Coupling jets to hydro: multiple scatterings and temperature gradients

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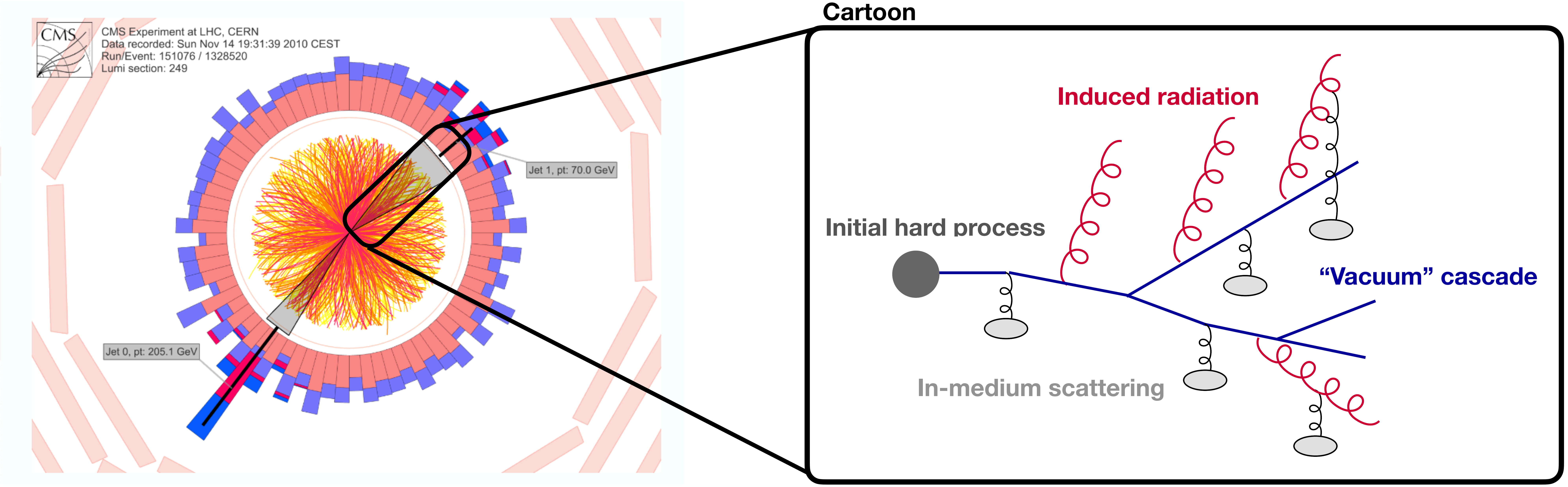
10th of December 2021, Zimányi School

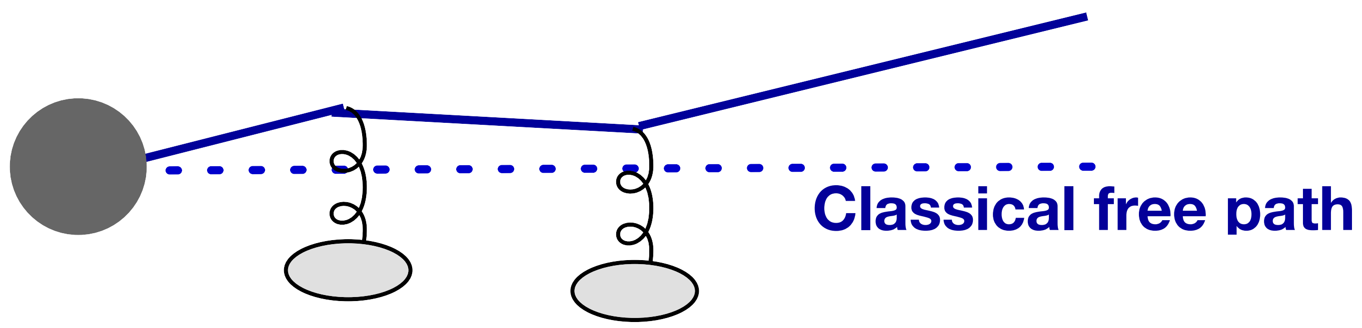
based on arXiv: 2112.xxxx with A. Sadofyev, C. Salgado

Jets in Heavy Ion Collisions



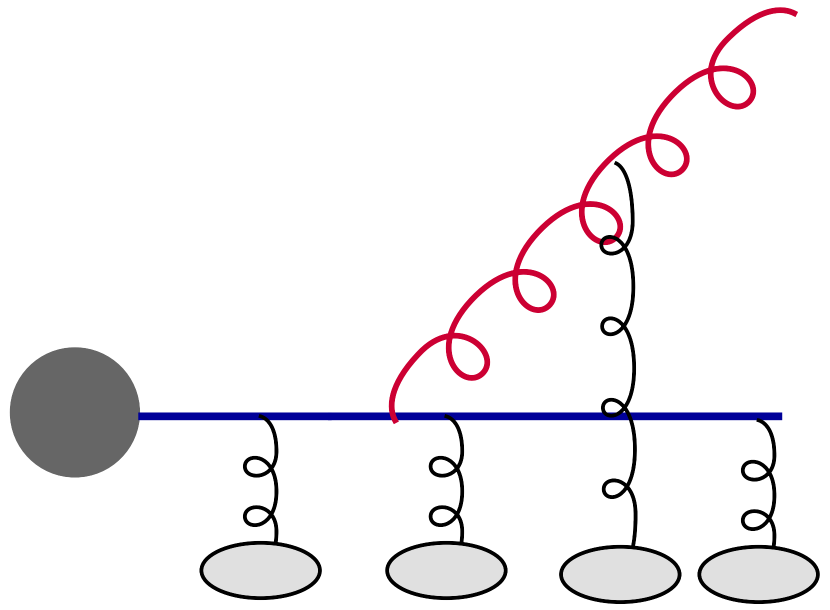
Computation of such processes is complicated; major focus on observables up to $\mathcal{O}(\alpha_s(Q_\perp \gg T))$

Single particle broadening $\mathcal{O}(\alpha_s^0)$



Today

Medium induced single gluon radiation $\mathcal{O}(\alpha_s)$



Jets in Heavy Ion Collisions

There are many assumptions going into these type of computations. Some are:

- The leading parton is assumed to be eikonal
- Recoiless background admits to be treated classically (stochatiscally)
- Medium is assumed to be static, homogeneous and infinitely long

Today

A recent extension showed how to treat anisotropic flowing media in the dilute regime

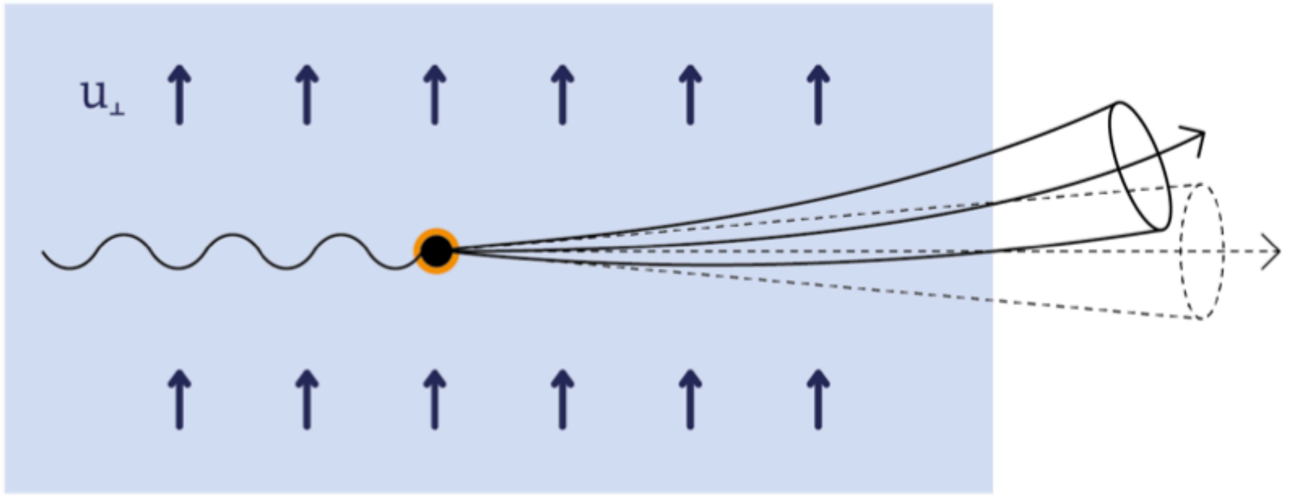
Today

2104.09513, A. Sadofyev, M. Sievert, I. Vitev

Such effects are sub-eikonal but enhanced by medium scale or are leading order effect

