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UNIVERSIDADE  
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**FCT**  
Fundação  
para a Ciência  
e a Tecnologia



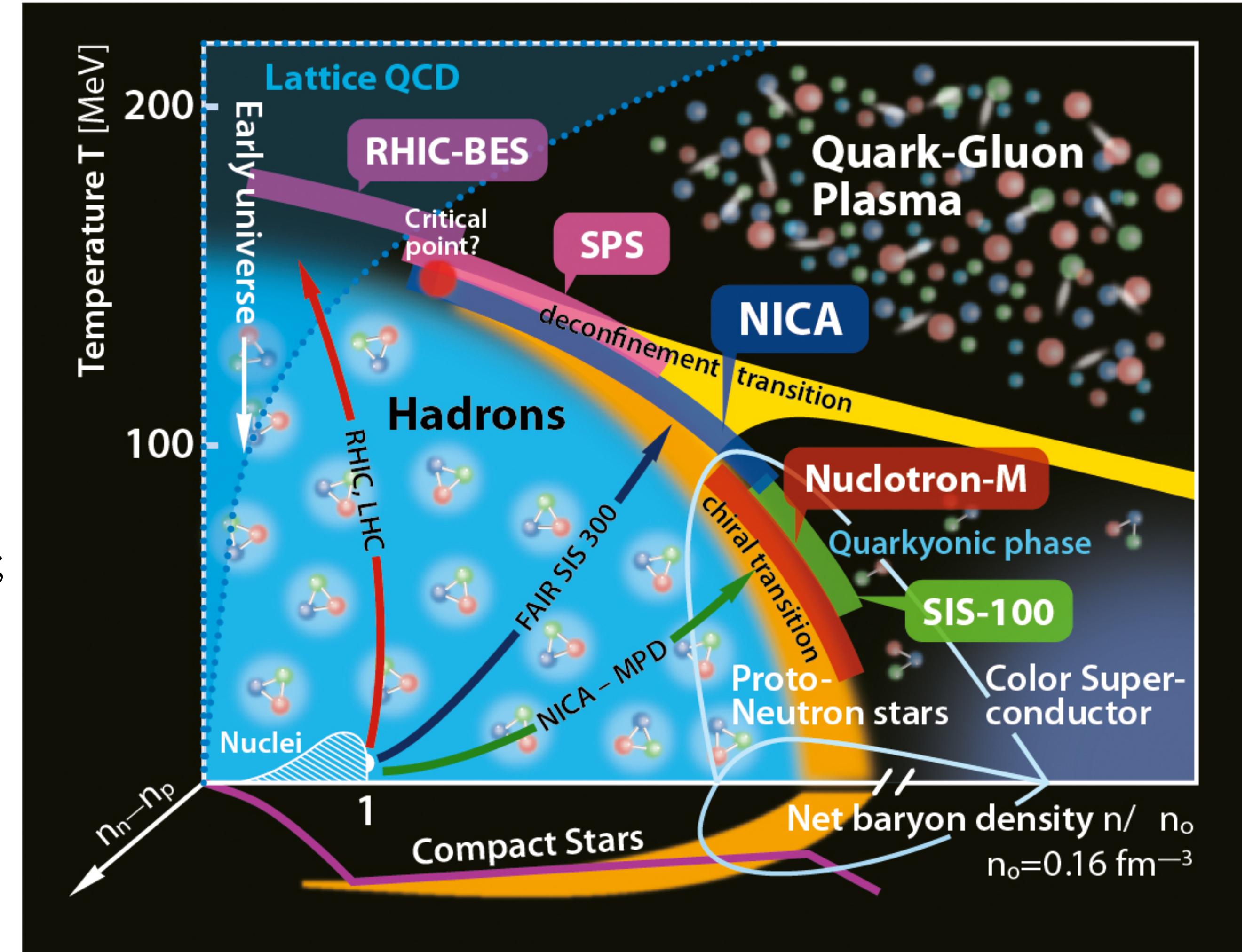
In collaboration with: V. Sagun, O. Ivanytskyi, C. Providência, T. Dietrich

# Probing strongly interacting matter properties with compact stars

# Strongly interacting matter phase diagram

## NEUTRON STARS (NS) PROPERTIES

- Highly asymmetric matter  
$$\delta = \frac{n_n - n_p}{n_n + n_p} > 0;$$
- Dense environment  $n_b \simeq 5 - 10 n_o$  ;
- Neutron stars (NS) are cold objects,  $T_{CO} \rightarrow 0$  MeV;
- During mergers,  $T \simeq 100$  MeV.



Credits: <http://indico.jinr.ru/event/csqcd2017>

# Equation of State (EoS) at T=0

Binding energy per nucleon can be expanded in the asymmetric matter case as:

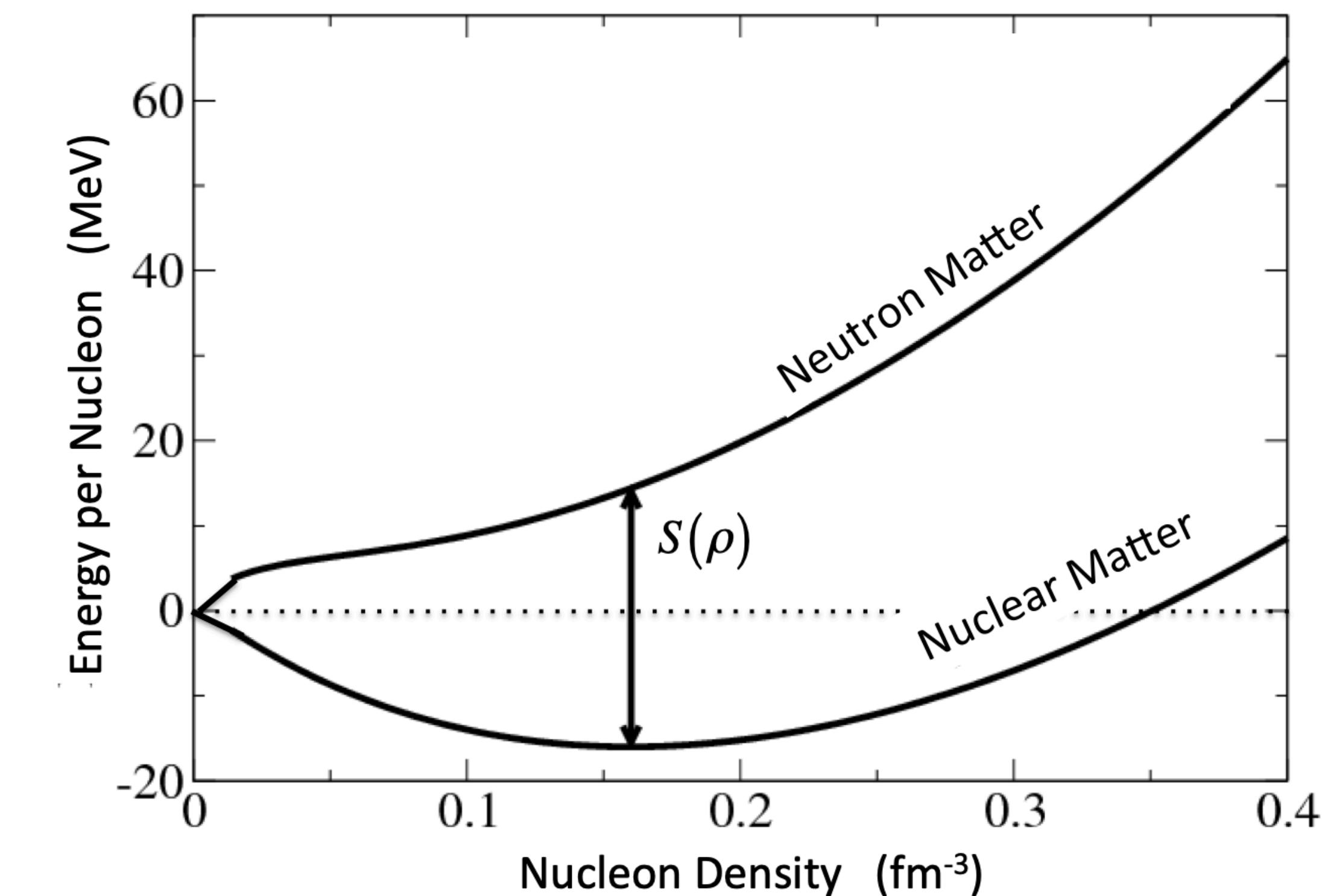
$$\frac{E(n_b, \delta)}{A} = \frac{E(n_b, 0)}{A} + S(n_b)\delta^2 + O(\delta^4) \quad \text{where} \quad \delta = \frac{n_n - n_p}{n_n + n_p} \quad \text{and} \quad \epsilon = \frac{n_b - n_0}{n_0}$$

- Binding energy per nucleon at  $n_0$ :

$$\frac{E(n_0 = 0.16 \text{ fm}^{-3})}{A} = \frac{E_0}{A} + \frac{1}{18} K_0 \epsilon^2 + \dots = -16 \pm 1 \text{ MeV};$$

- Incompressibility of the matter at  $n_0$ :

$$K_0 \equiv 9n_0^2 \left( \frac{\partial^2 E}{\partial n_b^2} \right)_{n_b=n_0, \delta=0} = 200 - 260 \text{ MeV};$$



Binding energy plot.  
[Sagawa 1906.00637 (2019)]

Stone, *Universe* **2021**, *7*(8), 257;

# Equation of State (EoS) at T=0

Symmetry energy can be expanded as:

