

Non-Equilibrium Transport of Conserved Charges in High-Energy Heavy Ion Collisions

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Introduction

Introduction

- ▶ HICs usually described by multistage models
 - For $\tau \geq \tau_{\text{hydro}} \sim 1 \text{ fm}/c$ the system is well described by hydrodynamics
 - For $\tau \ll \tau_{\text{hydro}}$ hydrodynamics breaks down since the created QGP is far from equilibrium
- ▶ Far-from-equilibrium dynamics well described by kinetic theory (see e.g. KØMPØST) [Kurkela et al., Phys.Rev.C (2019)]
 - Use non-equilibrium Greens Functions (GFs) to match initial state at τ_0 to the hydrodynamic phase at later times
- ▶ Strategy: Obtain GFs by analyzing the response of moments of macroscopic quantities to linear perturbations [Kamata et al., Phys.Rev.D (2020)]
 - Divide spacetime evolution in evolution of boost invariant homogeneous background and linear perturbations around this background

Theoretical Background

Theoretical Background - Boltzmann Equation

- ▶ At early times the QGP experiences rapid longitudinal expansion while transverse plane is initially at rest -> assume Bjorken flow
- ▶ Suitable coordinates for Bjorken flow are Milne coordinates

$$\tau = \sqrt{(x^0)^2 - (x^3)^2} \quad \eta = \operatorname{arctanh}(x^3/x^0) \quad g_{\mu\nu} = \operatorname{diag}(1, -1, -1, -\tau^2)$$

- ▶ As the background is assumed to be boost invariant & homogeneous the distribution function can be written as

$$f(x, p) = f_{BG}(\tau, p_T, |p_\eta|) =: \nu_g f_{g,BG}(\tau, p_T, |p_\eta|) + \nu_q \sum_a [f_{q_a,BG}(\tau, p_T, |p_\eta|) + \bar{f}_{q_a,BG}(\tau, p_T, |p_\eta|)] ,$$

$$f_a(x, p) = f_{a,BG}(\tau, p_T, |p_\eta|) =: \nu_q [f_{q_a,BG}(\tau, p_T, |p_\eta|) - \bar{f}_{q_a,BG}(\tau, p_T, |p_\eta|)] .$$

Theoretical Background - Boltzmann Equation

- ▶ Starting point: Boltzmann equation (BE) in relaxation time approximation (RTA)

$$\begin{aligned}\tau \partial_\tau f_{BG}(\tau, p_T, |p_\eta|) &= -\frac{\tau}{\tau_R} \left[f_{BG}(\tau, p_T, |p_\eta|) - f_{\text{eq}}\left(\frac{p^\tau}{T(\tau)}, \mu(\tau)\right) \right], \\ \tau \partial_\tau f_{a,BG}(\tau, p_T, |p_\eta|) &= -\frac{\tau}{\tau_R} \left[f_{a,BG}(\tau, p_T, |p_\eta|) - f_{a,\text{eq}}\left(\frac{p^\tau}{T(\tau)}, \mu(\tau)\right) \right].\end{aligned}$$

- ▶ Instead of solving BE numerically, study evolution of moments

$$E_l^m(\tau) = \tau^{1/3} \int \frac{dp_\eta}{(2\pi)} \int \frac{d^2\mathbf{p}}{(2\pi)^2} p^\tau Y_l^m(\phi_{\mathbf{p}}, \theta_{\mathbf{p}}) f_{BG}(\tau, p_T, |p_\eta|),$$

Spherical Harmonics

$$N_{al}^m(\tau) = \int \frac{dp_\eta}{(2\pi)} \int \frac{d^2\mathbf{p}}{(2\pi)^2} Y_l^m(\phi_{\mathbf{p}}, \theta_{\mathbf{p}}) f_{a,BG}(\tau, p_T, |p_\eta|).$$

$\tan\Phi_{\mathbf{p}} = p^1/p^2$, $\cos\Theta_{\mathbf{p}} = p_\eta/(\tau p^\tau)$

- ▶ Macroscopic quantities are obtained by low order moments
- ▶ Evolution equation for the moments

$$\begin{aligned}\tau \partial_\tau E_l^m &= b_{l,-2}^m E_{l-2}^m + b_{l,0}^m E_l^m + b_{l,+2}^m E_{l+2}^m - \frac{\tau}{\tau_R} \left[E_l^m - E_l^m|_{\text{eq}} \right], \\ \text{Numerical Factors} \\ \tau \partial_\tau N_{al}^m &= B_{l,-2}^m N_{al-2}^m + B_{l,0}^m N_{al}^m + B_{l,+2}^m N_{al+2}^m - \frac{\tau}{\tau_R} \left[N_{al}^m - N_{al}^m|_{\text{eq}} \right].\end{aligned}$$

Theoretical Background - Initial Conditions

- ▶ At early times the system cannot maintain considerable longitudinal momenta
 - Initial distribution naturally of the form that transverse momentum is much larger than longitudinal
- ▶ (Longitudinal) support in form of a Dirac delta function corresponds to non-equilibrium attractor of the kinetic equations (different initial conditions will approach same curve for later times)
- ▶ Initial conditions for the moments:

$$E_l^m(\tau_0) = \tau_0^{1/3} (e\tau)_0 y_l^m P_l^m(0) \delta^{m0},$$

$$N_l^m(\tau_0) = (n_a \tau)_0 y_l^m P_l^m(0) \delta^{m0}.$$

$$(e\tau)_0 := \lim_{\tau_0 \rightarrow 0} \tau_0 e(\tau_0) = \text{const},$$

$$(n_a \tau)_0 := \lim_{\tau_0 \rightarrow 0} \tau_0 n_a(\tau_0) = \text{const}.$$

