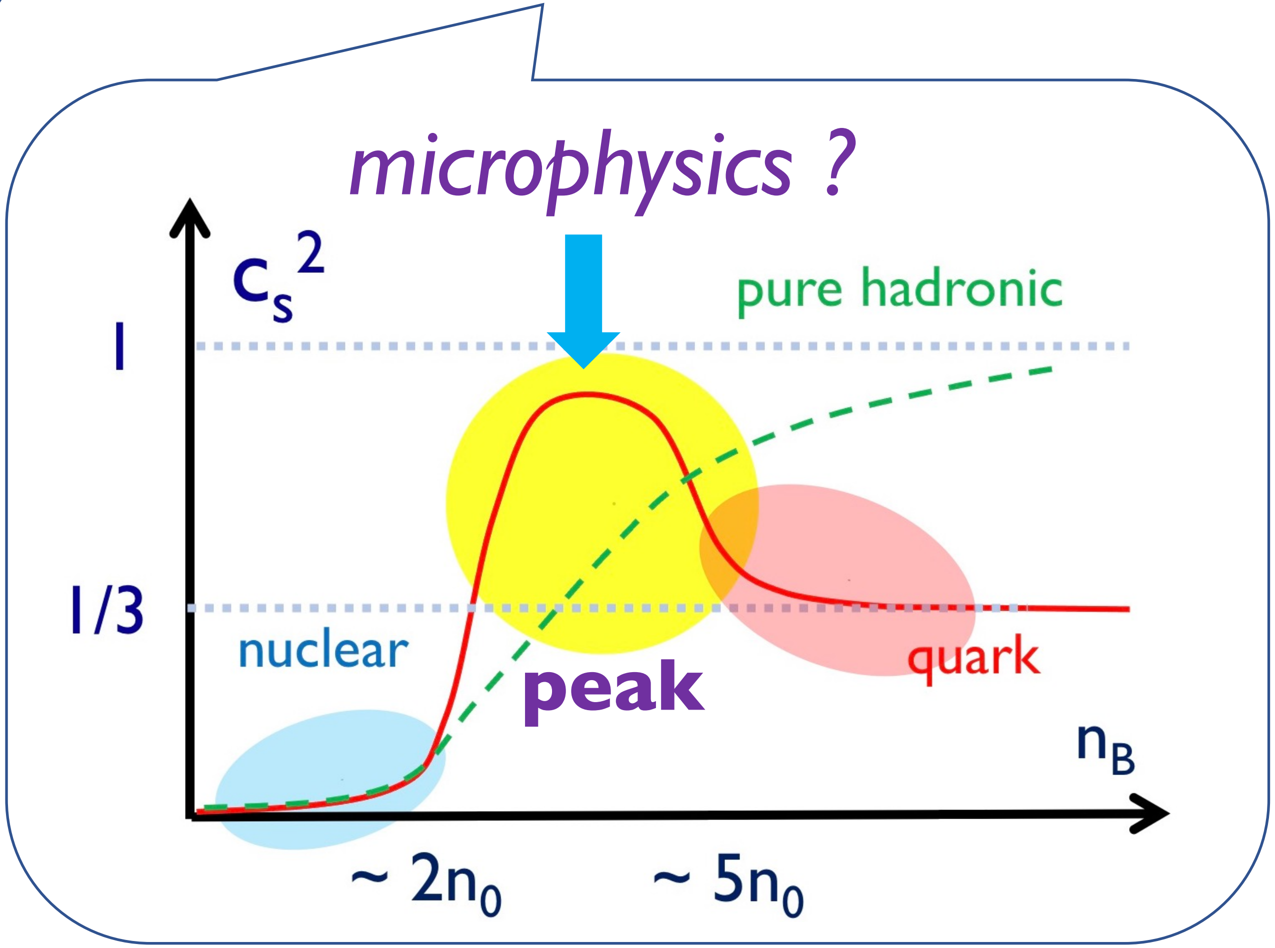




**baseline:** *quark-hadron continuity* **(QHC)**

# Crossovers & $c_s^2 = dP/d\varepsilon$



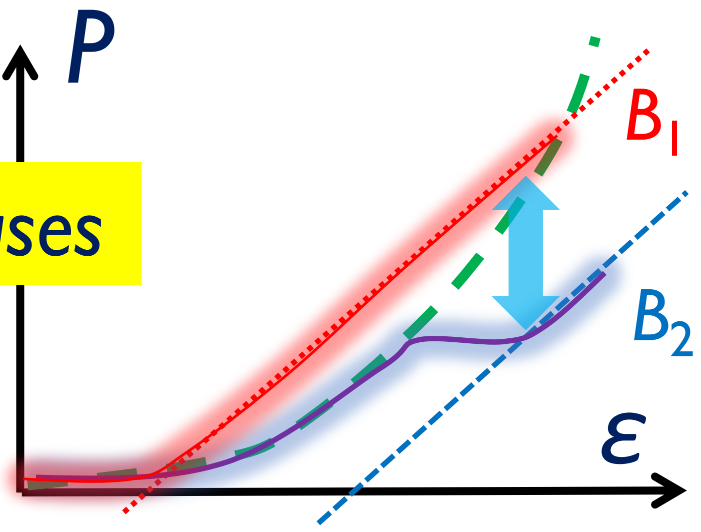



# *Quark-Hadron-Continuity EoS*

# Direct descriptions for 2-5$n_0$ ?

**confusing point:**




・ *Switching from baryonic to quark bases*

→ *a source of confusions in hybrid models*

(e.g. normalization of energy )

## Strategy

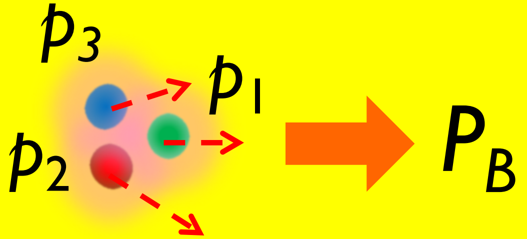
*Follow quark states from nuclear to quark matter*

(within a *single* model, e.g., percolation model, Fukushima-TK-Weise '20)

# Quarks in a baryon

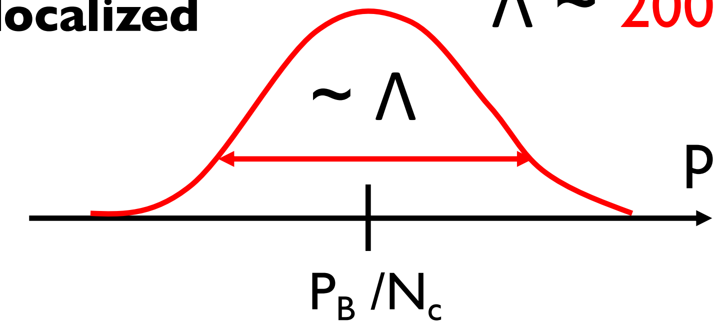
$N_c$ (=3): number of colors

probability density: $$Q_{\rm in}(\boldsymbol{p},\boldsymbol{P}_B) = \mathcal{N} e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2}$$




**localized**

$\Lambda \sim 200$ MeV




mean: $$\langle \boldsymbol{P}_B \rangle = N_c \int_{\boldsymbol{p}} \boldsymbol{p} Q_{\rm in}(\boldsymbol{p},\boldsymbol{P}_B)$$

variance: $$\left\langle \left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2 \right\rangle \sim \Lambda^2$$

*energetic !*

$$\langle E_q(\boldsymbol{p}) \rangle_{\boldsymbol{P}_B} = \mathcal{N} \int_{\boldsymbol{p}} E_q(\boldsymbol{p}) e^{-\frac{1}{\Lambda^2}\left(\boldsymbol{p} - \frac{\boldsymbol{P}_B}{N_c}\right)^2} \simeq \langle E_q(\boldsymbol{p}) \rangle_{\boldsymbol{P}_B = 0} + \frac{1}{6} \left\langle \frac{\partial^2 E_q}{\partial p_i \partial p_i} \right\rangle_{\boldsymbol{P}_B = 0} \left(\frac{\boldsymbol{P}_B}{N_c}\right)^2 + \cdots$$

average energy (quark)

x $N_c$ x $N_c$

$\sim N_c E_q$ $\gg$ $\sim P_B^2/(N_c E_q)$

baryon mass — baryon kin. energy

# A new **unified** model for **QHC**

cf) [TK '21, TK-Suenaga '21]

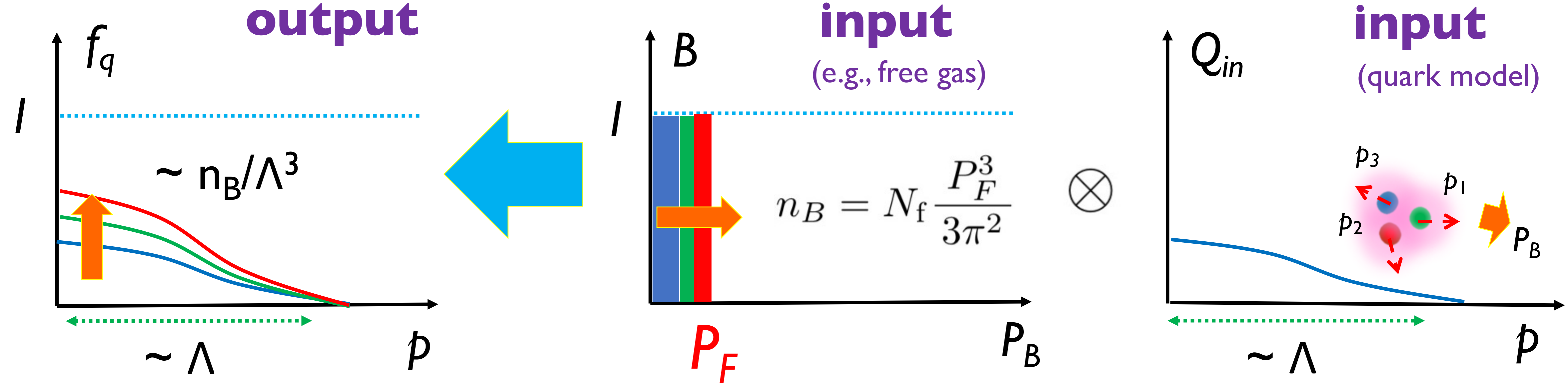
occupation **probability** of **quark** state with p