$$\langle \mathbf{p} \rangle_{\mathbf{u}\neq 0, \nabla \mathrm{T}=0} \propto \frac{u_\perp}{1-u_z} \frac{\mu^2 L}{E\lambda}$$

$$\langle \mathbf{p}\, \mathbf{p}^2 \rangle_{\mathbf{u}=0, \nabla \mathrm{T}\neq 0} \propto \left(\frac{\nabla T}{T} L \right) \frac{\mu^2 L}{E\lambda}$$



2104.09513, A. Sadofyev, M. Sievert, I. Vitev

Broadening in anisotropic non-flowing media

+

Dense regime:
Multiple medium-probe interactions

Today

Overview

1 The Opacity Expansion and BDMPS-Z formalisms

2 Broadening in a dense anisotropic medium

2.1 Opacity Expansion approach

2.2 BDMPS-Z approach

3 Final particle distribution

1 Medium model

More details in for example:

2104.09513, A. Sadofyev, M. Sievert, I. Vitev

1807.03799, M. Sievert, I. Vitev

The field generated by the medium reads

$$gA^{\lambda a}_{\text{ext}}(q) = -(2\pi)\, g^{\lambda 0} \sum_i e^{-i(\boldsymbol{q}\cdot\boldsymbol{x}_i + q_z z_i)}\, t^a_i\, \underline{v_i(q)}\, \underline{\delta\left(q^0\right)}$$

Model dependent elastic scattering potential for source i

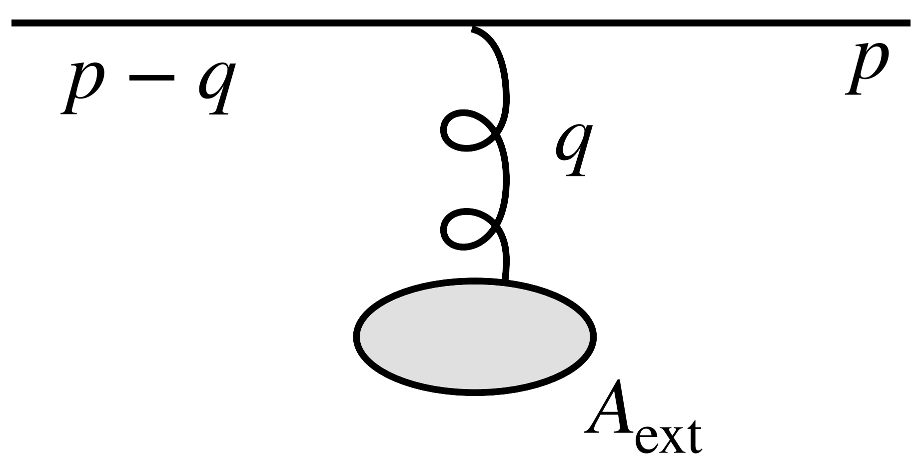
No energy transfer in each scattering: transverse t-channel gluon exchanges only

We consider the Gyulassy-Wang model for the potential

PRL 68, 1480 X.-N. Wang, M. Gyulassy

$$v_i(q) \equiv \frac{-g^2}{-q_0^2 + \boldsymbol{q}^2 + q_z^2 + \underline{\mu_i^2} - i\epsilon}$$

Debye mass for gluon i

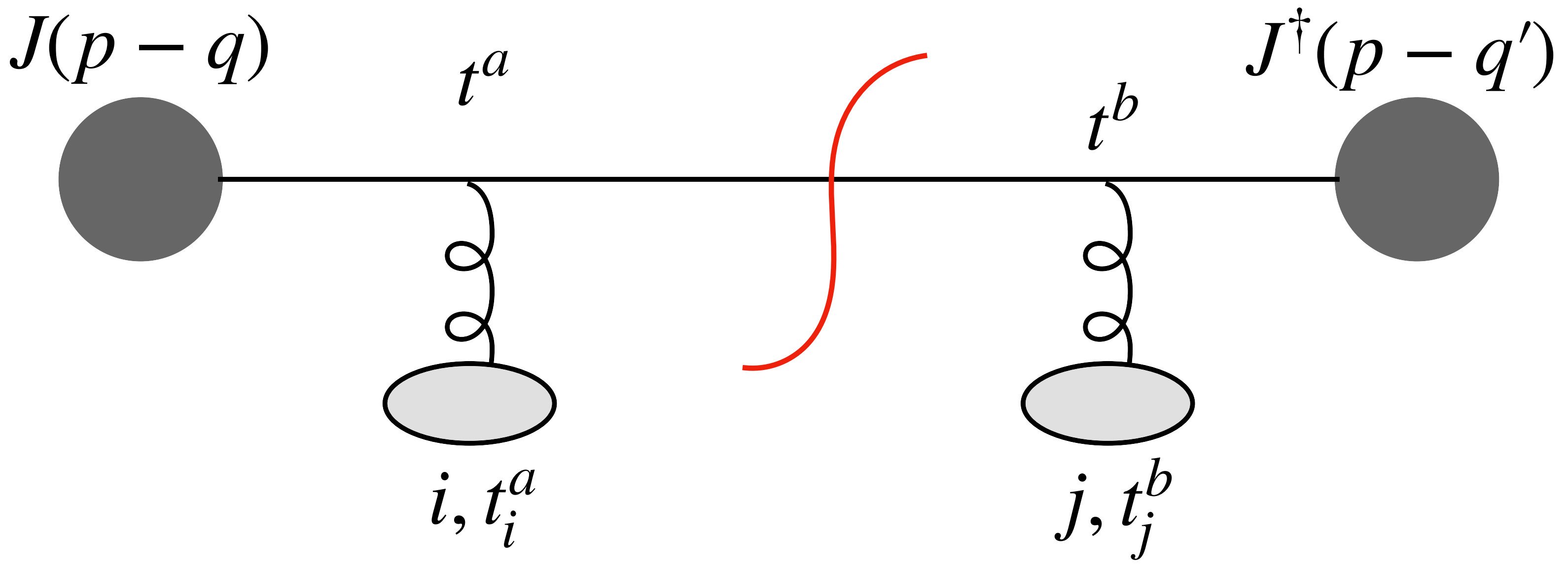


We further assume that the interactions satisfy color neutrality in the 2-gluon approximation

$$\underline{\left\langle t^a_i t^b_j \right\rangle} = \frac{1}{d_{\text{tgt}}} \text{tr}\left(t^a_i t^b_j\right) = \frac{1}{2C_{\bar{R}}} \underline{\delta_{ij}}\delta^{ab}$$

Only non-trivial correlator

Probe interacts with the same scattering center in amplitude and conjugate amplitude