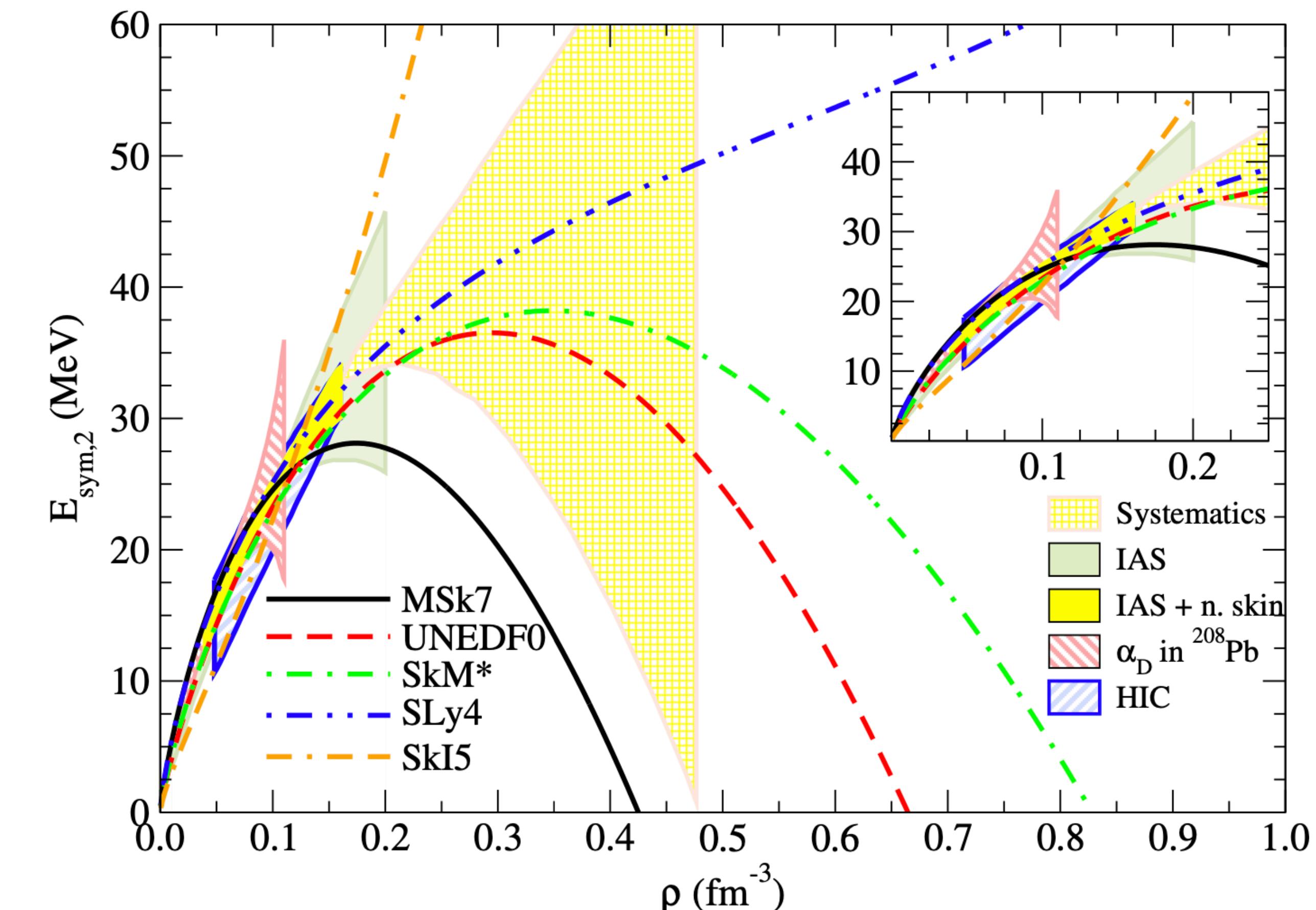
$$S(n_b) = S(n_o) + \frac{1}{3}L\epsilon + \frac{1}{18}K_{\text{sym}}\epsilon^2 + \dots$$

- Symmetry energy at  $n_0$ :

$$S(n_b) \equiv \frac{1}{2} \left( \frac{\partial^2 E}{\partial \delta^2} \right)_{n_b=n_o, \delta=0} = 30 \pm 4 \text{ MeV};$$

- Symmetry energy slope L at  $n_0$ :

$$L_0 \equiv 3n_o \left( \frac{\partial S(n_b)}{\partial n_b} \right)_{n_b=n_0, \delta=0} = 20 - 115 \text{ MeV}.$$

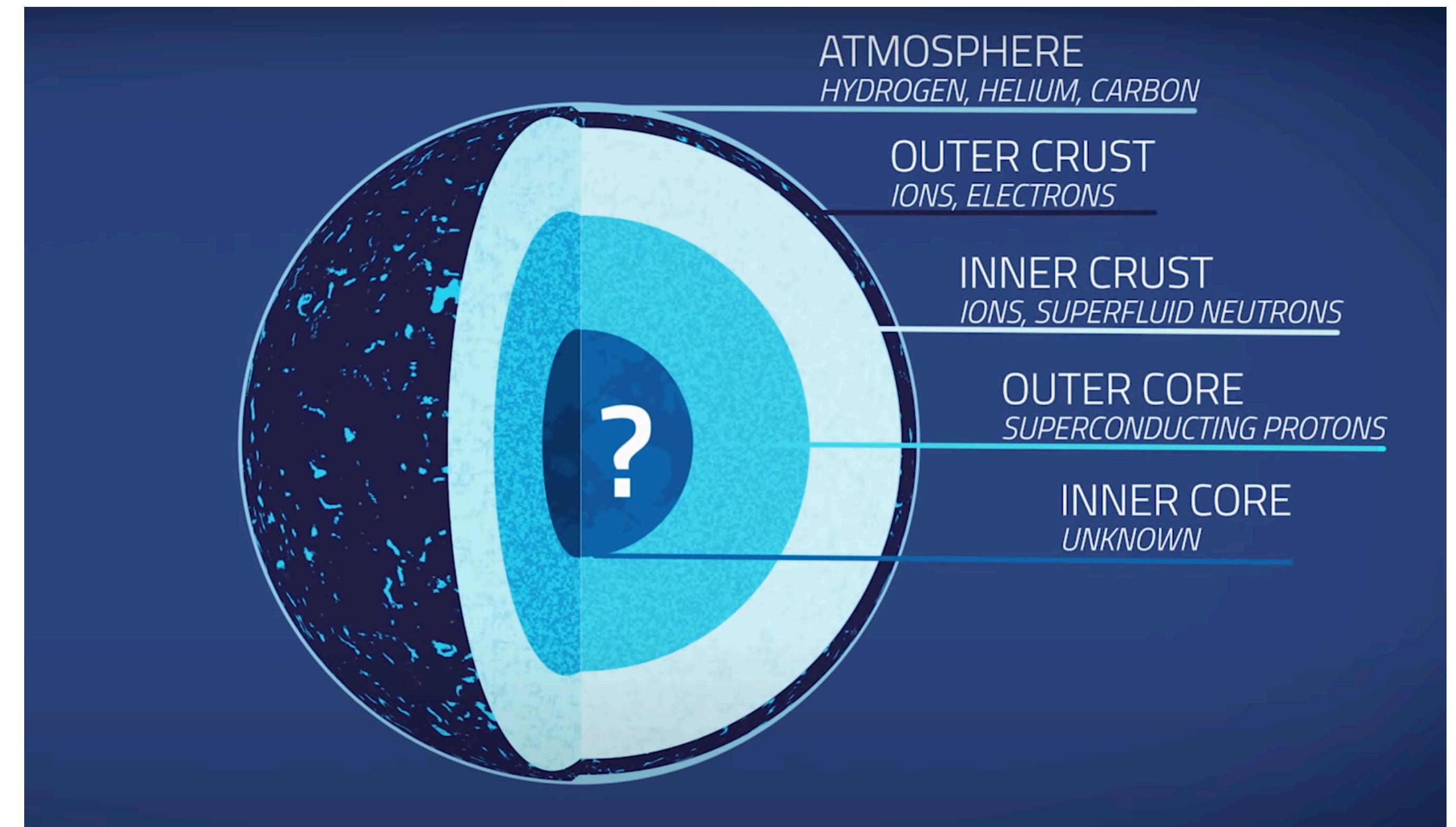


Symmetry energy plot.  
[Boquera, 2003.00490 (2019)]

# Constraints on the EoS

## GENERAL REQUIREMENTS

- Causality  $c_s^2 < 1$ ;
- Electric neutrality;
- Thermodynamic consistency;
- $\beta$ -equilibrium;
- Multicomponent characters ( $n, p, e^-, \dots$ );
- Realistic interactions between the components.



# Constraints on the EoS

## HIGH ENERGY PHYSICS

- Proton flow constraint;
- Hadron multiplicity;
- Nucleon-Nucleon scattering;

## ASTROPHYSICS

- Observed masses  $M_{NS} \sim 2 M_\odot$ ;
- NICER results on the radii and masses;
- Neutron star (NS) cooling;
- Observations of pulsars;

## NUCLEAR PHYSICS

- Nuclear ground state properties;

## GRAVITATIONAL PHYSICS

- Tidal deformability parameter  $\Lambda/k_2$ ;