Results for Conformal Background

Results for Conformal Background

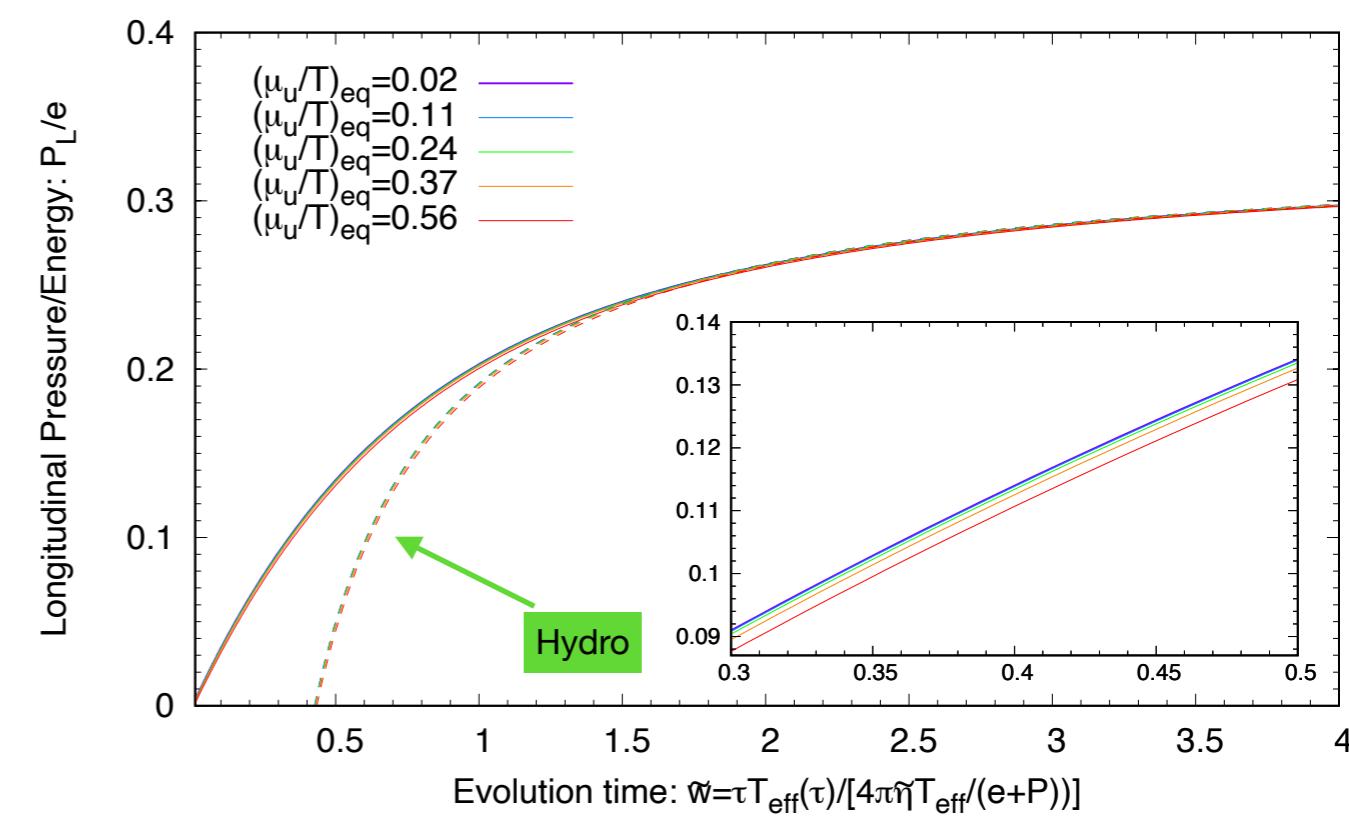
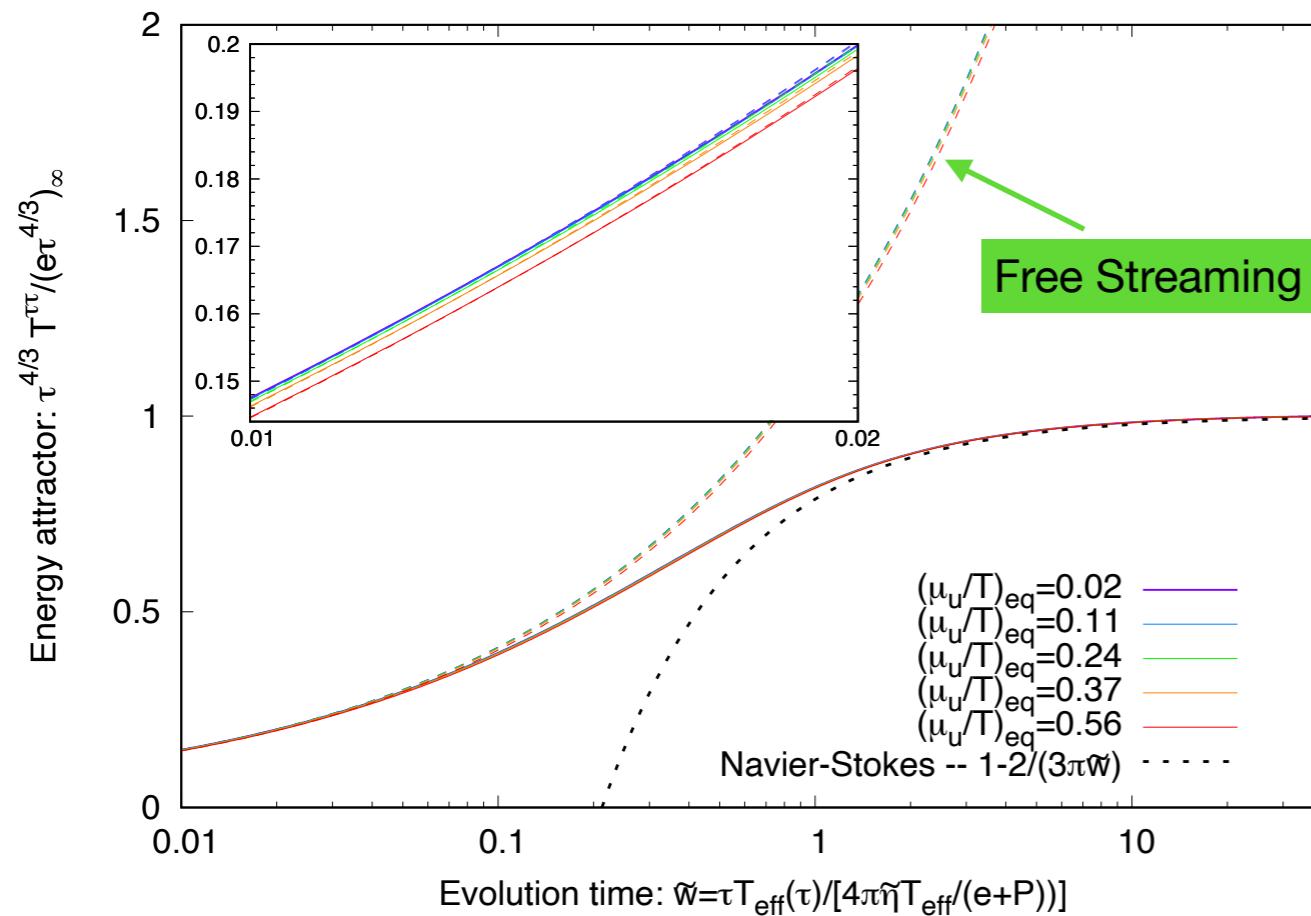
- Rewrite equation in terms of dimensionless time variable