occupation **probability** of **baryon** state with $P_B$

**quark** mom. distribution **in a baryon**

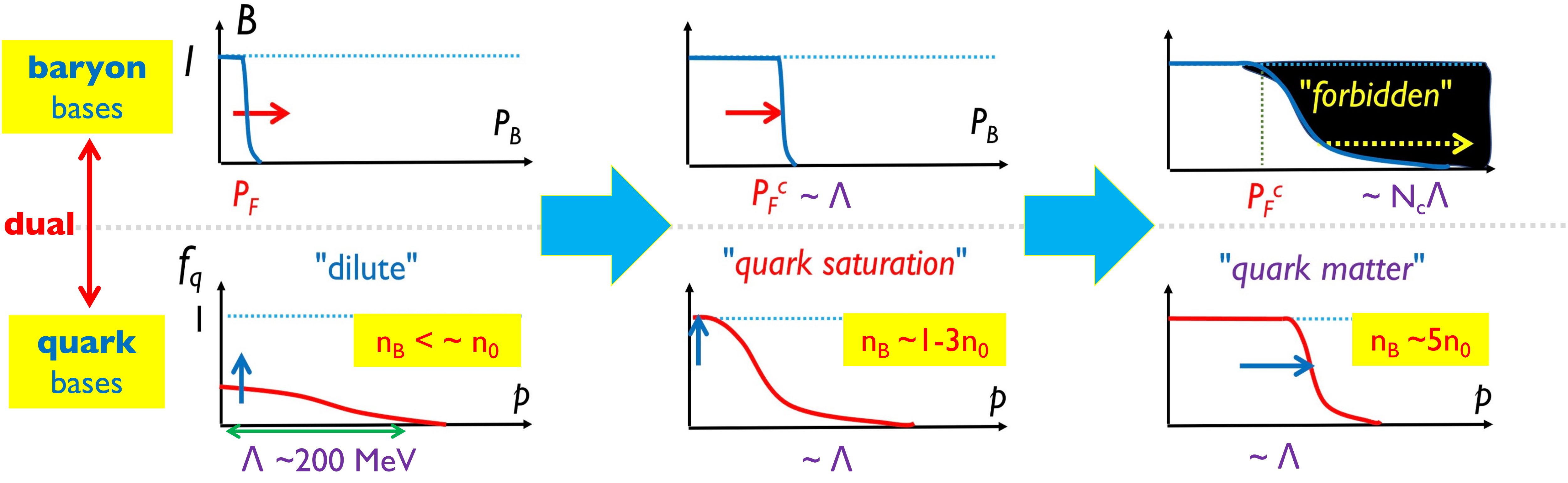
$$f_q(p; n_B) = \int_{\boldsymbol{P}_B} \mathcal{B}(P_B; n_B) Q_{\rm in}(\boldsymbol{p}, \boldsymbol{P}_B)$$

e.g.) in **ideal** baryonic matter



# Evolution of occ. probabilities

$$f_q(p;n_B) = \int_{\boldsymbol{P}_B} \mathcal{B}(P_B;n_B) Q_{\rm in}(\boldsymbol{p},\boldsymbol{P}_B)$$



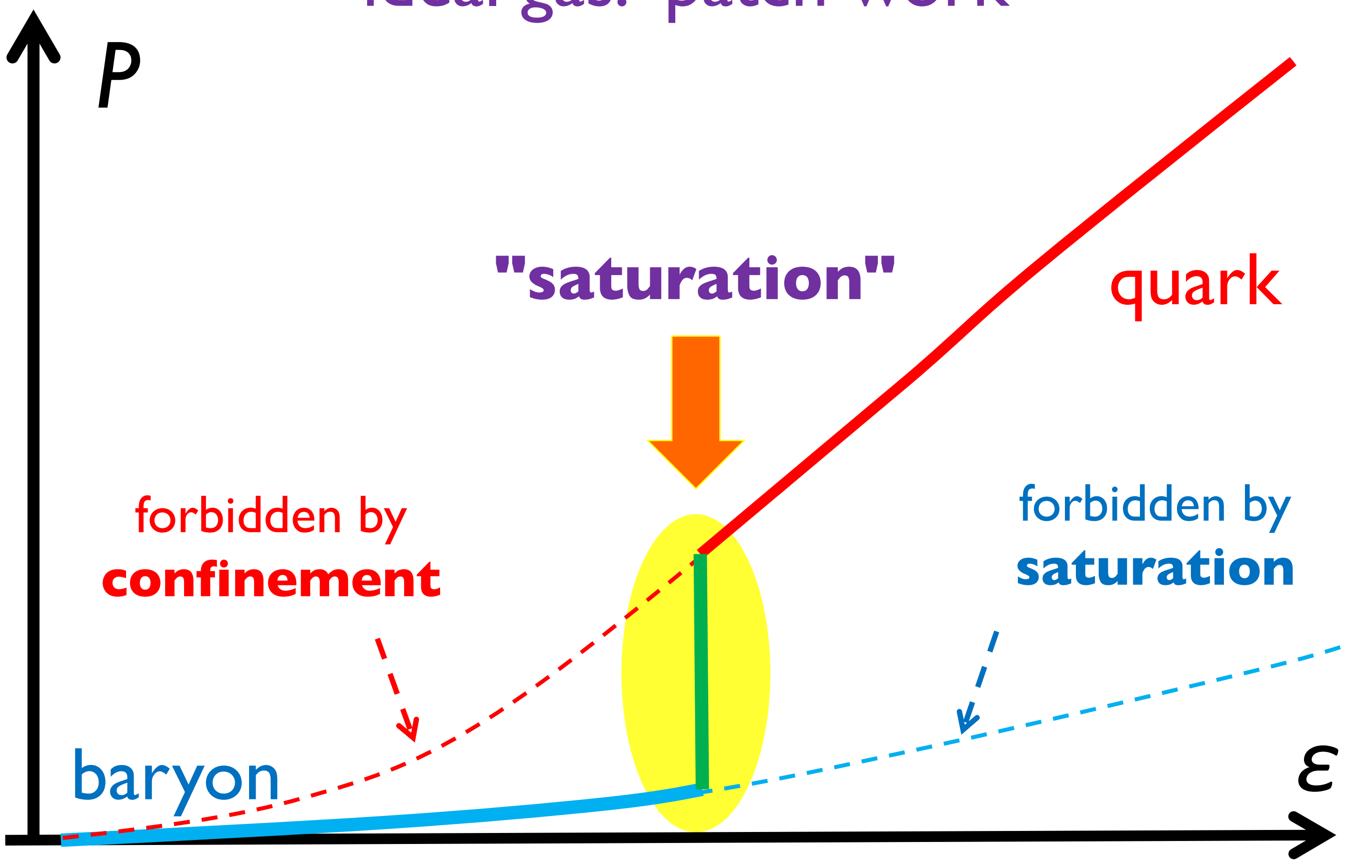

"**quark saturation**" constraint

→ **relativistic baryons at low density, $n_B$ ~ 1-3$n_0$ !**

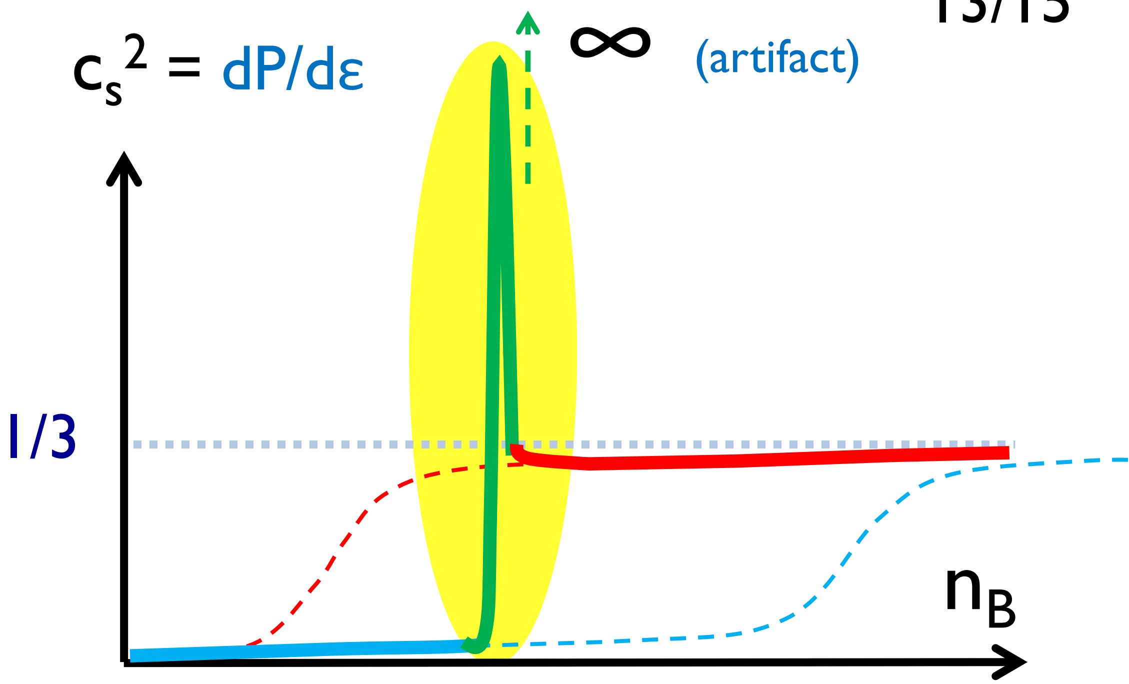
cf) McLerran-Reddy model (2018) of quarkyonic matter