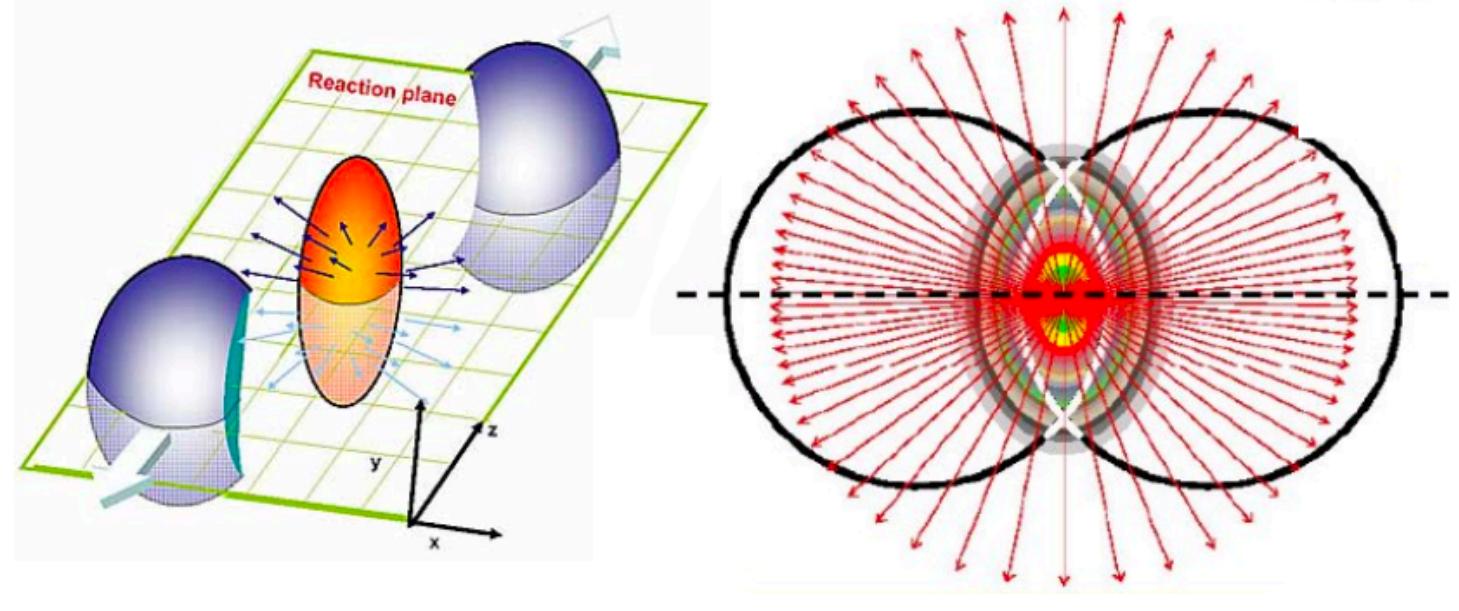


# Constraints on symmetric matter

## HYDRODYNAMIC RECONSTRUCTION OF FLOWS

Anisotropic expansion caused by a gradient of pressure;

P.Danielewicz et al., Science 198, 1593 (2002)



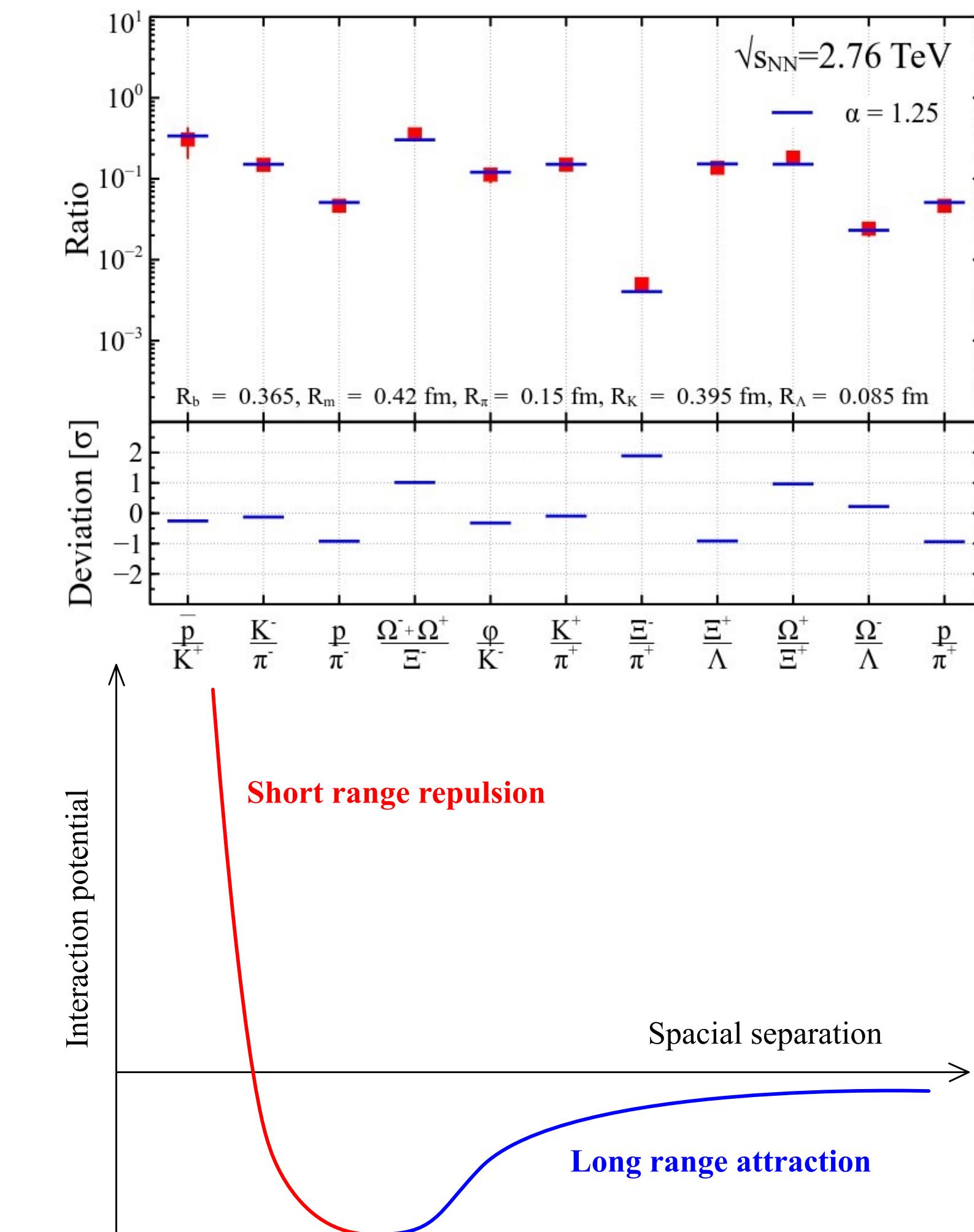
## HADRON MULTIPLICITIES

Hard core radii of hadrons control the rate of their production in thermal medium  $R = 0.3\text{-}0.5 \text{ fm}$ .

Andronic et al., Nucl.Phys.A772:167-199, (2006)

## NUCLEON-NUCLEON SCATTERING

Hard core radius of nucleons as a parameter of the microscopic interaction potential extracted as  $R = 0.5 \text{ fm}$ ;

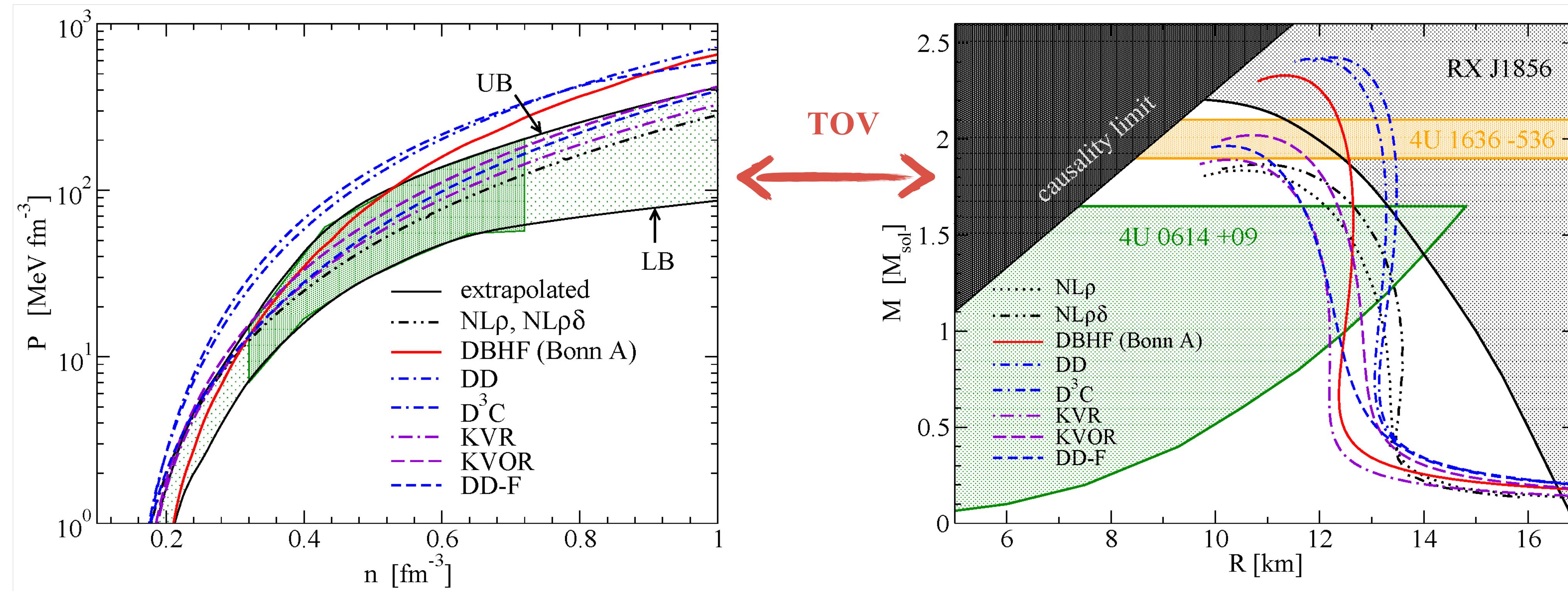


Measured hadron yield ratios compared to the thermal model calculations  
[Bugaev, et al., NPA 970, p. 133-155, (2018)]  
and typical microscopic interaction potential.

