Higher Order Corrections to $\hat{q}$ in a Weakly Coupled Quark Gluon Plasma

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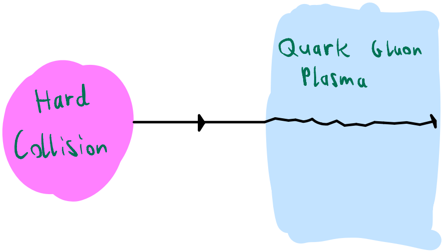
Outline

1. Physical Picture
2. Dichotomy of single scattering and multiple scattering
3. Calculating $\hat{q}$

Jet Propagation

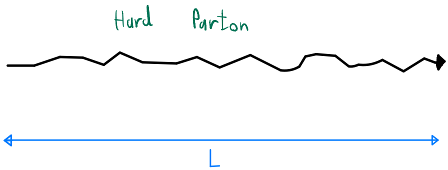
- Consider a nearly massless, highly energetic (hard) parton with energy, **E** produced in a heavy ion collision

- Parton undergoes collisions with medium constituents while propagating through the plasma



- Hard parton picks up transverse momentum, $\mathbf{k}_\perp \ll \mathbf{E}$ from collisions with medium constituents

- View as diffusion process and define diffusion coefficient, $\hat{\mathbf{q}}$ as $\mathbf{k}_\perp^2 \equiv \hat{\mathbf{q}}\mathbf{L}$



- The dominant mechanism for energy loss in the QGP is not the energy lost through these elastic collisions

- Instead, it comes from the bremsstrahlung induced from these elastic collisions

- Motivates the study of quantities such as particle bremsstrahlung rates

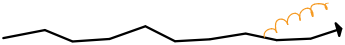
Outline

How exactly is bremsstrahlung triggered?

This depends on the quantum mechanical formation time, τ_f associated with the radiated gluon and can be crudely separated into two cases:

- Case 1: Radiated gluon with energy ω is triggered by just one rare collision with medium constituent
  - Known as Bethe-Heitler or single scattering regime

- Case 2: Many collisions with smaller momentum exchange add up to trigger gluon radiation with energy ω
  - Known as many scattering or harmonic oscillator regime (I will explain why shortly)
  - Requires LPM resummation



Single Scattering

- Formation time of radiated particle is given parametrically as [Blaizot and Mehtar-Tani, 2015]

$$\tau_f \sim \frac{\omega}{k_\perp^2} \tag{1}$$

- Let λ_{el} be the mean free path associated with collisions with the medium

- If $\tau_f \lesssim \lambda_{el}$, the so-called Bethe-Heitler spectrum turns out to be proportional to the number of elastic collisions, **N**

$$\omega \frac{dI}{d\omega} \simeq \frac{\alpha_s N_c}{\pi} N = \frac{\alpha_s N_c}{\pi} \frac{L}{\lambda_{el}} \tag{2}$$

- Now, assume that $\tau_f \gg \lambda_{el}$
- Use that radiate gluon undergoes transverse momentum kicks during formation time and picks up $k_\perp^2 \sim \hat{q}\tau_f$

$$\implies \tau_f = \sqrt{\frac{\omega}{\hat{q}}} \tag{3}$$

- Then, we can crudely say that if $\mathbf{N_{coh}} = \tau_f/\lambda_{el}$ is the number of coherent collisions that

$$\omega \frac{dI}{d\omega} \simeq \frac{\alpha_s N_c}{\pi} \frac{N}{N_{coh}} = \frac{\alpha_s N_c}{\pi} L \sqrt{\frac{\hat{q}}{\omega}} \tag{4}$$

Things to Note

- Spectrum is suppressed as ω is increased, which is a manifestation of the LPM effect
- Physics of these two regimes is very different
- In many scattering regime, many collisions need to be resummed via BDMPS-Z/AMY formalisms [Baier et al., 1995, Zakharov, 1997, Arnold et al., 2003]
- Within this formalism, analytical solutions can be found if harmonic oscillator approximation is made, where potential describing radiation-inducing soft interactions of hard partons with the medium $\propto \hat{q}x_\perp^2$

Outline

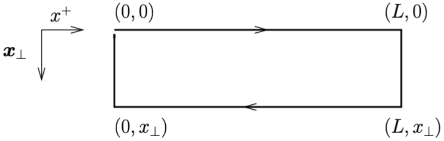
Why are we interested in $\hat{q}$?

