Kinetic theory for massive spin-1 particles in electromagnetic fields

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in collaboration with

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CRC-TR 211 Strong-interaction matter under extreme conditions

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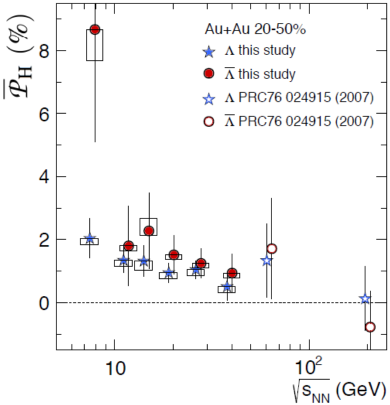
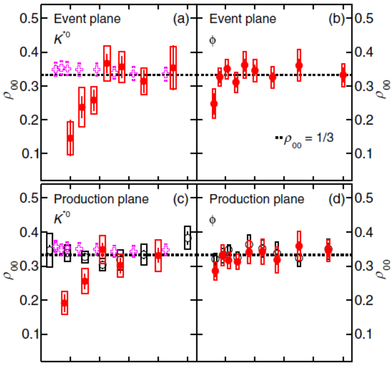
Motivation

- Heavy-ion collisions provide several polarization observables
  - Spin $1/2$: Decay of Λ-Hyperons
  - Spin 1: Decay of ϕ/K^{*0}-Mesons
- Both feature significant global polarization at lower energies

L. Adamczyk et al. (STAR), Nature 548 62-65 (2017)

S. Acharya et al. (ALICE), Physical Review Letters 125, 012301 (2020)





Charged vector fields

Maxwell-Proca Lagrangian

$$\mathcal{L} = \hbar\left(-\frac{1}{2}V^{*\mu\nu}V_{\mu\nu} + \frac{m^2}{\hbar^2}V^{*\mu}V_\mu\right) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - iqF_{\mu\nu}V^\mu V^{*\nu} \qquad (1)$$

Field strength tensors $V^{\mu\nu} := D^\mu V^\nu - D^\nu V^\mu\ , \quad F^{\mu\nu} := \partial^\mu A^\nu - \partial^\nu A^\mu$.

- Introduce (gauge-)covariant derivative $D^\mu := \partial^\mu + iq/\hbar A^\mu$
- Take into account magnetic moment $\mu := q\hbar/m\ (g = 2)$

H. C. Corben, J. Schwinger, Physical Review 58, no. 11, 953-968 (1940)
M. Napsuciale, S. Rodriguez E. G. Delgado-Acosta, M. Kirchbach, Physical Review D 77, no. 1, 430 (2008)

Constraint equation

$$D^\mu V_\mu = -i\frac{q\hbar}{m^2}J^\mu V_\mu\ . \qquad (2)$$

- Constraint equation removes one degree of freedom

Wigner function

- Idea: Introduce a quantum-mechanical analogue of the one-particle distribution function H. -W. Lee, Physics Reports 259, no. 3, 147-211 (1995)

Wigner function for vector fields

$$W^{\mu\nu}(x,k) := -\frac{2}{(2\pi)^4\hbar^5}\int \mathrm{d}^4v e^{-\frac{i}{\hbar}k^\alpha v_\alpha}\left\langle : V_+^{*\mu} U_{+-} V_-^\nu : \right\rangle \tag{3}$$

with

$$V_\pm^\mu := V^\mu\left(x \pm \frac{v}{2}\right), \tag{4}$$

$$U_{+-} := \hat{T}\exp\left[-i\frac{q}{\hbar}v^\alpha \int_{-1/2}^{1/2}\mathrm{d}t A_\alpha(x+tv)\right]. \tag{5}$$

- U_{+-} is the gauge link such that the Wigner function is gauge invariant

Decomposition of the Wigner function

- Use dynamics of V^μ to obtain equations of motion for $W^{\mu\nu}$
- Decompose $W^{\mu\nu}_{S/A} := (W^{\mu\nu} \pm W^{\nu\mu})/2$ to recover nine independent components