# EoS $\longleftrightarrow$ MR diagram

Through the Tolman–Oppenheimer–Volkoff (TOV) equation, mass and radius can be obtained with a one-to-one correspondence with the EoS.

$$\frac{dP}{dr} = -\frac{Gm}{r^2} (\rho + P) \left( 1 + \frac{4\pi r^3 P}{m} \right) \left( 1 - \frac{2Gm}{r} \right)^{-1}$$



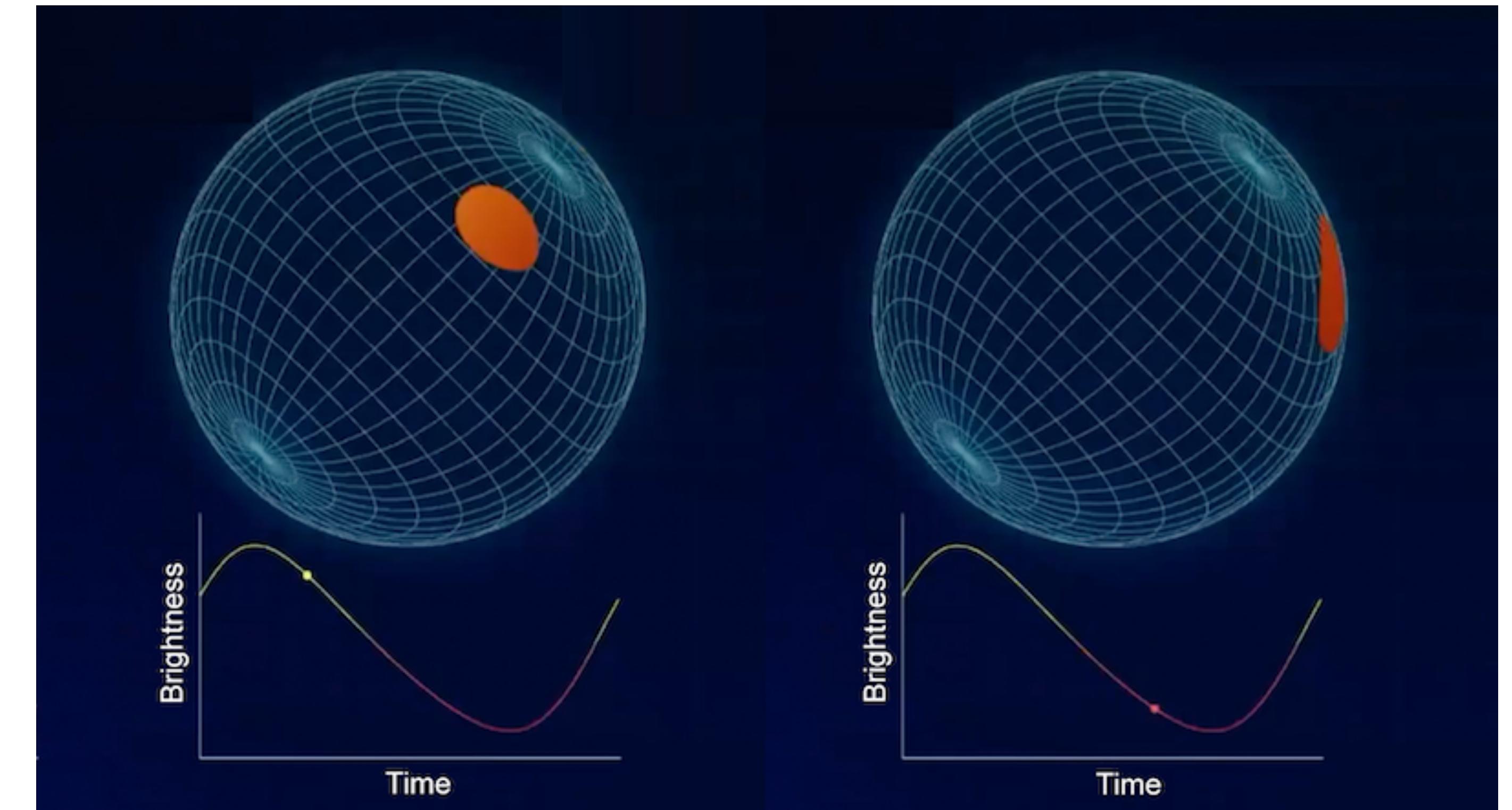
Correlation between EoS and M-R diagram, obtained using the TOV equation.  
[D.N. Voskresensky, Universe 2018, 4(2), 28]

# NICER constraints

- General relativistic effects of spacetime;
- Hot spots due to magnetic fields;
- Oscillations in the X-ray emission;
- Informations on compactness, hence mass and radius.

$$R = 13.7_{-1.5}^{+2.6} \text{ km for } M = 2.08 \pm 0.07 \text{ M}_\odot$$

$$R = 13.02_{-1.06}^{+1.24} \text{ km for } M = 1.44_{-0.14}^{+0.15} \text{ M}_\odot$$

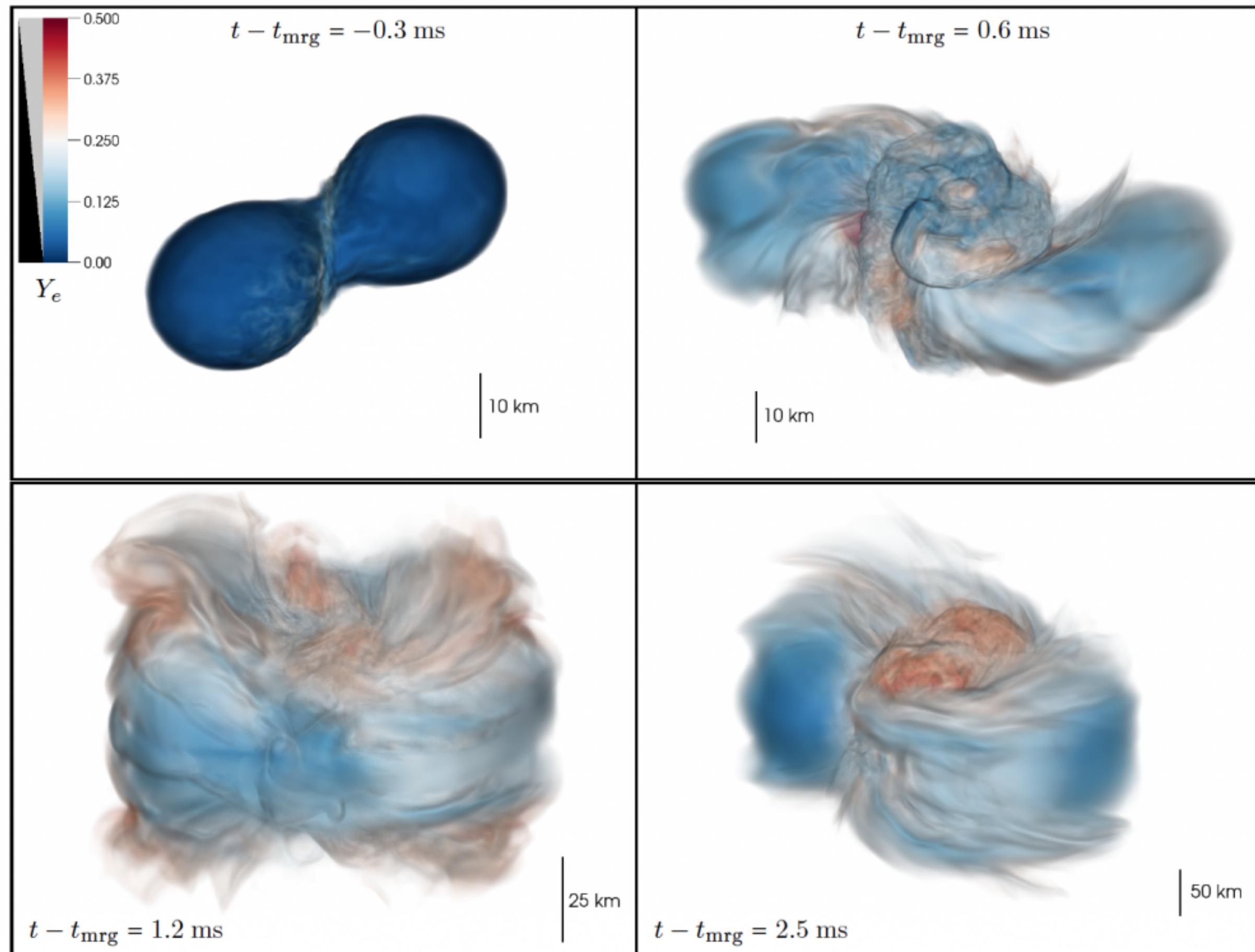


Credit: Goddard Space Flight Center

Miller et al., ApJL. 887:L24, (2019)

Miller et al., ApJL. 918:L28, (2021)

# Multi-messenger constraints GW170817



Tidal disruption in binary neutron star merger event.  
[Radice et al. ApJ 869:130 (2018)]