1 The Opacity Expansion: single particle broadening

More details in for example:

nucl-th/9306003, M. Gyulassy, X.-N. Wang

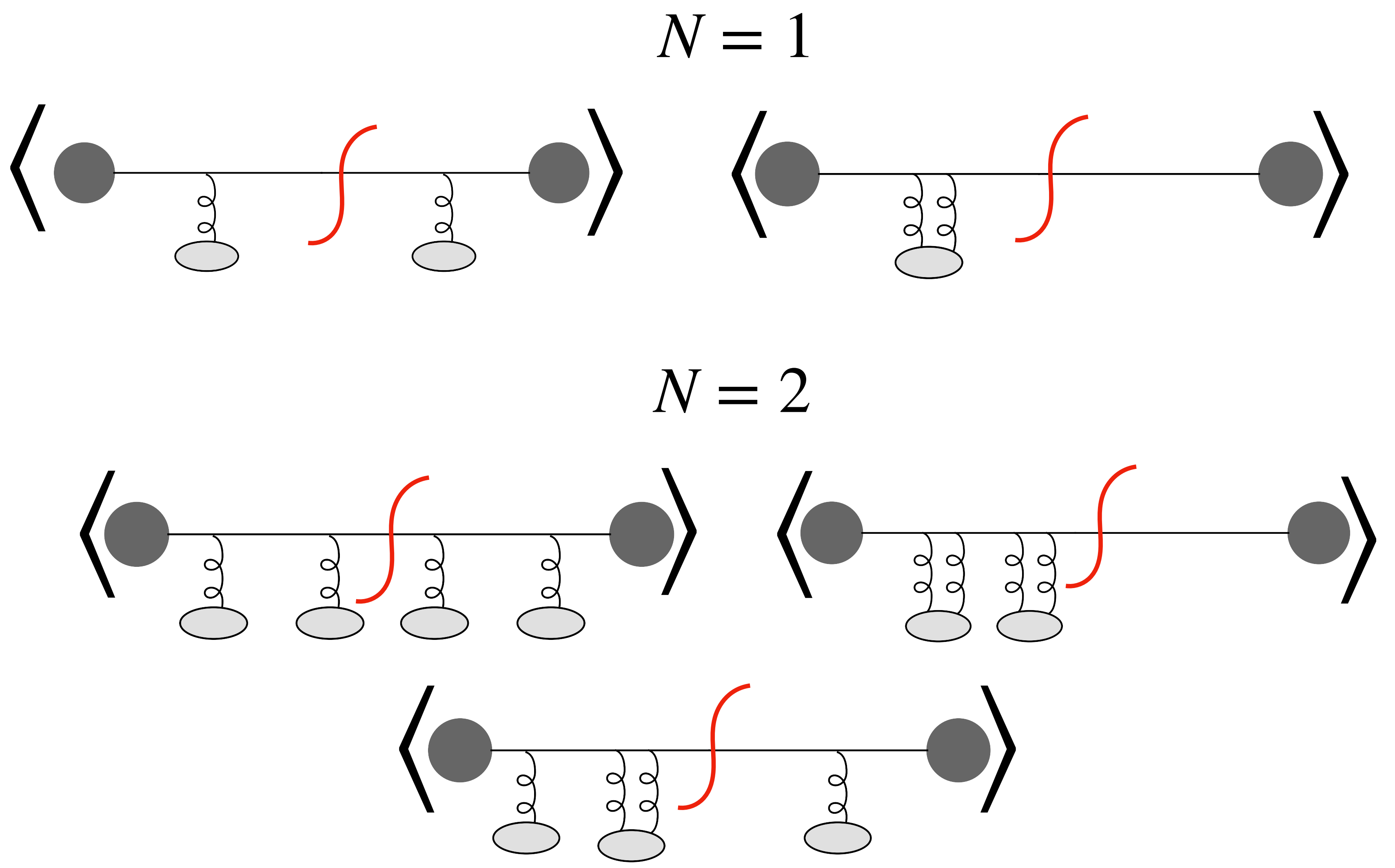
In the simple case of homogeneous media, the particle distribution can be computed as follows

1 Compute all diagrams up to $2N$ field insertions

For example, the diagram with $r = N$ insertions at distinct i_n reads $\quad Q_n \equiv \frac{\boldsymbol{p}_n^2 - \boldsymbol{p}_f^2}{2E}$

$$iM_r = \prod_{n=1}^{r} \left[\sum_{i_n} \int \frac{d^2\boldsymbol{q}_n}{(2\pi)^2} \, i t^a_{\text{proj}} t^a_{i_n} \, \theta_{i_n, i_{n-1}} \, e^{-i\boldsymbol{q}_n \cdot \boldsymbol{x}_{i_n}} \underbrace{e^{-iQ_n\left(z_{i_n} - z_{i_{n-1}}\right)}}_{\text{LPM phase factor}} v_{i_n}(q_n) \right] J\left(p_{in}\right)$$

$$p_n = p_f - \sum_{m=n}^{N} q_m, \; p_{in} = p_1$$



2 For each N, square and average the respective diagrams

The full squared amplitude is then obtained by summing over all N

$$\langle |M|^2 \rangle = \underbrace{\langle |M_0|^2 \rangle}_{N=0} + \underbrace{\langle |M_1|^2 \rangle + \langle M_2 M_0^* \rangle + \langle M_0 M_2^* \rangle}_{N=1} + \underbrace{\langle |M_2|^2 \rangle + \langle M_3 M_1^* \rangle + \langle M_1 M_3^* \rangle + \langle M_4 M_0^* \rangle + \langle M_0 M_4^* \rangle}_{N=2} + \ldots$$

The averaging is performed by taking the limit of continuous distribution in the medium

$$\sum_i f_i = \int d^2\boldsymbol{x} \, dz \, \rho(\boldsymbol{x}, z) \, f(\boldsymbol{x}, z) \xrightarrow[\nabla\rho = 0]{} \int d^2\boldsymbol{x}_n \, e^{-i(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)\cdot \boldsymbol{x}_n} = (2\pi)^2 \, \delta^{(2)}(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)$$

1 The Opacity Expansion: single particle broadening

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In the simple case of homogeneous media, the particle distribution can be computed as follows

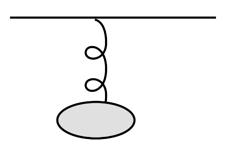
3 Resum the Opacity Series

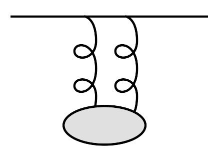
A detailed derivation shows that the square amplitude for $2N$ insertions has the form

$$\langle |M|^2 \rangle^{(N)} = \prod_{n=1}^{N} \left[(-1) \int_0^{z_{n+1}} dz_n \int \frac{d^2 \boldsymbol{q}_n}{(2\pi)^2} \mathcal{V}(\boldsymbol{q}_n, z_n) \right] |J(E, \boldsymbol{p}_{in})|^2$$

where we identify the effective scattering potential

$$\mathcal{V}(\boldsymbol{q}, z) \equiv -\mathcal{C}\, \rho(z) \left(\left| v(\boldsymbol{q}^2) \right|^2 - \delta^{(2)}(\boldsymbol{q}) \int d^2 \boldsymbol{l}\, \left| v(\boldsymbol{l}^2) \right|^2 \right)$$





The resummation in this case is simple:

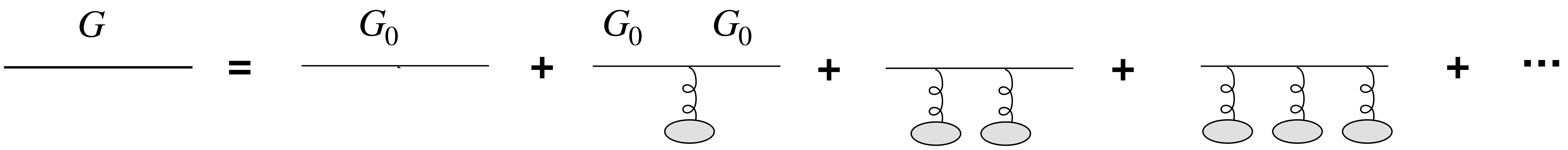
$$\frac{d\mathcal{N}}{d^2\boldsymbol{x} dE} = \sum_{N=0}^{\infty} \int \frac{d^2\boldsymbol{p}\, d^2\boldsymbol{r}}{(2\pi)^2} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{r})} \frac{(-1)^N \left[\mathcal{V}(\boldsymbol{r}) L\right]^N}{N!} \frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{r} dE} = e^{-\mathcal{V}(\boldsymbol{x})L} \frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x} dE}$$

The BDMPS-Z approach: single particle broadening

More details in for example:
1302.2579, Y. Mehtar-Tani, J. Milhano, K. Tywoniuk

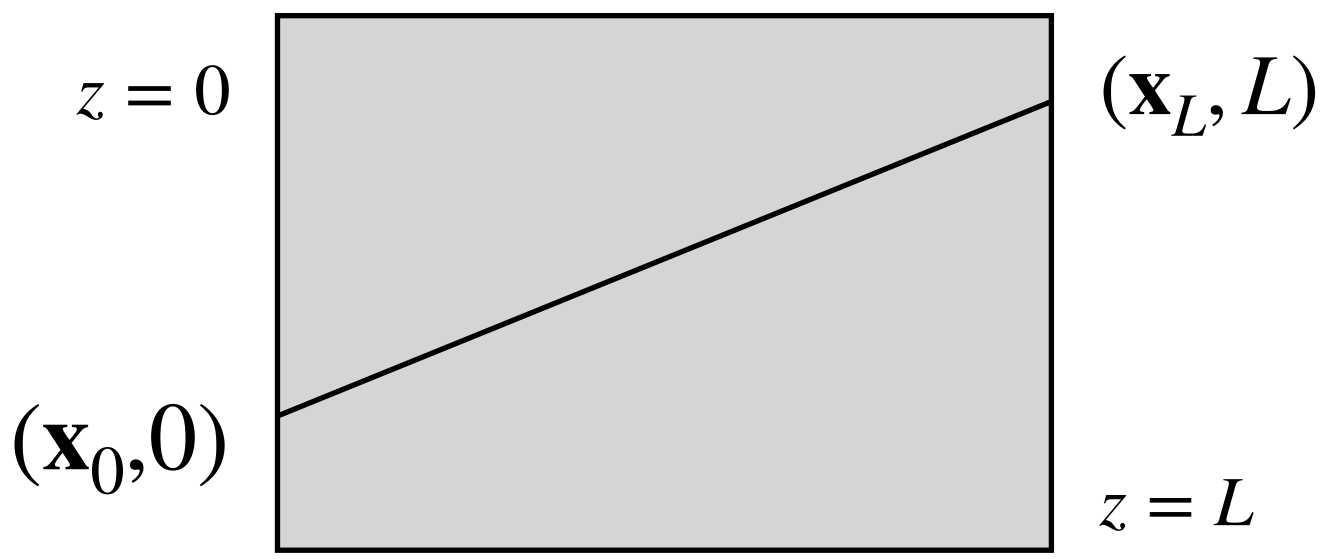
One can try to perform the resummation already at amplitude level. In this case the steps are

1 Compute an effective in-medium propagator



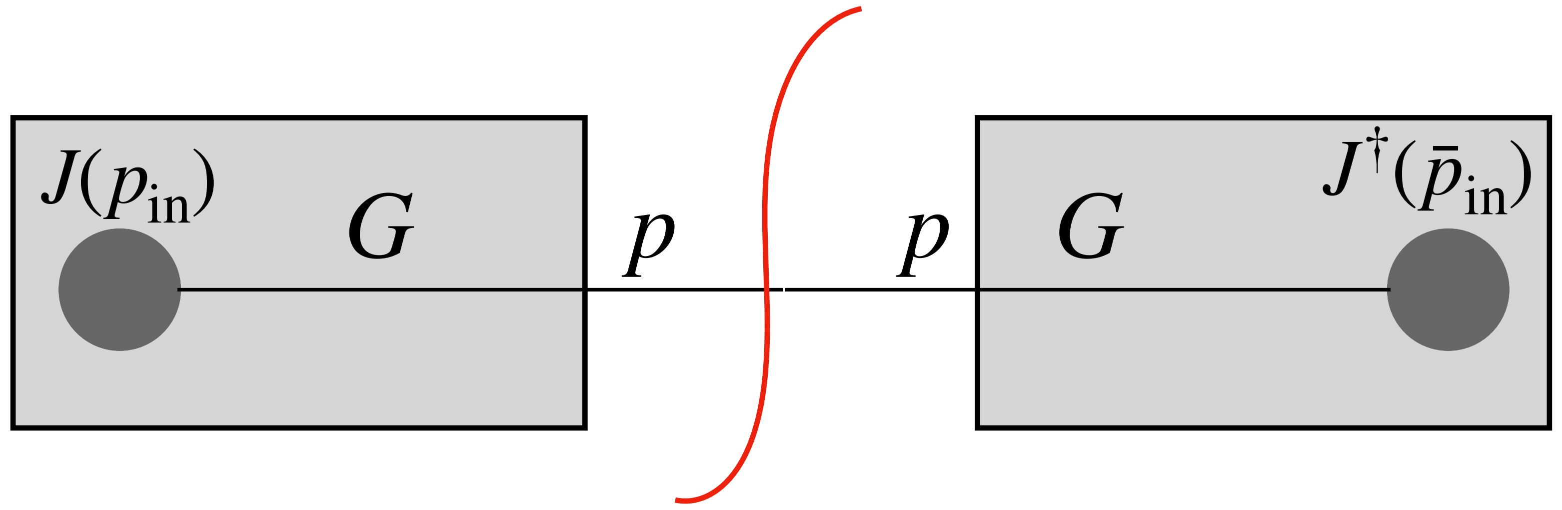
This results in an effective propagator G

$$G(\boldsymbol{x}_L, L; \boldsymbol{x}_0, 0) = \int_{\boldsymbol{x}_0}^{\boldsymbol{x}_L} \mathcal{D}\boldsymbol{r} \exp\left(\frac{iE}{2}\int_0^L d\tau\, \dot{\boldsymbol{r}}^2\right) \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\text{proj}} v^a(\boldsymbol{r}(\tau),\tau)\right)$$



2 Compute the relevant Feynman diagrams

$$\langle |M|^2 \rangle = \int \frac{d^2\boldsymbol{p}_{in} d^2\overline{\boldsymbol{p}}_{in}}{(2\pi)^4} \langle G(\boldsymbol{p}_f, L; \boldsymbol{p}_{in}, 0) G^\dagger(\boldsymbol{p}_f, L; \overline{\boldsymbol{p}}_{in}, 0)\rangle J(\boldsymbol{p}_{in}) J^*(\overline{\boldsymbol{p}}_{in})$$



1 The BDMPS-Z approach: single particle broadening

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One can try to perform the resummation already at amplitude level. In this case the steps are

3 Solve the remaining average of dressed propagators

$$gA^{\mu a}_{\rm ext}(q) = -(2\pi)\, g^{\mu 0}\, v^a(q)\, \delta\left(q^0\right) \qquad v^a(q) = \sum_i e^{-i\vec{q}\cdot\vec{x}_i}\, t^a_i\, v_i(q)$$

Two alternatives:

$$G(\boldsymbol{x}_L, L; \boldsymbol{x}_0, 0) = \int_{\boldsymbol{x}_0}^{\boldsymbol{x}_L} \mathcal{D}\boldsymbol{r} \exp\left(\frac{iE}{2}\int_0^L d\tau\, \dot{\boldsymbol{r}}^2\right) \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\rm proj} v^a(\boldsymbol{r}(\tau),\tau)\right)$$

