

Constraining quark matter inside hybrid stars

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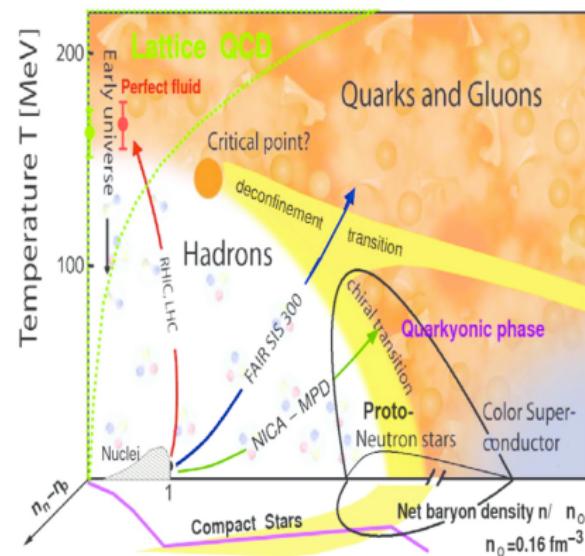


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Motivation: QCD and neutron stars

- ▶ We can not solve QCD at large densities from first principles due to the sign problem
- ▶ There are no experimental results in this region so far
- ▶ We may use effective models to try to describe strongly interacting matter
- ▶ Neutron stars may provide constraints for these models



Linear sigma model

Simple effective model that realizes global chiral symmetry:

$SU(1)$ linear sigma model:

$$\mathcal{L} = \bar{\Psi} [i\not{D} - g(\sigma + i\pi\gamma_5)] \Psi + \frac{1}{2} [(\partial_\mu\sigma)^2 + (\partial_\mu\pi)^2] - V(\sigma, \pi)$$

Mesonic potential: $V(\sigma, \pi) = \frac{\lambda}{4}(\sigma^2 + \pi^2 - f^2)^2$, $\lambda > 0$

Spontaneous symmetry breaking: $(\sigma, \pi) \rightarrow (f + \sigma, \pi)$

↪ generates mass for the fermion: $m_q = gf$ (Goldberger–Treiman relation)

↪ Nambu–Goldstone boson: $m_\pi = 0$, $m_\sigma = \sqrt{2\lambda f^2}$

Including **thermal contribution** from quarks → symmetry is restored at high temperature and density

↪ $SU(3)$ theories describe vacuum phenomenology and chiral phase transition successfully

eLSM Particle content

- **Vector** and **Axial-vector** meson nonets

$$V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \omega_S \end{pmatrix}^\mu \quad A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

$$\begin{aligned} \rho &\rightarrow \rho(770), K^* \rightarrow K^*(894) \\ \omega_N &\rightarrow \omega(782), \omega_S \rightarrow \phi(1020) \end{aligned}$$

$$\begin{aligned} a_1 &\rightarrow a_1(1230), K_1 \rightarrow K_1(1270) \\ f_{1N} &\rightarrow f_1(1280), f_{1S} \rightarrow f_1(1426) \end{aligned}$$

- **Scalar** ($\sim \bar{q}_i q_j$) and **pseudoscalar** ($\sim \bar{q}_i \gamma_5 q_j$) meson nonets

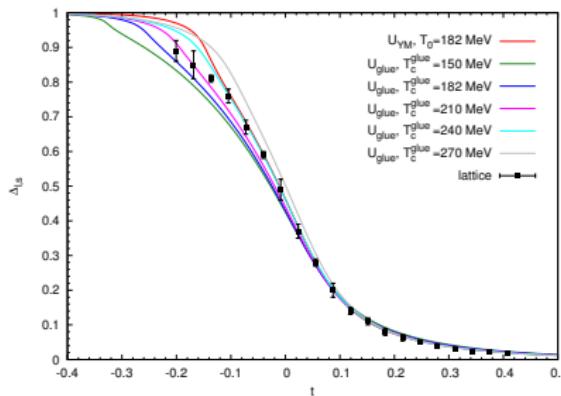
$$\Phi_S = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_0^{*+} \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_0^{*0} \\ K_0^{*-} & \bar{K}_0^{*0} & \sigma_S \end{pmatrix} \quad \Phi_{PS} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

multiple possible assignments
mixing in the $\sigma_N - \sigma_S$ sector

$\pi \rightarrow \pi(138), K \rightarrow K(495)$
mixing: $\eta_N, \eta_S \rightarrow \eta(548), \eta'(958)$

Spontaneous symmetry breaking: $\sigma_{N/S}$ acquire nonzero expectation values $\phi_{N/S}$
fields shifted by their expectation value: $\sigma_{N/S} \rightarrow \sigma_{N/S} + \phi_{N/S}$