$$x = \frac{\tau}{\tau_R} = \frac{\tau T_{\text{eff}}(\tau)}{5} \frac{(e + P)}{\tilde{\eta} T_{\text{eff}}} = \frac{4\pi}{5} \tilde{w}$$

Shear Viscosity Determined by Landau Matching

- Only small deviations for varying chemical potential

- Same relaxation time for every species
- Only small values of chemical potentials reached



Perturbations Around Background

Perturbations in Transverse Plane

- ▶ Small space-time perturbations around homogeneous background with $n_a=0$ (-> no cross-diffusion)
- ▶ Perturbations in transverse plane (in Fourier space)

$$\delta f_{\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|) = \nu_g \delta f_{q,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|) + \nu_q \sum_a [\delta f_{q_a,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|) + \delta \bar{f}_{q_a,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|)] ,$$

$$\delta f_{a,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|) = \nu_q [\delta f_{q_a,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|) - \delta \bar{f}_{q_a,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|)] .$$

- ▶ Definition of moments analogous to background moments

$$\delta E_{l,\mathbf{k}}^m(\tau) = \tau^{1/3} \int \frac{dp_\eta}{(2\pi)} \int \frac{d^2\mathbf{p}}{(2\pi)^2} p^\tau Y_l^m(\phi_{\mathbf{p}\mathbf{k}}, \theta_{\mathbf{p}}) \delta f_{\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|) ,$$

$$\delta N_{a,l,\mathbf{k}}^m(\tau) = \int \frac{dp_\eta}{(2\pi)} \int \frac{d^2\mathbf{p}}{(2\pi)^2} Y_l^m(\phi_{\mathbf{p}\mathbf{k}}, \theta_{\mathbf{p}}) \delta f_{a,\mathbf{k}}(\tau, \mathbf{p}, |p_\eta|) .$$

- ▶ Again macroscopic quantities can be obtained by low order moments
- ▶ Equations of motions obtained by computing derivatives of moments

Initial Conditions for Perturbations & Green's Functions

► Initial energy, momentum and charge perturbations

$$\delta E_{l,\mathbf{k}}^m(\tau_0) = \tau_0^{1/3} (-i)^m J_m(|\mathbf{k}| \tau_0) y_l^m P_l^m(0) (e\tau)_0,$$

Energy perturbation

$$\delta E_{l,\mathbf{k}}^{m\parallel}(\tau_0) = -i\tau^{1/3} (-i)^m [J_{m+1}(|\mathbf{k}| \tau_0) - J_{m-1}(|\mathbf{k}| \tau_0)] y_l^m P_l^m(0) (e\tau)_0,$$

Long. momentum perturbation

$$\delta E_{l,\mathbf{k}}^{m\perp}(\tau_0) = -\tau^{1/3} (-i)^m [J_{m+1}(|\mathbf{k}| \tau_0) + J_{m-1}(|\mathbf{k}| \tau_0)] y_l^m P_l^m(0) (e\tau)_0,$$

Trans. momentum perturbation

$$\delta N_{a,l,\mathbf{k}}^m(\tau_0) = (-i)^m J_m(|\mathbf{k}| \tau_0) y_l^m P_l^m(0) \alpha_a(n_a \tau)_0.$$

