FINITE VOLUME EFFECTS IN THE EXTENDED LINEAR SIGMA MODEL VIA LOW MOMENTUM CUTOFF

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- System size: motivations and implementations
- Extended linear sigma model
- Results
- Summary and outlook

What are the typical sizes?

- Typical size of the fireball in heavy ion collisions is a few fm.
- Neutron stars and compact stars built up from strongly interacting matter (with extra structure) with a size ~ 10 km.
- Several models with finite (different) size.
- In field theoretical calculations (LSM, NJL, DS, etc) usually the size is infinite.

Why does it matter?

- It can be seen that the properties of the system can change significantly.
- One example: In the phase diagram of QCD the CEP (and the first order region) might disappear.

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- Specially for strongly interacting systems: Studying the volume effects in our models.

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Models with **finite volume**: Straightforward.

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Models with finite volume: Straightforward.



There are already results in HRG, (P)NJL, (P)LSM, DS, etc. calculations. For example for the phase diagram:

\mathbf{LSM}

Palhares, Fraga and Kodama, J. Phys. G **38**, 085101 (2011)

PNJL

Bhattacharyya, Deb, Ghosh, Ray and Sur, Phys. Rev. D 87, no.5, 054009 (2013)

QM model FRG

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DS approach

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EXTENDED LINEAR SIGMA MODEL

Vector and axial vector meson Extended Polyakov Linear Sigma Model. Effective model to study the phase diagram of strongly interacting matter at finite T and μ . Phys. Rev. D 93, no. 11, 114014 (2016)

- Linear Sigma Model: "simple" quark-meson model
- Extended: Vector and Axial vector nonets (besides to Scalar and Pseudoscalar) Isospin symmetric case: 16 mesonic degrees of freedom.
- Polyakov: Polyakov loop variables give 2 order parameters Φ , $\overline{\Phi}$.
- The mesonic Lagrangian \mathcal{L}_m with chiral symmetry

 $SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A \to SU(2)_I \times U(1)_V$

broken explicitly (and spontaneously) and with the axial anomaly taken into account

ELSM

- \mathcal{L}_m contains the dynamical, the symmetry breaking, and the meson-meson interaction terms.
 - $U(1)_A$ anomaly and explicit breaking of the chiral symmetry.
 - Each meson-meson terms up to 4th order that are allowed by the chiral symmetry.
- Constituent quarks $(N_f = 2 + 1)$ in Yukawa Lagrangian

$$\mathcal{L}_Y = \bar{\psi} \left(i \gamma^\mu \partial_\mu - g_F (S - i \gamma_5 P) - g_V \gamma^\mu (V_\mu + \gamma_5 A_\mu) \right) \psi \tag{1}$$

In the 2016 version $g_V = 0$ was used.

Phys. Rev. D 104, 056013 (2021)

- SSB with nonzero vev. for scalar-isoscalar sector ϕ_N , ϕ_S . $\Rightarrow m_{u,d} = \frac{g_F}{2} \phi_N$, $m_s = \frac{g_F}{\sqrt{2}} \phi_S$ fermion masses in \mathcal{L}_Y .
- Mean field level effective potential \rightarrow the meson masses and the thermodynamics are calculated from this.

The grand potential

Thermodynamics: Mean field level effective potential:

- Classical potential.
- Fermionic one-loop correction with vanishing fluctuating mesonic fields.

$$\bar{\psi} \left(i \gamma^{\mu} \partial_{\mu} - \operatorname{diag}(m_u, m_d, m_s) \right) \psi$$

Functional integration over the fermionic fields. The momentum integrals are renormalized.