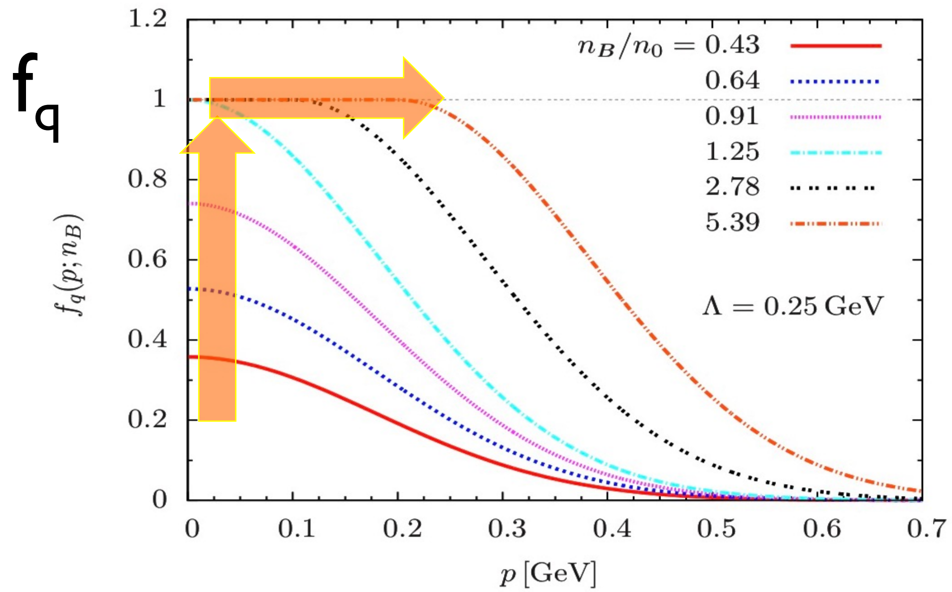
# Peak in sound velocity

ideal gas: patch work

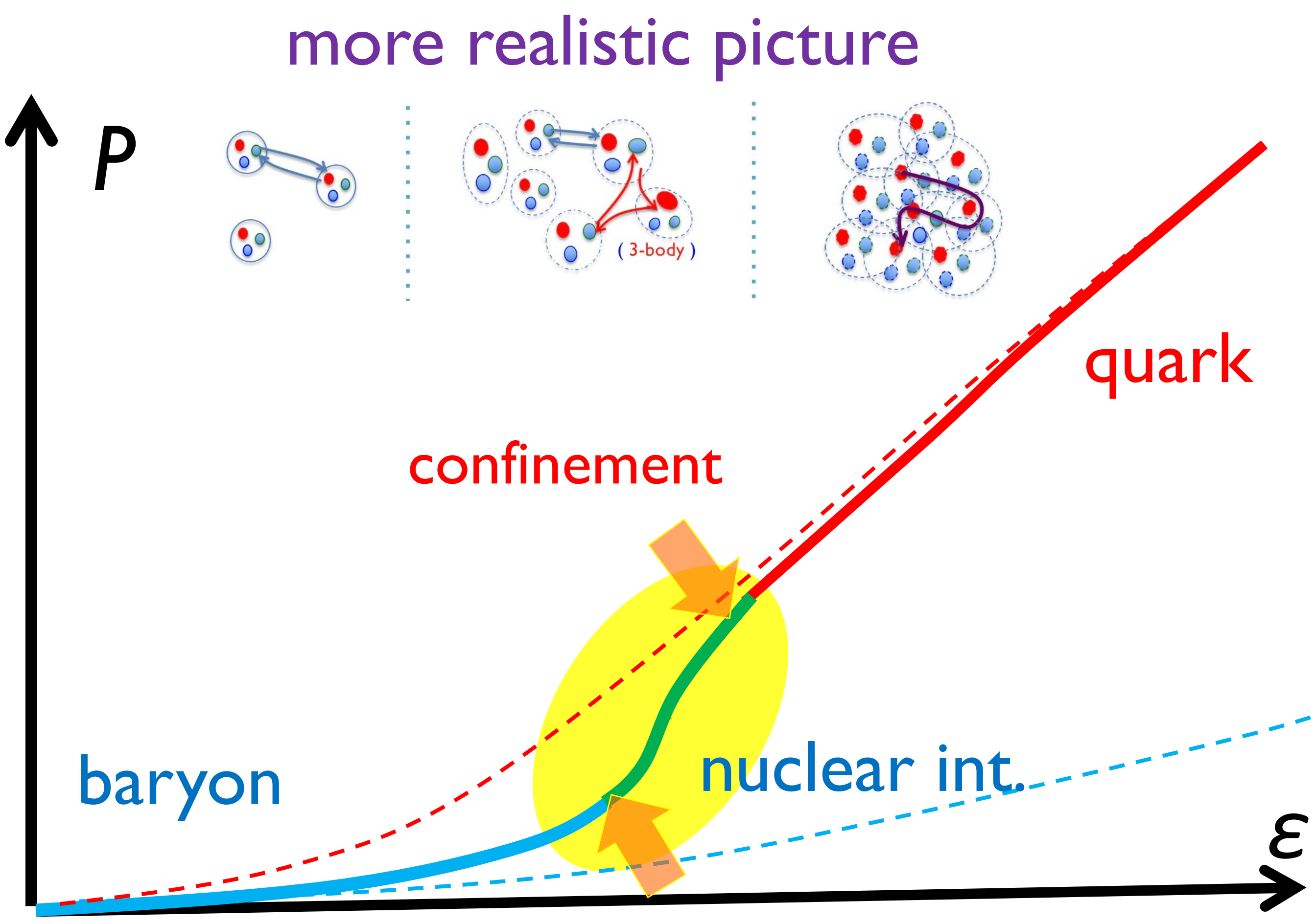



*"inevitable" stiffening*

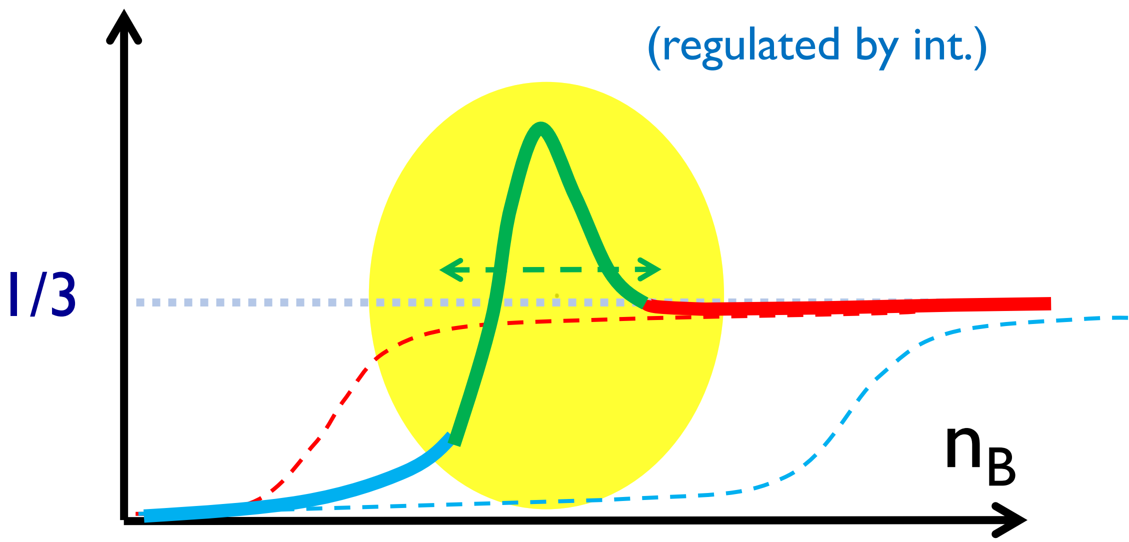


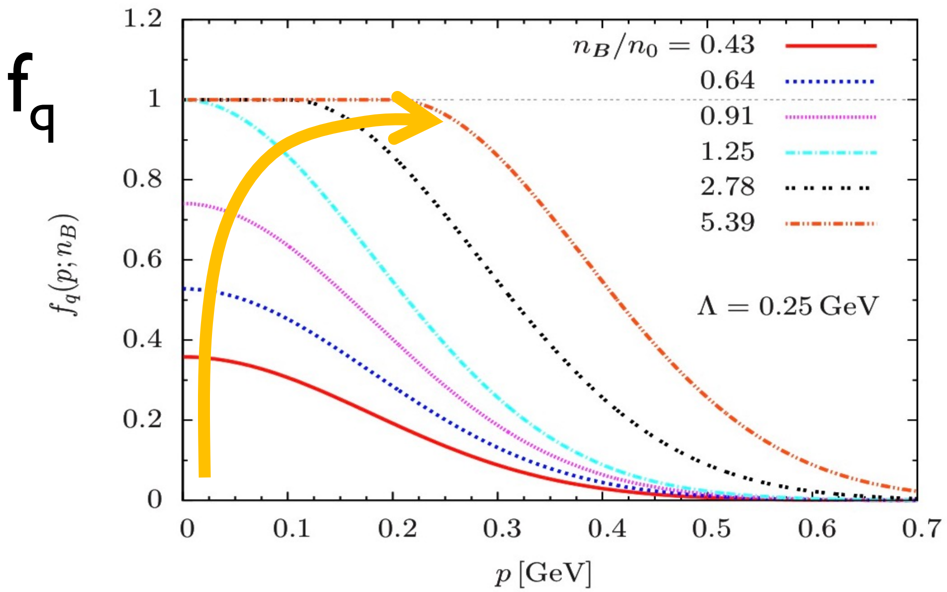



# Peak in sound velocity




$c_s^2 = dP/d\varepsilon$



# Summary

$R_{2.08} \sim R_{1.40}$ (!) → strong 1st order P.T. unlikely for $n_B \sim$ 2-5$n_0$

Quark-Hadron-Continuity : a good **baseline**

**Peak** in sound velocity → signature of quark matter formation

(quark saturation effects)

**Not explained in this talk:** (please see appendix)

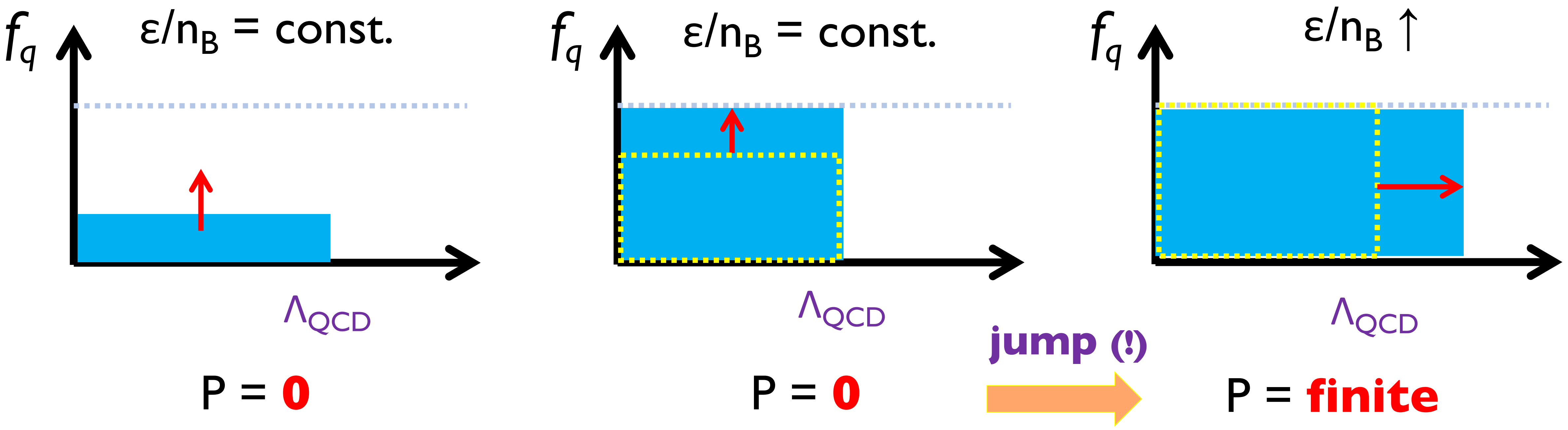
- effects of interactions
- quantum numbers (flavors, spins, …)
- the relation to quarkyonic matter models
- QHC18, 19, 21
- 2-color QCD examples

# Back up

# Jump in pressure : schematic picture

$$\mathcal{P} = n_B^2 \frac{\partial}{\partial n_B}\left(\frac{\varepsilon}{n_B}\right)$$

energy per particle




ε, $n_B$ are continuous ( $f_q$ continuous )

# Stiff quark EoS ? : *a guide*

cf) [TK-Powell-Song-Baym, '14]

kin. energy    interactions

$$\varepsilon(n) = a n^{4/3} + b n^{\alpha}$$

($n$: quark density)

ideal gas    interactions

$$\Rightarrow \quad P = \frac{\varepsilon}{3} + b\left(\alpha - \frac{4}{3}\right) n^{\alpha}$$

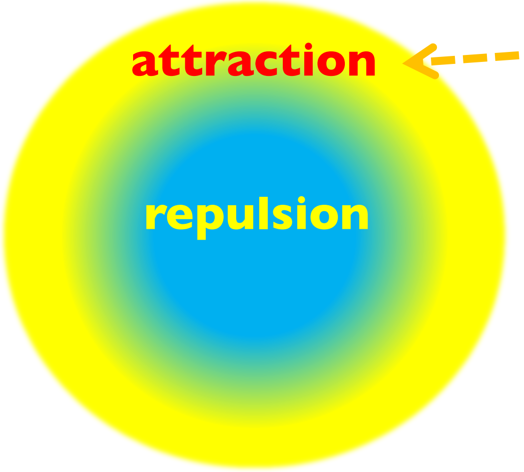
For stiff EoS:

(for large P)

for $\alpha > 4/3$:   b > 0   (e.g. bulk repulsion, ~ $+ n_B^{2}/\Lambda^2$)

for $\alpha < 4/3$:   b < 0   (e.g. surface pairings, ~ $- \Lambda^2 n_B^{2/3}$ )

quark
Fermi sea
(ideal combo)




2- or 3-quark correlations

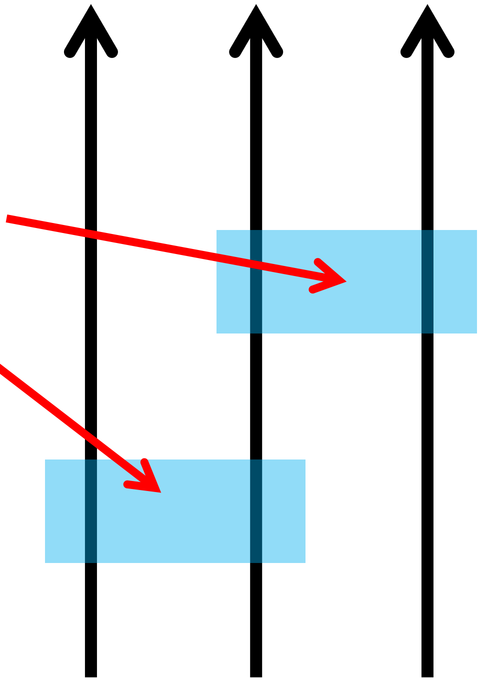
**"Exotic"** Fermi surface stiffens EoS !

