Corrections to the hadron resonance gas from lattice QCD and their effect on fluctuation-ratios at finite density

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in collaboration with

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The Hadron Resonance Gas (HRG) model

Importance:

- parameters of chemical freeze-out in experiments,
- non-critical baseline.

HRG: interacting gas of hadrons $\cong$ non-interacting gas of hadrons *and* resonances.

$$\frac{p}{T^4} = \frac{1}{T^4}\sum_{\mathrm{h}} p_{\mathrm{h}} = \frac{1}{VT^3}\sum_{\mathrm{h}} \log \mathcal{Z}_{\mathrm{h}}(T, \boldsymbol{\mu} = (\mu_B, \mu_Q, \mu_S))$$

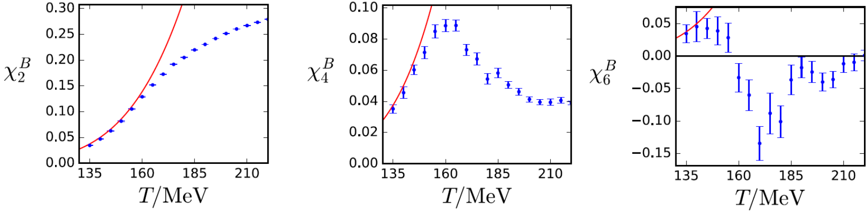
with

$$\begin{aligned}\log \mathcal{Z}_{\mathrm{h}} &= \mp \frac{d_{\mathrm{h}}}{2\pi^2}\frac{V}{T^3}\int_0^\infty \mathrm{d}p\, p^2 \log\left[1 \mp z_{\mathrm{h}} \exp\left(-\frac{\sqrt{m_{\mathrm{h}}^2 + p^2}}{T}\right)\right] \\ &= VTm_{\mathrm{h}}^2 \frac{d_{\mathrm{h}}}{2\pi^2}\sum_{n=1}^{\infty}\frac{(\pm 1)^{n+1}}{n^2} z_{\mathrm{h}}^n K_2\left(\frac{n m_{\mathrm{h}}}{T}\right)\end{aligned}$$

and fugacity factors: $z_{\mathrm{h}} = \exp[\beta(B_{\mathrm{h}}\mu_B + Q_{\mathrm{h}}\mu_Q + S_{\mathrm{h}}\mu_S)]$.

Discrepancies of HRG and the lattice

Mostly good match for $\boldsymbol{\mu} = (0, 0, 0)$ in the hadronic phase:



[WB: 1805.04445]
[Pisa: 1611.08285]
[HotQCD: 2001.08530]

Possible reasons for the deviation:

- incomplete resonance list,
- finite widths are missing,
- lack of non-resonant and repulsive interactions,
- restoration of chiral symmetry.

Generalised susceptibilities:

$$\chi_{lmn}^{BQS} = \frac{\partial^{l+m+n}(p/T^4)}{\partial\hat{\mu}_B^l \partial\hat{\mu}_Q^m \partial\hat{\mu}_S^n} ,$$

with $\hat{\mu}_i = \mu_i/T$.

Fugacity expansion of the QCD free energy

Fugacity expansion:

$$\frac{p(T,\hat{\mu}_B,\hat{\mu}_S)}{T^4} = \sum_{j,k} P^{BS}_{jk}(T)\cosh(j\hat{\mu}_B - k\hat{\mu}_S)$$

with $\mu_Q = 0$.

Sector coefficients: P^{BS}_{jk},

- contributions from Hilbert subspaces with fixed $B = j, S = k$;
- e.g. (at $\boldsymbol{\mu} = 0$)

$$\chi^B_4 = \sum_k P^{BS}_{1k} + \sum_k 32P^{BS}_{2k} + \sum_k 81P^{BS}_{3k} + \ldots$$