Charge perturbation

Bessel Functions

► Consider linear response functions (GFs) $G_{\alpha\beta}^{\mu\nu}$ (energy-momentum) and F_α^μ (charge)

$$\frac{\delta T_{\mathbf{k}}^{\mu\nu}(\tau)}{e(\tau)} = \frac{1}{2} G_{\alpha\beta}^{\mu\nu}(\mathbf{k}, \tau, \tau_0) \frac{\delta T_{\mathbf{k}}^{\alpha\beta}(\tau_0)}{e(\tau_0)},$$

$$\tau \delta N_{\mathbf{k}}^\mu(\tau) = F_\alpha^\mu(\mathbf{k}, \tau, \tau_0) \tau_0 \delta N_{\mathbf{k}}^\alpha(\tau_0).$$

No flavor dependence since we consider $n_a=0$

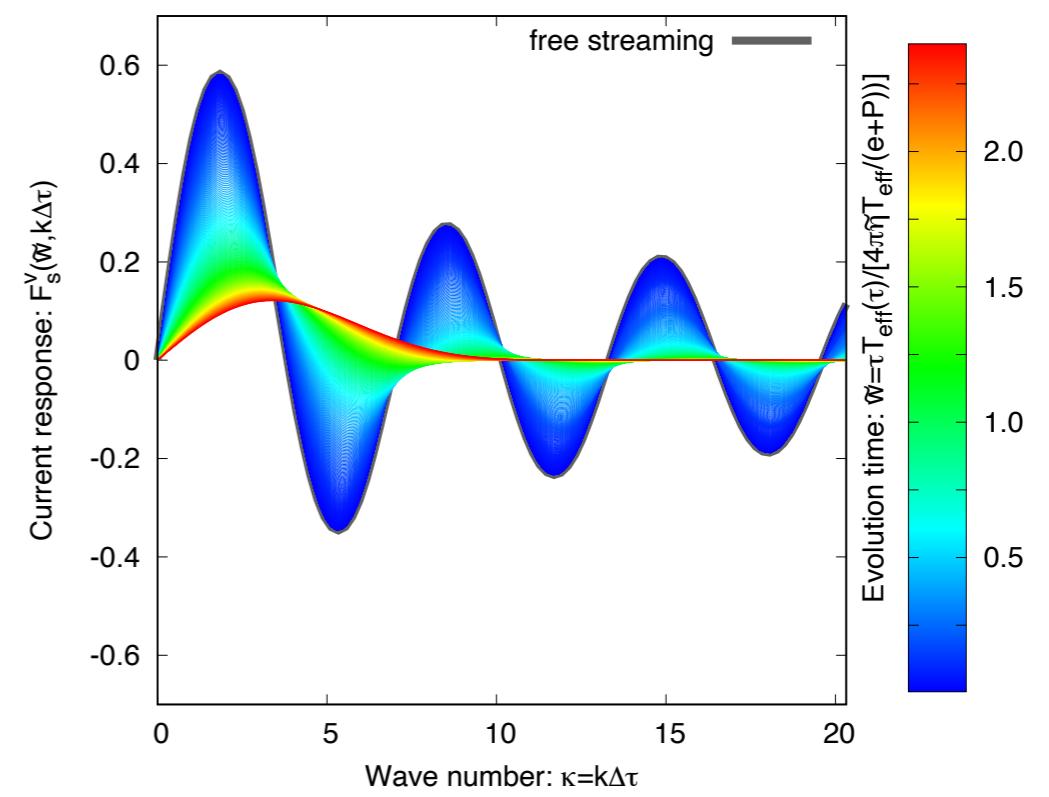
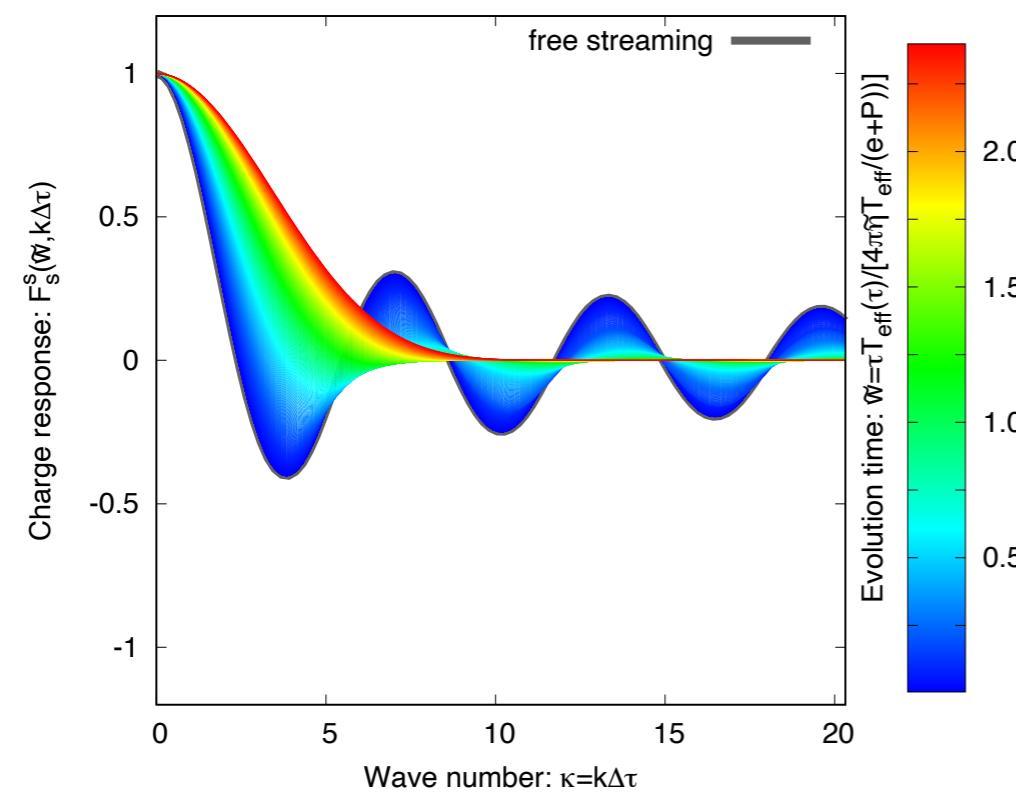
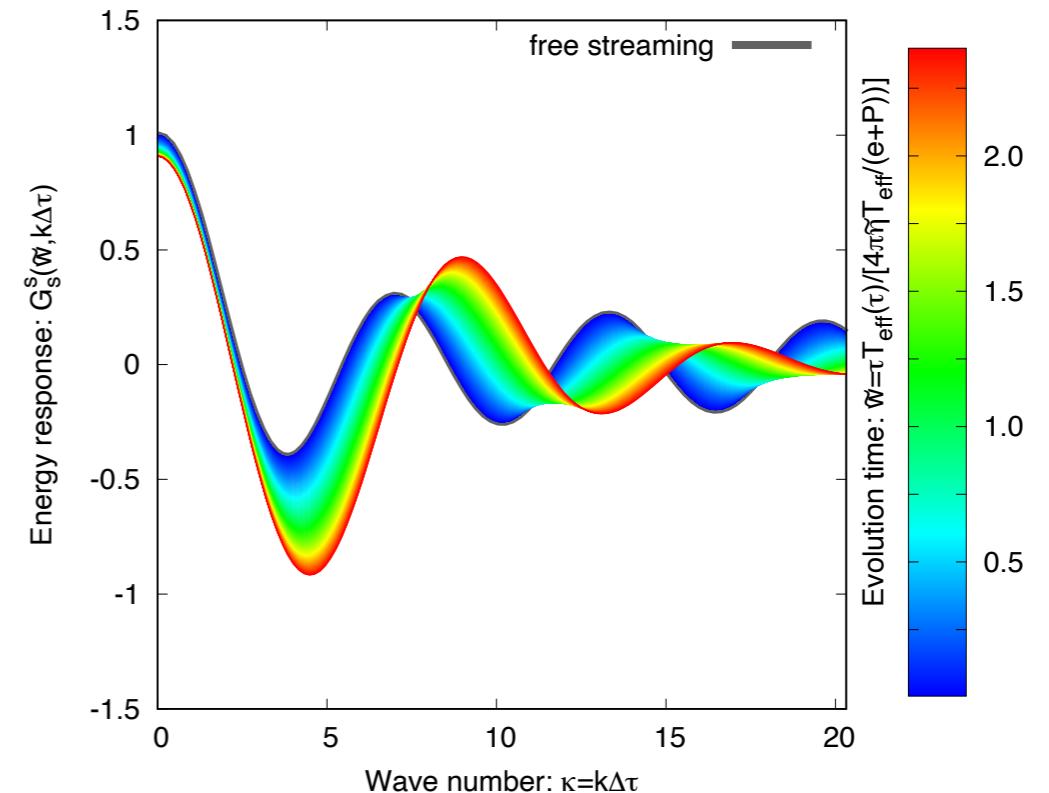
► Decompose GFs according to basis of scalars (s), vectors (v), tensors (t)