- Love's number:

$$k_2 = k_2(\beta, R_{\text{NS}});$$

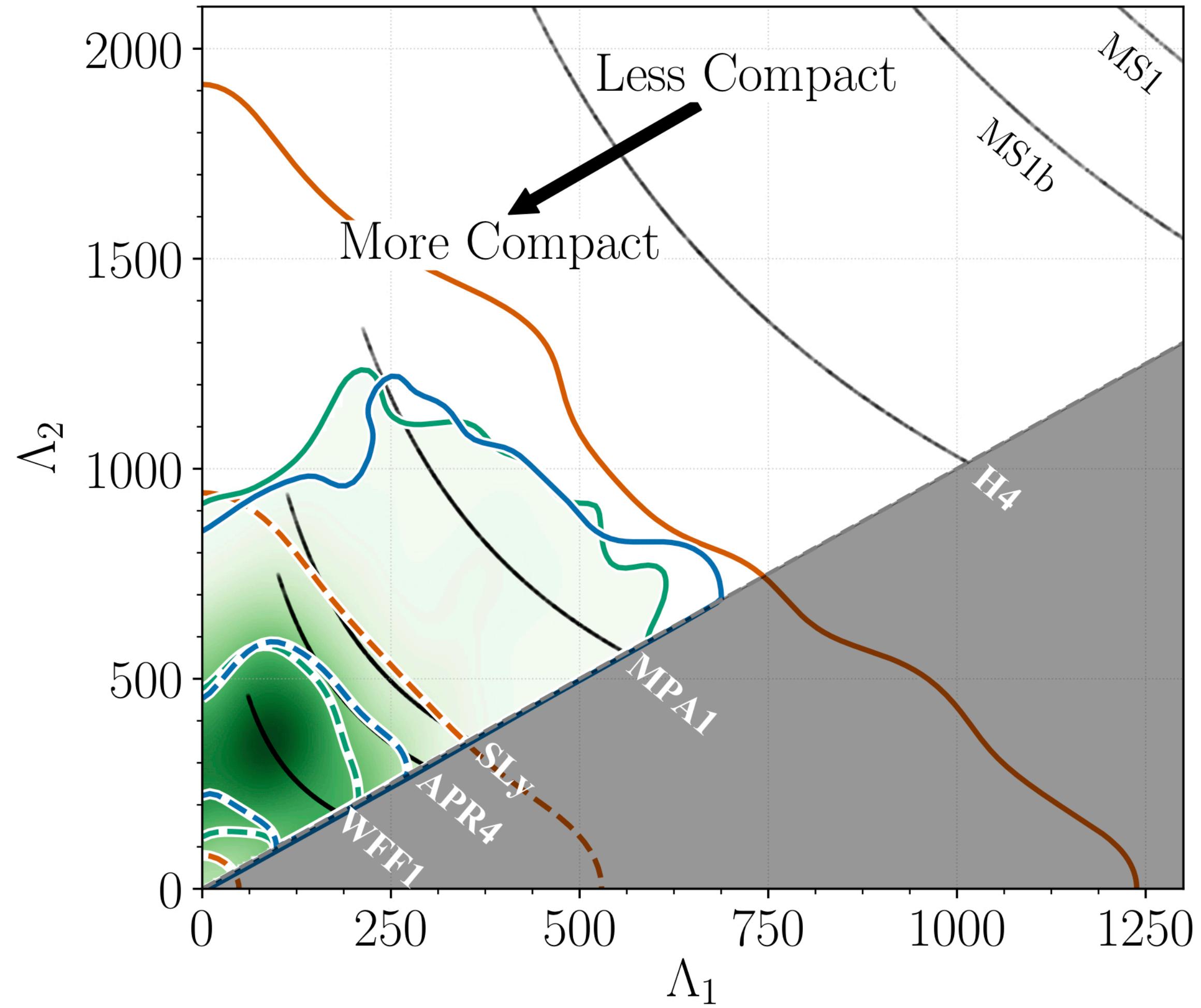
- Tidal deformability parameter:

$$\Lambda_i = \frac{2}{3} k_2 \left( \frac{R_i}{GM_i} \right)^5;$$

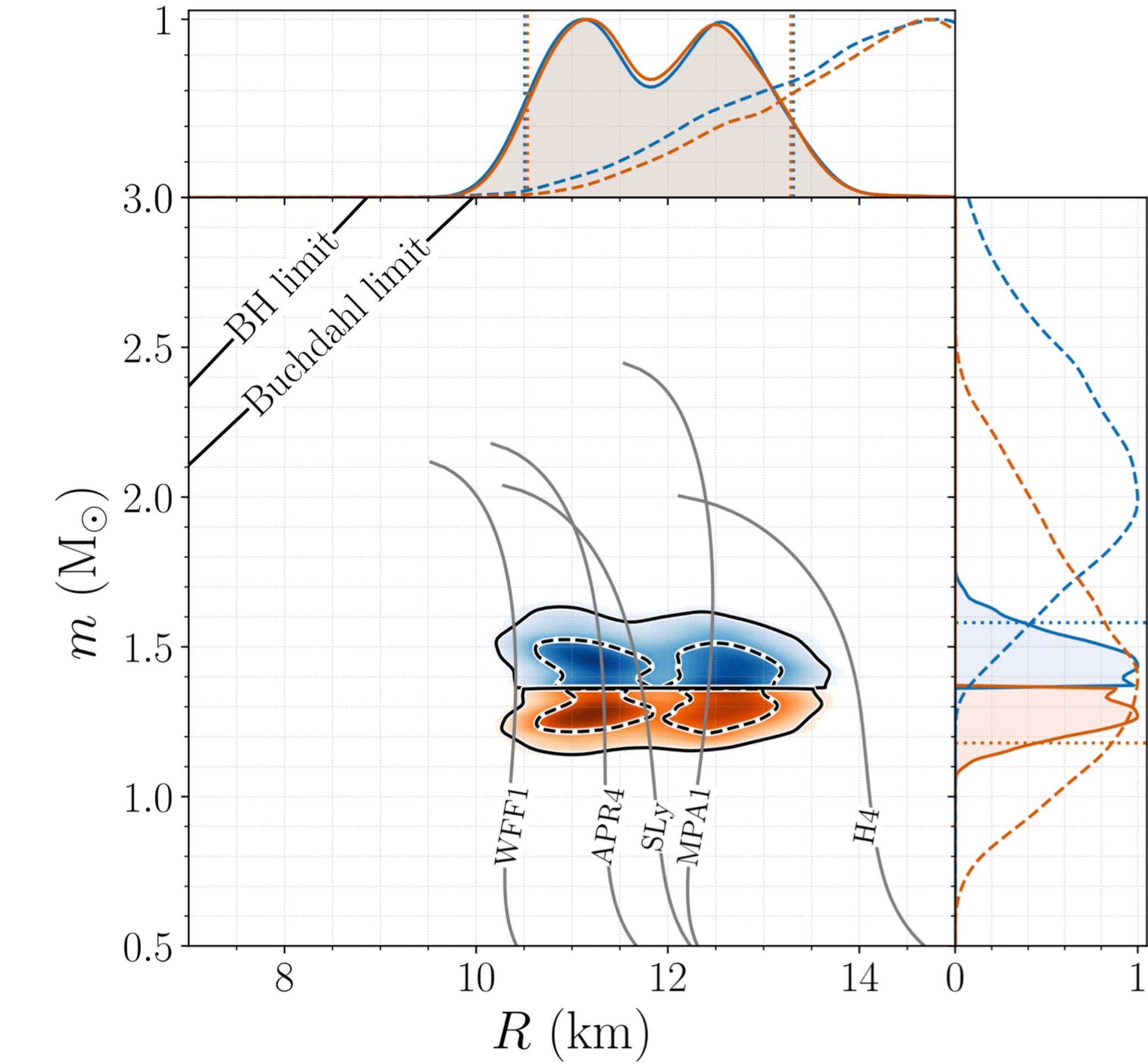
- Mass extracted from the gravitational wave;
- Informations on radii and  $k_2$ .

$$\begin{cases} \Lambda(1.4 \text{ M}_\odot) \leq 800 & \text{LIGO-Virgo Collab. PhysRevL. 121, 161101 (2018)} \\ R(1.4 \text{ M}_\odot) \leq 11.8^{+2.7}_{-3.3} \text{ km} \end{cases}$$

# Multi-messenger constraints GW170817



Constraint on the tidal deformability parameters obtained from GW170817.  
[LIGO-Virgo Collaboration Phys. Rev. X 9, 011001 (2019)]



Constraints on mass-radius relation from GW170817  
[LIGO-Virgo Collaboration, Phys. Rev. Lett. 121, 161101 (2018)]

# Induced Surface Tension (IST) EoS

The grand canonical phenomenological EoS with  $n, p, e^-$  has the following form:

$$\begin{cases} p = \sum_i p_{id}(T, \mu_i - pV_i - \sigma S_i + U_{at} \pm U_{sym}) + p_{id}(\mu_e) - p_{at} + p_{sym} \\ \sigma = \sum_i p_{id}(T, \mu_i - pV_i - \alpha\sigma S_i + U_0) R_i \end{cases}$$

Thermodynamical consistency:  $\frac{\partial p}{\partial n_{id}} = n_{id} \frac{\partial U(n_{id})}{\partial n_{id}}$ ;

Mean field potential parametrisation:  $U_{at} = -C_d^2 n_{id}^k$ ;

$$\text{Symmetry energy pressure: } P_{sym}(n) = \frac{A_{sym} n^2}{[1 + (B_{sym} n)^2]^2}.$$