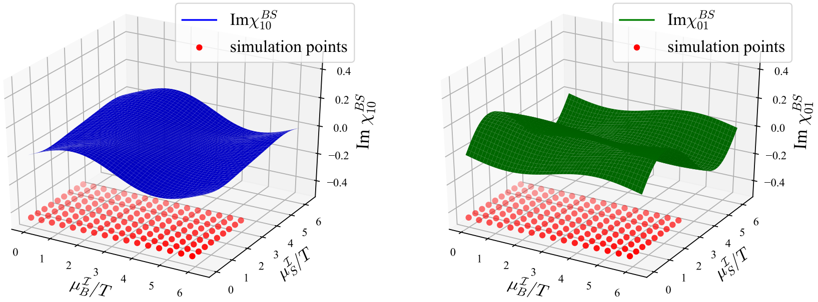
(B,S)	(0,0)	(1,0)	(0,1)	(1,1)	(1,2)	(2,0)	(0,2)	(2,1)
hadrons	π,η,ρ	p,Δ	K	Λ,Σ	Ξ	p-p	K-K	p-Λ, p-p-K

- (main) contributions to ideal-HRG from $P^{BS}_{00}, P^{BS}_{01}, P^{BS}_{10}, P^{BS}_{11}, P^{BS}_{12}$ and P^{BS}_{13},
- for $B \geq 2$ coefficients are close to zero $\sim$ deviation!

Lattice setup

- $N_\mathrm{f} = 2+1$; 4stout-improved staggered action. `[WB: 1507.04627]`
- $\boldsymbol{\mu} = i\boldsymbol{\mu}^{\boldsymbol{\mathcal{I}}} \sim \cosh(ix) = \cos(x) \sim$ Fourier series.
- $N_\tau = 8, 10, 12$; $LT \approx 3$; $T = 145\ \mathrm{MeV}, 150\ \mathrm{MeV}, 155\ \mathrm{MeV}$ and $160\ \mathrm{MeV}$.

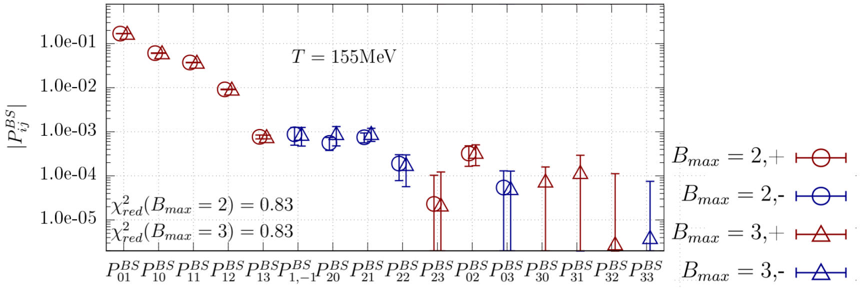
$$\mathrm{Im}\chi_{10}^{BS} = \sum_{j,k} j P_{jk}^{BS}(T)\sin(j\hat{\mu}_B^{\mathcal{I}} - k\hat{\mu}_S^{\mathcal{I}}) \qquad \mathrm{Im}\chi_{01}^{BS} = \sum_{j,k} (-k) P_{jk}^{BS}(T)\sin(j\hat{\mu}_B^{\mathcal{I}} - k\hat{\mu}_S^{\mathcal{I}})$$



Example: $T = 155$ MeV.

Sectors

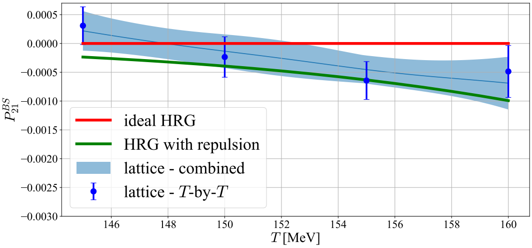
$N_\tau = 12$:



- Hierarchy: $P^{BS}_{01} > P^{BS}_{02} > P^{BS}_{03}$; $P^{BS}_{10} > P^{BS}_{20} > P^{BS}_{30}$; $P^{BS}_{10} > P^{BS}_{12} > P^{BS}_{13}$; etc.
- $B = 2$ sector: $P^{BS}_{2k} < 0$, and heavily underestimated in ideal HRG.