Results at zero chemical potential

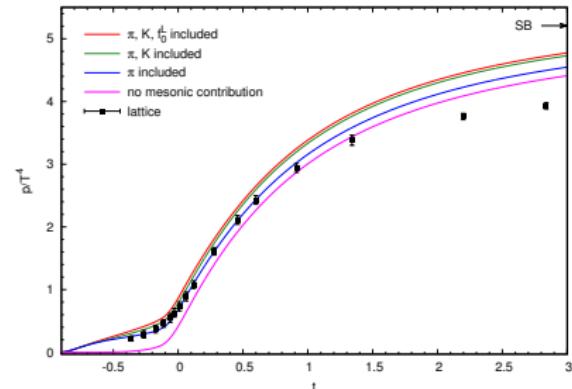


– subtracted chiral condensate:

$$\Delta_{ls} = \frac{\left(\Phi_N - \frac{h_N}{h_S} \cdot \Phi_S \right) \Big|_T}{\left(\Phi_N - \frac{h_N}{h_S} \cdot \Phi_S \right) \Big|_{T=0}}$$

– good agreement with the lattice result of [Borsányi et al., JHEP 1009, 073 \(2010\)](#)

- pions dominate the pressure at small T
- contribution of the kaons is important
- at high T the pressure overshoots the lattice data of [Borsányi et al., JHEP 1011, 077 \(2010\)](#)



Ingredients for hybrid stars 1

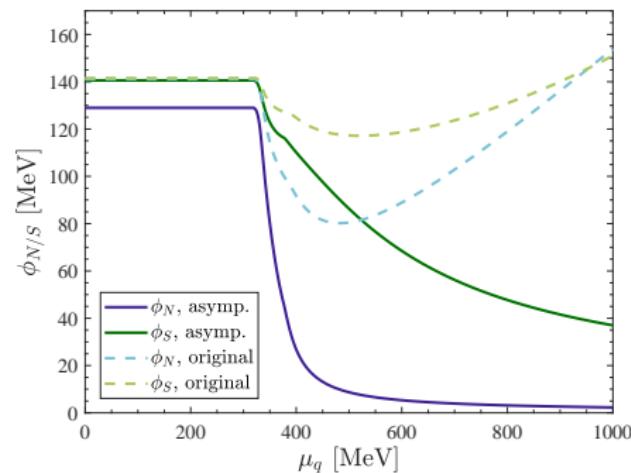
For **hybrid stars** we need the EoS at high density and $T = 0$:

- ▶ we need to introduce non-zero **vector condensates**
- ▶ free electron gas + β -equilibrium
- ▶ charge neutrality: $\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0$
- ▶ 5 field equations (no Polyakov-loop contribution)

↪ a naive parametrization →
chiral symmetry would be
broken at high densities

↪ investigating the **asymptotic**
behavior we get an additional
constraint for the parameters

↪ we get **$m_\sigma = 290$ MeV** from
parametrization



Ingredients for hybrid stars 2

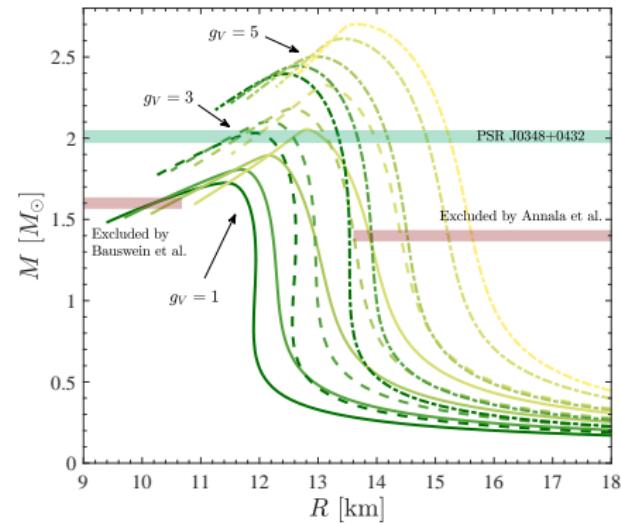
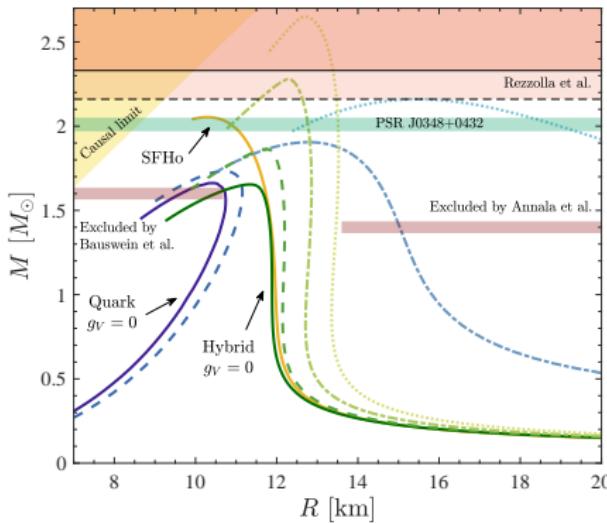
Hybrid stars also have a hadronic crust and outer core:

- ▶ at low densities we use hadronic EoS's (**SFHo** and **DD2**)
- ▶ we apply a smooth crossover between the two phases:
 $\varepsilon(n)$ interpolation

$$\begin{aligned}\varepsilon(n) &= \varepsilon_H(n)f_-(n) + \varepsilon_Q(n)f_+(n), \\ f_{\pm}(n) &= \frac{1}{2} \left(1 \pm \tanh \left(\frac{n - \bar{n}}{\Gamma} \right) \right)\end{aligned}$$

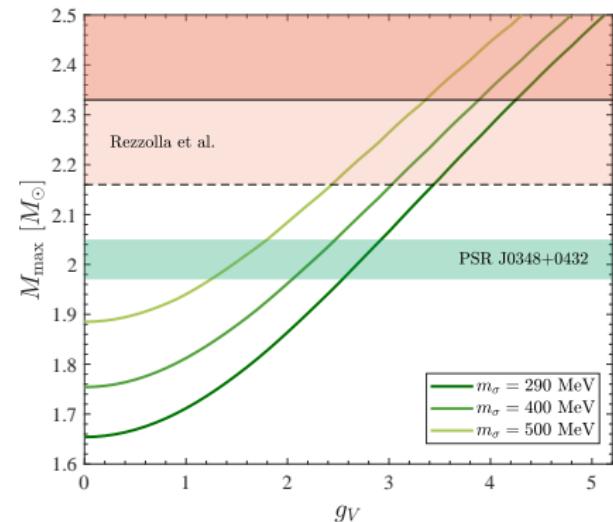
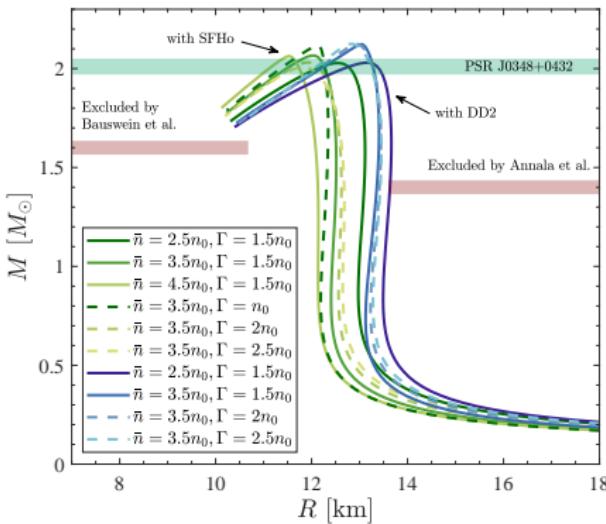
- ↪ we also try other interpolation methods
- ↪ 4 tunable parameters altogether: m_σ , g_V , \bar{n} , Γ
- ↪ we use the $\varepsilon(n)$ interpolation with $\bar{n} = 3.5n_0$ and $\Gamma = 1.5n_0$ as our standard choice

$M - R$ curves for different g_V 's



- ↪ larger vector couplings result in larger hybrid star masses
- ↪ maximum masses are increased due to the **intermediate density stiffening** of the hybrid EoS's
- ↪ large sigma masses (brighter tones) are excluded by upper radius constraints

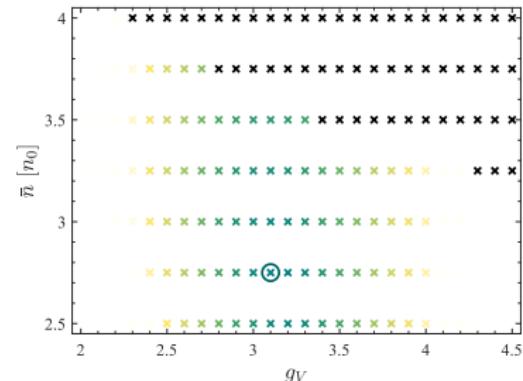
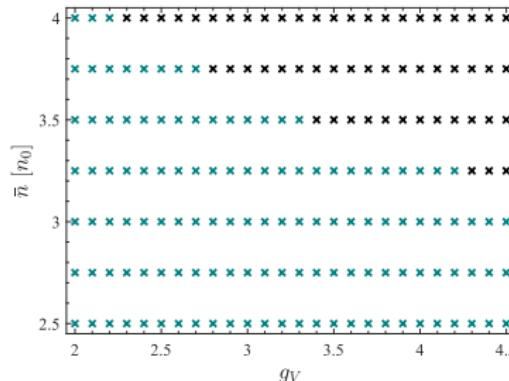
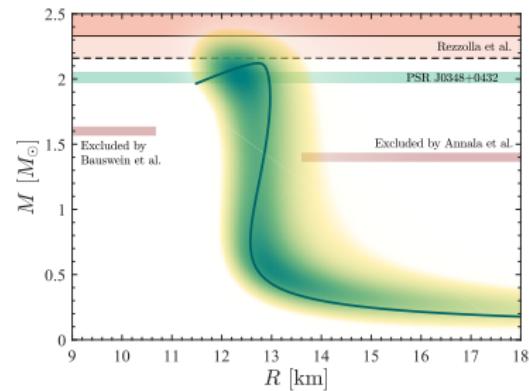
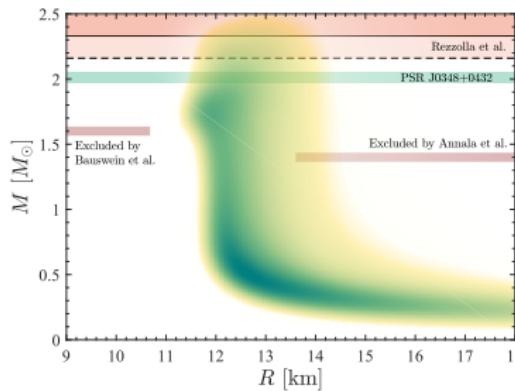
Effect of sigma mass and phase transition