Reminder: QCD int. are very **channel dependent**!

## a baryon in dilute regime

(color-singlet)

(always) color-**antisymmetric**



(attractive **electric** int.)

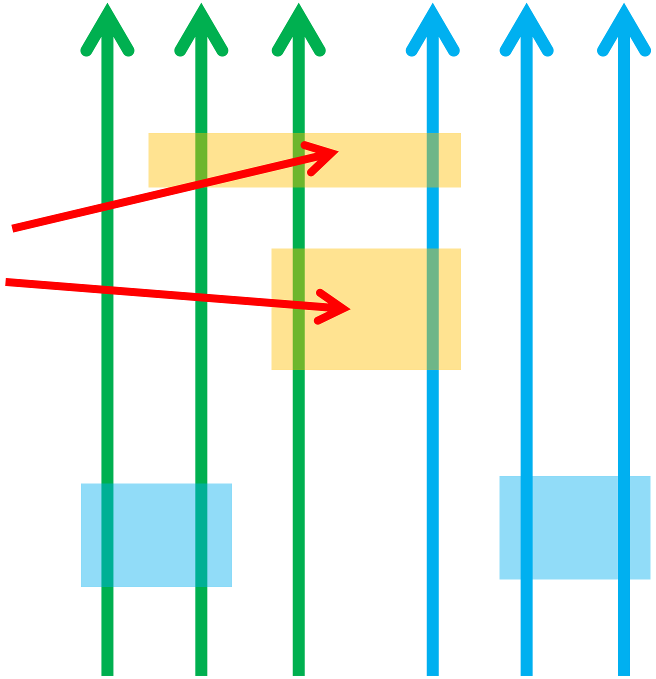
e.g., **nucleons**

$M_N \sim 3M_{q.} + \text{kin.} + \text{color-EM}$

~ 940MeV ~ 1100MeV - 150-200MeV

## in dense regime

sometimes color-**symmetric**



(repulsive)

**more chances to feel repulsion**

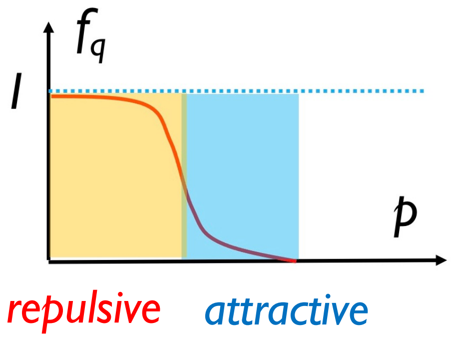
# quark energy; *parameterization of MF*

$$E_{\mathrm{CQM}}(\boldsymbol{k}) = \sqrt{M_q^2 + \boldsymbol{k}^2} - C_A + C_S\underline{[f_q(k)]^\beta}$$

for $f_q$ (p) << 1

$$\mathcal{V}_{\mathrm{CE}}[f_q] \simeq -C_E^{\mathcal{A}}$$

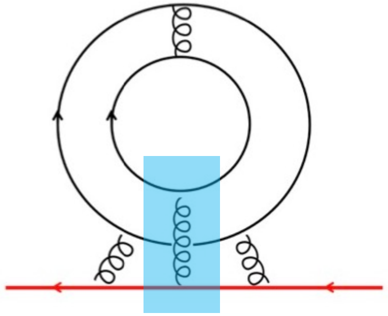
*dilute in momentum space*




for $f_q$ (p) ~ 1

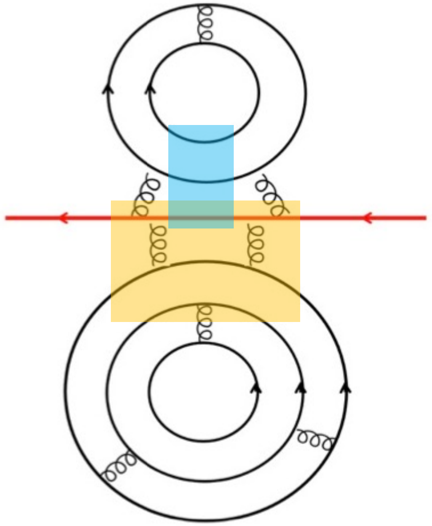
$$\mathcal{V}_{\mathrm{CE}}[f_q] \simeq C_E^{\mathcal{S}}$$

for *saturated levels*



color-*antisym.* channels dominate

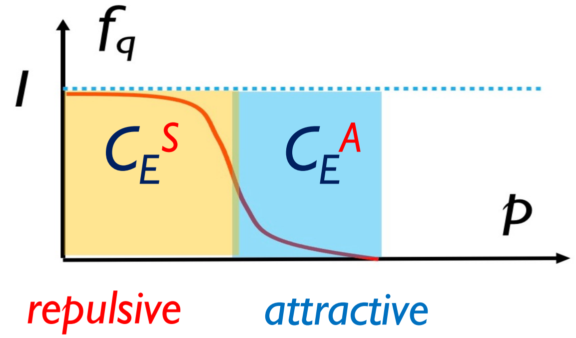
→ the quark feels *attractive* correlations

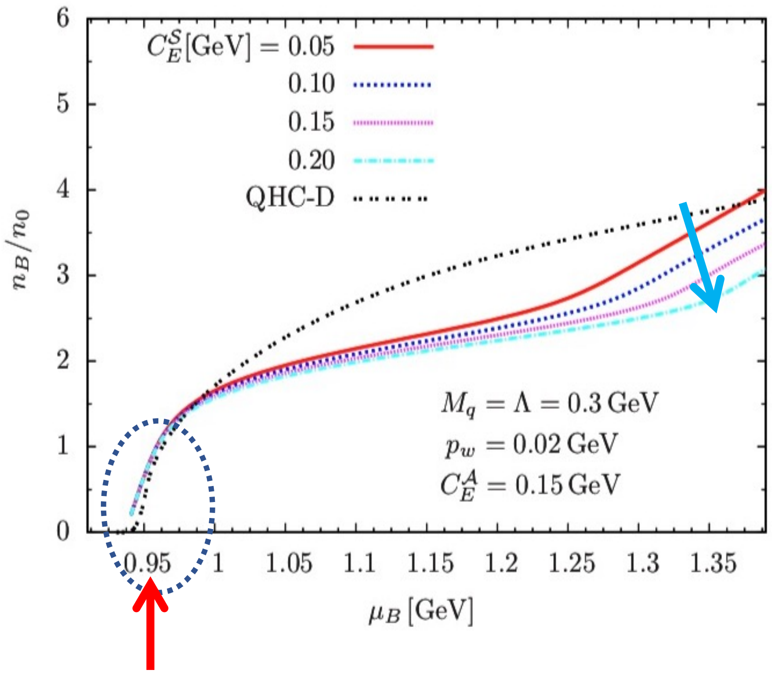


color-*sym.* channels also enter

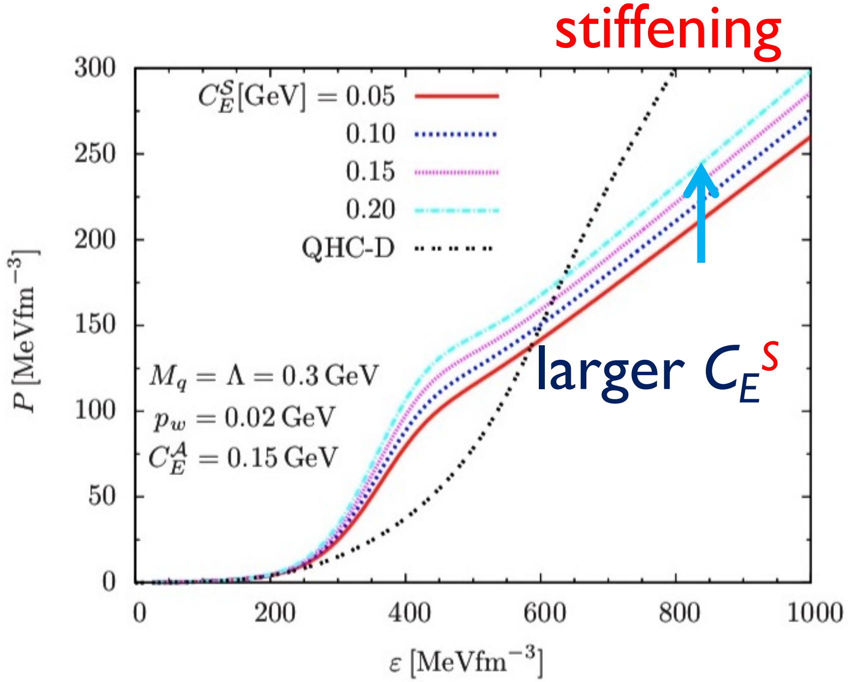
→ the quark feels *repulsive* correlations

