# One-loop HTL thermodynamics of magnetized QCD matter

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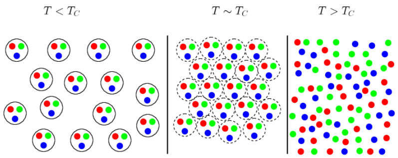
# Outline

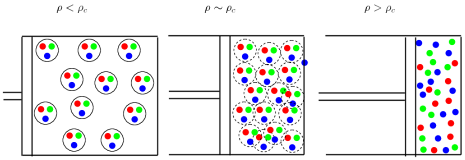
1. Quark gluon plasma
2. QGP in presence of magnetic field
3. General structure of gluon effective propagator
4. Thermodynamics of magnetized QGP

# Recipes for quark-gluon plasma

High temperature ($T_c \sim 160$ MeV)

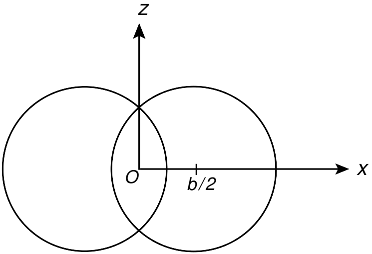



High baryon density



# Magnetic field in non-central HIC

- Magnetic field of strength upto $\sim 20m_\pi^2$ ($\sim 10^{14}$ T) can be created in non-central heavy ion collision.




- Magnetic field strength decreases with time .
- In a direction perpendicular to the reaction plane.

# Fermion propagator in magnetic field

**Approximations on the magnetic field**

- Homogeneous
- Time independent

Energy of fermion in presence of magnetic field
$E_n = \sqrt{k_z^2 + m_f^2 + 2nq_f B}.$

Fermion propagator in strong magnetic field is given as

$$S(K) = ie^{-\frac{k_\perp^2}{q_f B}} \frac{\not{K}_\shortparallel + m_f}{K_\shortparallel^2 - m_f^2} \Big(1 - i\gamma_1\gamma_2\Big).$$

# General structure of gluon effective propagator

# Formalism

1. The presence of heat bath breaks the Lorentz symmetry of the system.
2. Four velocity $u_\mu$ of heat bath is introduced because of presence of medium. We choose rest frame of heat bath $u_\mu = (1, 0, 0, 0)$.
3. We consider magnetic field along z direction $n_\mu = (0, 0, 0, 1)$.
4. Rotational symmetry is broken due to the presence of magnetic field.
5. Quarks are directly affected by magnetic field.
6. Gluons are affected via quark loop.

$$
\begin{aligned}
P_{\shortparallel}^{\mu} &= (P\cdot u)u^{\mu} - (P\cdot n)n^{\mu} = (p_0, 0, 0, p_z) \\
P_{\perp}^{\mu} &= P^{\mu} - P_{\shortparallel}^{\mu} = (0, p_x, p_y, 0)
\end{aligned}
$$

# General structure of gluon self energy

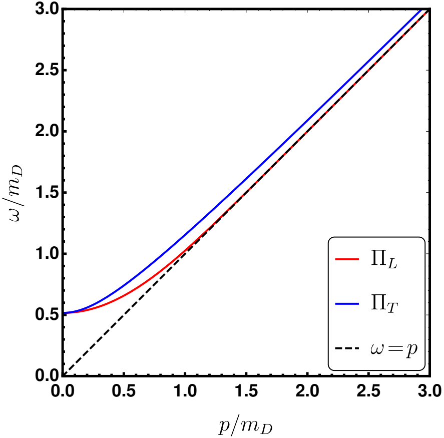
**Ward identity** $P_\mu \Pi^{\mu\nu} = 0$

- Vacuum

$$\Pi^{\mu\nu} = \Pi(P^2)\left(\eta^{\mu\nu} - \frac{P^\mu P^\nu}{P^2}\right) = \Pi(P^2)V^{\mu\nu}.$$

- Thermal
    1. Two linearly independent tensors needed.
    2. Define $\bar{u}^\mu = u_\nu V^{\mu\nu}$.
    3. Define $B^{\mu\nu} = \frac{\bar{u}^\mu \bar{u}^\nu}{\bar{u}^2}$ and $A^{\mu\nu} = V^{\mu\nu} - B^{\mu\nu}$.
    4. $\Pi^{\mu\nu} = \Pi_T A^{\mu\nu} + \Pi_L B^{\mu\nu}$.
    5. $\Pi_L = B^{\mu\nu}\Pi_{\mu\nu}, \ \Pi_T = \frac{1}{2} A^{\mu\nu}\Pi_{\mu\nu}$

