Polarization effects at finite temperature and magnetic field

Michał Szymański[1]

In collaboration with P. M. Lo, K. Redlich and C. Sasaki

Institute of Theoretical Physics, University of Wroclaw

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[1]michal.szymanski@uwr.edu.pl

Why study QCD at strong magnetic field?

- Important for phenomenology:
  - Non-central heavy-ion collisions
  - Neutron stars
- Additional parameter to study QCD under extreme conditions

Non-zero electric charge of quarks $\rightarrow$ Non-trivial interplay between strong interaction and magnetic fields possible

Theoretical tools

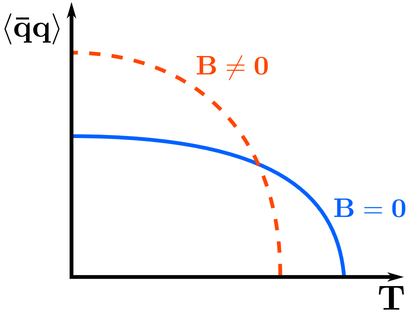
- LQCD $\rightarrow$ first-principle calculations, possibility to explore non-perturbative dynamics
- Effective models $\rightarrow$ Explanatory tools, complementary to LQCD

LQCD and strong magnetic fields

- Phase diagram affected by B

Two characteristic phenomena:

- Magnetic catalysis
  - $T = 0$ quark condensate increases with magnetic field
- Inverse magnetic catalysis
  - $B > 0$ condensate decreases faster with temperature



LQCD $\rightarrow$ IMC as result of competition between "sea" and "valence" contributions[1]

- Sea $\rightarrow$ suppresses condensate
- Valence $\rightarrow$ enhances condensate

Need to explore the role of in-medium effects

[1] F. Bruckmann, G. Endrodi, T. G. Kovacs, JHEP **04**, 112 (2013)

This talk → in-medium dressing of 4-quark interaction

Starting point → Chiral model inspired by Coulomb gauge QCD[2]

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ - m_0)\psi(x) + \int d^4y \rho^a(x) V^{ab}(x-y)\rho^b(y)$$

with

- $\rho^a(x) = \bar{\psi}(x)\gamma^0 T^a \psi(x)$ → color quark current
- $V^{ab}(x-y)$ → Interaction potential

Instantaneous & color-diagonal potential:

$$V^{ab}(x-y) = \delta(x_0 - y_0) \times \delta^{ab} \times V(\vec{x}-\vec{y})$$

$$S^{-1}(p) = p\!\!\!/ - m_0 - C_F \int \frac{d^4q}{(2\pi)^4} V(\vec{p}-\vec{q}) i\gamma^0 S(q)\gamma^0$$

$$\sum_{a=1}^{N_c^2-1} T^a T^a = C_F \mathcal{I}_{N_c\times N_c}\,, \qquad C_F = \frac{N_c^2-1}{2N_c}$$

[2] See e.g. P. M. Lo, E. S. Swanson Phys. Rev. D **81** 034030 (2010)

Contact model: $V(\vec{p}-\vec{q}) = V_0 \rightarrow$ Gap equation

$$M = m_0 + C_F V_0 \int \frac{d^3q}{(2\pi)^3} \frac{M}{2E} \left(1 - 2N_{th}(E)\right)$$

$$E = \sqrt{\vec{q}^2 + M^2}, \qquad N_{th}(E) = \frac{1}{e^{\beta E} + 1}$$

The same form as the NJL model if $C_F V_0 \rightarrow 4 N_c N_f (2 G_{NJL})$

However

- NJL $\rightarrow$ Scalar-scalar interaction

$$\mathcal{L}_{NJL} = \mathcal{L}_0 + G_{NJL}\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \vec{\tau}\psi)^2\right]$$

- Current model $\rightarrow$ Vector-vector interaction
  - Closely mimics the quark-gluon interactions of QCD
  - V_0 can be formally identified with the longitudinal gluon propagator
  - Allows for systematic improvements by taking into account properties of gluon propagators