# EoS with interactions

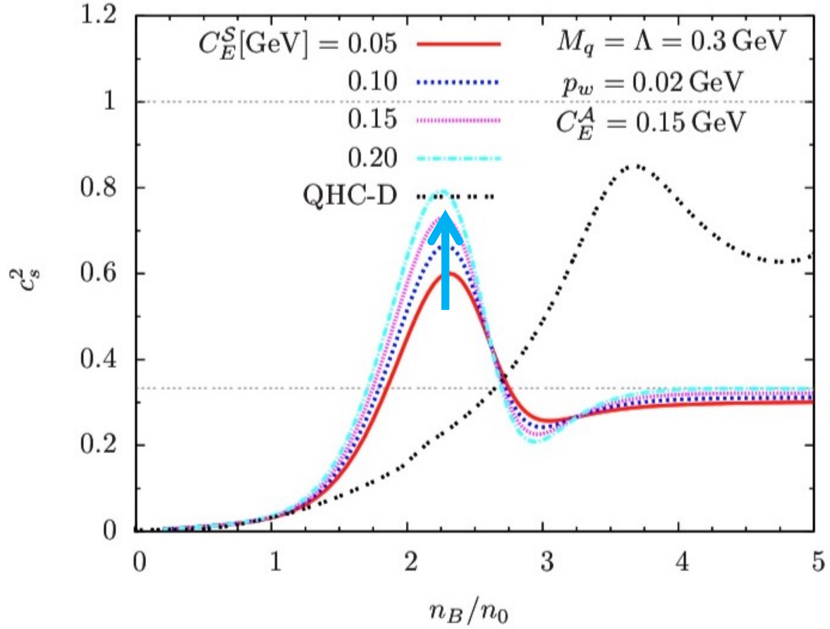



adjust $C_E{}^A$ (fit $M_B$ = 939 MeV)




high density stiffening



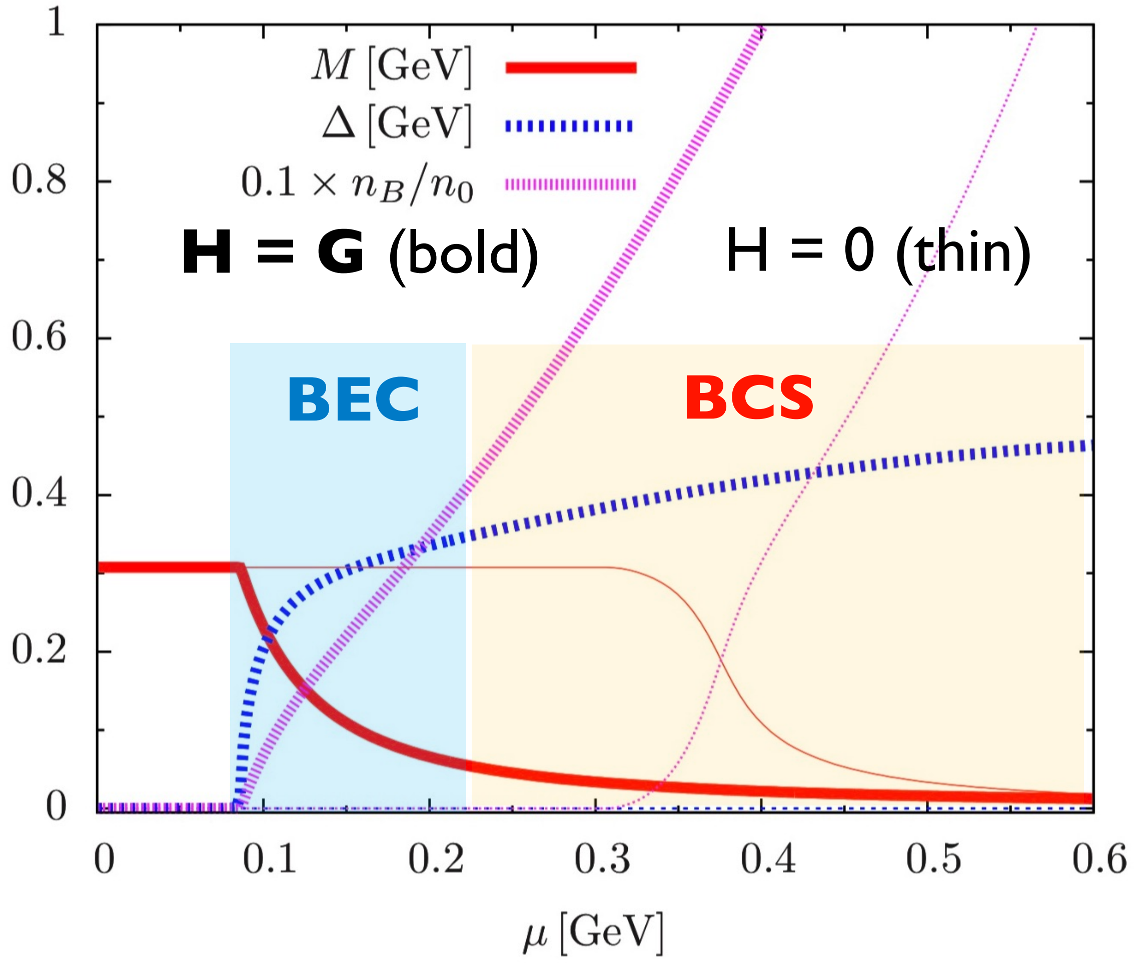

**peak in $c_s$**

# Example) 2-color NJL model

[TK-Suenaga '21]

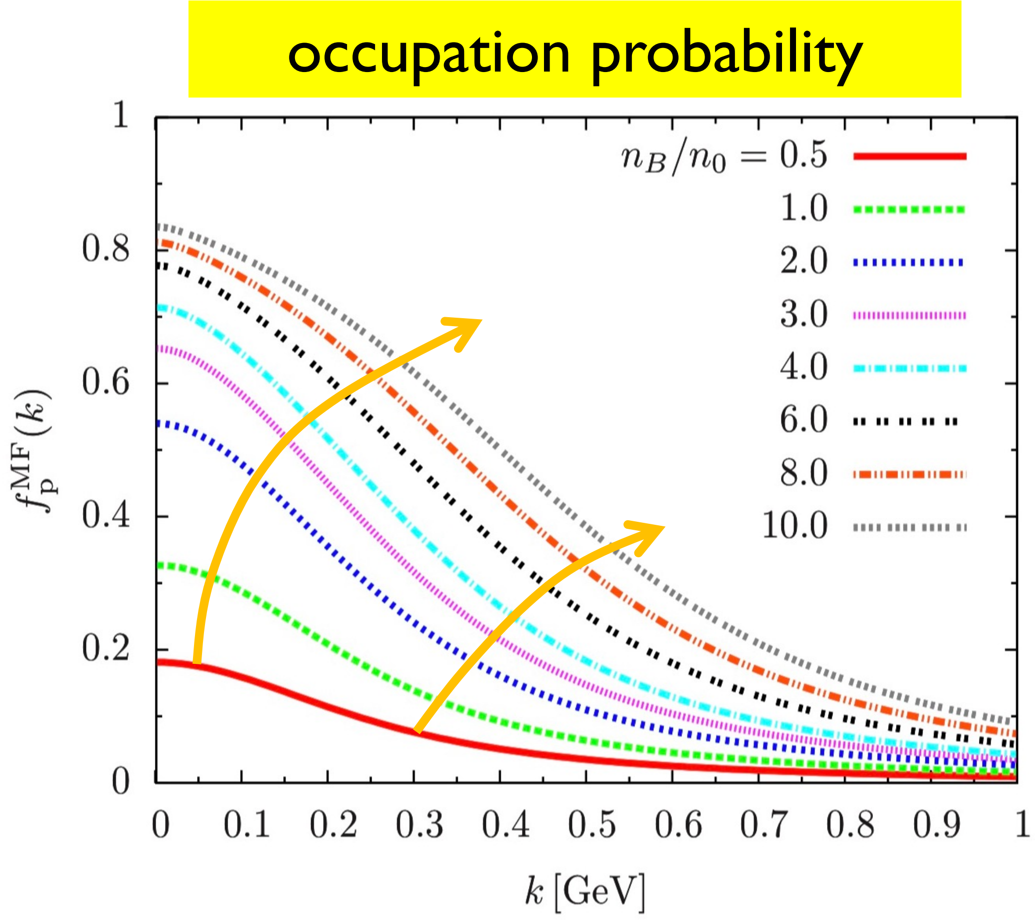
- baryons = diquarks
- diquark mass = $m_\pi$ << $M_q$
- BEC-BCS crossover (diquark condensate)

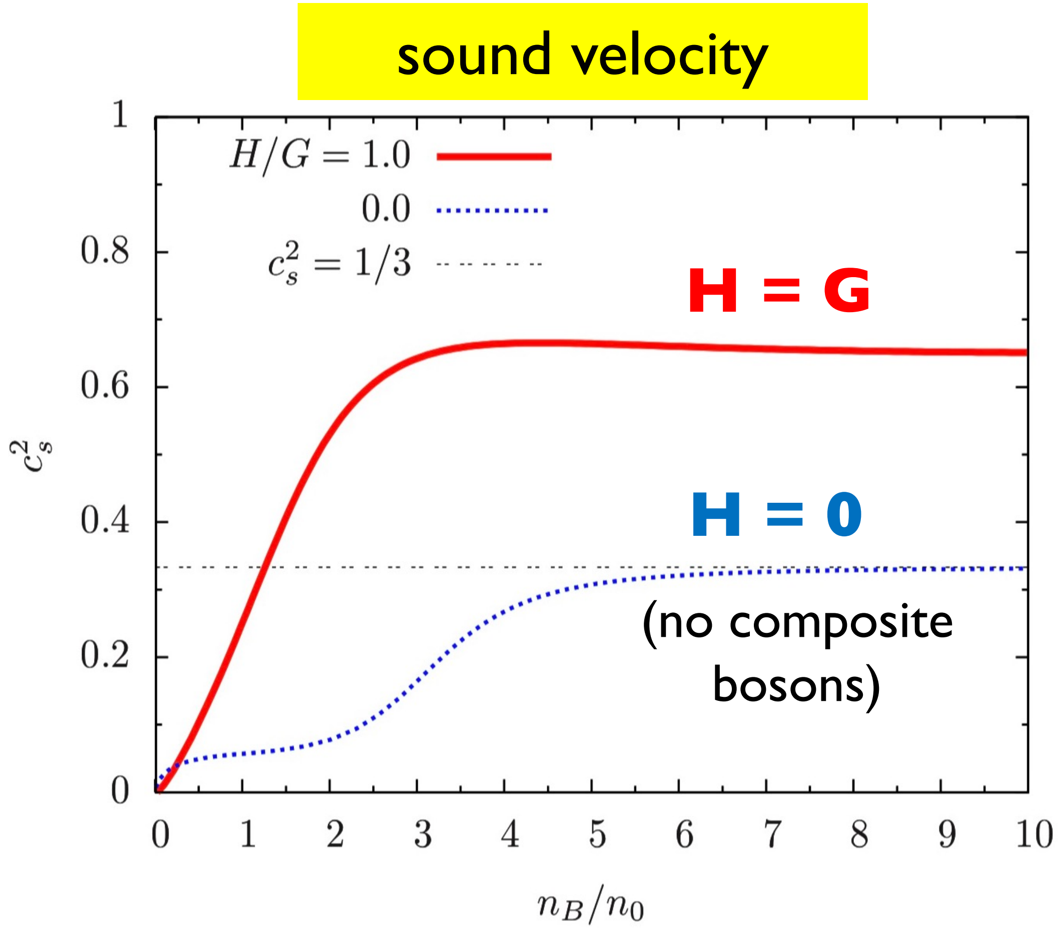
$$\mathcal{L}_4 = G\left[\,(\bar{q}\tau_a q)^2 + (\bar{q}\mathrm{i}\gamma_5\tau_a q)^2\,\right] + H\left[\,|\bar{q}\mathrm{i}\gamma_5\tau_2\sigma_2 q_C|^2 + |\bar{q}\tau_2\sigma_2 q_C|^2\,\right]$$