Decomposition with respect to k^μ

$$W_S^{\mu\nu} = E^{\mu\nu} f_E + K^{\mu\nu} f_K + \frac{k^{(\mu}}{2k} F_S^{\nu)} + F_K^{\mu\nu} \tag{6}$$

$$W_A^{\mu\nu} = \frac{k^{[\mu}}{2k} F_A^{\nu]} + \epsilon^{\mu\nu\alpha\beta} \frac{k_\alpha}{m} G_\beta \tag{7}$$

- $F_S^\mu k_\mu = F_A^\mu k_\mu = G^\mu k_\mu = F^\mu_{K,\mu} = 0\,, \quad k_\mu F_K^{\mu\nu} = 0\,, \quad F_K^{\mu\nu} = F_K^{\nu\mu}$

$E^{\mu\nu} := k^\mu k^\nu / k^2\,, \quad K^{\mu\nu} := g^{\mu\nu} - E^{\mu\nu}$

Perturbative solution

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- Goal: Solve equations of motion perturbatively
  $\rightarrow$ Expansion $W^{\mu\nu} = W^{(0),\mu\nu} + \hbar W^{(1),\mu\nu} + \cdots$
- Zeroth order: Wigner function on shell, $W^{\mu\nu} \propto \delta(k^2 - m^2)$
  - $f_E^{(0)} = F_A^{(0),\mu} = F_S^{(0),\mu} = 0$
  - Independent components follow evolution equations

Equations of motion at order $\mathcal{O}(\hbar^0)$

$$0 = k^\alpha \hat{\nabla}_\alpha^{(0)} f_K^{(0)} \tag{8}$$

$$0 = k^\alpha \hat{\nabla}_\alpha^{(0)} G^{(0),\mu} - qF^{\mu\nu} G_\nu^{(0)} \tag{9}$$

$$0 = k^\alpha \hat{\nabla}_\alpha^{(0)} F_K^{(0),\mu\nu} - qF_{K,\alpha}^{(0),(\mu} F^{\nu)\alpha} \tag{10}$$

- Eq. (8) has transparent interpretation
- What about the others?

$\hat{\nabla}_\mu^{(0)} := \partial_\mu - qF_{\mu\alpha} \frac{\partial}{\partial k_\alpha}$

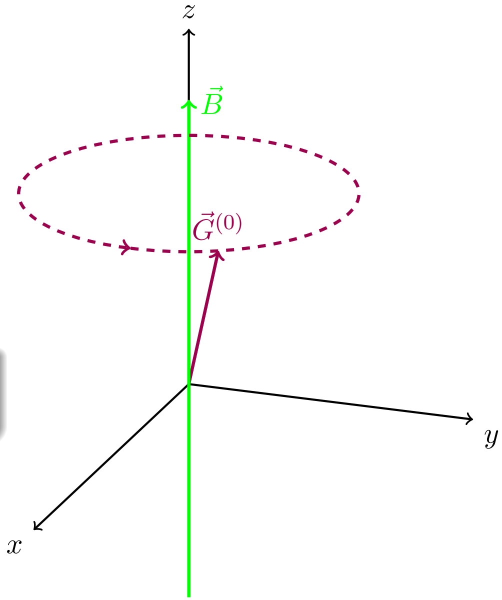
Evolution of vector polarization

- Particle rest frame: Familiar BMT equation
- Describes rotation of vector polarization around the (rest-frame-) magnetic field