This talk $\rightarrow$ dressing by polarization

This talk

- Dressing by polarization
- Ring diagram approximation

$$\tilde{V}_0^{-1} = V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p}) \quad \Rightarrow \quad \tilde{V}_0 = \frac{1}{V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p})}$$

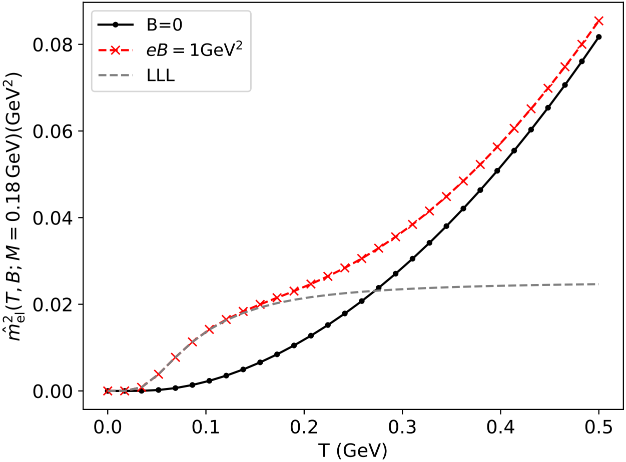
$$\Pi_{00}(p_0, \vec{p}) = \frac{1}{\beta} \sum\!\!\!\!\!\!\int \mathrm{Tr}\left(\gamma^0 S(q) \gamma^0 S(q+p)\right)$$

Static limit

$$m_{el}^2 = -\frac{1}{2} N_f \times \Pi_{00}(p_0 = 0, \vec{p} \to 0)$$

External magnetic field

- Landau quantization
- m_{el} becomes B-dependent



- $m^2,\ |q_f B| \ll T^2 \rightarrow m_{el}^2 \sim \frac{1}{2} N_f T^2$
- $|q_f B| \gg T^2 \rightarrow$ LLL dominates

Coupling to the Polyakov loop → Statistical confinement

- Pure gluon system → Deconfinement order parameter
- Effective models → Accounts for non-preturbative gluon dynamics

$$N_{th}(E) \to N_{th}(E,\ell) = \frac{1}{3}\frac{3\ell e^{-\beta E} + 6\ell e^{-2\beta E} + 3e^{-3\beta E}}{1 + 3\ell e^{-\beta E} + 3\ell e^{-2\beta E} + e^{-3\beta E}}$$
$$= \begin{cases} \dfrac{1}{1+e^{3\beta E}}\,, & \ell = 0\,, \quad \text{baryon-like} \\ \dfrac{1}{1+e^{\beta E}}\,, & \ell = 1\,, \quad \text{quark-like} \end{cases}$$

Gap equation for Polyakov loop

$$\frac{\partial}{\partial \ell}\left(\mathcal{U}_G + \mathcal{U}_Q\right) = 0$$

- $\mathcal{U}_G$ – pure gauge potential
- $\mathcal{U}_Q$ – quark-gluon interaction

Final set of gap equations:

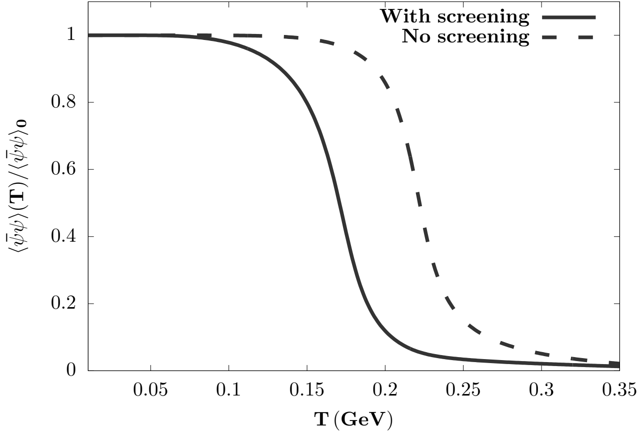
$$M = m_0 + C_F \tilde{V}_0(M,\ell)\, M \left(I_{vac} - \int \frac{d^3q}{(2\pi)^3} \frac{1}{2E} \times 2N_{th}(E,\ell) \right)$$

$$\tilde{V}_0(M,\ell) = \frac{1}{V_0^{-1} + m_{el}^2(T,M,\ell)}$$

$$\frac{\partial}{\partial \ell} \left(\mathcal{U}_G + \mathcal{U}_Q \right) = 0$$