$$G_s^s(\kappa, x) = \frac{\delta T_{\mathbf{k}}^{\tau\tau}(x)}{e(x)} \quad F_s^s(\kappa, x) = \tau \delta N_{\kappa}^\tau(x) \quad F_s^v(\kappa, x) = i \frac{\mathbf{k}_i}{|\mathbf{k}|} \tau \delta N_{\kappa}^i(x).$$

$\kappa = |\mathbf{k}_T| (\tau - \tau_0)$

Results Green's Functions

- ▶ When hydrodynamics becomes applicable, only long wavelength-modes will survive
- ▶ Energy/momentum unchanged to case without charges (because equations decouple when $n_a=0$)
- ▶ Damping higher for charges than for energy/momentum



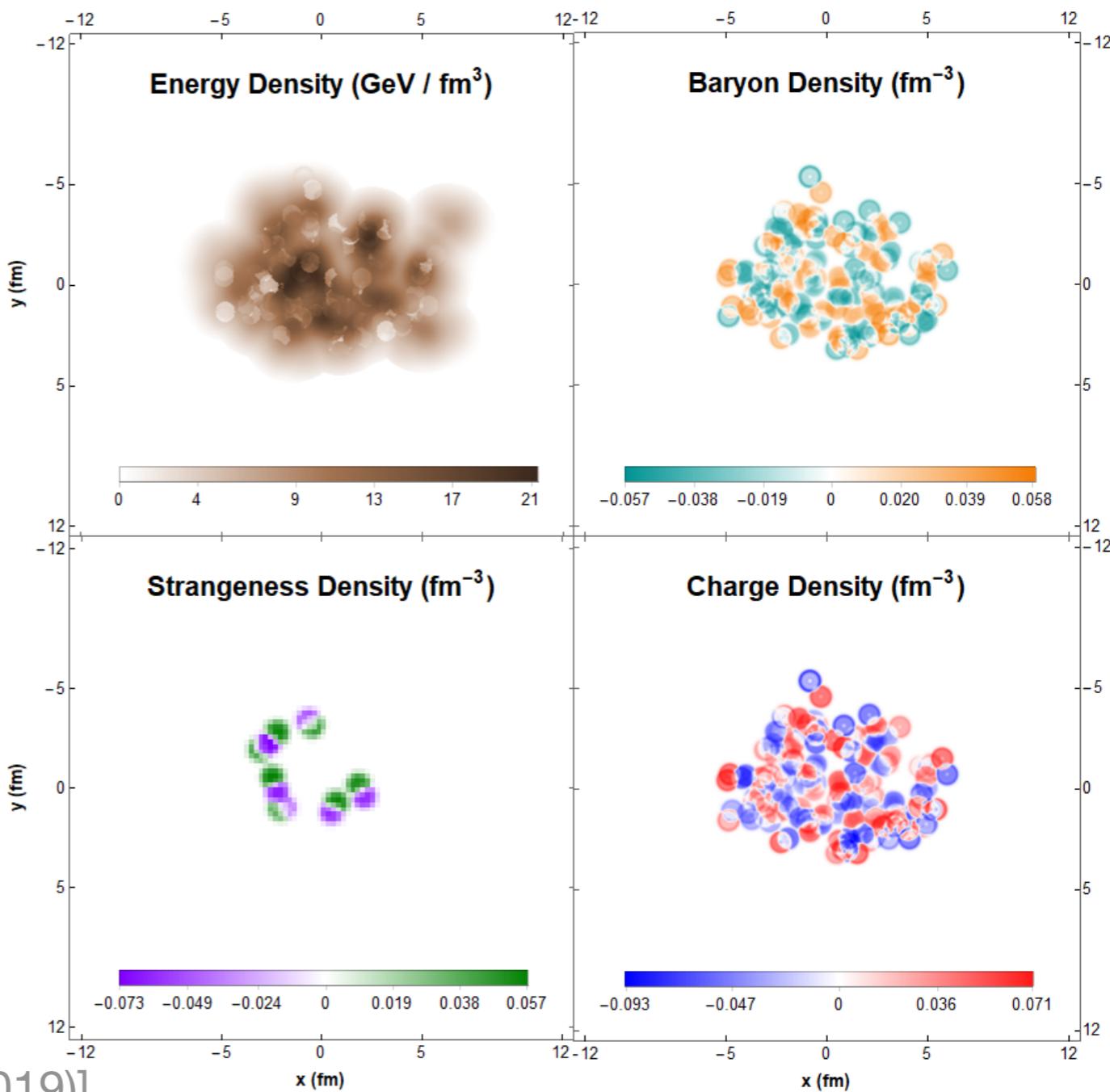
Hydrodynamics: $\tilde{w} \geq 1$

Conclusion and Outlook

Conclusion

- ▶ Green's Functions provide a powerful tool to analyze early time dynamics in Heavy Ion Collisions
- ▶ In RTA background evolution does not show a huge dependence on the presence of charges
 - More realistic picture: consider different relaxation times for different species and higher chemical potentials
- ▶ For vanishing background charges ($n_a=0$) equations of motion for the moments decouple and energy-momentum response is unchanged
- ▶ Green's Functions for charges show strong damping

- ▶ Consider evolution for $n_a \neq 0$
- ▶ Compute Green's Functions for charges in kinetic theory to come closer to real QCD dynamics [Du, Schlichting, Phys.Rev.Lett. 127 (2021)]
- ▶ Combine ICCING (Initial Conserved Charges in Nuclear Geometry) and non-equilibrium Green's function
 - Treat differences in energy/ charge density after gluon splitting as perturbations that propagate via Green's functions



More on ICCING:

[Carzon et al., arxiv:1911.12454 (Pre-print), (2019)]

Thank You

Backup

Backup - Bjorken Flow

- ▶ Main idea: Space-time evolution looks the same in all frames shortly after the collision
 - Results in a symmetry, which is taken into account in the initial condition
- ▶ For central collisions: Expansion at early only longitudinal and homogeneous -> problem reduced to 1+1 dim. case
- ▶ Fluid will move in longitudinal distance z with velocity $v_z = z/t$
- ▶ Assumption breaks down at later times when transverse expansion cannot be neglected anymore

Background

$$e(\tau) = \frac{\sqrt{4\pi}}{\tau^{4/3}} E_0^0(\tau),$$

$$P_T(\tau) = \frac{\sqrt{4\pi}}{\tau^{4/3}} \left[\frac{1}{3} E_0^0(\tau) - \sqrt{\frac{1}{45}} E_2^0(\tau) \right],$$

$$P_L(\tau) = \frac{\sqrt{4\pi}}{\tau^{4/3}} \left[\frac{1}{3} E_0^0(\tau) + \sqrt{\frac{4}{45}} E_2^0(\tau) \right].$$

$$n_a(\tau) = \frac{\sqrt{4\pi}}{\tau} N_{a0}^0(\tau)$$

Perturbations

$$\tau^{4/3} \delta T_{\mathbf{k}}^{\tau\tau} = \sqrt{4\pi} \delta E_{0,\mathbf{k}}^0,$$