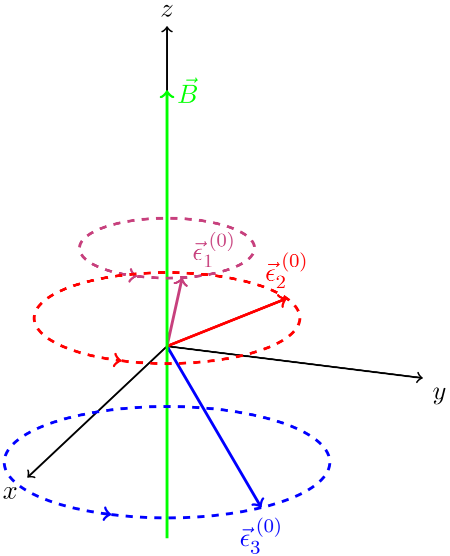
BMT equation

$$\dot{\vec{G}}^{(0)} = q\vec{G}^{(0)} \times \vec{B} \tag{11}$$

$G^{(0),\mu} \equiv (G^{(0),0}, \vec{G}^{(0)})$



- Decompose $F_K^{(0),\mu\nu}$ into three orthogonal main axes $\vec{\epsilon}_i^{\,(0)}, i = 1, 2, 3$
- All main axes fulfill BMT equation separately



Boltzmann equation for spin-1 particles

- Introduce scalar distribution function for (anti-)particles of spin λ

$$W^{\mu}_{\text{on-shell},\mu} = \frac{1}{(2\pi\hbar)^3}\frac{1}{3}\sum_{\lambda=-1}^{1}\sum_{e=\pm}\Theta(ek^0) f^e_\lambda \tag{12}$$

Scalar Boltzmann equation to order $\mathcal{O}(\hbar)$

$$\sum_{\lambda,e}\Theta(ek^0)\delta\left(k^2-m^2\right)\left[k\cdot\hat{\nabla}^{(0)}+\frac{q\hbar\lambda}{2}(\partial^\gamma F^{\rho\sigma})\partial_{k,\gamma}\Sigma_{\rho\sigma}\right] f^e_\lambda = 0 \tag{13}$$

- Contains free-streaming and Vlasov terms as well as Mathisson force (twice as large as in spin-1/2 case)
  - $\rightarrow$ Naive picture of spinning balls with twice the spin magnitude holds up to this order
- However, dynamics of polarization still different

$f^e_\lambda := f^{(0),e}_\lambda + \hbar f^{(1),e}_\lambda$

Global equilibrium

- Two kinds of equilibrium:
  - Local: Collision term in Boltzmann equation vanishes
  - Global: Local+streaming term (other side of Boltzmann equation) vanishes
- Collision term not yet included [$\rightarrow$ (near) future]
- Use Ansatz for local equilibrium distribution function, Boltzmann equation then determines conditions for global equilibrium

  $\rightarrow$ Induced vector polarization by thermal vorticity and magnetic fields

  $\rightarrow$ No induced tensor polarization to first order in $\hbar$

Local equilibrium distribution function

$$f_\lambda^{\text{eq}} = \left(e^{g_\lambda} - 1\right)^{-1} \;, \quad g_\lambda := a_\lambda + k_\mu \beta^\mu + \lambda \frac{\hbar}{2} \Omega_{\mu\nu} \Sigma^{\mu\nu} \tag{14}$$

Conserved quantities

- Action invariant under $\mathrm{SO}^{+}(1,3) \times \mathbf{R}^{1,3} \times \mathrm{U}(1)$
- Non-conservation of spin tensor $S^{\lambda\mu\nu}$
- Conservation of
  - Energy-momentum tensor $T^{\mu\nu}$
  - Electric current J^{μ}

(Non-)conservation equations

$$\partial_\mu J^\mu = 0 \tag{15}$$

$$\partial_\mu T^{\mu\nu} = 0 \tag{16}$$

$$\hbar \partial_\lambda S^{\lambda\mu\nu} = T^{\nu\mu} - T^{\mu\nu} \tag{17}$$

- Can be expressed via Wigner functions

Conserved currents: Energy-momentum tensor

Energy-momentum tensor

$$T^{\mu\nu} = \sum_{\lambda=-1}^{1} \sum_{e=\pm} \int \mathrm{d}K^{(\lambda,e)} k^\mu k^\nu f_\lambda^e + H^{\mu\alpha} F_\alpha{}^\nu - \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \tag{18}$$