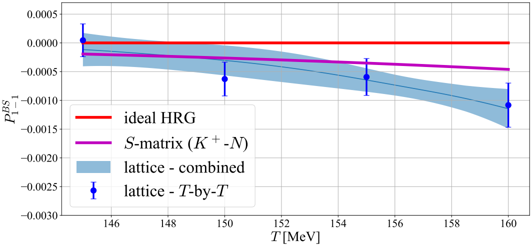
Systematic errors $\sim B_{\max} = 2$ or 3.

Subleading sectors – examples



P_{21}^{BS}

- more negative for higher T,
- e.g. N-Λ or N-Σ interactions,
- ideal HRG deviates,
- repulsive mean field HRG,
  [1708.00879]
- excluded volume HRG.
  [1708.02852]



P_{1-1}^{BS}

- e.g. N-K^+ scattering,
- S-matrix formalism applicable.
  [1806.02177]

Fluctuation ratios at finite density

Extrapolation to finite μ_B using truncated

$$\frac{p}{T^4} = \sum_{j,k} P^{BS}_{jk}(T) \cosh\left[j\hat{\mu}_B - k\hat{\mu}_S(\hat{\mu}_B)\right]$$

with **strangeness neutrality**:
$\chi_1^S = \sum_{j,k}(-k)P^{BS}_{jk}(T)\sinh(j\hat{\mu}_B - k\hat{\mu}_S) \stackrel{!}{=} 0.$

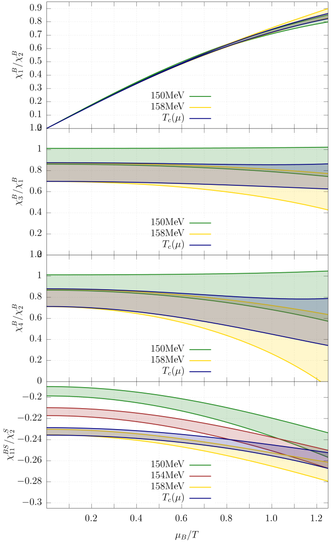
Ratios:

- $\chi_1^B/\chi_2^B \sim$ proxy for μ_B,
- χ_3^B/χ_1^B, $\chi_4^B/\chi_2^B \sim$ baryon thermometer,
- $\chi_{11}^{BS}/\chi_2^S \sim$ strangeness thermometer

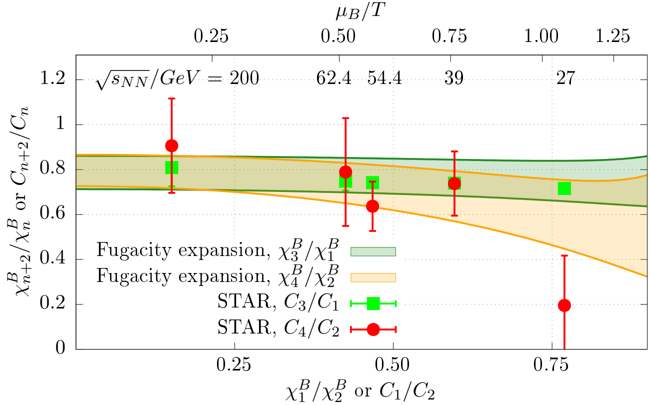
and

experimental proxy: $\sigma_\Lambda^2/(\sigma_\Lambda^2 + \sigma_K^2)$.
[WB: 1910.14592]

Crossover line: $T_c(\mu_B) \approx T_c^0(1 - \kappa_2\hat{\mu}_B^2)$
[WB: 2002.02821]



Comparison with experiment



- Consistent with: `[HotQCD: 2001.08530]`, `[WB: 1805.0444]`.
- Using C_i net-proton cumulants from STAR Experiment.
- Assuming that the crossover and chemical freeze-out lines are close to each other.
- Ignoring ceaveats of comparison.

Summary and conclusion

- Scanning of the QCD free energy in imaginary μ_B-μ_S plane $\sim$ separation of sectors.
- Possible separation of processes like K-K or p-p scattering.
- Continuum estimates for P_{jk}^{BS} sector coefficients and fluctuation ratios.
- Consistency with STAR data for small μ_B.
- Deviation of χ_4^B/χ_2^B from ideal HRG is due to $B = 2$ effects.
- Our lattice results could provide help in
  1. phenomenology,
  2. construction of more realistic models.

Thank you for your attention.

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