# Example) 2-color NJL model

[TK-Suenaga '21]

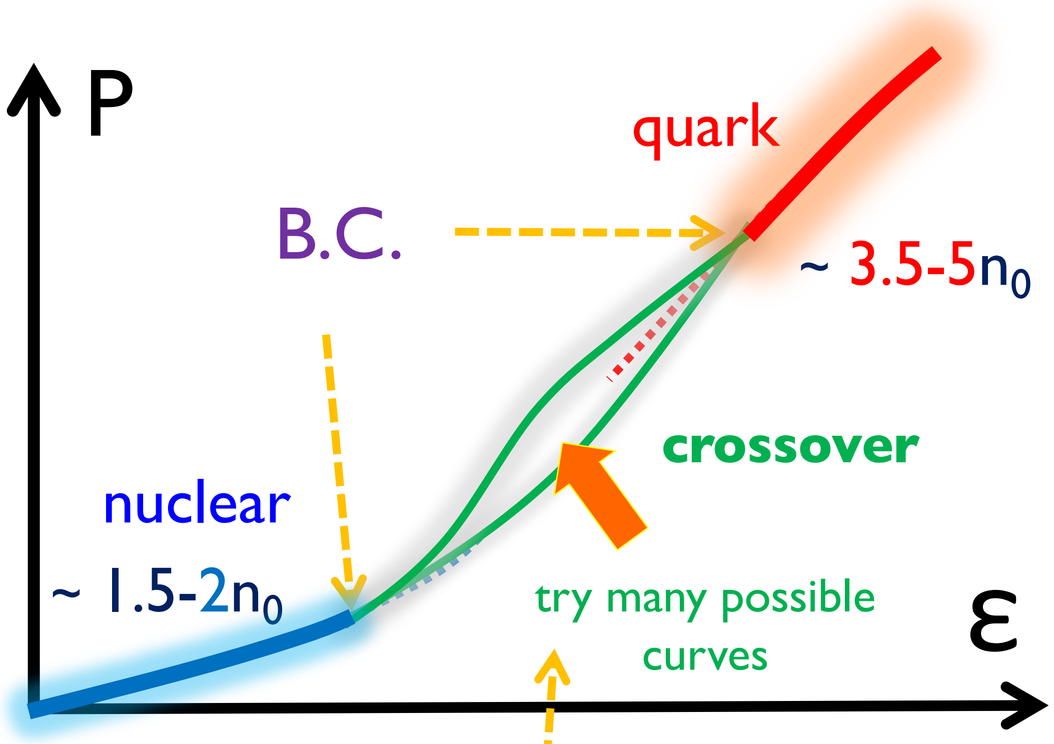



BEC → BCS & $c_s^2$↑ occur at $0.5\text{-}1 n_0$ (early stiffening)

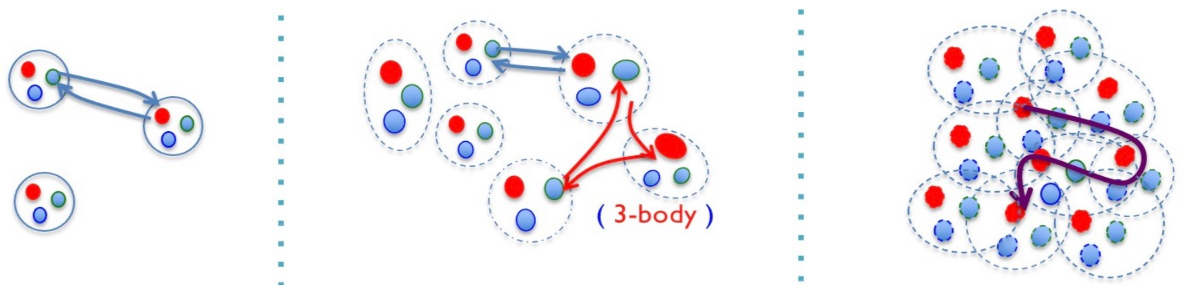
# QHC EoS (18, 19, 21)

## 3-window modeling




$c_s^2 = dP/d\varepsilon < 1$ (causality)

Masuda+('12), TK+('14), Baym+('18, 19), TK('21)




quark model template

chiral | color-mag. | nB-nB int.

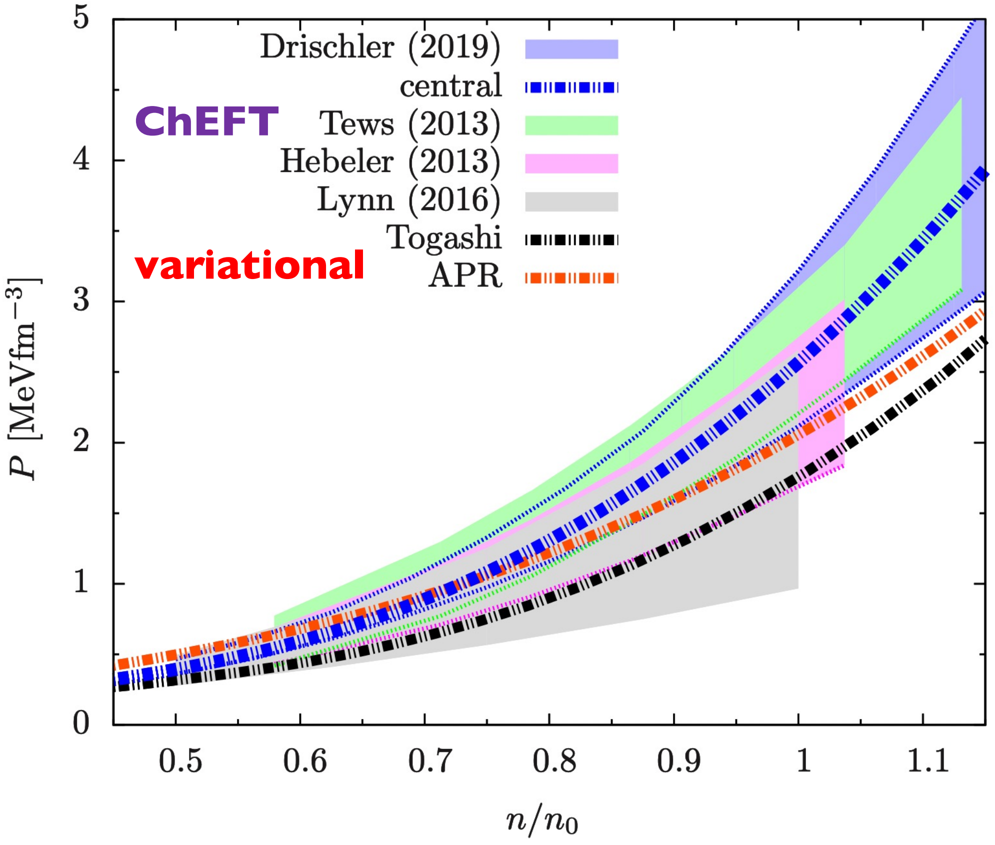
$$\mathcal{H} = \mathcal{H}_{\mathrm{NJL}} - \underline{H} \sum_A (q\Gamma_A q)(\bar{q}\Gamma_A \bar{q}) + \underline{g_V} (\bar{q}\gamma_0 q)^2$$

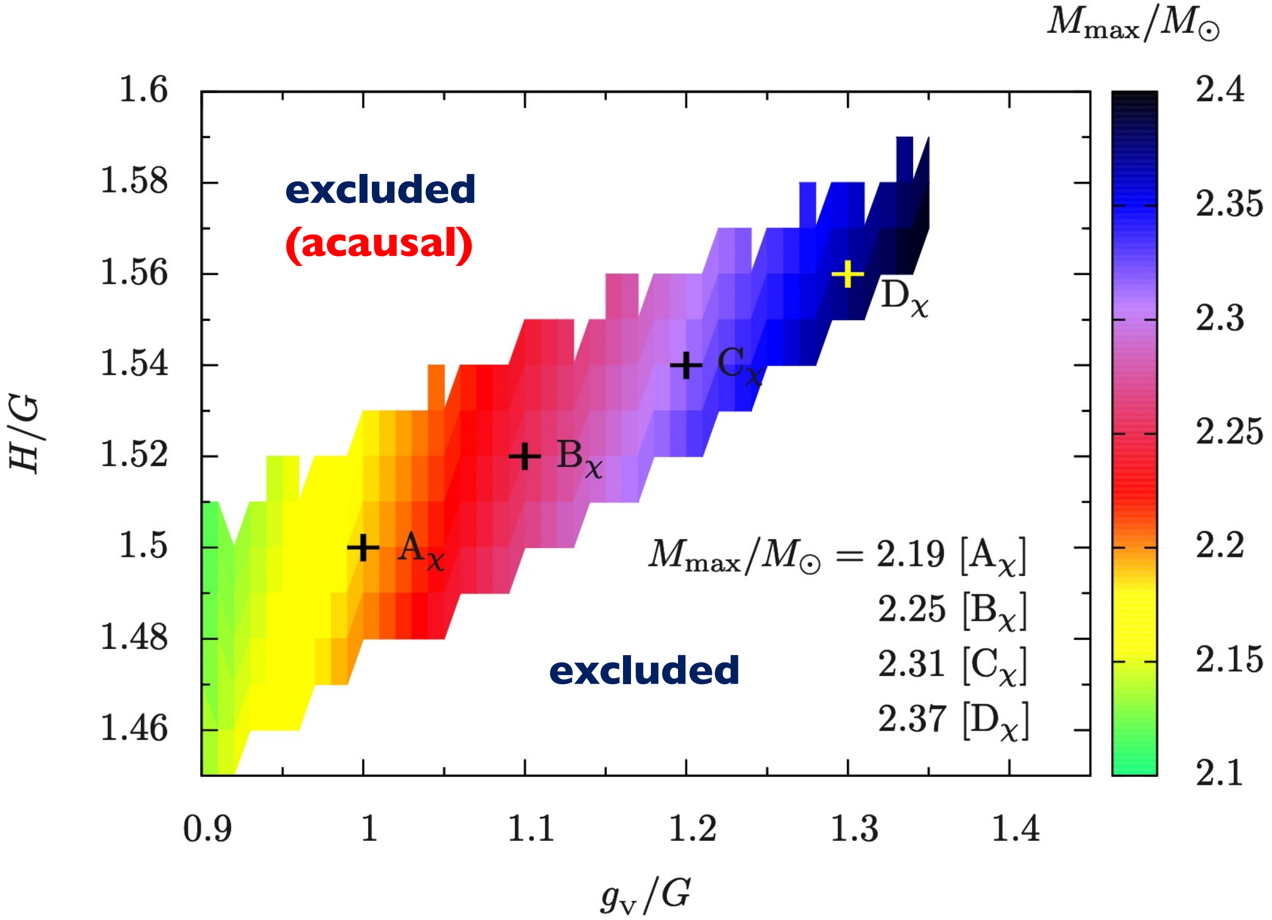
# An exercise: survey for ($g_V$ , H)@3.5-5n0 [Baym+ '19, TK '21]