• Polyakov loop potential.

$$\Omega(T,\mu_q) = U_{Cl} + \operatorname{tr} \int_K \log\left(iS_0^{-1}\right) + U(\Phi,\bar{\Phi}) \tag{2}$$

Field equations (FE):

$$\frac{\partial\Omega}{\partial\bar{\Phi}} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\phi_N} = \frac{\partial\Omega}{\partial\phi_S} = 0 \tag{3}$$

Curvature meson masses:

$$M_{ab}^{2} = \left. \frac{\partial^{2} \Omega}{\partial \varphi_{a} \partial \varphi_{b}} \right|_{\{\varphi_{i}\}=0} \tag{4}$$

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Functional integration over the fermionic fields.

The momentum integrals are renormalized. Low momentum cut!

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The model parameters are fitted with χ^2 method, using ~ 30 physical quantities like meson masses, decay widths or the pion decay constant.

The parameters:

 $m_0^2, m_1^2, g_1, g_2, \lambda_1, \lambda_2, h_1, h_2, h_3, \delta_S, c, g_F, h_N, h_S$

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Instead of the external fields h_N, h_S one may use the meson condensates: ϕ_N, ϕ_S The field equations directly connect ϕ_N with h_N and ϕ_S with h_S .

 $\Rightarrow \phi_N$ and ϕ_S are in the parametrization while h_N and h_S are calculated with the FEs

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In the finite volume calculation

Fix h_N , h_S at $L = \infty$ Fix ϕ_N , ϕ_S at $L = \infty$ Calculate h_N , h_S with FEs Calculate ϕ_N , ϕ_S with FEs \rightarrow Fixed external fields with volume dependent f_{π} and m_q .

RESULTS

Meson and constituent quark masses



If the external fields are fixed $\phi_{N,S}$ scales with the size below ~ 5 fm.





The scaling of the pion mass shift $R = (m_{\pi}(L) - m_{\pi}(\infty)) / m_{\pi}(\infty)$ compared to the results shown in Phys. Rept. 707-708, 1-51 (2017)









SUMMARY AND OUTLOOK

- Finite volume effects on thermodynamics and the phase diagram of strong interaction was studied via a low momentum cutoff.
- The meson masses and other physical quantities start to significantly change with the system size under ~ 5 fm.
- A decreasing trend in the pseudocritical temperature (using fixed external fields) was found.
- The critical end point moves to lower temperature and higher chemical potential with the decreasing size.
- The CEP and the first order region disappear at a small finite size (~ 2.5 fm for fixed external fields).
- Further study of the thermodynamics.
- Further study of the scaling of physical quantities.
- Open questions about the phase diagram.

THANK YOU!



BACKUP: LAGRANGIAN

The Mesonic Lagrangian built up from the

$$L^{\mu} = \sum_{a} \left(V_{a}^{\mu} + A_{a}^{\mu} \right) T_{a}, \quad R^{\mu} = \sum_{a} \left(V_{a}^{\mu} - A_{a}^{\mu} \right) T_{a}, \quad M = \sum_{a} \left(S_{a} + i P_{a} \right) T_{a},$$