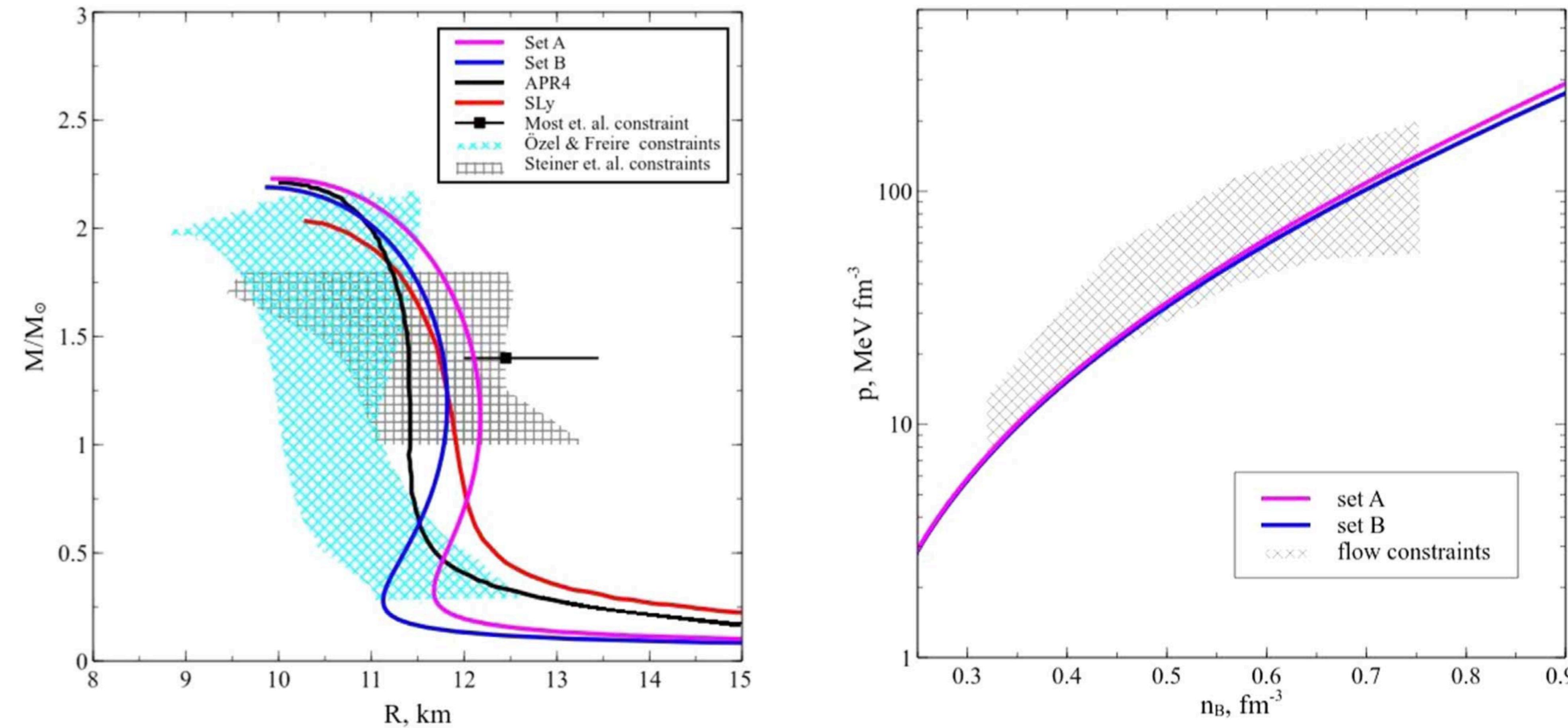
Sagun et al., arXiv:2002.12209 (2020)

Ivanytskyi et al., PRC 97, 064905 (2018)

Reed et al., Phys. Rev. Lett. 126, 172503 (2021)

$\sigma$	Induced surface tension parameter
$V_i$	Excluded volume
$S_i$	Surface
$R_i$	Hard core radius of the particle
$p_{id}$	Ideal pressure for QM
$U_0 = 162.87 \text{ MeV}$	Model parameter
$\alpha = 1.245$	Model parameter
$A_{sym} = 138.30 \text{ MeV} \cdot \text{fm}^3$	Model parameter
$B_{sym} = 16.0 \text{ fm}^3$	Model parameter
$C_d^2 = 146.30 \text{ MeV} \cdot \text{fm}^{3\kappa}$	Model parameter
$\kappa = 0.25$	Model parameter
$L_0 = 93.19 \text{ MeV}$	Symmetry energy slope
$K_0 = 201.02 \text{ MeV}$	Incompressibility parameter

# Induced Surface Tension (IST) EoS



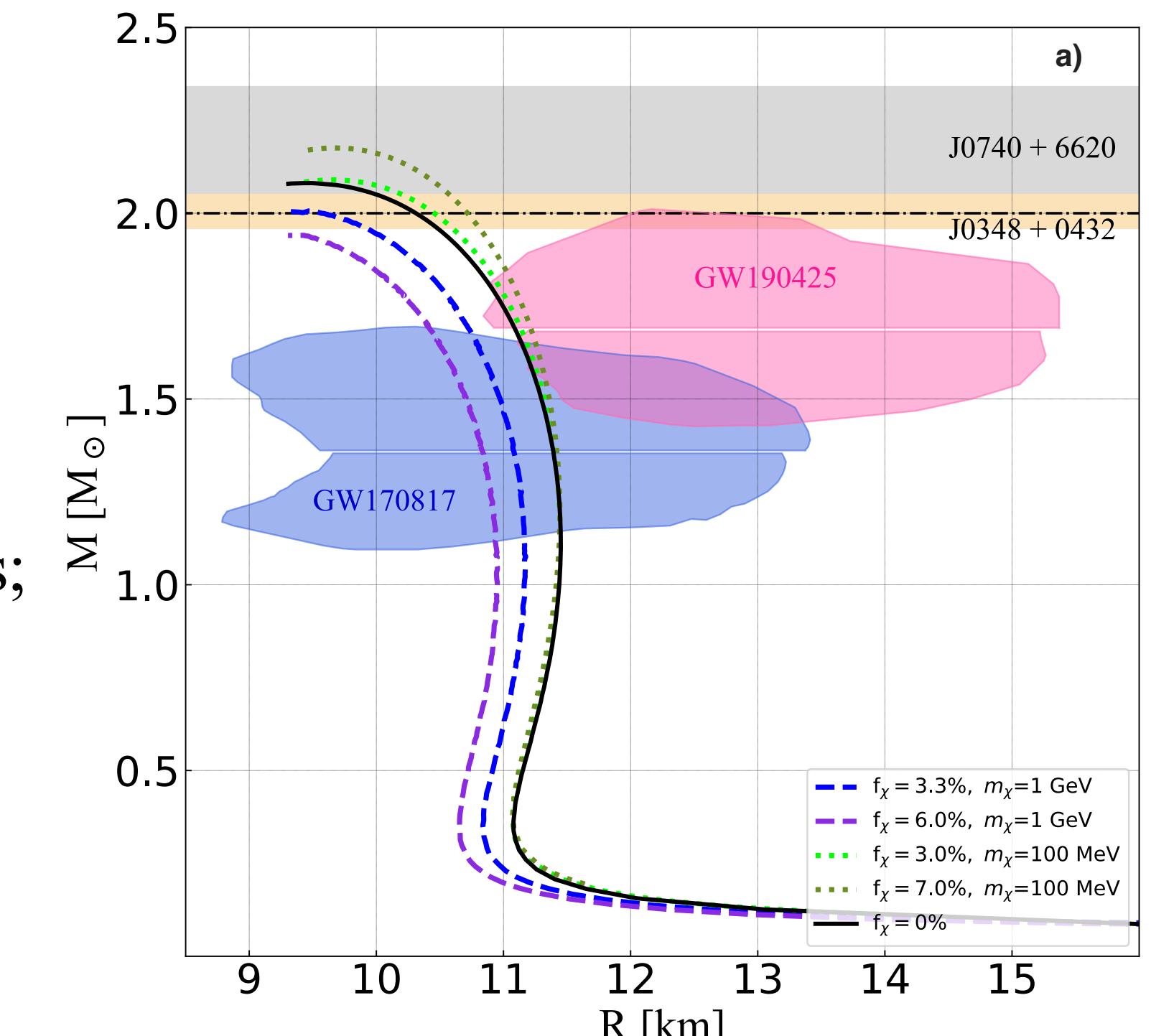
MR diagram and EoS of the IST EoS with two different sets of the model parameters.  
[Sagun et al., ApJ 871:157 (2019)]

# Present and future works

- Self-interacting fermionic and bosonic dark matter (DM);
- Fitting all the constraints coming from every corner of physics;
- General relativistic magneto-hydrodynamical simulations of NSs and their mergers;
- Constraining ever further the DM research.



Sagun et al., arXiv:2111.13289, (2021)



MR diagram of a NS with DM content  
(outer core or diluted halo)



# Köszönöm!

(Please, don't ask me how to pronounce this)

# Mean field interaction for nuclear matter

- Thermodynamic consistency provides identity  $\frac{\partial p}{\partial \mu} = n$ :

$$P(\mu) = P_{id}(\mu - U(x)) + p_{int}, \text{ where } p_{int}(x) = \int_0^x dx' x' \frac{\partial U(x')}{\partial x'}$$

being  $x$  any quantity ( $\rho, \delta, \dots$ ).

K. A. Bugaev and M. I. Gorenstein, Z. Phys. C 43, 261 (1989)  
D.H. Rischke, et al. Z. Phys. C 51, 485 (1991)

- Long range attraction (negative contribution to pressure)

$$P_{at}(x) = -\frac{\kappa}{\kappa+1} C_d^2 x^{1+\kappa}, \text{ where } x = n_n^{id} + n_p^{id}$$

where  $\kappa < 1$  and  $C_d^2$  fitted to flow constraint and properties of ground state;

A. Ivanytskyi et al., PRC 97, 064905 (2018)

- Repulsion due to symmetry energy (positive contribution to pressure)

$$E_{sym} = a_{sym} \frac{(N-Z)^2}{A}, a_{sym} = 30 \pm 4 \text{ MeV}$$

$$P_{sym}(n) = \frac{A_{sym} n^2}{[1 + (B_{sym} n)^2]^2}$$

where the parameters are fitted to  $a_{sym}$  and the slope of symmetry energy at ground state.