# Dispersive modes of gluon in thermal medium



## Magnetized medium

1. Constituent tensors constructed out of $\eta_{\mu\nu}, P_\mu P_\nu, u_\mu u_\nu, n_\mu n_\nu, P_\mu u_\nu + u_\mu P_\nu, P_\mu n_\nu + n_\mu P_\nu, u_\mu n_\nu + n_\mu u_\nu$.
2. Four linearly independent tensors needed.
3. Define $\bar{n}^\mu = n_\nu A^{\mu\nu}$.
4. Define $Q^{\mu\nu} = \frac{\bar{n}^\mu \bar{n}^\nu}{\bar{n}^2}$, $N^{\mu\nu} = \frac{\bar{u}^\mu \bar{n}^\nu + \bar{n}^\mu \bar{u}^\nu}{\sqrt{\bar{u}^2}\sqrt{\bar{n}^2}}$ and $R^{\mu\nu} = A^{\mu\nu} - Q^{\mu\nu}$.
5. Gluon self energy $\Pi^{\mu\nu} = bB^{\mu\nu} + cR^{\mu\nu} + dQ^{\mu\nu} + aN^{\mu\nu}$.
6. $b = B^{\mu\nu}\Pi_{\mu\nu},\ c = R^{\mu\nu}\Pi_{\mu\nu},\ d = Q^{\mu\nu}\Pi_{\mu\nu},\ a = \frac{1}{2}N^{\mu\nu}\Pi_{\mu\nu}$.

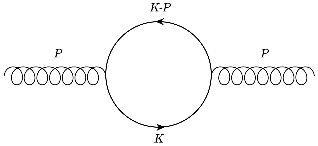



Figure: Quark loop of gluon self energy $\Pi^{\mu\nu}$

# Effective gluon propagator

Dyson Schwinger equation $D_{\mu\nu}^{-1} = (D_{\mu\nu}^{0})^{-1} - \Pi_{\mu\nu}$

$$\begin{aligned} D_{\mu\nu} &= \frac{\xi P_\mu P_\nu}{P^4} + \frac{(P^2-d)B_{\mu\nu}}{(P^2-b)(P^2-d)-a^2} + \frac{R_{\mu\nu}}{P^2-c} \\ &+ \frac{(P^2-b)Q_{\mu\nu}}{(P^2-b)(P^2-d)-a^2} + \frac{aN_{\mu\nu}}{(P^2-b)(P^2-d)-a^2} \end{aligned}$$

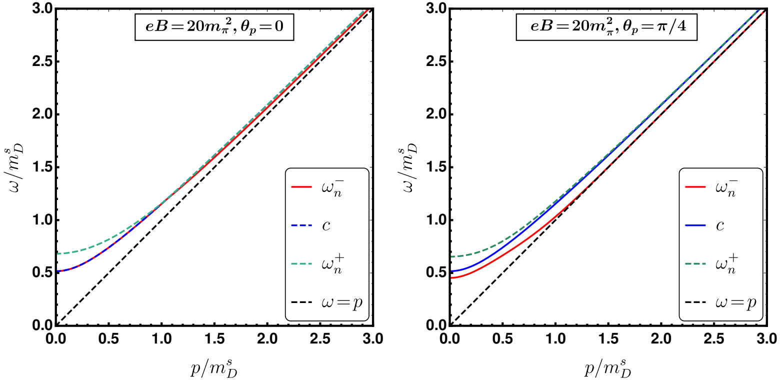
- Leads to three dispersive modes of gluon.
- Reduces to thermal effective propagator at $eB = 0$.

**Three dispersive modes**

$$\begin{aligned} P^2 - c &= 0, \\ (P^2-b)(P^2-d)-a^2 &= (P^2-\omega_n^+)(P^2-\omega_n^-) = 0 \end{aligned}$$

# Dispersion relation of gluon

**SFA**




Eur.Phys.J.C 79 (2019) 8, 658

# Conclusion

1. Three non-degenerate dispersive modes of gluon.
2. One transverse and two mixed modes in SFA.

# Thermodynamics of magnetized quark-gluon plasma

# Anisotropic pressure in presence of strong magnetic field

- $F = \epsilon - Ts - eB \cdot M$.
- Longitudinal pressure $P_z = -F$.
- Transverse pressure $P_\perp = -F - eB \cdot M = P_z - eB \cdot M$.
- Magnetization $M = -\frac{\partial F}{\partial (eB)}$.
- Longitudinal pressure of free quarks and gluons in presence of strong magnetic field

$$P_z^{\mathrm{i}} = N_c N_f \sum_f q_f B \frac{T^2}{6}\left(1 + 12\hat{\mu}^2\right) + (N_c^2 - 1)\frac{\pi^2 T^4}{45}$$

- Transverse pressure

$$P_\perp^{\mathrm{i}} = (N_c^2 - 1)\frac{\pi^2 T^4}{45} = \text{gluon pressure}$$