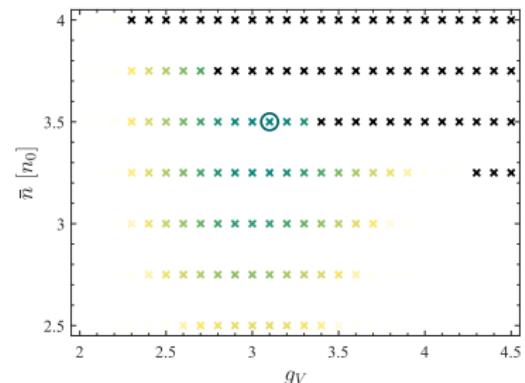
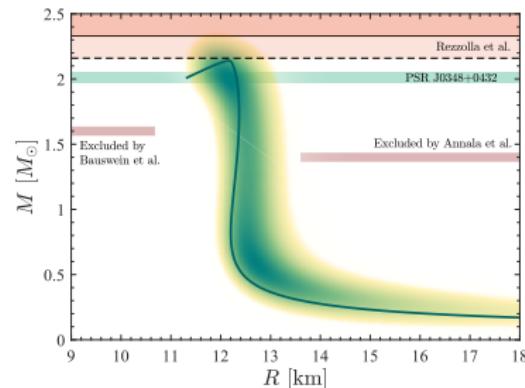
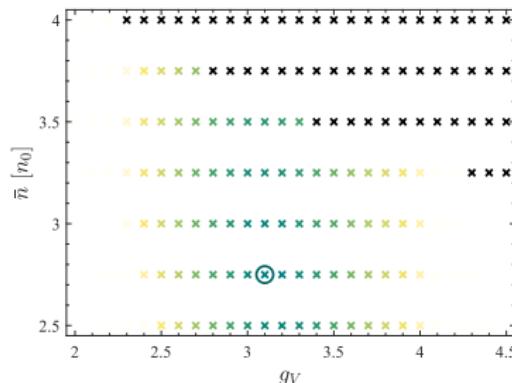
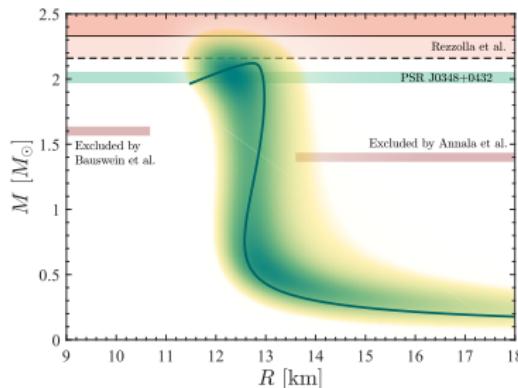
- ↪ maximum mass hybrid stars seem to reside in a small region, independent of the phase transition parameters¹
- ↪ with $m_{\sigma} = 290$ MeV g_V is constrained to $2.6 < g_V < 4.3$

¹similar results were found in Cierniak & Blaschke, EPJ ST 229, 3663 (2020)

Results from Bayesian analysis



Results from Bayesian analysis



Summary

Conclusions

- ▶ we developed a model that describes **vacuum phenomenology** and **finite temperature behaviour** accurately
- ▶ we found that the **maximum neutron star mass** can be used to **constrain the parameters** of the model
- ▶ a very low sigma meson mass is preferred ($m_\sigma = 290$ MeV), while $2.6 < g_V < 4.3$
- ▶ from our Bayesian analysis we found that a **narrow phase transition** is optimal with its **center at $3 - 4n_0$**
- ▶ check out our paper: arXiv:2111.06127

Discussion

- ▶ without upper mass constraint we get $g_V \approx 5$
- ▶ using a self-consistent parametrization², we also get $g_V \approx 5$

²Gy. Kovács et al., Phys. Rev. D 104, 056013 (2021)

Thank you for your attention!