**Step1)** Prepare **realistic** nuclear EoS up to **1.5-2**n0

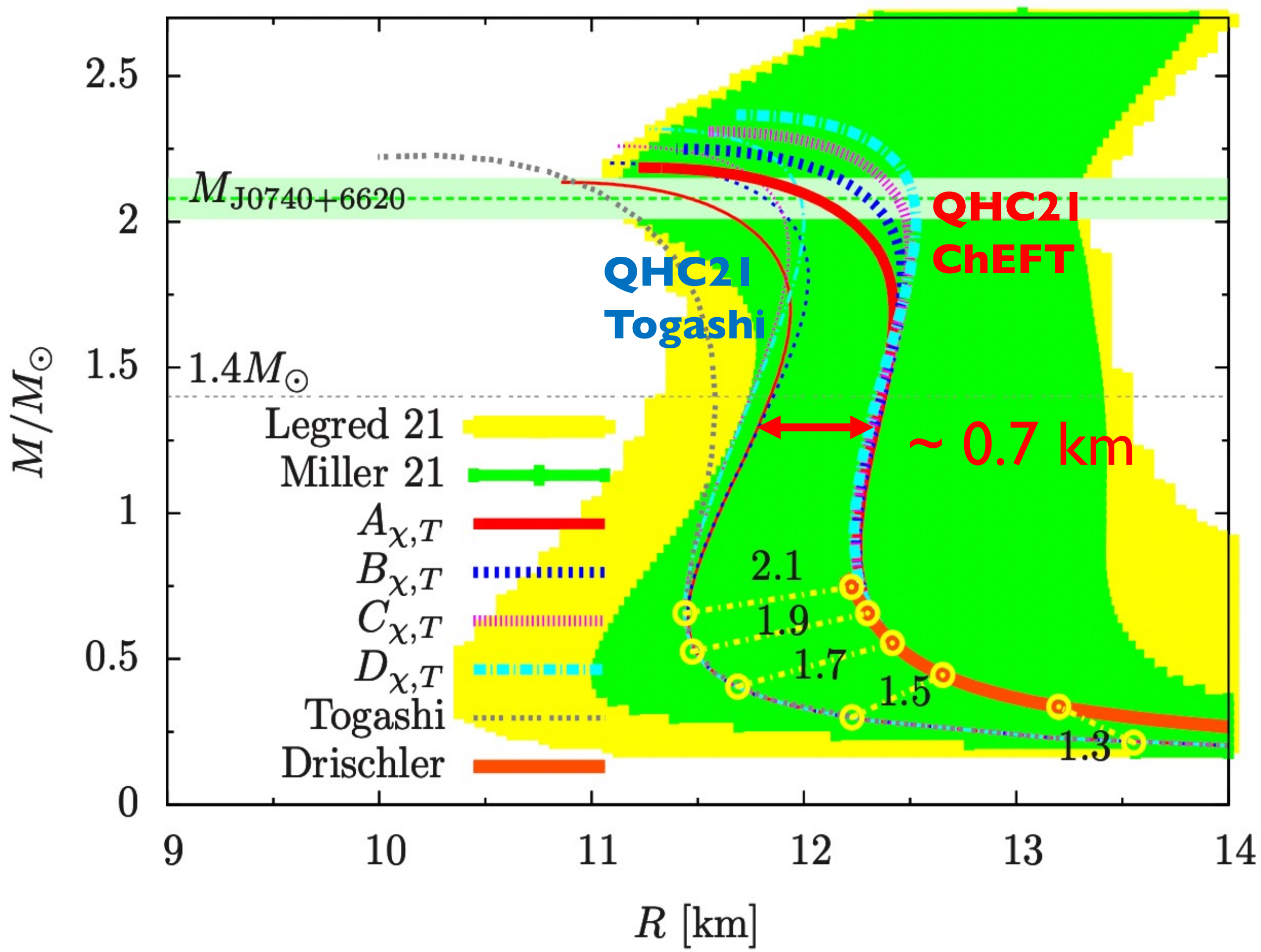
[e.g. Akmal+1998, **Togashi+2017**, **ChEFT**, ...] ➡ 30-40% uncertainties @~n0

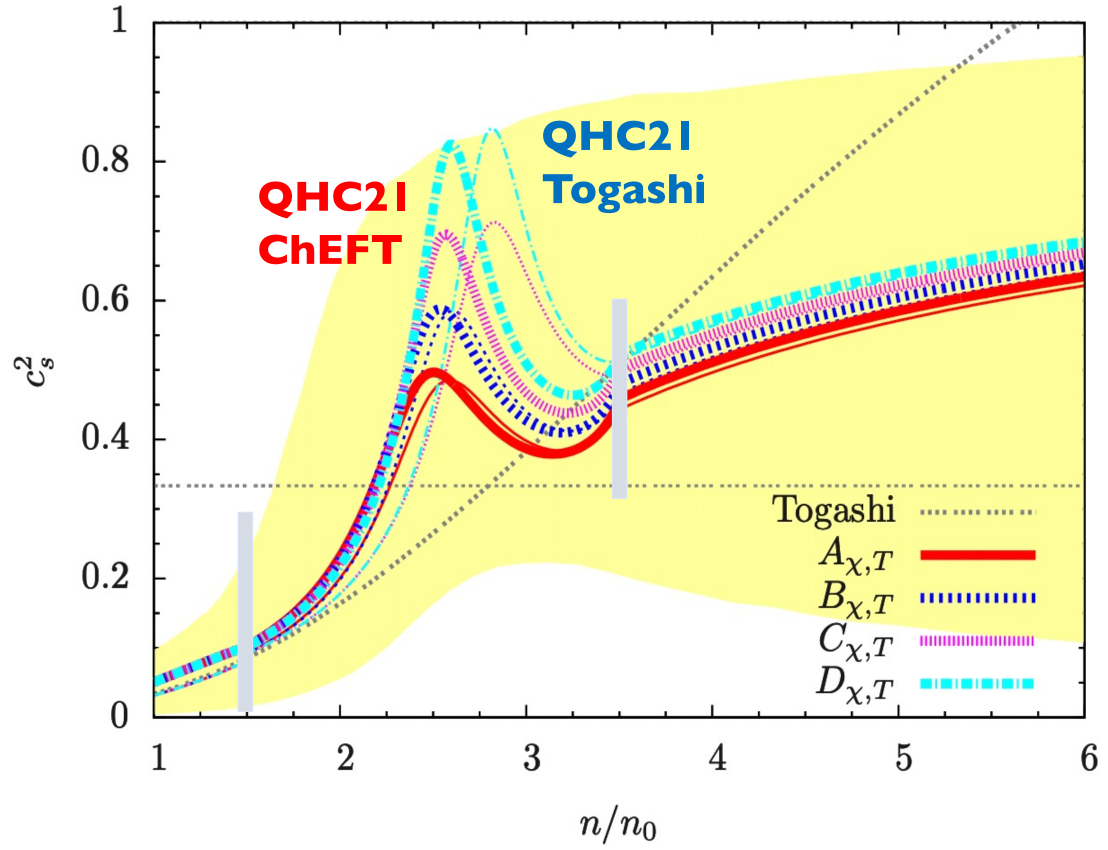
**Step2)** Survey the range of ($g_V$ , H) compatible with **causality & stability**



# An exercise: survey for $(g_V, H)_{@3.5\text{-}5n0}$ [Baym+ '19, TK '21]




nuclear uncertainties → $\Delta R_{1.4}$ ~ 0.7 km, the peak in $c_s^2$ robust

# Trends found in this exercise (for quark matter part)

for quark EoS consistent with all constraints

- bottom line: $(g_V, H)_{@3.5\text{-}5n0} \sim (G_s)_{@vac}$

  *interactions remain non-perturbative (!)*

- Slow chiral restoration

  at $5n_0$: $M_u \sim M_d \sim 50$ MeV >> ~ 5 MeV, $M_s \sim 300$ MeV >> ~100 MeV

- Pairing effects important

  at $5n_0$: $\Delta_{CFL} \sim$ **200** MeV (!)

- For allowed range of $(g_v, H)$, $M_{max} \sim 2.4\ M_{sun}$

# Inversion problem: motivations to study $B$

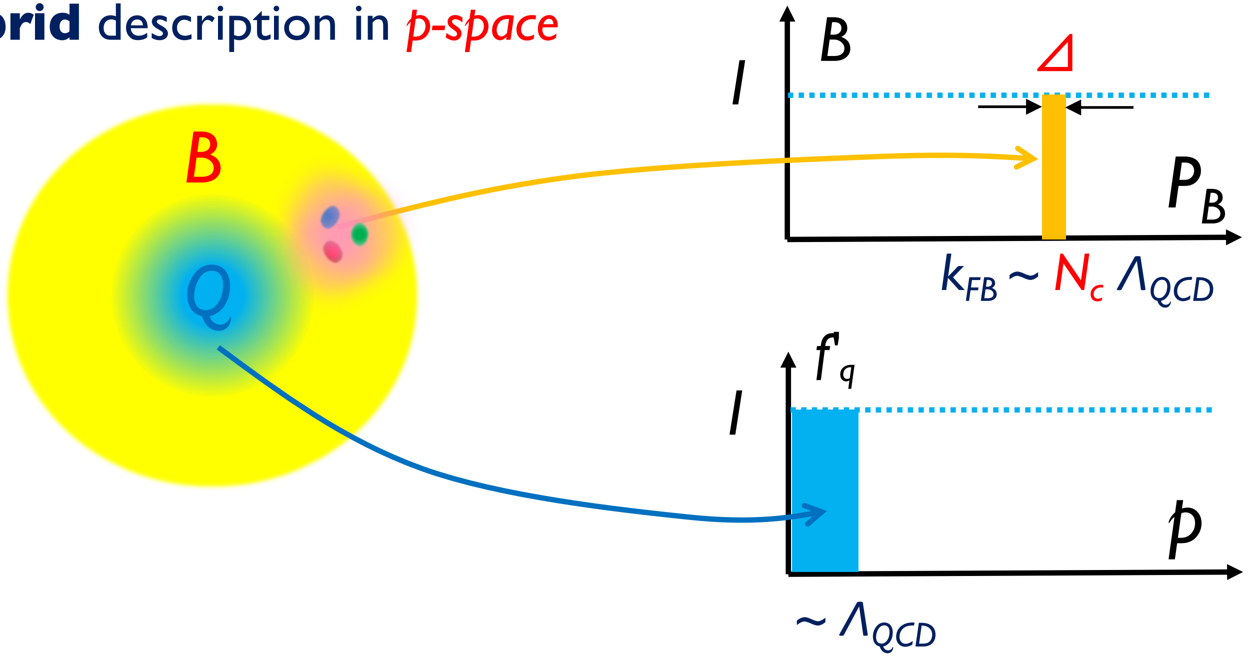
- perhaps convenient to *use the baryonic bases* for *low E* physics