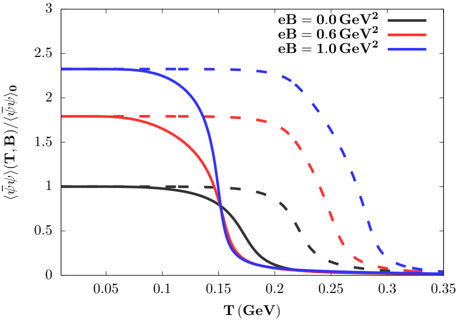
Regularization

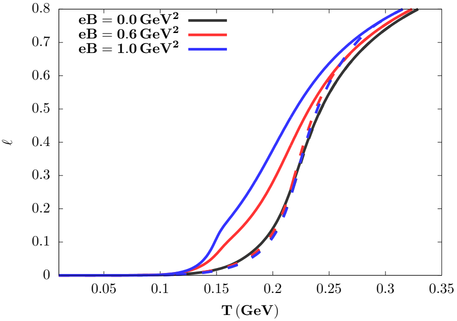
$$I_{vac} = \int \frac{d^3q}{(2\pi^3)} \frac{M}{2E} \to \int_{1/\Lambda^2} \frac{ds}{16\pi^2} \frac{1}{s^2} e^{-M^2 s}$$



Screening $\rightarrow$ pseudocritical temperature reduced

- $T_c \sim 230\,\mathrm{MeV} \rightarrow \sim 160\,\mathrm{MeV}$
- No additional modification of model parameters necessary


$\mathrm{eB = 0.0\,GeV^2}$
$\mathrm{eB = 0.6\,GeV^2}$
$\mathrm{eB = 1.0\,GeV^2}$
$\langle\bar{\psi}\psi\rangle(\mathbf{T},\mathbf{B})/\langle\bar{\psi}\psi\rangle_0$
0
0.5
1
1.5
2
2.5
3
0.05
0.1
0.15
0.2
0.25
0.3
0.35
T (GeV)


$\mathrm{eB = 0.0\,GeV^2}$
$\mathrm{eB = 0.6\,GeV^2}$
$\mathrm{eB = 1.0\,GeV^2}$
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0.5
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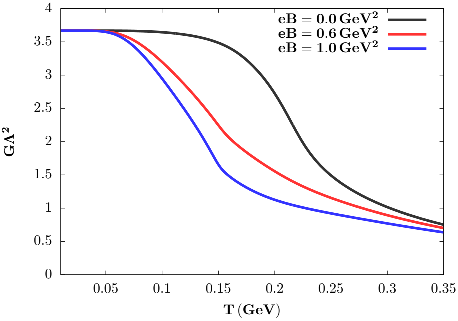
0.7
0.8
0.05
0.1
0.15
0.2
0.25
0.3
0.35
T (GeV)


$\mathrm{eB = 0.0\,GeV^2}$
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$\mathrm{eB = 1.0\,GeV^2}$
$\mathbf{G\Lambda^2}$
0
0.5
1
1.5
2
2.5
3
3.5
4
0.05
0.1
0.15
0.2
0.25
0.3
0.35
T (GeV)

Conclusions and outlook

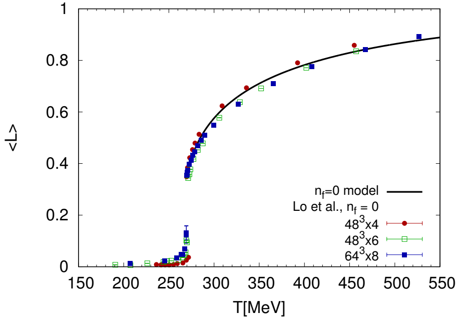
- Dressing of 4-quark interaction has profound effect on chiral phase transition
  - $B = 0$: $T_C^{no\ screening} \approx 230\,\text{MeV} \to T_C^{screening} \approx 160\,\text{MeV}$
  - $B \neq 0$: IMC due to screening
  - No need for artificial rescaling of parameters or fitting the coupling
- Future prospects
  - $\mu_B \neq 0 \to$ investigation of fluctuations
  - Momentum dependence of gluon propagators $\to$ going beyond contact interaction