QCD and chiral symmetry

The QCD Lagrangian:

$$\mathcal{L} = \sum_f \bar{q}_f (i\gamma^\mu D_\mu - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

↪ has local $SU(3)^{\text{color}}$ gauge symmetry

Introducing $q_L = \frac{1-\gamma_5}{2} q$ and $q_R = \frac{1+\gamma_5}{2} q$ left- and right-handed fields the fermionic terms become:

$$\begin{aligned}\bar{q} i \not{D} q &\rightarrow \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R \\ \bar{q} m q &\rightarrow \bar{q}_R m q_L + \bar{q}_L m q_R\end{aligned}$$

↪ in the limit $m \rightarrow 0$ we have an additional $U(3)_L \times U(3)_R$
 $\simeq U(3)_V \times U(3)_A$ global symmetry

QCD and chiral symmetry

$$\bar{q}i\cancel{D}q \rightarrow \bar{q}_L i\cancel{D}q_L + \bar{q}_R i\cancel{D}q_R$$
$$\bar{q}mq \rightarrow \bar{q}_R mq_L + \bar{q}_L mq_R$$

↪ in the limit $m \rightarrow 0$ we have an additional $U(3)_L \times U(3)_R$
 $\simeq U(3)_V \times U(3)_A$ global symmetry

Symmetry breaking:

- ▶ Nontrivial QCD vacuum: Nambu–Goldstone bosons → pseudoscalar mesons
- ▶ Explicit symmetry breaking by the **quark masses**: $SU(3)_A$ is broken, pseudoscalar mesons have nonzero masses
- ▶ With $m_u = m_d \neq m_s$: $U(3)_V \rightarrow U(1)_V \times SU(2)_V$
- ▶ $U(1)_A$ symmetry is broken due to the 't Hooft instanton mechanism: large mass of the η' meson
- ▶ $U(1)_V$ symmetry corresponds to baryon number conservation

eLSM Lagrangian I.

The **matter** and **external** fields are

$$\begin{aligned}\Phi &= \sum_{i=0}^8 (\Phi_{S,i} + i\Phi_{PS,i}) T_i, & H &= \sum_{i=0}^8 h_i T_i & T_i : U(3) \text{ generators} \\ R^\mu &= \sum_{i=0}^8 (V_i^\mu - A_i^\mu) T_i, & L^\mu &= \sum_{i=0}^8 (V_i^\mu + A_i^\mu) T_i, & \Delta &= \sum_{i=0}^8 \delta_i T_i \\ \Psi &= (u, d, s)^T\end{aligned}$$

Non strange – strange basis:

$$\xi_N = \sqrt{2/3}\xi_0 + \sqrt{1/3}\xi_8,$$

$$\xi_S = \sqrt{1/3}\xi_0 - \sqrt{2/3}\xi_8, \quad \xi \in (\Phi_{S,i}, \Phi_{PS,i}, V_i^\mu, A_i^\mu, h_i)$$

Broken symmetry: non-zero condensates $\langle \sigma_{N/S} \rangle \equiv \bar{\sigma}_{N/S}$

eLSM Lagrangian

\mathcal{L} constructed based on linearly realized global $U(3)_L \times U(3)_R$ symmetry and its explicit breaking

$$\begin{aligned}
 \mathcal{L} = & \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\
 & + \textcolor{red}{c_1 (\det \Phi + \det \Phi^\dagger)} + \textcolor{teal}{\text{Tr}[H(\Phi + \Phi^\dagger)]} - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\
 & + \text{Tr} \left[\left(\frac{m_1^2}{2} \mathbb{I} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + i \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
 & + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\
 & + \bar{\Psi} i \gamma_\mu D^\mu \Psi - g_F \bar{\Psi} (\Phi_S + i \gamma_5 \Phi_{PS}) \Psi - g_V \bar{\Psi} (\gamma^\mu (V_\mu + \gamma_5 A_\mu)) \Psi
 \end{aligned}$$

$$\begin{aligned}
 D^\mu \Phi &= \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu[T_3, \Phi], \\
 L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\}, \\
 R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\}, \\
 D^\mu \Psi &= \partial^\mu \Psi - iG^\mu \Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a.
 \end{aligned}$$