$$\delta^{ij} \frac{i\mathbf{k}^i}{|\mathbf{k}|} \tau^{4/3} \delta T_{\mathbf{k}}^{\tau j} = -i \sqrt{\frac{2\pi}{3}} (\delta E_{1,\mathbf{k}}^{+1} - \delta E_{1,\mathbf{k}}^{-1}),$$

$$\epsilon^{ij} \frac{i\mathbf{k}^i}{|\mathbf{k}|} \tau^{4/3} \delta T_{\mathbf{k}}^{\tau j} = -\sqrt{\frac{2\pi}{3}} (\delta E_{1,\mathbf{k}}^{+1} + \delta E_{1,\mathbf{k}}^{-1}),$$

$$\tau^{4/3} (-\tau) \delta T_{\mathbf{k}}^{\tau\eta} = \sqrt{\frac{4\pi}{3}} \delta E_{1,\mathbf{k}}^0,$$

$$\delta^{ij} \tau^{4/3} \delta T_{\mathbf{k}}^{ij} = \sqrt{\frac{16\pi}{9}} \delta E_{0,\mathbf{k}}^0 - \sqrt{\frac{16\pi}{45}} \delta E_{2,\mathbf{k}}^0,$$

$$\frac{\mathbf{k}^i \mathbf{k}^j}{\mathbf{k}^2} \tau^{4/3} \delta T_{\mathbf{k}}^{ij} = \sqrt{\frac{4\pi}{9}} \delta E_{0,\mathbf{k}}^0 - \sqrt{\frac{4\pi}{45}} \delta E_{2,\mathbf{k}}^0 + \sqrt{\frac{2\pi}{15}} (\delta E_{2,\mathbf{k}}^{+2} + \delta E_{2,\mathbf{k}}^{-2}),$$

$$\epsilon^{lj} \frac{\mathbf{k}^i \mathbf{k}^l}{\mathbf{k}^2} \tau^{4/3} \delta T_{\mathbf{k}}^{ij} = -i \sqrt{\frac{2\pi}{15}} (\delta E_{2,\mathbf{k}}^{+2} - \delta E_{2,\mathbf{k}}^{-2}),$$

$$\delta^{ij} \frac{i\mathbf{k}^i}{|\mathbf{k}|} \tau^{4/3} (-\tau) \delta T_{\mathbf{k}}^{\eta j} = -i \sqrt{\frac{2\pi}{15}} (\delta E_{2,\mathbf{k}}^{+1} - \delta E_{2,\mathbf{k}}^{-1}),$$

$$\epsilon^{ij} \frac{i\mathbf{k}^i}{|\mathbf{k}|} \tau^{4/3} (-\tau) \delta T_{\mathbf{k}}^{\eta j} = -\sqrt{\frac{2\pi}{15}} (\delta E_{2,\mathbf{k}}^{+1} + \delta E_{2,\mathbf{k}}^{-1}),$$

$$\tau^{4/3} \tau^2 \delta T_{\mathbf{k}}^{\eta\eta} = \sqrt{\frac{16\pi}{45}} \delta E_{2,\mathbf{k}}^0 + \sqrt{\frac{4\pi}{9}} \delta E_{0,\mathbf{k}}^0.$$

$$\tau \delta N_{a,\mathbf{k}}^\tau = \sqrt{4\pi} \delta N_{a0,\mathbf{k}}^0,$$

$$\delta^{ij} \frac{i\mathbf{k}^i}{|\mathbf{k}|} \tau \delta N_{a,\mathbf{k}}^j = -i \sqrt{\frac{2\pi}{3}} (\delta N_{a1,\mathbf{k}}^{+1} - \delta N_{a1,\mathbf{k}}^{-1}),$$

$$\epsilon^{ij} \frac{i\mathbf{k}^i}{|\mathbf{k}|} \tau \delta N_{a,\mathbf{k}}^j = -\sqrt{\frac{2\pi}{3}} (\delta N_{a1,\mathbf{k}}^{+1} + \delta N_{a1,\mathbf{k}}^{-1}),$$

$$\tau (-\tau) \delta N_{a,\mathbf{k}}^\eta = \sqrt{\frac{4\pi}{3}} \delta N_{a1,\mathbf{k}}^0.$$

Backup - Scale Factor

$$a(x) = 1 + \frac{\tau \partial_\tau T}{T} = 1 - \frac{\chi_u \chi_d \chi_s \tilde{a}(x) e + 3n_u^2 \chi_d \chi_s + 3n_d^2 \chi_u \chi_s + 3n_s^2 \chi_u \chi_d}{9n_u^2 \chi_d \chi_s + 9n_d^2 \chi_u \chi_s + 9n_s^2 \chi_u \chi_d - 4e \chi_u \chi_d \chi_s},$$

Susceptibilities

$$\tilde{a}(x) := \left[-\frac{4}{3} + b_{0,0}^0 E_0^0 + b_{0,+2}^0 \frac{E_2^0(x)}{E_0^0(x)} \right]$$