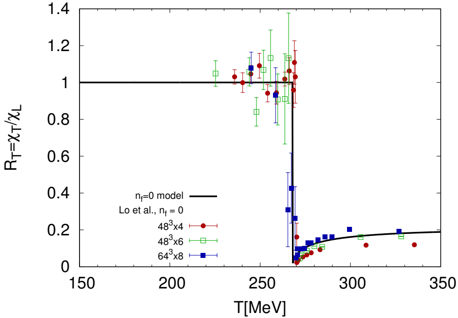
Appendix

Pure gauge part → Polyakov loop potential[1]

$$\frac{\mathcal{U}_G}{T^4} = -\frac{1}{2}a(T)\ell\bar{\ell} + b(T)\ln M_H(\ell,\bar{\ell}) + \frac{1}{2}c(T)(\ell^3+\bar{\ell}^3) + d(T)(\ell\bar{\ell})^2$$

- Polyakov loop & fluctuations determined from LQCD





Quark-gluon interaction

$$\mathcal{U}_Q = -2T\int\frac{d^3q}{(2\pi)^3}2\ln\left(1+3\ell e^{-\beta E}+3\ell e^{-2\beta E}+e^{-3\beta E}\right)$$

[1] *P. M. Lo, B. Friman, O. Kaczmarek, K. Redlich and C. Sasaki, Phys. Rev. D* **88***, 074502 (2013)*

Electric mass

$$m_{el}^2 = -\frac{1}{2} N_f \times \Pi_{00}(p_0 = 0, \vec{p} \to 0) = \frac{1}{2} N_f \times \int \frac{d^3q}{(2\pi)^3} 4\beta N_{th}(1 - N_{th})$$

External magnetic field $\to$ Landau quantization

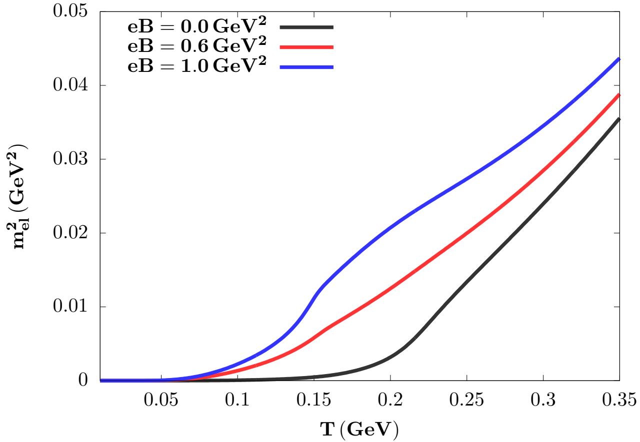
$$2 \int \frac{d^3p}{(2\pi)^3} \to \frac{|qB|}{2\pi} \sum_{k=0}^{\infty} (2 - \delta_{k,0}) \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}$$

$${E_k}^2 = m^2 + p_z^2 + 2k|q_f B| \,,$$

Electric mass (per flavor)

$$m_{el}^2 = \frac{1}{2} \frac{|q_f B|}{2\pi} \sum_{k=0}^{\infty} (2 - \delta_{k,0}) \int \frac{dq_z}{2\pi} 4\beta N_{th}(E_k)(1 - N_{th}(E_k))$$

$$\approx \frac{1}{2} \frac{|q_f B|}{4\pi} \int \frac{dq_z}{2\pi} \frac{4\beta e^{\beta\sqrt{(q_z)^2 + m^2}}}{(e^{\beta\sqrt{(q_z)^2 + m^2}} + 1)^2} \,, \qquad |q_f B| \gg T^2$$



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0.05
0.04
0.03
0.02
0.01
0
$\mathbf{m_{el}^2\,(GeV^2)}$
0.05
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$\mathbf{T\,(GeV)}$