+ Polyakov loop potential

Features of our approach

- ▶ D.O.F's:
 - scalar, pseudoscalar, vector, and axial-vector nonets
 - u, d, s constituent quarks ($m_u = m_d$)
 - Polyakov loop variables $\Phi, \bar{\Phi}$ with $\mathcal{U}_{\log}^{\text{YM}}$ or $\mathcal{U}_{\log}^{\text{glue}}$
- ▶ no mesonic fluctuations, only fermionic ones, grand potential approximated as: $\Omega(T, \mu_q) = \mathcal{U}_{\text{meson}}^{\text{tree}}(\langle M \rangle) + \Omega_{\bar{q}q}^{(0)}(T, \mu_q) + \mathcal{U}_{\log}(\Phi, \bar{\Phi})$, $\tilde{\mu}_q = \mu_q - iG_4$
- ▶ fermionic vacuum and thermal fluctuations included in the (pseudo)scalar curvature masses used to parameterize the model

$$\Delta \hat{m}_{ab}^{2,(X)} \equiv \left. \frac{d^2 \Omega_{\bar{q}q}^{(0)}(T, \mu_q)}{dX_a dX_b} \right|_{\min}, \quad X \in \{S, P\}$$

- ▶ tree-level (axial)vector masses
- ▶ 4 coupled T/μ_B -dependent field equations for the condensates $\phi_N, \phi_S, \Phi, \bar{\Phi}$
- ▶ thermal contribution of π, K, f_0^L included in the pressure, however their curvature mass contains no mesonic fluctuations

Determination of the parameters

- ▶ PCAC → 2 physical quantities: f_π, f_K
- ▶ Curvature masses → 16 physical quantities:
 $m_u/d, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{K_1},$
 $m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- ▶ Decay widths → 12 physical quantities:
 $\Gamma_{\rho \rightarrow \pi\pi}, \Gamma_{\Phi \rightarrow KK}, \Gamma_{K^* \rightarrow K\pi}, \Gamma_{a_1 \rightarrow \pi\gamma}, \Gamma_{a_1 \rightarrow \rho\pi}, \Gamma_{f_1 \rightarrow KK^*}, \Gamma_{a_0}, \Gamma_{K_S \rightarrow K\pi},$
 $\Gamma_{f_0^L \rightarrow \pi\pi}, \Gamma_{f_0^L \rightarrow KK}, \Gamma_{f_0^H \rightarrow \pi\pi}, \Gamma_{f_0^H \rightarrow KK}$
- ▶ Pseudocritical temperature T_c at $\mu_B = 0$

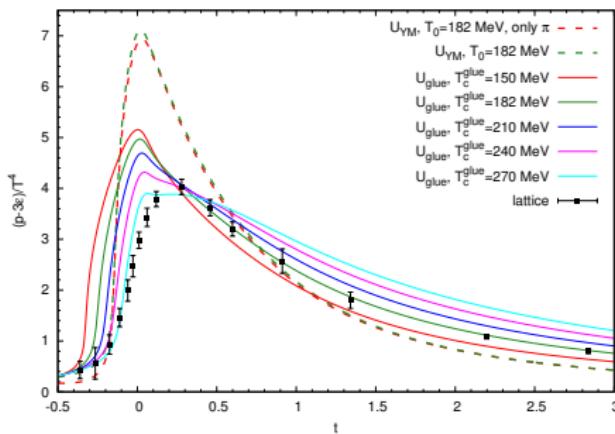
14 unknown parameters ($m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F$) → determined by the min. of χ^2 :

$$\chi^2(x_1, \dots, x_N) = \sum_{i=1}^M \left[\frac{Q_i(x_1, \dots, x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2,$$

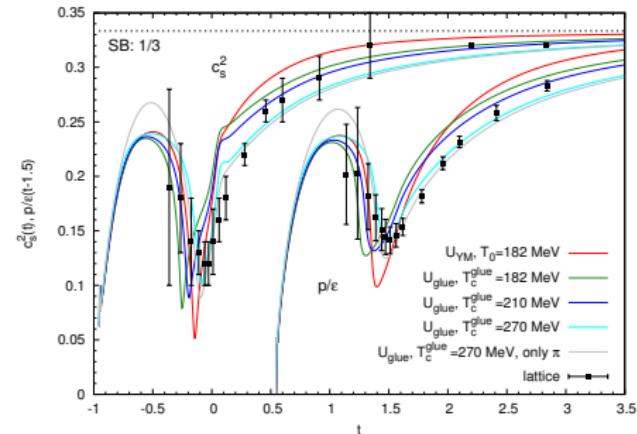
$(x_1, \dots, x_N) = (m_0, \lambda_1, \lambda_2, \dots)$, $Q_i(x_1, \dots, x_N)$ → from the model, Q_i^{exp} → PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$
multiparametric minimalization → MINUIT