$$P(\mu_B)|_{\beta\text{-eq}} \Rightarrow P(\mu_B, \mu_Q, T, \dots)$$

*extensions* of the *quark-hadron continuity*

- relations to the *McLerran-Reddy* (*MR*) model

**hybrid** description in *p-space*




**important** parameter

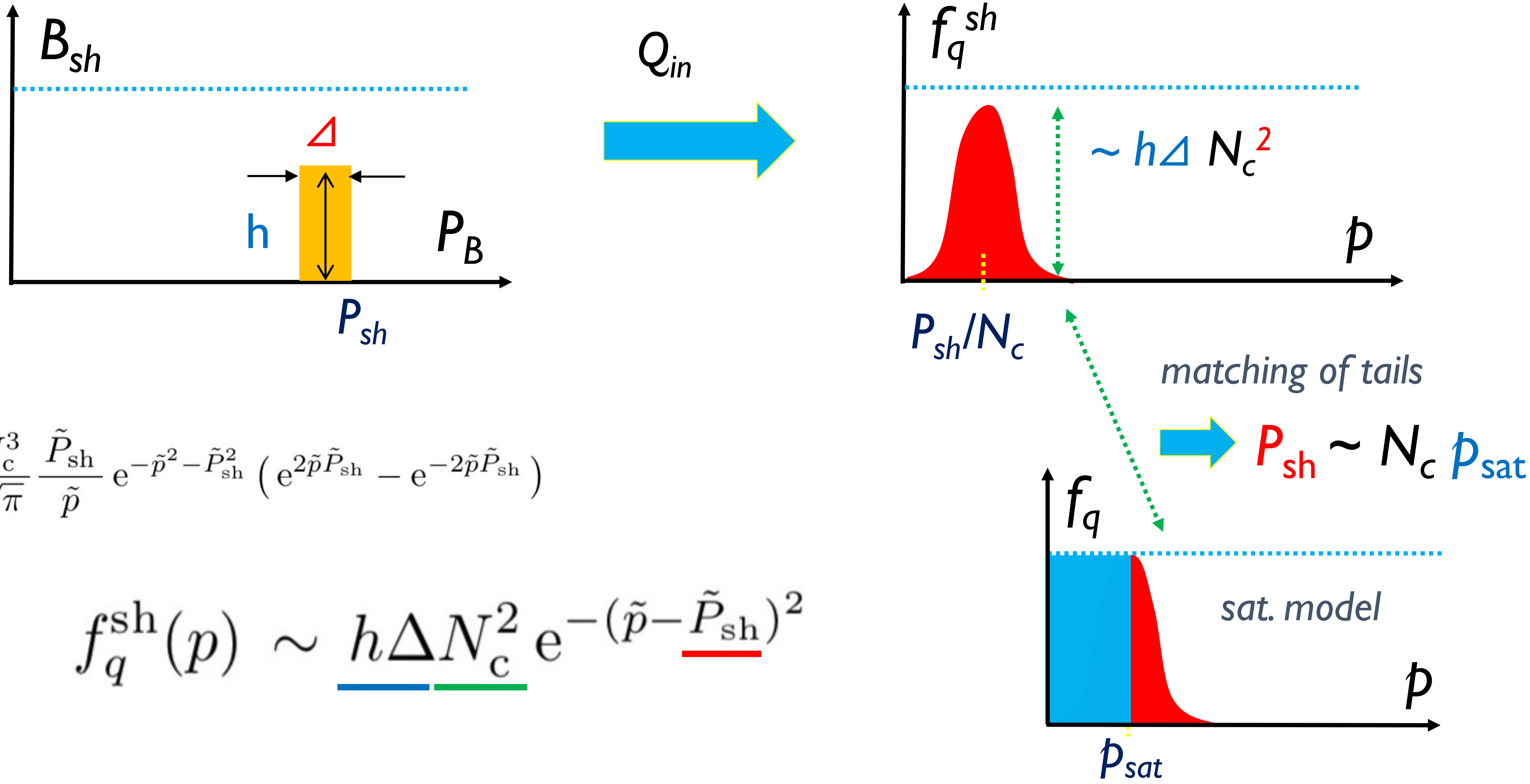
$$\Delta = \frac{\Lambda^3}{k_{\rm FB}^2} + \kappa \frac{\Lambda}{N_c^2}$$

*why this form?*

- phenomenological [McLerran-Reddy, PRL '19]
- derivation in excluded vol. model [Jeong-McLerran-Sen, '19]

# A trial: *shell form*

$$\mathcal{B}^{\rm sh}(P_B;P_{\rm sh}) = h\theta(P_{\rm sh}-P_B)\theta(P_B-P_{\rm sh}-\Delta)$$




$$f_q^{\rm sh}(p) \simeq h\Delta \frac{N_c^3}{\sqrt{\pi}}\frac{\tilde{P}_{\rm sh}}{\tilde{p}} e^{-\tilde{p}^2-\tilde{P}_{\rm sh}^2}\left(e^{2\tilde{p}\tilde{P}_{\rm sh}} - e^{-2\tilde{p}\tilde{P}_{\rm sh}}\right)$$

$$f_q^{\rm sh}(p) \sim h\Delta N_c^2\, e^{-(\tilde{p}-\tilde{P}_{\rm sh})^2}$$

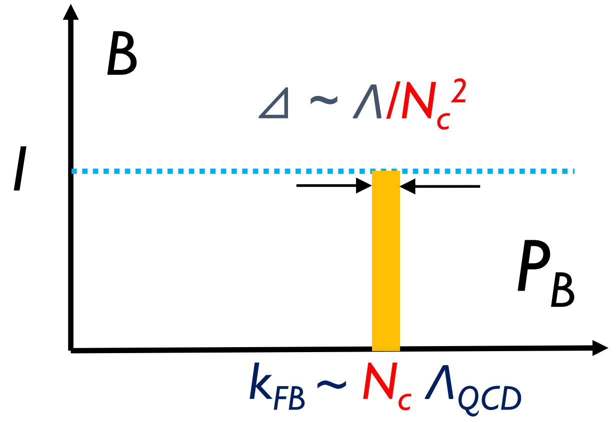
$P_{sh} \sim N_c \Lambda$

# Constraints from $f_q$ ( for $P_{sh} \sim N_c \Lambda$ )

$$f_q^{\rm sh}(p) \sim h\Delta N_c^2 \, e^{-(\tilde{p}-\tilde{P}_{\rm sh})^2}$$

constraint : $f_q^{sh} < 1$ ➡ $h\Delta < \Lambda / N_c^2$

a possible scaling form: $[h\Delta](P_{\rm sh}) \sim c_0 \Lambda \left( \frac{\Lambda^2}{P_{\rm sh}^2} + \frac{c_1}{N_c} \frac{\Lambda}{P_{\rm sh}} + \frac{c_2}{N_c^2} \right)$




MR-model (thin shell model)

$h = 1$ & $\Delta = \frac{\Lambda^3}{k_{\rm FB}^2} + \kappa \frac{\Lambda}{N_c^2}$ ($c_1 = 0$)

# MR-model: *EoS*

$P_{sh} \sim N_c \Lambda$ baryon relativistic

but $n_B^{(shell)} \simeq \frac{h}{\pi^2}\left(P_{\rm sh}^3 - (P_{\rm sh} - \Delta)^3\right) \sim \underline{h \Delta P_{\rm sh}^2}$ $n_B^{(bulk)} \sim \Lambda^3$

$$\simeq c_0 \Lambda^3 + c_1 \Lambda^2 \frac{P_{\rm sh}}{N_{\rm c}} + c_2 \Lambda \left(\frac{P_{\rm sh}}{N_{\rm c}}\right)^2$$

$n_B \sim \Lambda^3$ (!) << $(N_c \Lambda)^3$

(kin.) energy density:

$$\varepsilon - m_B n_B \sim h\Delta \times [\, E(P_{sh}) - m_B \,] \times 4\pi P_{sh}^2$$

consistent with quark's

$$\sim \Lambda/N_c^2 \times (N_c\Lambda)^2/m_B \times (N_c\Lambda)^2 \sim N_c \Lambda^4$$

*relativistic* pressure ~ $N_c \Lambda^4$ within $n_B \sim \Lambda^3$ → *stiff* EoS

# Quantum numbers ?

quark quantum numbers; $N_c$, $N_f$, 2-spins (for a given spatial w.f.)

how many baryon species are needed to saturate quark states?

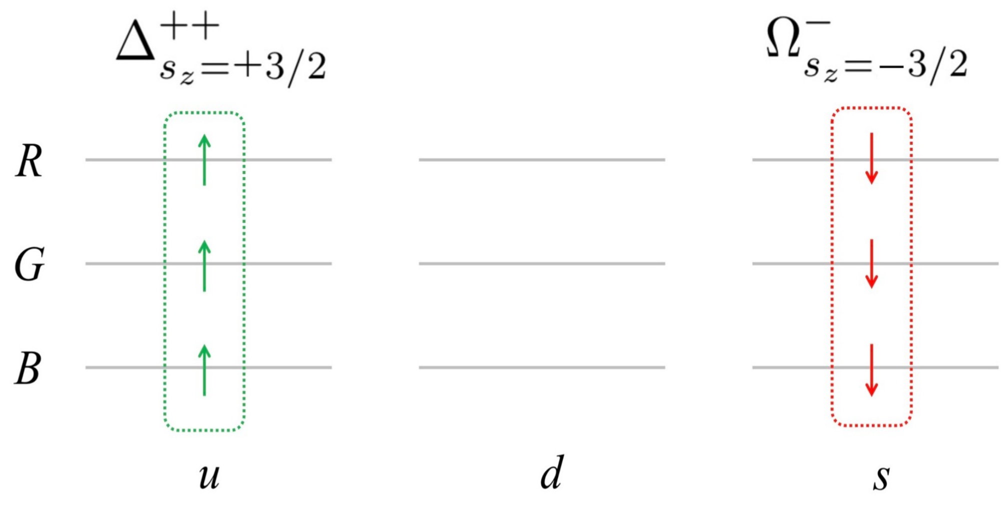
$\rightarrow$ we need only **$2N_f$ = 6** species for $N_f$ = 3

(full members of singlet, octet, decuplet are NOT necessary)

convenient **color-flavor-spin** bases

[ neglect N-Δ splitting etc. for simplicity ]

$$\Delta^{++}_{s_z=\pm 3/2} = [u_R\!\uparrow u_G\!\uparrow u_B\!\uparrow],\quad [u_R\!\downarrow u_G\!\downarrow u_B\!\downarrow],$$
$$\Delta^{-}_{s_z=\pm 3/2} = [d_R\!\uparrow d_G\!\uparrow d_B\!\uparrow],\quad [d_R\!\downarrow d_G\!\downarrow d_B\!\downarrow],$$
$$\Omega^{-}_{s_z=\pm 3/2} = [s_R\!\uparrow s_G\!\uparrow s_B\!\uparrow],\quad [s_R\!\downarrow s_G\!\downarrow s_B\!\downarrow],$$



**Total 20 min + 5 min**

**Introduction** (7 slides): 9 min

| | |
|---|---|
| 1, title | [0.5 min] |
| 2, general context | [1.0 min] |
| 3, a picture | [1.5 min] |
| 4, M-R vs EOS | [1.0 min] |
| 5 NICER | [1.5 min] |
| 7 soft-stiff | [1.5 min] |
| 8, T vs mu | [1.5 min] |

**main** (8 slides): 10.5 min

| | |
|---|---|
| 1, strategy | [1.5 min] |
| 2, quarks in B | [2.0 min] |
| 3, quarks in B-mat | [1.5 min] |
| 4, evo | [2.0 min] |
| 5, saturation3 | [1.5 min] |
| 6, saturation4 | [1.5 min] |