V. Sagun et al., ApJ 871:157 (2019)

# EoS with hard core repulsion

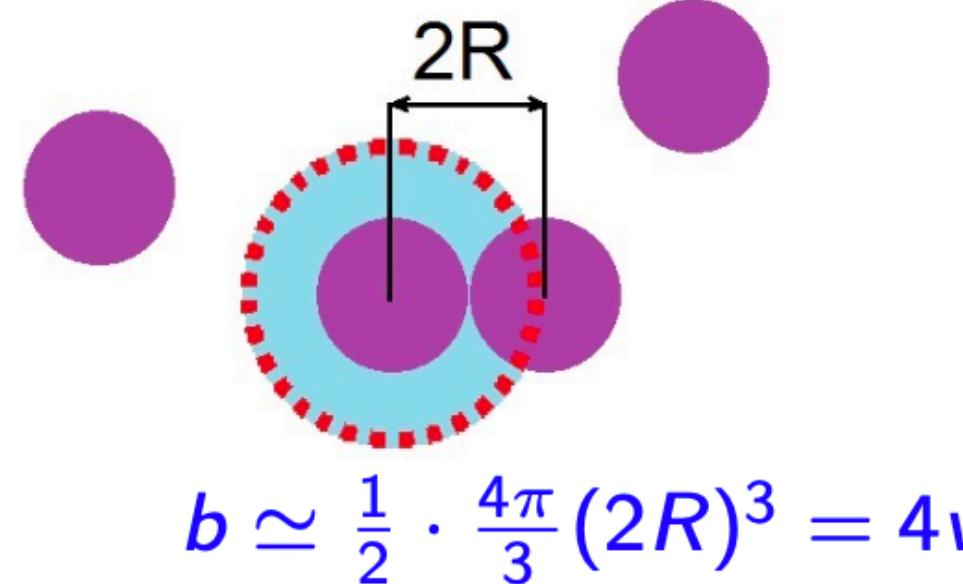
- **Hard core** reduces volume available for motion of particles by

$$V_{excl} = Nb, \text{ hence: } V \rightarrow V - V_{excl} \implies p = nT = \frac{NT}{V} \rightarrow \frac{NT}{V - Nb} = \frac{nT}{1 - nb}$$

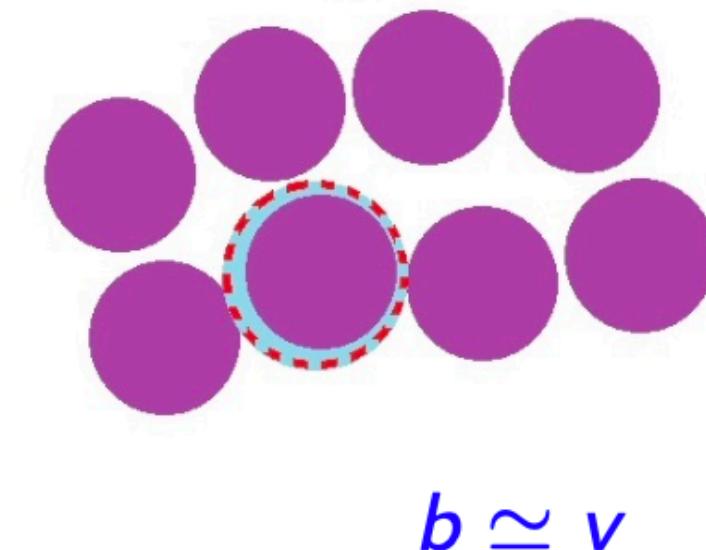
- **Van der Waals EoS**, with  $b=\text{const}$  in the Grand Canonical Ensemble

$$\begin{cases} p = p(T, \mu) \\ n = \frac{\partial p}{\partial \mu} \end{cases} \implies p = T \int_{\vec{k}} \exp \frac{\mu - pb - \sqrt{m^2 + k^2}}{T} = p_{id}(T, \mu - pb)$$

**Low densities**



**High densities**



- Excluded volume (per particle depends on density ( $b \neq \text{const}$ ));

# EoS with hard core repulsion

- From Boltzmann ideal gas

$$p = nT, \quad n = \sum_i n_i^{id}, \quad n_i^{id} = \frac{p_i^{id}}{T}$$

- Virial expansion for one particle species

$$\frac{p}{T} = n + a_2 n^2 + a_3 n^3 + a_4 n^4 + \dots$$

- Adding the possibility of many particle species:

$$\frac{p}{T} = \overbrace{\sum_i n_i^{id}}^{\simeq n} - \overbrace{\sum_{i,j} a_2^{ij} n_i^{id} n_j^{id}}^{\simeq a_2 n^2} + \dots = \sum_i \frac{p_i^{id}}{T} \left( 1 - v_i \sum_j \frac{p_j^{id}}{T} - s_i \sum_j \frac{p_j^{id}}{T} R_j \right) + \dots$$

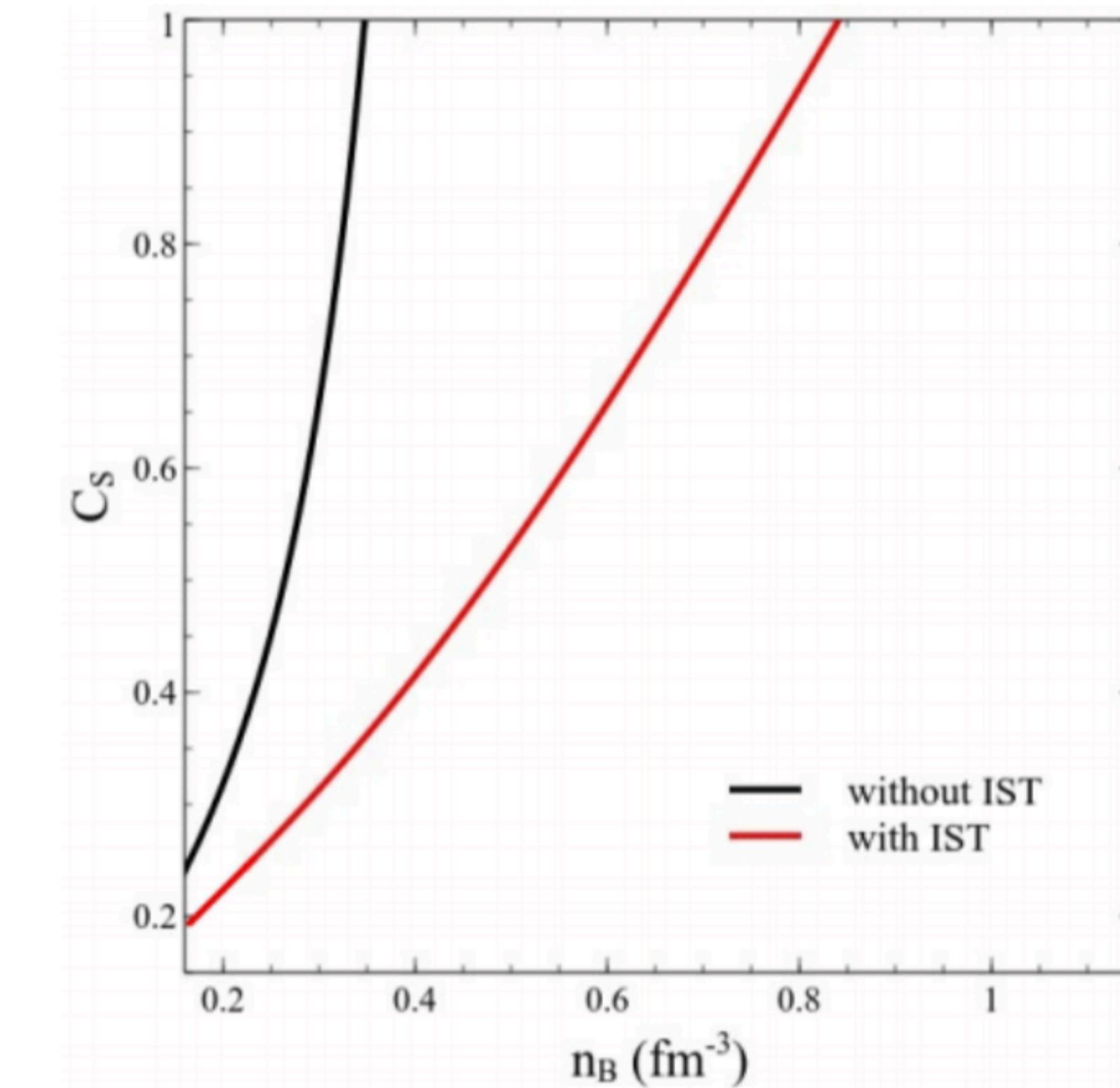
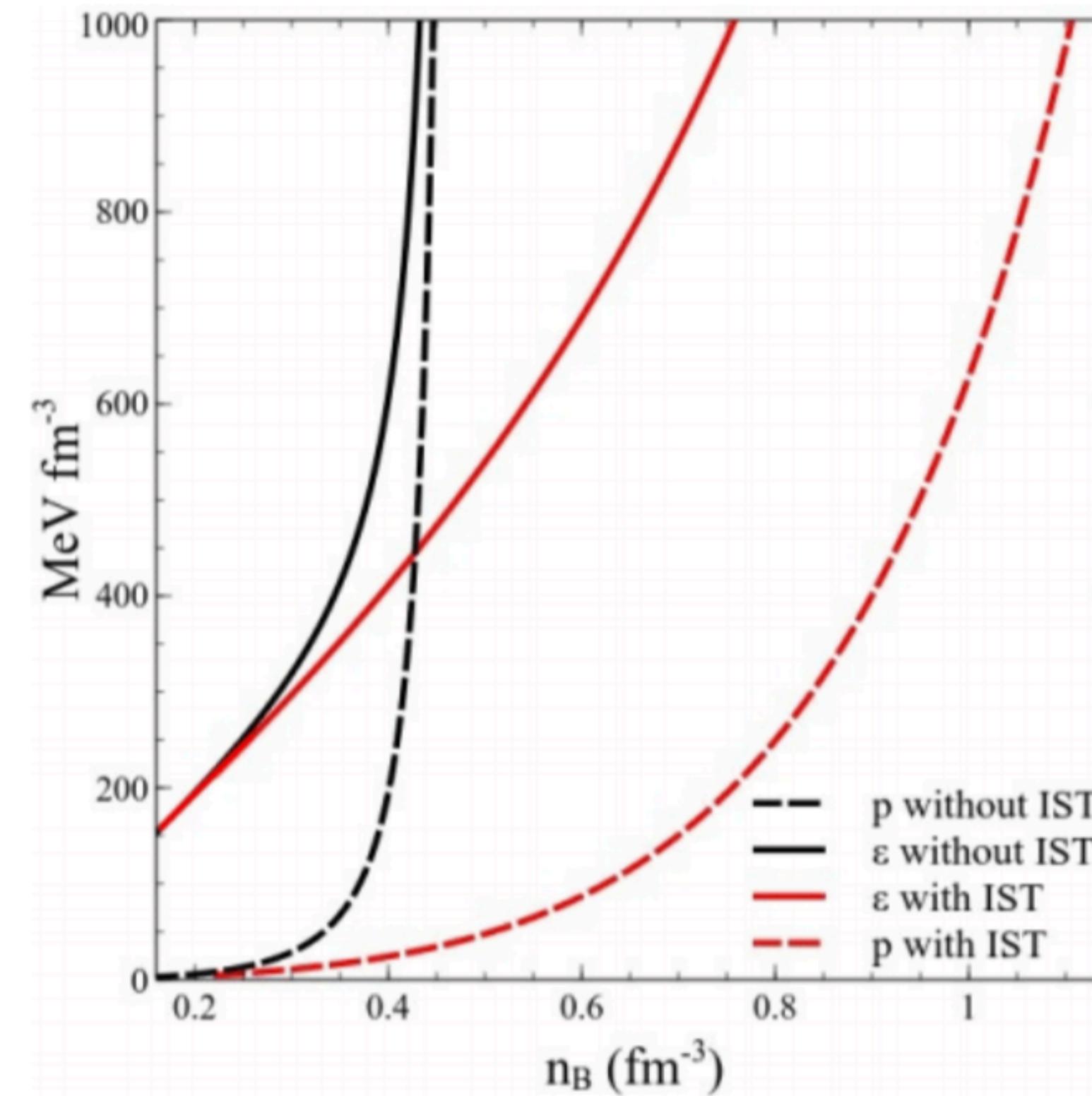
- $\Sigma$  is conjugated to  $s_i$  - **induced surface tension (IST)**
- And (finally), the high density extrapolation