Results at zero chemical potential

Scaled interaction measure

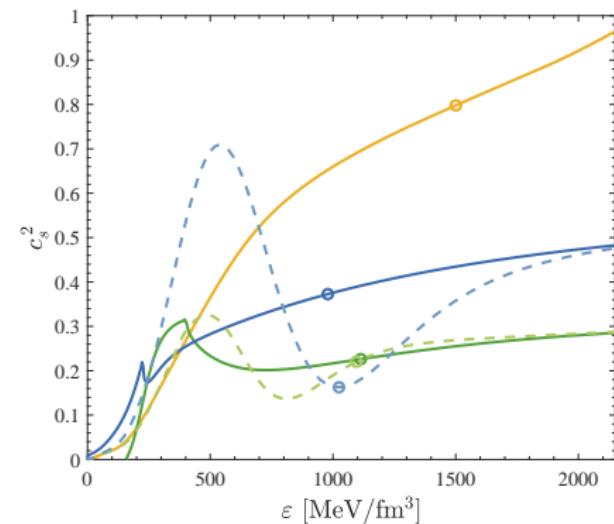
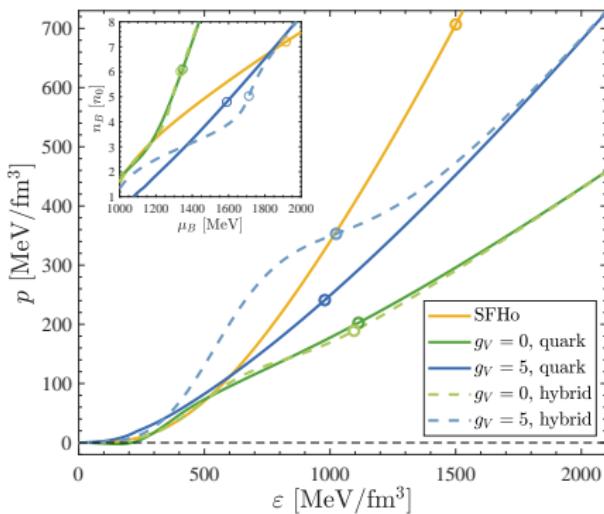


Speed of sound and p/ϵ



For large temperature the speed of sound approaches the conformal limit $c_s^2 \rightarrow 1/3$

Equation of state



- ↪ the concatenation results in a **stiff intermediate density region** for larger vector couplings
- ↪ the maximum density inside hybrid stars resides in the crossover region

Self-consistent Gaussian approximation

$$\Delta \hat{m}_{ab}^{2,(X)} \equiv \left. \frac{d^2 U_f(\phi, \xi)}{dX_a dX_b} \right|_{\xi=0}, \quad X \in \{S, P\}$$

$$\Delta \hat{M}_{\mu\nu,ab}^{2,(Y)} \equiv - \left. \frac{d^2 U_f(\phi, \xi)}{dY_a^\mu dY_b^\nu} \right|_{\xi=0}, \quad Y \in \{V, A\}, \quad \xi \in \{X_a, Y_a\}, \quad \phi \in \{\phi_N, \phi_S, \Phi, \bar{\Phi} \dots\}$$

where U_f is the fermionic contribution to the effective potential,

$$U_f(\phi, \xi) = i \text{Tr}_D \int_K \log(i\mathcal{S}^{-1}(K; \xi))|_{\xi=0} - \frac{i}{2} \text{Tr} \int_K \log \left(i\mathcal{D}_{(\mu\nu),ab}^{-1}(K) - \Pi_{(\mu\nu),ab}(K) \right)$$

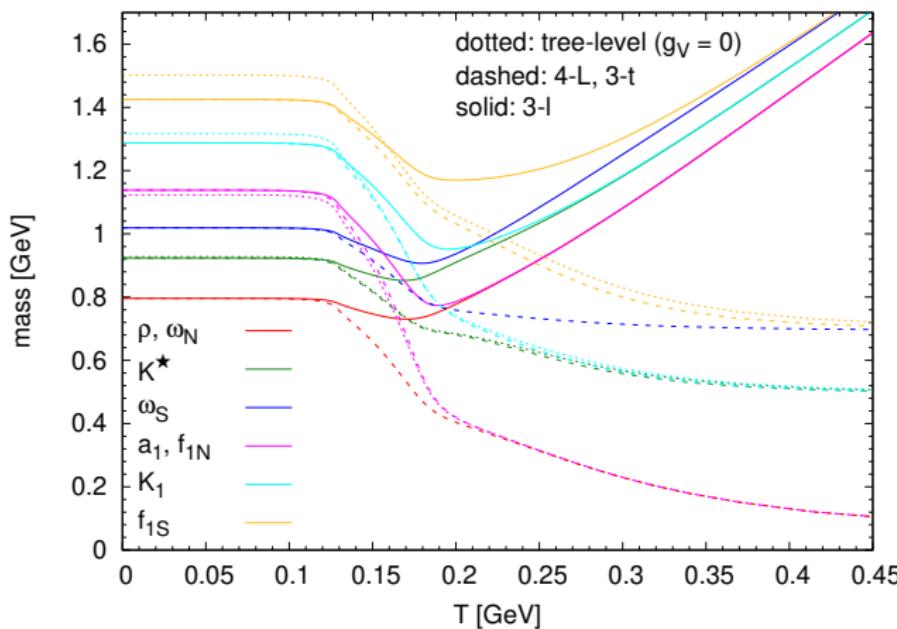
On the other hand: The curvature masses are the one-loop self-energies at vanishing momentum:

$$\Pi_{ab}^{(V/A)\mu\nu}(Q) = i2N_c g_V^2 \int_K \frac{g^{\mu\nu}(\pm m_a m_b - K^2 + K \cdot Q) + 2K^\mu K^\nu - K^\mu Q^\nu - Q^\mu K^\nu}{(K^2 - m_a^2)((K - Q)^2 - m_b^2)}$$

- At $T = 0 \rightarrow$ vacuum self-energy \rightarrow renormalization \rightarrow dimensional regularization
- At $T \neq 0 \longrightarrow$ matter part (with statistical function)

\rightarrow Wick rotation, Matsubara sum, $\int_K \rightarrow iT \sum_n \int \frac{d^3 k}{(2\pi)^3}$

T dependence of (axial)vector masses



From the χ^2 fit the vector coupling: $g_V \approx 5$
(Gy. Kovács, P. Kovács, Zs. Szép, arXiv:2105.12689)

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\text{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\text{Tr}_c \bar{L}(\vec{x})}{N_c}$ with
 $L(x) = \mathcal{P} \exp \left[i \int_0^\beta d\tau G_4(\vec{x}, \tau) \right]$

→ signals center symmetry (\mathbb{Z}_3) breaking at the deconfinement transition

low T : confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$

high T : deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

- ▶ Polyakov gauge: $G_4(\vec{x}, \tau) = G_4(\vec{x})$, plus gauge rotation to diagonal form in color space
- ▶ further simplification: \vec{x} -independence

$$\hookrightarrow L = e^{i\beta G_4} = \text{diag}(a, b, c) \left(\in SU(3)^{\text{color}} \right); \quad a, b, c \in \mathbb{Z}$$

→ use this to calculate partition function of free quarks

Form of the potential

I.) Simple polynomial potential invariant under \mathbb{Z}_3 and charge conjugation: R.D.Pisarski, PRD 62, 111501

$$\frac{\mathcal{U}_{\text{poly}}^{\text{YM}}(\Phi, \bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2$$

with $b_2(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \frac{T_0^2}{T^2} + a_3 \frac{T_0^3}{T^3}$

II.) Logarithmic potential coming from the $SU(3)$ Haar measure of group integration K. Fukushima, Phys. Lett. **B591**, 277 (2004)

$$\frac{\mathcal{U}_{\log}^{\text{YM}}(\Phi, \bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi\bar{\Phi} + b(T) \ln \left[1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2 \right]$$

with $a(T) = a_0 + a_1 \frac{T_0}{T} + a_2 \frac{T_0^2}{T^2}, \quad b(T) = b_3 \frac{T_0^3}{T^3}$

$\mathcal{U}^{\text{YM}}(\Phi, \bar{\Phi})$ models the free energy of a pure gauge theory

Result of the parametrization

- 40 possible assignments of scalar mesons to the scalar nonet states
- 3 values of M_0 are used \implies 120 cases to investigate
for each case $5 \cdot 10^4 - 10^5$ configurations are used for the χ^2 minimization
- lowest χ^2 obtained for $M_0 = 0.3$ GeV $\chi^2 = 18.57$ and $\chi^2_{\text{red}} \equiv \frac{\chi^2}{N_{\text{dof}}} = 1.16$
assignment: $a_0^{\bar{q}q} \rightarrow a_0(980)$, $K_0^{*,\bar{q}q} \rightarrow K_0^*(800)$, $f_0^{L,\bar{q}q} \rightarrow f_0(500)$, $f_0^{H,\bar{q}q} \rightarrow f_0(980)$
- problems: $m_{a_0} < m_{K_0^*}$, $m_{f_0^{H/L}}$ too light
- by minimizing also for M_0 we obtain using $\mathcal{U}_{\log}^{\text{YM}}(\Phi, \bar{\Phi})$ with $T_0 = 182$ MeV:

Parameter	Value	Parameter	Value
ϕ_N [GeV]	0.1411	g_1	5.6156
ϕ_S [GeV]	0.1416	g_2	3.0467
m_0^2 [GeV 2]	$2.3925 \cdot 10^{-4}$	h_1	27.4617
m_1^2 [GeV 2]	$6.3298 \cdot 10^{-8}$	h_2	4.2281
λ_1	-1.6738	h_3	5.9839
λ_2	23.5078	g_F	4.5708
c_1 [GeV]	1.3086	M_0 [GeV]	0.3511
δ_S [GeV 2]	0.1133		