- In the many scattering regime, $\hat{q}$ appears very naturally in expression used for calculating bremsstrahlung rate

- Turns out that $\hat{q}$ can be related to the transverse scattering rate, $\mathcal{C}(k_\perp)$

$$\hat{q} = \int \frac{d^2k_\perp}{(2\pi)^2} k_\perp^2 \mathcal{C}(k_\perp) \quad (5)$$

- $\mathcal{C}(k_\perp)$ can in turn be related to a certain Wilson loop



Taken from [Ghiglieri and Teaney, 2015]

LO and NLO contributions to $\hat{q}$

- Leading order contributions calculated coming from **soft scale** $k_\perp \sim gT$ and **hard scale** $k_\perp \sim T$ calculated by [Aurenche et al., 2002] and [Arnold and Xiao, 2008] respectively

- Next to leading order contributions in g calculated by [Caron-Huot, 2009] coming from **soft scale** with sizeable numerical prefactors

Logarithmic Enhancements

- $\mathcal{O}(g^2)$ correction found to have double logarithmic and single logarithmic enhancements by [Liou et al., 2013](LMW) and separately by [Blaizot et al., 2014](BDIM)

- These are **radiative** corrections and come from the single scattering regime

- Both of these calculations were done by making use of the harmonic oscillator approximation

- However, as was mentioned before, the harmonic oscillator approximation is only well-suited to the many scattering regime!

Current Dilemma

- Which is larger: $K\mathcal{O}(g)$ or $\ln^2(\#)\mathcal{O}(g^2)$? Hard to say...

- $\#$ has been calculated by LMW and BDIM but in different settings, taking into account various approximations

- The strategy of our work is to try and pin down this $\#$, starting from the single scattering regime in a weakly coupled quark gluon plasma without making any kind of harmonic oscillator approximation

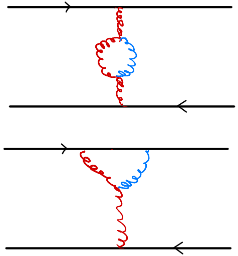
What are we doing now?

- Assume infinite medium and send $E \to \infty$ so that hard parton's behaviour eikonalizes

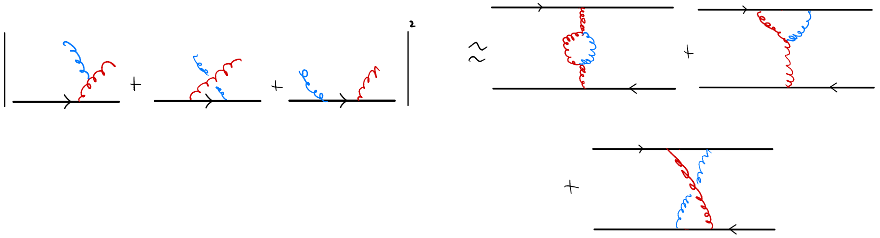
- Similar to Caron-Huot calculation, use that $\mathcal{C}(k_\perp)$ can be related to a Wilson loop, which in turn can be computed using techniques from real-time finite temperature quantum field theory

Some Wilson loop diagrams

- Can think of sticking together amplitude and conjugate amplitude to get diagrams on the right
- **Black lines** represent hard parton in the amplitude and conjugate amplitude
- Red gluons are bremsstrahlung, represented by thermal propagators
- Blue gluons are those that are exchanged with the medium and are represented by Hard Thermal Loop propagators



Where do these diagrams come from?



- Up to this point, we have recovered the LMW result by starting our computation in the many scattering regime and expanding so that we move towards the boundary of the single scattering regime

- Upon doing entire calculation in the single scattering regime, find different results although still need to understand exactly what the bounds should be on our $k_\perp$ integral
  Reminder:

$$\hat{q} = \int \frac{d^2 k_\perp}{(2\pi)^2} k_\perp^2 \mathcal{C}(k_\perp) \tag{6}$$

Thanks for listening!

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Relating $\mathcal{C}(k_\perp)$ to the Wilson Loop

Wilson loop defined, in the $x^- = 0$ plane in as

$$\langle W(x_\perp, 0)\rangle = \frac{1}{N_c}\,\mathrm{Tr}\langle [0, x_\perp]_- \mathcal{W}^\dagger(x_\perp)[x_\perp, 0]_+ \mathcal{W}(0)\rangle, \tag{7}$$

where

$$\mathcal{W}(x_\perp) = \mathcal{P}\exp\left(ig\int_{-\frac{L}{2}}^{\frac{L}{2}} dx^+ A^-(x^+, x_\perp)\right) \tag{8}$$

One can show that [D'Eramo et al., 2011, Benzke et al., 2013]

$$\lim_{L\to\infty}\langle W(x_\perp)\rangle = \exp(-\mathcal{C}(x_\perp)L) \tag{9}$$

where

$$\mathcal{C}(x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2}(1 - e^{i\mathbf{x}_\perp\cdot\mathbf{k}_\perp})\mathcal{C}(k_\perp) \tag{10}$$

Parametric form of $\hat{q}$

$$\hat{q} \sim \alpha_s^2 T^3 \{C_1 \ln \frac{T}{m_D} + C_2 \ln \frac{\mu}{T} + C_3 + Kg + \alpha_s (C_5 \ln^2(\#) + C_6 \ln \#' + \ldots)\} \tag{11}$$