1) Solve first the path integrals and then average

In practice this option, implies performing resummation as in the Opacity Series approach

2) Perform the average before integration

$$\langle |M|^2\rangle = \int \frac{d^2\boldsymbol{p}_{in} d^2\overline{\boldsymbol{p}}_{in}}{(2\pi)^4} \langle G(\boldsymbol{p}_f, L; \boldsymbol{p}_{in}, 0) G^\dagger(\boldsymbol{p}_f, L; \overline{\boldsymbol{p}}_{in}, 0)\rangle J(\boldsymbol{p}_{in}) J^*(\overline{\boldsymbol{p}}_{in})$$

In practice, by solving the remaining integrals one performs the resummation of averaged quantities directly

The key step is to use the fact that the color average of potential at different positions

$$\langle t^a_{\rm proj} v^a(\boldsymbol{r},\tau) t^b_{\rm proj} v^{\dagger b}(\overline{\boldsymbol{r}},\overline{\tau})\rangle = \mathcal{C}\, g^4 \int dz\, d^2\boldsymbol{x}\, \rho(\boldsymbol{x},\, z) \int \frac{d^2\boldsymbol{q}\, dq_z\, d^2\overline{\boldsymbol{q}}\, d\overline{q}_z}{(2\pi)^6} \frac{e^{i\boldsymbol{q}\cdot(\boldsymbol{r}-\boldsymbol{x})} e^{-i\overline{\boldsymbol{q}}\cdot(\overline{\boldsymbol{r}}-\boldsymbol{x})} e^{iq_z(\tau-z)} e^{-i\overline{q}_z(\overline{\tau}-z)}}{(\boldsymbol{q}^2 + q_z^2 + \mu^2(\boldsymbol{x},z))(\overline{\boldsymbol{q}}^2 + \overline{q}_z^2 + \mu^2(\boldsymbol{x},z))}$$

implies for $\nabla\rho = 0$

$$\left\langle \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\rm proj} v^a(\boldsymbol{r}(\tau),\tau)\right) \mathcal{P}\exp\left(i\int_0^L d\overline{\tau}\, t^b_{\rm proj} v^b(\overline{\boldsymbol{r}}(\overline{\tau}),\overline{\tau})\right)\right\rangle = \exp\left\{-\int_0^L d\tau \quad \mathcal{V}\left(\boldsymbol{r}(\tau) - \overline{\boldsymbol{r}}(\tau)\right)\right\}$$

No need to deal with double and single scattering diagrams explicitly

2 Broadening in anisotropic media: Opacity Expansion approach

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Previously when averaging in 2 we used

$$\sum_i f_i = \int d^2\boldsymbol{x}\, dz\, \rho(\boldsymbol{x}, z)\, f(\boldsymbol{x}, z) \xrightarrow[\nabla\rho = 0]{} \underline{\int d^2\boldsymbol{x}_n\, e^{-i(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)\cdot \boldsymbol{x}_n}} = (2\pi)^2\, \delta^{(2)}(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)$$

For anisotropic media this no longer holds

We perform a gradient expansion for the 2 relevant parameters: ρ and μ

$$\rho(\boldsymbol{x}, z) \approx \rho(z) + \boldsymbol{\nabla}\rho(z)\cdot \boldsymbol{x} \qquad \mu^2(\boldsymbol{x}, z) \approx \mu^2(z) + \boldsymbol{\nabla}\mu^2(z)\cdot\boldsymbol{x}$$

So that when averaging instead of a momentum space Dirac delta one obtains

$$\underline{\int d^2\boldsymbol{x}_n\, x_n^\alpha\, e^{-i(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)\cdot \boldsymbol{x}_n}} = i\,(2\pi)^2 \frac{\partial}{\partial(q_n \pm \bar{q}_n)_\alpha}\delta^{(2)}(\boldsymbol{q}_n \pm \bar{\boldsymbol{q}}_n)$$

With this modification, we find that the N order squared contribution now reads

$$\langle |M|^2\rangle^{(N)} = \prod_{n=1}^{N}\left[(-1)\int_0^{z_{n+1}} dz_n \int \frac{d^2\boldsymbol{q}_n}{(2\pi)^2}\mathcal{V}(\boldsymbol{q}_n, z_n)\right] |J(E, \boldsymbol{p}_{in})|^2$$

$$\langle |M|^2\rangle^{(N)} = \prod_{n=1}^{N}\left[\int_0^{z_{n+1}} dz_n \int \frac{d^2\boldsymbol{q}_n}{(2\pi)^2}\right]\left(1 + \frac{1}{E}\sum_{m=1}^{N}(z_m - z_{m-1})\boldsymbol{p}_m \cdot \sum_{k=m}^{N}\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho_k} + \boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu_k^2}\right)\right)(-1)^N \mathcal{V}_1(\boldsymbol{q}_1)\ldots\mathcal{V}_N(\boldsymbol{q}_N)|J(E, \boldsymbol{p}_{in})|^2$$

$$p_n = p_f - \sum_{m=n}^{N} q_m,\; p_{in} = p_1$$

2 Broadening in anisotropic media: Opacity Expansion approach

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Proceeding as in [3] we find that

$$\mathcal{V}'(\boldsymbol{x}) \equiv \frac{\partial}{\partial \mu^2}\mathcal{V}(\boldsymbol{x})$$

$$\frac{d\mathcal{N}^{(N)}}{d^2\boldsymbol{x}dE} = \int \frac{d^2\boldsymbol{p}\,d^2\boldsymbol{r}}{(2\pi)^2} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{r})}(-1)^N[\mathcal{V}(\boldsymbol{r})L]^N\left\{\frac{1}{N!} + \frac{L}{E(N+1)!} \times \sum_{m=1}^{N}\left[(N+1-m)\boldsymbol{p}\cdot\left(\frac{\mathcal{V}'(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}\boldsymbol{\nabla}\mu^2 + \frac{1}{\rho}\boldsymbol{\nabla}\rho\right) + i(N+1-m)^2\frac{\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{r})}{\rho\,\mathcal{V}(\boldsymbol{r})}\cdot\boldsymbol{\nabla}\rho\right.\right.$$

$$\left.\left. + i(N+1-m)\left(\frac{\boldsymbol{\nabla}\mathcal{V}'(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})} + (N-m)\frac{\mathcal{V}'(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}\frac{\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{r})}{\mathcal{V}(\boldsymbol{r})}\right)\cdot\boldsymbol{\nabla}\mu^2\right]\right\}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{r}dE}$$

Resumming the opacity series then leads to the compact expression

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} = \sum_{N=0}^{\infty}\int\frac{d^2\boldsymbol{p}\,d^2\boldsymbol{r}}{(2\pi)^2}e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{r})}\frac{(-1)^N\left[\mathcal{V}(\boldsymbol{r})L\right]^N}{N!}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{r}dE} = e^{-\mathcal{V}(\boldsymbol{x})L}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}$$

$$\nabla\rho = \nabla\mu = 0$$

$$\Big\downarrow\ \nabla\rho, \nabla\mu \neq 0$$

$$\boxed{\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} = e^{-\mathcal{V}(\boldsymbol{x})L}\left\{\left[1 - i\frac{\mathcal{V}(\boldsymbol{x})L^3}{6E}\left(\frac{\mathcal{V}'(\boldsymbol{x})}{\mathcal{V}(\boldsymbol{x})}\boldsymbol{\nabla}\mu^2 + \frac{1}{\rho}\boldsymbol{\nabla}\rho\right)\cdot\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x})\right]\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE} + i\frac{\mathcal{V}(\boldsymbol{x})L^2}{2E}\left(\frac{\mathcal{V}'(\boldsymbol{x})}{\mathcal{V}(\boldsymbol{x})}\boldsymbol{\nabla}\mu^2 + \frac{1}{\rho}\boldsymbol{\nabla}\rho\right)\cdot\boldsymbol{\nabla}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right\}}$$