- Spin- and energy-dependent mass-shell
  $\mathrm{d}K^{(\lambda,e)} := \mathrm{d}^4k\delta\left(k^2 - m^2 - \hbar\lambda q F^{\alpha\beta}\Sigma_{\alpha\beta}\right)\Theta(ek^0)$
- Familiar objects from macroscopic electrodynamics:
  - Displacement tensor $H^{\mu\nu} := F^{\mu\nu} - M^{\mu\nu}$
  - Dipole tensor $M^{\mu\nu} := 2q\hbar \int \mathrm{d}K W^{\mu\nu}_{A,\text{on}-\text{shell}}$
- Energy-momentum tensor of fluid and electromagnetic field split in a familiar way

W. Israel, General Relativity and Gravitation 9, no. 5, 451-468 (1978)

Conclusion and future perspectives

- Work done:
  - Formulated kinetic theory for massive spin-1 particles in electromagnetic fields
  - Computed global equilibrium
  - Clarified connection to conserved currents
- Next steps:
  - Formulate extended phase space $(x, k) \rightarrow (x, k, \mathfrak{s})$, include collisions
    - Nearly finished

      DW, N. Weickgenannt, E. Speranza, D. Rischke, in preparation
    - Next logical step towards dissipative hydrodynamics

      N. Weickgenannt, DW, E. Speranza, D. Rischke, in preparation
    - Compare to spin-1/2

      N. Weickgenannt, E. Speranza, X.-L. Sheng, Q. Wang, D. Rischke, arXiv: 2103.04896 (2021)
  - Consider gauge fields

    X. -G. Huang, P. Mitkin, A. V. Sadofyev, E. Speranza, J. High Energ. Phys. 2020, 117 (2020)

Appendix

Pseudo-gauge transformations

- Conserved currents not unique, determined up to pseudo-gauge transformations

  E. Speranza, N. Weickgenannt, The European Physical Journal A 57, 155 (2021)

- Idea: Find a spin tensor conserved in the absence of interactions
  - Reasoning: Spin and angular momentum should only be exchanged in interactions

Electric current

$$J^\mu = \frac{2q}{\hbar}\int \mathrm{d}K \left(k^\mu V + \frac{q\hbar}{2} F^{\alpha\beta}\partial_k^\mu \bar{\Sigma}_{\alpha\beta} + \hbar\partial_\nu \bar{\Sigma}^{\mu\nu} \right) + \mathcal{O}(\hbar^2) \tag{19}$$

- Recover magnetization current (separately conserved)

$\bar{\Sigma}^{\mu\nu} := -\epsilon^{\mu\nu\alpha\beta}\frac{k_\alpha}{m}G_\beta$, $\mathrm{d}K := \mathrm{d}^4k\,\delta(k^2 - m^2)$

Conserved currents: spin tensor

- Comparison to spin-$1/2$:

  E. Speranza, N. Weickgenannt, The European Physical Journal A 57, 155 (2021)

Spin tensors

$$\mathrm{Spin}-1: \qquad S^{\lambda\mu\nu}_{s=1} = i g^{[\mu}_{\alpha} g^{\nu]}_{\beta} \int \mathrm{d}^4 k\, k^\lambda W^{\alpha\beta}_{(s=1)} \tag{20}$$

$$\mathrm{Spin}-\frac{1}{2}: \qquad S^{\lambda\mu\nu}_{s=1/2} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]_{ab} \int \mathrm{d}^4 k\, k^\lambda W^{ab}_{(s=1/2)} \tag{21}$$

- Connection between spin-1 and spin-$1/2$: generators of $\mathrm{SO}^+(1,3)$ in respective representations (as expected)
- Spin tensor not conserved only due to electromagnetic interactions

Global equilibrium: polarization

- Assumption: polarization at least of order $\mathcal{O}(\hbar)$
  $\rightarrow$ No large initial polarization
- Idea: Split first-order polarization into free and induced parts
  - Free parts follow BMT equation
  - Induced parts determined from force terms in kinetic equations
- Induced parts determined by $V = \frac{1}{(2\pi\hbar)^3}\frac{1}{3}\sum_{\lambda,e}\Theta(ek^0)f_\lambda^e$
- Induced vector polarization by thermal vorticity and magnetic fields
- No induced tensor polarization to first order in $\hbar$