# HTL Pressure of QGP in magnetized medium

- Free energy is calculated using the effective propagator of quarks and gluons in magnetized medium.

$$
\begin{aligned}
F_q &= -N_c N_f \sum_f \sum\!\!\!\!\!\!\!\int_{\{P\}} \ln\left(\det[S_{eff}^{-1}]\right) \\
&= -N_c N_f \sum_f \sum\!\!\!\!\!\!\!\int_{\{P\}} \ln\left[P_{\shortparallel}^4\left(1+\frac{4\alpha^2-4\beta^2+4\alpha p_0+4\beta p_3}{P_{\shortparallel}^2}\right)\right]
\end{aligned}
$$

- where $S_{eff}^{-1} = \not{P} + \Sigma$.

$$
F_g = (N_c^2-1)\left[\frac{1}{2}\sum\!\!\!\!\!\!\!\int_P \ln\left[\det(D_{\mu\nu}^{-1})\right] - \sum\!\!\!\!\!\!\!\int_P \ln(-P^2)\right].
$$

# Pressure of QGP in magnetized medium

- Soft ($P \sim gT$) and hard($P \sim T$) quasiparticle

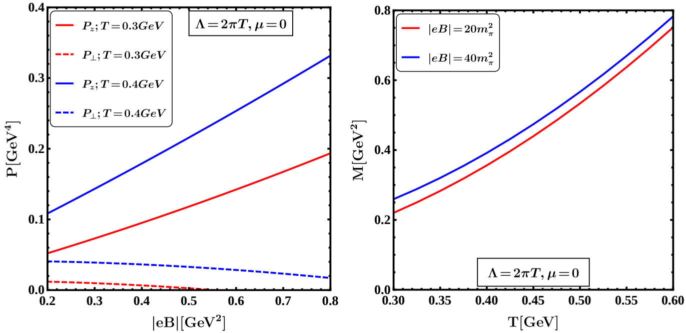
For quarks

- Lowest matsubara frequency $\sim T$.
- We keep terms upto $\mathcal{O}[g^4]$ and $\mathcal{O}[(\mu/T)^4]$ .

For gluons

- For hard quasiparticles ($P \sim T$) we make a high temperature expansion of the 'Log' term.
- We keep terms upto $\mathcal{O}[g^4]$.
- For soft gluons ($P \sim gT$), we put $p_0 = 0$ because this is the lowest Matsubara mode.
- $g^3$ term appears from soft part.

# Results in SFA

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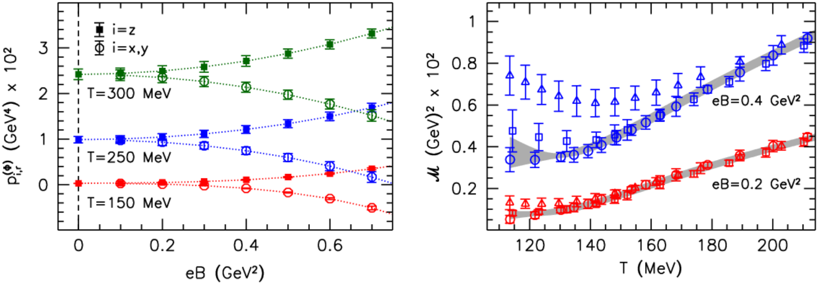
# Lattice results

JHEP 1408 (2014) 177

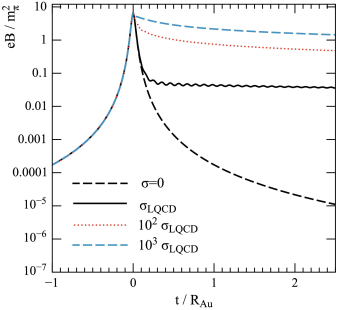


# Conclusion

1. Pressure of QGP becomes anisotropic in presence of strong magnetic field.
2. Longitudinal pressure is greater than the transverse pressure.
3. The magnetized medium shows paramagnetic nature.
4. Qualitative nature of one loop pressure matches with lattice.
5. Results can be improved by performing higher loop order calculations.


thank you

# Time dependence of the magnetic field

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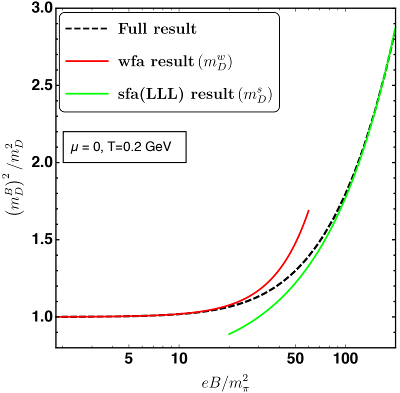
# Debye screening mass

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