fields as

$$\mathcal{L}_{m} = + \operatorname{Tr}\left[\left(D_{\mu}M\right)^{\dagger}\left(D^{\mu}M\right)\right] - m_{0}\operatorname{Tr}\left(M^{\dagger}M\right) - \lambda_{1}\left[\operatorname{Tr}\left(M^{\dagger}M\right)\right]^{2} - \lambda_{2}\left[\operatorname{Tr}\left(M^{\dagger}M\right)^{2}\right] + c\left(\det M + \det M^{\dagger}\right)^{2} + \operatorname{Tr}\left[H\left(M + M^{\dagger}\right)\right] - \frac{1}{4}\operatorname{Tr}\left[L_{\mu\nu}L^{\mu\nu} + R_{\mu\nu}R^{\mu\nu}\right] + \operatorname{Tr}\left[\left(\frac{m_{1}^{2}}{2} + \Delta\right)\left(L_{\mu}L^{\mu} + R_{\mu}R^{\mu}\right)\right] + \frac{h_{1}}{2}\operatorname{Tr}\left(\phi^{\dagger}\phi\right)\operatorname{Tr}\left[L_{\mu}L^{\mu} + R_{\mu}R^{\mu}\right] + h_{2}\operatorname{Tr}\left[\left(MR_{\mu}\right)^{\dagger}\left(MR^{\mu}\right) + \left(L_{\mu}M\right)^{\dagger}\left(L^{\mu}M\right)\right] + 2h_{3}\operatorname{Tr}\left[R_{\mu}M^{\dagger}L^{\mu}M\right] - 2g_{2}\operatorname{Tr}\{L_{\mu\nu}\left[L^{\mu}, L^{\nu}\right]\} + \operatorname{Tr}\{R_{\mu\nu}\left[R^{\mu}, R^{\nu}\right] + g_{4}\{\operatorname{Tr}\left[L^{\mu}L_{\mu}L^{\nu}L_{\nu}\right] + \operatorname{Tr}\left[R^{\mu}R_{\mu}R^{\nu}R_{\nu}\right]\} + g_{5}\operatorname{Tr}\left[R^{\mu}R_{\mu}\right]\left[L^{\nu}L_{\nu}\right] + g_{6}\{\operatorname{Tr}\left[L^{\mu}L_{\mu}\right]\left[L^{\nu}L_{\nu}\right] + \operatorname{Tr}\left[R^{\mu}R_{\mu}\right]\left[R^{\nu}R_{\nu}\right]\},$$

where

$$\begin{split} D^{\mu} &= \partial^{\mu} M - ig_1(L_{\mu}M - MR_{\mu}) - ieA^{\mu} \left[T_3, M\right] \\ L^{\mu\nu} &= \partial^{\mu} L^{\nu} - ieA^{\mu} \left[T_3, L^{\nu}\right] - \left\{\partial^{\nu} L^{\mu} - ieA^{\nu} \left[T_3, L^{\mu}\right]\right\} \\ R^{\mu\nu} &= \partial^{\mu} R^{\nu} - ieA^{\mu} \left[T_3, R^{\nu}\right] - \left\{\partial^{\nu} R^{\mu} - ieA^{\nu} \left[T_3, R^{\mu}\right]\right\} \end{split}$$

BACKUP: FIELD EQUATIONS

The explicit form of the field equations (FEs)

$$-\frac{d}{d\Phi}\left(\frac{U\left(\Phi,\bar{\Phi}\right)}{T^{4}}\right) + \frac{6}{T^{3}}\sum_{f}\int\frac{d^{3}p}{(2\pi)^{3}}\left(\frac{e^{-\beta E_{f}^{-}(p)}}{g_{f}^{-}(p)} + \frac{e^{-2\beta E_{f}^{+}(p)}}{g_{f}^{+}(p)}\right) = 0$$
$$-\frac{d}{d\bar{\Phi}}\left(\frac{U\left(\Phi,\bar{\Phi}\right)}{T^{4}}\right) + \frac{6}{T^{3}}\sum_{f}\int\frac{d^{3}p}{(2\pi)^{3}}\left(\frac{e^{-\beta E_{f}^{+}(p)}}{g_{f}^{+}(p)} + \frac{e^{-2\beta E_{f}^{-}(p)}}{g_{f}^{-}(p)}\right) = 0$$
(5)
$$m_{0}^{2}\phi_{N} + \left(\lambda_{1} + \frac{\lambda_{2}}{2}\right)\phi_{N}^{3} + \lambda_{1}\phi_{N}\phi_{S}^{2} - \frac{c}{\sqrt{2}}\phi_{N}\phi_{S} - h_{0N} + \frac{1}{2}g_{F}\sum_{l=u,d}\langle\bar{q}_{l}q_{l}\rangle_{T} = 0$$
$$m_{0}^{2}\phi_{S} + (\lambda_{1} + \lambda_{2})\phi_{S}^{3} + \lambda_{1}\phi_{N}^{2}\phi_{S} - \frac{\sqrt{2}c}{4}\phi_{N}^{2} - h_{0S} + \frac{1}{\sqrt{2}}g_{F}\langle\bar{q}_{s}q_{s}\rangle_{T} = 0$$