Global equilibrium: polarization

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Wigner function in global equilibrium

$$W^{\mu\nu}_{S,\text{eq,on-shell}} = K^{\mu\nu}\left(V^{(0)} + \hbar V^{(1)}\right) + \hbar \Phi^{\mu\nu} \tag{22}$$

$$\begin{aligned} W^{\mu\nu}_{A,\text{eq,on-shell}} &= -i\hbar\Bigg[\varpi_{\mu\nu} V^{(0)\prime} + \frac{q}{2k^2} F_{\mu\nu} V^{(0)} \\ &\quad + \frac{1}{2} E^{\alpha}_{\ [\mu}\left(\varpi_{\nu]\alpha} V^{(0)\prime} + \frac{2q}{k^2} F_{\nu]\alpha} V^{(0)}\right) + \Xi^{\mu\nu}\Bigg] \end{aligned} \tag{23}$$

$$W^{\mu\nu}_{S,\text{eq,off-shell}} = 0 \tag{24}$$

$$W^{\mu\nu}_{A,\text{eq,off-shell}} = -i\hbar q F_{\alpha}^{\ [\mu} K^{\nu]\alpha} V^{(0)} \tag{25}$$

- $\Phi^{\mu\nu}$, $\Xi^{\mu\nu}$ follow BMT equations, unconstrained otherwise
- Terms $\propto E^{\alpha}_{\ \mu}$ do not contribute to polarization density P^{μ}_{eq}

$$P^{\mu}_{\text{eq}}(x,k) \propto \frac{i}{2m} \epsilon^{\mu\nu\alpha\beta} k_{\nu} W_{A,\text{eq},\alpha\beta} \tag{26}$$

F. Becattini, arXiv:2004.04050v2 (2020)

Kinetic equations to order $\mathcal{O}(\hbar)$

- Definitions:
  - $3V := K_{\mu\nu} W^{\mu\nu}_{\text{on-shell}}$
  - $\bar{\Sigma}^{\mu\nu} := -iW^{\mu\nu}_{A,\text{on-shell}} - \frac{2q\hbar}{k^2} E^{[\mu}_{\alpha} F^{\nu]\alpha} V + \frac{q\hbar}{k^2} F^{\mu\nu} V$
  - $\mathcal{F}^{\mu\nu}_{K} := K^{\mu\nu}_{\alpha\beta} W^{\alpha\beta}_{\text{on-shell}}$
- Modifications arise due to constraint equations

Kinetic equations to order $\mathcal{O}(\hbar)$

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Combined kinetic equations

$$0 = \delta(k^2 - m^2)\left[k \cdot \hat{\nabla}^{(0)} 3\left(V + \frac{1}{3}\frac{q\hbar}{4k^2}F^{\mu\nu}\bar{\Sigma}_{\mu\nu}\right) + \frac{q\hbar}{2}(\partial^\gamma F^{\alpha\beta})\partial_{k,\gamma}\bar{\Sigma}_{\alpha\beta}\right] \tag{27}$$

$$0 = \delta(k^2 - m^2)\left[k \cdot \hat{\nabla}^{(0)}\bar{\Sigma}^{\mu\nu} - qF_\rho{}^{[\mu}\bar{\Sigma}^{\nu]\rho} - \frac{q\hbar}{2}(\partial^\gamma F_\rho{}^{[\mu})\partial_{k,\gamma}\left(\mathcal{F}_K^{\nu]\rho} + g^{\nu]\rho}\mathcal{F}_K\right) - \frac{q}{k^2}J_\alpha \mathcal{F}_K^{\alpha[\mu}k^{\nu]}\right] \tag{28}$$

$$0 = \delta(k^2 - m^2)\left[k \cdot \hat{\nabla}^{(0)}\mathcal{F}_K^{\rho\sigma} + qF_\alpha{}^{(\rho}\mathcal{F}_K^{\sigma)\alpha} + \frac{q\hbar}{2}K_{\mu\nu}^{\rho\sigma}(\partial^\gamma F^\mu{}_\alpha)\partial_{k,\gamma}\bar{\Sigma}^{\nu\alpha}\right] \tag{29}$$