$$\chi_a = \frac{\nu_q}{6} \left(\frac{3\mu_a^2}{\pi^2} + T^2 \right)$$

Backup - Equations of Motion - Perturbed Energy Moments

$$\begin{aligned}
\tau \partial_\tau \delta E_{l,\mathbf{k}}^m &= b_{l,-2}^m \delta E_{l-2,\mathbf{k}}^m + b_{l,0}^m \delta E_{l,\mathbf{k}}^m + b_{l,+2}^m \delta E_{l+2,\mathbf{k}}^m \\
&\quad - \frac{i|\mathbf{k}|\tau}{2} \left[u_{l,-}^m \delta E_{l-1,\mathbf{k}}^{m+1} + u_{l,+}^m \delta E_{l+1,\mathbf{k}}^{m+1} + d_{l,-}^m \delta E_{l-1,\mathbf{k}}^{m-1} + d_{l,+}^m \delta E_{l+1,\mathbf{k}}^{m-1} \right] \\
&\quad - \frac{\tau}{\tau_R} \left[\delta E_{l,\mathbf{k}}^m + \frac{\delta T_{\mathbf{k}}}{T} (E_{\text{eq}}^{(1,0)})_l^m - \sum_a \delta \mu_{a,\mathbf{k}} \left[(E_{q_a,\text{eq}}^{(0,1)} + \bar{E}_{q_a,\text{eq}}^{(0,1)})_l^m \right] \right] \\
&\quad - \frac{\tau}{\tau_R} \frac{\delta T_{\mathbf{k}}}{T} \frac{T(\tau)}{\tau_R} \frac{\partial \tau_R}{\partial T} (E_{\text{eq}} - E)_l^m \\
&\quad - \frac{\tau}{\tau_R} \frac{\delta u_{\mathbf{k}}^\parallel}{2} \left[u_{l,-}^m (E_{\text{eq}} - E + E_{\text{eq}}^{(1,0)})_{l-1}^{m+1} + u_{l,+}^m (E_{\text{eq}} - E + E_{\text{eq}}^{(1,0)})_{l+1}^{m+1} \right. \\
&\quad \left. + d_{l,-}^m (E_{\text{eq}} - E + E_{\text{eq}}^{(1,0)})_{l-1}^{m-1} + d_{l,+}^m (E_{\text{eq}} - E + E_{\text{eq}}^{(1,0)})_{l+1}^{m-1} \right] \\
&\quad - \frac{\tau}{\tau_R} \frac{\delta u_{\mathbf{k}}^\perp}{2i} \left[u_{l,-}^m (E_{\text{eq}} - E + E_{\text{eq}}^{(1,0)})_{l-1}^{m+1} + u_{l,+}^m (E_{\text{eq}} - E + E_{\text{eq}}^{(1,0)})_{l+1}^{m+1} \right. \\
&\quad \left. - d_{l,-}^m (E_{\text{eq}} - E + E_{\text{eq}}^{(1,0)})_{l-1}^{m-1} - d_{l,+}^m (E_{\text{eq}} - E + E_{\text{eq}}^{(1,0)})_{l+1}^{m-1} \right],
\end{aligned}$$

Backup - Equations of Motion - Perturbed Charge Moments

$$\begin{aligned}
\tau \partial_\tau \delta N_{al,\mathbf{k}}^m &= B_{l,-2}^m \delta N_{al-2,\mathbf{k}}^m + B_{l,0}^m \delta N_{al,\mathbf{k}}^m + B_{l,+2}^m \delta N_{al+2,\mathbf{k}}^m \\
&\quad - \frac{i|\mathbf{k}|\tau}{2} \left[u_{l,-}^m \delta N_{al-1,\mathbf{k}}^{m+1} + u_{l,+}^m \delta N_{al+1,\mathbf{k}}^{m+1} + d_{l,-}^m \delta N_{al-1,\mathbf{k}}^{m-1} + d_{l,+}^m \delta N_{al+1,\mathbf{k}}^{m-1} \right] \\
&\quad - \frac{\tau}{\tau_R} \left[\delta N_{al,\mathbf{k}}^m + \frac{\delta T_{\mathbf{k}}}{T} (N_{a,\text{eq}}^{(1,0)})_l^m - \delta \mu_{a,\mathbf{k}} (N_{a,\text{eq}}^{(0,1)})_l^m \right] \\
&\quad - \frac{\tau}{\tau_R} \frac{\delta T_{\mathbf{k}}}{T} \frac{T(\tau)}{\tau_R} \frac{\partial \tau_R}{\partial T} (N_{a,\text{eq}} - N_a)_l^m \\
&\quad - \frac{\tau}{\tau_R} \frac{\delta u_{\mathbf{k}}^\parallel}{2} \left[u_{l,-}^m (N_{a,\text{eq}} - N_a + N_{a,\text{eq}}^{(1,0)})_{l-1}^{m+1} + u_{l,+}^m (N_{a,\text{eq}} - N_a + N_{a,\text{eq}}^{(1,0)})_{l+1}^{m+1} \right. \\
&\quad \quad \left. + d_{l,-}^m (N_{a,\text{eq}} - N_a + N_{a,\text{eq}}^{(1,0)})_{l-1}^{m-1} + d_{l,+}^m (N_{a,\text{eq}} - N_a + N_{a,\text{eq}}^{(1,0)})_{l+1}^{m-1} \right] \\
&\quad - \frac{\tau}{\tau_R} \frac{\delta u_{\mathbf{k}}^\perp}{2i} \left[u_{l,-}^m (N_{a,\text{eq}} - N_a + N_{a,\text{eq}}^{(1,0)})_{l-1}^{m+1} + u_{l,+}^m (N_{a,\text{eq}} - N_a + N_{a,\text{eq}}^{(1,0)})_{l+1}^{m+1} \right. \\
&\quad \quad \left. - d_{l,-}^m (N_{a,\text{eq}} - N_a + N_{a,\text{eq}}^{(1,0)})_{l-1}^{m-1} - d_{l,+}^m (N_{a,\text{eq}} - N_a + N_{a,\text{eq}}^{(1,0)})_{l+1}^{m-1} \right].
\end{aligned}$$