2 Broadening in anisotropic media: BDMPS-Z approach

To linear order in gradients from $\boxed{3}$ **we find now**

$$\langle t^a_{\rm proj} v^a(\boldsymbol{r},\tau) t^b_{\rm proj} v^{\dagger b}(\overline{\boldsymbol{r}},\overline{\tau})\rangle = \mathcal{C}\, g^4 \int dz\, d^2\boldsymbol{x}\, \rho(\boldsymbol{x},z) \int \frac{d^2\boldsymbol{q}\, dq_z\, d^2\overline{\boldsymbol{q}}\, d\overline{q}_z}{(2\pi)^6} \frac{e^{i\boldsymbol{q}\cdot(\boldsymbol{r}-\boldsymbol{x})} e^{-i\overline{\boldsymbol{q}}\cdot(\overline{\boldsymbol{r}}-\boldsymbol{x})} e^{iq_z(\tau-z)} e^{-i\overline{q}_z(\overline{\tau}-z)}}{(\boldsymbol{q}^2+q_z^2+\mu^2(\boldsymbol{x},z))(\overline{\boldsymbol{q}}^2+\overline{q}_z^2+\mu^2(\boldsymbol{x},z))}$$

$$\langle t^a_{\rm proj} v^a(\boldsymbol{r},\tau) t^b_{\rm proj} v^{\dagger b}(\overline{\boldsymbol{r}},\overline{\tau})\rangle \simeq \left(1+\frac{\boldsymbol{r}(\tau)+\overline{\boldsymbol{r}}(\tau)}{2}\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\right)\mathcal{C}\,\delta(\tau-\overline{\tau})\,\rho\, g^4\int\frac{d^2\boldsymbol{q}}{(2\pi)^2}\frac{e^{i\boldsymbol{q}\cdot(\boldsymbol{r}-\overline{\boldsymbol{r}})}}{(\boldsymbol{q}^2+\mu^2)^2}$$

One can still show that the 2-point correlator exponentiates

$$\left\langle \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\rm proj} v^a(\boldsymbol{r}(\tau),\tau)\right)\mathcal{P}\exp\left(i\int_0^L d\overline{\tau}\, t^b_{\rm proj} v^b(\overline{\boldsymbol{r}}(\overline{\tau}),\overline{\tau})\right)\right\rangle = \exp\left\{-\int_0^L d\tau \quad \mathcal{V}(\boldsymbol{r}(\tau)-\overline{\boldsymbol{r}}(\tau))\right\}$$

$$\left\langle \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\rm proj} v^a(\boldsymbol{r}(\tau),\tau)\right)\mathcal{P}\exp\left(i\int_0^L d\overline{\tau}\, t^b_{\rm proj} v^b(\overline{\boldsymbol{r}}(\overline{\tau}),\overline{\tau})\right)\right\rangle = \exp\left\{-\int_0^L d\tau\left[1+\underbrace{\frac{\boldsymbol{r}(\tau)+\overline{\boldsymbol{r}}(\tau)}{2}}_{\text{Center of mass of dipole}}\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\right]\mathcal{V}(\underbrace{\boldsymbol{r}(\tau)-\overline{\boldsymbol{r}}(\tau)}_{\text{Dipole size}})\right\}$$

Combining all the results one just needs to compute

$$\langle |M|^2\rangle = \int \frac{d^2\boldsymbol{p}_{in} d^2\overline{\boldsymbol{p}}_{in}}{(2\pi)^4}\langle G(\boldsymbol{p}_f,L;\boldsymbol{p}_{in},0)G^\dagger(\boldsymbol{p}_f,L;\overline{\boldsymbol{p}}_{in},0)\rangle J(\boldsymbol{p}_{in})J^*(\overline{\boldsymbol{p}}_{in})$$

$$\langle G(\boldsymbol{x}_L,L;\boldsymbol{x}_0,0)G^\dagger(\overline{\boldsymbol{x}}_L,L;\overline{\boldsymbol{x}}_0,0)\rangle = \int_{\boldsymbol{x}_0}^{\boldsymbol{x}_L}\mathcal{D}\boldsymbol{r}\int_{\overline{\boldsymbol{x}}_0}^{\overline{\boldsymbol{x}}_L}\mathcal{D}\overline{\boldsymbol{r}}\,\exp\left\{\frac{iE}{2}\int_0^L d\tau\,\left[\dot{\boldsymbol{r}}^2-\dot{\overline{\boldsymbol{r}}}^2\right]\right\}\quad\exp\left\{-\int_0^L d\tau\left[1+\frac{\boldsymbol{r}(\tau)+\overline{\boldsymbol{r}}(\tau)}{2}\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\right]\mathcal{V}(\boldsymbol{r}(\tau)-\overline{\boldsymbol{r}}(\tau))\right\}$$

2 Broadening in anisotropic media: BDMPS-Z approach

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$$\boldsymbol{w} \equiv \frac{\boldsymbol{r}+\boldsymbol{r}'}{2}$$

$$\boldsymbol{u} \equiv \boldsymbol{r} - \boldsymbol{r}'$$

To solve the path integral it is convenient to go the center of mass coordinates

$$\langle G(\boldsymbol{x}_L, L; \boldsymbol{x}_0, 0) G^\dagger(\overline{\boldsymbol{x}}_L, L; \overline{\boldsymbol{x}}_0, 0)\rangle = \int_{\boldsymbol{u}_0}^{\boldsymbol{u}_L} \mathcal{D}\boldsymbol{u} \int_{\boldsymbol{w}_0}^{\boldsymbol{w}_L} \mathcal{D}\boldsymbol{w} \, \exp\left\{ iE \int_0^L d\tau \, \dot{\boldsymbol{u}} \cdot \dot{\boldsymbol{w}} \right\} \exp\left\{ -\int_0^L d\tau \left[1 + \boldsymbol{w} \cdot \left(\boldsymbol{\nabla}\rho \frac{\delta}{\delta\rho} + \boldsymbol{\nabla}\mu^2 \frac{\delta}{\delta\mu^2} \right) \right] \mathcal{V}\left(\boldsymbol{u}(\tau)\right) \right\}$$

Since the action is linear in w one can solve it exactly

$$\langle G(\boldsymbol{x}_L, L; \boldsymbol{x}_0, 0) G^\dagger(\overline{\boldsymbol{x}}_L, L; \overline{\boldsymbol{x}}_0, 0)\rangle = \left(\frac{E}{2\pi L}\right)^2 \exp\left\{ iE \left(\boldsymbol{w} \cdot \dot{\boldsymbol{u}}_c \right) \Big|_0^L \right\} \exp\left\{ -\int_0^L d\tau \, \mathcal{V}\left(\boldsymbol{u}_c(\tau)\right) \right\}$$

$$E\ddot{\boldsymbol{u}} = i \left(\boldsymbol{\nabla}\rho \frac{\delta}{\delta\rho} + \boldsymbol{\nabla}\mu^2 \frac{\delta}{\delta\mu^2} \right) \mathcal{V}(\boldsymbol{u}(\tau))$$