where

$$\langle \bar{q}_{f}q_{f}\rangle_{T} = 4N_{c}m_{f}\left[-\frac{m_{f}^{2}}{16\pi^{2}}\left(\frac{1}{2}+\ln\frac{m_{f}^{2}}{M_{0}^{2}}\right)+T_{f}
ight]$$
(6)

The curvature meson masses are calculated from the grand potential.

For the (pseudo)scalars: **Tree-level** + **fermionic vacuum** + **fermionic matter**

$$M_{ab}^{2} = \frac{\partial^{2}\Omega}{\partial\varphi_{a}\partial\varphi_{b}} = m_{ab}^{2} + \Delta m_{ab}^{2} + \delta m_{ab}^{2} \tag{7}$$

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For the (axial) vectors only tree-level contribution

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Tree-level : S-V and P-A mixing in the quadratic (after SSB) part of the Lagrangian

Shift the A/V fields and a "wavefunction renormalization factor" for the P/S fields. \Rightarrow The S/P masses get an extra factor $M^2 \rightarrow Z^2 M^2$. The fremionic vacuum part with cutoff and renormalization:

$$\begin{split} \Omega_{\rm ferm}^V &= -2N_c \sum_{f=u,d,s} \int \frac{d^3 p}{(2\pi)^3} E_f(p) \Theta(\Lambda - p) \Theta(p - \lambda) = -6 \frac{4\pi}{(2\pi)^3} \sum_{f=u,d,s} \int_{\lambda}^{\Lambda} dp \; p^2 E_f(p) \\ &= -\frac{9}{4\pi^2} \Lambda^4 - \frac{3g^2}{8\pi^2} \left(\phi_N^2 + \phi_S^2 \right) \Lambda^2 + \frac{3g^2}{64\pi^2} \log \left(2\Lambda e^{-1/4} \right) \left(\phi_N^4 + \phi_S^4 \right) \\ &- \frac{3}{8\pi^2} \sum_{f=u,d,s} \left[m_f^4 \log \left(\lambda + \sqrt{\lambda^2 + m_f^2} \right) - \lambda \sqrt{\lambda^2 + m_f^2} \left(2\lambda^2 + m_f^2 \right) \right] \\ &\xrightarrow{R} - \frac{3}{8\pi^2} \sum_{f=u,d,s} \left[m_f^4 \log \left(\frac{\lambda + \sqrt{\lambda^2 + m_f^2}}{M_0} \right) - \lambda \sqrt{\lambda^2 + m_f^2} \left(2\lambda^2 + m_f^2 \right) \right], \end{split}$$
(8)

$$\int \frac{d^3 p}{(2\pi)^3} \mathcal{I}(E) \Theta(p-\lambda) = \frac{4\pi}{(2\pi)^3} \int_{\lambda}^{\infty} dp \ p^2 \mathcal{I}(E) = \frac{4\pi}{(2\pi)^3} \int_{\sqrt{\lambda^2 + m^2}}^{\infty} dE \frac{p}{E} \mathcal{I}(E)$$
(9)



Figure: The integrand for cquad with m = 1, T = 0.1 and $\lambda = 0.0$, 0.2 GeV respectively.

$$\Omega_{\bar{q}q}^{(0)m}(T,\mu) = -2T \int \frac{d^3p}{(2\pi)^3} \left[\log g_f^+(p) + \log g_f^-(p) \right] \Theta(p-\lambda) = -\frac{T}{\pi^2} \int_{\lambda}^{\infty} dp p^2 \left[\log g_f^+(p) + \log g_f^-(p) \right]$$
(10)