$$\begin{cases} p = \sum_i p_i^{id} (\mu_i - p v_i - \Sigma s_i) \\ \Sigma = \sum_i p_i^{id} (\mu_i - p v_i - \alpha \Sigma s_i) R_i \end{cases}$$

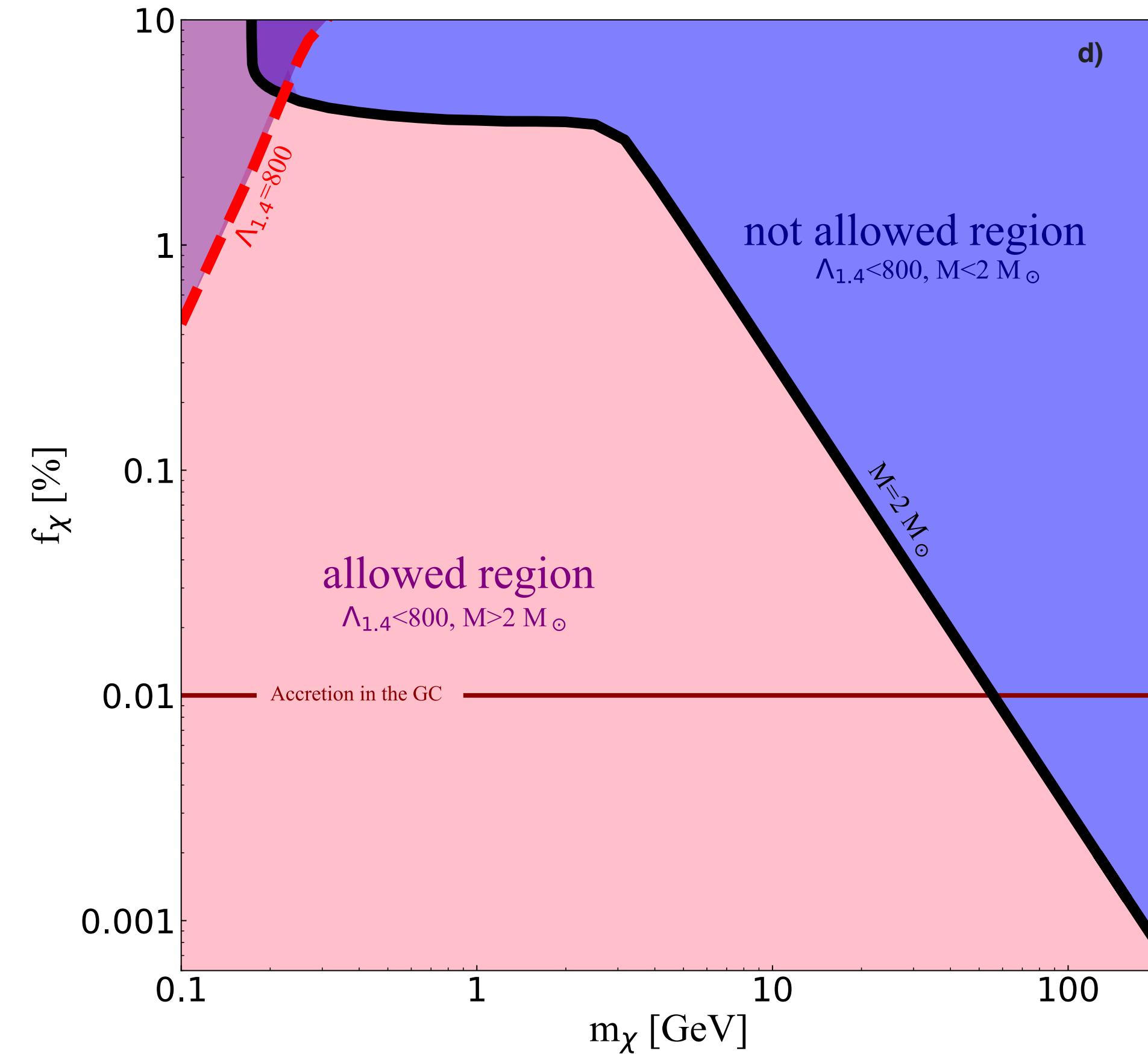
Sagun et al., NPA 924, 24 (2014)



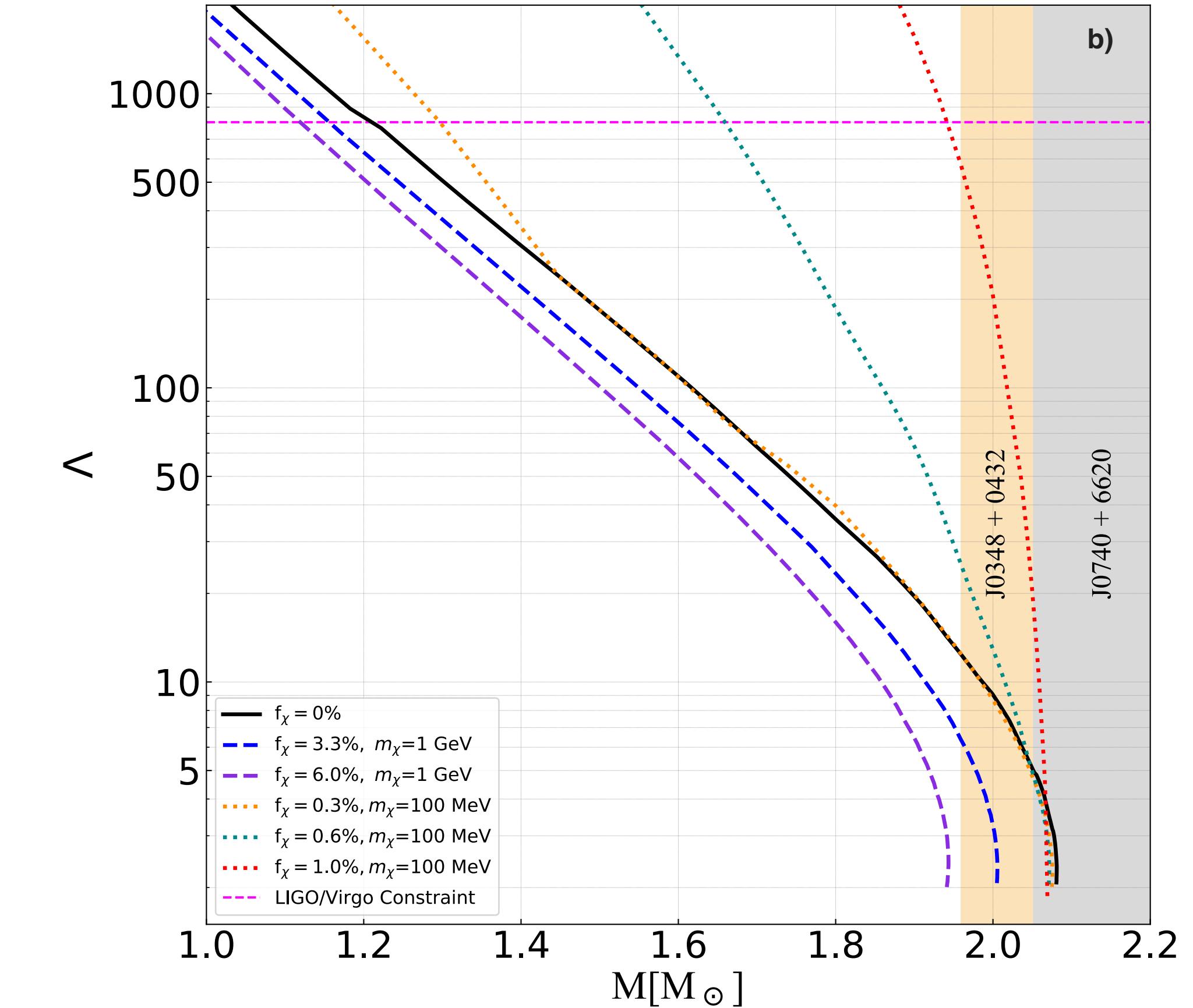
# Effects of IST



# Tidal deformability parameter



Constraints for DM coming from astrophysics and gravitational observations (GW170817)



Tidal deformability parameter related to NS total mass.