At leading gradient accuracy we can solve the eom perturbatively

$$\mathbf{u}_c = \mathbf{u}_c^{(0)} + \mathbf{u}_c^{(1)}$$

$$\boldsymbol{u}_c^{(0)}(\tau) = \frac{\boldsymbol{u}_L - \boldsymbol{u}_0}{L}\tau + \boldsymbol{u}_0$$

$$\boldsymbol{u}_c^{(1)}(\tau) = \frac{i}{E} \left(\boldsymbol{\nabla}\rho \frac{\delta}{\delta\rho} + \boldsymbol{\nabla}\mu^2 \frac{\delta}{\delta\mu^2} \right) \left\{ \int_0^\tau d\zeta \int_0^\zeta d\xi \, \mathcal{V}\left(\boldsymbol{u}^{(0)}(\xi)\right) - \frac{\tau}{L} \int_0^L d\zeta \int_0^\zeta d\xi \, \mathcal{V}\left(\boldsymbol{u}^{(0)}(\xi)\right) \right\}$$

The relevant correlator then reads

$$\langle G(\boldsymbol{p}_f, L; \boldsymbol{p}_{in}, 0) G^\dagger(\boldsymbol{p}_f, L; \overline{\boldsymbol{p}}_{in}, 0)\rangle = \frac{(2\pi)^2}{L^2} \int d^2\boldsymbol{u}_0 \, d^2\boldsymbol{u}_L \, e^{-i\boldsymbol{p}_f \cdot \boldsymbol{u}_L} e^{i\boldsymbol{u}_0 \cdot \frac{\boldsymbol{p}_{in} + \overline{\boldsymbol{p}}_{in}}{2}} \quad \delta^{(2)}\left(\dot{\boldsymbol{u}}_c(L)\right) \delta^{(2)}\left(\boldsymbol{p}_{in} - \overline{\boldsymbol{p}}_{in} - E\dot{\boldsymbol{u}}_c(0)\right) \exp\left\{ -\int_0^L d\tau \, \mathcal{V}\left(\boldsymbol{u}_c(\tau)\right) \right\}$$

2 Broadening in anisotropic media: BDMPS-Z approach

The eom are further constrained to give $E\ddot{\boldsymbol{u}} = i\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho} + \boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}(\boldsymbol{u}(\tau))$ $\quad \delta^{(2)}\left(\dot{\boldsymbol{u}}_c(L)\right)$

$$\boldsymbol{u}_c(\tau) = \boldsymbol{u}_L + \frac{i}{E}\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho} + \boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}\left(\boldsymbol{u}_L\right)\left\{\frac{(\tau-L)^2}{2}\right\}$$

Contracting with the initial currents we find

$$\langle|M|^2\rangle = \frac{1}{(2\pi L)^2}\int d^2\boldsymbol{P}_{in}\, d^2\boldsymbol{u}_0\, d^2\boldsymbol{u}_L\, e^{-i\boldsymbol{p}_f\cdot\boldsymbol{u}_L} e^{i\boldsymbol{P}_{in}\cdot\boldsymbol{u}_0} \quad \delta^{(2)}\left(\dot{\boldsymbol{u}}_c(L)\right)\exp\left\{-\int_0^L d\tau\,\mathcal{V}\left(\boldsymbol{u}_c(\tau)\right)\right\} J\left(\boldsymbol{P}_{in} + \frac{E}{2}\dot{\boldsymbol{u}}_c(0)\right) J^*\left(\boldsymbol{P}_{in} - \frac{E}{2}\dot{\boldsymbol{u}}_c(0)\right)$$

Such that for real J this leads to

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} \simeq \exp\left\{-\mathcal{V}\left(\boldsymbol{x}\right)L\right\}\left\{\left[1 - \frac{iL^3}{6E}\boldsymbol{\nabla}\mathcal{V}\left(\boldsymbol{x}\right)\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho} + \boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}\left(\boldsymbol{x}\right)\right]\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE} + \frac{iL^2}{2E}\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho} + \boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}\left(\boldsymbol{x}\right)\cdot\boldsymbol{\nabla}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right\}$$

Same result as in Opacity Expansion approach

Final particle distribution

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The final distribution has the form

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} \simeq \exp\left\{-\mathcal{V}(\boldsymbol{x})\,L\right\}\left\{\left[1-\frac{iL^3}{6E}\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x})\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}(\boldsymbol{x})\right]\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}+\underline{\frac{iL^2}{2E}\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}(\boldsymbol{x})\cdot\boldsymbol{\nabla}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}}\right\}$$

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} = \underline{\mathcal{P}(\boldsymbol{x})}\,\underline{\hat{\mathfrak{T}}(\boldsymbol{x})}\,\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}$$

In the literature this is sometimes referred to as single particle broadening distribution (when Fourier transformed)

Usually a unit operator, but now it acts with ∇ on initial distribution

Effective factorization no longer holds in general due to operator nature

Still
$$\int d^2\boldsymbol{p}\,\frac{d\mathcal{N}}{d^2\boldsymbol{p}\,dE} = \left.\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE}\right|_{\boldsymbol{x}=0} = \left.\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right|_{\boldsymbol{x}=0} = \int d^2\boldsymbol{p}\,\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{p}\,dE}$$

We consider first the case of a source with finite width

$$E\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{p}\,dE} = \frac{f(E)}{2\pi w^2}e^{-\frac{\boldsymbol{p}^2}{2w^2}}$$

It is possible to show that even though $\langle\boldsymbol{p}\rangle = 0$

higher odd moments can be generated, for example

$$\langle p^\alpha\,\boldsymbol{p}^2\rangle = \underbrace{\frac{w^2L^2\mu^2}{E\,\lambda}\frac{\nabla^\alpha\rho}{\rho}\ln\frac{E}{\mu}}_{N=1} + \underbrace{\frac{L^3\mu^4}{6E\,\lambda^2}\frac{\nabla^\alpha\rho}{\rho}\left(\ln\frac{E}{\mu}\right)^2}_{N=2}$$

Higher N terms dominate due to diverging potential at large momenta

Coulomb logarithm

3 Final particle distribution

The final distribution has the form $$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} \simeq \exp\{-\mathcal{V}(\boldsymbol{x})\,L\}\left\{\left[1-\frac{iL^3}{6E}\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x})\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}(\boldsymbol{x})\right]\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}+\frac{iL^2}{2E}\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}(\boldsymbol{x})\cdot\boldsymbol{\nabla}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right\}$$

If we neglect initial state effects, then we are left with $\chi = \frac{\mathcal{C}g^4\rho}{4\pi\mu^2}L$ medium opacity

$$\mathcal{P}(\boldsymbol{p}) = \int d^2\boldsymbol{x}\, e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}e^{-\mathcal{V}(\boldsymbol{x})L}\left[1-\frac{iL^3}{6E}\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x})\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}(\boldsymbol{x})\right]$$

where for GW model $\boldsymbol{p}^2 \gg \mu^2$ $$\frac{4L}{\chi}\mathcal{V}^{\rm GW}(\boldsymbol{x}) = \mu^2\boldsymbol{x}^2\log\frac{4e^{1-2\gamma_E}}{\mu^2\boldsymbol{x}^2}+\mathcal{O}\left(\mu^4\boldsymbol{x}^4\right)$$

In the hard region where $\boldsymbol{p}^2 \gg \chi\,\mu^2$ it can be written in a closed form

$$\mathcal{P}(\boldsymbol{p}) \simeq \frac{4\pi\mu^2\chi}{\boldsymbol{p}^4}+\frac{16\pi\mu^4\chi^2}{\boldsymbol{p}^6}\left(\log\frac{\boldsymbol{p}^2}{\mu^2}-2\right)+\frac{4\pi\mu^4\chi^2L}{3E}\left[\frac{\boldsymbol{\nabla}\rho}{\rho}\left(\log\frac{\boldsymbol{p}^4}{\mu^4}-4\right)-\frac{\boldsymbol{\nabla}\mu^2}{\mu^2}\right]\cdot\frac{\boldsymbol{p}}{\boldsymbol{p}^6}$$

Coulomb tail

In the complementary region where $\mu^2 \ll \boldsymbol{p}^2 \leq \chi\,\mu^2$ one has $\frac{4L}{\chi}\mathcal{V}^{\rm GW}(\boldsymbol{x}) \simeq \mu^2\boldsymbol{x}^2\left(\log\frac{Q^2}{\mu^2}+\log\frac{4e^{1-2\gamma_E}}{Q^2\boldsymbol{x}^2}\right)$

$$\mathcal{P}(\boldsymbol{p}) = \frac{4\pi}{\chi\mu^2\log\frac{Q^2}{\mu^2}}\left[1+\frac{L}{6E}\frac{\boldsymbol{p}^2-2\chi\mu^2\log\frac{Q^2}{\mu^2}}{\chi\mu^2\log\frac{Q^2}{\mu^2}}\left(\frac{\boldsymbol{\nabla}\rho}{\rho}-\frac{1}{\log\frac{Q^2}{\mu^2}}\frac{\boldsymbol{\nabla}\mu^2}{\mu^2}\right)\cdot\boldsymbol{p}\right]e^{-\frac{\boldsymbol{p}^2}{\chi\mu^2\log\frac{Q^2}{\mu^2}}}$$

Usual Gaussian distribution

3 Final particle distribution

For the full GW model we obtain

$$\mathcal{P}(\boldsymbol{p}) = \int d^2\boldsymbol{x}\, e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} e^{-\mathcal{V}(\boldsymbol{x})L}\left[1 - \frac{iL^3}{6E}\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x})\cdot\left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho} + \boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)\mathcal{V}(\boldsymbol{x})\right]$$

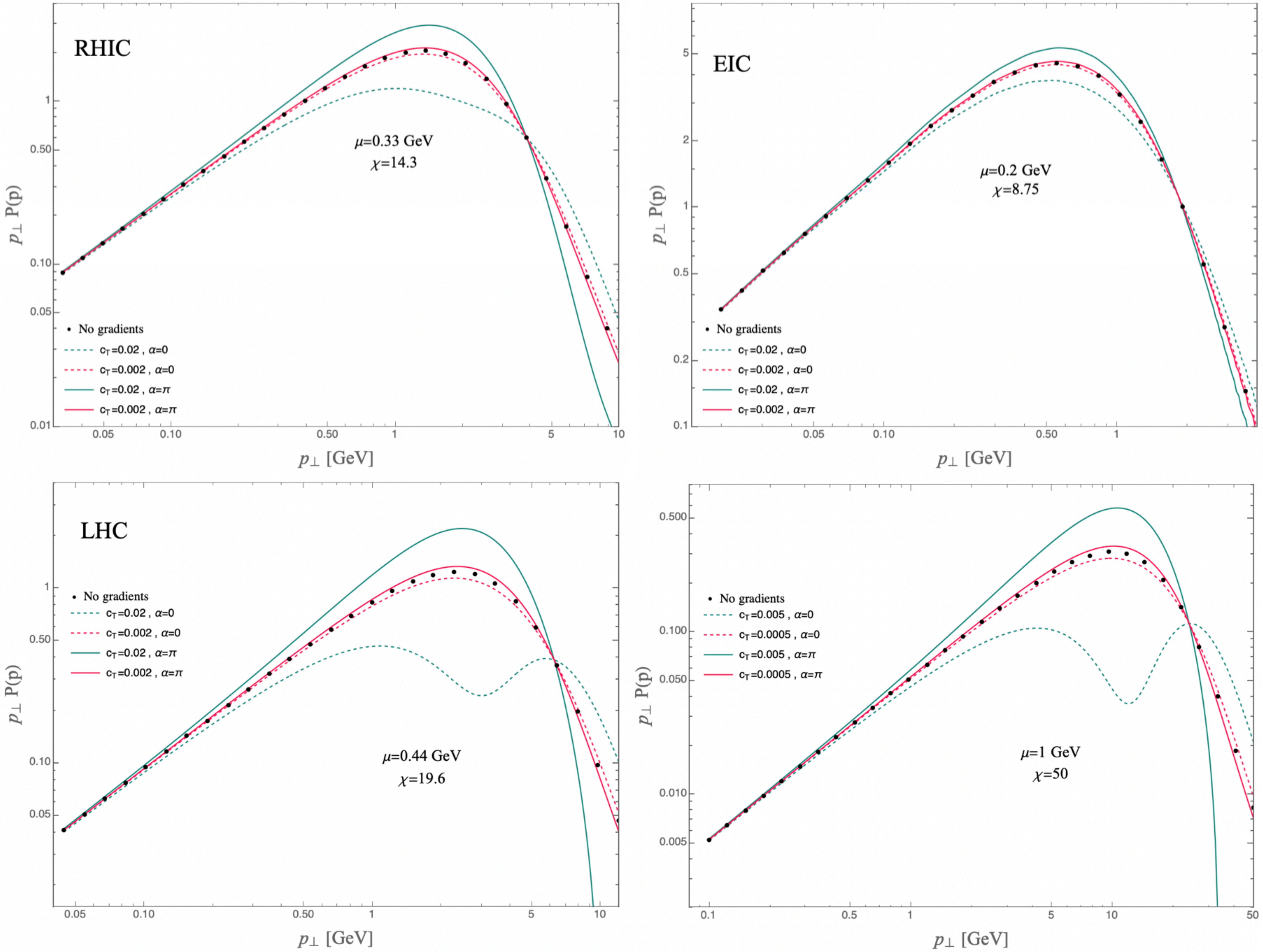
$$\mathcal{P}(\boldsymbol{p}) = \int_{\boldsymbol{x}} \mathrm{e}^{-i\boldsymbol{p}\cdot\boldsymbol{x}}\,\mathrm{e}^{-\mathcal{V}^{\mathrm{GW}}(\boldsymbol{x})L}\left\{1 - i\frac{\chi\mu^2L^2}{6E}K_0(|\boldsymbol{x}|\mu)\right.$$
$$\left.\times\left(\mathcal{V}^{\mathrm{GW}}(\boldsymbol{x})\frac{\boldsymbol{\nabla}\rho}{\rho} - \frac{\chi}{L}\left[1 - \frac{\mu^2\boldsymbol{x}^2}{2}K_2(|\boldsymbol{x}|\mu)\right]\frac{\boldsymbol{\nabla}\mu^2}{\mu^2}\right)\cdot\boldsymbol{x}\right\}$$

We use the parametric relation

$$\frac{\boldsymbol{\nabla}\rho}{\rho} \sim 3\frac{\boldsymbol{\nabla}T}{T}\,,\qquad \frac{\boldsymbol{\nabla}\mu^2}{\mu^2} \sim 2\frac{\boldsymbol{\nabla}T}{T}$$

To rewrite the full distribution in terms of the angle α and parameter $c_T \equiv \left|\frac{\boldsymbol{\nabla}T}{ET}\right|$

$$\mathcal{P}(\boldsymbol{p}) = 2\pi\int_0^\infty dx\, x\,\mathrm{e}^{-\mathcal{V}^{\mathrm{GW}}(x)L}\left\{J_0(p_\perp x) - \frac{\chi\mu^2L^2c_T}{6}xK_0(x\mu)J_1(p_\perp x)\right.$$
$$\left.\times\left(3\mathcal{V}^{\mathrm{GW}}(x) - \frac{\chi}{L}\left[2 - x^2\mu^2K_2(x\mu)\right]\right)\cos(\alpha)\right\}$$



Conclusions and Outlook

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- We computed the **broadening distribution** for **non-flowing anisotropic** media
- We derived it in two jet quenching formalisms: **Opacity Expansion** and **BDMPS-Z**
- Final distribution generates **leading odd moments**
- **Future plans:**
  - Perform the resummation for gluon production

    Ongoing, J.B., X. Mayo, A. Sadofyev, C. Salgado

  - Impact on jet substructure observables
  - Extensions to EIC set up